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Editorial office: Budapest II., Keleti Károly utca 5–7. Postal address: P.O.B. 51. Budapest, 1525.
Phone: 345-6528, Telefax: 345-6783
E-mail: statszemle@ksh.x400gw.itb.hu
Publishing office: Central Statistical Office, Budapest II., Keleti Károly utca 5–7.
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EKS INDEX AND INTERNATIONAL COMPARISONS*

PÁL KÖVES[1]

SUMMARY

The *EKS* (*Éltető – Köves – Szulc*) index formula (hereinafter referred to as X) is in official use in the comparison of per capita real GDP and purchasing power parity of the OECD countries. Early versions of formula X can be found in Fisher's and Gini's works.

According to the author, X can be regarded as the result of a multi-situational crossing. It is the geometric mean of the 'generalized F' and its factor-antithesis. The three-member index family obtained this way shows a close relationship to *J. van Yzeren's* three indices (family **Y**). The paper presents an empirical procedure to measure the similarity of the behaviour and performance of the different index formulae. The author comes to the conclusion that the members of families **X** and **Y** walk around index X.

KEYWORDS: Index formulae; International comparisons.

Different index formulae are used to measure the real value of GDP and the temporal change of price level. If the target is not the measurement of temporal change but the spatial comparison of different countries for a given period (year) then a special use should be considered when choosing the index formula. We know many volume-index formulae that are suitable for comparing the real value of per capita GDP of a fixed number (m) of countries. Consequently there are many price-index formulae that can be used to compare the purchasing power of currencies of different countries.

One of the index formulae used for international comparison is *EKS* volume and price index which was introduced separately but at the same time by the two Hungarians *Éltető* and *Köves* (1964) and the Polish *Szulc* (1964). Both propositions use Fisher's (F) bilateral 'ideal' non-transitive indices to produce transitive indices.[2]

$$X_{t/b} = \left(\prod_{i=1}^{m} F_{t/i} \cdot F_{i/b}\right)^{\frac{1}{m}} = \left(F_{t/b}^{2} \cdot \prod_{i \neq t/b}^{m} F_{t/i} \cdot F_{i/b}\right)^{\frac{1}{m}} \qquad /1/$$

* The abridged and revised version of article *Köves* (1995) originally published in Hungarian.

[1] Professor Emeritus.

[2] Transitivity means that equations of type $I_{2/1} \cdot I_{3/2} = I_{3/1}$ hold.

In the index

$$F_{t/b} = \left(L_{t/b} \cdot P_{t/b}\right)^{\frac{1}{2}} \qquad /2/$$

is the geometric mean of the respective bilateral Laspeyres (L) and Paasche (P) indices.[3]

It was proved by *Éltető* and *Köves* (1964) that $X_{t/b}$ also meets the requirement

$$\sum_{t=1}^{m}\sum_{b=1}^{m}(\log F_{t/b} - \log X_{t/b})^2 \rightarrow \min. \qquad /3/$$

This means that replacing the non-transitive indices $F_{t/b}$ by the transitive $X_{t/b}$ indices minimizes the overall replacement error in a certain sense. The relative magnitude of the indices are represented by their logarithms. /3/ requires, that the sum of squares of differences between the logarithms of F and X indices should be minimum for all indices calculated from all t/b relation. Formula /1/ means that country t and country b are compared sequentially through the mediation of each county (i=1,…m) and then the geometrical mean of the indirect results is considered as a final result. Countries t and b are also taken as 'mediator' countries, consequently the direct comparison carries a double weight in the comparison.

To justify formula /1/ Szulc states that a mean value of the direct and indirect comparisons should be calculated. His initial formula differs from the second form of /1/ in considering the direct comparison $F_{t/b}$ on the first power. To assure the transitivity of these results an iterative infinitesimal process is suggested which approaches the indices $X_{t/b}$.

In addition to transitivity, X also passes one of the most important index tests, the time reversal and factor reversal tests.[4] It does not pass the proportionality test,[5] but the degree of its violation is negligible in practice. (This was proved by *Köves* (1995) on pages 23-26, by giving both some theoretical and numerical arguments.)

The absence of additivity is usually considered a disadvantage. (The sum of values corresponding to the partial indices does not equal to the value corresponding to the general index.)

In the first phase of ICP an experimental comparison has been made where the 1970 figures of ten countries and four index calculation methods have been used (*Kravis* et al., 1975). One of these was formula X, as it appears in /1/. It was named *EKS* after the authors *Éltető* and *Köves* (1964) and *Szulc* (1964).

[3] Here $L_{t/b}(t,b=1,\ldots m)$ which relates country t to country b, can stand for the volume index $L^q_{t/b} = \sum p_b q_t / p_b q_b$ or the price index $L^p_{t/b} = \sum p_t q_b / \sum p_b q_b$ (purchasing power parity). (Here and in all further formulae the summing up limits by commodities will be neglected.) Similarly, $P_{t/b}$ can be made concrete by volume index $P^q_{t/b}$ or price index $P^p_{t/b}$. (For numerical examples see Appendix II.)

[4] The time reversal test requires the equation $I_{t/b} \cdot I_{b/t} = 1$ to be valid, that is the indices calculated with a given formula by reversing the two periods of time must be in reciprocal relationship. The factor reversal test requires that the product of the volume and price indices be equal to the value index $V_{t/b} = \sum p_t q_t / \sum q_b p_b$.

[5] The proportionality test (commonly referred to as average test) requires that the aggregate index be I=c if each individual index equals to a constant c.

The three other methods used were

1. the Geary–Khamis method *GK* [*Khamis* (1972), *Kravis* et al. (1975)],
2. the Walsh price index *W* [*Kravis* et al. (1975)],
3. J. van Yzeren's 'balanced' method *Y* [*Yzeren* (1956)].

All four methods are results of some multi-situational crossing (*Köves*, 1983 a, b). Only *GK* provides additive results because its calculation assumes the existence of certain average prices. Price index *W* is arrived at by calculating the weighted geometrical mean of the individual indices, where the weight attached is the arithmetical mean of the value shares of the smallest units over the countries. The *W*-type volume index is the ratio of the value index and price index *W*. Index *W* could only be made transitive by fixing the basis (USA).

The common data set of *X* and *Y* is a matrix which contains the volume indices in all relations. If m=2, both methods produce the Fisher index. The numerical results of *X* and *Y* are very close to each other, but *X* has a direct formula, while the *Y* results can be obtained by an iterative process, similarly to *GK*.

Drechsler,[6] the well-known Hungarian ICP expert played a major role in recommending to name formula *X EKS*. As one of the three persons whose initials produce *EKS*, I must mention that the birth of the *Éltető* and *Köves* (1964) paper should also be attributed to Drechsler's inspiration or 'order'. An important step in this process was the publication of Drechsler's book (1962), the appendix of which written by Éltető, contains formula /1/.

The afore-mentioned 'competition' of the four formulae was won by *GK* and thus it was used to produce all published indices of the first five ICP phases. Its recognition was due, on the one hand, to the fact that its equation system reflected attractive economic considerations and that on the other hand, it met the requirement of additive consistency. Not much later, however, it was considered a growing disadvantage that *GK* volume index systematically appreciated countries with a low GDP, and the price index did the same with the currencies of these countries.

After the regionalisation of ICP in 1980, a separate European comparison has been carried out. ECP'90, began to use primarily *EKS*[7] in the view of the criticism raised against GK. In addition to the official *EKS* results, *GK* results appeared only as secondary figures[8]. (See *Hill*, 1997; *Khamis*, 1984; *Khamis*, 1996; *Prasada*, 1997 too.)

1. Irving Fisher and formula *X*

Fisher (1922) considered the time reversal test and the factor reversal test of utmost importance. The reason why these two tests are so important for Fisher is that they have a 'formula discovering' function. The tool of discovery is the antithesis index, and the result is a new crossed index, which meets a requirement that the original index does not.

If we calculate an index by some formula *I* not passing the time reversal test, by reversing the two time periods, the reciprocal of this is the time-antithesis. The geometrical

[6] *L. Drechsler* was the director of ICP in 1985–1989. He died in 1990.
[7] Purchasing Power Parities and Real Expenditures. *EKS* Results, 1990. OECD. Paris. 1992.
[8] Purchasing Power Parities and Real Expenditures. *GK* Results, 1990. OECD Paris. 1993.

mean of the original index and the antithesis is then the crossed index. If we calculate a volume index by using a price index formula not passing the factor reversal test, and then the value index is divided by this index, we obtain the factor antithesis. The geometrical mean of the original price index and the antithesis is the crossed index. (This is shown in Table App. 1.)

Fisher (1922) states on page 416. that any index formula that passes the two most important tests can be modified so as to pass the circularity test as well, so that the formula be transitive for three periods of time or spatial units. The method can also be extended to more than three situations.

The modification to be made is

$$I'''_{2/1} = \frac{I''_{2/1}}{(I''_{2/1} \cdot I''_{3/2} \cdot I''_{3/1})^{\frac{1}{3}}} \qquad /4/$$

where

I is the initial formula,
I' is the index crossed with factor antithesis,
I'' is the index crossed with both antitheses, and
I''' is the index which also passes the circularity test, in addition to the other two ones.

The point here is that in two special cases (if I=L or I=P) I''' will be identical to X for three countries. What Fisher stresses is that circularity (transitivity) can be attained by applying a multi-stage process to any formula. Thus, we can say that Fisher produced the earliest version of the *EKS* formula.

2. Corrado Gini and formula *X*

Gini (1924 and 1931) proposed several transitive index formulae for the purpose of spatial comparisons. His propositions included both the generalised Edgeworth–Marshall-type price index

$$\overline{E}^{p}_{t/b} = \frac{\sum \sum_{i=1}^{m} p_t q_i}{\sum \sum_{i=1}^{m} p_b q_i}, \qquad /5/$$

and the generalised Fisher index

$$\overline{F}^{p}_{t/b} = \left(\prod_{i=1}^{m} \frac{\sum p_t q_i}{\sum p_b q_i} \right)^{\frac{1}{m}}. \qquad /6/$$

(For numerical illustrations see Appendix II.) Another proposed formula is:

$$\overline{I} = \left(\prod_{i=1}^{m} \frac{I_{t/i}}{I_{b/i}} \right)^{\frac{1}{m}} \qquad /7/$$

If I is replaced by F in /7/, we will have X. Since Gini (just like Fisher) did not state requirement /3/, he could not possibly attach a 'high rank' to /1/. At the same time, it seemed natural for him that the better the underlying two-situational I was, the better /7/ became. This is also shown by the fact that in his numerical example he made I concrete by using F.

Gini (1931) compared the prices of five Italian cities in eight periods of time. He calculated the price index for all relations by applying 14 different formulae. This was much to the delight of researchers (also) interested in the empirical testing of index formulae. One of these was X, so the first application of *EKS* can be found in *Ginis'* paper. Several decades had to pass until the next application.

Italian authors *Biggeri et al.* (1987) propose that the name of X should begin with a letter G. It is of course a reasonable wish, however one must not forget about Fisher either. (An overview of the history of formula X is in Appendix I.)

3. An obvious derivation of formula *X*

Requirement /3/ or the iterative procedure suggested by *Szulc* (1964) is not necessarily the most natural way of developing formula X. Gini's formula signals the route from two-situational indices to X. Gini gave formula /7/, which is more general than X, since he left out a link in the chain of deriving it. This missing link is formula /8/. On page 249. *Szulc* (1964) gives the entire logical route between F and X. (See also *Köves*, 1975, p. 1199. and *Köves*, 1983, p. 150.) The multi-situational generalisation of F produces $\overline{F}$. Since $\overline{F}$ does not pass the time reveral test, I produce its factor antithesis by dividing the value index by the volume index corresponding to price index formula /6/:

$$\overline{F}^{pa}_{t/b} = \frac{V_{t/b}}{\overline{F}^{q}_{t/b}} = \frac{V_{t/b}}{\left(\prod_{i=1}^{m} \frac{\sum p_i q_t}{\sum p_i q_b}\right)^{\frac{1}{m}}} \qquad /8/$$

The geometric mean of the original formula /6/ and its antithesis /8/ is then

$$X^{p}_{t/b} = \left(\overline{F}^{p}_{t/b} \cdot \overline{F}^{pa}_{t/b}\right)^{\frac{1}{2}} \qquad /9/$$

which is identical to /1/. (See the numerical example in Appendix II.)

It must be mentioned here that out of these three indices price index /6/ and the cor-res-ponding volume indices pass the proportionality test because they are the means of aggregate indices, with different weights but identical price relatives, which pass the test. In antithesis /8/ the guarantee of passing the test vanishes, and formula /9/=/1/ inherits this deficiency.

4. Two families of index formulae

J. van Yzeren (1956) described three closely related methods of which we have only touched method III or the balanced method. These three methods are closely related to the triad /6/, /8/ and /9/. This relationship can also be expressed by the symbols used. The indices of family **Y** are Y', Y'' and Y, while family **X** consists of X', X'' and X. Table App. 4 contains the equations referring to country j (j=1, …m) of the corresponding equation systems.

The equations combine the non-transitive Laspeyres indices (L) with transitive exchange rates. The first two methods are slightly biased, which is balanced by the third one. If any of the three methods is applied to two countries, $Y' = Y'' = Y = F$, and the same holds for family **X**.

Here we can again demonstrate the close relationship between the two families. If the expressions with the summation sign in Table App. 4 are replaced by their logarithms, the operations will give the logarithms of the indices of family **X**. Consequently, a direct formula can be given instead of the iterative process.

Let us take out the indices of the two families corresponding to relation B/A from Table App. 3:

Y''		Y		Y'	X''		X		X'
1.0076	<	1.0098	<	1.0120	1.0075	<	1.0097	<	1.0118

It is not accidental that the results of method III are in the centre, and that the order of magnitude for the relation picked is identical in both families.

W. D. Heller (1982) points out that the matrix of empirical indices which is necessary to calculate the 'van Yzeren-type' indices may consist not only of L, but also of P or F indices.

In my paper (*Köves*, 1995. p. 20–22), I completed Heller's calculations and consideration, and extend the investigation to family **X**. The conclusions can be summarised as follows.

If we use matrix P instead of matrix L for calculating the indices of family **Y**, replacing Y' by Y'' it would produce similar results as matrix L. If this exchange of matrices is made for family **X**, we obtain not only similar but the same results by replacing method I by method II. If this is done with matrix F, then $X' = X'' = X$.

The function of the contrasting pairs is that one member relies on the proportionality test in terms of prices, while the other in terms of quantities. The 'balanced index' (in both families) produces a satisfactory situation in both respects.

A further conclusion is that the duality of volume and price indices is added to the duality of methods I and II, and matrixes L and P. (Thus far, we have always dealt with the price index only – in a direct way.) We can also speak of the duality of positive or negative correlation between the volume and price relatives.

An even shorter summary is that X stands in the focus of the narrower and broader system of 'van Yzeren-type' indices and the elements of the system 'are dancing around' it.

5. Shall we weight the weights? Further formulae

In the first phase of ICP *GK* was chosen because the additive relations can be retained by taking 'international average prices' to calculate the volume index. The term 'international average price' seems reasonable since it is calculated as the weighted mean of the converted national prices. It appears that attaching different weights to the weights also deserves recognition: the average price appears more realistic. However, the weights of the volume indices are not the prices but the relative prices. (The relative prices of large and small countries should be considered equal if we are to compare them by index numbers.)

In index calculation the absolute price merely 'wears' the relative price as a model wears a dress. If the index expert selects a 'good' price and not a 'good' relative price, he acts as a designer who selects the most beautiful model instead of the most beautiful dress. Weighting in the process of averaging price weights is not the implementation of some economic requirements but the order of the model which includes arbitrary elements as well.

The Geary-Khamis index has two shortcomings. The first one is a common property of average-price indices: the bias due to the negative correlation between the volume and price relatives (the so-called Gerschenkron effect). It is well-known, that $L>P$ in case of negative correlation. The price structure of some countries is close to the average, while that of others is very far from it. Thus, the index of the former countries will be quasi P, while that of the latter will be quasi L. The other shortcoming of GK can be explained with weighting the weights.[9]

Of the average-price methods Gerardi's formula (*Ge*) (*Gerardi*, 1982; *Köves*, 1983 a p. 156.) embodies the principle of unweighted averaging the most consistently. The weights of the *Ge* volume index come into being as a simple geometric mean of unconverted unit prices expressed in different currencies. The price index is the quotient of the value index and the volume index.

If any two-situational crossed formula (e.g. *F*) does not require average prices, there may be a hidden but verifiable average price in the background. (This is shown for *F* by van *Yzeren*,1952.) For multi-situational formulae that do not pass the average test, the average prices cannot be calculated, and they do not even exist. Only the price level is fixed for the aggregates given in publications.

Of the average-price formulae, attention must be made of Iklé's formule (*Ik*), which can be calculated from a model similar to *GK* applying an iterative method. Here, however, weighting the weights is much more fortunate. (See *Köves*, 1983 b).

In *Balk's* (1996) calculations, the weighted versions of *X* and *Y* also appear. (In this case value weights referring to countries rather than weights within the weights are used.) The unjustifiable weighting only slightly deteriorated the quality of the 'unweighted' *X* and *Y* (see *Köves*, 1995. p. 16.). I think I may discard these results.

6. The competition of index formulae

Table 5 in Appendix II. shows the purchasing power parities taken from the results of the first ICP phase (*Kravis et al*., 1975).

So that we can assess the similarity between the results obtained by different formulae, we calculated a correlation coefficient from the logarithms of the parities produced

[9] Suppose, we calculated volume indices from the data of many countries, which were weighted by average prices but were free from the special bias of formula *GK*. If these values are plotted against the values obtained by some good index formula (e.g. *EKS*) then the points will approach a convex parabola of second degree. Countries could be found with an 'average'price structure around the minimum point of the parabola. The special bias of the *GK* formula would be expressed by a declining line. Thus the joint effect of the bias of these two kinds could be described by a combination of the parabola and the line.

by each pair of the formulae and then subtracted their squared values from 1 (residual component). Table 1 shows these results[10] multiplied by 10^6.

Table 1

Residual components referring to all pairs of the formulae, $10^6 \cdot (1\text{-}r^2)$

Formulae	Symbol	*GK*	*W*	*X*	*Y*	Total
Geary-Khamis	*GK*	–	721	565	571	1857
Walsh	*W*	721	–	153	161	1035
EKS	*X*	565	153	–	1	719
van Yzeren	*Y*	571	161	1	–	733

The strikingly closest relationship is between *X* and *Y*, while *GK* and *W* are the furthest apart. The last column shows the totals of the figures in each row (or column), which reflects how similar numerical result each formulae produced in relation to the others. The first two places are taken by *X* and *Y*.

J. van Yzeren (1987) illustrated the indices that he had proposed with a further schematic example, which is similar to the one contained in *Yzeren* (1956). *Balk* (1996) calculated a series of further indices using the figures of this example. Appendix II includes some of the calculations made by the two authors. (*Köves*, 1995 presents all the results together with their residual components.)

Table 2 shows the most important 'competition results' of the 9 formulae, which I regard as the most important ones.

Table 2

Residual components, $10^6 \cdot (1\text{-}r^2)$

Formula	*Y*	*X*	*Y′*	*Y″*	*X′*	*X″*	*Ge*	*Ik*
GK	665	662	854	505	848	504	589	801
Y		0	13	13	12	13	8	7
X			13	12	12	12	7	8
Y′				50	0 a	51 a	40 b	6 b
Y″					49 a	0 a	1 b	31 b
X′						49	39 c	5 c
X″							1 c	34 c
Ge								26

It can be seen that the relationship of *GK* to the others is strikingly bad. We can also see that *X* and *Y* are the closest to each other: under the given accuracy the corresponding residual could be rounded to zero, i.e. the correlation coefficient to 1. The two rounded residual figures which characterize the relationship between members of families **X** and **Y** in the same position are also zero. The two zeros are shown on the diagonal line in box

[10] In the calculation of the coefficient between *GK* and *W* the first pair of values is: lg8.01, lg8.76. The value of the coefficient is: 0.9996395. This produces 721 as residual variance if multiplied by 10^6.

'a'. The other diagonal is taken by relatively high values. This means, the one-prime member of one family 'does not like' the two-prime member of the other.

Boxes 'b' and 'c' are almost identical. The one-prime members of both families show a close relationship with formula *Ge*, while the two-prime members have a slightly looser relationship with formula *Ik*. The 'other' diagonal also reflects a fairly friendly relationship.

I think *Ge* and *Ik* must be taken into account in addition to *X* when the next application is considered. However, in the light of the test calculations formulae X' and X'' can also be regarded as leaders. Table 3 may give some help to the consideration. (The residuals are taken from Table 2.)

Table 3

Comparison of five price index formulae

Formula	Symbol	Residual sum	Way of calculation	Proportionality test		Additivity
				price	volume	
EKS	X	393	direct	no	no	no
Gerardi	Ge	800	direct	no	yes	yes
Iklé	Ik	805	iteration	no	yes	yes
Generalised *F*	X'	1171	direct	yes	no	no
Antithesis of X'	X''	1085	direct	no	yes	no

The term 'direct' can, of course, mean a simple or a complex calculation. The numerical extent of not passing the test and of the lack of additivity can also differ.

Just like in the first phase of ICP, it seems advisable to implement an experimental phase again.

APPENDIX I

AN OVERVIEW OF THE HISTORY OF FORMULA *X*

1. Fisher (1922) develops an adjustment formula that guarantees circularity for three situations, which leads to *X* when starting from formula *L* or *P*.

2. C. Gini (1924) creates the generalised *F* ($\overline{F}$).

3. C. Gini (1924) constructs a general crossed formula, which gives *X* if the initial formula is *F*.

4. C. Gini (1931) publishes the results of his calculations obtained with 14 formulae, including *X*.

5. Ö. Éltető (in the Appendix of *L. Drechsler's* book, published in 1962), unaware of the antecedents in 1–4., introduces formula *X* by an intuitive explanation.

6. B. Szulc (1964) creates the antithesis of *F* and *X* by crossing. (Items 6-8 were published in independent studies, which appeared at the same time.)

7. B. Szulc (1964) shows an iterative method leading to *X* using an intuitive reasoning (and infinitesimal verification).

8. Ö. Éltető and *P. Köves* (1964) establish the minimum square property of *X*.

9. ICP makes the 1970 comparisons experimenting with four formulae. One of the four formulae is *X*, which here gets the name *EKS*.

10. P. Köves (1975) gives a general overview of multi-situational crossing, and places *X* in this context. He reveals the close relationship between 'index families' *X* and *Y*. (The symbol *X* originates from this paper.)

11. OECD's official results for 1990 are produced by using the *EKS* formula.

APPENDIX II

TABLES, CALCULATIONS

1. Crossing the index formulae

Table App. 1

Two types of crossing

Test	Original index	Antithesis	Crossed index	Check of the test
Time reversal	$I^p_{t/b}$	$\dfrac{1}{I^p_{b/t}}$	$\left(I^p_{t/b}\cdot\dfrac{1}{I^p_{b/t}}\right)^{\frac{1}{2}}$	$\left(I^p_{t/b}\cdot\dfrac{1}{I^p_{b/t}}\right)^{\frac{1}{2}}=\dfrac{1}{\left(I^p_{b/t}\cdot\dfrac{1}{I^p_{t/b}}\right)^{\frac{1}{2}}}$
Factor reversal	$I^p_{b/t}$	$\dfrac{V}{I^q_{t/b}}$	$\left(I^p_{t/b}\cdot\dfrac{V}{I^q_{t/b}}\right)^{\frac{1}{2}}$	$\left(I^p_{t/b}\cdot\dfrac{V}{I^q_{t/b}}\right)^{\frac{1}{2}}\left(I^q_{t/b}\cdot\dfrac{V}{I^p_{t/b}}\right)^{\frac{1}{2}}=V$

2. A numerical example

The following example has been elaborated by van Yzeren and shows fictional data of 4 countries (*A, B, C, D*).

Table App. 2

$\sum p_i q_j$ *data for four countries*

i \ j	*A*	*B*	*C*	*D*
A	5 800	27 175	1 206	1 396
B	5 950	26 925	1 234	1 407
C	74 240	345 200	12 108	14 144
D	15 570	71 175	2 490	2 718

Source: *Yzeren* (1987).

Using the data of Table App 2 formulae /1/–/9/ are computed as follows:

/1/ $$X_{B/A}=\left[1.0082^2\cdot(0.08916\cdot 11.3362)\cdot(0.4425\cdot 2.2862)\right]^{\frac{1}{4}}=1.0097\ ,$$

where

/2/ $L^p_{B/A}=5950/5800=1.0259$ $P^p_{B/A}=26925/27175=0.9908$ $F^p_{B/A}=(1.0259\cdot 0.9908)^{\frac{1}{2}}=1.0082\ .$

/5/ $$\overline{E}^p_{B/A}=\frac{5950+26925+1234+1407}{5800+27175+1206+1396}=0.99829\ .$$

/6/ $$\overline{F}^p_{B/A}=\left(\frac{5950\cdot 26925\cdot 1234\cdot 1407}{5800\cdot 27175\cdot 1206\cdot 1396}\right)^{\frac{1}{4}}=1.01184\ .$$

/8/ $$\overline{F}^{q}_{B/A}=\left(\frac{27175\cdot 26925\cdot 345200\cdot 71175}{5800\cdot 5950\cdot 74240\cdot 15570}\right)^{\frac{1}{4}}=4.60748 \qquad V_{B/A}=\frac{26925}{5800}=4.64224,$$

therefore

$$\overline{F}^{pa}_{B/A}=\frac{4.64224}{4.60748}=1.00754\ .$$

/9/ $$X^{p}_{B/A}=(1.01184\cdot 1.00754)^{\frac{1}{2}}=1.0097\ .$$

Table App. 3

The purchasing power parities (price indices) of currencies of four countries obtained by different index formulae from the data of Table App. 2

Formula	*B/A*	*C/A*	*D/A*
F	1.0082	11.3362	2.2862
GK	1.0166	10.3526	2.0387
Y	1.0098	11.3689	2.2767
X	1.0097	11.3698	2.2760
Y'	1.0120	11.3293	2.3062
Y''	1.0076	11.4098	2.2478
$X'=\overline{F}$	1.0118	11.3405	2.3057
$X''=\overline{F}^{a}$	1.0075	11.3995	2.2467
GE	1.0078	11.4142	2.2543
Ik	1.0058	11.6295	2.3036

Source: *Yzeren* (1987), *Köves* (1995), *Balk* (1996).

The indices F^p not given in Table App. 3:

$$F^{p}_{B/C}=0.08916 \qquad F^{p}_{B/D}=0.4425 \qquad F^{p}_{C/D}=5.0303$$

3. Two families of index formulae

Table App. 4

The comparison of families **X** *and* **Y**

Method	Family **Y**			Family **X** Direct formula
	Equations	Name in Yzeren (1956)	Name in Yzeren (1987)	
I.	$\sum_{i=1}^{m} L_{j/i}\frac{Y'_i}{Y'_j}=s$	Method of heterogeneous groups	q-combining method	$X'=\overline{F}$
II.	$\sum_{i=1}^{m} L_{i/j}\frac{Y''_j}{Y''_i}=s$	Method of homogeneous groups	p-combining method	$X''=\overline{F}^{a}$
III.	$\sum_{i=1}^{m} L_{j/i}\frac{Y_i}{Y_j}=\sum_{i=1}^{m} L_{i/j}\frac{Y_j}{Y_i}$	Balanced method	Balanced method	$X=\left(X'\cdot X''\right)^{\frac{1}{2}}$

4. Empirical comparison of index formulae

Table App. 5

Purchasing power parity indices in 1970 applying four formulae (United States = 1)

Country	Currency	Geary–Khamis *GK*	Walsh *W*	*EKS* *X*	van Yzeren *Y*
Columbia	P	8,01	8,76	8,42	8,41
France	Fr	4,48	4,46	4,35	4,33
Federal Republic of Germany	DM	3,14	3,24	3,16	3,16
Hungary	Ft	16,07	15,92	15,93	15,90
India	Re	2,16	2,46	2,47	2,47
Italy	L	483	470	457	457
Japan	Y	244	247	240	239
Kenya	Sh	3,74	4,17	3,80	3,79
United Kingdom	£	0,308	0,291	0,291	0,291

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ON THE DEPRIVATION-SENSITIVE MEASUREMENT OF POVERTY

OTTÓ HAJDU[1]

SUMMARY

Measuring the overall degree of poverty in a society, any poverty index is desired to depend on the feeling of relative deprivation of the poor. Investigating this problem, present paper deals with the sensitivity of the overall poverty measurement to the change in the overall degree of relative deprivation felt by the poor. According to the approach applied, a decrease in the overall degree of relative deprivation is considered ceteris paribus as a poverty measurement reducing factor no matter what additional change occurred in the income configuration of the poor. The paper discusses a poverty measure which explicitly involves the value of an overall deprivation index of the poor and hence is deprivation sensitive in the sense described above.

KEYWORDS: Poverty measurement; Relative deprivation; Transfer sensitivity.

In order to measure the poverty in a society of n persons with $Y_i \geq 0$ $(i=1,2\ldots,n)$ incomes, we first of all need to identify who the poor are. In this identifying process the population is split into two groups by fixing an appropriate poverty line z, which is common and given for all individuals. People with incomes not higher than (or strictly below) this income threshold constitute π, the q-member set of 'the poor'. Once this subset of the society has been distinguished, a comprehensive, brief P poverty index is needed to aggregate the information about the poverty gaps of the poor.[2] This overall poverty index should reflect the relative number of the poor (q,n), the distance of the poor from the poverty line (δ_π) and the degree of dispersion among the poor (σ_π) while its value is expected (but not necessarily) to fall into the [0,1] interval with $P=1$ when everyone in the society has zero income and $P=0$ when no poor are in the society at all: $0 \leq P = f(q,n,\delta_\pi,\sigma_\pi) \leq 1$.[3] It was *Amartya Sen* (1976) who in his pioneering work, argued for taking into account the feeling of relative deprivation as the σ_π component of poverty. In addition, he introduced some basic axioms (reasonable requirements) to be satisfied by any poverty index. One of these axioms is the minimal regressive transfer axiom which

[1] Associate Professor, Budapest University of Economic Sciences.
[2] The poverty gap is the shortfall of an individual from the poverty line.
[3] The 'π' subscript indicates that the measure concerned is evaluated for the subgroup of the poor.

states that the poverty index should increase due to the transfer when a poor person gives a positive amount of his (her) income to a ‘richer’ poor, who, however, remains poor despite the transfer. Without doubt such a transfer raises the degree of inequality, but not necessarily that of relative deprivation, which is also a straightforward aspect of the income dispersion. Consequently, when an overall relative deprivation measure is involved in the poverty index, its reduction due to a regressive transfer may occasionally, *ceteris paribus,* reduce the degree of poverty. Nevertheless, despite this reducing effect, the level of the overall poverty index does not necessarily decrease. This is the basic problem to be discussed in this paper. Thus, the paper is structured as follows. Section 1 summarizes the basic principles of measuring poverty. In Section 2, the deprivation impact of the regressive transfer is investigated based on some *Paul* type relative deprivation measure including a newly defined deprivation index as well. Based on these deprivation indices, Section 3 introduces a new poverty index, which inherits the properties of the deprivation indices applied.

