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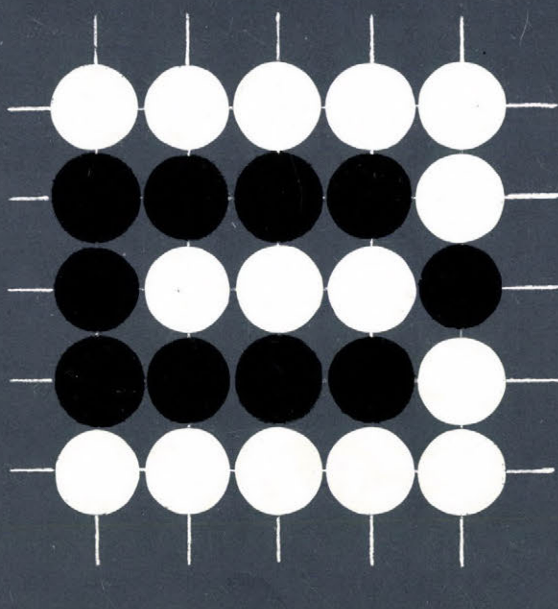

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CONTENTS

Procedings of the Computer and Automation Institute
Hungarian Academy of Sciences
Vol. 23.

СОДЕРЖАНИЕ

Труды Исследовательского Института
Вычислительной Техники и Автоматизации
Венгерской Академии Наук
Выпус 23.

MTA Számítástechnikai és Automatizálási Kutató Intézete, **Közlemények** 23/1979.

ON THE CARDINALITY OF SELF–DUAL CLOSED CLASSES IN k-VALUED LOGICS

J. Demetrovics – L. Hannák

Introduction

Let $E_k = \{0,1,\dots,k-1\}$. By a k-valued function we shall mean a function $f\colon E_k^n \to E_k$, and by P_k we denote the set of all those functions. If A is a subset of P_k, [A] will denote the set of all superpositions over A. The definition of a superposition over A is the following:

1. $f \in A$ is a superposition over A

2. If $g_0(x_1\dots x_n)$, $g_1(x_{11},\dots,x_{1m_1}),\dots,$ $g_n(x_{n1},\dots,x_{nm_n})$ are either superpositions over A, or $g_i(x_{i1},\dots,x_{im_i}) = x_{ij}$ then $g_0(g_1(x_{11},\dots,x_{1m_1})\ \dots\ g_n(x_{n1},\dots,x_{nm_n}))$ is a superposition over A.

The set A is closed if A = [A] . Let s be a permutation of 0,1,...,k-1. We say, that $f \in P_k$ preserves s, if

$$f(x_1,\dots,x_n) = s^{-1}\,[f(s(x_1)\ \dots\ s(x_n))]\ .$$

We shall denote by $\mathfrak{c}$ the cardinality of the continuum.

Ju. I. Janov and A.A. Mučnik [5] have proved, that if $k \geq 3$, then the cardinality of the set of all closed sets in P_k is continuum. E.Post's general result implies that there are countably many closed sets in P_k for k = 2.

It is well known, [see [4], [8]], that there exist 6 types of maximal closed sets in P_k. The characterisation of these sets can be found in [8]. J.Demetrovics and J.Bagyinszki have proved in [2] that the linear classes in P_k (k prime) contain a finite number of closed classes. J.Bagyinszki and A.Szendrei [1], [9] have proved that if k is square-free, then there are also finitely many closed linear classes in P_k. D.Lau in [6] have shown, that the cardinality of the so-called quasi-linear closed classes is countable. In [3] the authors have proved, that the so-called central, k-regular, monotonous and equivalence-preserving maximal classes in P_k, for $k \geq 3$ contain as many as $\mathfrak{c}$ closed classes. In this paper it is also shown that the maximal classes, which preserve a permutation s, conatain $\mathfrak{c}$ closed classes provided k is not prime. Marcenkov in [7] has proved that for all $k \in \{13,14,16,17,\}$ and for all permutation $s : E_k \to E_k$ there exist a set of closed classes preserving s with cardinality $\mathfrak{c}$. In the case k=2. E.Post's result ([10]) implies that there are finitely many closed classes pereserving a permutation of E_2.

The purpose of this paper is to show that for all $k \geq 3$ and for all permutation $s : E_k \to E_k$ /except for two cases, namely $k=3$ and $s = (012)$ or $k=4$, $s = (0123)$/ there exist $\mathfrak{c}$ closed sets in P_k preserving s. We shall also prove that for all $k \geq 3$ there is at least a countable number of closed sets preserving s, for all permutation $s: E_k \to E_k$.

§.1.

A permutation s of E_k can be written as a product of disjoint cycles. Such a cycle will be denoted by C_i. If

$$s = C_1.C_2 \dots C_m \quad \text{and} \quad \begin{array}{l} C_1 = (a_{11},\dots,a_{n_1 1}) \\ \vdots \\ C_m = (a_{1m},\dots,a_{n_m m}), \text{ then} \end{array}$$

$|C_i|$ will denote the number of the elements of the set

$$\{a_{1i},\dots,a_{n_i i}\}$$

Lemma 1. Let $k \geq 3$, s a permutation in the form $s = C_1.C_2.\dots.C_m$. If $m > 1$ and there are $i,j \leq m$ such that $i \neq j$, $|C_i| = k_1$, $|C_j| = k_2$ and k_1 / k_2 then it can be constructed $\mathfrak{c}$ closed classes preserving s.

Proof. We can assume that $s = C_1.C_2.\dots.C_m$, where

$C_1 = (0,\dots,a_{m_1})$ $\quad C_2 = (1,2,\dots,a_{m_2})$ and $|C_1| / |C_2|$

We shall prove, that there is a set $\{f_i\} = F$ of functions such that for all $f_i \in F$, $f_i \notin [F \setminus f_i]$ and all f_i preserve s. This is sufficient since in this case all subsets of F generate a closed class, and $H_1 \subset F$, $H_2 \subset F$ $H_1 \neq H_2$ implies $[H_1] \neq [H_2]$.

Let $f_m(x_1,x_2,\dots,x_m)$, $m \geq 3$ be defined as follows:

$$f_m(a_1,\dots,a_m) = \begin{cases} b \in C_2, & \text{if } (a_1,\dots,a_m) \subset C_2 \ \ |\{i/a_i = b\}| = 1 \\ & \text{and all } a_i \neq b \text{ are equal to } s(b); \\ d \in C_1, & \text{if } \{a_1,\dots,a_m\} \subset C_1 \cup C_2 \text{ and the previous} \\ & \text{condition does not hold;} \\ a_1 & \text{, in all other cases.} \end{cases}$$

One can easily see that since $/C_1/ \,|\, /C_2/$, $f_m(x_1,\dots,x_n)$ preserves s.

Let us suppose, that $f_k(x_1,\dots x_k) \in [F \setminus f_k]$. This means that

$$f_k(x_1,\dots,x_k) = \mathfrak{A}(x_1,\dots,x_k)$$

where $\mathfrak{A}$ is a superposition over $F \setminus f_k$.

Let $f_s(x_{i_1},\dots,x_{i_s})$ be a function in $\mathfrak{A}$.

If $s < k$, then we can find an x_ℓ such that $x_\ell \notin \{x_{i_1},\dots,x_{i_s}\}$

If $x_\ell = 1$, and all $x_i = 2$ $(i \neq \ell)$, then – by the definition – $f_k(x_1,\dots,x_k) = 1$.

If we choose $(x_1,\dots,x_k)$ as above, then $f_s(x_{i_1},\dots,x_{i_s}) \in C_1$ that is $\mathfrak{A}$ cannot be equal to 1. (f_m preserves the set $C_1 \cup C_2$ and if $\{a_1,\dots,a_m\} \cap C_1 \neq \emptyset$ then $f_m(a_1,\dots,a_m) \in C_1$.) If $s > k$, then we have at least one pair x_{i_k}, x_{i_ℓ} such that $i_k = i_\ell$.

Let $x_{i_k} = x_{i_\ell} = 1$, and all $x_j = 2$ $(j \neq i_k)$. In this case $f_s(x_{i_1},\dots,x_{i_s}) \in C_1$ and $f_k(x_1,\dots,x_k) = 1$. This is a contradiction, thus Lemma 1 is proved.

Corollary:

1. if k is not prime, then in the maximal closed class S_k of P_k there exists $\mathfrak{C}$ closed classes. (S_k denotes the class of all functions pre-

serving a permutation π; π is the product of cycles C_i of length p, where p is prime.)

2. if π is a permutation of the form $\pi = (1)\ C_1 \ldots\ldots C_m$ then there is a continuum cardinality set of closed classes pereserving π.

<u>Lemma 2.</u> Let $k \geq 5$, let s be a permutation consisting of one cycle of length k. Then we can construct a set of closed classes in P_k of cardinality $\mathfrak{c}$ which preserves s.

<u>Proof.</u> We can assume, that

$$s = (01234\ \ldots).$$

Analogously to the proof of Lemma 1 we shall give a set $\{g_i\} = G$ of functions so that $g_i \notin [G \setminus g_i]$ and g_i preserves s.

We define g_i, $i>3$ on the set $\{0,1,2\}^i$. It can be easily verified that the definition does not contradict the assumption that g_i preserves s.

Let:

$$g_k(a,\ldots,a) = a$$

$$g_k(\{\{0,1\}^k \setminus (1,\ldots,1)\}) = 0$$

$$g_k(\{\{0,2\}^k \setminus (2,\ldots,2)\}) = 0$$

$$g_k(\{\{1,2\}^k \setminus (2,\ldots,2)\}) = 1$$

and for $\{0,1,2\}^k \setminus \{0,1\}^k \setminus \{0,2\}^k \setminus \{1,2\}^k$:

$$g_k(a_1,\ldots,a_k) = \begin{cases} 1, & \text{if } /\{a_i/a_i = 0\}/ = 1 \\ & \phantom{\text{if }} /\{a_i/a_i = 2\}/ = 1 \\ & \phantom{\text{if }} /\{a_i/a_i = 1\}/ = k-2; \\ 0, & \text{in all other cases.} \end{cases}$$

A vector $(a_1,\ldots,a_k) = \underline{a} \in \{0,1,2\}^k$ is called characteristic if

$/\{a_i/a_i = 0\}/ = 1$,
$/\{a_i/a_i = 2\}/ = 1$, and
$/\{a_i/a_i = 1\}/ = k-2$.

Let us suppose $g_k \in [G \setminus g_k]$, that is $g_k(x_1,\ldots,x_k) = \mathfrak{A}(x_1,\ldots,x_k)$.

If $g_k(x_1,\ldots,x_k) = \mathfrak{A}$ then there exists at least one superposition over $G \setminus g_k$ such that $g_k(x_1,\ldots,x_k)$ is equal to this superposition on the characteristic vectors. Hence we can choose a minimal formula $\mathfrak{A}^*$ which equals $g_k(x_1,\ldots,x_k)$ on the characteristic vectors. The minimality of $\mathfrak{A}^*$ means that if $\mathfrak{A}^* = g_m(\mathscr{L}_1,\ldots,\mathscr{L}_m)$ then $\mathscr{L}_1,\ldots,\mathscr{L}_m$ cannot be equal to $g_k(x_1,\ldots,x_k)$ on the characteristic vectors.