1. Poverty index constructions

Let us consider a population of n persons with their $\mathbf{Y}=(Y_1 \leq ... \leq Y_i \leq ... \leq Y_j \leq ... \leq Y_n)$ ordered discrete income distribution ($Y_1 \geq 0$) and the corresponding $P(\mathbf{Y},z)$ poverty index the value of which indicates the overall degree of poverty associated with the z poverty line. The following definitions are of our interest in relation with the income distribution.

– *Regressive transfer*. $\mathbf{Y}^t$ is obtained from $\mathbf{Y}$ by a single regressive transfer if this transfer reduces the i person’s income by a Δ positive amount and simultaneously gives this income to a richer (at least not poorer) person so that $Y^t_i=Y_i-\Delta$ and $Y^t_j=Y_j+\Delta$ while all other incomes remain unchanged.

– *Weak definition of the poor*. Any person with income less than the poverty line is considered poor.

– *Strong definition of the poor*. Any person with income not higher than the poverty line is considered poor.

– *Truncated income distribution*. The $\mathbf{Y}_\pi$ distribution is truncated by the poverty line if the nonpoor incomes are omitted from the income distribution.

– *Censored income distribution*. The $\mathbf{Y}^c$ distribution is censored if the incomes above the poverty line are replaced by the level of the poverty line itself (*Hamada* et al., 1978).

Concerning the properties of poverty measures, a basic framework of eight ‘core’ axioms has been distinguished by *Zheng* (1997).[4]

1. *Focus axiom*: the poverty measure is required to be independent of the income distribution of the nonpoor.

2. Symmetry axiom: this axiom says that the poverty measure is invariant to any permutation of the income recipients.

[4] These axioms are ‘core’ axioms in the sense that ‘they are independent and jointly they can formulate other reasonable axioms’ such as weak (strong) monotonicity, nonpoverty growth, monotone sensitivity, progressive transfer, normalization, decomposability axioms, etc. For further details see *Sen* (1976), *Takayama* (1979), *Thon* (1979), *Blackorby* and *Donaldson* (1980), *Kakwani* (1980a,b), *Clark* et al. (1981), *Kundu* and *Smith* (1983), *Foster* et al. (1984), *Hagenaars* (1987), etc.

3. Replication invariance axiom: $P(\mathbf{Y},z)=P(\mathbf{Y}',z)$ whenever $\mathbf{Y}'=(\mathbf{Y},\mathbf{Y},...,\mathbf{Y})$ is obtained by some replications of $\mathbf{Y}$.

4. Continuity axiom: the poverty measure is a continuous function of Y for any given z.

5. Increasing poverty line axiom: $P(\mathbf{Y},z)<P(\mathbf{Y},z')$ whenever $z<z'$.

6. Regressive transfer axiom: $P(\mathbf{Y},z)<P(\mathbf{Y}',z)$ whenever $\mathbf{Y}'$ is obtained from $\mathbf{Y}$ by a regressive transfer with at least the donor being poor.

7. Weak transfer sensitivity axiom: $P(\mathbf{Y}',z)>P(\mathbf{Y}'',z)$ whenever $\mathbf{Y}'$ and $\mathbf{Y}''$ are obtained from $\mathbf{Y}$ by transferring a $\Delta>0$ income from poor person i to j and from poor person k to l respectively with $Y_j-Y_i=Y_l-Y_k>\Delta$, $Y_k>Y_i$ with no one crossing the poverty line after the transfers.

8. Subgroup consistency axioms: considering a grouped population the poverty index, ceteris paribus, must decrease by the decrease in the poverty level of a subgroup.

From the deprivation point of view, the so-called minimal regressive transfer axiom is the focus of our attention in the following approach.[5]

Minimal regressive transfer axiom: the poverty index must increase whenever, with other given things, a poor person gives a positive amount of his (her) income to a 'richer' poor, who 'however' remains poor despite this transfer.

A poverty index which is sensitive to the relative number of the poor and simultaneously insensitive to the incomes above the poverty line can be constructed applying either the so-called truncated or the censored income distribution. In the case of the truncated distribution P must be an explicit function of q and n, while using the censored income distribution P implicitly possesses this insensitivity property.

The most common measure of poverty is the $H=q/n$ '*head-count ratio*' which is the relative number of the poor proportional to the total population. Another fundamental poverty measure is the '*income-gap ratio*' I which is the percentage shortfall of the average income of a poor from the poverty line. Its value indicates the δ_π distance of the poor from the poverty line: $I=1-\sum^{q}_{i=1} r_i/q$ where $r_i=Y_i/z$ is the *relative income* of the poor person i, proportional to the poverty line. In addition, when $\sigma_\pi=0$, i.e. all the poor have the same income level, the $H{\cdot}I$ product of these two factors – termed '*normalized poverty value*' – is an appropriate measure of poverty.

Clearly, both H, I and $H{\cdot}I$ are completely insensitive to any change in the income configuration when the average income of the poor remains unchanged. Nevertheless, creating an I_σ distribution-sensitive version of the income-gap ratio, it yields a distribution-sensitive $H{\cdot}I_\sigma$ *poverty value.*[6] This way we get one of the basic definitions of the poverty level.[7]

[5] Any poverty index, which satisfies the minimal transfer axiom, is termed *distribution-sensitive* poverty measure in the literature.

[6] The subscript σ indicates distribution-sensitive measure.

[7] Other definitions of P applied by the literature are as follows: (*i*) Poverty is the normalized weighted sum of some function of the individual $g_i=z-Y_i$ poverty gaps (for all $i\in\pi$) with the $P=H{\cdot}I$ normalization requirement when there is no dispersion among the poor at all. (*ii*) Poverty is the weighted average of the 'head count ratio' and the 'normalized poverty value' with weights E_π and $(1-E_\pi)$ respectively where $0\le E_\pi\le 1$ is the degree of inequality of the incomes of the poor. (*iii*) Poverty is the inequality index of the censored income distribution. (*iiii*) Poverty is the distribution-sensitive income (welfare) -*gap ratio* of the censored income distribution $P=I_\sigma(\mathbf{Y}^c)$. For the most important poverty indices proposed by the literature so far see e.g. *Foster* (1984) and *Zheng* (1997).

At this stage our purpose below is to characterize the parameter σ_π by measuring the feeling of relative deprivation among the poor in relation to other poor persons, based on some appropriate individual deprivation function.[8]

Deprivation function: Individual i feels $d_{i<j} = d_{ij}>0$ deprivation in relation to any j person with income $Y_j > Y_i$ and $d_{i \geq j}=d_{ij}=0$ deprivation otherwise. Any j person with $d_{ij}>0$ deprivation is considered as a reference person of i and they constitute the reference group of i. The overall measure D of relative deprivation is some function of the individual $d(Y_i)$ deprivation functions: $D=f\{d(Y_i)\}$.

Paul (1991) argued that an aggregate index of relative deprivation had to be based on such an individual deprivation function which is sensitive to income transfers among those who are richer than him and satisfies the following desirable axioms.

– *Deprivation axiom 1*: the incomes of all others remaining the same, the deprivation of person i declines with the increase in his own income, i.e., $\partial D(Y_i)/\partial Y_i < 0$.

– *Deprivation axiom 2*: an increase in the income of person j causes an increase in the relative deprivation of person i, i.e., $\partial D(Y_i)/\partial Y_j > 0$.

– *Deprivation axiom 3*: the deprivation of person i increases less than proportionately with the increase in the income of person j, i.e., $\partial^2 D(Y_i)/\partial Y_j^2 < 0$.

– *Deprivation axiom 4*: if Y_i, Y_j, Y_k, Y_l and Y_m are the incomes of persons i, j, k, l and m, such that $Y_i < Y_j < Y_k < Y_l < Y_m$, then a transfer of income, say $\Delta>0$ from person m to person l will cause less deprivation to person i than the transfer of Δ from person k to j, i.e., $\partial^3 D(Y_i)/\partial Y_j^3 > 0$.

– *Deprivation axiom 5*: the deprivation of person i decreases less than proportionately with the increase in its own income. In other words, marginal deprivation of person i increases with the increase in its own income, i.e., $\partial^2 D(Y_i)/\partial Y_i^2 > 0$.

– *Deprivation axiom 6*: the higher the income of person i is, the lower the increase in the marginal deprivation is, i.e., $\partial^3 D(Y_i)/ \partial Y_i^3 < 0$.

2. Measuring the relative deprivation of the poor

The question arising at this stage is how a regressive transfer influences the D overall degree of relative deprivation. Let us divide the population into two subgroups: on the one hand we have the set of people unaffected by the transfer, on the other hand we have a two-member group of the donor and the receiver.

It is obvious that among those with unaffected incomes, the degree of relative deprivation remains unchanged, while between the donor and the receiver it increases. However, the overall impact of the regressive transfer is ambiguous, because some of the remaining individual deprivations decrease while others increase simultaneously. While deprivations felt in relation to the donor and those felt by the receiver decrease, deprivations felt by the donor and those felt in relation to the receiver increase. Besides, when

[8] In accordance with Runciman's criteria (*Runciman*, 1966), a person is relatively deprived of *X*, when: '(*i*) he does not have *X*, (*ii*) he sees some other person or persons, which may include himself at some previous or expected time, as having *X*, (*iii*) he wants *X*, (*iiii*) he sees it as feasible that he should have *X*.' The relativity of the concept is introduced by (*ii*) and (*iiii*), and the feeling of deprivation is defined by (*i*) and (*iii*). Runciman's criteria suggest that people compare themselves with some reference group within the society rather than with the whole society. Those people to whom person *i* compares himself constitute the reference group *i*.

the income rank of the donor or the receiver changes, the reference group of the donor will broaden and the reference group of the receiver will narrow. Altogether, as a joint result of these factors, the overall degree of relative deprivation can either increase or decrease.

Now with the reference group of a poor person defined, several methods can be applied for measuring the *overall degree* of relative deprivation.

Firstly, let us define the deprivation function felt by person *i* with respect to person *j* as

$$d_{i<j} = (Y_j / Y_i)^{1/\beta} - 1$$

and the relative deprivation of person *i* as

$$D(Y_i) = (1/q) \sum_j d_{i<j}$$

where β>0 is the deprivation aversion parameter.[9] Then, the Paul-index of the relative deprivation, which satisfies all the deprivation axioms listed previously, is as follows[10]

$$D_{(\beta)} = (1/q) \sum^q_{i=1} D(Y_i).$$

As β increases, the degree of the relative deprivation of the society decreases.

While characterizing the relative deprivation of the poor, it is a serious disadvantage of index $D_{(\beta)}$ that it is not defined for zero incomes, although this is a realistic situation among the poor. Furthermore there is no upper limit to express an extreme degree of being deprived, but $D_{(\beta)}$=0 when all incomes of the poor are equal.

Considering the case of a regressive transfer – as a consequence – the Paul-index always indicates an increase of the overall degree of relative deprivation and hence $D_{(\beta)}$ satisfies the *Dalton-Pique* principle of transfer. This is because the rising effect felt in relation to the receiver of the transfer dominates the reducing factor felt in relation to the donor, and the rising effect felt by the donor dominates the reducing factor felt by the receiver.

On the other hand – as it has been investigated in the literature – a relative deprivation index does not necessarily increases as a result of the regressive transfer. So it is, from this point of view that we will discuss further on a relative deprivation index which is inversely related to the Paul's function and allows the deprivation rising factor to dominate the reducing factor and hence sometimes indicates a decrease rather than an increase in the overall degree of relative deprivation.

For this reason let us consider alternatively the

$$d_{i<j} = (1 - Y_i/Y_j)^{1/\beta}$$

deprivation function (β>0) which is defined for zero Y_i incomes as well. Using this function, the relative deprivation of person *i* is

$$Q(Y_i) = (1/q) \sum_j d_{i<j}$$

[9] Recall that for $d_{i<j}$, $Y_i < Y_j$.

[10] We do not use the *Chakravarty-Chakraborty* (1984) general relative deprivation index here, because it does not satisfy the deprivation axioms 3–6.

and the overall measure is

$$Q_{(\beta)} = [\ (1/q)\ \Sigma^{q}_{i=1}\ Q(Y_i)\ ]^{\beta}.$$

Obviously, $0 \le Q_{(\beta)} \le (q\text{-}1)/q$ with $Q_{(\beta)}=0$ when all the poor have equal incomes and $Q_{(\beta)}=(q\text{-}1)/q$ when there is a perfect inequality among the poor. Index $Q_{(\beta)}$ can also be interpreted as the average proportion of the individual incomes which is not shared by the deprived poor.

Index $Q_{(\beta)}$ satisfies the deprivation axioms 1-2 for $\beta>0$, axiom 3 for $\beta>(0<c_1<1)$, axiom 4 for $\beta>(0<c_2<1)$, axiom 5 for $0<\beta<1$ and axiom 6 for $0<\beta<0.5$ and $\beta>1$. As β approaches infinity, $Q_{(\beta)}$ becomes zero.

From the point of view of the regressive transfer the following properties of $Q_{(\beta)}$ can be summarized.

– $Q_{(\beta)}$ always increases when the donor is the poorest and the receiver is the richest among the poor because there is no reducing effect in this case at all.

– A reduction in someone's deprivation felt in relation to the donor always dominates his (her) increment felt in relation to the receiver.

– Assuming $\beta=1$ unit deprivation aversion, the fall and rise in the deprivation of the receiver and the donor felt respectively in relation to a common reference person equalize one another. Hence, in this case, the change of $Q_{(\beta)}$ will be a decrease when the decrease of deprivation felt in relation to the donor dominates the joint effect of the remaining increasing factors, namely the deprivation felt by the donor in relation to nonrichers than the receiver, and felt by nonrichers than the receiver.

– Assuming $\beta>1$ deprivation aversion, the reducing factor felt by the receiver exceeds the rising factor felt by the donor in relation to the reference persons of the receiver and the reduction in someone's deprivation felt in relation to the donor becomes more dominant to his/her increment in relation to the receiver.

– Assuming $0<\beta<1$ deprivation aversion, the reducing factor felt by the receiver is dominated by the rising factor felt by the donor in relation to the reference persons of the receiver. Meanwhile, the reduction in someone's deprivation felt in relation to the donor becomes less dominant to his (her) increment in relation to the receiver.

3. Poverty indices based on Paul-type deprivation measures

At the outset let us consider the $r_1 \le r_2 \le \ldots \le r_q$ truncated distribution of the relative incomes and define the[11]

$$r_{(\beta)} = [\ (1/N)\cdot \Sigma_{j=1,\ldots,q}\ (q+1\text{-}j)(1/r_j)^{1/\beta}\ ]^{|\beta|}$$

weighted average deprivation of these relative incomes felt in relation to the *unit poverty line*, where $N=q(q+1)/2$ means the sum of the $(q+1\text{-}j)$ weights and $\beta\neq0$.[12] This index represents the overall degree of being deprived among the poor, where all the reference persons have an income level corresponding to the poverty line. Clearly $1/r_{(1)}$ is the

[11] As earlier defined, $r_i=Y_i/z$.
[12] Obviously, this is a slightly modified version of the Paul's deprivation index.

weighted harmonic mean and $r_{(-1)}$ is the weighted arithmetic mean of the relative incomes. In other words, $1/r_{(1)}$ is a deprivation preserving representative (average) relative income, which preserves the $r_{(1)}$ degree of the relative deprivation felt by the average income level in relation to the poverty line.

Furthermore, let us consider the

$$1 - Q_{|\beta|}$$

complement of index Q, termed '*relative satisfaction*'.[13] Apparently $1–Q_{(1)}$ represents the proportional amount of income of the richer poor owned by the deprived persons in an average sense. Based on this relative satisfaction we specify the following $H{\cdot}I_{\sigma}$ type index which depends on the overall degree of the relative deprivation felt among the poor:

$$P_{(\beta)} = H{\cdot}I_{(\beta)} = H \cdot [\ 1 - (1/r_{(\beta)})^{\beta/|\beta|}\ (1- Q_{|\beta|})\ ]$$

where $\beta \neq 0$.

Recalling the interpretation of $1–Q_{(1)}$ and $1/r_{(1)}$, the meaning of the $(1–Q_{(1)})/r_{(1)}$ quantity is the proportional satisfaction of the 'representative deprived poor person' who is deprived of the deprivation-preserving relative income level. So, $I_{(1)}$ represents a *deprivation gap-ratio* in relation to the unit poverty line.

Unfortunately $P_{(\beta>0)}$ is not defined for zero incomes. However, this problem can be eliminated by choosing $\beta<0$ values for which $r_{(\beta<0)}$ is the $1/\beta$-order moment of the relative incomes and is defined for zero incomes as well.

Obviously, $0 \leq P_{(\beta<0)} \leq 1$ and $0 \leq P_{(\beta>0)} < 1$ with $P_{(\beta<0)}=1$ when each member of the society has a zero income and $P_{(\beta<0)}=P_{(\beta>0)}=0$ when there are no poor in the society at all. In addition, ceteris paribus, the higher the weighted (parametric) average of the relative incomes is, the lower the degree of poverty is, and the higher the overall relative deprivation among the poor is, the higher the level of poverty is.

Since $1/r_{(\beta>0)}$ and $r_{(\beta<-0.5)}$ always decrease as a result of a regressive transfer, this decrement – depending on the value of β - can dominate a simultaneous decrement of Q. So choosing an appropriate value for β (putting more weight on the lower tail of the income distribution) an increment of $P_{(\beta)}$ is expected in the case of a minimal regressive transfer.

Apparently index $P_{(\beta)}$ satisfies the focus, symmetry, increasing poverty line and transfer sensitivity axioms, but it is not continuous at the poverty line level and it is not replication invariant.[14] Finally, its deprivation-sensitivity (similarly to the problem of the regressive transfer) may conflict with its subgroup consistency.

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[13] This terminology has been introduced by *Yitzhaky* (1979).

[14] Although Q is replication invariant, this is not valid for r.

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STOCK RETURN DISTRIBUTIONS: A SURVEY OF EMPIRICAL INVESTIGATIONS*

JÓZSEF VARGA[1]

SUMMARY

In this paper we give a brief survey of the empirical investigations of the distribution of stock returns and some detailed discussion of Hungarian and German stock returns as well as the DAX using refined methods. As a conclusion the stable law hypothesis for the stock returns is rejected and procedures requiring much weaker distributional assumptions are suggested instead of the more traditional techniques.

KEYWORDS: : Pareto-distribution; Extremal value theory; Tail index.

Financial researchers have long been interested in studying the empirical distribution of stock returns. This interest can be explained by the fact that the return distribution has a direct bearing on the descriptive validity of theoretical models in financial economics.

A still open debate concerns the analysis of the asymmetry of return distributions. The issue is relevant for portfolio theory and management, because of the importance of the distribution of stock returns in designing profitable investment strategies. The important role of skewness in explaining security returns is demonstrated by *Jean* (1971,1973), and *Levy* and *Sarnat* (1972). Several researchers (*Samuelson*, 1970; *Rubinstein*, 1973) argue that in order to ignore the third and higher moments, at least one of the following three conditions must be true:

(*i*) The return distribution has negligible variation, therefore any moments beyond the first are zero.

(*ii*) The derivatives of the applicable utility function are zero for the third and higher moments.

(*iii*) The asset returns have normal distributions, or the investors' utility functions are quadratic.

Some researchers support the use of quadratic approximation for utility functions in practical problems, assuming that the risk taken by the investor is small compared to his

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[1] Associate Professor, Janus Pannunius University, Pécs, Hungary.

total wealth. Others, however, remain skeptical of its application. *Hanoch* and *Levy* (1970) point out that the quadratic utility function implies increasing absolute risk aversion (which is contrary to the normal assumption of decreasing absolute risk aversion). As a consequence of these investigations more attention has been directed to the existence and importance of skewness in portfolio selection. In the case of restricting the investment decision to a finite time interval, portfolio selection based on the mean-variance approximation becomes inadequate and higher moments become more relevant (*Samuelson* 1970). *Jean* (1971, 1973) derives the risk premium for higher moments similar to that for the two-moment case, extending portfolio analysis to three or more parameters. Empirical studies show that the three-moment CAPM fits the return distribution better than the two-moment model (*Kraus* and *Litzenberger* 1976). *Markowitz* (1991) finds that the mean-variance model may approximately maximize expected utility for relatively small deviations in rates of return even if distributions are not normal.

Stable distributions have often been used to explain the stochastic behaviour of stock prices because of the following statistical properties: (*1*) only stable distributions have domains of attraction (generalized central limit theorem), and (*2*) stable distributions belong to their own domain of attraction (stability). These properties are consistent with economic price theory and capable of explaining the observed leptokurtosis and skewness in return distributions.

The logarithm of the characteristic function $\varphi(t)$ of any *stable* random variable X is: $\log\varphi(t) \equiv \log \mathrm{E}\left[\mathrm{e}^{\mathrm{it}\,X}\right] = i\delta t - \gamma|t|^{\alpha}\left[1 - i\beta\,\mathrm{sgn}(t) tan(\alpha\pi/2)\right]$, where $(\alpha,\beta,\gamma,\delta)$ are the four parameters that characterize each stable distribution.

In the above formula $\alpha \in (0,\ 2]$ is the *exponent*, $\beta \in (-\infty,\ \infty)$ is the *skewness index*, $\gamma \in (0,\ \infty)$ is the *scale parameter*, and $\delta \in (-\infty,\ \infty)$ is said to be the *location parameter.*

When $\alpha = 2$, the stable distribution reduces to normal. As α decreases from 2 to 0, the tail areas of the distribution become increasingly 'fatter' than that of the normal. When $\alpha \in (1,\ 2)$, the stable distribution has a finite mean given by δ, but when $\alpha \in (0,\ 1]$, even the mean is infinite. The parameter β measures the symmetry of the stable distribution; when $\beta = 0$ the distribution is symmetric, and when $\beta < 0$ (or $\beta > 0$) the distribution is skewed to the left (or right). When $\beta = 0$ and $\alpha = 1$ we have the Cauchy distribution, and when $\alpha = 1/2,\ \beta = 1,\ \delta = 0,$ and $\gamma = 1$ we have the Bernoulli distribution.

Detailed description of stable laws can be found in *Feller* (1971) and *DuMouchel* (1971). Discussions of their applicability in economic analysis are in *Mandelbrot* (1963), *Fama* (1965) and *McCulloch* (1978).

The following part of this paper is organized as follows. Section 1 summarizes the results of previous discussions as well as the methods used for investigations. In Section 2 some more detailed discussion is presented analysing the appropriateness of stable laws for German and Hungarian stock returns using refined methods. The last section provides concluding remarks.

1. Methods and results of previous discussions

Stable laws other than the normal distribution share the features of fat tails and high peak at the mean (leptokurtosis) observed in data. Stability under addition seems to be a necessary property for daily, weekly etc. data when succesive high frequency price changes are assumed to be independent and identically distributed random variables. The reasoning outlined in the literature seemed to be so persuasive that researchers accepted the stable laws as evidence without the further testing of fit. *Teichmoeller* (1971) and *Simkowitz* and *Beedles* (1980) examined stock returns, *McFarland*, *Pettit* and *Sung* (1980) and *So* (1987a) investigated exchange rate changes, *Cornew*, *Town* and *Crowson* (1984) and *So* (1987b) studied futures returns without the testing of fit.

Some indications for the violation of the stability-under-addition property expressing itself as time dependence of α over daily, weekly etc. data motivated others to query the stable law hypothesis (*Hsu*, *Miller* and *Wichern* 1974, *Upton* and *Shannon* 1979, *Friedman* and *Bandersteel* 1982, and *Hall*, *Brorson* and *Irwin* 1989).

Despite the a priori plausibility of stable distributions, several empirical studies have found some evidence against the hypothesis that stock returns can be characterized by stable distributions (*Officer* 1972, *Blattberg* and *Gonedes* 1974, *Hsu*, *Miller*, and *Wichern* 1974). Most of these studies have been restricted to the symmetric case because the parameter estimation as well as the economic analysis have been considerably facilitated when $\beta = 0$. More recent investigations by *Simkowitz* and *Beedles* (1980), *Rozelle* and *Fielitz* (1980) and *Fielitz* and *Rozelle* (1983) have shown, however, that empirical return distributions are in most cases significantly skewed and only the asymmetric stable laws can be used as probability models of stock returns. *Peccati* and *Tibiletti* (1993) suggest a possible reading-key to the interpretation of the skewness of stock return distributions. This key relies on the fact that the asymmetry of a sum of random variables depends not only on that of the random addenda, but also on their dependence structure. The conclusion of the empirical investigations on the skewness of stock return distribution is that the introduction of the asymmetry in the mean-variance framework serves as a useful tool for describing the ex-post equilibrium of the financial markets; however, it does not seem to be a proper ex-ante tool for selecting profitable portfolio strategies.

Akgiray and *Booth* (1988) investigate the stable-law hypothesis for stock returns discussing the empirical tail shapes instead of testing the overall fit of stable distributions to data. This approach is based on the notion that the tails of stable distributions and finite-variance distributions are distinctly different. (The rate at which the tail probability $Pr(|x| > n)$ converges to 0 as $n \to \infty$ is proportional to n^k.) For infinite-variance stable distributions $k < \alpha < 2$ and $k \geq 2$ for finite-variance distributions.

The analysis of tails relies on the following result of the extremal value theory: consider a stationary sequence of independent and identically distributed random variables $X_1, X_2, ..., X_n$ and define the order statistics

$$M_n = \max (X_1, X_2, \ldots, X_n) . \qquad /1/$$

As it can be shown, the limiting distribution of M_n is appropriately scaled and converges to one of the max-stable distributions (for details see *Leadbetter* et.al., 1983).

The relevant distribution is fat-tailed without finite endpoint given by the cumulative distribution function

$$F(x) = \begin{cases} 0 & if \quad x \leq 0 \\ exp\left(-x^{-\alpha}\right) & if \quad x > 0 \end{cases} \qquad /2/$$

This means that $Pr\{a_n(M_n - b_n) \le x\}$ converges (weakly) to $F(x)$ with normalizing constants a_n and b_n. The characteristic exponent of a stable distribution coincides with the tail index α of the limiting extremal value distribution $F(x)$. Taking into account that the tail index is not restricted to the interval (0, 2] but may assume any positive value, the characteristic exponent estimated by the fractile method (or other methods) may not coincide with the tail index when the underlying distribution is not a stable one.

ARCH processes may obey limit laws characterized by indices greater than 2. De *Haan* et al. (1989) show how the tail index depends on the parameters of ARCH processes. As an other example, the Student *t* distribution converges to a limiting extremal distribution with tail index identical to the number of degrees of freedom. It means that the tail indices for the Student family extend from 1 to infinity. All these alternatives are nested in the tail estimation procedure. The relationship between the number of existing moments and the tail index also gives some useful information for the analysis. All moments smaller than the tail index exist, whereas higher moments exhibit do not converge.

Studies of proposed estimation techniques for the tail index α have favoured the estimator introduced by *Hill* (1975) as the most effective one. This estimation procedure gives a consistent estimate of the inverse of α by calculating :

$$\gamma_H = 1/\alpha_H = \frac{1}{m}\sum_{i=1}^{n}\left[\log x_{(i)} - \log x_{(m)}\right], \qquad /3/$$

where *n* is the sample size, *m* is the number of observations located in the tail of the distribution and the elements of the sample are in descending order:

$$x_{(1)} \ge x_{(2)} \ge \ldots \ge x_{(m)} \ge \ldots \ge x_{(n)}.$$

It can be shown that the variable $(\gamma_H - \gamma)\sqrt{m}$ follows asymptotically normal distribution with zero mean and variance γ^2. This result can be used to test the hypothesis of identity of limit laws across stocks as well as the equality of lower and upper tails in the same sample. The main problem connected with the application of tail index estimations is the decision about an appropriate tail size, i.e. determining the number of observations *m* used in the calculation of α_H. The choice of tail size *m* necessarily involves judgement or maintenance of a specific hypothesis on the true α. The higher (lower) α itself is, the thinner (fatter) the tails will be and the fewer (more) elements will belong to the tail region. One can realize that choosing a too large value for *m* will result in a contamination of the tail region with elements of the central parts of the distribution when the true α assumes a relatively high value.

Tail index estimation has only recently been applied in the financial literature. *Koedijk*, *Schafgans* and de *Vries* (1990) and *Kähler* (1993) analyse European exchange rates quoted against the US dollar. *Koedijk* et al. cannot reject the hypothesis of a tail index within the realm of the stable laws while *Kähler's* estimates lie within the interval 3-5 allowing rejection of $\alpha < 2$. *Dewachter* and *Gielens* (1991) point to biases in the estimates of *Koedijk* et.al. and report upward corrected tail indices. *Akgiray*, *Booth* and *Seifert* (1988) and *Koedijk*, *Stork* and *de Vries* analyse Latin-American black market exchange rates. In the study of Akgiray et.al. (1988) a less efficient maximum likelihood estimator was used giving values within the interval 0.5 to 7. *Koedijk* et al. revised the results using the *Hill* estimator for the same data. The revision resulted a narrower interval of α values (about 1.2 to 3.2). *Koedijk* and *Kool* (1993) investigate the East European exchange rates against the US dollar finding α values within the interval (2, 3). The studies of US and German stock prices performed by *Akgiray* and *Booth* (1988) and *Akgiray*, *Booth* and *Loistl* (1989) respectively were based on maximum likelihood estimation. Jensen and *de Vries* (1991) found the α values in the range of 3.2 and 5.2 considering daily returns of 10 US stocks.

The next section of the paper analyses the appropriateness of stable laws for German and Hungarian stock returns. Data used for the analysis cover the period from 1 January, 1988 to 9 September, 1994 of thirty of the most frequently traded German stocks forming the DAX share price index and the period from 6 January, 1993 to 31 August, 1995 for the most frequently traded Hungarian stocks (*Lux* and *Varga*, 1996).

2. Analysis of return distributions for major German and Hungarian stocks

Returns are calculated as differences of the logarithms of daily closing prices. First chi-squared tests with 10 and 25 equiprobable cells are applied to test the fit of the estimated distributions. The 25-cell test for the German individual stocks rejects the stable Paretian hypothesis in 12 (16) cases at 1 percent (5%) significance level, whilst for the DAX index the hypothesis is not rejected.

(Nearly the same results have been obtained for the Hungarian stock market, but the sample is not as large as the German one, therefore the inference may be questionable). The 10-cell test even rejects 8 more cases at 1 percent and also rejects the stable distribution for the DAX at 5 percent level. Some of the rejections of the 10-cell test may be due to certain non-robustness against some slight misspecification of the location parameter α and therefore the 25-cell variant may be considered more reliable. The interpretation of this standard test is ambiguous: partly because for about one third of all cases the stable laws are rejected at 1 percent level, and partly because the results also show that many of the empirical distributions seem to be described quite well by stable distributions. The computational results confirm the picture available from many other previous studies that there is at least some overall similarity in the shapes of empirical distributions and that of the stable distributions and the estimated characteristic exponents lie in a relatively limited interval around 1.5.

As a counter-check the tails of the empirical distributions are considered. To investigate whether upper and lower tails are identical, the *Hill*-estimator was used to the lowest and highest 5 percent of observations. Point estimates denoted by α^+ and α^- for German stocks forming the DAX and the most often traded Hungarian stocks were calculated. The test of hypothesis $\alpha^+ = \alpha^-$ relies on the approximate normality of $1/\gamma_H$. As a consequence, the sum

$$Q = \left(\frac{\gamma^+ - \gamma}{\sigma_\gamma}\right)^2 + \left(\frac{\gamma^- - \gamma}{\sigma_\gamma}\right)^2 = \left(\alpha / \alpha^+ - 1\right)^2 m + \left(\alpha / \alpha^- - 1\right)^2 m \qquad /4/$$

follows chi-squared distribution with two degrees of freedom, and m is the number of observations located in the tail of the distribution as in /3/ $\left(\sigma_{\gamma}^{2}=\gamma^{2}/m\right)$. The results of the computations show that in all cases there exists a broad range of hypothetical α values for which the hypothesis $\alpha^{+} = \alpha^{-}$ is not rejected. Simultaneously it is obtained that $\alpha^{+} > \alpha^{-}$ in 25 out of 30 cases for German stocks. This result could raise doubts about the appropriateness of the assumption of identical tail behavior on the left and right tails of the distributions. To make this point clear a simple symmetry test can be used. The hypothesis $\alpha^{+} = \alpha^{-}$ for all stocks implies

$$\Pr(\alpha^{+} > \alpha^{-}) = \Pr(\alpha^{+} < \alpha^{-}) = 0.5.$$

The number of k observations with $\alpha^{+} > \alpha^{-}$ under the hypothesis $\alpha^{+} = \alpha^{-}$ follows a binomial distribution $B(30, 0.5)$. Only individual stocks are considered because the DAX is a linear combination of its constituent elements. The probability of observing $k \leq 5$ or $k \leq 25$ under $H_0: \quad \alpha^{+} = \alpha^{-}$ is only 0.003. It tells us that a significant asymmetry between upper and lower tail indices seems to exist considering the 30 stocks as a whole. The 'mini-crash' in October 1989 (the Gulf crisis, the Russian putsch) may be responsible for this asymmetry. Omitting relevant data and recalculating the upper and lower 5 percent tail indices reduces asymmetry and gives a statistically insignificant result. This means that the asymmetry between left and right tails was caused by an extreme event. This extreme event affected all stocks in a rather uniform way (individual stocks fell by 6 to 25 percent and the DAX declined by 13 percent that day). The conclusion that no systematic differences in the extremal behaviour of left and right tails exist can be accepted. To obtain the point estimates and some insight into variation with sample size, the two-sided Hill-estimator was computed for the stocks and the DAX and BUX at tail sizes of 15, 10, 5 and 2.5 percent. The results for the German market are shown in Table 1. Monte Carlo simulations show that the 15 percent tail size would be appropriate for stable family members with characteristic exponent $1 < \alpha < 2$, whilst the thinner tails would apply to Student distributions with 3 to 5 degrees of freedom. The point estimates are either rather uniform using different choices of the number of tail observations or tend to increase slightly. It can be seen that the point estimates are outside the region characterizing the stable distribution family in all cases. Even if the point estimates and confidence intervals given in Table 1 form already strong evidence against the Paretian model, it seems useful to investigate whether the different tail sizes chosen are all appropriate or not.

It is interesting to test whether the respective tails really follow an extreme value distribution of type /2/ with the estimated parameter α. Under distribution /2/ the random variable $u_i = \alpha \cdot i \cdot \log \frac{x_{(i+1)}}{x_{(i)}}$ follows exponential distribution with origin 0 and parameter 1, i.e. $Exp(x;0, 1)=1-exp(-x)$, where $x_{(1)} \geq x_{(2)} \geq \dots x_{(i)} \geq \dots x_{(m)}$ denote the m largest observations and α is the parameter of the distribution /3/ (see *Hill*, 1975). The appropriateness of the tail size can be tested by performing the goodness-of-fit test for u_i. The rejection of the exponential distribution for u_i implies the rejection of convergence of the original sample to the limit law at the tail sizes considered. Standard chi-squared test with 20, 16, 10 and 8 equiprobable cells has been implemented for this test procedure. Only very few cases led to the rejection of the exponential distribution.

Table 1

Estimates of tail indices

Stocks and the index	$\alpha_{15\%}$	$\alpha_{10\%}$	$\alpha_{5\%}$	$\alpha_{2.5\%}$
DAX	2.654 (2.326, 2.982)	2.723 (2.310, 3.137)	2.964 (2.327, 3.602)	2.622 (1.819, 3.425)
Allianz	2.601 (2.279, 2.923)	2.746 (2.330, 3.163)	2.766 (2.171, 3.137)	2.446 (1.697, 3.194)
BASF	2.508 (2.198, 2.819)	2.641 (2.241, 3.042)	2.670 (2.119, 3.281)	2.961 (2.055, 3.868)
Bayer	2.571 (2.253, 2.889)	2.845 (2.413, 3.276)	3.320 (2.606, 4.034)	2.879 (1.998, 3.760)
Bayer Hypobank	2.456* (2.153, 2.760)	2.598** (2.204, 2.993)	3.266 (2.563, 3.968)	3.696 (2.565, 4.828)
BMW	2.494 (2.186, 2.803)	2.501 (2.122, 2.881)	2.753 (2.161, 3.346)	2.784 (1.932, 3.636)
Bayer Vereinsbank	2.602 (2.280, 2.924)	3.101 (2.631, 3.571)	3.380 (2.653, 4.107)	2.852 (1.979, 3.726)
Commerzbank	2.849 (2.497, 3.202)	2.996 (2.541, 3.450)	3.018 (2.369, 3.667)	2.896 (2.010, 3.783)
Continental	2.829 (2.479, 3.179)	3.062 (2.598, 3.527)	3.312 (2.599, 4.024)	3.565 (2.474, 4.656)
Daimler	2.732* (2.394, 3.070)	3.094 (2.625, 3.563)	2.882 (2.262, 3.502)	3.099 (2.150, 4.047)
Dt. Babcock	2.622 (2.298, 2.946)	2.848 (2.416, 3.279)	3.211 (2.520, 3.902)	3.141 (2.179, 4.102)
Deutsche Bank	2.698 (2.365, 3.032)	2.917 (2.474, 3.359)	3.132 (2.458, 3.806)	3.096 (2.148, 4.044)
Degussa	2.836 (2.485, 3.187)	3.119 (2.646, 3.592)	3.294 (2.585, 4.002)	3.131 (2.173, 4.090)
Dresdner Bank	2.450 (2.191, 2.809)	2.818 (2.390, 3.245)	2.786* (2.186, 3.385)	3.625 (2.515, 4.734)
Henkel	2.699 (2.339, 2.999)	2.899 (2.459, 3.338)	3.113 (2.443, 3.783)	2.768 (1.921, 3.616)
Hoechst	2.544 (2.229, 2.858)	2.832 (2.403, 3.262)	2.708 (2.125, 3.291)	3.777 (2.621, 4.933)
Karstadt	2.647 (2.320, 2.975)	2.772 (2.351, 3.192)	2.830 (2.221, 3.439)	3.365 (2.335, 4.395)
Kaufhof	2.887 (2.529, 3.243)	3.374** (2.823, 4.370)	3.597 (2.823, 4.370)	3.852 (2.673, 5.031)
Lufthansa	2.503 (2.194, 2.813)	3.073 (2.607, 3.539)	3.335 (2.617, 4.052)	4.011 (2.783, 5.238)
Linde	2.652 (2.324, 2.980)	2.845 (2.414, 3.277)	3.941 (3.093, 4.789)	3.510 (2.435, 4.584)
MAN	2.691 (2.358, 3.024)	2.913 (2.471, 3.355)	3.054 (2.397, 3.711)	3.136 (2.176, 4.096)
Metallges.	2.371 (2.078, 2.664)	2.682 (2.275, 3.089)	2.563 (2.011, 3.114)	2.422 (1.680, 3.163)
Mannesm.	2.654 (2.326, 2.983)	2.865 (2.430, 3.299)	2.858 (2.243, 3.472)	2.756 (1.193, 3.600)
Preussag	2.207 (1.934, 2.480)	2.753 (2.335, 3.170)	3.210 (2.519, 3.900)	3.462 (2.402, 4.522)
RWE	2.526 (2.213, 2.838)	2.829 (2.400, 3.259)	3.044 (2.389, 3.699)	3.286 (2.280, 4.292)
Schering	2.495 (2.187, 2.804)	2.819 (2.391, 3.246)	3.094 (2.429, 3.760)	3.891 (2.700, 5.082)
Siemens	2.450 (2.147, 2.753)	2.947 (2.450, 3.394)	2.901 (2.277, 3.525)	3.031 (2.103, 3.959)
Thyssen	2.805 (2.458, 3.152)	3.006 (2.550, 3.462)	3.365 (2.641, 4.089)	3.524 (2.445, 4.602)
VEBA	2.550 (2.235, 2.865)	2.707 (2.297, 3.118)	2.842 (2.230, 3.453)	3.140 (2.179, 4.102)
Viag	2.460 (2.156, 2.765)	2.789 (2.366, 3.212)	3.179 (2.495, 3.863)	3.274 (2.272, 4.276)
VW	3.033 (2.658, 3.409)	3.016 (2.559, 3.474)	3.121 (2.450, 3.792)	3.160 (2.193, 4.128)
Number of observations in the tail	251	167	83	41

Note. The signs * and ** indicate rejection at 5 or 1 percent level of the extreme value distribution /2/ for the sample of observations in the tail.

In most cases cell frequencies are very close to their hypothetical values have been at all tail sizes considered, hence good convergence to the extreme value distributions can be accepted. To demonstrate the difference in prediction of extremal events, exceeding probabilities per year for certain threshold values were calculated using the estimated stable distributions as well as the semi-parametric tail index estimates. The results for the DAX using the estimated stable parameters α = 1.737 and c = 0.651 are summarized in Table 2.

Table 2

Annual exceeding probabilities

$\lvert r\rvert$	$\alpha_{5\%}$=2.964	$\alpha_{2.5\%}$=2.622	Stable d. with α_S=1.737	Number of observations in data
$\lvert r\rvert$>0.006	0.5254	0.8734	0.7544	6
$\lvert r\rvert$>0.10	0.1101	0.2372	0.4405	1
$\lvert r\rvert$>0.15	0.0323	0.0834	0.2329	0
$\lvert r\rvert$>0.20	0.0136	0.0396	0.1479	0

Exceeding probabilities are calculated from *Deckers* and *de Haan's* upper quantile estimation formula /4/, stable law probabilities are obtained by interpolation from *Du Mouchel's* tabulation using the scale parameter c = 0.651 (obtained for DAX), and for comparability α = 0 is assumed.

For simplicity of comparison α = 0, i.e. the symmetry is assumed. In the case of estimated stable distribution, the probability of at least one extreme return exceeding in absolute value a certain threshold is computed by interpolation using DuMouchel's tabulation (*DuMouchel*, 1971). For the semi-parametric estimation procedure the consistent estimator of upper quantiles proposed by *Decker* and *de Haan* (1989) has been applied. This estimator is given by:

$$x_p = \left(\frac{k\cdot m}{2p\cdot n}\right)^{\gamma H}\left(1-2^{-\gamma H}\right)^{-1}\left(x_{(n-m/2)} - x_{(n-m)}\right) + x_{(n-m/2)}, \qquad /5/$$

where x_p denotes the p-quantile, k is the number of observations per year (here it is 250, the number of trading days per year), n and m are the sample size and the number of observations in the tail region respectively, and γ_H is the inverse of the tail index estimate α_H. Given the x_p value, the probability can be obtained by solving equation /5/. Significant differences in the valuation of the most extreme events exist. As an example, the probability of absolute returns exceed the level of 0.20, is equal to 0.1479 under the stable law hypothesis (it was not rejected by the goodness-of-fit test). Considering the lower one of the tail index estimates, $\alpha_{2.5\%}$ =2.622, the corresponding probability is 0.0396, and for $\alpha_{5\%}$ = 2.964 this probability is 0.0136. If the stable law hypothesis holds, returns of this magnitude are expected to occur once within six to seven years, while the Hill tail index estimates predict occurence of such large absolute returns only once within twenty-five or even once within seventy-five years. This example shows that conclusions drawn from the stable model concerning large absolute returns are misleading.

The point estimates of the various stocks for a given tail size lie within a relatively limited range. Any inference related to the homogeneity or heterogeneity with respect to the likelihood of extreme returns across stocks is of paramount interest to questions of risk management and portfolio selection. In order to answer these questions the identity of limit laws has been tested. Formally it means the test of hypothesis $\alpha_1 = \alpha_2 = \ldots = \alpha_{30} = \alpha$. Using normality of $1/\alpha_H$ the statistic

$$Q = \sum_{i=1}^{30} \left(\alpha / \alpha_{H,i} - 1\right)^2 m \qquad /6/$$

is approximately chi-squared distributed with 30 degrees of freedom under the null hypothesis of identical α values. The test results show that the hypothesis of identical extreme value distributions cannot be rejected.

Table 3

Uniformity test of limit laws across stocks

Tail size	15	10	5	2.5
	percent			
Lower bound	2.460	2.629	2.699	2.618
Upper bound	2.722	3.089	3.364	3.610

The lower and upper bounds determine the intervals of α-values for which the hypothesis $\alpha_1 = \alpha_2 = \ldots = \alpha_{30}$ cannot be rejected at 1 percent level.

The analysis of extreme value distributions confirms that there are no stocks with more pronounced inclination for extreme changes than the average. This result may suggest that macroeconomic shocks may have similar impacts on the formation of their returns.

3. Conclusions

This paper gives a survey of the empirical studies investigating stock return distributions and the detailed analysis of the most recent results for the main German stocks and some conclusions of the investigations for the Hungarian stock market. It has been found that the stable model seems to fit well for most of the stocks when the standard goodness-of-fit test was applied. Counterchecking this result with a semi-parametric analysis of extreme value distributions led to the rejection of the stable law hypothesis. These findings are in accordance with the results reported in the literature indicating that empirical distribution shapes of stock returns are similar to the Pareto-Levy distributions at first sight, while refined methods of analysis point out that they are generated by other distributions. Stable distributions make theoretical modelling difficult. (Closed form expressions for the density functions of stable random variables are available for only three special cases: the normal, the Cauchy and the Bernoulli distributions.) Standard finance theory almost always requires finite second moments of returns, and often finite higher moments as well. Stable distributions also have some counterfactual implications. First, they imply that simple estimates of the variance and higher moments of returns will tend to increase as the sample size increases, whereas in practice these estimates seem to converge. Secondly, they imply that long-horizon returns will be just as non-normal as short-horizon returns. (Long-horizon returns are sums of short-horizon returns, and these distributions are stable under addition). In practice the evidence for non-normality is much weaker for long-horizon returns than for short-horizon returns. We suggest that returns should be modelled as drawn from a fat-tailed distribution with finite higher moments, such as the *t* distribution, or as drawn from a mixture of distributions. The return might be conditionally normal, conditional on a variance parameter which in itself is random. Then the unconditional distribution of returns is a mixture of normal distributions, some with small conditional variances that concentrate mass around the mean and others with large conditional variances that put mass in the tails of the distribution. This yields a fat-tailed unconditional distribution with a finite variance and finite higher moments. Since all moments are finite, and long-horizon returns will tend to be closer to the normal distribution than short-horizon returns just as the the Central Limit Theorem implies. The most convenient and widely acceptable paradigm postulates that returns are normally distributed which means that asset prices follow lognormal distributions. Both modern portfolio theory and the Black-Scholes methodology of pricing derivative assets are founded on such a paradigm.

The uncertainty of speculative prices, as measured by the variances and covariances, are changing through time. Explicit modelling time variation in second- or higher-order moments is also proposed as an alternative to the analysis. One of the most prominent tools that has emerged for characterizing variances is the Autoregressive Conditional Heteroskedasticity (ARCH) model of *Engle* (1982) and its various extensions. Since the introduction of the ARCH model a lot of research papers applying this model strategy to financial time series data have already appeared.

The randomness of asset price changes hypothesized by the Efficient Market Hypothesis (EMH) naturally leads to questions about the behaviour of the variance of such changes. If price changes are induced by changes in information, can shocks in fundamental factors affecting the economy explain the price volatility? Or, is the variance of price changes due to other factors? The literature of this topic documents that prices are too volatile and although this evidence does not imply rejection of the EMH, it raises the question of what factors other than fundamental shocks could explain such evidence of high volatility. A nonlinear deterministic methodology, chaotic dynamics, as an alternative to linear stochastic models can clarify the relation between price variability and speculation as well as explain why the empirical studies of the time series properties of asset prices are ambiguous and inconclusive. *Baumol* and *Benhabib* (1989), and *Boldrin* and *Woodford* (1990) used various single variable chaotic maps as a metaphor to illustrate the intellectual possibilities of the deterministic approach.

Another finding of the analysis was that the uniformity of extreme value distributions across the sample indicates a high degree of cooperation among the German firms forming the DAX index, and one may conclude the same on some lower level for the stocks forming the BUX. This is the reason for reporting only the computational results of the German market analysis. It is also likely that the differences in results between previous and present investigations are caused by the previously used less efficient methods. (As some examples: *Akgiray*, *Booth* and *Loistl* (1989) found tail indices in the range (3, 13), *Lux* and *Varga* (1996) reported the interval (2, 4)).

One may conclude that the uniformity of limit laws may be a more general phenomenon and it is worth searching for the reason of this behaviour.

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THE ROLE OF INTERNATIONAL COMPARISONS IN THE HUNGARIAN STATISTICS

VERA NYITRAI[1]

SUMMARY

International comparisons have had a long tradition in Hungarian CSO since its foundation. The autor's aim is to draw the attention of the readers to the growing importance of the participation in the multilateral comparisons directed by different international bodies and at the same time to continue the traditional bilateral comparisons which have a special role from the point-of-view of the users. Recommendation of using models, underlining the user-friendly publication, finding the most suitable topics and partners are also presented in the paper.

KEYWORDS: Multilateral international comparisons; ICP; Bilateral comparisons.

Being a small and open country, international comparisons have had a long tradition in Hungarian statistics. *Károly Keleti*, the founding father of the Hungarian Statistical Office introduced an independent statistical service in our country which meant the beginning of not only the collection and aggregation of the most important economic and social data of the country, but also of their comparison to the published data of some more developed countries. This process has continued during our century as well.

The Hungarian Central Statistical Office (HCSO) has played a significant role in the programmes of different international statistical organizations both in the Eastern and in the Western parts of Europe; as well as in the scope of the UN Statistical Commission and in its regional bodies. This means that Hungarian statisticians have taken part in international multilateral comparisons started and directed by UN statistical or other bodies. For example HCSO is one of the first 16 countries which began the International Comparison Project at the end of the sixties; Hungarian demographers have taken part in the programme of the International Statistical Institute which has dealt with several important topics of the demography since the end of the previous century.

In the practice of the HCSO, not only multilateral international comparisons have been on the agenda, but several bilateral economic comparisons have also been initiated by the office. These kinds of comparisons have had an important side-effect on the Hun-

[1] Doctor of economic sciences, honorary professor.

garian statistics. I am trying to draw the readers' attention to the idea of bilateral comparisons, because based on my several decade-long practice, it seems clear to me that from the point of view of our users these can give more practical results than most multilateral comparisons in which the Hungarian data are at the lower part of the countries' list and it is very difficult to interpret the factors influencing our special situation. It 'however' has to be emphasized that bilateral comparisons have never replaced the multilateral ones.

Let us see an outstanding example: in the seventies, Austrian and Hungarian industrial statisticians performed a productivity comparison. This was detailed according to industrial branches and aimed to compare primarily those kinds of activities which could be found in both countries. During the bilateral work, statisticians of both countries had an opportunity to get acquainted with the statistical methodology used in the partner-country. Analysing the results made it possible to indicate the effect of the different methods used in the interested countries to the final data and to eliminate or reduce – if possible – some disturbing factors. This procedure cannot generally be adopted by multilateral comparisons especially in cases when one statistically developed country is charged with the co-ordination of the comparison and the common method is only more or less (in many cases less) acceptable to some participants.

The history of the international statistical comparisons (both multilateral and bilateral ones) would need a more detailed analysis. The aim of this very short introduction was to illustrate the Hungarian traditions in this respect and – if possible – to arise the interest of the reader.

1. The present situation

At the end of the 20th century, macroeconomic and social processes are becoming more and more globalized, which means there is an urgent need to elaborate one or more new 'scenarios' in order to study the place and role of the country in the world. These scenarios depend on the attained economic and social levels of the country.

In the case of Hungary international comparisons could be very useful for the decision-makers, especially in this transition period of ours when preparing the country for the EU membership.

The HCSO has tried several experiments in this field during the last decade. In my opinion the most important steps have been the following.

– Yearbooks, monthly bulletins and some special publications contain regular international data in aggregated forms. These kinds of information make it possible for scientific researchers to compare, and analyse the Hungarian situation in Europe or in the world. It, however, has to be mentioned that the degree of comparability and the effect of some distorting factors cannot be found out from the very aggregated data – as we have seen during the last years. Some of the users of our data have had the courage to compare several kinds of information with rather different contents (that means the courage of the blinkers-forge-man!). In my opinion, publishing data of foreign countries, country-groups or regions cannot replace analyses made by statistical experts. Data publication is a necessary step but not a sufficient one.

– Some interesting comparisons have appeared based on the 1990, 1993 and recently the 1996 rounds of the International Comparison Programme (ICP) of which the Purchasing Power Parity (PPP) is the most important component. With the help of PPP it has become possible to eliminate the differences of the price systems of individual countries and the comparisons are more realistic than with the use of official exchange rates. Though PPP is not a method without any possible distorting factors,[2] still in my opinion, it is the best among the world-wide used methods nowadays. This more than 30-year old and regularly renewed programme is suitable for comparing the economic performance of our country with that of others.

This programme has had a growing world-co-ordinator role during the last decade (relying on professional participations of interested countries). At the same time, however, it must be mentioned that the standard of quality of ICP work was disputed by the thirtieth session of the UN Statistical Commission (1–5 March, 1999) and different opinions were presented on factors influencing the credibility of the ICP, especially in some details. Several important new recommendations were accepted which could make this programme more useful, among others: 'It is important to provide greater transparency, to define the role of National Statistical Offices and to ensure that there is a proper co-ordination among regions and between regional co-ordinators and participating NSO-s.'

Without any doubt ICP does have some limits in analysing details (for example: real consumption of several products) but nowadays this is the most significant and most frequently cited international comparison which can be forecasted and made up-to-date by the users. The new recommendations adopted at the thirtieth session of the UN Statistical Commission can help in refining the results. My personal opinion is that the role of the so-called bridge-countries (linking one region's data with those of another region) must not be underestimated. According to this interpretation, bridge-countries can connect the more or less homogeneous parts of the world with other ones. For example Austria is a typical example of 'bridge-making' between the Eastern and Western parts of Europe. The use of several ones is better than using only *one* world-co-ordinator country. (This was the first step of the ICP in the USA.)

– During the last years a few bilateral and multilateral comparisons have been published by the HCSO, but regarding their topics one cannot recognize any kind of consistent priorities. I suppose I am not mistaken in stating that the topics of these international comparisons depended primarily either on the interest of some heads of departments or on the statistical interest of several experts of the HCSO. I am sure that this is not a misconclusion only an interperation of the present situation.

Comparing the Hungarian statistics with the work of other transition countries, especially with those who are standing, together with us, on the threshold of the EU, Hungary's situation in the scope of international comparisons seems to be more developed.

2. The aims and possibilities of international comparisons nowadays

If we try to consider the new situation in the world, it is doubtless that the international bodies are more and more interested in world-wide international economic as well

[2] See the paper of *György Szilágyi*: Reflections on Purchasing Power Parities and Real Values (in this issue).

as social comparisons. EUROSTAT, OECD, the International Monetary Found (IMF), the World Bank, several special organizations of the United Nations (ILO, WHO, WTO, etc.) are generally publishing data-collections, analyses, some of which are special country reports or analyses of selected issues as well. The problem to be considered is whether it is necessary to devote a lot of energy and to allocate expenditures in order to compile several new international comparisons or not. My answer is a clear *yes*, it certainly is necessary from the following points of view.

The practice of national statistical offices has shown in the past years that all of the above mentioned international comparisons have had the aim of presenting a distant picture on the countries involved in the comparison, but if we approach this picture, it becomes evident that this could be more or less deformed. An example for illustrating this statement is the paper containing comments to the ICP made and presented at the thirtieth session of the UN Statistician Commission by the Australian Bureau of Statistics. I only mention Australian example to make the significance of a world-level comparison perceptible, as the ICP depends on the interest of the users of statistical comparisons which could be aroused by the producers of the comparisons who could be for or against these laborious processes. (It seems from this paper that Australian statisticians are far from being in favour of the ICP.) About 20-30 incredible per capita volumes of products or other dubious data were presented by the Australian paper, naturally from the point of view of the Australian statisticians. They mentioned that local users in Australia were not interested is these kinds of data. They consequently suggested undertaking a cost-benefit study before continuing the ICP work. I am sure that this example is an extreme one, since in Europe most countries are interested in continuing the ICP and so is Hungary.

Nevertheless this paper is useful in illustrating the fact that the comparisons on world- or Europe-level could present only large-scale information and are not suitable for small-scale ones like that of a branch, or that of a special topic (living standard, quality of life etc.).

In my opinion this means that in the future we will also have to take part in the comparisons directed by the international statistical organizations. Our interest is to publish their data as soon as possible along with the comments of the Hungarian statisticians involved in the process. (They can inform the users on the possible incredibilities, on the quality of data, on the lack of representativeness).

Besides these kinds of international comparisons it seems necessary to compile bilateral comparisons together with the interested partner-countries and the multilateral comparisons which could be based on the published data of OECD, EUROSTAT, CEFTA and which are produced on the basis of the same methodology.

Being in a nearly similar phase of economic development as that of our neighbouring countries, comparison with the CEFTA region (Central European Free Trade Association, which includes the Czech Republic, Hungary, Poland, Romania, Slovenia and Slovakia) seems practical and useful. It must be emphasized that during the last years, there have been regular quarterly publications based on their harmonized methods.

A special advantage of these publications is that the country in charge of compiling data and publishing the bulletin is every year a different one, so the responsibility of the comparability of information is divided among the interested countries. This bulletin contains some data and methodological comments but no analysis. I think that the series of these publications can serve as a fairly reliable basis for comparing the actual situation and its modifications in the particular countries within the region. The analyses could be further deepened by using data other than the ones in the CESTAT bulletin (this publication is a quaterly one containing the most important data of the mentioned CEFTA countries) only such kinds of country-data which are more or less comparable and can explain factors influencing the actual position of a country or its changes. Having in mind the fact that data have been available for 20 or more quarters it is not too risky a recommendation to construct models, describing the economy of this region, making it possible to deduce some regularities and at the same time to determine the differences in the process of development of the countries. It is beyond all doubt that this kind of study requires absorbing work: first of all it is not easy to develop an appropriate model (including values of different parameters); to select and choose the factors which influence to the greatest extent the economic development is also a responsible task. Last but not least, to get acquainted with the statistical methodology used in the different countries and recognise their differences also belongs to the task of making a sufficiently correct and authentic model. Without doubt this task is well worth trying especially at the end of this century before gaining the membership of total right in the EU. This model can mainly help in the decision-making process by analysing the route followed during the transition period and this could make it possible to forecast the development of the next 3-5 years as well. Before overestimating our possibilities, however, it has to be stated that several economic data and only a few suitable and comparable social data can be taken into consideration in this kind of model because of data-insufficiency in some of the countries.

Another good basis is gained from the different country reports published by the World Bank or the IMF which only requires some reanalysing work from Hungary's point of view and that of the Hungarian statistics. It must not be left out of consideration that sometimes the international bodies are rather bold, which is understandable from their points of view, but compiling an authoritative bilateral comparison requires not only more details but more control as well.

3. Special role of bilateral comparisons

Relying on my personal experiences of several decades and trying to take into consideration the new circumstances and expanding possibilities, it seems to me that today's bilateral comparisons are facing new prospects. The harmonized and detailed methods recommended by the EUROSTAT contribute to this by a great extent. The countries are interested in using these methods because in a lot of cases the financial interest of a given country is connected with the level of several indicators. The growing role of this kind of comparisons is influenced by the expansion of the economic globalization process which makes the earlier member countries of the EU competitors with the new ones, particularly in some special branches and territories.

We have to be aware of the fact that bilateral comparisons deserve special attention if they deal with some unique topics and branches. This is the case when we compare the level of productivity, that of the total factor productivity, to the unit-cost in some rapidly developing branches for example in high-tech oriented engineering. The case of agriculture along with food industry can also be mentioned as a sensitive territory of the EU. It is well known that on the world-market less developed countries are competitors in this field and any new member of the EU has to provide proof of their advantages.

This means that before becoming a fully qualified member of the EU several bilateral agricultural as well as food industrial comparisons could be of outstanding interest to the decision-makers and the experts preparing a long-run strategy for the agrarian-sector.