We shall prove that such an $\mathfrak{A}^*$ cannot exist. $\mathfrak{A}^*$ can be written in the form $g_m(\mathscr{L}_1,\ldots,\mathscr{L}_m)$ where $\mathscr{L}_i = x_{ij}$ or $\mathscr{L}_i$ is a superposition over $G \setminus g_k$.

a./ if all $\mathscr{L}_i$ are superpositions over $G \setminus g_k$ then all $\mathscr{L}_i$ equal 1 or 0 on the characteristic vectors.
$g_\ell(\{\{0,1,2\}^\ell \ (2,\ldots,2)\}) \subseteq \{0,1\}$
Since $\mathfrak{A}^*$ is minimal /in the above sence/, there is exists a characteristic vector $\underline{c}$ such that $\mathscr{L}_1(\underline{c}) = 0$ that is $\mathfrak{A}^*(\underline{c}) = 0$. On the other hand $g_\ell(\underline{c}) = 1$ holds. This is a contradiction;

b./ We have seen, that there is a $\mathscr{L}_\ell = x_q$ in the superposition $\mathfrak{A}^* = g_m(\mathscr{L}_1,\ldots,\mathscr{L}_m)$.
Let $\underline{x} = x_1,\ldots,x_k$ be a characteristic vector so that $x_q = 0$, and $x_n = 2$. If $x_n \neq \mathscr{L}_1$, $x_n \neq \mathscr{L}_2, \ldots$ $x_n \neq \mathscr{L}_m$ then all $\mathscr{L}_i$ are equal to 1 or 0 on this characteristic vector, and hence $\mathfrak{A}^*(\underline{x}) = 0$.
$[(\mathscr{L}_1(x),\ldots,\mathscr{L}_m(x))] \neq (1,1,\ldots,1)$ and by the definition
$g_m(\{\{0,1\}^m \setminus (1,\ldots,1)\}) = 0.)$ This is also a contradiction.

c./ By a/ and b/ $\mathfrak{A}^*$ can be written in the form
$$g_m(\mathscr{L}_1,\ldots,\mathscr{L}_q, \ x_1,\ldots x_k) .$$
The assumption that $\mathfrak{A}^*$ is minimal implies that $\mathscr{L}_1$ cannot be equal to 1 on all characteristic vectors. Let $\underline{x}$ be a characteristic vector so that $\mathscr{L}_1 = 0$.

In this case, $\mathscr{L}_2, \ldots \mathscr{L}_q = 0$ or 1, and there is one $x_j = 0$. Since $(\mathscr{L}_1, \ldots \mathscr{L}_q, x_1, \ldots, x_k) \notin \{1,2\}^n$ and it cannot be characteristic, $\mathfrak{A}^*(\underline{x}) = \emptyset$. This implies that $\mathfrak{A}^* = g_m(x_{i_1}, \ldots, x_{i_m})$. If $m < k$, then there is a $x_q \notin \{x_{i_1}, \ldots, x_{i_m}\}$. On the characteristic vector $x_q = 2$, $x_{i_1} = 0$, $x_j = 1$ $(j \neq q,\ j \neq i_1)$, the statements $g_k = 1$ and $\mathfrak{A}^* = 0$ hold. If $m > k$ then there exists at least one pair i_ℓ, i_j such that $i_\ell = i_j$. In this case let $x_{i_\ell} = 0$, $x_j = 2$ $(j \neq i_\ell)$ and $x_t = 1$ $(t \neq j,\ t \neq i_\ell)$. On this characteristic vector $g_k(x_1 \ldots x_k) = 1$ and $\mathfrak{A}^* = 0$ hold. This is also a contradiction, thus lemma 2 is completely proved.

<u>Lemma 3.</u> Let $k = 5$ and π a permutation of the form $C_1 . C_2$ where $/C_1/ = 2$, $/C_2/ = 3$ or let $k = 7$ and π be a permutation of the form $C_1 . C_2$ where $/C_1/ = 3$, $/C_2/ = 4$. Then there is a set of closed sets in P_5 or in P_7 preserving π which has cardinality $\mathfrak{c}$.

It is easy to see that it is sufficient to consider the cases when

$\pi = (03)(124)$ and

$\pi = (034)(1256)$

The definition of g_m in Lemma 2 does not contradict the property g_m preserves π.

If we define h_m so that $h_m(a_1, \ldots, a_n) = g_m(a_1, \ldots, a_n)$ on the set $\{o,1,2\}^m$ and h_m preserves π, then $H = \{h_m / m \geq 3\}$ is a set with the property $h_m \notin [H \setminus h_m]$. Thus analogously to Lemma 1 $H^* = \{[S] / S \subset H\}$ is a set consisting of closed classes preserving π, and the cardinality of H^* is $\mathfrak{c}$.

<u>Theorem 1:</u> Let $k \geq 2$ and π be a permutation of E_k. If

$\pi \neq (a_1 a_2 a_3)$ for $k=3$ and

$\pi \neq (a_1 a_2 a_3 a_4)$ for $k=4$

then there are as many as $\mathfrak{c}$ closed classes in P_k preserving π.

Proof.: If π contains a cycle C such that $|C| \geq 5$, the statement is implied by Lemma 2.

If π contains a cycle C such that $|C| = 1$ or two cycles with equal lengths, then the statement follows from Lemma 1. If π contains at least 4 cycles with lengths 2,3,4 then two of them have equal lengths.

Thus we have the following cases:

$$\pi = C_1 . C_2, \quad /C_1/ = 2, \ /C_2/ = 3 \ \text{or}$$
$$/C_1/ = 3, \ /C_2/ = 4$$

$$\pi = C_1 . C_2 . C_3 \quad /C_1/ = 2, \ /C_2/ = 3, \ /C_3/ = 4$$

The first case is treated in Lemma 3.
In the second case $/C_1/ \mid /C_3/$, therefore the assumptions of Lemma 1 hold. Thus the proof of Theorem 1 is complete.

§. 2.

In §.1. we have seen, that for all but three permutations π $\mathfrak{c}$ closed sets in P_k $(k \geq 2)$ preserving π can be constructed.

In the case k=2 there is only a finite number of closed sets in P_2 which preserve (01) ([10]). In the cases k=3, π = (012) and k=4, π = (0123) we cannot give an "independent" set of functions with cardinality $\aleph_0$. However we can prove.

Theorem 2: For all k>2 and all permutations π there is at least a countably many closed sets in P_k that preserve π.

Proof: It is sufficient to consider the following two cases: k=3 and π = (012); k=4 and π = (0123). We will construct a set $\{t_i\} = T$ of functions such that $t_i \notin [\bigcup_{j>i} \{t_j\}] = T_i$, and t_i preserves π.

If we have such a family of functions, then the set $\{T_i \mid i \in \omega\}$ contains countably many closed classes, and it can be ordered as

$$T_1 \supset T_2 \supset T_3 \supset \dots$$

We define t_i as follows:

$$t_m(a_1,\dots,a_m) = \begin{cases} b, \text{ if } (a_1,\dots,a_m) = b \text{ or} \\ \quad a_1,\dots,a_{j-1},a_{j+1}\dots a_m = b \text{ and} \\ \quad a_j = \pi^{-1}(b); \\ \pi^{-1}(b), \text{ if} \\ \quad a_1,\dots,a_m \in \{\pi^{-1}(b),b\}^m \quad \text{and} \\ \quad \{a_i/a_i = b\} < m-1; \\ a_1 \quad \text{otherwise.} \end{cases}$$

A vector $\underline{a} = (a_1,\dots,a_m)$ is called characteristic, if $|\{i/a_i = 0\}| = 1$ and $|\{i/a_i| = 1\}| = m-1$. The definition implies that t_m preserves π. Let us suppose, that

$$t_m(x_1,\dots,x_m) = \mathfrak{A},$$

where $\mathfrak{A}$ is a superposition over T_i.

We can choose - analogously to Lemma 2 - a minimal formula $\mathfrak{A}^*$ which equals 1 on all characteristic vectors. This $\mathfrak{A}^*$ cannot be equal to x_i, that is $\mathfrak{A}^*$ can be written in the form

$$t_s(\mathcal{L}_1,\dots,\mathcal{L}_s) \quad \text{where} \quad s > m$$

Denote by y_j the characteristic vector with $x_j = 0$. Let us consider the matrix

$$\begin{vmatrix} \mathcal{L}_1(y_1) & \dots & \mathcal{L}_s(y_1) \\ \mathcal{L}_1(y_2) & \dots & \mathcal{L}_s(y_2) \\ \vdots & & \\ \mathcal{L}_1(y_m) & \dots & \mathcal{L}_s(y_m) \end{vmatrix}$$

By the minimality of $\mathfrak{A}^*$ every column of the matrix contains at least one 0. $s > m$ implies, that at least one row in the matrix contains two or more 0's. If the e'th row in the matrix contains at least two 0 - elements then $\mathfrak{m}^*(y_\ell) = 0$. This is a contradiction, since $t_m(y_i) = 1$ for all $i \in \{1,2,\ldots,m\}$.

Thus Theorem 2 is proved.

R E F E R E N C E S

[1] J.Bagyinszki, The lattice of closed classes of linear functions over a finite ring of square free order. Dep.of Mat. Karl Marx University of Economics 1979.

[2] J.Demetrovics, J.Bagyinszki, The lattice of linear classe in prime-valued logics. Banach Center Publications, Warsaw, PWN, 8/1979 (in press)

[3] J.Demetrovics, L.Hannák, The cardinality of closed sets in pre-complete classes in k-valued logics. Acta Cybernetcia (in press).

[4] S.V.Jablonski, Functional constructions in k-valued logics (Russian) Trudy Mat.Inst.Steklov. 51/1958. 5-142.

[5] Ju.I.Janov, A.A.Mučnik, Existence of k-valued classes without a finite bases. Dokl.Akad.Nauk SSSR (Russian) 127, 1 (1959), 44-46.

[6] D.Lau, Über die Anzahl von abgescholossenen Mengen von linearen Funktionen der n-wertigen Logik, EIK 14(1978) 11.

[7] S.S.Marčenkov, On closed classes of self-dual functions in k-valued logics (Russian) Prob.Kibern. 36/1979/ (in press).

[8] I.G.Rosenberg, Structure des fonctions de plusieurs variables sur un ensemble fini. C.R. Acad. Sci. Paris, 260/1965/ 3817-19.

[9] A.Szendrei, On closed sets of linear operations over a finite set of square-free cardinality. EIK 14(1978) 11,553-565.

[10] E.Post, Introduction to a generel theory of elementary propositions. Amer.J.Math. 93/1921/ 163-185.

Ö s s z e f o g l a l ó

A k-értékü logika önduális osztályairól

J.Demetrovics - L. Hannák

A jelen dolgozatban a szerzök bebizonyitják, hogy $\forall s(x) \in P_k$, $k \geq 3$ - kivéve, ha $s(x) = (0\ 1\ 2)$ ill. $s(x) = (0\ 1\ 2\ 3)$, - ($s(x)$-permutáció) az önduális zárt osztályok száma kontinuum.

Ha $s(x) = (0\ 1\ 2)$ ill. $s(x) = (0\ 1\ 2\ 3)$, akkor is legalább megszámlálható sok önduális osztály van.

Резюме

О мощностях самодейственных заминутных классов в P_k

Я. Деметрович, Л. Ханнак

В настоящей работе авторы изучают самодвойственные замкнутные классы в P_k $(k \geq 3)$. Они доказывают, что

а/ для любого $S(x) \in P_k$ /где $S(x)$ - перестановка; $S(x) \neq (0\ 1)$; $S(x) \neq (0\ 1\ 2)$ и $S(x) \neq (0\ 1\ 2\ 3)$/, существует континуум самодвойственных замкнутых классов относительно $S(x)$;

б/ если $S(x) = (0\ 1\ 2)$ из P_3 или $S(x) = (0\ 1\ 2\ 3)$ из P_4, то существует по крайней мере счетное число самодвойственных замкнутых классов относительно $S(x)$.

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CDC 3300 COMPUTER OPERATING SYSTEM: FILE ENVIRONMENT HANDLING METHOD

Ákos Radó

Our task is to enlighten the file environment handling method of the CDC 3300 computer operating system, the search and retrieval of user files in the user files' maintenance system and after the analysis optimize the work of the computer.

The efficiency of MASTER 4.1 operating system's time sharing and multiprogramming depends on optimum random access of mass storage.

The MASTER system operates in an environment in which all files have an identical basic structure. MASTER provides the user with a broad range of functions for manipulating the file definitions.

Functions that manage file definitions include allocation and release of space, modification of labels, expansion of defined file size, and opening and closing of files.

The system files required for MASTER to handle user files are File Label Directory (FLD) and Identifier File (IDF).