Another example could be taken from the topic of social statistics, like the quality of life of some special strata of the country. During the last few years, the situation of the poor, jobless people has been in the searchlight of analyses not only in Hungary and some other transition countries, but in some wealthier ones as well. The study of their problems has been connected to the level of unemployment, especially to the education and qualification structure of the unemployed. Several studies have dealt with the regional structure of poverty within the countries and also analysed the connections in relation to the number of children, the attitude of local governments etc.

It seems to me that two conclusions can be drawn from the fact mentioned above. First: there are enough country-based studies at the disposal of the interested statisticians and their linking-up does not need too much supplementary work. The second one is that relaying on the studies made in this topic, it is high time to prepare a more detailed bilateral comparison indicating the relation of cause and effect, as well as taking into consideration the special economic situations of the countries.

Another important topic – which to the best of my knowledge has not been studied deeply enough in any of the countries – is the analysis of the families whose living standard have been greatly influenced by their economic activities. This is not a homogeneous stratum, it consists of young managers, administrators, university professors etc. It can be seen that the first step, namely defining the stratum to be studied is not an easy task. No kind of international recommendation could be found in this field though the situation of this group of the population can determine the future state of the economy. It is beyond doubt that bilateral comparison of the position of these families could be interesting to other countries and statistical services. The above mentioned stratum was developed in the traditional market economies, but its progress needed a relatively long time. In the transition countries – to my opinion – we are at the beginning of this process and the way of development is painful for the strata which played a more important role in the economy several years ago (for example skilled workers, administrators with only special knowledge etc.). I am sure this can be an interesting topic in the near future. (I decided to analyse this process along with the mutual effects between the development and/or modifications of the economy and society, I have raised the topic to illustrate a brand new possibility for the international comparisons.)

The above mentioned possible topics for bilateral comparisons are far from completeness, my aim was only to present some special topics to illustrate the abundant choice of interesting and useful bilateral comparisons. Usefulness must be emphasized in choosing the suitable topics because in the past it was not unusual or exceptional that the personal interest of the statistical experts taking part in or initiating any kind of bilateral comparisons was the dominant factor.

In my opinion the whole statistical service including the experts in charge of making bilateral comparisons, has to be more user-friendly. This means that before selecting the topic of a not very cheap bilateral comparison, the experts (the leaders as well) of the statistical service must be well-informed not only about the most actual topics of the senior country managers, but also the topics on the agenda of the economy and society as a whole for the next 3-5 years.

4. The role of finding the most suitable partner-country

Selecting the topic could be the first step, the next and not the least negligible one is to find the country which is interested in the topic to be studied, the statistical service of which has suitable experiences and an international reputation in this area. It is also desirable that the level of development in the given country of the field to be compared should be higher than that of Hungary. To satisfy all the above mentioned requirements is not easy but could be solved because in most cases Hungarian statisticians have enough practice and could take control of the work.

It has to be taken into consideration that the statistical services of countries interested in investing in the Hungarian economy are more convincible of the necessity of a bilateral comparison than those of other ones.

Apart form the main purpose of this kind of bilateral comparison the possibility of getting acquainted with methods used by the partner-country can be of importance to both collaborators. The Canada–USA bilateral comparison based on PPP and PPP adjusted macroeconomic variables can be mentioned as a new and an excellent example. They have a long tradition including acceptable as well as inconvenient experiences in using PPP for multilateral comparisons. There was a need, however, by the statistical services of both countries to concentrate on a research-oriented bilateral programme for controlling the quality of the figures used earlier, for supplementing or replacing some of them if necessary. Several results arose from this bilateral comparison, among others an agreement on the list of indicators and their exact descriptions to be used; agreement on aggregation and on regional break-down etc. Most parts of these results could also be used by other interested countries.[3]

I have mentioned this example to illustrate the interest of highly developed countries in compiling bilateral comparisons dealing with well-known topics and their control. I hope that the former description could convince the reader of the necessity of jointly made bilateral comparisons, especially in topics which could be influenced by different factors in the interested countries.

I am sure that one of the elements of a bilateral comparison of highest value is presenting the opportunity of analysing the differences of the levels of the indicators studied. This needs not only extremely precise work and the knowledge of the topic, of the circumstances within the country, but it also requires the ability to get acquainted with some special up-to-date situations and the most important elements of the factors influencing the differences studied. The latter one is the most difficult one because it can differ from the given situation in the partner-country. There is no doubt that this kind of analysis could be made easier with a well structured model for both of the interested countries. I am sure, this has to be the way of the near future; experiments in this respect will have to

[3] See: *Ian Castles* (OECD): Review of the OECD–EUROSTAT PPP Program Paris OECD 1997, and the report presented to the 30th session of the UN Statistical Commission, March 1999.

be started during the next 2-3 years perhaps not only by the statistical services themselves but together with some research institutes, in the case of Hungary together with the ECOSTAT institute which deals with internal models nowadays as well.

5. Presentation of bilateral comparisons

After the exhausting work of producing the results of a bilateral comparison, the question which arises is how to publish them. The main purpose of the bilateral comparison is to give a detailed information to the decision-makers at different levels about the development of the Hungarian economy and/or society compared with a country with better results in the topic studied and to highlight the factors influencing the differences. This, however, is only one of the important aspects. It is the task of qualified statisticians to give basic data as well as analysis to researchers and under the new condition to describe the real situation to the interested population via the media, which involves helping them to understand some new features of the development, providing further convincing reasons to facilitate the adaptation of the managers of small and medium-size enterprises to the new macroeconomic environment. This task demands more attention from the statistical service to the form and easy understanding of the presentation.

It is clear that at our level of development the traditional publication with several graphic illustrations plays the leading role. It cannot be substituted only supplemented by CD-ROM, Internet and other new forms of publication. I am sure that this is a personal (perhaps too traditional) point of view of an old statistician, which could be disputed.

Dealing with the presentation of the bilateral comparisons it seems to be useful to find some international forms to make the results available for other countries as well. The most evident solution is to present the summarized form of the results at a meeting of the UN Statistical Commission or at one of the EUROSTAT meetings (high-level or expert-meetings) if the topic of the comparison is one of the agenda (or if it is possible to find a topic similar enough on the agenda). Other suitable forums could be the regular sessions of the International Statistical Institute or of the IAOS. These provide larger audiences, make the discussions of the results or the methodology used by the comparison possible.

The above mentioned kinds of forums belong to the ‘producer-side’, but if a broader achievement of the results is our target, the ‘user-side’ must also be aimed at if possible not only from within the participating countries of the comparison, but also from those parts of the world where similar problems could arise. This aim needs more effort to publish the results in several internationally well-known economic journals, or to find a possibility of making the results available by the help of some intermediate bodies (World Bank, IMF or other special UN institutions).

Publishing the results of a bilateral comparison has no purpose in itself. This can offer an opportunity to get reactions from responsible policy-makers, experts of the particular topic examined (not only from statistical experts) and nowadays these could also encourage the expansion of the bilateral work to a multilateral one.

Examining the necessity as well as the possibilities of making an up-to-date PR (Public Relations) work of the bilateral comparisons, it has to be added that the publication of the methods used (and sometimes different from the international ones) is suitable for the statistical experts, but only the most important factors of the methods should be made available for the users. This is very important when using models for the comparison, namely a long, detailed mathematical deduction can discourage the potential users.

6. Country comparisons connected with traditional international comparisons

The ideal case would be if the selected topic of the bilateral comparison were also studied by an international statistical body (OECD, EUSOSTAT, IMF or other). In this case the bilateral comparison could support or discuss statements of the international comparison. From the point of view of the decision-makers, this could be useful in offering more arguments for or against forecasting the reached results.

This target could be achieved if national statistical services were involved in the comparison-work of the international organizations. During the last few years particular emphasis has been placed on making joint studies as well as publications by different international organizations (IMF, OECD, World Bank, in some topics the Bank for International Settlements was included) but it could give a new and additional element for users if national statisticians were involved. At the same time it can help the participant statisticians in getting acquainted with some details of statistical analytical work made by the interested international organizations.

7. Some final remarks

Readers of this paper can easily recognize that the author has been in favour of making more bilateral comparisons keeping in mind their many-sided effects to the reputation and development of the Hungarian statistics. It has to be added that this is one of the best ways of teaching the next generation of statisticians, giving them not only tasks but the opportunity of ‘learning by making’ as well. This can lead in a short time to excellent results as well as to some fiascos, which are also factors of being educated experts of statistics in the near future.

REFLECTIONS ON PURCHASING POWER PARITIES AND REAL VALUES

GYÖRGY SZILÁGYI[1]

SUMMARY

The article investigates the nature of purchasing power parities and real values in the framework of multilateral international comparisons. It discusses the modification in the definition of these terms in the conditions of multilateral exercises. The relevant properties of these calculations, transitivity and base country invariance are also displayed.

On the basis of these properties, discussion focuses on the term 'numeraire'. Various versions of this 'artificial monetary unit' is presented, that differ in terms of two characteristics: to what extent exchange rates be used and which currency be chosen to derive the numeraire.

KEYWORDS: International comparisons; Purchasing power parities.

In this article special issues of international comparisons in terms of Purchasing Power Parities (PPP) are discussed. These comparisons are often labelled as ICP (International Comparison Project) or ECP (European Comparison Programme). Methods in general of those exercises do not belong to the scope of present investigations as they have been discussed to large extent in the reports of actual comparisons as well as in special studies. The focus of the analysis below is the economic and statistical interpretation of the results.

From among the rich set of numerical information provided by the comparison, two are being considered:

1. purchasing power parities of currencies,

2. real values, i.e. major economic magnitudes, as GDP, expressed in real terms (i.e. in a common currency).

Speaking in general terms, Purchasing Power Parities convert data of national currency into real value. In case of country J

$$NATIONAL\ VALUE\ (J) \times PPP = REAL\ VALUE\ (J) \qquad /1/$$

[1] Professor, doctor of economic sciences, president of the Hungarian Statistical Association.

Country-to-country (spatial) volume ratios of any two countries (*J* and *K*), referred to as volume indices, equal the ratio of the respective real values

$$\frac{REAL\ VALUE\ (J)}{REAL\ VALUE\ (K)} = VOLUME\ INDEX\ (J/K) \qquad /2/$$

In this paper, however, less attention is paid to volume indices, as their interpretation is not exposed to the ambiguities discussed in connection to PPP and real value.

1. Basic and modified interpretation

A generally accepted interpretation of Purchasing Power Parity is the following: a given sum of money, when converted into a different currency at PPP rate, will buy the same basket of commodities in one country as in another one. This simple and transparent interpretation, however, undergoes some modifications in multilateral comparison systems, especially in those carried out in the framework of international groupings (e.g. EU, OECD). In these systems a set of consistency criteria are set against multilateral aggregation schemes. Authors as well as actual exercises differ in number and composition of the desired properties, in the present context it is sufficient to mention two of them: transitivity and base country invariance.

Transitivity requires that each index, whether a price or quantity one, should be a number on a continuous scale, one that will meet comparisons between the indexes of members of any group of countries:

$$INDEX\ (J/K) = INDEX\ (J/L)\ /\ INDEX\ (K/L) \qquad /3/$$

Base country invariance means that it makes no difference for the volume ratios which country is chosen as the base. It makes the system *neutral* in the sense that none of the countries get any privilege in the calculations.

Transitivity requirement is met if the same price system is used all throughout the comparison. To satisfy base country invariance, it is necessary to introduce an international average price system. This (in most cases implicit) price system means that the weighting pattern of the volume indices corresponds to an average of all participating countries. In this way PPP is no longer a parity between two currencies, but parity between a currency and the international average.

This is the reason why the initial interpretation of PPP as given above shall be modified in the following way: when the expenditure of different countries is converted into a common currency by means of PPP, it is expressed in the same set of international prices.

There are a considerable number of aggregation schemes that meet the two criteria, transitivity and base country invariance. From among them *Geary-Khamis* (*GK*) and *Éltető-Köves-Szulc* (*EKS*) formulae are of the broadest (almost exclusive) use. Both have been discussed in comparison reports and other studies, it is therefore sufficient to give a short reminder here. The *GK* method consists of a linear equation system with *N*+*M* variables and equations, *N* being the number of items, *M* the number of participating countries. Its most relevant variable in the present context is $PPP_{GK}(J)$, the parity between national currency of country *J* and the international price system. *EKS* is a compound average parity $PPP_{EKS}(J/K)$, that makes the parity of two countries J and K depend – in addition to direct binary ratio – also on all indirect ratios through all participating countries ($J/L : K/L$, with L=1,...,M).

2. Numeraire

International average prices, discussed in the previous section, need a numeraire, i.e. an actually existing currency. Due to base country invariance the choice of the numeraire does not effect the relative position of the countries in terms of volume ratios, but PPPs, real values and their interpretation depend on the choice. Several variants of choice or creation of numeraire have been in use so far. They can be presented in terms of two characteristics.

– Do exchange rates have to do with the numeraire or not?
– Is the currency of a participating country used or not?

By the combination of these characteristics, three variants can be described.

1. No exchange rate – country numeraire. In this variant the numeraire country and base country are the same. This has been the case in 'European Comparison Programme – Group 2' with Austria as base country and the Schilling as numeraire. PPPs can be understood as real parities between national currencies and the numeraire (e.g. Bulgarian Leva to Austrian Schilling). But this Schilling constitutes the unit of an international price system expressed in Schilling.

A substantial property of this scheme is the following: the overall GDP of the base country is the same in national currency as in international price system. Let us label this property as 'Equality of Totals' (ET). It is valid for the overall figure, but not necessarily for the positions of the breakdown (consumption, capital formation etc.).[2]

Real values of participating countries – in accordance with formula /1/ – are expressed in the currency of the base country (in our example Schilling), but in terms of international price system. As in formula /2/ the ratio of these real values for any two countries produces volume index.

2. Exchange rate – international numeraire. In this scheme no base country is chosen. The function of a base country is transferred to the total and the average of the participating countries. Such a procedure fits to comparisons carried out in the framework of a community. It is, indeed, the European Union practice where this method is used.

Instead of a base country, the numeraire is a sort of combination of the currencies. In the case of EU this the ECU. Since however the purchasing power of this currency exists only as the combination of the purchasing power of other currencies, the only way to get an international numeraire is by the use of exchange rates. The ECU in real terms is called Purchasing Power Standards (PPS).

In other words, parities are expressed in an arbitrary unit, the Purchasing Power Standard. PPS is defined in such a way that for each individual aggregate the EU total (EU15)

[2] In the case of *EKS* yes, in the case of *GK* not.

– obtained from converting the values in national currencies with the PPP – is equal to the EU total for that individual aggregates in ECU.

Consequently, ET property is met at the level of countries' total and average, so that the EU15 figures are the same in terms of nominal values (national currencies converted by exchange rates) and in real values (converted by PPP).

$$EU15^{ECU} = EU15^{PPS}\text{ , but } COUNTRY\ J^{ECU} \neq COUNTRY\ J^{PPS} \qquad /4/$$

As an illustration, the following examples of the 1996 EU comparison demonstrate equalities and inequalities relating to per capita GDP:

EU15	18 113 ECU, (18 113 PPS)
Denmark	26 185 ECU, (21 342 PPS)
Spain	11 736 ECU, (14 068 PPS)

In this version the use of exchange rate seems to be unavoidable, nevertheless it does not fit the philosophy of the comparison project, the basic idea of which is getting rid of the exchange rates. (This drawback becomes more explicit in connection with the next variant.) Fortunately this use of exchange rates does not affect the comparison of volumes.

3. Exchange rate - country numeraire. The third type is a kind of a hybrid of the two variants discussed above. Similarly to the second one, no base country, rather the total and average of all the countries serve as basis. However, the currency of one of the countries is chosen as numeraire. This has been the procedure of the comparison carried out by OECD, with US dollar as numeraire.

The procedure starts by converting all price and expenditure data to US dollars in terms of exchange rates and further calculations are made with these converted figures. OECD total is obtained by the sum of exchange rate converted national data. ET property is valid at countries' total, as in the previous version, i.e. (not in terms of US and USD).

The dissociation of base and numeraire leads to a sort of ambiguity. There is a set of real values expressed in international prices, the numeraire of which is dollar, but the data differs from USD. It is rather hard to find a proper label for this peculiar international currency. It could be 'dollar', as its numeraire is USD. But the overall purchasing power of this 'dollar' differs from the purchasing power of the numeraire. It could be 'international currency', as it actually is, but such a label does not express the magnitude of the numeraire. 'International dollar' seems to be closer to the properties of this unit, but it is still misleading. My choice is therefore 'OECD dollar', as a working term and an analogy to PPS in the EU practice. (Despite the fact that it has never been called so by OECD.) So in the following formulae

– USDN shall indicate nominal values in dollars, calculated with the help of exchange rates (in the case of US the national currency),
– USDR shall mean real values in US dollars,
– OED shall label real values in OECD dollars as defined above.

$$\begin{aligned} &OECD28^{USDN} = OECD28^{OED} \neq OECD28^{USDR} \\ &USA^{USDN} = USA^{USDR} \neq USA^{OED} \\ &COUNTRY\ J^{USDN} \neq COUNTRY\ J^{USDR} \neq COUNTRY\ J^{OED} \end{aligned} \qquad /5/$$

Some illustrative examples taken from the 1996 OECD comparison:

	GDP per capita			Ratio (USDR/OED)
	USDN	USDR*	OED	
OECD28	21 903	19 900	21 903	0.907
USA	27 831	27 831	30 694	0.907
Denmark	33 229	23 100	25 521	0.907
Spain	14 892	15 300	16 822	0.907
Hungary	4 431	9 300	10 281	0.907

* Rounded to 100 $, except for the US.

So the drawback of this procedure is the difficulty in interpreting the ten percent difference between the purchasing power of the international currency (OED) and the purchasing power of the numeraire (both being 'dollar'). The ratio USD/International price is the same for each country, but different by headings for example:

	USDR/OED
GDP	0.907
Private final consumption	0.870
Government final consumption	1.135
Gross fixed capital formation	0.875

3. Concluding review of the variants

Three variants of numeraire, real value and purchasing power parity have been put forward. The following table provides an overview on their differences and similarities in terms of the following characteristics.

1. Actual comparison in 1996.
2. The numeraire.
3. The base country or group of countries, at the level of which the 'Equality of totals' property is met.

	Exchange rate *no*	Exchange rate *yes*
Country numeraire	1. European 'Group 2' 2. ATS 3. Austria	1. OECD 2. USD 3. OECD total (OECD28)
International numeraire		1. EU 2. ECU (PPS) 3. EU total (EU15)

The lower left box is empty. Symmetry would require filling it, but – for the time being – there is no exercise with international numeraire where an exchange rate is not used. Perhaps when EURO enters the comparison systems?

INCOME INEQUALITY AND POVERTY IN HUNGARY IN THE MID 1990S*

ÖDÖN ÉLTETŐ[1] – ÉVA HAVASI[2]

SUMMARY

The first part of the paper, on the basis of the 1996 Income Survey of the HCSO, presents the income distribution and income inequality in Hungary in the mid 1990s and makes comparisons with data referring to the before transition period. The second part of the study discusses poverty issues such as how to measure the poverty, the various possibilities of choosing the poverty line and its consequences on poverty measures and the composition of the poor. The paper ends with some concluding remarks.

KEYWORDS: Income distribution; Inequality; Concept of poverty; Poverty lines; Measure of poverty.

Since 1963 till 1988, every fifth year the Hungarian Central Statistical Office (HCSO) carried out a large scale – comprising 16-27 thousand households – income survey based on a probability sample of households (dwellings). In this period of 25 years the data of the income surveys reflected reliably the income situation of the Hungarian society, the income differences within it, the relative income position of various social strata and regions, as well as the level and underlying causes of the income inequality. Several factors contributed to the success of these surveys. Although answering the questions has always been voluntary, the response rate in this period was very high – about 95 percent –, thus nonresponse did not affect the reliability of the data to any perceptible extent. Furthermore, and perhaps this was the crucial point, in case of employed persons and members of agricultural and other co-operatives, data on their earnings and other incomes were available not only from the respondents, but also from their employers. As in this period the number of self-employed persons was still very low and earnings and incomes received from employers amounted to 63–87 percent of all incomes of active households, the reliability of earnings-data contributed to a great extent to the appropriate representativity of the surveys. Moreover, based on detailed and reliable macrostatistical data on the produce of plant cultivation and horticultural products as well as on live animals and animal products in household plots and small farms, it was

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[1] Retired deputy head of department of the HCSO.
[2] Sociologist of the HCSO.

possible to adjust incomes from agricultural activity at household level in such a way that expanded aggregates of products corresponded to the respective macrostatistics. Finally, it has to be mentioned as a further favourable factor that the general personal income tax (PIT) was introduced only in 1988, thus it did not yet have a negative effect on the data of the 1988 income survey with 1987 as reference year. The fact that the average disposable household income according to the 1988 survey was only 3.5 percent less than that shown by official national account computations characterises well the reliability of the survey results.

More detailed data and the evaluation of the results of the income surveys in this period can be found apart from the respective publications of the HCSO, in *Atkinson-Micklewright* (1992) and *Éltető* (1997), as well.

The situation in respect of an income survey in the transition period became by far less favourable. The introduction of the PIT itself made people much more suspicious to questions inquiring as to their incomes. Besides, there was a general fall in the rate of confidence towards official statistics contributing to considerably lower response rates in all kinds of household surveys. Employers are not obliged to inform the HCSO on the earnings of individual employees, thus income surveys can now rely on answers of the respondents only in respect of earnings, just as in case of any other types of incomes. Moreover, the fact that the number of employers and self-employed persons has increased outstandingly since 1990 and still keeps increasing also renders the execution of a good income survey more difficult, because it is a well-known experience in many countries that employers and self-employed persons tend to underreport their incomes substantially. (This holds by the way to their tax return, too, in Hungary.) Finally, the macrostatistical data necessary to carry out a household level adjustment of agricultural incomes are also greatly missing these years.

In 1996, however, a new, advantageous opportunity arose for the HCSO to carry out an income survey in connection with the 2 percent microcensus that year. Moreover, the data of this survey yielded a sound basis for researches dealing with poverty in Hungary in the mid 1990s.

In section 1 the characteristic features of this 1996 income survey are briefly discussed. Section 2 shows the relative income differences between various social groups and regions in the mid 1990s and compares them with those before the transition. Section 3 presents the very substantial changes in the income inequality during the eight year period in question. Sections 4, 5 and 6 deal with poverty issues. First, section 4 with the concept and measurement of poverty, then section 5 discusses the problem of choosing the poverty line and shows some of its consequences using Hungarian data. Characteristics of the poor from various aspects and the use of different poverty lines are discussed in section 6. Finally the paper ends with some concluding remarks.

1. Description of the 1996 income survey

In April 1996 the HCSO carried out a 2 percent microcensus. It was obligatory by law to answer the questions of this microcensus. Three 25 percent subsamples of the microcensus sample have been used for various accompanying voluntary surveys, one of which was the income survey.

The sample of the income survey comprised of nearly 4900 census enumeration districts (EDs) in more than 440 towns and villages. The number of the interviewed households was over 18 thousand. In case of 16.5 percent of the households the interview was unsuccessful due to total or partial refusal or other reasons. However, even for these households the data of the microcensus were available, i.e. we disposed of data like the size and composition of the household, age, sex, educational attainment and economic activity of their members, occupation, economic branch of the earners, etc. In the course of the data processing, these available personal and household data were used to impute income to nonresponding persons by selecting randomly a person with identical or very similar characteristics from among the respondents and imputing all income items of this person to the corresponding nonrespondent. Due to the rather large sample we could match each nonrespondent person with a similar respondent one, this way the biasing effect of nonresponse could be eliminated and the average per capita disposable income in the sample increased by more than 25 percent.

However, imputation itself does not solve the problem caused by not reporting or underreporting incomes on behalf of many respondents. Even after imputation the income level in the sample was considerably below that shown in macrostatistics. Naturally, micro and macro income averages can not be expected to match, not only because of sampling errors, but also due to differences in content and coverage. Still, it is indispensable to strive for an agreement as close as possible. Therefore, after imputing, necessary corrections were made on the data of the 1996 income survey in respect to those income items for which reliable external information were available. Such external information sources were:

– a large scale (of almost 100 thousand element) sample of anonymized 1995 tax returns obtained from the tax authorities;
– certain estimates of national accounts;
– social security reports on various social incomes;
– a thorough study on types and size of incomes from the underground economy;
– the household budget survey in respect of consumption from own production (a significant income source of Hungarian households, especially in villages).

Not all types of incomes were corrected either due to the lack of reliable external information or because in some cases (e.g. pensions, family allowances, etc.) the agreement with macrostatistical data was quite acceptable.

The correction procedures generally involved allocating a certain amount of income to randomly selected persons with given characteristics or increasing some of their incomes by a given amount or percentage and then determining the disposable income (net of income taxes and social security contributions) of the person applying the rules of the PIT act valid for 1995.

After carrying out the corrections the average disposable income in the sample increased by nearly 20 percent, thus providing a much better estimate of the respective macrostatistics.

We firmly believe that after the imputation and the correction, the data of the 1996 income survey represent the income situation of the Hungarian households, the existing income differences in 1995 appropriately thus making the comparison between the data of the 1988 and 1996 income surveys feasible.

2. Changes in relative income differences between population groups and regions in the period 1987–1995

Considerable structural changes took place in the Hungarian society in the period investigated. While in 1987 three-quarters of the population lived in households where the head was an active earner and only one quarter in households with an inactive earner or a dependant head, in 1995 the latter proportion increased to 43 percent and among them almost 8 percent lived in households with an unemployed head. Unemployment – at least officially – did not practically exist before the transition in Hungary, like in other former socialist countries. In the mid 1990s, on the other hand, more than 10 percent of potential earners were unemployed. Unemployment in Hungary has the peculiar feature of being higher among men than among women. As a consequence, the proportion of unemployed heads among all household heads – disregarding the pensioners – is greater than the unemployment rate it is almost 12 percent because household heads are in the great majority men.

Another significant change regarding the structure of the society is that the proportion of households with manual workers as heads has sunk from 45 percent to less than 30 percent, while there was only 1 percentage point decrease in the proportion of households of non-manual workers.

The changes in the structure of the society are also reflected in the composition of incomes according to major sources. The share of social benefits in cash increased from 23.5 percent to more than 29 percent, and among them that of pensions from 16 to 19.5 percent. Incomes from entrepreneurial and self-employed activity more than doubled in this eight-year period amounting to nearly 13 percent of the total gross income in 1995, while the share of incomes from agriculture decreased from 17 to 8 percent. It is to be noted that the percentages in 1987 refer to shares in net incomes, because no PIT existed yet in that year, while those in 1995 to shares in gross income.

According to a general agreement in the first half of the 1990s, income differences in Hungary should have been increased considerably as compared to what was typical of the before transition period, but owing to the lack of reliable statistical data, opinions as to the extent of the increase varied significantly. The first reliable source of information in this respect was the 1996 income survey of the HCSO. Income differences can be investigated from two aspects: what relative income differences exist between the income averages of various social or demographic groups of households or between different regions of the country and what the level of the inequality of the income distribution within the total population is. In this section the first aspect is discussed, while the second aspect will be investigated in the next section.

Restricting our investigations to groupings with more or less identical contents in the two respective income surveys, it is obvious from Table 1 that group averages except one case dispersed to a considerably greater extent in 1995 than they did in 1987.

Not only the coefficients of variation of group averages increased considerably, but also the differences between group extremes. The per capita disposable income of households of leaders and directors exceeded that of unskilled workers by 75 percent in 1987, while in 1995 by 114 percent, and the proportions are similar – though a little bit less stressed – in equalized[3] income as well. The per capita and equalized, respectively income of active households without any dependant child amounted to 209 and 160, percent of that of households with three or more dependant children in 1987 and these differences increased to 222 and 175 percent, respectively for 1995. In 1987 the equalized disposable income of heads of households with university or college qualification exceeded those with at most elementary educational attainment by 51 percent whereas in 1995 by 87 percent. The average income in the poorest county was 86 percent as compared to the richest county in 1987, while only 78 percent in 1995.

Table 1

Coefficients of variation of group averages of incomes in percent in 1987 and 1995

Basis of grouping	Per capita disposable income 1987	Per capita disposable income 1995	Equalized disposable income 1987	Equalized disposable income 1995
Type of activity of the head of household	12.7	20.6	12.1	19.9
Educational attainment of the head of household	12.4	24.5	12.4	23.0
Age of the head of household within active and inactive households	15.7	18.9	7.1	13.0
Number of dependant children under 20 in active households	22.2	24.9	11.9	7.9
Size of active households	18.4	25.5	3.4	5.1
Counties	8.7	13.5	6.6	10.8
Regions	6.2	11.0	4.4	9.1
Per capita income deciles	45.0	58.3	34.9	52.2

Data source here and in subsequent tables: Publications of the HCSO on the results of the 1988 and 1996 income surveys.

These data corroborate the assertion that relative income differences between population groups considerably increased from 1987 to 1995. As an illustration, Table 2 shows the relative income differences and some household characteristics for households of different size within active households in the two years, investigated.

As it can be seen from the data, not only the relative income differences increased, but also the composition of households changed considerably in the sense that there was a 12 percent decline in the number of active earners and a more than 50 percent increase in the number of inactive earners living in households headed by active earners. The relatively high unemployment rate – an entirely new pheomenon of the transition period – contributed significantly to both changes. As a consequence, while in 1987, 100 active earners had to provide for 63 dependants in their households, this number increased to 68 in 1995.

The worsening of the earner–dependant ratio can be observed in all household sizes except of course in single person households as well as in household groups of other types of classification.

[3] Equalized disposable income of a household is defined as the household income divided by the amount of units attached to the household by the equivalance scale used.

Table 2

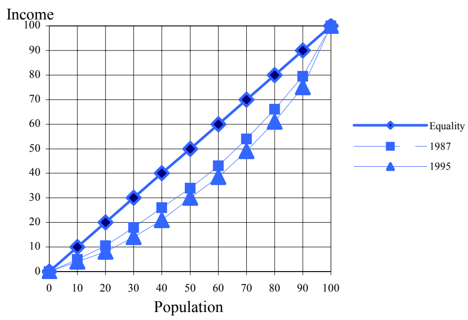
Disposable income and some household characteristics
by size of households of active earners in 1987 and 1995

Size of households (persons)	Distribution of		Per capita	Equalized	Number of		
					active	inactive	dependants
	households	persons	disposable income as per-cent of the average		earners		
					per 100 households		
				1987			
1	7.3	2.2	155.5	120.7	100	0	0
2	19.8	11.8	132.1	116.6	156	23	21
3	25.3	22.7	110.3	106.3	191	25	84
4	32.8	39.2	93.9	97.3	206	25	169
5	10.5	15.7	83.9	89.3	222	53	225
6 or more	4.3	8.4	71.1	78.0	248	94	304
All active households	100.0	100.0	100.0	100.0	188	28	119
				1995			
1	12.8	4.0	186.0	147.0	100	0	0
2	17.8	11.0	136.7	124.3	145	28	27
3	24.9	23.3	108.1	105.1	169	41	90
4	31.0	38.7	90.4	93.2	187	42	171
5	9.6	14.9	80.9	86.4	185	91	224
6 or more	3.9	8.1	65.2	71.8	211	151	294
All active households	100.0	100.0	100.0	100.0	165	43	113

3. The inequality of income distributions in 1987 and 1995

An increase in the relative differences of group averages already indicates a greater dispersion of incomes, but total income inequality may increase to a larger or smaller extent depending on the behaviour of inequality within the groups. Several studies tried to estimate the income disparities in Hungary in the transition period based on the data of either the regular Household Budget Survey of the HCSO or the Hungarian Household Panel Survey (see e.g. *Katuman* and *Redmond*, 1997).

Figure 1. Lorenz curves for distributions
of per capita disposable income, in 1987 and 1995
(percent)



However, these surveys did not intend to estimate the cross-sectional income distribution and for various reasons their income data are not reliable enough for that purpose. As explained earlier in Section 1., only the 1996 income survey could be considered to yield reliable data on the income distributions in 1995 and thus make it possible to compare the income inequality before and after the transition.

As expected, a significant increase in income differentials took place in the period investigated. This can clearly be seen in Figure 1 depicting the respective Lorenz curves of the per capita disposable income distributions of persons in 1987 and 1995. It is to be noted that the distribution of equalized incomes or that referring to larger population groups changed in a similar degree. This is evident from the figures of Table 3 showing the shares of quintiles (deciles) of persons in total disposable income and from Table 4 containing the values of various inequality measures.

Table 3

Shares of quintiles and extreme deciles of persons ranked by the size of per capita and equalized, disposable income in 1987 and 1995

Type of household year	1st decile	1st quintile	2nd quintile	3rd quintile	4th quintile	5th quintile	10th decile
				Per capita disposable income			
Active households							
1987	4.4	10.3	14.4	17.7	22.2	35.3	21.3
1995	3.4	8.3	12.7	16.4	21.5	41.1	26.5
Inactive households							
1987	4.8	11.1	15.2	18.4	22.5	32.9	19.3
1995	3.3	8.6	14.5	18.9	23.1	34.9	20.9
All households							
1987	4.5	10.5	14.6	17.9	22.2	34.8	20.9
1985	3.3	8.3	13.4	17.3	21.9	39.1	25.0
				Equalized disposable income			
Active households							
1987	4.5	10.6	14.6	17.9	22.2	34.8	21.0
1995	3.4	8.4	12.7	16.5	21.5	40.9	26.3
Inactive households							
1987	4.9	11.2	15.2	18.5	22.5	32.7	19.2
1995	3.4	8.7	14.6	19.0	24.0	34.7	20.7
All households							
1987	4.6	10.7	14.7	17.9	22.2	34.4	20.7
1995	3.4	8.5	13.5	17.3	21.8	38.9	24.8

As it can clearly be seen from the figures above, the shares of the top deciles and quintiles, resp., increased from 1987 to 1995 and that of the bottom deciles and quintiles, declined. It is worth mentioning that both types of income are significantly more evenly distributed among households of inactive earners than among active or all households in both years. It is interesting to note, furthermore, that quintile distributions and the share of extreme deciles indicate only a slightly smaller inequality of equalized incomes than that of per capita incomes among all household groups and in both years.

To characterize the inequality of an income distribution, the value of six inequality measures were calculated though, unfortunately, in some cases the necessary data were not available. These inequality indicators are the following:

G – Gini coefficient
T – Theil measure of inequality
HIM – Hungarian Inequality Measure which is the ratio of the average income of those above the mean income to that of those below the mean (see Éltető–Frigyes, 1968)
E – maximum equalization percentage or called alternitavely as Robin-Hood index, i.e. the sum of percentages above 10 percent of deciles with shares exceeding 10 percent
*D*10/*D*1 – ratio of the mean income in the top decile to that in the bottom decile
*Q*5/*Q*1 – ratio of the mean income in the 5th quintile to that in the first quintile

Table 4

Inequality measures in 1987 and 1995

Type of household year	*G*	*T*	*HIM*	*E*	*D*10/*D*1	*Q*5/*Q*1
	Per capita disposable income					
Active households						
1987	0.244	0.1042	2.01	17.4	4.80	3.37
1995	0.315	0.1870	2.38	22.6	7.79	4.95
Inactive households						
1987	0.213	0.0934	1.85	15.2	3.86	2.89
1995	0.257	0.1676	....	18.9	6.33	4.06
All households						
1987	0.236	0.1029	1.99	17.0	4.68	3.32
1985	0.296	0.1768	2.36	21.0	7.54	4.69
	Equalized income					
Active households						
1987	0.235	.	1.98	17.0	4.67	3.28
1995	0.312	.	.	22.5	7.67	4.87
Inactive households						
1987	0.210	.	1.84	15.2	3.92	2.92
1995	0.253	.	.	17.7	6.03	3.56
All households						
1987	0.231	0.0870	1.95	16.6	4.50	3.21
1995	0.292	0.1701	2.31	20.5	7.34	4.59

The data of Table 4 support what was said relating to Table 3. Moreover, the various inequality indicators differ in the degree of increase in inequality. It seems that the Gini coefficient, *HIM* and *E* are not very sensitive, they indicate only a 20-30 percent increase between 1987 and 1995. According to the Theil measure, on the other hand, the distribution of per capita disposable incomes in 1995 was more unequal than in 1987 by 70-80 percent. Finally, the ratios of incomes in extreme deciles and quintiles, resp., show a 40-60 percent increase in income inequality. To sum it up, we do not seem to be far from the truth by saying that the income inequality in 1995 was about 50 percent higher than in

1987 which is a very remarkable increase even if taking into account that before the transition incomes dispersed rather moderately in Hungary (though less moderately than in the 1970s or in the early 1980s).

In many studies on income inequality, the distribution of household income is investigated. We do not consider the household income an appropriate indicator of either the welfare level of a household or of its poverty level. Nevertheless, there are a few data available also on the distribution of household income in both periods, especially among households of economically active heads. In their cases the Robin Hood index increased from 18,5 in 1987 to 22,9 in 1995 and the ratio of the mean income in the top decile to that in the bottom decile increased by more than 50 percent. The inequality of household income of inactive households, on the other hand, decreased somewhat as it was rather high already before the transition. This can be attributed mostly to the fact that in the late 80s, 95 percent of inactive households consisted of single persons or two persons and the income of a single pensioner – they get a pension mostly on a widow's right – was only a small portion of that of a household where both husband and wife were pensioners on their own right.

Naturally, the question which arises is what is behind the increase in inequality. Although this issue needs further researches, a few of the underlying causes are rather evident. First of all it is important to point out that at least till 1995, the increase in income inequality was not brought about by greater earning differentials. As it is shown by Éltető (1996/a and 1996/b), the inequality of net earnings of employees hardly increased in the first half of the 1990s. This may perhaps sound surprising, but may be explained primarily by the fact that the majority of those who became unemployed in this period were low earning unskilled workers and this may have levelled off the impact of a significant increase in earnings of top managers and high ranked officials mainly in the sphere of financial services.

The rather high unemployment rate – 11 percent in 1995 – must have contributed markedly to higher income inequality, because mostly not well off families were affected by it and thus many poor families became even poorer. The increase in social benefits in cash did not follow the rate of inflation and this again afflicted mostly pensioners and households with several children the majority of whom had already been in the lower deciles. The spectacular growth of the private sector and the number of entrepreneurs and self-employed persons affected, on the other hand, mostly the upper tail of the income distribution giving rise to a considerable increase in the number of really well off households.

To be able to draw more exact and well-founded conclusions about the causes of the higher inequality of incomes in 1995 than before the transition requires further and more detailed investigation.

4. The concept and measurement of poverty[4]

There are two important questions emerging at this point, namely who are the poor and how to measure poverty.

[4] In this chapter we used the internal working papers prepared on this topic at the Household, Income and Consumption Statistical Division of the HCSO.

What is poverty?

Various authors use the concept of poverty in many ways. However, not always in an explicit way, ideologies, political and ethical standpoints and commitments are reflected in various definitions. The concept of poverty is strongly influenced by the given age and the given social and economic situation. There is no never-changing and absolutely true definition. Many kinds of correct definitions exist, and which one is used is always influenced, in addition to ideology, by the given situation and the goal of cognition.

In the initial stage of poverty examinations some kind absolute income threshold was generally defined, with an underlying recognition of certain general human – biological, physiological and social – needs.

Many people describe poverty as a certain degree of and as different types of inequality. That is the poor have unfairly less of something that is available to the majority. The representatives of the deprivation theories belong also to this school, including the multi-dimension objective-relative deprivation theory of Peter Townsend.

The structural school of Titmuss also grasps poverty in the framework of social inequality, but in a wider sense, in its complexity. The main thought of the school is that the reason of poverty should not be sought for in the individual but in the society. In their book 'The future of inequality' Miller and Roby wrote the following: 'Poverty is not economic scarcity, but social and political exclusion as well. We believe that in any society where inequalities are significant, the minimum program of the government should not only include an increase of the minimum level of income, assets and fundamental services, but this has to be set as an objective also in the area of self-esteem, chances of education and social mobility as well as the participation in the various forms of decision-making.'

The concept of exclusion lays emphasis on the social approach of poverty, and it explains poverty with the absence of social identity and the possibility for social integration. Robert Castel writes the following: 'There is of course family integration. There is also school, professional, social, political, cultural and many other kinds of integration... But work is a catalyst that imbues all these spheres, work is a principle, paradigm, something that can be found in all integrations mentioned and concerned, and which thus enables the integration of the various integrations without eliminating differences or conflicts.'(Castel, 1996) After these various possible approaches of scientific value, let us look at the definition of poverty adopted by the Europe Council.

By virtue of the decision of the Europe Council dated 19 November 1984 'a person, family or group of people must be considered poor in case the (financial, cultural and social) resources available to them are restricted for the extent that they exclude them from the minimally required way of life in the country where they live'.

This definition includes several important thoughts. One of them is that someone can be poor not only on an individual or family basis, but also on the basis of belonging to a group. Thus someone can be poor not only as the result of a given lifecycle, but also as being a member of an excluded group of the society. The other essential element of the definition is that poverty is not simply and necessarily of financial nature. The criterion of poverty whereby it does not make a 'minimally required way of life' possible in a given society should also be emphasized. That is, poverty is relative. Someone who is poor in a given country is not necessarily one in another country. The definition also includes the concept of exclusion.

The separation of poverty on the basis of exclusion from the minimally required way of life makes it also necessary to define the financial and non-financial factors which are clear, unambiguous and measurable. When we select the different non-financial indicators in the measurement of poverty, we need to choose from an endless sea of assets and services. In general, the indicators of housing and the equipment of the household, as well as the access to education and health care are taken into account, but the lists of international organisations vary in this respect and are immature in many cases.

The measurement of poverty means also that the poor must be identified in a given society so that they can be distinguished from other groups. Furthermore, an indicator must be constructed which can be used to do this.

The difficulty starts when surveys concerning the entire population often do not include the people and groups of people who are certainly poor. Even obligatory censuses and micro-censuses struggle with such problems. This is particularly true for surveys based on voluntary participation, which are occasionally time-consuming and require regularity, and an ability and willingness to co-operate. Of these, from the aspect of poverty analyses, household budget surveys are the most important, which contain detailed expenditure and consumption data in addition to the income data, and at the same time they also have data concerning the housing condition and equipment of households. We also used some supplementary information from the 1997 household budget survey in order to grasp the idea of poverty more precisely. The main source of this study, however, was the 1996 income survey that supplemented the micro-census, thus not only our income data could be more precise, but we also had the possibility to use the background information of the microcensus.

The information base providing the maximum of possibilities, such as the two large household surveys of the HCSO are far from the optimum from the aspect of the representative description of poverty. Those who are extremely poor and excluded and have no housing – the homeless – are absolutely excluded from the scope of household surveys. As no targeted, representative research concerning the homeless has been carried out so far, we do not have any exact or statistically manageable information. We have only extreme estimates differing in order of magnitude.

Thus the study relates only to the poor who could be included in the household surveys, although occasionally under-represented.

The other major restriction of the analysis is that we could only consider, at least in a ‘direct’ manner, poverty indicators and specifics characterising poverty if we had statistical data. For instance, the gypsies who represent the group most endangered by poverty, appear as poor only indirectly in the statistical surveys, but in the absence of data, they cannot be quantified and analysed directly as a socially segregated poverty group.

After acknowledging all these restrictions, we need to make a decision on the measure of poverty, we need to set the demarcation line between the poor and non-poor, and we need to decide on the unit of analysis and our relation to the equivalence scales.

Choosing the measure of poverty

We have already referred to the fact that choosing the measure is not an easy task, as poverty is a multi-dimensional, social phenomenon which, in addition to the absence of financial means, also includes the exclusion from non-financial assets. Although the concept of poverty has changed much since Rowntree till today, its measurement method has remained essentially the same. 'On the theoretical side, the deprivation concept of poverty has been elaborated as a supplement and alternative to the traditional concept of minimum subsistence level, the poverty line based on income has remained the most important tool for the relative measurement of poverty, irrespective of the concept used. The only difference between the various measurement methods is simply the level where the poverty line is set.'

Despite the fact that the notion of poverty is approached in many different ways, with the exception of a few attempts, income is still used, directly or indirectly, as its measure.

Although everyone is aware of the fact that the measure, the measuring indicator must be constructed for the concept intended to be measured, it is extremely difficult to put it in practice. Income is a sensitive area all over the world. People do not like to speak about their incomes, they do not often intend to state their income-position honestly. Even if the income is correct, with its use we can underestimate or even overestimate poverty. 'Income is useful inasmuch as consumer goods can be purchased for it on the market. However, what we purchase does not depend exclusively on our income, but also on other factors that affect our use of income.' In different periods of life with different cultural backgrounds, under different market conditions and wider circumstances, different qualities of life or 'welfare level' can be achieved with a given income level. In addition to a current income, wealth condition is also a very essential element that can result in completely different chances of life even in the case of the same income, just as in the case of health status, the position on the labor market, working conditions and social integration which are all factors related to the income level.

Many consider the indicator of consumption, that is expenditure as an efficient measure for avoiding some of these problems. Without wanting to enumerate a series of counter-arguments against this measure, we only want to note that consumption resolves only seemingly the difficulties mentioned in the use of income, moreover it adds new ones to them. Such are cultural difficulties and those resulting from lifecycle or local purchase possibilities, appearing in consumption habits.

We can analyse people's incomes in several ways. We can start from the personal incomes of individuals, the total income of families and households or also from the per capita income or the income per consumption unit of families.

The use of per capita income is still generally accepted in Hungarian practice but we have made calculations using equivalence scales as well.

5. Poverty lines used for measuring poverty in Hungary

We have used objective and subjective as well as official poverty lines for the assessment of the size of poverty and the characteristics of the poor. We have also considered it important to present their relationships.

Minimum subsistence level is the domestic equivalent of absolute poverty line. 'Minimum subsistence level is a forint amount that ensures the fulfillment of very modest – conventionally qualified fundamental – needs related to a continuous conduct of life.'[5]

[5] Létminimum, 1997. *Központi Statisztikai Hivatal*, Budapest. 1998.

It is determined in two steps. After determining the norms of food consumption, they are used for calculating the normative value of total personal expenditure, that is the minimum subsistence level.

For the norms of food consumption we have used a food basket compiled on the basis of the nutriment recommendations of the National Institute of Nutrition and Dietetics, which covers the physiological nutrition needs of an adult in active age. The normative food requirements of household members of different age and economic activity are calculated using defined 'keys' (in fact equivalence scales). The value of minimum subsistence level is the average consumption expenditure of those households of the survey use, in case of which the actual food consumption corresponds mostly or completely to the calculated norm. As a consequence of its nature, the minimum subsistence level can be interpreted primarily for consumption units, but it can also be calculated for one person. Its monthly value for a consumption unit was HUF16 435 in 1995, HUF19 425 in 1996 and HUF23 709 in 1997. The national average per person was HUF12 906 in 1995, HUF15 172 in 1996 and HUF18 574 in 1997. Naturally, as a result of the logic of the minimum subsistence level, it can be interpreted also for different types of households and consumption units, and the HCSO publishes these values in its annual publications Létminimum (Minimum Subsistence Level). Compared to the food norm the minimum subsistence value was 2.3 to 2.4-fold in the second half of the nineties.

The measuring of minimum subsistence level in Hungary is very similar to the calculation method used in the United States which can be considered a 'relative absolute' poverty line based on its calculation method. Considered either for a consumption unit or for one person, the value of minimum subsistence level represents an increasing proportion of average income compared to 1987.

From the *relative poverty lines* we used two indicators, one being the upper limit of the per capita income of those belonging to the 5 percent income quantile and the other to the bottom income decile. We get these values by separating the bottom 5 or 10 percent of the total population after having ranked them on the basis of their per capita net income, and we determine the poverty line by the maximum income of those belonging there.

The 5 percent of the population with the lowest income is particularly interesting, because as the income limit is extremely low, it does not achieve half of the minimum subsistence level. The income of those belonging to the lowest income decile is closer to the minimum subsistence level, but its value still falls short of it.

For the definition of the poor many, particularly in international comparisons, use *half of the median income*. In the 1996 income survey this value coincided with the maximum of the bottom income decile, so this was the value we considered the relative poverty line. So the relative poverty line was not only the upper limit of the bottom income decile, but at the same time also the half of the median income.

In addition to relative poverty lines, the 'official' poverty line is also decisive and has practical importance, as this is the income level that political decision-makers accept as the minimal necessary income. This provides the threshold for the eligibility of social allowances. Today in Hungary this threshold although not qualified, but at least as regards its function considered more or less official, is the amount of *minimum own right old-age pension*. The eligibility limit for social allowances is somewhere between the minimum subsistence level and the relative poverty line.

From the household budget surveys we have also information on the amount that, in the opinion of the population, households must have in order to maintain a continuous conduct of life on a very poor level. Both in 1995 and 1997 the subjective poverty level was around 65 percent of the overall average of per capita net income.

Table 5

The poverty lines examined expressed in HUF and as a percentage of the per capita disposable income in 1995 and 1997

Year	Households below			
	the upper limit of the bottom income		the minimum old-age pension	the subjective poverty line
	5 percent quantile	decile		
	HUF/year			
in 1995	75 164	93 147	100 800	139 883
in 1995 at 1997 prices	109 890	136 181	147 370	204 509
in 1997	93 045	120 904	138 000	199 776
	Percent of per capita disposable income			
in 1995	34.8	43.2	46.7	64.8
in 1997	30.1	39.1	44.6	64.6

Finding out whether people consider themselves or their family poor is also a very essential aspect of *subjective poverty examinations*. We will discuss in detail this later, here we would only note that in 1997, 12.5 percent of the households considered themselves ‘very poor’ and a further 34 percent ‘poor’.[6]

The size and intensity of poverty in Hungary using different poverty lines

The lower we set the poverty line, logically the fewer poor people we find. The order, valid for the circumstances in Hungary today – the lower 5 percent, decile, quintile, minimum old-age pension, subjective poverty line and minimum subsistence – is not a necessary hierarchy of the poverty lines regardless of place and time. On a higher level of welfare, the official level may coincide with or even exceed the absolute poverty level (calculated minimum subsistence). However, Hungarian realities show something different.

The subjective poverty level is a function of several factors; apart from the actual income, it is strongly influenced by the picture formed by people about income inequalities, the past situation and the future prospects of individuals and families.

Besides the several advantages of relative poverty lines, their huge disadvantage lies in the fact that the sphere of the poor is determined by their actual income conditions and income distribution. Thus, the increase in poverty as a consequence of the general deterioration of the living standard cannot be followed. While the relative poverty line has been calculated with a constant poverty rate on the basis of the absolute poverty line – the minimum subsistence – the number and proportion of the poor have increased.

[6] The data originate from the 1997 household budget survey. In: Családi költségvetés, 1997. KSH. Budapest. 1998.

The relative poverty gap measures the intensity of poverty, its value is between 0 and 1 (or expressed as a percentage between 0 and 100). The further the value of the indicator from 0 is, the deeper the poverty within the examined group is. Consequently, the higher we set the poverty line, the greater the average distance of the poor in the group from the line is, which is also the poverty gap. Even in case of a poverty level set at the lowest level, the relative distance of the income of the poor from the line exceeded 20 percent.

Table 6

The proportion of households and persons living below the poverty lines examined, the size of the poverty gap and the relative poverty gap in 1995

Denomination	Households below			
	the upper limit of the bottom income		the minimum old age pension	the subjective poverty line
	5 percent quantile	decile		
Proportion of people (percent)	5.0	10.0	12.1	27.7
Proportion of households (percent)	3.4	7.1	8.7	20.9
Poverty gap (HUF)	16 561	21 681	24 799	38 493
Relative poverty gap (percent)	22.0	23.3	24.6	27.5

6. Characteristics of the poor in case of the different poverty lines

In order to get closer to the problem of poverty, we need to get a picture of who the group of the poor consists of. Another important question to be answered is how the group of the poor changes, how its composition modifies when different poverty lines are used.

The income structure of the poor

The lower we set the poverty line, the smaller the share of income from work in the income of the households is and the greater the role of social allowances is. While in 1995 the share of cash social allowances was in average 29.3 percent in the per capita gross income, in case of the households belonging to the lowest income 5 percent, this proportion was 63 percent. The income data estimated for 1997 show the same trend.

From within the cash social allowances, primarily the shares of changes in the family allowance, unemployment benefit and the income supplementing allowance of the unemployed, the child care fee, child care allowance and child care benefit as well as pensions, reveal a lot about the living conditions of the households.

While in the bottom 5 percent income, the family allowance is a quarter of the total income, at the poverty level, set at the subjective poverty line, its share is only 12 percent and in the latter case the main source of income – within the sphere of social incomes – is pension. In 1995, in the bottom 5 percent unemployment benefits represented 13, while choosing the subjective poverty line, ‘only’ 7 percent. The share of maternity benefits reduced from 10 to 6 percent.

The examination of the income of the poor by sources allows us to conclude that the majority of those living in deep poverty are families with children where many of the adults do not have any earnings as they are unemployed, or mothers with small children getting their child care allowances, or older single pensioners.

Composition of households below the different poverty lines

More than one third of the household members are active earners in Hungary today. As regards to those living below the different poverty lines, the lower we put the line, the less the proportion of active earners is. The proportion of active earners in the households belonging to the bottom decile is approximately half of the national average. In 55 percent of the households in the bottom income 5 percent there was no earner at all. This ratio was 48 percent in the bottom income decile and 41 percent in case of the households living below the subjective poverty line. Compared to 1995, there has been no positive shift in the past two years.

With the economic stabilization, those living in poor households could get back to the labour market to a lesser extent than those living in average households. The proportion of the unemployed was twice their own weight in the bottom income decile in 1995 and two and a half times that in 1997.

The fact that the proportion of non learning dependant children over the age of 15 in the bottom income decile was four times the national average in 1995, just as in 1997 according to our estimates clearly shows that we inherited a bad labour market position.

Among the poor there are many unemployed and people of working age with no earnings; and at the same time, the proportion of children who do not finish primary school in due time is also greater among them, the proportion of primary school pupils over the age of 15 is four times that of the national average. This also calls our attention to the risk of inheriting an unfavourable situation.

The number and proportion of pensioners among the poor is far from what would be justified by their weight. In spite of this, as we will see later, those who still fall below the poverty limit are very poor.

The size of poor households is higher than the national average. This does not result from the greater number of families including several generations, but obviously from the higher number of children. According to the national average, 4.4 percent of households have at least three dependant children below the age of 20, at the same time their proportion was 26 percent in the bottom income 5 percent, 23 percent in the bottom income decile, 22 percent among those living below the minimum old age pension in 1995, and according to our estimates these figures became even higher in 1997.

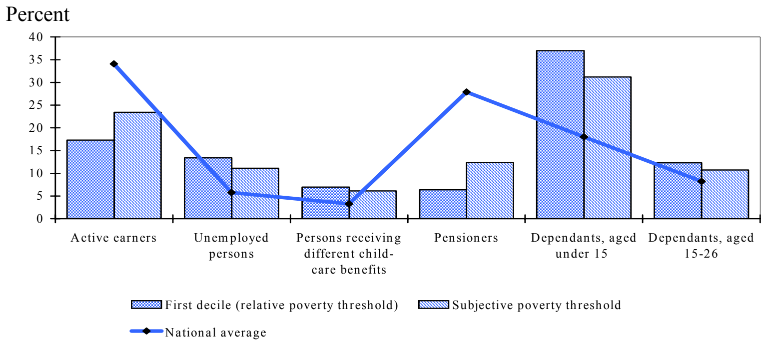
In 1997, on the basis of the estimated income data, the risk[7] for households with 3 and more children of falling below the lowest poverty line was nearly five times, but even the probability of falling below the bottom income decile was nearly four times the national average.

The poverty risk factor is even greater in the case where the head of the household is unemployed. If there is an unemployed in the household, but it is not the head, then – choosing any poverty line – the probability of falling below the poverty line is one and a half times the national average. If the head of the household is unemployed, the risk of

[7] For the concept and measurement of risk see *Teekens* (1996) and *Hajdu* (1997).

poverty was fourfold for the bottom income 5 percent, 3.4 fold for the bottom income decile, 3.2 fold for the level of minimum old age pension, and 2.4 fold for the subjective poverty line in 1995, and according to our estimates these values were even higher in 1997.

Figure 2. The proportions of household members of different economic activity in the groups of households belonging to the bottom income decile and falling below the subjective poverty line in 1995



The role of low qualification and type of activity of the head of the household in respect to the poverty of the household

As we have already referred to it earlier, the qualifications of the household members and particularly that of the head of household play a role in becoming and remaining poor. The impact made by low qualification on the income situation and poverty affects primarily the households of heads in economically active age, but even if we consider all households, the low qualification of the head of the household increases the risk of getting into the poorest income 5 percent one and a half times.

While at the national level in nearly half of the households the highest qualification of the head of the household is 8 grades or less, among active households their proportion is 21 percent. For households with active heads having low qualifications the probability of getting into the bottom income 5 percent or the bottom income decile is one and a half times the average of all active households.

In active households, the role of the type of activity of the head of the household – as regards the risk of the household getting below the poverty line – is similar to that of the qualification level. The lower the position of the head of the household is, the greater the probability for the household of getting below the poverty limit is. In case of unskilled worker household heads, the probability for the household of getting into the bottom income 5 percent or the bottom income decile is two and a half times of what would result from their weight becoming and remaining poor.

The role of the type of settlement, the place of residence in case of the different poverty lines

While the type of settlement has a significant role in bringing about income inequalities, its role played in poverty is far from being that considerable.

In the capital and in other larger towns in general the risk of poverty is smaller than in small settlements, but the difference is not poignant. On the basis of the estimated data it seems that the probabilities of poverty risk did not change from 1995 to 1997.

Table 7

The risk of getting below the poverty level, according to the character of the place of residence, in case of different poverty lines in 1995

Residence	Households below			
	the upper limit of the bottom income		the minimum old-age pension	the subjective poverty line
	5 percent quantile	decile		
Budapest	0.9	0.9	0.9	0.7
Large towns (over 50 thousand inhabitants)	0.8	0.8	0.8	0.9
Settlements with 1000-50000 inhabitants	1.2	1.2	1.1	1.1
Small villages with less than 1000 inhabitants	0.9	1.0	1.0	1.1

The poverty risk of single-parent families and old people living alone

In contrary to expectations, the poverty risk of old people, whether living alone or not, is not too high, if we choose any of the poverty lines, it stays below the national average. At the same time, those old people who live alone, are generally very poor.

Among 75-year-old or older singles, the relative shortfall of the income of the poor from the poverty level is 40 percent or higher in the bottom 5 percent, the bottom decile just as below the official poverty line. This reveals the great poverty of old people living alone. When we speak about the relatively favourable income position of old people, we must not leave out of consideration those who live in poverty, because the intensity of their poverty is extremely high. When we used the equalized income instead of the per capita income in the analysis, the income position of pensioner households consisting of one person proved to be more unfavourable.

The other endangered group is the ‘single parent with child(ren)’ household. In accordance with the expectation, the poverty risk of this type of household is nearly one and a half times the national average. However, the intensity of poverty is not outstanding, that is many of them live near the different poverty lines, their poverty is shallow. They easily get below the poverty line, but they also get over it relatively easily.

The housing characteristics of the poor in case of the different poverty lines

70 percent of those living in ‘very bad’ housing conditions live below the subjective poverty line. The poverty risk of those living in very bad housing conditions is nearly 7 fold in case of the bottom 5 percent, 5.5 fold for the bottom decile and 5 fold for the ‘official’ subsidy threshold, that is, the poverty line set at the minimum old-age pension.

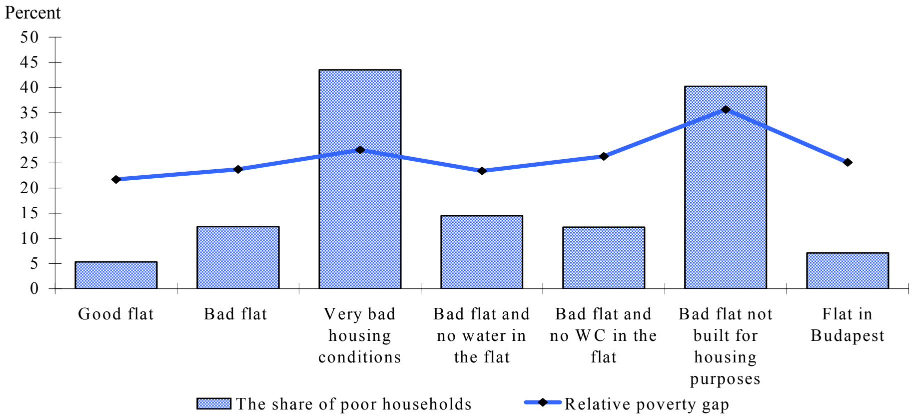
‘Bad housing conditions’ are also in close relationship with poverty, but the real difference is represented by ‘very bad housing conditions’.