The FLD contains a complete description of each file known to the system. Each file has one file label entry written in the FLD of minimum 53 words in size (depending on the number of the segments of the user file). Each file has one two-word entry in the IDF. The first word is a 24 bit hash value calculated with "EXCLUSIVE OR" from the first ten words of the FLD. It is the remainder resulting from dividing the 40 characters of file identification (owner, filename, edition) by the largest prime number which will fit into 24 bits (8.388.593). Word two is the block number of the label's FLD entry. The block size of this file is set by a parameter of install time. The number of blocks is determined in the following manner:

a./ The IDF file consists of two parts, a number of blocks which comprise the main body of the file, and an additional number of blocks which comprise an overflow section.

b./ The number of blocks in the main body is always a prime number. To arrive at this number, the maximum file count is increased by 10 % and divided by the number of entries per block (e.g. a 64-word block size has 32 entries). The next highest number in the list of prime numbers is selected as the number of blocks for the main body.

c./ The overflow section is calculated as 10 % of the number of blocks in the main body of the file. For example, with a maximum file count of 1000, and a block size of 64 words, the main body contains 59 blocks, and the overflow contains 6 blocks, making a total of 65 blocks.

An entry in the IDF is made by dividing the remainder mentioned above by the number of blocks in the main body of the file. The remainder from this division plus one yields an IDF block number. If there is room in this block, the entry is placed here. Otherwise, the entry is placed in the first empty shot in the overflow area.

To reference a label, the owner, file name, and edition are divided by 8.388.593. This remainder is in turn divided by the prime number of blocks in the main body of the IDF. If no match can be made with any entry in this block, the overflow area is searched until the desired entry is found.

At allocation time the information about the file to be written into the FLD gets into the highest available block of the file. If the FLD is full, we try to find an empty block - due to a previous deletion of a file - that is the FLD is not compressed after deletions.

In the IDF the searching algorithm depends on the fact whether we allocate a new file or we look for an existing file (opening, deletion, nodification). In the latter case the algorithm is as follows: the overflow area is searched serially, if not found, the primary area is searched.

In the first case we begin to search an empty slot in the primary area, then it not found in the overflow area. The search in the primary area is as follows: in the block, the number of which counted from the hash code, serially (bucket!) look for an empty slot or a slot where previously an already deleted file resided. Deleting a file the first word of its entry in the IDF is zeroed, the second word (reference to the block in the FLD) remains unchanged (reason to be seen later).

In case of allocation it may happen, that we find however an empty slot in the primary or overflow area but the FLD is full (because it is not compressed). In this case we examine one by one the blocks of the IDF, whether there is an entry having its first word zeroed. If found, the information about the newly allocated file written into the FLD block, whose number was just found in the second word of the IDF entry. The IDF entry´s second word is zeroed as well, and the two word entry is placed into the primary or the overflow area.

The primary hash funciton of the addressing algorithm is the division method. The secondary function has three phases: open addressing linear search in a bucket of the primary area; if it is not successful, then open addressing linear search serially in the blocks of the overflow area; if it is not successful, open addressing linear search serially in the other blocks of the overflow area.

By this relatively simple, but sometimes too long algorithm ensured, that the algorithm always finds an empty slot.

The MASTER 4.1 operating system has been installed to handle maximum 4291 files (block number of FLD). To enlarge the FLD needs new installation of the whole operating system (about 50 hours computer time).

To define the IDF size, the installation guide book suggests the following: at first we fix the block size (bucket size) which should be divisible by two and less than 160 words (one sector on magnetic disc - if greater, then data transfer between the disc and the memory (30 msec) at least would increase by two).

In the existing installation block size is 160 words, that is one block can accomodate 80 entries.
Block number is defined as follows: increase the maximum file count (4291) by 10 % (4720) then divide by the number of entries (80) and from the following prime numbers (3,7,11,31,59,127,503,1019,2039,4091,8191,16319,32719, 65519) which is closest one but less than the result of the previous division (in our case: 59). The form of these prime numbers is

$$4j + 3 \qquad (j \text{ integer})$$

which ensures a uniform distribution of the files over the available addresses.

These 59 blocks consist of the primary area. The overflow area is 10 % of the primary area (in our case 6).

According to the installation instructions the bucket size is between 2-160 words. Because the IDF and FLD reside on magnetic disc, our task is to minimize the data transfer between the memory and the magnetic disc (memory cycle time by four magnitudes less than data transfer time!).

To execute the search algorithm - excluding data transfer - we need about 1 msec (it varies with loading factor of the files).

The CDC 3300 computer operates in three shifts, 5 day a week (300-400 jobs per day) and about 50 allocation and deletion occurs daily. File data modifications are about 10-15 daily. File openings are about 1000 daily.

To analyze the quality of the system, the following data are of particular interest:

- (α) loading factor at the overflow area
- dislocation (displacement distance from the "originally" appointed block)
- the effect of insertion/deletion cycles on the previous data

Our system to be analyzed had 4182 files (loading factor 97,5 %). In the course of the first measurement no deletions occured.

If $\alpha < 90$ % no entries were occupied in the overflow area. If $\alpha = 90$ %, one primary block became full, that is the probability that an overflow entry becomes occupied equals $\frac{1}{59} = 0.0169$.

This result well coincides with the theoretical approximation. The effect of insertion/deletion cycles was investigated in the [0.1;0.9] interval of α. The 40 characters of new files were generated by random number generator. One note before we evaluate the results: in optimal case one access to a file needs two data transfers (60 msec). This time is about 0.1 % of the daily work of the computer. Table 1 shows the effect of insertion/deletion cycles at different loading factors on the percentage increase of data transfers.

Insertion/deletion cycles

α	1000	2000	3000	4000	5000	6000	7000	8000	9000	10000
0.7	0.8	0.9	1.9	2.8	3.3	3.9	4.1	4.3	4.4	4.6
0.8	2.1	2.3	4.7	6.7	8.0	9.4	9.9	10.2	10.2	11.0
0.9	1.5	2.9	5.9	8.2	9.9	11.7	12.3	12.6	13.2	14.6
0.975	23.8	35.7	43.0	71.4	78.5	85.7	87.0	88.1	94.0	100.0

In the α [0.7;0.9] interval no essential increase. If $\alpha > 0.9$, and cycle number is in the magnitude or larger than the table size the data transfers are doubled. No full overflow area and for this reason dislocation and overflow loading factor could be easily deducted from data in Table 1.

The effective operation of the almost full IDF and FLD could be assured by the occasional run of *SF4 operating system program which insures the compression of the FLD and IDF.

According the above results, if the increase of data transfers exceeds 70 %, we have to run our program (at present about once in 5 weeks).

However the compression of the two files apparently increases slightly the system´s throughput, but taking into account, that job execution is suspended while data transfers executed, and the two file is single accessible, the effect of the compression is larger.

Ö s s z e f o g l a l ó

A CDC 3300 számitógép operációs rendszere file kezelő módszere

Radó Ákos

Analizáljuk a MASTER 4.1. operációs rendszer file kezelő módszerét. Vizsgáljuk a file behelyezés/kivétel ciklusok hatását a túlcsordulási terület kitöltöttségére és a file bejegyzések diszlokációjára. A rendszer hatékony Müködéséhez szükséges file bejegyzések átrendezésének gyakoriságát a mérési eredmények alapján adjuk meg.

Резюме

Метод обработки файловой системы операционной системы ЭВМ CDC 3300

Акош Радо

Анализируется метод обработки файловой системы операционной системы MASTER 4.1 ЭВМ CDC 3300. Качество системы кодирования hash измеряется оценкой влияния циклонов ввода/вывода на сдвиг и на коэффициент нагрузки области переполнения. Для оптимальной работы ЭВМ диапазон времени между двумя разборками файлов системы определяется.

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ON THE GENERATION OF BINARY VECTORS BY BOOLEAN FUNCTIONS

Hans-Dietrich Gronau

Dedicated to Prof. Dr. W. Engel on the
occasion of his 5 0th birthday.

1. Introduction and notation

This is the last paper in a series of four. In (2) the author began studies in the following direction.

Let k be an integer, $k \geq 2$.

Let $V_k = \left\{ \begin{pmatrix} a_1 \\ \\ a_k \end{pmatrix} , a_1, \ldots, a_k \in \{0,1\} \right\}$ and $M = \{ X_1, \ldots, X_n \} \subseteq V_k$.

Then we define $f(M)$, where f is a Boolean function and

$X_i = \begin{pmatrix} a_{i1} \\ \vdots \\ a_{ik} \end{pmatrix}$ for $i = 1, \ldots, n$, by

$$f(M) = f(X_1, \ldots, X_n) = \begin{pmatrix} f(a_{11}, \ldots, a_{n1}) \\ \vdots \\ f(a_{1k}, \ldots, a_{nk}) \end{pmatrix}$$

For a set K of Boolean functions we define the closure $[M]_K$ of M with respect to K.

Definition. *Let a sequence* $M_K^i \subseteq V_k$ *defined by*

1^0 $M_K^0 = M$ and

2^0 $M_K^{i+1} = M_K^i \cup \{ X : \exists\, f \in K,\ X_1, \ldots, X_n \in M_K^i : X = f(X_1, \ldots, X_n) \}$

for $i = 0,1,2, \ldots$

Then let $[M]_K = \lim\limits_{i \to \infty} M_K^i$.

We notice that the successor of M_K^i is a superset of M_K^i and all members of this sequence are subsets of V_k. Hence, starting by some M_K^a this sequence has to be constant. This M_K^a is denoted by $\lim\limits_{i \to \infty} M_K^i$ or by $[M]_K$, accordingly.

We will investigate the following problems:

1. Find K-conditions for M such that M is *K-complete*, i.e. $[M]_K = V_k$.

2. Find the cardinality of a *K-base*, i.e. M is K-complete, but any proper subset of

M is not K-complete. If there are K-bases of different cardinalities, find the minimal and the maximal cardinality of K-bases.

In (2),(3) and (4) we solved these problems for some closed sets of Boolean functions, namely for all closed sets of nonmonotonic functions. In (2), (3) and (4) we used M_K^1 for the closure of M with respect to K. Without loss of generality these restrictions are possible, because $M_K^1 = [M]_K$ was proved for closed sets K in (4). Moreover, in this paper (section2) we will prove $[M]_K = [M]_{[K]}$ for arbitrary sets K of Boolean functions, where $[K]$ is the usual closure of functions. Hence, in order to solve our problems, we only have to solve the problems for closed sets K. All closed sets of Boolean functions are known. For a survey and notations of these closed sets see (1).

In section 3 we give a survey of the results for all closed sets. In section 4 we prove these results.

2. A theorem

In this section we will prove the following

Theorem 1. *Let M be an arbitrary subset of V_k and let K be an arbitrary set of Boolean functions.*
Then $[M]_K = [M]_{[K]}$.

We give the following version of the definition of the closure $[K]$ which we will use in the proof of Theorem 1.

If $id \epsilon K$, $[K]$ is defined by (1), p.4, 1.,2.,3., and 4!:

Definition. *Let $id \epsilon K$. Let $K^i (i = 0,1, \ldots)$ be a sequence of sets of Boolean functions as follows.*

1^0 *If a function f belongs to K^i $(i = 0,1, \ldots)$, all functions which can be generated by f by adding fictive variables to f, identification of variables belong to K^i too.*

2^0 – $K^0 = K$

– $K^{i+1} = K^i \cup \{ f : \exists g, g_1, \ldots, g_m \epsilon K^i : f = g(g_1, \ldots, g_m) \}$
if $i = 0,1, \ldots$ *and*

– $[K] = \bigcup_{i=0}^{\infty} K^i$.

We notice that we only need a finite set of Boolean functions for the generation of $[M]_{[K]}$, i.e. there is an integer a with

(1) $[M]_{[K]} = [M]_{K^a}$.

Moreover, we only need functions of K^a with a finite number of variables. Finally it is worthy of remark that only K^0 has to contain all functions obtained by 1^0. This property we will use.

Proof of Theorem 1.

a) Let $id \epsilon K$. First we prove $[M]_{[K]} \subseteq [M]_K$. Let b be an integer, $b \geqslant 1$. Then there is an integer q satisfying

(2) $M^q_{K^b} = [M]_{K^b}$

by our remarks at the definition of the closure of M with respect to K.

We prove

(3) $[M]_{K^{b-1}} \supseteq M^i_{K^b}$

by induction on $i (i \geqslant 1)$, for all integers $b \geqslant 1$.