Table 8

The risk of getting below the poverty level in case of the different poverty lines in 1995

Housing conditions	Households below: the upper limit of the bottom income 5 percent quantile	the upper limit of the bottom income decile	the minimum old-age pension	the subjective poverty line
Very good	0.5	0.4	0.5	0.5
Good	0.6	0.6	0.7	0.8
Average	1.1	1.1	1.1	1.0
Bad	1.7	1.7	1.7	1.5
Very bad	6.6	5.6	4.8	2.9
Very bad housing conditions and				
no WC in the flat	2.6	2.2	2.1	1.6
no water in the flat	2.5	2.7	2.4	1.9
unfavourable environment	3.1	2.6	2.3	1.7
flat not built for housing purposes	8.9	5.8	4.8	3.0

Figure 3. The incidence of poverty and the gap of the poor according to some major characteristics of the household among those living in the lowest income decile



Households that have 'bad housing conditions' and their flat 'not built for housing purposes' are with great probability below the poverty line. In 1995, nearly 90 percent of such households lived below the minimum subsistence level and their proportion was also 40 percent in the bottom income decile.

The relationship of objective and subjective poverty

Subjective poverty line is calculated on the basis of the income expectations of the population. Its size is the amount of income considered necessary for a very poor level of subsistence in the opinion of the population.

The sense of reality of the population is reflected by the fact that each year the subjective poverty line is between the income limit considered 'official' for eligibility for regular social subsidies, that is the amount of minimum old age pension and the absolute poverty line, namely the minimum subsistence level.

Beside the costs of subsistence, the subjective poverty line depends on a number of other factors as well. The higher the income level of a household is, the higher the amount necessary in their opinion for a very poor subsistence is.[8]

The size of this amount is also influenced by the previous financial position of the households, as everyone adjusts their expectations to their usual living standard. If the household used to live permanently at a higher living standard, then its structure of consumption corresponds to it, and this is what determines their needs. The picture formed by people about society also influences their expectations. In case of large income inequalities and permanent poverty, people tend to consider their own situation natural and adjust their expectations accordingly. 'Deep-rooted inequalities often flourish', says Amartya Sen, 'because they find passive allies in subordinated people'. (*Sen, A.*, 1990) One of the criteria of classical poverty is that those living in poverty for a long time do not consider themselves poor.

On the basis of the data of the 1997 household budget survey, we also had the opportunity to compare the opinions of households about their own level of subsistence with their actual financial position.

42 percent of the households belonging to the bottom income decile considered themselves very poor and a further 36 percent poor. Among those living under the minimum subsistence level these proportions were 33 and 37 percent. In the national average 13 and 34 percent of the households admitted themselves as very poor and poor, respectively.

Table 9

Opinion of households belonging to the bottom income decile and living below the minimum subsistence level in 1997 on the development of their financial position in the previous three years

(percent)

The financial position	In the bottom income decile	Below the minimum subsistence level	On national level
Deteriorated substantially	61.2	53.0	31.2
Deteriorated a bit	22.8	28.8	39.6
Did not deteriorate	14.0	15.9	24.0
Improved a bit	2.0	1.9	4.5
Improved substantially	0.0	0.4	0.7
Total	*100.0*	*100.0*	*100.0*

As we referred to it earlier, in the judgement of their own situation and present position, people also assess their past situation and future prospects. In a period when – as a result of the economic recession – the subsistence conditions and living standards of families deteriorated significantly by the middle of the nineties, the start of the economic uplift affected the individuals, families and households of different labour market positions in very different ways.

[8] The household statistical publications mentioned earlier and appear quarterly, include very valuable data and calculations on this topic. See the most detailed data for 1997 in volume 8.

7. Conclusions

The first reliable income survey after the transition carried out by the HCSO in 1996 supplied ample evidences supporting everyday empirical experiences that incomes in the mid 1990s dispersed to a much greater extent than before the transition in Hungary. Both the marked increase in unemployment and the restructuring of the economy by branches and ownership contributed considerably to the significant increase in income inequality. There are still some other factors for further research which may have been part of the process. It seems that primarily households of unemployed persons, pensioners living on widow's or very low pension and families with several children are the loosers of the restructuring of the income distribution. It is hoped that a continuing economic growth of the country may stabilize the level of income differences or these may even decrease to some extent if the new government turns greater attention to the well-being of families with children and some social groups and regions being now in a disadvantageous situation. This may, however, decrease the number and the proportion of people in poverty.

Nevertheless, the rather moderate level of income inequality of the decades before the transition can not be expected to return.

The elimination of the poverty of those living around the income level of the poverty line calculated on the basis of the minimum subsistence level can be managed along with the improvement of the economic situation of the country, the increase of the supporting power of earnings and the stability of social benefits.

The low income level of the groups of households below the threshold of eligibility for regular social subsidies and below relative poverty lines, living in deep poverty, is coupled by a series of economic and social disadvantages that the automatism resulting from the improving economic situation is unable to solve within the framework of the present social policy.[9]

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EDUCATIONAL CHOICES IN HUNGARY

ERZSÉBET BUKODI[1]

SUMMARY

This study is devoted to the investigation of the impact of the childhood material and cultural resources of individuals on their schooling successes. To answer research questions concerning educational career, I apply the rational action theory and the cultural reproduction thesis as competing theoretical explanations. These provide the background for interpreting my findings based on a multinomial logit analysis of the school-transition processes of males and females. The reason for applying a multinomial model instead of the traditional Mare's logit analysis is that the Hungarian school system contains parallel branches of education which can be seen as different alternative career lines with different odds of continuing one's studies. As for the data, I have used the Hungarian General Youth Survey from 1995 as a source. According to my results when using different characteristics of parents to examine trends in schooling process, I have found that childhood material and cultural capital have a significant and independent impact on educational success. In sum, my findings leave no doubt that in investigating educational careers both the rational action theory and the cultural reproduction thesis are relevant, but a resource transmission process appears to be gendered. Girls are more likely to follow the social reproduction 'route', but boys' choices are controlled by the rational action approach.

KEYWORDS: Education; Multinomial regression model.

The role of educational attainment in the reproduction of social order has been a crucial topic in the social stratification literature from classical writings to current ones. In modern societies the process of industrialization and modernization has enhanced the value of formal education, skills and qualifications have become the pre-determinants of job opportunities. As a consequence of educational expansion in economically advanced societies – like in Hungary – the primary and some forms of secondary education have become universal, the average level of educational attainment has substantially risen. Given this trend, one can expect a decline in the impact of social background on schooling opportunities. However, empirical research from a wide range of societies shows that the variation in educational attainment between social classes has hardly changed across successive birth cohorts. For the United States, *Featherman* and *Hauser* (1978) concluded that the effect of social origin on schooling remained quite stable during the first

[1] Statistician (HCSO) and Ph.D. candidate.

half of the century. More specifically, if one investigates schooling careers as a series of transitions (*Mare* 1980, 1981), the association between social origin and educational transitions is more or less unchanged across cohorts. (For an international confirmation of this finding see *Shavit* and *Blossfeld,* 1993). It means that children from less advantageous families are more likely to leave the educational system earlier than children with more advantageous social origins. Even if they do continue their studies, they generally choose courses which – due to the qualifications provided – reduce their chances of continuing at higher educational levels. It is commonly recognized by social scientists that social characteristics of parents play a significant role in evoking opportunities for children. More advantaged families set a higher value on education and are better equipped to encourage and promote school success than parents from less advantaged classes. This phenomenon is a part of social reproduction involving the persistence of inequalities from one generation to another as well as the role of educational attainment in transmitting these inequalities. Thus, while schooling is expected to provide opportunities for upward mobility, it helps to maintain socio-economic differences among families.

The studies on educational inequalities rely primarily on conventional characteristics of parents (father's education, father's occupation) in order to measure social origin. However, research on the schooling process requires to develop a more refined model. In the sociological literature two additional 'resources' have been considered: the economic situation of the family and the cultural resources of parents. It is obvious that school success depends on the material resources of parents, because parents with high income status are able to finance participation in higher education more, and they can 'protect' their children from the downward status mobility. At the same time, literature on cognitive development suggests that early influences of cultural values of family affect the child's later development and their school success (*Alwin* and *Thorton*, 1984). In this respect the activities of parents engaged with their children are crucial. By reading to a child, by talking about things related to school, by going to the theatre and museum, by encouraging a child to participate in extra-curriculum activities – in sum, by possessing cultural capital – parents help to develop a certain cultural lifestyle which improves educational aspirations as well as performance at school.

Several researchers have included variables concerning material or cultural resources of parents or both in their educational attainment models. Among the prior studies *DiMaggio's* research should be emphasized which indicates the crucial role of cultural resources provided by parents in the course of the academic careers of children (*DiMaggio* and *Mohr*, 1985). According to DiMaggio's findings the familiarity with high-culture activities strongly affects the educational attainment. The more cultural capital one possesses, the more likely he/she is able to obtain a higher degree of education. For all school transitions the effect of cultural resources has been larger than that of any predictor variables, except a child's ability. It means that cultural capital, as an indicator of status-culture participation provided by parents for their children increases children's chances for success at school independently of the class positions of parents.

P. de Graaf (1986) tried to test two hypotheses derived from the social reproduction theory for the Netherlands: whether the effect of the financial resources of parents on school outcomes has declined over time or not; and if the impact of cultural resources has increased across successive birth cohorts. According to his results, the influence of family resources disappeared for the younger cohorts, thus the relationship between social background and school success is no longer attributable to material inequalities. The effect of cultural capital, which used to be small for the older cohorts, has become even smaller in the last decades, so parents with more high-culture activities do not provide a better educational climate for their children than poorly cultured families.

Mateju (1989) prepared an analysis in which he compared the educational attainment process in Czechoslovakia, the Netherlands and Hungary. As for the effect of cultural resources, his results are in line with those of Graaf's: cultural capital of parents is far from being the most important predictor of educational inequalities in these countries. He found that the impact of material resources on school outcomes was largest in Hungary.

For Hungary, *Róbert* (1991) investigated the role of material and cultural resources in educational attainment models. According to his findings the effect of cultural capital follows a curvilinear trend across successive cohorts. The greatest increase in the impact of the cultural status of parents was observed for the fifties and it was accompanied by a strong decline in the effect of material resources. In this period the Hungarian educational system underwent a major structural reform aiming to fulfill one of the most important political considerations of the socialist regime: to abolish the economic constraints of the educational opportunities. For the younger cohorts the effect of cultural resources became smaller and smaller. As for the material status of parents, there was no clear trend after the early socialist period, the estimates were insignificant.

So far very few studies have applied the notions of material and cultural capital to investigate the life-cycle variation in educational career. It is obvious from the list of the above referred studies that most researches have been interested in historical variations in the schooling process. This article does not deal with birth cohort differences, but focuses on life-course variations in the effect of childhood material and cultural resources on educational choices.

In this study I deal with the following research questions.

– Considering the father's social class: to what extent do material and cultural resources influence the school choices?

– Does the magnitude of the effects of material–cultural resources of parents differ across school transitions (secondary and tertiary level) and according to educational track (academic versus vocational)?

– Are there any differences in the impacts of material and cultural capital according to social class background?

– Are these effects different in the achievements of men and women?

The aim of this study is not to estimate the complete model of the educational attainment process, but to predict the extent of the impacts of material resources and cultural capital on school success.

To answer these questions I analyze the data-set of the General Youth Survey from 1995, which was conducted by the Hungarian Central Statistical Office interviewing persons aged 15 to 29. To seek an explanation for the effect of the resources of parents on school career, I have applied the so-called discrete choice model of educational decisions and I have used two competing economic and sociological theories, namely the cultural reproduction thesis and the rational action theory (RAT). These have provided the background for interpreting my findings gained from the analysis of the propensity of males and females to choose among the different educational options.

The paper is organized as follows. The first section presents the important properties of the discrete choice models, the next one outlines the theoretical framework for understanding how different factors influence educational careers. Next the data and research designs are described, which is followed by an empirical analysis. The final section closes with a summary and some conclusions.

1. Educational decisions – discrete choice model

Much of the previous research on the relationship of educational career and family background effects has either focused on the highest grade completed or on a single school transition, such as college entry among secondary school graduates. In contrast, *Mare* (1980, 1981) conceptualized educational attainment as a sequence of grade transition probabilities. By dividing the educational process into stages, this model disaggregates differentials in overall schooling attainment into differentials in transition rates at various stages. In other words, the Mare model concentrates on the probabilities of making the transition from school level k to level $k+1$, conditional on having attained level k. These probabilities are taken to be the linear functions of different exogenous variables the values of which vary with individuals and school transitions. This feature of the model makes it possible to take into account the theoretically grounded hypotheses of the differences in the effects of explanatory variables at different points of educational careers, e.g. the idea of age-decreasing influence of social origin characteristics (*Shavit* and *Blossfeld*, 1993).[2] In addition, since the Mare model is based on the odds ratios, the parameters of this model are not affected by the expansion of the educational system, thus, it captures the 'pure' effects of explanatory variables. In spite of these attractive properties of the Mare schooling-transition model, it is obvious that the assumption of the sequential nature of the educational career can be challenged. At certain branching points of a school system, pupils (or their parents) face qualitatively different possibilities of choices. In Hungary – on the secondary level and above – the academic track of schooling process runs parallel to lower and higher vocational branches of studies. A consequence of it is that students have different future labor market prospects depending on which educational branch they follow. Thus a model which takes into account the structure of a school system gives a better explanation to why educational decisions vary according to gender, family background or other explanatory variables, and '…such a model is more appropriate for identifying at which transition the impact of such variables is the greatest'. (*Breen* and *Jonsson*, 1998 p. 8.). In practical terms, it seems to be reasonable to replace the sequential schooling-transition model by the discrete choice model of educational decisions.

A general study of the discrete choice behaviour was developed by *McFadden* (1974), and is described by *1*) the set of alternatives available to decision-makers, *2*) the characteristics of decision-makers, and *3*) the model of individual choice. Assume $\boldsymbol{X}$ denotes the universe of alternatives of choices and $\boldsymbol{S}$ the vectors of observed attributes of

[2] Several researches have also applied Mare's approach to study schooling transitions for Hungary (*Simkus* and *Andorka*, 1982; *Róbert*, 1991; *Szelényi* and *Aschaffenburg,* 1993).

decision-makers. An individual – drawn randomly from the population – has an attribute vector $\boldsymbol{s} \in \boldsymbol{S}$, and faces a set of available alternatives denoted by $\boldsymbol{B} \subseteq \boldsymbol{X}$. Let $P(\boldsymbol{x} \,/\, \boldsymbol{s}, \boldsymbol{B})$ denote the conditional probability that an individual chooses alternative $\boldsymbol{x}$, conditional on having characteristics $\boldsymbol{s}$ and existing alternative set $\boldsymbol{B}$. Suppose that an individual behaviour rule is a function $\boldsymbol{f}$ which maps the individual's attributes $\boldsymbol{s}$ and alternative set $\boldsymbol{B}$ into a chosen member of $\boldsymbol{B}$. A model of individual behaviour is a set of behaviour rules $\boldsymbol{F}$. There is a probability π defined on a subset of F specifying the distribution of behaviour rules in the population. The probability that an individual chooses x, given traits s and alternative set B, equals the probability of occurrence of a decision rule resulting in this choice:

$$\boldsymbol{P}(\boldsymbol{x} \mid \boldsymbol{s}, \boldsymbol{B}) = \pi[\ \{\ \boldsymbol{f} \in \boldsymbol{F} \mid \boldsymbol{f}(\boldsymbol{s}, \boldsymbol{B}) = \boldsymbol{x}\ \}\ ].$$

In my analysis on educational career $\boldsymbol{f}$ is the behaviour rule which can be derived either from the maximization of a specific utility function or from the reproduction of social status. The former approach leads to the application of a *rational action theory* for educational decisions, the latter one means the application of *cultural reproduction thesis* in schooling research.

2. Competing theoretical explanations – cultural reproduction thesis

The cultural reproduction thesis (*Bourdieu* and *Passeron* 1977, *Collins* 1971) – similar to the modernization theory – emphasizes the importance of the educational system to job opportunities in modern societies. The theory claims that educational certificates have a particular role in the explanation of persistent inequalities in social stratification. Selection in the labor market determined by education helps to maintain the privileges of dominant social classes. In Bourdieu's conception, cultural capital operates as a principle of cumulative advantage from the perspective of those who possess it. Offsprings from more educated families are likely to acquire the abilities, knowledge, language skills which are rewarded by schools quickly, because they are already familiar with them – they already possess more cultural capital by the time they enter the educational system than children from less educated families do. Pupils with less advantaged social background, however, have more difficulties in learning the values and rewarded skills due to the lack of the abilities normally transmitted by the parental family. Hence, school selection favours children from families already possessing the dominant cultural resources. In other words, educational credentials represent the class structure and help to legitimate the inequalities of occupational attainment. According to the social reproduction model, the whole process is driven by early socialization, and the cultural participation in later life-stages is a direct consequence of early cultural socialization. And since there are children who begin their educational career with more cultural capital and thus will be culturally advantaged throughout their whole schooling processes over those who begin their educational career without it, the effect of cultural capital on school success is constant over life-course.

Besides emphasizing the direct effect of the cultural resources of parents on school success, the social reproduction model argues that the efficacy of cultural capital depends on the attributes of its possessor. Individuals with higher social-class backgrounds possess a more authentic relationship to the dominant culture than those from less advantageous classes. In methodological terminology, this effect implies a positive interaction between social background and cultural resources. Persons have more chances for school success if they are from more advantageous families and they possess a larger stock of cultural capital, and they 'combine' the appropriate social background with a higher amount of cultural resources.

I have translated the implication of the social reproduction model into a set of hypotheses regarding educational process:

$\boldsymbol{H_1}$: Cultural resources acquired in childhood have a positive net impact on the probability of making all schooling transitions compared to leaving the educational system.

$\boldsymbol{H_2}$: The positive effect of cultural capital is greater for the academic track of schooling compared to vocational tracks.

$\boldsymbol{H_3}$: The magnitude of the effect of cultural resources is constant over the educational career, both for academic and vocational tracks.

$\boldsymbol{H_4}$: There is a significant positive interaction term between social class and cultural capital of parents across all school transitions compared to dropping out of the educational system.

3. Competing theoretical explanations – rational action theory

There is a theoretical explanation which emphasizes the important role of social origin on the schooling process in another sense. While the social reproduction thesis focuses on cultural elements of the family, the economic constraint approach concentrates on the material parameters of social background (*Boudon*, 1974). According to this explanation education must be financed by family resources which include, on the one hand direct costs (learning materials, fees), on the other hand child's foregone earnings. This means that educational attainment depends on the material resources of the family. Although, due to the school expansion, more and more children from less advantaged families continue at a higher level of the school system, for poor families it means larger sacrifices. Boudon introduces the notion of the 'primary' and 'secondary' effects that serve to stra-tify educational attainment. Primary effects – which are expressed by the association between children's class origin and their average level of academic achievement – create differentials at the lower school levels. In this case Boudon acknowledges the importance of the cultural environment of the family. In the case of secondary effects, however, he concentrates on the educational choices about the transitions to higher levels as being determined by the evaluation that parents or children make about predictable costs and benefits. He argues that secondary effects produce differences in school success even among those children who reach similar educational standards in their early school career – because of their different social origins. Thus economic inequalities between the families lead to educational inequalities among their children.

Boudon's argument was Goldthorpe's starting-point for developing an explanation of the educational attainment based on the rational action theory (*Goldthorpe*, 1996). He criticized the liberal approach of educational process in some respects. Acknowledging the declining influence of the costs in explaining the educational growth in modern societies, he underlines the fact that the decision-making procedure regarding schooling is conditioned by the social situation of the different families, and this is likely to lead to a different evaluation of costs and benefits. In this theoretical framework, families in less advantaged social position view the higher educational possibilities for their children in a different way compared to the families in advantaged status. In the former case, less ambitious and less costly educational alternatives are 'appropriately' adequate to the goal of maintaining the stability of social status. By contrast, parents in more advantaged classes will encourage their children to continue at higher educational levels and to preserve the intergenerational stability of their social position. It means that different social groups adjust their ambitions and plans to the unequal distribution of resources, opportunities, constraints of class-society by rational adaptive strategies.

According to a simple wealth-maximizing model of schooling transitions, the optimal educational level for an individual with $\boldsymbol{s}$ vector of attributes is the solution to the maximization problem (*Cameran* and *Heckman*, 1998):

$$\max_{j} \{R(j) - c(j|\boldsymbol{s})\}, \qquad j = 1 \ldots k \text{ (schooling alternatives).}$$

In this equation $c(j|\boldsymbol{s})$ denotes the cost of schooling. In choosing from among several educational alternatives parents and their children take into account three factors of the cost (*Breen* and *Goldthorpe*, 1997). The first of these is the direct cost of education (fees, materials), the second one concerns the foregone and postponed earnings. The third factor is the likelihood (subjective belief) of the success if a student chooses a particular type of schooling. $R(j)$ denotes the lifetime return to schooling which means the utility that is attached to the different educational outcomes by children or their parents.

As *Goldthorpe* (1996) noted, the interest of families is to avoid the downward mobility of their children which is achieved by the maximization of their offsprings' probability of access to an at least as advantageous position as what they are in. Probably there are differences in subjective beliefs about the 'value' of (return to) education according to characteristics of family. There are differences in the direct and indirect costs of education according to social origin as well. These arguments lead to the following hypotheses on the role of family resources in the schooling-decision process:

$\boldsymbol{H_1}$: Material and cultural capital of parents (as well as father's social class) have positive net impacts on making all educational transitions as opposed to leaving the school system.

$\boldsymbol{H_2}$: The positive effect of childhood material (and cultural) resources is greater for the academic track of education as opposed to vocational tracks.

(For children from an advantageous status position the optimal schooling strategy is to choose the academic type of education because following this track they can maximize the probability of access to the best labour market positions.)

It is obvious that discrete educational choices are embedded in a sequence of schooling-decisions. Thus social origin differences in choices at a lower educational level are influenced by expectations about choices that will be made at higher schooling levels. And these higher level choices show less variation by the attributes of the parental family than earlier ones because '…more ambitious educational option now carries with it no risk of downward mobility for working-class pupils [for pupils with less advantageous class position – *E. B.*]'. (*Breen* and *Goldthorpe*, 1997 p. 289). In other words:

$\boldsymbol{H_3}$: The magnitude of the effect of the material (and cultural) resources of parents should be lessened at higher educational levels.

In addition I hypothesize:

$\boldsymbol{H_4}$: The less advantageous the father's class position, the greater the influence of material (and cultural) resources of parents is on the transition to higher (academic) education (negative interaction term between parents' social class and paternal material resources).

This is partly because the academic type of education for these children is more expensive: the subjective belief about school success is lower; and partly because the benefits from choosing more ambitious educational options – in terms of avoiding the risk of social demotion – are higher for children from more advantageous class positions. Under these circumstances the parental resources (first of all the material capital) for children from less advantageous class positions, can serve as the effective minimizer of costs and effective maximizer of benefits.

4. Measurements and research design

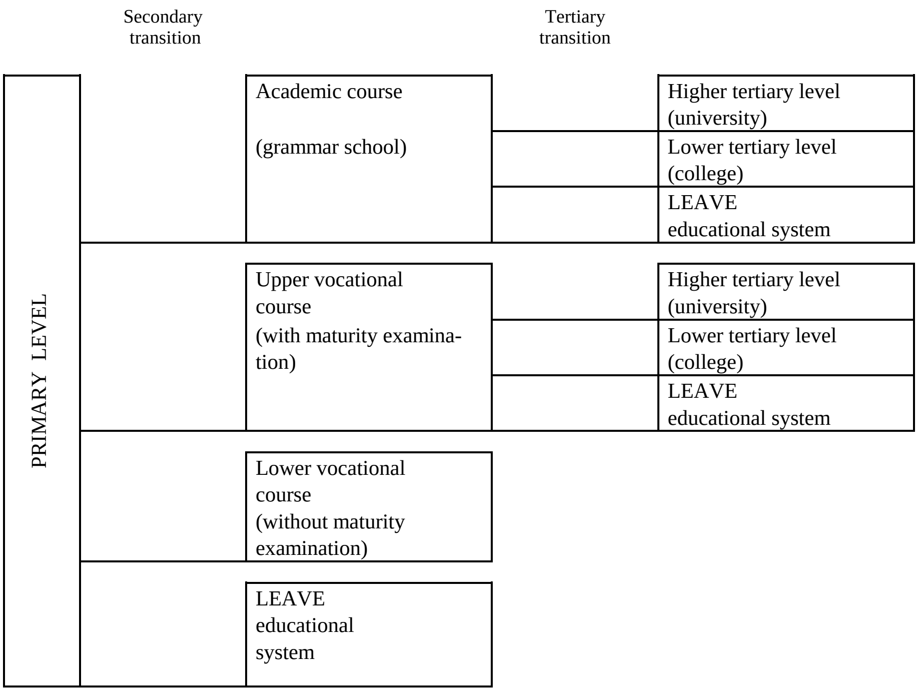
Data source

The data for this analysis is derived from the 1995 General Children and Youth Survey, a national representative survey of the Hungarian population aged 0 to 29 which contains detailed information on the social backgrounds and attainments of young people. To investigate the relative significance of the different factors of family environment-capital, I selected respondents who were born between 1966 and 1980 as the target group population (N=5378). The information on children's educational attainment is longitudinal starting with the first choice at age 14 – this decision is made when pupils leave the elementary school – involving alternatives to leave the school system, to begin study at an academic track, or at a vocational track. The second piece of information is whether individuals continue at tertiary level or not, if so, in which branch (academic versus vocational) they start to study.

Variables in the analysis

The aim of my analysis is to assess the impact of the different factors of social origin on the educational decisions. As it has been emphasized previously, I focus on the question of how the effect of material and cultural capital vary according to the educational path followed. Figure 1. shows the simple structure of the Hungarian educational decision tree which has two transition levels (T_1 : secondary, T_2 : tertiary), and at each point there are several possible choices. At the secondary level there are four variants: an academic course, a lower vocational or a higher vocational course versus leaving the school. At the second transition (tertiary level) there are also four paths which students might follow to arrive at the choice of different types of tertiary education. These are as follows: academic secondary – higher tertiary; academic secondary – lower tertiary; higher vocational secondary – higher tertiary; higher vocational secondary – lower tertiary. In addition, of course, students have an option to leave the school system after finishing secondary education. It is important to emphasize that pupils with lower vocational school certificate are not eligible to continue their educational career at tertiary level, thus in the analysis of the tertiary education I do not include individuals with this kind of qualification (likewise I also omit people who have not continued their school career after primary education). In sum, this kind of an educational decision tree requires the extension of the traditional Mare's schooling transition model in order to take into account the qualitatively different nature of the possible educational paths, and – in methodological terms – it is necessary to replace the binary logit model by the multinomial logit model[3] (*Agresti*, 1990; *Hendrickx* and *Ganzeboom*, 1998).

Figure 1. The educational decision tree



In this study I have included the following explanatory variables in order to assess educational success.

Father's social class (CLASS): This variable contains four classes which are defined according to the principle of the Hungarian Occupational Class schema including the fol-

[3] The multinomial logit model has been estimated in SPSS, using a macro program prepared by *John Hendrickx*.

lowing categories: service class, intermediate class which contains routine nonmanuals and self-employed fathers, skilled workers and unskilled workers.

Material resources of parents (MAT): The material capital of families is measured by the family's possession of the following items: colour television, freezer, automatic washing machine, hi-fi stereo equipment, camera and video cassette recorder, automobile, personal computer and child's own room. I have created a set of dummy variables which takes value 1 indicating the possession of a particular item, otherwise it is 0. In the second step I have normalized these observed variables, then I have added up these standardized values (Z-scores). This kind of measure of material capital is a better proxy to capture the economic environment under which the child grows up than income measures, since it takes into account the accumulation time of these items and thus the material possession reflects a continuous level of wealth throughout childhood.

Cultural resources of parents (CUL): The cultural climate of families is measured by the following cultural activities of the child (his/her parents): attendance at theatres, concerts and exhibitions (it is important to emphasize, that these pieces of information refer to the activities in which parents are engaged with their children); and possession of more than 200 books. In addition, I have considered some information on childhood educational climate, namely participating in extra-curriculum activities (attending language and music courses). Similar to the construction of the variable on material resources, I have normalized the dummy variables concerning different types of cultural activities, then I have added up these standardized values resulting in a synthetic measure of childhood cultural capital.

Path-way (PATH): As it was shown in Figure 1, individuals may follow different paths to arrive at tertiary education, and it may be assumed that pupils who follow paths which do not include an academic component at secondary level have a lower probability of entering any kind of tertiary education. To test this hypotheses, in the analysis of tertiary education I have included a dummy variable which takes value 1 if respondent graduates from any academic type of secondary school and it sets 0 if they graduate from any vocational type of secondary school.

To assess the developments of educational attainment over the investigated time period, I have included in the analysis the respondents' *birth cohort* (BIRTH) measured in single years. This variable has been rescaled for the analysis, so the most recent cohort (1980) takes a value of 14 and the oldest cohort (1966) takes a value of 0.

Developing nested models for educational career

To estimate the effect of the different resources on school success, several nested models have been tested. To evaluate whether the effect of parents' social class was biased in previous analyses on educational success, first I have estimated the baseline models which omit the material and cultural capital variables (Model 1). For the *i*th individual and the *k* educational category at transition level *t*, this baseline model is as follows:

$$\ln \frac{p_{ikt}}{\sum_{j \neq k}^{K} p_{ijt}} = \alpha + \beta_f \,\mathrm{PATH}_i + \beta_h \mathrm{BIRTH}_i + \sum_{l=1}^{L-1} \beta_l \mathrm{CLASS}_{il}$$

for k=1,…K-1 and t=1 (secondary transition), 2 (tertiary transition).

Here p_k is the probability of choosing school branch k. BIRTH represents the single-year birth cohort effect and CLASS means a set of dummy variables for L-1 out of L class categories. The variable PATH is included in the model only for the analysis at tertiary education. In the multinomial logit model the kth school alternative represents the omitted choice which is always the 'leave education' option, α is the intercept term and the parameter estimates are denoted by β. The model is fitted for each transition (secondary and tertiary) level.

Model 2 adds up material and cultural resource variables:

$$\ln \frac{p_{ikt}}{\sum_{j \neq k}^{K} p_{ijt}} = \text{Model } 1 + \beta_m \text{MAT}_i + \beta_r \text{CUL}_i .$$

Since the relative influences of material and cultural capital may depend on the father's social class, Model 3 includes interactions of the father's class with childhood material and cultural resources:

$$\ln \frac{p_{ikt}}{\sum_{j \neq k}^{K} p_{ijt}} = \text{Model } 2 + \sum_{s=1}^{S-1} \beta_s \text{CLASS}_{is} \times \text{MAT}_i + \sum_{u=1}^{U-1} \beta_u \text{CLASS}_{iu} \times \text{CUL}_i .$$

The interaction effects (β_s , β_u) tell whether the effect of material and cultural resources are stronger or weaker for respondents whose fathers belong to a particular social class compared to respondents with fathers from the reference social class category.

As noted earlier, it can be assumed that there is some path dependence in educational careers. We can hypothesize that the greater the influence of the different factors of social background on the transition to tertiary education is, the more difficult is to follow the path. In other words, transition to third level education is dependent on the type of secondary school (academic versus vocational) the individual graduates from. To test the hypothesis of a positive relationship between the difficulty of secondary school path-way and the choice of the different types of tertiary education, I have introduced interactions between the social origin parameters and the dummy variable PATH:

$$\ln \frac{p_{ikt}}{\sum_{j \neq k}^{K} p_{ijt}} = \text{Model } 3 + \beta_v \text{BIRTH}_i \times \text{PATH}_i + \sum_{x=1}^{X-1} \beta_x \text{CLASS}_{ix} + \beta_y \text{MAT}_i \times \text{PATH}_i +$$

$$+ \beta_z \text{CUL}_i \times \text{PATH}_i + \sum_{w=1}^{W-1} \beta_w \text{CLASS}_{iw} \times \text{MAT}_i \times \text{PATH}_i + \sum_{q=1}^{Q-1} \beta_Q \text{CLASS}_{iq} \times \text{CUL}_i \times \text{PATH}_i .$$

5. Empirical results

In the following subsections the empirical findings will be presented.

Comparison of nested models

Subsequently, investigating predictor factors of the choice of different kinds of educational paths, 4 nested models have been tested. Comparisons of these models, based on the differences in log-likelihood-ratio statistics, are given in Table 1.1 for males and Table 1.2 for females.

Table 1.1

Log-likelihood ratio chi-square tests
for multinomial logistic regression models for educational choices
(Males, born 1966–1980.)

Model/Comparison	Log-likelihood ratio		Degrees of freedom	
	secondary level	tertiary level	secondary level	tertiary level
Models				
1. (Baseline, school path-way, linear birth cohort, father’s class)	590.0*	119.7*	12	10
2. (Model 1 + material resources, cultural resources)	971.5*	129.4*	18	14
3. (Model 2 + interaction of father's class with material and cultural resources)	1105.6*	163.4*	36	26
4. (Model 3 + interaction of family background variables with school path-way)	–	188.9*	–	50
Comparisons				
Model 2 versus Model 1	381.5*	9.7*	6	4
Model 3 versus Model 2	134.1*	34.0*	18	12
Model 4 versus Model 3	–	25.5	–	24

* p<.05.

Note: Fit assessed by -2 log (L_0 / L_1), where L_1 is the likelihood of the fitted model and L_0 is the likelihood of the intercept models in the first four rows (Model 1 through 4) and comparison of different models can be found in the last three rows.

Table 1.2

Log-likelihood ratio chi-square tests
for multinomial logistic regression models for educational choices
(Females, born 1966–1980.)

Model/Comparison	Log-likelihood ratio		Degrees of freedom	
	secondary level	tertiary level	secondary level	tertiary level
Models				
1. (Baseline, school path-way, linear birth cohort, father's class)	555.4*	157.5*	12	10
2. (Model 1 + material resources, cultural resources)	848.8*	180.5*	18	14
3. (Model 2 + interaction of father's class with material and cultural resources)	975.3*	220.0*	36	26
4. (Model 3 + interaction of family background variables with school path-way)	–	248.8*	–	50
Comparisions				
Model 2 versus Model 1	293.4*	23.0*	6	4
Model 3 versus Model 2	126.5*	39.5*	18	12
Model 4 versus Model 3	–	28.8	–	24

* $p < .05$

The first two models are the main-effect models, that is they do not include any interaction terms. Model 1 (base model) specifies birth cohort and father's social class effects; Model 2 displays the impact of material and cultural resources of parents. A comparison of Models 1 and 2 shows that the inclusion of childhood material and cultural capital significantly improves the fit of the models for both educational levels, both for men and women.

In Model 3 and Model 4, I have incorporated two interaction-term models into my analysis which are modifications of Model 2. Model 3 hypothesizes that the effect of the material and cultural resources varies among individuals with different social class backgrounds. The overall fit statistics of this model confirm this hypothesis for both sexes and for all kinds of school transitions. Finally I have developed a model which tests whether or not the effect of the different measures of family background on the tertiary educational choice varies from the secondary school path-way. According to the results, Model 4 has not attained a significantly better fit compared to Model 3 either for males or females. In sum, one can say that the impact of the attributes of family on the likelihood of choosing tertiary education of any kind does not depend on the type of secondary school where the individual comes from. In conclusion, Model 3 seems to be the 'best' one because it fits to the data more parsimoniously than Model 4 does. Thus I have decided to present in the subsequent section the parameter estimates derived from Model 1 to Model 3.

Interpretation of parameter estimates

Differences in educational career according to father's social class. Starting with secondary level, class origin appears to have a markedly stronger effect on academic studies compared to lower and higher vocational tracks. Males with service class origins are the most likely to choose academic types of study as opposed to leaving the educational system. As we go down the social class ladder, the propensity to choose an academic branch of education decreases. For females the pattern is the same. In other words, there is a striking trend towards following an academic track of education at a higher extreme of the social class hierarchy. As for the likelihood of going on to an upper vocational type of secondary schooling – which makes pupils eligible to going on to tertiary level as well –, is largest for individuals with an intermediate class background. Lower vocational training seems to be the best option for pupils with skilled worker origins. These results underline the great differences in the social class recruitment of various branches of studies at secondary level.

Class differences are generally smaller at tertiary level, indicating that the most important decision is to choose among the different types of secondary studies. In spite of this fact, as parameter estimates show in Table 3, a service class origin is important for the choice of an academic (university) versus vocational (college) type of education similarly to secondary schooling. For females, there is one more statistically significant social class effect regarding tertiary education. As revealed by coefficients, the propensity to study at college level is the strongest for women with intermediate class backgrounds.

Table 2

Coefficients for multinomial logistic regression of choosing among different types of secondary education on family resource variables: Model 1

(Standard errors in parentheses)

	Males			Females		
	Lower vocational track	Upper vocational track	Academic track	Lower vocational track	Upper vocational track	Academic track
Linear birth cohort	.078***	.082***	.092***	.093***	.118***	.082***
	(.016)	(.017)	(.019)	(.017)	(.093)	(.017)
Father's class						
service	.766	2.24***	3.98***	1.11	2.16***	4.15***
	(.474)	(.241)	(.374)	(.656)	(.261)	(.517)
intermediate	.809	2.74***	2.51***	.937	3.01***	2.40***
	(.431)	(.368)	(.265)	(.567)	(.524)	(.260)
skilled worker	.777***	1.42***	1.31***	.844***	1.48***	1.26***
	(.145)	(.168)	(.204)	(.152)	(.159)	(.164)
unskilled worker (reference)	.000	.000	.000	.000	.000	.000
Constant	.428***	-.842***	-1.65***	-.186**	-.891***	-.742***
	(.129)	(.158)	(.194)	(.138)	(.152)	(.151)
Number of cases	483	721	1177	823	797	718

* *p* < .05; ** *p* < .001; *** *p* <.001.
Note. Reference category is 'drop out'.

Table 3

Coefficients for multinomial logistic regression of choosing among different types of tertiary education on family resource variables: Model 1

(Standard errors in parentheses)

	Males		Females	
	Lower tertiary (college)	Higher tertiary (university)	Lower tertiary (college)	Higher tertiary (university)
Secondary path: academic (ref.: vocational)	1.09***	2.03***	1.37***	1.99***
	(.206)	(.304)	(.184)	(.324)
Linear birth cohort	.059	.079	.061*	.018
	(.034)	(.044))	(.027)	(.038)
Father's class				
service	.187	1.89***	.781	1.12**
	(.331)	(.565)	(.476)	(.347)
intermediate	.353	1.01	.859**	.344
	(.323)	(.603)	(.265)	(.385)
skilled worker	.671	.353	.272	- .267
	(.587)	(.323)	(.247)	(.366)
unskilled worker (reference)	.000	.000	.000	.000
Constant	-1.28***	-3.44***	-1.96***	-3.56
	(.307)	(.592)	(.261)	(.426)
Number of cases	83	145	93	216

* *p* < .05; ** *p* < .001; *** *p* <.001.
Note. Reference category is 'drop out'.

An interesting feature of the model at the tertiary level of education is that it controls the effect of secondary school path-way. It is obvious from the coefficients that the odds of making a tertiary level transition differs markedly according to the type of secondary education: the choice of any kind of tertiary schooling is much more likely in case of pupils graduating from an academic type of secondary education. And it may be assumed that the distribution of class origins is quite different in the two branches of secondary schooling. Consequently, following this research strategy, we may realise the 'pure' impact – namely the effect after filtering the influence of secondary school path-way – of the attributes of family on the choice of tertiary education.

Impact of childhood material and cultural capital on educational choices. For secondary school transition, the effect of the material resources of parents appears to be substantial, statistically significant and independent of the conventional parameters of social origin.

Table 4

Coefficients for multinomial logistic regression of choosing among different types of secondary education on family resource variables: Model 2
(Standard errors in parentheses)

	Males			Females		
	Lower vocational track	Upper vocational track	Academic track	Lower vocational track	Upper vocational track	Academic track
Linear birth cohort	.016	.018	.020	.039	.031	.006
	(.018)	(.020)	(.023)	(.019)	(.019)	(.020)
Father's class						
service	.231	1.43***	2.50***	.594	1.31**	2.58***
	(.382)	(.254)	(.396)	(.571)	(.275)	(.535)
intermediate	.296	1.60***	1.60***	.509	1.71***	1.43***
	(.241)	(.386)	(.282)	(.279)	(.443)	(.275)
skilled worker	.579***	1.09***	.973***	.609***	1.05***	.794***
	(.148)	(.175)	(.214)	(.158)	(.167)	(.172)
unskilled worker reference)	.000	.000	.000	.000	.000	.000
Material resources	1.02***	1.27***	1.31***	1.21***	1.49***	1.50***
	(.138)	(.143)	(.149)	(.156)	(.156)	(.158)
Cultural resources	.385	1.04***	1.34***	.234	.439**	.621***
	(.201)	(.201)	(.203)	(.163)	(.151)	(.150)
Constant	1.61***	.882***	.168	.852***	.719***	.910***
	(.200)	(.223)	(.254)	(.209)	(.216)	(.216)
Number of cases	483	721	1177	823	797	718

* $p < .05$; ** $p < .001$; *** $p < .001$.
Note. Reference category is 'drop out'.

For men, a one-unit increase in childhood material status is associated with a 270 percent increase ($100[e^{1.31} - 1]$) in the odds of choosing an academic track of secondary education and a 256 percent increase in the likelihood of going on to a higher vocational type of secondary school, and a 177 percent increase in the odds of choosing a lower vocational branch, as opposed to leaving the school system. For women these estimates are as follows: 348, 344, and 235 percent, respectively. As these figures show, secondary school choices of females are more dependent on the financial credentials of parents than the decisions of males about secondary school careers. Although secondary educational choices appear to be markedly influenced by childhood material conditions, the magnitude of the material resource effects is weaker for ‘easier’ options, more precisely, for vocational tracks of education.

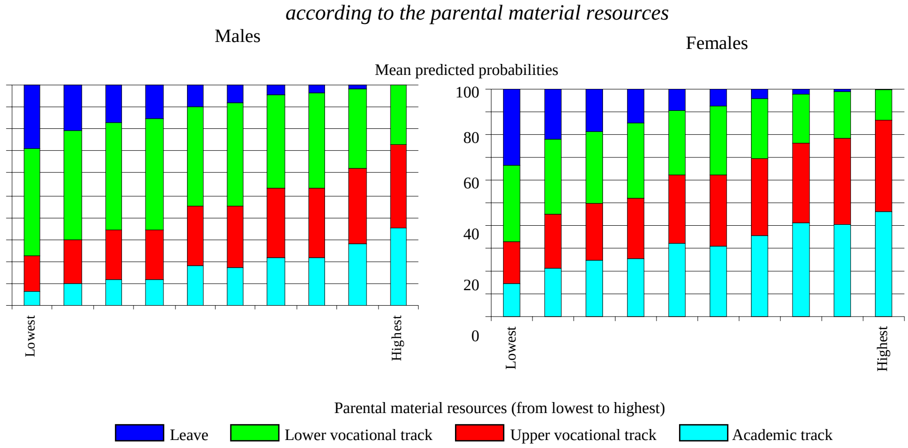
The influence of cultural resources follow a similar trend to the impact of material status: the more ‘difficult’ the educational option is, the larger the magnitude of this effect is for both sexes. For men, a one-point increase in childhood cultural resources is associated with a 280 percent increase in the odds of going on an academic type of secondary school and 183 percent increase in the propensity to choose an upper vocational track – as opposed to dropping out. However – as the parameter estimate suggests – the choice of a lower vocational type of secondary level does not depend significantly on the cultural capital of males’ family. For women, the trend concerning the impact of cultural capital on secondary school decisions is similar, although the magnitude of the effects appears to be smaller: in the case of academic track 86, for upper vocational study 55 percent, for lower vocational type the effect of cultural capital is statistically insignificant.

The findings about the impact of childhood material and cultural ‘climate’ on the educational decisions concerning secondary level can be summarized as follows.

a) The more ambitious the school choice is, the greater the effect of the resources of parents is.

b) The influence of material capital seems to be greater than the effect of cultural resources for the choice of continuing an educational career after primary school versus dropping out. This trend is illustrated in Figure 2.1. For instance, for males coming from the poorest material status, the predicted probability of leaving the educational system after primary school is about 30 percent, while for boys with the poorest cultural climate this probability is ‘only’ 20 percent. For females these predictions are 35 and 20 percent, respectively. (Note, that these probabilities are calculated on the bases of Model 2, in other words, these are controlled for birth cohort and father’s social class as well as cultural resources.)

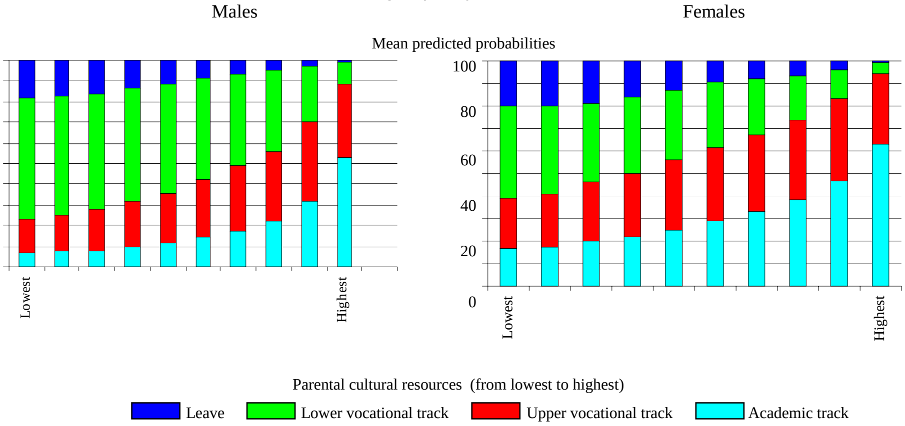
Figure 2.1. Predicted probabilities of educational choices at secondary level according to the parental material resources



Remark: Controlled for birth cohort, father’s social class, cultural capital.

c) On the contrary, the effect of the cultural capital of parents appears to be crucial for the decision on choosing between two types of upper secondary studies, which make pupils eligible for tertiary education: the more cultural resources someone possesses, the more likely it is that they will choose the academic branch of study, as opposed to the vocational track. According to the predicted probability of continuing on the academic track, almost 50 percent of boys with the largest amount of cultural capital choose this kind of secondary study as opposed to 5 percent of boys with the poorest cultural background. For females these probabilities are 60 and 18 percent rexpetively (see Figure 2.2.).

Figure 2.2. Predicted probabilities of educational choices at secondary level according to family cultural resources



Remark: Controlled for birth cohort, father's social class, material capital.

The next branching point is the choice of lower or upper tertiary education versus leaving the school system with the General Certificate of Education obtained. In this case there are important gender-specific differences in the effect of childhood material and cultural resources. For males only parental financial credentials exert statistically significant effect on the choice of an academic type of tertiary study (the choice of lower tertiary track is not influenced by either material or cultural capital stock). For females only the possession of cultural capital influences the decision on going to tertiary level or not, and if so, which branch of study to choose.

As for the parameter estimates, a one-point increase in the material capital is linked to a 27 percent increase in the odds of university enrollment for men. Remember that at secondary level the large amount of financial credentials increased the likelihood of enrollment in academic track by 270 percent. It means that the importance of childhood material conditions is smaller when males decide their academic type of tertiary education than when they choose the academic branch of secondary study. For females a one-point increase in childhood cultural resources is associated with an 83 percent increase in the propensity to choose university level, and a 47 percent increase in the likelihood of choosing lower tertiary level. Comparing these figures to the coefficients of cultural capital derived from the model on secondary school decision, an interesting trend is revealed: unlike the effect of financial resources for men, for women the impact of childhood cultural 'climate' appears to be stable over their whole educational careers.

Table 5

Coefficients for multinomial logistic regression of choosing among different types of tertiary education on family resource variables: Model 2
(Standard errors in parentheses)

	Males		Females	
	Lower tertiary (college)	Higher tertiary (university)	Lower tertiary (college)	Higher tertiary (university)
Secondary path: academic (ref.: vocational)	1.03***	1.95***	1.35***	1.94***
	(.208)	(.306)	(.185)	(.327)
Linear birth cohort	.069	.094	.068*	.022
	(.044)	(.065)	(.028)	(.041)
Father's class				
service	- .031	1.56**	.312	.417*
	(.346)	(.578)	(.403)	(.208)
intermediate	.237	.839	.651**	- .053
	(.329)	(.608)	(.277)	(.407)
skilled worker	.628	- .154	.169	- .488
	(.589)	(.312)	(.252)	(.377)
unskilled worker (reference)	.000	.000	.000	.000
Material resources	.162	.237*	.189	.206
	(.139)	(.111)	(.129)	(.129)
Cultural resources	.155	.204	.383*	.604***
	(.092)	(.108)	(.086)	(.098)
Constant	-1.15***	-3.25***	-1.78***	-3.36***
	(.322)	(.604)	(.279)	(.442)
Number of cases	83	145	93	216

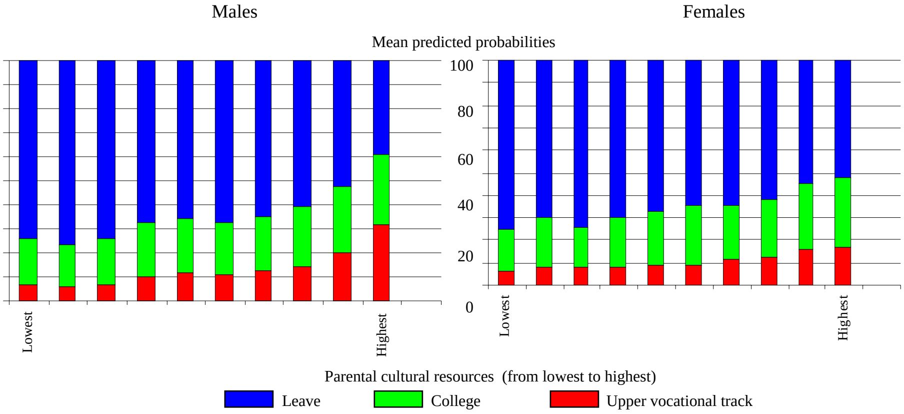
* $p < .05$; ** $p < .001$; *** $p <.001$.
Note. Reference category is 'drop out'.

The major results concerning the importance of different kinds of family resources on the choice of tertiary education are as follows.

a) The more resources one possesses, the more likely they are to choose the academic (higher) branch of tertiary education.

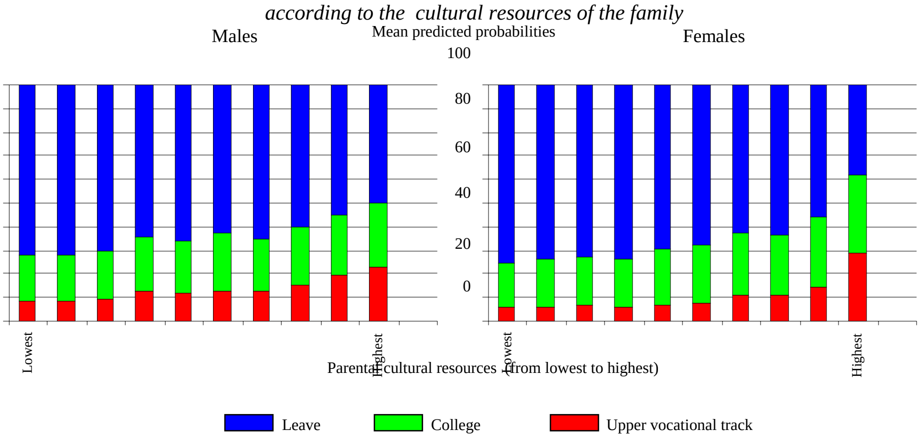
b) For males it is the material capital possession, for females the cultural capital possession which exerts a substantial impact on educational choices at tertiary level. It is well-supported by the predicted probabilities of Figures 3.1. and 3.2. For males with the highest amount of material resources the propensity to continue on tertiary level is about 60 percent, while for females with the same attributes it is 'only' 40 percent. On the contrary, in the case of childhood cultural capital, for girls with the richest cultural background the predicted probability of going on to universities or colleges is about 60 percent, for boys with the same traits it takes the value of 45 percent. (Again these probabilities are controlled for birth cohort, father'1s social class and the secondary school path way.)

Figure 3.1. Predicted probabilities of educational choices at tertiary level according to the material resources of the family



Remark: Controlled for path-way, birth cohort, father's social class, cultural capital.

Figure 3.2. Predicted probabilities of educational choices at tertiary level according to the cultural resources of the family



Remark: Controlled for path-way, birth cohort, father's social class, material capital.

c) For men the importance of childhood material conditions is smaller at tertiary level than at secondary level.

d) For women the reliance on the cultural resources of parents appears to be constant over their educational careers.

Do parental material and cultural resources mediate the relationship between class origin and educational attainment or are the effects of these resource variables independent of father's social class? Although both resource variables have a significant net effect on school choices, the reduction in the impact of class backgrounds is not substantial when it is controlled for childhood material and cultural capital (more precisely, the magnitude of class effects is smaller, but the statistical power of parameter estimates remains the same). Hence, those studies which exclude the material as well as the cultural credentials of parents do not overestimate the impact of social class background. This result, namely that estimates on class background are weakly affected by adding childhood material and cultural resources, suggests that family variables are not completely substituted by each other.

Family class background differences in the effect of material and cultural resources on schooling career

To test the controversial hypotheses derived from the social reproduction and the rational action theories concerning the relationship between social class background and material as well as cultural resources of parents, I have incorporated the interaction terms of these variables into my model. It is evident from the regression coefficients that at secondary level the effects of material resources vary according to class origin in the case of both sexes. For men 6, for women 3 parameter estimates out of 9 are significant.

Do the relative size of resources effects support the hypothesis of the rational action that is, the less advantageous background an individual has, the stronger these effects are? All interaction terms are negative, which means that the greatest importance of childhood material capital for making secondary school transition is found for individuals with unskilled class background (the reference category of class origin). For a male who comes from service class a one-point increase in material resources is associated with an 80 percent less increase in the odds of choosing an academic branch of study than for a male with an unskilled-worker background. For females this estimate is similar, 84 percent. As for the interactions between class origin and childhood cultural capital, coefficients appear to be insignificant for both sexes. In sum, in the secondary educational level parameter estimates confirm the prediction of the rational choice theory.

Table 6

Coefficients for multinomial logistic regression of choosing among different types of secondary education on parental resources variable: Model 3
(Standard errors in parentheses)

	Males			Females		
	Lower vocational track	Upper vocational track	Academic track	Lower vocational track	Upper vocational track	Academic track
Linear birth cohort	.017	.017	.019	.041	.034	.005
	(.018)	(.020)	(.023)	(.029)	(.020)	(.020)
Father's class						
service	- .026	1.29***	2.27***	.764	2.02**	2.81***
	(.482)	(.480)	(.487)	(.767)	(.739)	(.731)
intermediate	- .138	1.08***	1.14**	.039	.890*	1.03**
	(.317)	(.321)	(.344)	(.379)	(.369)	(.369)
skilled worker	.722*	1.19***	1.07**	.157	.578*	.337
	(.323)	(.332)	(.352)	(.285)	(.283)	(.286)
unskilled worker (reference)	.000	.000	.000	.000	.000	.000
Material resources	1.42***	1.69***	1.73***	1.58***	1.85***	2.01***
	(.226)	(.247)	(.269)	(.249)	(.259)	(.261)
Cultural resources	.108	.782*	.895**	.105	.622*	.543*
	(.301)	(.307)	(.317)	(.279)	(.264)	(.261)

(Continued on the next page.)

(Continuation.)

	Males			Females		
	Lower vocational track	Upper vocational track	Academic track	Lower vocational track	Upper vocational track	Academic track
Father's class × material resources						
service	-1.37**	-1.56**	-1.59***	-1.47*	-1.65**	-1.81**
	(.491)	(.489)	(.299)	(.647)	(.607)	(.600)
intermediate	-1.01**	-1.01**	-1.12**	-.528	-.521	-.646
	(.348)	(.379)	(.359)	(.482)	(.480)	(.481)
skilled worker	- .313	- .247	- .302	- .639	- .664	- .806
	(.316)	(.335)	(.357)	(.337)	(.350)	(.440)
Father's class × cultural resources						
service	.606	1.154	1.269	-.931	-.989	-.622
	(.885)	(.867)	(.870)	(.582)	(.573)	(.364)
intermediate	.097	.204	.233	-.285	-.124	-.357
	(.489)	(.498)	(.499)	(.535)	(.488)	(.489)
skilled worker	.516	.791	.836	-.099	-.077	-.187
	(.503)	(.496)	(.514)	(.386)	(.367)	(.372)
Constant	1.76***	1.03***	.312	1.20***	1.06***	1.24***
	(.251)	(.267)	(.293)	(.261)	(.264)	(.265)
Number of cases	483	721	1177	823	797	718

* $p < .05$; ** $p < .001$; *** $p <.001$.
Note. Reference category is 'drop out'.

An additional support for the hypotheses of the stronger material capital effect of individuals from less advantageous class backgrounds can be comprehended, by the estimates of material resources of those choosing the academic track of tertiary education (university); however, only for males. In this case, for pupils with a service class father a one-point increase in childhood financial credentials results in a 98 percent less increase with the likelihood of continuing in universities than for pupils who come from the unskilled class. As one can remember, at the secondary level this figure was 'only' 80 percent, thus for males with the least advantageous class background the importance of the material capital of parents appears to be more vital for making a tertiary transition as opposed a transition to a lower level of the educational hierarchy.

Table 7

Coefficients for multinomial logistic regression of choosing among different types of tertiary education on family resources variables: Model 3
(Standard errors in parentheses)

	Males		Females	
	Lower tertiary (college)	Higher tertiary (university)	Lower tertiary (college)	Higher tertiary (university)
Secondary path: academic (ref.: vocational)	1.08***	2.06***	1.34***	1.96***
	(.213)	(.317)	(.187)	(.332)
Linear birth cohort	.071	.114	.070**	.020
	(.046)	(.087)	(.029)	(.041)

(Continued on the next page.)

(Continuation.)

	Males		Females	
	Lower tertiary (college)	Higher tertiary (university)	Lower tertiary (college)	Higher tertiary (university)
Father's class				
service	- .397	1.75	.089	.737
	(.396)	(.928)	(.470)	(.427)
intermediate	- .283	.982	.961*	- .764
	(.377)	(.956)	(.397)	(.526)
skilled worker	.679	- .597	.429	- .857
	(.942)	(.354)	(.380)	(.449)
unskilled worker (reference)	.000	.000	.000	.000
Material resources	1.09	3.32**	.173	.286
	(.675)	(1.08)	(.554)	(.490)
Cultural resources	- .123	-1.19	.554*	1.23*
	(.265)	(1.23)	(.244)	(.619)
Father's class × material resources				
service	-1.41	-4.11***	- .083	-1.29
	(.848)	(1.17)	(.559)	(.683)
intermediate	- .973	-3.24**	.741	- .841
	(.533)	(1.11)	(.545)	(.726)
skilled worker	- .562	-2.41*	.637	-1.13
	(.541)	(1.12)	(.528)	(.708)
Father's class × cultural resources				
service	.310	1.53	.588	.959*
	(.302)	(1.27)	(.314)	(.470)
intermediate	.681	1.49	.374	.524*
	(.354)	(1.23)	(.274)	(.250)
skilled worker	.129	1.52	.320	.626
	(.375)	(1.278)	(.320)	(.609)
Constant	- .82***	-3.52***	-2.00***	-3.00***
	(.358)	(.931)	(..390)	(.474)
Number of cases	83	145	93	216

* $p < .05$; ** $p < .001$; *** $p < .001$.
Note. Reference category is 'drop out'.

For females parameter estimates concerning tertiary education confirm the prediction of the cultural reproduction theses. The importance of childhood cultural capital for making an academic type of tertiary transition is the greatest for students with the most advantageous class backgrounds. Namely, for a woman who comes from the service class a one-point increase in her cultural resources is linked to a 161 percent greater increase in her propensity to go on to university than for her counterparts with unskilled worker backgrounds.

6. Conclusions

The aim of this study was to investigate the effect of childhood material and cultural capital on the educational career applying a discrete choice approach and testing several hypotheses derived from the social reproduction thesis and the rational action theory.

When using different characteristics of parents to examine trends in the schooling process, I have found that childhood material and cultural capital have significant and independent impacts on educational success. The predictions of the two theoretical approaches on the effect of these parental resources on educational career are the same. Namely, one unit increase in childhood material and cultural capital is linked to the increase in the likelihood of continuing the educational career compared to leaving the school system. Results confirm this hypothesis, however, a gender-specific variation emerges. While for males childhood material credentials appear to have a particular importance in influencing the odds of educational attainment through the whole school career, for females cultural capital exerts a greater effect on educational success, at least at tertiary level. It suggests that for men the rational action theory, for women the cultural reproduction thesis provide the most appropriate explanation for educational behaviour.