1. $i = 1$. Then $M^1_{K^b} = M^0_{K^b} \cup \{X : \ \exists f \epsilon K^b, \exists X_1, \ldots, X_n \epsilon M : X = f(X_1, \ldots, X_n)\}$.

Let us assume there is a vector $X \epsilon M^1_{K^b} \setminus [M]_{K^{b-1}}$.

Then there is a function $f \epsilon K^b$ and there are vectors $X_1, \ldots, X_n \epsilon M = M^0_{K^b}$ with $X = f(X_1, \ldots, X_n)$. If $f \epsilon K^{b-1}$ then $X \epsilon [M]_{K^{b-1}}$, which is a contradiction to our assumption. Hence, $f \epsilon K^b \setminus K^{b-1}$. Then there are functions $g, g_1, \ldots, g_m \epsilon K^{b-1}$ with $f = g(g_1, \ldots, g_m)$, i.e. $X = g(g_1(X_1, \ldots, X_n), \ldots, g_m(X_1, \ldots, X_n))$. By $g_j \epsilon K^{b-1}$ and $X_l \epsilon M$ $(j = 1, \ldots, m;\ l = 1, \ldots, n)$ it follows $X_j = g_j(X_1, \ldots, X_n) \epsilon [M]_{K^{b-1}}$

Hence using $g \epsilon K^{b-1}$, we obtain $X = g(X_1, \ldots, X_m) \epsilon [M]_{K^{b-1}}$, which is also a contradiction to our assumption.

2. We have $M^{i+1}_{K^b} = M^i_{K^b} \cup \{X : \exists f \epsilon K^b, \exists X_1, \ldots, X_n \epsilon M^i_{K^b} : X = f(X_1, \ldots, X_n)\}$.

Let us consider an arbitrary vector $x \epsilon M^{i+1}_{K^b}$. If $X \epsilon M^i_{K_b}$ then $X \epsilon [M]_{K^{b-1}}$ follows by induction assumption.

Let $X \epsilon M^{i+1}_{K^b} \setminus M^i_{K^b}$. Then there is a function $f \epsilon K^b$ and there are vectors $X_1, \ldots, X_n \epsilon M^i_{K^b}$ with $X = f(X_1, \ldots, X_n)$. By the induction assumption we have

$X_1, \ldots, X_n \epsilon [M]_{K^{b-1}}$. $f \epsilon K^b$ implies by the definition of K^b that there are functions

$g, g_1, \ldots, g_m \epsilon K^{b-1}$ with $f = g(g_1, \ldots, g_m)$. Hence $X'_j = g_j(X_1, \ldots, X_n) \epsilon [M]_{K^{b-1}}$

$(j = 1, \ldots, n)$ and finally $X = g(g_1(X_1, \ldots, X_n), \ldots, g_m(X_1, \ldots, X_n)) =$

$= g(X'_1, \ldots, X'_m) \epsilon [M]_{K^{b-1}}$. Therefore (3) is proved for arbitrary $i \geqslant 1$ and arbitrary fixed $b \geqslant 1$. Inparticular (3) is proved for $i = q$, where q is defined as in (2). Using (2) we have

$$[M]_{K^{b-1}} \overset{\supset}{=} [M]_{K^b}$$

for arbitrary integer $b \geq 1$.

We observe $[M]_K = [M]_{K^0}$ (The vectors, which can be generated by functions of $K^0 \setminus K$, we also obtain by functions of K, what follows by the definition of the closure $[M]_K$.).

By induction we get for arbitrary integer $b \geq 1$:

$$(4) \qquad [M]_{K^b} \overset{\subset}{=} [M]_K .$$

In particular, (4) holds for $b = a$, where a is defined in (1), i.e.

$$[M]_{[K]} \overset{\subset}{=} [M]_K .$$

Clearly, $K = [K]$ implies the converse direction

$$[M]_K \overset{\subset}{=} [M]_{[K]} .$$

If $id \in K$, the theorem is proved.

b) Let $id \notin K$. By the definition of $[M]_K$ we have $[M]_{K'} = [M]_{K' \cup \{id\}}$ for all sets of functions K'.
If c_0 and c_1 are the constant functions,
$K \overset{\subset}{=} \{c_0, c_1\}$ implies $[K] = K$ and $[K \cup \{id\}] = [K] \cup \{id\}$ and $K \not\subseteq \{c_0, c_1\}$ implies $id \in [K]$ and $[K \cup \{id\}] = [K] \cup \{id\}$ too.
Hence, using part a), we obtain

$$[M]_K = [M]_{K \cup \{id\}} = [M]_{[K \cup \{id\}]} = [M]_{[K] \cup \{id\}} = [M]_{[K]} .$$

q.e.d.

3. A survey on results

In this section we will give the answers to our problems for each closed set of Boolean functions.

Let the closed sets of Boolean functions denote by the notation by Post, see (1).

Further we use the following notations:

- $\underline{0} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} \in V_k$, $\underline{1} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \in V_k$.

- $e_i (i = 1, \ldots, k)$ denotes the vector of V_k containing a 1 exactly in the i-th component.

If $X \in V_k$, $\overline{X}$ denotes the vector of V_k which does not coincide with X in any component.

– If $M \subseteq V_k$, we consisder M also as a matrix. We say M has the property $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$, if and only if for each pair (i,j), $1 \leq i < j \leq k$, the 2-rows-matrix M_{ij}, whose first row is the i-th row of M and the second row is the j-th row of M, has a column $\binom{0}{1}, \binom{1}{0}, \binom{0}{0}, \binom{1}{1}$, respectively. Further M has the property $\boldsymbol{C}(\mu), \boldsymbol{D}(\mu)$, $\mu \geq 2$, if and only if every matrix consisting of μ rows of M has a column

$$\begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix},$$

respectively. Let $\boldsymbol{P}$ be a Boolean function of $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$. Then M has the property $\boldsymbol{P}^i$, $i \in \{0,1\}$, if and only if M has the property $\boldsymbol{P}$ and does not contain, in addition, rows consisting only of i's. Accordingly, let $\boldsymbol{P}^{01}$ defined as $\boldsymbol{P}^0 \wedge \boldsymbol{P}^1$.

Now we are able to formulate the main results.

2. **A theorem** *Let* $K \in \{0_i, S_i, P_i, L_i\}$. $M \subseteq V_k$ *is*

1^0 *K-complete, if and only if* M *satisfies the condition of table* 1,

2^0 *a K-base, if and only if* M *is K-complete and has the cardinality given in table* 1.

set	criterion of completeness	m
0_1	$M = V_k$	2^k
$0_2, 0_5$	$M \supseteq V_k \setminus \{1\}$	$2^k - 1$
$0_3, 0_6$	$M \supseteq V_k \setminus \{0\}$	$2^k - 1$
0_4	$\forall X \in V_k$ we have $X \in M$ or $\overline{X} \in M$	2^{k-1}
$0_7, 0_8$	$M = V_k \setminus 0,1$	$2^k - 2$
0_9	$\forall X \in V_k \setminus 0,1$ we have $X \in M$ or $\overline{X} \in M$	$2^{k-1} - 1$
S_1, S_3	$M \supseteq \{\underline{0}, e_1, \ldots, e_k\}$	$k+1$
S_5, S_6	$M \supseteq \{e_1, \ldots, e_k\}$	k
P_1, P_3	$M \supseteq \{\underline{1}, \overline{e}_1, \ldots, \overline{e}_k\}$	$k+1$
P_5, P_6	$M \supseteq \{\overline{e}_1, \ldots, \overline{e}_k\}$	k
L_1	$\exists X_1, \ldots, X_{k-1} \in M$: $rg(X_1, \ldots, X_{k-1}, \underline{1}) = k$	$k-1$
L_2	$\exists X_1, \ldots, X_k \in M$: $rg(\overline{X}_1, \ldots, \overline{X}_k) = k$	k
L_3	$\exists X_1, \ldots, X_k \in M$: $rg(X_1, \ldots, X_k) = k$	k
L_4	$\exists X_1, \ldots, X_k \in M$: $rg(X_1, \ldots, X_k) = k$ and $X \in M \setminus [\{X_1, \ldots, X_k\}]_{L_4}$	$k+1$
L_5	$\exists X_1, \ldots, X_{k-1} \in M$: $rg(X_1, \ldots, X_{k-1}) = k-1$ $\exists X \in M \setminus [\{X_1, \ldots, X_{k-1}\}]_{L_4}$ and $\nexists$ even number of vectors of $X_1, \ldots, X_{k-1}, X$ with sum 1	k

Table 1.

These results are proved in (2), (3) and (4).

Theorem 3. *Let* $K \in \{C_i, D_i, A_i, F_i^\mu, F_i^\infty\}$. $M \subseteq V_k$ *is K-complete, if and only if* M *satisfies the condition* $\boldsymbol{P}$ *of table* 2. *The K-bases have the minimal cardinality* m *and the maximal cardinality* p, *given in table* 2.

In table 2 let

1. 1. $a \in \{0,1\}$,
   2. $]x[\, = \min (y\colon y \in N,\ y \geqslant x)$,
   3. $\varphi_1(k) = x \in N \leftrightarrow \binom{x}{[\frac{x}{2}]} \geqslant k > \binom{x-1}{[\frac{x-1}{2}]}$,
   4. $\varphi_2(k) = x \in N \leftrightarrow \binom{x}{[\frac{x-1}{2}]} \geqslant k > \binom{x-1}{[\frac{x-2}{2}]}$.
   5. *) For $\mu = 3$ we do not give an explicit formula; see the remark at the end of this section.

K		$\boldsymbol{P}$	m	p
C_1		$\boldsymbol{A} \vee \boldsymbol{B}$	$]\log_2 k[$	$k-1$
C_2		$(\boldsymbol{A} \vee \boldsymbol{B})^1$	$]\log_2 (k+1)[$	k
C_3		$(\boldsymbol{A} \vee \boldsymbol{B})^0$	$]\log_2 (k+1)[$	k
C_4		$(\boldsymbol{A} \vee \boldsymbol{B})^{01}$	$]\log_2 (k+2)[$	$k+1$
D_1		$((\boldsymbol{A} \vee \boldsymbol{B})(\boldsymbol{C} \vee \boldsymbol{D}))^{01}$	$]\log_2 (k+1)[+ 1$	$k+1$
D_2		$\boldsymbol{ABCD}$	$\varphi_2(k) + 1$	$\begin{cases} 2k & 2 \leqslant k \leqslant 4 \\ \binom{k}{2} & k \geqslant 5 \end{cases}$
D_3		$(\boldsymbol{A} \vee \boldsymbol{B})(\boldsymbol{C} \vee \boldsymbol{D})$	$]\log_2 k[+ 1$	k
A_1, A_2, A_3, A_4		$\boldsymbol{AB}$	$\varphi_1(k)$	$\begin{cases} 2k-2 & 2 \leqslant k \leqslant 6 \\ [\frac{k^2}{4}] & k \geqslant 7 \end{cases}$
F_1^μ	$(\mu < k)$	$((\boldsymbol{A} \vee \boldsymbol{B})\boldsymbol{C}(\mu))^0$	$]\log_2 (k+1)[+ 1$	$\begin{cases} \binom{k}{\mu} & 2 \leqslant \mu \leqslant k-2 \\ k+1 & \mu = k-1 \end{cases}$
F_5^μ	$(\mu < k)$	$((\boldsymbol{A} \vee \boldsymbol{B})\boldsymbol{D}(\mu))^1$		
F_{2+a}^μ	$(\mu < k)$	$\boldsymbol{ABC}(\mu)$	$\begin{cases} \varphi_2(k) & \mu = 2 \\ \varphi_1(k) + 1, & \mu \geqslant 4 \end{cases}$	$\begin{cases} 2k-1 \begin{cases} 2 \leqslant \mu = k-1 \leqslant 5 \\ 2 = \mu = k-2 \end{cases} \\ [\frac{k^2}{4}] + 1 & 6 \leqslant \mu \leqslant k-1 \\ \binom{k}{\mu} & 2 \leqslant \mu \leqslant k-2 \geqslant 3 \end{cases}$
F_{6+a}	$(\mu < k)$	$\boldsymbol{ABD}(\mu)$		
F_4^μ	$(\mu < 2)$	$(\boldsymbol{A} \vee \boldsymbol{B})\boldsymbol{C}(\mu)$	$]\log_2 k[+ 1$	$\binom{k}{\mu}$
F_8^μ	$(\mu < k)$	$(\boldsymbol{A} \vee \boldsymbol{B})\boldsymbol{D}(\mu)$		
$F_1^\infty,\ F_1^\mu$	$(\mu \geqslant k)$	$((\boldsymbol{A} \vee \boldsymbol{B})\boldsymbol{C}(k))^0$	$]\log_2 (k+1)[+ 1$	$k+1$
$F_5^\infty,\ F_5^\infty$	$(\mu \geqslant k)$	$((\boldsymbol{A} \vee \boldsymbol{B})\boldsymbol{D}(k))^1$		
$F_{2+a}^\infty,\ F_{2+a}^\mu$	$(\mu \geqslant k)$	$\boldsymbol{ABC}(k)$	$\varphi_1(k) + 1$	$\begin{cases} 2k-1 & 2 \leqslant k \leqslant 6 \\ [\frac{k^2}{4}] + 1 & k \geqslant 7 \end{cases}$
$F_{6+a}^{-\infty},\ F_{6+a}^\mu$	$(\mu \geqslant k)$	$\boldsymbol{ABC}(k)$		
$F_4^\infty,\ F_4^\mu$	$(\mu \geqslant k)$	$(\boldsymbol{A} \vee \boldsymbol{B})\boldsymbol{C}(k)$	$]\log_2 k[+ 1$	k
F_8^∞, F_8^μ	$(\mu \geqslant k)$	$(\boldsymbol{A} \vee \boldsymbol{B})\boldsymbol{D}(k)$		

4. Proof of Theorem 3

1. Completeness

By Theorem 2 of (3) we have only to consider the following closed sets of Boolean functions: $C_1, C_3, C_4, D_1, D_2, D_3, A_1, A_3, A_4, F_i^{\mu}, F_i^{\infty} (i = 1,2,3,4)$. The problems were solved for the sets $C_1, C_3, C_4, F_i^{\mu}, F_i^{\infty}$ $(i = 1,4)$ in (3) and for the sets D_1 and D_3 in (2). So we have to prove the statement of Theorem 3 for the closed sets

$$K \epsilon \{ D_2, A_1, A_3, A_4, F_2^{\mu}, F_2^{\infty}, F_3^{\mu}, F_3^{\infty} \} .$$

1. First we show that the conditions of table 2 are necessary. Let M be K-complete.

1.1 The monotony of the functions of K implies that M satisfies $\boldsymbol{AB}$ (i.e. $\boldsymbol{A} \wedge \boldsymbol{B}$). To show this, let (i,j) be a pair with $i,j \epsilon \{1, \ldots, k\}$ and $i \neq j$ such that $M_{i,j}$ does not contain a column $\binom{1}{0}$. Then it is impossible to generate vectors, having the i-th component 1 and the j-th componenet o, by monotonic functions, i.e. $[M]_K \neq V_k$.
Hence M has to satisfy $\boldsymbol{B}$ and, in analogy, $\boldsymbol{A}$ too.

1.2. If $K = D_2$, M has to satisfy $\boldsymbol{CD}$ too. To show this, let (i,j) be a pair with $i,j \epsilon \{1, \ldots, k\}$ and $i \neq j$ such that M_{ij} does not contain a column $\binom{1}{1}$. Let M'_{ij} be a matrix of the same type as M_{ij}, whose elements of the first row coincide with the correspondent elements of the first row of M_{ij}, while this does not hold for any element of the second row of M_{ij}. Then M'_{ij} does not contain a column $\binom{1}{0}$, it is impossible to generate the vector $\binom{1}{0}$ by M'_{ij} and by a monotonic function. Thus, it is impossible to generate vectors, having 1 as the i-th and j-th component, by M_{ij} and by functions of $K = D_2$.
Hence, M has to satisfy $\boldsymbol{D}$ and, in analogy, $\boldsymbol{C}$ too.

1.3. Let $K \epsilon \{F_i^{\mu}, F_i^{\infty}\}$ $(i = 2,3)$. Then M has the property $\boldsymbol{C}(\mu)$ for $\mu < k$ and $\boldsymbol{C}(k)$ for $\mu \geqslant k$ and $\mu = \infty$. Either $\underline{0} \epsilon M$ or $\underline{0} \notin M$.
In the first case M satisfies $\boldsymbol{C}(\mu)$ and in the second case there is a function $f \epsilon K$ with $f(M) = \underline{0}$. Now the statement follows by the definition of the functions of F_i^{μ} or F_i^{∞}.

1.4. If $K \epsilon \{ A_3, A_4, F_2^{\mu}, F_2^{\infty} \}$, $f(o,o, \ldots, o) = o$ holds for each function $f \epsilon K$. Thus, M does not contain rows consisting of o's only.

1.5. If $K \epsilon \{A_4, F_2^{\mu}, F_2^{\infty}\}$ we obtain, in analogy to 1.4., that M has no rows consisting of 1's only.
We notice that M satisfying $\boldsymbol{AB}$ implies M has no rows consisting of o's or 1's only.

2. In order to show that the conditions of table 2 are sufficient, let M be a matrix having the property $\boldsymbol{P}(K)$ of table 2. Denote the rows of M by α_i and let

$$\underline{a} = \begin{pmatrix} a_1 \\ \vdots \\ a_k \end{pmatrix}$$

be an arbitrary chosen vector of V_k. Then we give a function $f \epsilon K$ satisfying $f(M) = \underline{a}$. If $\beta = (b_1, \ldots, b_t)$ and $\gamma = (c_1, \ldots, c_t)$, $\beta < \gamma$ means that $b_i \leqslant c_i$ for $i = 1, \ldots, t$, and at least for one i we have the inequality.

2.1 Let $K \epsilon \{A_1, A_3, A_4, F_2^{\mu}, F_2^{\infty}, F_3^{\mu}, F_3^{\infty}\}$.

Then

$$f(\alpha) = \begin{cases} a_i & \text{if } \alpha = \alpha_i, \\ 0 & \text{if there is a } \alpha_i \text{ with } \alpha < \alpha_i, \\ 1 & \text{otherwise.} \end{cases}$$

2.2 Let $K = D_2$.
Then

$$f(\alpha) = \begin{cases} a_i & \text{if } \alpha = \alpha_i, \\ \overline{a_i} & \text{if } \alpha = \alpha_i, \\ 0 & \text{if there is a } \alpha_i \text{ with } \alpha < \alpha_i \text{ or } \alpha < \overline{\alpha_i}, \\ 1 & \text{if there is a } \alpha_i \text{ with } \alpha > \alpha_i \text{ or } \alpha > \overline{\alpha_i}, \\ 0 & \text{for all other } \alpha \text{ with } \alpha = (0, \sim), \\ 1 & \text{for all other } \alpha \text{ with } \alpha = (1, \sim). \end{cases}$$

Thus this part is proved.

2. Cardinality of bases

If we consider the matrices M as an incidence matrix of a family F of k subsets of an r-element set R, the determination of m is equivalent to the determination of the maximal cardinality $n(r)$ of families of a finite set satisfying a certain K-condition, according to $m = \min\{x : x \epsilon N,\ n(x) \geqslant k\}$.

The following conditions for M and F are equivalent:

- $\boldsymbol{AB} \leftrightarrow X \not\subset Y$ for all different $X, Y \epsilon F$,
- $\boldsymbol{CD} \leftrightarrow X \cap Y \neq 0, X \cup Y \neq R$ for all $X, Y \epsilon F$,
- $\boldsymbol{C}(\mu) \leftrightarrow \bigcup_{i=1}^{\mu} X_i \neq R$ for all $X_1, X_2, \ldots, X_{\mu} \epsilon F$.

The maximal cardinality of families satisfying the conditions related to ***AB***, ***ABCD***, ***ABC***(2), ***ABC***(μ) $\mu \geqslant 4$ was determined by Sperner [12], Katona [9] and Schönheim [11] and Brace and Daykin [7], Milner [10], the author [5], respectively.

Fraknl [8] and the author [5] solved this problem in the ***ABC***(3) case for sufficiently large r. These maximal cardinalities have different structures for even and odd r. So we did not give an explicit formula in table 2 in this case.

The values of p were determined by the author in [6].

References

[1] S.W. Jablonski, G.P. Gawrilow, W.B. Kundrjawzew, Boolesche Funktionen und Postsche Klassen, Akademie-Verlag Berlin, 1970.

[2] H.-D. Gronau, Erzeugung dualer Vektoren durch selbstduale Funktionen, *Wiss. Zeitschrift der Univ. Rostock, Math.-Nat.* Reihe, **23** (1974), 9, 791-799.

[3] H.-D. Gronau, Erzeugung dualer Vektoren durch gewisse abgeschlossene Mengen Boolesher Funktionen, *Rostocker Math. Kolloquium* 3 (1977), 45-56.

[4] H.-D. Gronau, On the generation of binary vectors by some closed sets of Boolean functions (linear functions and alternatives), Proceedings of the 4th Winterschool on t the Theory of Operating Systems – "Visegrád" 1978, Computer and Automation Institute of the Hungarian Academy of Science, to appear.

[5] H.-D. Gronau, On Sperner familoes in which no k sets have an empty intersection, *J. Combinatorial Theory A,* to appear.

[6] H.-D. Gronau, Minimale Familien, in: Extremale Familien von Teilmengen einer endlichen Menge und die Erzeugung von dualen Vektoren durch Boolesche Funktionen, Dissertation, Wilhelm-Pick-Universitat Rostock, 1978, 33-67.

[7] A. Brace, D.E. Daykin, Sperner type theorems for finite sets, *Combinatorics* (Proc. Conf. Combinatorial Math. Inst. Oxford, 1972), 18-37.

[8] P. Frankl, On Sperner families satisfying an additional condition, *J. Combinatorial Theory* ***A*** **20** (1976), 1-11.

[9] G.O.H. Katona, Two applications of Sperner type theorems (for search theory and truth functions), *Period. Math. Hung.* **3** (1973), 19-26.

[10] E.C. Milner, A combinatorial theorem on systems of sets, *J. London Math. Soc.* **43** (1968), 204-206.

[11] J. Schönheim, On a problem of Purdy related to Sperner systems, *Canad, Math. Bull.* **17** (1974), 135-136.

[12] E. Sperner, Ein Satz über Untermengen einer endlichen Menge, *Math. Z.* **27** (1928), 544-548.

Összefoglaló

Bináris vektoroknak Boole függvényekkel való generálásáról

Hans-Dietrich Gronau

Legyen $M \subset \{0,1\}^k$, ahol k természetes szám. Jelölje K a Boole függvények egy zárt halmazát. Az összes zárt Boole függvényhalmazra megadja a szerző annak szükséges és elégséges feltételét, hogy M K-teljes legyen, azaz hogy M K-lezárása megegyezzen a $\{0,1\}^k$ halmazzal. Továbbá meghatározza $\{0,1\}^k$ K-bázisainak lehetséges minimális és maximális számosságát, ahol M K-bázis ha minimális a K-tejességre nézve.

Резюме

О порождении бинарных векторов булевым функциям

Ханц-Дитрих Гронау

Пусть $M \leq \{0,1\}^k$, где k натуральное число, и K замкнутое множество Булевых функций. Автор дает необходимые и достаточные условия K-полности множества M. Под K-полностью понимается, что замыкание по K множества M равно множеству $\{0,1\}^k$. В дальнейшем будут определены возможные минимальные и максимальные мощности K-базисов множества $\{0,1\}^k$, где M является K-базисом, если оно минимально относительно K-полноты.

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MATHEMATICAL MODELLING OF THE R 12 "UNTER"

István Kun – Gábor Farkas

MTA SzTAKI – SzKI

"UNTER" is the abbreviation of Universal Terminal System. It has been developed in the SZKI (Coordination Institute for Computing Techniques).

The main task of the system is to organize the simultaneous work of a few terminals. These terminals are controlled by a R 12 minicomputer. (OS12, partition F1). Their job is to establish a direct interactive communication between an IBM 370/125 computer and the users. Of course the R 12 is not completely exploited by the satellite functions (e.g. preprocessing, postprocessing). In the remaining time it can act as an independent computer.

A special technique, called SPOOL, is applied to perform the input–output operations of the R 12. The functioning of SPOOL is roughly as follows:

A record, entering the system, first goes through a space compression, after which it is stored on a disk file. On the other hand a record, leaving the system, goes through a space decompression (it is restored in the original form), after which it is passed through a channel. Both READ and WRITE operations consist of two parts: a fast processor action initiates a slow peripherial action. Space compression decompression is again a processor job. When a file is transferred, only one of its records can move at the same time. That is this record, until it arrives, blocks the route for the next record of its file. The advantage of this organization is quite evident. The processor can quickly change from one channel to the other, to look for a channel the records of which have a free route to go on, while on the other busy channels the slow peripherial actions are being performed simultaneously. When the processor does not find any channel waiting for it, some other computational work can be done.

For the time being there are 22 channels handled by the R 12. These channels are:

Nr.	Direction	Equipment
0	I/0	asynchron line to/from the IBM 370/125
1	I/0	console
2	I/0	
.		
.		user terminals (VT 340 displays)
.		
9	I/0	
10	I	card reader
11	0	line printer
12	I	BG card reader
13	0	BG line printer
14		idle
15	I/0	
.		
.		user libraries (on the disk)
.		
22	I/0	

As we have seen before, while a record gets through the core memory, the processor is needed three times. Therefore it is necessary to register for each channel whether and at which phase the processor is expected. This registration is made by SPOOL in three double words:

Bit Nr. / Task	0	1	2	3	4	. . .	21	22
READ						. . .		
COMPR/DECOMPR						. . .		
WRITE						. . .		

The usual value of each bit is 0. A change to 1 means a request for the processor. Such a request arises when either a new record enters the system or a peripherial action is finished. Of course there may be no more than one 1 in a column.

The service principle is as follows: The first row is inspected in natural order. If there is a request, it will be served and the inspection recommences from the first bit. The second line comes only when there is no more unsatisfied request in the first row. The satisfaction of a second row request is followed by a return to the first bit of the first row. The third row is inspected only when there is no more unsatisfied request in the first two rows. Which means that priority decreases from left to right, the last bit of a word being followd by the first bit of the next word.

Of course no request can avoid detection and satisfaction: as we can see from the description of SPOOL, no more than one, often no 1 appears in a column, the latter for much longer periods. Therefore columns with higher priority cannot permanently "capture" the processor.

Mathematical modelling of the system needs some simplifying assumptions:

1. Channels operate in one direction only.
2. Buffers can accept only one record at the same time, as we have supposed. (Actually: five records).
3. In every channel files arrive according to Poisson processes. File lengths are geometrically, processor and peripherial times are exponentially distributed. All these variables are independent among themselves. (According to experience, some of the service times may be constant, a good approximation of which is the convolution of several exponential distributions. This might be interpreted simply as an increase in the number of exponential service phases.)

Possible states in the case of two channels, with the parameters of the exponential distributions:

		Channel 1		Channel 2
1.	α_1	Interarrival time	β_1	Interarrival time
2.		Wait		Wait
3.	α_3	Read/CPU	β_3	Read/CPU
4.	α_4	Read/Periph.	β_4	Read/Periph.
5.		Wait		Wait
6.	α_6	Compr./Decompr.	β_6	Compr/Decompr.
7.		Wait		Wait
8.	α_8	Write/CPU	β_8	Write/CPU
9.	α_9	Write/Periph.	β_9	Write/Periph.

States 2,5 and 7 represent the cases when work is interrupted because the processor is engaged with the other channel.

For a given channel the possible states remain the same even if the number of channels is increased.

For N channels the number of possible different states of the system is of course not 9^N but

(1) $\quad N * 3 * 6^{N-1} + 3^N$

where the first member gives the number of states when the processor is busy, while the second member gives the number of states when the processor is idle.

Following the method well–known in queuing theory (see e.g. [3]) we can easily set up the birth–and–death type differential equations. The resulting system of equations has the form

$$(2) \qquad \dot{\underline{P}}(t) = \underline{\underline{A}} * \underline{P}(t)$$

where $\underline{P}(t)$ is the vector of the probabilities of possible states and $\underline{\underline{A}}$ is a matrix with constant elements. We know also that

$$(3) \qquad \|\underline{P}(t)\|_{i_1} = 1 \qquad\qquad 0 \leqslant t < +\infty .$$

Since (2) is a finite system of linear differential equations with constant coefficients, (2) and (3) give the existence of

$$(4) \qquad \begin{aligned} &\lim_{t \to +\infty} \underline{P}(t) = \underline{P} \\ &\|\underline{P}\|_{i_1} = 1 \\ &\lim_{t \to +\infty} \dot{\underline{P}}(t) = \underline{0} \end{aligned}$$

so (2) becomes with t

$$(5) \qquad \underline{0} = \underline{\underline{A}} * \underline{P}$$

so a system of linear algebraic equations remains to be solved.

Instead of the detailed description of (5), we try to get some more compact information. For $N = 2$ denote by $P_i(Q_i)$ the probability that the first (second) channel is in the state i, and by $E(F)$ the parameter of the file length distribution in the first (second) channel. Then

$$(6) \qquad \begin{aligned} 0 &= -\alpha_1 P_1 + \alpha_9 (1 - E) P_9 \\ 0 &= \alpha_1 P_1 - \alpha_3 P_3 + \alpha_9 E P_9 \\ 0 &= \alpha_3 P_3 - \alpha_4 P_4 \\ 0 &= \alpha_4 P_4 - \alpha_6 P_6 \\ 0 &= \alpha_6 P_6 - \alpha_8 P_8 \\ 0 &= \alpha_8 P_8 - \alpha_9 P_9 \end{aligned}$$

and

$$
\begin{aligned}
0 &= -\ \beta_1 Q_1 + \beta_9 (1-F) Q_9 \\
0 &= \ \beta_1 Q_1 - \beta_3 Q_3 + \beta_9 F Q_9 \\
0 &= \ \beta_3 Q_3 - \beta_4 Q_4 \\
0 &= \ \beta_4 Q_4 - \beta_6 Q_6 \\
0 &= \ \beta_6 Q_6 - \beta_8 Q_8 \\
0 &= \ \beta_8 Q_8 - \beta_9 Q_9
\end{aligned}
\tag{7}
$$

Every equation of (6) and (7) is the sum of a few equations in (5). The role of the channels is not symmetrical, due to the priority rule, therefore the identity in the structures of (6) and (7) must have a more general reason. Let us take e.g. the first channel. Let us fix for the stationary distribution

$$
w = \frac{\textit{waiting time}}{\textit{total time}} \tag{8}
$$

and consider the remaining $P_i - s$ under the condition (8). If we observe the system only outside the waiting periods, a (2) – type relation for $\underline{P} = (P_1, P_3, P_4, P_6, P_8, P_9)^T$ can be found. (6) will correspond to (5) and of course (8) to the second relation in (4). This reasoning is independent from N, which proves the general validity of (6) for any channel among the N ones.

Of course (5) cannot be eliminated; since (6) gives only the ratio of the $P_i - s$. With $N = 22$ (1) gives a number, which is hopeless for a computer as the dimension of (5). Again some further simplification is necessary.

Suppose that the loadings of the uniform channels 2 – 9 are identical and the same holds for the uniform channels 15 – 22. We try to compress 8 uniform channels into one. This is possible because the peripherial time is 20 – 40 times longer than the processor time. The number of the simultaneously busy channels follows a binomial distribution with expectation K. In the compressed common channel the length of a peripherial period has an exponential distribution with parameter $K\alpha$ where α is the individual parameter. In the compressed common channel the processor period will have a rough but reasonable approximation. We suppose it to be a variable being the product of a negative binomial distribution with expectation K and a constant T, which is the expextation of the individual processor time. Again this distribution has a good approximation by an exponential distribution. This kind of compression works well if for an individual channel the different processor and peripherial periods have nearly the same parameter in the exponential distribution. Of course other ideas of compression may also be used.

The compressed system has 5 channels, which – by means of (1) – involves $5 * 3 * 6^{5-1} + 3^5 \approx 20\,000$ equations, each of which have about 10 non–zero elements. This can already be handled with available computers.

Bibliography

[1] E. G. Coffman: Computer and job–shop scheduling theory. John Wiley and Sons, New York, 1976.

[2] C. Everling: Exercises in computer systems analysis Lecture Notes in Operations Research, No. 65. Springer Verlag, Berlin – Heidelberg – New York (1972).

[3] L. Takács: "Priority Queues", Operations Research. 12 (1964).

Összefoglaló

Az R12 "UNTER" matematikai modellezése

Kun István – Farkas Gábor

A cikk sorbanállási modellt ad az SZKI-ban kifejlesztett R12 számitógép perifériáinak müködését szabályozó rendszerre. Az egzakt modell a rendszer lehetséges állapotainak száma miatt megoldhatatlan. A cikkben tárgyalt kielégitő pontosságu közelitésekkel azonban a modellt megoldható méretüvé lehet redukálni.

Резюме

Математическое моделирование Р12 "УНТЭР"

Иштван Кун - Габор Фаркаш

Настоящая статья дает модель массового обслуживания к системе, развитой в Институте координации вычислительной техники, управляющей ходом периферий вычислительной машины Р12. Эгзактная модель неразрешима из-за большого числа возможных состояний системы. Но при помощи приближений удовлетворяющей точности, дискретных в статье, модель может уменьшатся до решаемого размера.

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CONDITIONAL MONOTONOUS FUNCTIONS OVER A FINITE SET. PART I.

Gustav Burosch, Klaus-Dieter Drews, Walter Harnau, Dietlinde Lau

1. Introduction

Let $E_k = \{0,1,\dots,k-1\}$ where k is an integer with $k \geq 2$, $P_k^{(n)}$ the set of all functions $f(x_1,x_2,\dots,x_n)$ of n variables defined whenever all the $x_i \in E_k$ and with values in E_k and $P_k = \bigcup_{n\geq 1} P_k^{(n)}$. The operation of superposition (composition) and the closure $[M]$ of a subset M of P_k are introduced in the usual manner (see e.g. [3] and [4]).

Let r an arbitrary partial order on E_k. Let $M_r^{(n)}$ the set of all $f(x_1,x_2,\dots,x_n) \in P_k^{(n)}$ satisfying the following condition:

$(a_1,a_2,\dots,a_n),(b_1,b_2,\dots,b_n) \in E_k^n$, $a_i \; r \; b_i$ $(i=1,2,\dots,n)$ implies $f(a_1,a_2,\dots,a_n) \; r \; f(b_1,b_2,\dots,b_n)$.

Let $M_r = \bigcup_{n\geq 1} M_r^{(n)}$. M_r is the set of all r-monotonous functions of P_k, because $M_r = [M_r]$ holds.

These closed classes M_r are very interesting not only with respect to the manifold applications but also with respect to the difficulty of the mathematical problems concerning these classes (e.g. the number of functions in $M_r^{(n)}$ or the problem of the existence of a finite base for M_r).

Because every partial order r is a binary relation, for h = 2 the connection between M_r and r is a special case of the concept Polρ of an arbitrary h-ary relation

$$\rho = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{h1} & a_{h2} & \cdots & a_{hm} \end{pmatrix} .$$

Let $Pol\rho = \bigcup_{n\geq 1} Pol^{(n)}\rho$, where $Pol^{(n)}\rho$ consists of all that functions $f \in P_k^{(n)}$, for which the row-wise application of f to n arbitrary column of ρ produces a column of ρ again.

The particularity of this paper is, that (for the first time) a weaker conception of the monotonity (the conditional monotonity) is investigated. We intend to explain the character of these weakening on the following example.

Let for $a \in E_3$ the relation $\{\tilde{\vec{a}}\}$ defined by

$E_3 \times \{a\} \cup \{(0,0), (1,1), (2,2)\}$.

We consider the set M of all functions $f \in P_3^{(n)}$, n = 1,2,..., satisfying the condition

$$\left.\begin{array}{l} \text{For all } a \in E_3 \text{ holds: If } (b_1,b_2,\ldots,b_n),\ (c_1,c_2,\ldots,c_n) \in E_3^n, \\ f(b_1,b_2,\ldots,b_n)=a \text{ and } b_i\ \{\tilde{\vec{a}}\}\ c_i \text{ for } i = 1,2,\ldots,n, \text{ then} \\ f(b_1,b_2,\ldots,b_n)\ \ \{\tilde{\vec{a}}\}\ f(c_1,c_2,\ldots,c_n). \end{array}\right\} \quad (1)$$

Let r_a the partial order $\begin{pmatrix} 0 & 1 & 2 & b & c \\ 0 & 1 & 2 & a & a \end{pmatrix}$, where $\{a,b,c\} = E_3$, so holds obviously $\bigcap_{a\in E_3} M_{r_a} \subseteq M$. If we consider the function g(x,y) (given table 1), so we see, that $\bigcap_{a\in E_3} M_{r_a} \subset M$ holds. The difference between $\bigcap_{a\in E_3} M_{r_a}$ and M is the particularity of the conception of the conditional monotonity.

x \ y	0	1	2
0	0	2	2
1	1	1	2
2	2	2	2

Table 1.

In addition to the definition (1) we investigate in this paper other conditions too.

Ju. I. Shurawljow was attentive to the functions with the property (1) working on the theory of noncorrect algorithms (see [5] and a paper prepared by him for Problems of Cybernetics (russian) Vol. 33). He regards the so-called correcting functions, which are such functions of $P_k^{(n)}$, which the results of the working of n algorithms on a set of m objects with respect to a measure of divergence, given from practical aspects, approximate as well as possible to an a priori given m-tuple of values of a certain predicate on this m obejcts. The fascination of this investigations is the following: If you make only few conditions on the correcting functions, so it is relatively easy in the arising voluminous set of functions of P_k to find an optimal correcting function in the sense of Shurawljow. Compared to it the in the practice relevant correcting functions are satisfying additional conditions. In the through it restricted set of functions it is more difficult to find optimal correcting functions. In particular to it you do need knowledges on the set of functions satisfying such conditions. Ju. I. Shurawljow said us certain of such conditions. Other conditions we added in result of discussions with him.

Here now our work begins. We investiagete the sets of functions which are given by certain of these conditions. In this paper we restrict us to $k = 3$ and to certain collections of the by Shurawljow named conditions, where essential differences to the usual monotonity here always appear. In general the set of the conditional monotonous functions do not be closed (with respect to the operation of superposition).

In this paper we investigate the with respect to the inclusion partial ordered set of the sets, which are defined by the various combinations of our conditions and show that some of these sets are equal $[\{x\}]$ (the set of the selector-funcitons). In two other papers, which will be published in Rostocker Mathematisches Kolloquium, we investigate the closure of sets of conditional monotonous functions and the clique-number of the graph defined by the threee partial orders r_o, r_1, r_2 and E_3^n.

We remark still, that we see an other application of the conditional monotonous functions in the mathematical description of votes too, if we allow the abstention from voting. We intend to facilitate by this example an interpretation of the essential contents of the by us investigated conditions. To it we consider the following situation. n persons vote in open election. On the base of their results a chairman has to give a result of the vote.

Let 0,1 resp. 2 the denotation of the results "No", "Yes" resp. "abstention from voting". A consequent chairman takes into consideration certainly the following rules. If all n persons vote with the same "a", $a \in E_3$, then he votes with "a" too. If nobody of the n persons votes with "a", $a \in E_2$, then he has to vote with "b", $b \in E_3 \setminus \{a\}$. If the chairman votes on the base of a concrete situation of vote with "a", $a \in E_2$, then he has to vote with "a" too, if in a second vote only one of the n persons changed his mind to "a". By these and similar considerations we receive the by us in the second paragraph defined conditions.

We are obliged to Ju. I. Shurawljow, who us refered by his request, to investigate the by us as conditional monotonous functions denoted types of functions of P_k, to a new type of questions in the k-valued logic P_k and to an interesting application of the k-valued logic.

2. Basic types of sets of functions

We define here some essential types of sets of functions over E_3. Let $E_3 = \{0,1,2\}$, $P_3^{(n)}$ the set of all functions $f(x_1,x_2,\ldots x_n)$ of n variables defined whenever all the $x_i \in E_3$ and with values in E_3 and $P_3 = \bigcup_{n\geq 1} P_3^{(n)}$. The operation of superposition and the closure $[M]$ of a subset M of P_3 are introduced in the usual manner (see e.g. [3]).

Let $R = \{\{0,1\},\{0,2\}, \{1,2\}\}$. We consider now for functions $f(\underline{x}) \in P_3^{(n)}$ with $\underline{x} = (x_1,x_2,\ldots,x_n)$ for arbitrary integer $n\geq 1$ the following conditions:

Condition 1. $\bigwedge\limits_{a \in E_3} f(a,a,\ldots,a) = a.$

Condition 2. $\bigwedge\limits_{M \in R\setminus\{0,1\}} \quad \bigwedge\limits_{\underline{\alpha} \in M^n} f(\underline{\alpha}) \in M.$

Condition 2'. $\bigwedge\limits_{M \in R} \quad \bigwedge\limits_{\underline{\alpha} \in M^n} f(\underline{\alpha}) \in M.$

Definition 1. For every nonempty proper subset M of E_3 let $\underset{M}{\simeq}$ the relation

$((E_3 x M) \cup \{(0,0),(1,1),(2,2)\})$ and $\underset{M}{\leq}$ the relation $((\overline{M}xM) \cup \{(0,0),(1,1),(2,2)\})$ where $\overline{M} = E_3 \setminus M$ is. If $\underline{\alpha} = (\alpha_1,\alpha_2,\ldots,\alpha_n)$, $\underline{\beta} = (\beta_1,\beta_2,\ldots,\beta_n) \in E_3^n$, then $\underline{\alpha} \underset{M}{\tilde{=}} \underline{\beta}$ or $\underline{\alpha} \underset{M}{\leq} \underline{\beta}$ holds, iff for $i = 1,2,\ldots,n$ $\alpha_i \underset{M}{\tilde{=}} \beta_i$ or $\alpha_i \underset{M}{\leq} \beta_i$ holds.

Example. $(1,0,0,2) \underset{\{1,2\}}{\tilde{=}} (1,1,0,2)$ and $(1,0,0,2) \underset{\{1,2\}}{\leq} (1,1,0,2)$, $(2,0,0,1,2) \underset{\{1,2\}}{\tilde{=}} (1,1,0,2,2)$ but $(2,0,0,1,2) \underset{\{1,2\}}{\not\leq} (1,1,0,2,2)$!

Obviously are the relations $\underset{\{a\}}{\tilde{=}}$ and $\underset{\{a\}}{\leq}$ for all $a \in E_3$ equal.

Condition 3. $\bigwedge_{a \in E_3} \bigwedge_{\underline{\alpha},\underline{\beta} \in E_3^n} (f(\underline{\alpha}) = a \Rightarrow (\underline{\alpha} \underset{\{a\}}{\tilde{=}} \underline{\beta} \Rightarrow f(\underline{\alpha}) \underset{\{a\}}{\tilde{=}} f(\underline{\alpha})))$.

Condition 3'. $\bigwedge_{a \in E_3} \bigwedge_{\underline{\alpha},\underline{\beta} \in E_3^n} (\underline{\alpha} \underset{\{a\}}{\tilde{=}} \underline{\beta} \Rightarrow f(\underline{\alpha}) \underset{\{a\}}{\tilde{=}} f(\underline{\beta}))$.

Condition 4. $\bigwedge_{M \in R} \bigwedge_{\underline{\alpha},\underline{\beta} \in E_3^n} (f(\underline{\alpha}) \in M \Rightarrow (\underline{\alpha} \underset{M}{\tilde{=}} \underline{\beta} \Rightarrow f(\underline{\alpha}) \underset{M}{\tilde{=}} f(\underline{\beta})))$.

Condition 4'. $\bigwedge_{M \in R} \bigwedge_{\underline{\alpha},\underline{\beta} \in E_3^n} (\underline{\alpha} \underset{M}{\tilde{=}} \underline{\beta} \Rightarrow f(\underline{\alpha}) \underset{M}{\tilde{=}} f(\underline{\beta}))$.

Condition 5. $\bigwedge_{M \in R} \bigwedge_{\underline{\alpha},\underline{\beta} \in E_3^n} (f(\underline{\alpha}) \in M \Rightarrow (\underline{\alpha} \underset{M}{\leq} \underline{\beta} \Rightarrow f(\underline{\alpha}) \underset{M}{\leq} f(\underline{\beta})))$.

Condition 5'. $\bigwedge_{M \in R} \bigwedge_{\underline{\alpha},\underline{\beta} \in E_3^n} (\underline{\alpha} \underset{M}{\leq} \underline{\beta} \Rightarrow f(\underline{\alpha}) \underset{M}{\leq} f(\underline{\beta}))$.

Let K the set of all functions of P_3 satisfying the condition 1. K^s denotes for $s \in \{2,2', 3,3', 4,4', 5,5'\}$ the set of all functions of K satisfying the condition s. If $M \subseteq \{2,2', 3,3', 4,4', 5,5'\}$, then $K^M := K$, iff M is the empty set. In all other cases $K^M := \bigcap_{s \in M} K^s$.

Let finally $\mathcal{K} := \{K^M \mid M \subseteq \{2,2', 3,3', 4,4', 5,5'\}\}$ and $|\mathcal{K}|$ the cardinality of $\mathcal{K}$. Obviously $|\mathcal{K}| \leq 2^8 = 256$ holds.

In the next paragraph we'll show, that $|\mathcal{K}|$ is rather less than 2^8 and investigate the partial ordered set $(\mathcal{K},\leq)$.

3. The invesitagtion of $\mathcal{K}$

It is obvious, that the lemma 1 holds.

Theorem 3. $X \in K$ is a subalgebra (with respect to the operation of superposition) of P_3, iff $X \in \{K, K^2, K^{2'}, K^{3'}\}$.

Proof. It follows directly by their definitions, that the sets K, K^2, $K^{2'}$ and $K^{3'}$ are subalgebras of P_3. We have to show still, that $X \subset [X]$ holds for $X \in \{K^3, K^{2,3}, K^{2',3}\}$. $K^3 \supset K^{2,3} \supset K^{2',3}$ holds by tehorem 1. Therefore it is enough to find a function $f(x,y) \in K^{2',3}$ with $f(f(x,y),z) \in K^3$. $f_6(x,y)$, given by table 2, is such a function. ■

Remark. We'll investigate the subalgebras, generated by K^3, $K^{2,3}$ or $K^{2',3}$, in the part II of this paper.

Theorem 4. $K^{3'} = [\{x\}]$.

Proof. The following statement holds ([1]): If $A = [A] \subseteq P_k$ and $[A \cap P_k^{(\max(k,3))}] = [A \cap P_k^{(1)}]$, then $A \subseteq [P_k^{(1)}]$.
By the tehorem 3 we know $K^{3'} = [K^{3'}]$. We have to prove still, that $f(x_1,x_2,x_3) = x_i$ $(i \in \{1,2,3\})$ holds for all $f(x_1,x_2,x_3) \in K^{3'}$. Let $f(x_1,x_2,x_3) \in K^{3'}$ and without loss of generality $f(0,1,2) = 0$.
By our conditions and the theorem 1 we know, that we receive for all $(a,b,c) \in E_3^3$ $f(a,b,c) \in \{a,b,c\}$. By lemma 5 $K^{4'} = K^{3'}$ holds.
Therefore we receive $(0,a,b) \underset{\{1,2\}}{\overset{\sim}{\to}} (0,1,2)$ and $f(0,a,b) = 0$ for all $a,b \in E_3$.
Let now $\{c,d\} = \{1,2\}$. Then $(0,c,c) \underset{\{d\}}{\overset{\sim}{\to}} (d,c,c)$, $(d,c,0) \underset{c}{\overset{\sim}{\to}} (d,c,c)$ and $f(d,c,c) = f(d,c,0) = d$ hold. Now we receive $(d,a,b) \underset{\{0,c\}}{\overset{\sim}{\to}} (d,c,0)$ and $f(d,a,b) = d$ for all $a,b \subseteq E_3$. ■

Let $\rho = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$, $\rho_2 = \begin{pmatrix} 0&0&0&0 \\ 1&1&1&1 \\ 2&2&2&2 \\ 0&0&2&2 \\ 1&2&1&2 \end{pmatrix}$ $\rho_{2'} = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 1&1&1&1&1&1&1&1 \\ 2&2&2&2&2&2&2&2 \\ 0&0&0&0&1&1&1&1 \\ 0&0&2&2&0&0&2&2 \\ 1&2&1&2&1&2&1&2 \end{pmatrix}$

and $\rho_{3'} = \begin{vmatrix} 0&0&1&2 \\ 1&2&0&1 \end{vmatrix}$.

Theorem 5. $K = \mathrm{Pol}\rho$, $K^2 = \mathrm{Pol}\rho_2$, $K^{2'} = \mathrm{Pol}\rho_{2'}$ and $K^{3'} = \mathrm{Pol}\rho_{3'}$.

Proof. The first three statements are direct conclusions of our conditions. In [2] $\mathrm{Pol}\rho_{3'} = [\{x\}]$ is proved. ■

Lemma 1. If $i \in \{2,3,4,5\}$, then $K^{i'} \subseteq K^{i}$. ■

Therefore we receive $|K| \leq 3^4 = 81$.

Lemma 2. $K^{4'} = K^4$,

Proof. Let $M \in R$. If $f(\underline{\alpha}) \notin M$, then $f(\underline{\alpha}) \underset{M}{\tilde{\rightarrow}} f(\underline{\beta})$ holds for every $\underline{\beta} \in E_3^n$. ■

Therefore we receive $|K| \leq 2.3^3 = 54$.
In the same way we are able to prove the

Lemma 3. $K^{5'} = K^5$.

Therefore $|K| \leq 2^2.3^2 = 36$ holds.

Lemma 4. $K^{3'} = K^{5'}$. ■

Proof. Let $\{a,b,c\} = E_3$. Then $\underset{\{a,b\}}{\leq}$ is the relation $\begin{pmatrix} c & c & a & b & c \\ a & b & a & b & c \end{pmatrix} = \rho_{a,b}$ and $\underset{\{c\}}{\tilde{\rightarrow}}$ is the relation $\begin{pmatrix} a & b & a & b & c \\ c & c & a & b & c \end{pmatrix} = \rho_c$. It is obvious that $Pol\rho_{a,b} = Pol\rho_c$ holds. Because $K^{3'} = Pol\rho_o \cap Pol\rho_1 \cap Pol\rho_2$ and $K^{5'} = Pol\rho_{1,2} \cap Pol\rho_{o,2} \cap Pol\rho_{o,1}$ hold, we receive $K^{3'} = K^{5'}$. ■

Therefore $|K| \leq 2.3^2 = 18$ holds.

Lemma 5. $K^{3'} = K^{4'}$.

Proof. Let $\{a,b,c\} = E_3$. Then $\underline{\alpha} \underset{\{a\}}{\tilde{\rightarrow}} \underline{\beta}$ holds, iff $\underline{\alpha} \underset{\{a,b\}}{\tilde{\rightarrow}} \underline{\beta}$ and $\underline{\alpha} \underset{a,c}{\tilde{\rightarrow}} \underline{\beta}$. Let $f(\underline{x}) \in K^{4'}$ and for $\underline{\alpha},\underline{\beta} \in E_3^n$ let $\underline{\alpha} \underset{\{a\}}{\tilde{\rightarrow}} \underline{\beta}$. Then $f(\underline{\alpha}) \underset{\{a,b\}}{\tilde{\rightarrow}} f(\underline{\beta})$ and $f(\underline{\alpha}) \underset{\{a,c\}}{\tilde{\rightarrow}} f(\underline{\beta})$ hold. Therefore we receive: If $f(\underline{\alpha}) = a$, then $f(\underline{\beta}) \in \{a,b\}$ and $f(\underline{\beta}) \in \{a,c\}$, that means $f(\underline{\beta}) = a$, if $f(\underline{\alpha}) = b$, then $f(\underline{\beta}) \in \{a,b\}$ and if $f(\underline{\alpha}) = c$, then $f(\underline{\beta}) \in \{a,c\}$. That means, that $f(\underline{\alpha}) \underset{\{a\}}{\tilde{\rightarrow}} f(\underline{\beta})$ holds. Therefore $K^{4'} \subseteq K^{3'}$ holds. Let now $f(\underline{x}) \in K^{3'}$ and $\underline{\alpha} \underset{\{a,b\}}{\tilde{\rightarrow}} \underline{\beta}$. Let $\underline{\gamma} = (\gamma_1,\gamma_2,\ldots,\gamma_n)$ the following element of E_3^n. For $i=1,2,\ldots,n$ we set $\gamma_i = \alpha_i$, iff $\alpha_i = \beta_i$, and $\gamma_i = a$ else. Then $\underline{\alpha} \underset{\{a\}}{\tilde{\rightarrow}} \underline{\gamma} \underset{\{b\}}{\tilde{\rightarrow}} \underline{\beta}$ holds. If now $f(\underline{\alpha}) \in \{a,b\}$, then we receive $f(\underline{\gamma}) \in \{a,b\}$ and $f(\underline{\beta}) \in \{a,b\}$. That means, that $f(\underline{x}) \in K^4 = K^{4'}$ (lemma 2) holds. Therefore $K^{3'} \subseteq K^{4'}$ holds too. ■

Therefore $|K| \leq 3^2 = 9$ holds.

Lemma 6. $K^{3'} \subseteq K^{2'}$.

Proof. Let $f(\underline{x}) \in K^{3'} = K^{4'} = K^4$ (lemma 2 and 5), $a,b \in E_3$, $a \neq b$, $\underline{\beta} \in \{a,b\}^n$ and $\underline{\alpha} = (a,a,\ldots,a)$. Then $f(\underline{\alpha}) = a$, $\underline{\alpha} \underset{\{a,b\}}{\overset{\sim}{-}} \frac{\beta}{2'}$ and $f(\underline{\alpha}) \underset{\{a,b\}}{\overset{\sim}{-}} f(\underline{\beta})$ hold. Therefore we receive $f(\underline{\beta}) \in \{a,b\}$ and $f(\underline{x}) \in K^{2'}$. ■

Therefore $K \leq 7$ and $K = \{K, K^2, K^{2'}, K^3, K^{3'}, K^{2,3}, K^{2',3}\}$ hold.

By the lemma 1-6 the following inclusions are valued:

$$K^{3'} \subseteq K^{2',3} \subseteq K^{2,3} \subseteq K^3 \subseteq K \tag{1}$$

$$K^{2,3} \subseteq K^2 \subseteq K \tag{2}$$

$$K^{2',3} \subseteq K^{2'} \subseteq K^2 \tag{3}$$

The functions $f_i(x,y)$ for $i = 1,2,3,4,5,6$ are given by the table 2.

x	y	$f_1(x,y)$	$f_2(x,y)$	$f_3(x,y)$	$f_4(x,y)$	$f_5(x,y)$	$f_6(x,y)$
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
0	1	0	2	0	1	2	1
1	0	0	0	0	1	2	2
0	2	1	0	0	1	2	2
2	0	0	0	0	1	2	2
1	2	2	1	1	1	1	1
2	1	0	1	1	1	1	2

Table 2.

By the table 3 the function $g(x,y,z)$ is given.

z \ x	0	1	2	0	1	0	2	1	2
y	0	1	2	1	0	2	0	2	1
0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2

Table 3.

Now the following relations, given by table 4, hold.

	because		because
$K \supset K^3$	$f_1 \in K\backslash K^3$	$K^{2'} \supset K^{2',3}$	$g \in K^{2'}\backslash K^{2',3}$
$K^3 \supset K^{2,3}$	$f_4 \in K^3\backslash K^{2,3}$	$K^2 \subset K^3$	$f_2 \in K^2\backslash K^3$
$K^{2,3} \supset K^{2',3}$	$f_5 \in K^{2,3}\backslash K^{2',3}$	$K^3 \subset K^2$	$f_4 \in K^3\backslash K^2$
$K^{2',3} \supset K^{3'}$	$f_3 \in K^{2',3}\backslash K^{3'}$	$K^{2'} \subset K^3$	$g \in K^{2'}\backslash K^3$
$K \supset K^2$	$f_1 \in K\backslash K^2$	$K^3 \subset K^{2'}$	$f_4 \in K^3\backslash K^{2'}$
$K^2 \supset K^{2,3}$	$f_2 \in K^2\backslash K^{2,3}$	$K^{2'} \subset K^{2,3}$	$g \in K^{2'}\backslash K^{2,3}$
$K^2 \supset K^{2'}$	$f_2 \in K^2\backslash K^{2'}$	$K^{2,3} \not\subset K^{2'}$	$f_5 \in K^{2,3}\backslash K^{2'}$

Table 4.

We receive therefore, together with the relations (1) - (3), the

Theorem 1. (i) $|K| = 7$

(ii) $(K,\subseteq)$ is given by mapping 1. ∎

$$\begin{tikzcd}
& K \arrow[dl] \arrow[dr] & \\
K^2 \arrow[d] \arrow[drr] & & K^3 \arrow[d] \\
K^{2'} \arrow[dr] & & K^{2,3} \arrow[dl] \\
& K^{2',3} \arrow[d] & \\
& K^{3'} &
\end{tikzcd}$$

Mapping 1.

$X \rightarrow Y$ denotes for $X,Y \in K$ in this mapping , that $X \supset Y$ holds and for all $Z \in K$ with $X \supseteq Z \supseteq Y$ holds $X = Z$ or $Z = Y$, and $X \supset Y$ holds, iff an integer n with $n \geq 2$ exists with $X_1,X_2,\ldots,X_n$, $X_1 = X$, $X_n = Y$ and $X_i \rightarrow X_{i+1}$ for $i = 1,2,\ldots,n-1$.

By the mapping 1 you are able easy to prove the

Theorem 2. $(K,\subseteq)$ is a distributive lattice. ∎

R e f e r e n c e s

[1] Harnau, W., Eine Verallgemeinerung der Vertauschbarkeit in der k-wertigen Logik. EIK 12 (1976) 1/2, 33-43.

[2] Harnau, W., Über antidiagonale Relationen. Rostock 1976 (unpublished).

[3] Яблонский, С.В., Лупанов, О.Б., /редакторы/, Дискретная математика и математические вопросы кибернетики, Том 1. Москва 1974.

[4] Rosenberg, I. Über die funktonale Vollständigkeit in mehrwertigen Logiken. Praha 1970.

[5] Журавлев, Ю.И., Корректные алгебры над множествоми некорректных /эвристических/ алгоритмов. I. Кибернетика, Киев, 1977, 4, 14-21.

Ö s s z e f o g l a l ó

Feltételesen monoton függvények véges halmazon. I.

Gustav Burosch, Klaus-Dieter Drews, Walter Harnau, und Dietlinde Lau

Legyen $E_3:=(\{0,1,2\};\leq)$ részben rendezett halmaz. Az összes feltételesen monoton $f:E_3^n \to E_3$ $n=1,2,\ldots$ függvényeknek a nyolc Zsuravljov feltétel lehetséges kombinációinak eleget tevő részhalmazai a tartalmazásra, mint részben rendezésre nézve 7 elemü disztributiv hálót alkotnak.

A feltételesen monotonitás gyengébb feltétel mint a szokásos monotonitási. A fenti részhalmazok közül egy éppen a szelektor függvények kompozicióra nézve zárt részhalmaza.

A Zsuravljov feltételek mint szavazási szabályok értelmezhetők.

Резюме

Условно монотонные функции на конечных множествах. Часть I

Густав Бурош, Клаус-Дитер Древс, Валтер Харнау

Пусть $E_3 = (\{0,1,2\};\leq)$ частичное упорядоченное множество. Подмножество условно монотонных функций $f:E_3^n \to E_3$ $n=1,2,\ldots$ удовлетворяющее возможным сочетаниям восьми условий Журавлева составляют дистрибутивную сеть с мощностью семь.

Условное множество является более слабым условием, чем монотонность в обычном смысле. Одно из вышеуказанных подмножеств является замкнутым, - относительно композиции - подмножество селекторных функций.

Условия Журавлева могут быть представлены как правила голосования.