As for the second set of hypotheses, both theoretical approaches have the same predictions: the net effects of family resources are the largest in the case of an academic track of education. My results support this hypothesis indicating that, on the one hand more prestigious educational branches require more 'inherited' cultural capital, on the other hand they provide 'easier' access to the best labour market positions.

The next branch of my findings concerns the life-course variation in the effect of childhood material and cultural capital on educational career. In this case the gender-specific differences are striking again: for males the influence of childhood material capital as well as the impact of cultural capital is smaller at tertiary level as compared to secondary transition, operating as the rational action theory predicts. However, for females it is true only for material credentials; as for the cultural capital, its effect appears to be stable over the whole educational career confirming the hypothesis derived from the cultural reproduction theory.

According to my findings, the interactions between the social class origins of individuals and their material-cultural resources are significant, however, operating in the reverse directions for males and females. The prediction of the rational action theory confirms the parameter estimates for men, that is, the importance of childhood material capital is the most vital for pupils with the least advantageous class background. For women, findings support the predictions of the cultural reproduction theory, at least at tertiary level, namely the effect of cultural capital is the greatest for individuals with the most advantageous class origins.

In sum, my findings leave no doubt that investigating school careers both the rational action theory and the cultural reproduction thesis are relevant, but the resource transmission process appears to be gendered. Girls are more likely to 'follow' the social reproduction 'route', but boys' behaviour is 'commanded' by a rational action approach.

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ON SOME PROPERTIES OF MORTALITY RATES

EMIL VALKOVICS[1]

SUMMARY

The first part of the paper underlines the necessity to consider in the analysis of mortality the double nature of general age-specific mortality rates: they determine with the number and age distribution of persons exposed to the risk of dying the number and the age distribution of the deceased. An attempt is made to separate the impact of these two roles.

The second part of the contribution describes the method of decomposition of the differences between the life expectancies at birth (and at higher ages) elaborated and used in the Demographic Research Institute of the HCSO, based on the evidence that the life expectancy at birth may be defined, among others, as the mean age of all the deceased of the life table and this mean age is equal to the weighted arithmetic mean of the mean ages of victims of different causes of death.

KEYWORDS: Mortality rates; Life expectancies; Causes of death.

Changing age-specific mortality rates always lead to the change of all the other life-table functions. The intensity of the phenomenon studied (i.e. mortality) remains equal to unity in all cases and the distribution of the deceased of the life table by ages changes in all cases. Life expectancy at birth remains equal among others to the mean age of the deceased of the life-table in all cases and this mean age remains equal to the weighted mean of the mean ages of victims of different causes of death in all cases. The decomposition of the differences between the two life expectancies is therefore the decomposition of the differences between the two weighted arithmetic means in all cases.

Several methods of decomposing the differences between the life-expectancies at birth have already been elaborated and published. The general age-specific mortality rates and the age- and cause-specific mortality rates have a certain role in all of them, but solely or almost solely in the distribution of the gains (or losses) in the number of person-years by causes of death studied. Their influence on the number and distribution by age and causes of death of the deceased of the life-tables compared is entirely neglected in all of them.

The method elaborated and used for this purpose in the Demographic Research Institute of the HCSO starts from distributing the deceased in the death function of the life-

[1] D. Sc., Scientific adviser of the Demographic Research Institute of the HCSO.

table compared by causes of death. We are therefore highly interested in studying the already known and unknown or simply neglected properties of general age-specific mortality rates and of age- and cause-specific mortality rates influencing the distribution of the deceased of the life table by age and causes of death studied.

1. The double nature of the general age-specific and age- and cause-specific mortality rates

The general age-specific mortality rates with the number and age distribution of persons exposed to the risk of dying, immediately determine the number and the age distribution of the deceased. They have therefore a double nature. If we consider an age interval with a given number of those exposed to the risk of dying, a higher value of the corres-ponding age-specific mortality rate produces a higher number and a lower value a lower number of the deceased. If the number of those exposed to the risk of dying is given for all the age groups, it is easy to establish which from the two series of age-specific mortality rates produces a higher or a lower number of the deceased. In a separate age group a higher rate produces more and a lower rate produces less of them. This is not true if we consider the sum of general age-specific mortality rates. A higher sum may produce the same or a lower and a lower sum the same or a higher total number of deceased persons because the number of the deceased does not only depend on the level of the rates, but it also depends on some other still neglected properties of them. It is obviously true that if in one of the series of the age-specific mortality rates all the values are lower than in the other one, the number of the deceased and the number of years they lived in different age groups and the total number of the deceased and of years they lived will be lower the and inversely. Nevertheless it may happen that the lower the values of all the rates, and the lower the value of their sum, then a lower number of deceased and a lower number of years they lived in all the age groups is connected with a higher number of years per one deceased (the total number of years lived divided by the total number of deceased). Such a situation is presented in Table 1.

Column (1) of Table 1 shows the age groups, column (2) the mean ages at death in different age groups (calculated by using an appropriate weighting procedure), column (3) the number of those exposed to the risk of dying in different age groups (equal in this case to the number of years in different age groups ($n^{(M)} = n^{(F)}$), columns (4) and (5) the general age-specific mortality rates of Hungarian males and females in 1966, columns (6) and (7) the number of deceased males and females. Column (8) shows that the number of deceased males is higher in all the age-groups, columns (9) and (10) present the number of years lived by the deceased males and females. It is clear that the total number of years lived by deceased females is lower than that lived by deceased males, nevertheless the total number of years divided by the total number of the deceased is higher in the case of females (84.200390 > 83.077122). This fact may only be explained by an until now neglected property of the series of general age-specific mortality rates: that is the ratios of the values of neighbouring rates in these series are different. The values of the rates experienced after childhood at higher ages exceed much more the rates experienced at younger ages by females. More precisely: their descent during the years of early childhood and their ascent after the minimum value attained is quicker than in the case of males.

Table 1

The double role of general age-specific mortality rates using the data of the Hungarian male and female population for 1966

Age groups (years) $x, x+n$	$\bar{x}$	$n^{(M)} = n^{(F)}$	${}_n m_x^{(M)}$	${}_n m_x^{(F)}$	$n\,{}_n m_x^{(M)}$	$n\,{}_n m_x^{(F)}$	$n\,{}_n m_x^{(F)} - n\,{}_n m_x^{(M)}$	$\bar{x}\,n\,{}_n m_x^{(M)}$	$\bar{x}\,n\,{}_n m_x^{(F)}$	$\bar{x}(n^{(F)}-n^{(M)})\,{}_n m_x^{(M)}$	$\bar{x}(n\,{}_n m_x^{(F)} - n\,{}_n m_x^{(M)})$	(11)+(12)
(1)	(2)	(3)	(4)	(5)	(6)=(3)·(4)	(7)=(3)·(5)	(8)=(7)-(6)	(9)=(12)·(6)	(10)=(2)·(7)	(11)	(12)=(2)·(8)	(13)
0	0.13935	1	0.045125	0.036906	0.045125	0.036906	-0.008219	0.006288	0.005143	0	-0.001145	-0.001145
1–4	2.50210	4	0.001235	0.001009	0.004940	0.004036	-0.000904	0.012360	0.010098	0	-0.002262	-0.002262
5–9	7.49737	5	0.000448	0.000269	0.002240	0.001345	-0.000895	0.016794	0.010084	0	-0.006710	-0.006710
10–14	12.63333	5	0.000438	0.000255	0.002190	0.001275	-0.000915	0.027667	0.016107	0	-0.011560	-0.011560
15–19	18.10476	5	0.000913	0.000502	0.004565	0.002510	-0.002055	0.082648	0.045443	0	-0.037205	-0.037205
20–24	22.87185	5	0.001373	0.000560	0.006865	0.002800	-0.004065	0.157015	0.064041	0	-0.092974	-0.092974
25–29	27.78945	5	0.001436	0.000723	0.007180	0.003615	-0.003565	0.199528	0.100459	0	-0.099069	-0.099069
30–34	33.10997	5	0.001819	0.000909	0.009095	0.004545	-0.004550	0.301135	0.150485	0	-0.150650	-0.150650
35–39	37.75633	5	0.002551	0.001495	0.012755	0.007475	-0.005280	0.481582	0.282229	0	-0.199353	-0.199353
40–44	42.80283	5	0.003399	0.002314	0.016995	0.011570	-0.005425	0.727434	0.495229	0	-0.232205	-0.232205
45–49	47.69929	5	0.004844	0.003319	0.024220	0.016595	-0.007625	1.155277	0.791570	0	-0.363707	-0.363707
50–54	52.68968	5	0.008590	0.005305	0.042950	0.026525	-0.016425	2.263022	1.397594	0	-0.865428	-0.865428
55–59	57.67475	5	0.013779	0.008137	0.068895	0.040685	-0.028210	3.973502	2.346497	0	-1.627005	-1.627005
60–64	62.65640	5	0.023267	0.013425	0.116335	0.067125	-0.049210	7.289132	4.205811	0	-3.083321	-3.083321
65–69	67.63221	5	0.037467	0.023770	0.187335	0.118850	-0.068485	12.669880	8.038088	0	-4.631792	-4.631792
70–74	72.59156	5	0.058186	0.042243	0.290930	0.211215	-0.079715	21.119063	15.332426	0	-5.786637	-5.786637
75–79	77.52282	5	0.092015	0.073663	0.460075	0.368315	-0.091760	35.666311	28.552817	0	-7.113494	-7.113494
80–84	82.40913	5	0.146590	0.125405	0.732950	0.627025	-0.105925	60.401772	51.672585	0	-8.729187	-8.729187
85–	89.44615	15	0.236192	0.218551	3.542880	3.278265	-0.264615	316.896976	293.228183	0	-23.668793	-23.668793
Total	–	100	–	–	5.578520	4.830677	-0.747843	463.447386	406.744889	0	-56.702497	-56.702497
Average	–	–	–	–	–	–	–	83.077122	84.200390	0	1.123268	1.123268

Source: Here and in the following tables the data of the Hungarian male and female population for 1966 are used.

Table 2

The influence of general age-specific mortality rates on the change of the age structure of the deceased

Age groups (years) x, $x+n$	$\bar{x}$	$n^{(M)} = n^{(F)}$	$_n m_x^{(M)}$	$_n m_x^{(F)}\left(\frac{\sum n_n m_x^{(M)}}{\sum n_n m_x^{(F)}}\right)$	$n_n m_x^{(M)}$	$n_n m_x^{(F)}\left(\frac{\sum n_n m_x^{(M)}}{\sum n_n m_x^{(F)}}\right)$	$n_n m_x^{(F)}\left(\frac{\sum n_n m_x^{(M)}}{\sum n_n m_x^{(F)}}\right) - n_n m_x^{(M)}$	$\bar{x}\, n_n m_x^{(M)}$	$\bar{x}\, n_n m_x^{(F)}\left(\frac{\sum n_n m_x^{(M)}}{\sum n_n m_x^{(F)}}\right)$	$\bar{x}(n^{(F)} - n^{(M)})\, n_n m_x^{(M)}$	$\bar{x}\left[n_n m_x^{(F)}\left(\frac{\sum n_n m_x^{(M)}}{\sum n_n m_x^{(F)}}\right) - n_n m_x^{(M)}\right]$	(11)+(12)
(1)	(2)	(3)	(4)	(5)	(6)=(3)·(4)	(7)=(3)·(5)	(8)=(7)-(6)	(9)=(12)·(6)	(10)=(2)·(7)	(11)	(12)=(2)·(8)	(13)
0	0.13935	1	0.045125	0.042620	0.045125	0.042620	-0.002505	0.006288	0.005939	0	-0.000349	-0.000349
1-4	2.50210	4	0.001235	0.001165	0.004940	0.004660	-0.000279	0.012360	0.011662	0	-0.000698	-0.000698
5-9	7.49737	5	0.000448	0.000311	0.002240	0.001555	-0.000687	0.016794	0.011643	0	-0.005151	-0.005151
10-14	12.63333	5	0.000438	0.000294	0.002190	0.001470	-0.000718	0.027667	0.018596	0	-0.009071	-0.009071
15-19	18.10476	5	0.000913	0.000580	0.004565	0.002900	-0.001666	0.082648	0.052486	0	-0.030162	-0.030162
20-24	22.87185	5	0.001373	0.000647	0.006865	0.003235	-0.003631	0.157015	0.073968	0	-0.083047	-0.083047
25-29	27.78945	5	0.001436	0.000835	0.007180	0.004175	-0.003005	0.199528	0.116021	0	-0.083507	-0.083507
30-34	33.10997	5	0.001819	0.001050	0.009095	0.005250	-0.003846	0.301135	0.173794	0	-0.127341	-0.127341
35-39	37.75633	5	0.002551	0.001726	0.012755	0.008630	-0.004123	0.481582	0.325913	0	-0.155669	-0.155669
40-44	42.80283	5	0.003399	0.002672	0.016995	0.013360	-0.003634	0.727434	0.571889	0	-0.155545	-0.155545
45-49	47.69929	5	0.004844	0.003833	0.024220	0.019165	-0.005056	1.155277	0.914109	0	-0.241168	-0.241168
50-54	52.68968	5	0.008590	0.006126	0.042950	0.030630	-0.012319	2.263022	1.613938	0	-0.649084	-0.649084
55-59	57.67475	5	0.013779	0.009397	0.068895	0.046985	-0.021912	3.973502	2.709733	0	-1.263769	-1.263769
60-64	62.65640	5	0.023267	0.015503	0.116335	0.077515	-0.038818	7.289132	4.856936	0	-2.432196	-2.432196
65-69	67.63221	5	0.037467	0.027450	0.187335	0.137250	-0.050086	12.669880	9.282453	0	-3.387427	-3.387427
70-74	72.59156	5	0.058186	0.048783	0.290930	0.243915	-0.047017	21.119063	17.706025	0	-3.413037	-3.413037
75-79	77.52282	5	0.092015	0.085067	0.460075	0.425335	-0.034741	35.666311	32.973091	0	-2.693219	-2.693219
80-84	82.40913	5	0.146590	0.144819	0.732950	0.724095	-0.008854	60.401772	59.672121	0	-0.729651	-0.729651
85-	89.44615	15	0.236192	0.252385	3.542880	3.785775	0.242897	316.896976	338.623177	0	21.726201	21.726201
Total	–	100	0.679667	0.645263	5.578520	5.578520	0.000000	463.447386	469.719494	0	6.266109	6.266109
Average	–	–	–	–	–	–	–	83.077122	83.200378	0	1.123256	1.123256

Table 3

Sex differences, sex ratios, magnitudes related to the lowest values and ratios between neighbouring values of general age-specific mortality rates

Age groups (years) $x, x+n$	${}_nm_x^{(M)}$	${}_nm_x^{(F)}$	${}_nm_x^{(M)} - {}_nm_x^{(F)}$	${}_nm_x^{(M)} / {}_nm_x^{(F)}$	$\frac{{}_nm_x^{(M)}}{{}_5m_{10}^{(M)}} x100$	$\frac{{}_nm_x^{(F)}}{{}_5m_{10}^{(F)}} x100$	(6)-(7)	(6)/(7)	$\frac{{}_nm_{x+n}^{(M)}}{{}_nm_x^{(M)}}$	$\frac{{}_nm_{x+n}^{(F)}}{{}_nm_x^{(F)}}$	(10)-(11)	(10)/(11)
(1)	(2)	(3)	(4)=(2)-(3)	(5)=(2/3)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
0	0.045125	0.036906	0.008219	1.222701	10 303	14 473	-4 170	0.711846	0.027368	0.027340	0.000029	1.001050
1–4	0.001235	0.001009	0.000226	1.223984	282	396	-114	0.712594	0.362753	0.266601	0.096152	1.360659
5–9	0.000448	0.000269	0.000179	1.665428	102	105	-3	0.969598	0.977679	0.947955	0.029723	1.031355
10–14	0.000438	0.000255	0.000183	1.717647	100	100	0	1.000000	2.084475	1.968627	0.115847	1.058847
15–19	0.000913	0.000502	0.000411	1.818725	208	197	12	1.058847	1.503834	1.115538	0.388296	1.348079
20–24	0.001373	0.000560	0.000813	2.451786	313	220	94	1.427409	1.045885	1.291071	-0.245187	0.810091
25–29	0.001436	0.000723	0.000713	1.986169	328	284	44	1.156331	1.266713	1.257261	0.009452	1.007518
30–34	0.001819	0.000909	0.000910	2.001100	415	356	59	1.165024	1.402419	1.644664	-0.242246	0.852708
35–39	0.002551	0.001495	0.001056	1.706355	582	586	-4	0.993426	1.332419	1.547826	-0.215407	0.860832
40–44	0.003399	0.002314	0.001085	1.468885	776	907	-131	0.855173	1.425125	1.434313	-0.009188	0.993594
45–49	0.004844	0.003319	0.001525	1.459476	1 106	1 302	-196	0.849695	1.773328	1.598373	0.174955	1.109458
50–54	0.008590	0.005305	0.003285	1.619227	1 961	2 080	-119	0.942701	1.604075	1.533836	0.070239	1.045793
55–59	0.013779	0.008137	0.005642	1.693376	3 146	3 191	-45	0.985870	1.688584	1.649871	0.038713	1.023464
60–64	0.023267	0.013425	0.009842	1.733110	5 312	5 265	47	1.009002	1.610306	1.770577	-0.160271	0.909481
65–69	0.037467	0.023770	0.013697	1.576231	8 554	9 322	-767	0.917668	1.552993	1.777156	-0.224163	0.873864
70–74	0.058186	0.042243	0.015943	1.377412	13 284	16 566	-3 281	0.801918	1.581394	1.743792	-0.162398	0.906871
75–79	0.092015	0.073663	0.018352	1.249135	21 008	28 887	-7 879	0.727236	1.593110	1.702415	-0.109305	0.935794
80–84	0.146590	0.125405	0.021185	1.168933	33 468	49 178	-15 710	0.680543	1.611242	1.742761	-0.131519	0.924534
85–	0.236192	0.218551	0.017641	1.080718	53 925	85 706	-31 781	0.629185	–	–	–	–

Table 4

The difference quotients and curvatures of the empirical curves of general age-specific mortality rates

Age groups (years) x, $x+n$	Approximate values of difference quotients		(2)-(3)	(2)/(3)	The angles calculated from difference quotients (DEG)		The differences between the neighbouring angles		The mean arc length		The mean curvatures	
	in the case of males	in the case of females			in the case of males	in the case of females	in the case of males	in the case of females	in the case of males	in the case of females	in the case of males	in the case of females
(1)	(2)	(3)	(4)	(5)	(6)=arc tan (2)	(7)=arc tan (3)	(8)	(9)	(10)	(11)	(12)=(8)/(10)	(13)=(9)/(11)
0	-18.576	-15.193	-3.383	1.223	93.0814	93.7658	77.9401	77.8155	43.954	35.975	1.773	2.163
1–4	-0.158	-0.148	-0.010	1.068	171.0215	171.5813	8.8639	8.2468	5.057	5.050	1.753	1.633
5–9	-0.002	-0.003	0.001	0.667	179.8854	179.8281	-174.9132	-177.2515	5.136	5.136	-34.056	-34.512
10–14	0.087	0.045	0.042	1.933	4.9722	2.5766	0.5114	-1.8891	5.492	5.477	0.093	-0.345
15–19	0.096	0.012	0.084	8.000	5.4836	0.6875	-4.7388	1.2026	4.789	4.767	-0.990	0.252
20–24	0.013	0.033	-0.020	0.394	0.7448	1.8901	3.3734	0.1145	4.918	4.920	0.686	0.023
25–29	0.072	0.035	0.037	2.057	4.1182	2.0045	4.8603	5.1769	5.334	5.324	0.911	0.972
30–34	0.158	0.126	0.032	1.254	8.9785	7.1814	0.5581	2.0205	4.704	4.683	0.119	0.431
35–39	0.168	0.162	0.006	1.037	9.5366	9.2020	6.8994	2.1630	5.117	5.113	1.348	0.423
40–44	0.295	0.201	0.094	1.468	16.4361	11.3650	20.4705	10.3375	5.105	5.091	4.010	2.031
45–49	0.751	0.398	0.353	1.887	36.9065	21.7026	9.2443	7.8940	6.240	5.371	1.481	1.470
50–54	1.041	0.568	0.473	1.833	46.1508	29.5966	16.1527	17.0987	7.196	5.733	2.245	2.983
55–59	1.905	1.061	0.844	1.795	62.3035	46.6953	8.3868	17.6171	10.716	7.265	0.783	2.425
60–64	2.854	2.079	0.775	1.373	70.6903	64.3124	5.8493	10.6605	15.047	11.479	0.389	0.929
65–69	4.178	3.725	0.453	1.122	76.5395	74.9729	5.1667	6.1081	21.304	19.127	0.243	0.319
70–74	6.860	6.372	0.488	1.077	81.7063	81.0809	3.1775	3.5242	34.187	31.805	0.093	0.111
75–79	11.169	10.589	0.580	1.055	84.8837	84.6051	0.6257	1.0746	54.793	51.972	0.011	0.021
85–	12.733	13.237	-0.504	0.962	85.5094	85.6797	–	–	89.878	93.411	–	–

If we multiply the series of general age-specific mortality rates of females by a constant that makes them produce as many deceased females as the number of deceased males (5.578520/4.830677 = 1.154811), the number of deceased females in the oldest age group will already be higher than that of the males, but the total number of years lived by deceased females divided by the number of deceased females will differ from the value of this indicator just like in the previous situation. (See Table 2.) If we use another multiplier (e.g. 0.5) the result will be the same. It is clear that the series of general age-specific mortality rates for females produce, ceteris paribus, a higher mean age of the deceased. The age-structure of the sum of these rates is also different: it is older in the case of females and younger in the case of males.

In the past a considerable number of authors along with the United Nations Secretariat considered only the differences between the corresponding elements of general age-specific mortality rates e.g. $({}_n m_x^{(M)} - {}_n m_x^{(F)})$, the sex differences of rates and their ratios e.g. $({}_n m_x^{(M)} / {}_n m_x^{(F)})$, the sex ratios of rates. In our case they are shown in columns (4) and (5) in Table 3. The following columns of Table 3 already show the properties of general age-specific mortality rates which have been neglected up to now.

Columns (6) and (7) in Table 3 show that if we divide all the rates by the lowest rate in both series, i.e. the rate for 10–14 years of age and multiply the results of the division by 100, the rates for females obtained this way will be higher at younger ages and mainly at higher ages than the rates for males despite the fact that in reality the general age-specific mortality rates in all the age groups are lower in the case of females (See Figure 1.) The differences and ratios of these artificially created figures rise in both directions from the age-group 10–14.

Figure 1. The general age-specific mortality rates for Hungarian males and females related to their lowest values in the age group 10–14, 1966

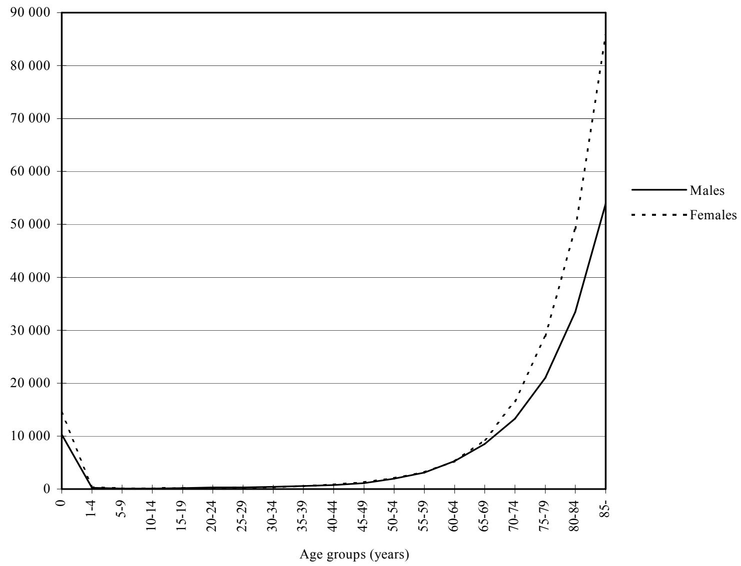


Table 5

Sums, cumulated values, the structure of the sums and cumulated values, the distance of the cumulated values from the sums and the multipliers which assure that the sums reach the general age-specific mortality rates

Age groups (years) x $x+n$	$_n m_x^{(M)}$	$_n m_x^{(F)}$	The percentage distribution of the sums of the values of		The cumulated values of the		The cumulated values of the		The distance of the cumulated values of		The multipliers of the cumulated values of	
			$_n m_x^{(M)}$	$_n m_x^{(F)}$	$_n m_x^{(M)}$	$_n m_x^{(F)}$	$_n m_x^{(M)}$	$_n m_x^{(F)}$	$_n m_x^{(M)}$	$_n m_x^{(F)}$	$_n m_x^{(M)}$	$_n m_x^{(F)}$
							calculated form the percentage distributions		from their sums		assuring the achievement of their sums	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
0	0.045125	0.036906	0.808906	0.763992	–	–	–	–	1.000000	1.000000	–	–
1–4	0.004940	0.004036	0.088554	0.083549	0.045125	0.036906	0.808906	0.763992	0.991911	0.992360	122.623712	129.891373
5–9	0.002240	0.001345	0.040154	0.027843	0.050065	0.040942	0.897460	0.847542	0.991025	0.991525	110.425547	116.988301
10–14	0.002190	0.001275	0.039258	0.026394	0.052305	0.042287	0.937614	0.875385	0.990624	0.991246	105.653666	113.235510
15–19	0.004565	0.002510	0.081832	0.051960	0.054495	0.043562	0.976872	0.901778	0.990231	0.990982	101.367557	109.891993
20–24	0.006865	0.002800	0.123061	0.057963	0.059060	0.046072	1.058704	0.953738	0.989413	0.990463	93.455130	103.850603
25–29	0.007180	0.003615	0.128708	0.074834	0.065925	0.048872	1.181765	1.011701	0.988182	0.989883	83.619188	97.843448
30–34	0.009095	0.004545	0.163036	0.094086	0.073105	0.052487	1.310473	1.086535	0.986895	0.989135	75.308324	91.035685
35–39	0.012755	0.007475	0.228645	0.154740	0.082200	0.057032	1.473509	1.180621	0.985265	0.988194	66.865207	83.701168
40–44	0.016995	0.011570	0.304651	0.239511	0.094955	0.064507	1.702154	1.335361	0.982978	0.986646	57.749092	73.886090
45–49	0.024220	0.016595	0.434165	0.343534	0.111950	0.076077	2.006805	1.574872	0.979932	0.984251	48.830460	62.497207
50–54	0.042950	0.026525	0.769917	0.549095	0.136170	0.092672	2.440970	1.918406	0.975590	0.980816	39.967320	51.126608
55–59	0.068895	0.040685	1.235005	0.842221	0.179120	0.119197	3.210887	2.467501	0.967891	0.975325	30.144038	39.526834
60–64	0.116335	0.067125	2.085410	1.389557	0.248015	0.159882	4.445892	3.309722	0.955541	0.966903	21.492672	29.214014
65–69	0.187335	0.118850	3.358149	2.460318	0.364350	0.227007	6.531302	4.699279	0.934687	0.953007	14.310882	20.279859
70–74	0.290930	0.211215	5.215183	4.372369	0.551685	0.345857	9.889451	7.159597	0.901105	0.928404	9.111785	12.967267
75–79	0.460075	0.368315	8.247259	7.624501	0.842615	0.557072	15.104633	11.531965	0.848954	0.884680	5.620485	7.671549
80–84	0.732950	0.627025	13.138789	12.980065	1.302690	0.925387	23.351893	19.156466	0.766481	0.808435	3.282308	4.220170
85–	3.542880	3.278265	63.509318	67.863469	2.035640	1.552412	36.490682	32.136531	0.635093	0.678635	1.740426	2.111724
Total	5.578520	4.830677	100.000000	100.000000	5.578520	4.830677	100.000000	100.000000	0.000000	0.000000	0.000000	0.000000

Columns (10) and (11) in Table 3 show the ratios of neighbouring rates separately in both series $({}_n m_{x+n}^{(M)}/{}_n m_x^{(M)})$ and $({}_n m_{x+n}^{(F)}/{}_n m_x^{(F)})$. The conclusions remain the same as before.

Columns (2) and (3) in Table 4 show the so called difference quotiens (i.e. the differences between the ordinate values divided by the differences between the abscissa values) calculated separately in both series. If we had the possibility to work with continuous and differentiable functions of age-specific mortality rates, it would be possible to calculate the differential coefficients or derivatives with respect to *x* (the age) so as to work with tangents at different points to the curves instead of secants. Nevertheless it is possible even when working with secants, – i.e. straight lines joining two points of the curves – to calculate the slopes and the differences between the tangents and angles made by these lines with the age axis and calculate the curvatures – i.e. the rates of the changes of the angles between the tangents with respect to the different arcs of the curves – and show that the curvature is e.g. at older age higher in the case of females than in the case of males. We may easily separate the monotonically descending and ascending segments of the empirical curves and distinguish the parts of the curves which are concave downwards and concave upwards even in our case.

Even in our case we would need a good approximation the length of the arc of the empirical curves. It is possible to demonstrate that in the case of females we obtain a curve, in the oldest age group, the arc of which is linked with a higher mean age of all the deceased of the life-table in question. This, however, does not mean a higher total length of life.

Columns (2) and (3) in Table 5 contain the values of the general age-specific mortality rates multiplied by the length of the age groups. The age-specific probabilities of surviving and dying of corresponding life tables may immediately be calculated by using the simple exponential formula ${}_n p_x = exp(-n_n m_x)$ and ${}_n q_x = 1 - {}_n p_x = 1 - exp(-n_n m_x)$, or the formulae of *Reed* and *Merrell*, of *Greville*, of *Keyfitz* and *Frauenthal*, etc. The same is true for the probabilities of surviving and dying from the exact age 0 to the exact age *x*, i.e. practically all life-table functions may already be calculated by using them. The sum of the multiplied general age-specific mortality rates is smaller in the case of females than in the case of males.

Columns (4) and (5) of Table 5 and Figure 2 show the age-structure (the percentage distribution) of the sums of these two series of multiplied rates. The elements of this distribution for younger ages are smaller and for older ages higher in the case of females.

Columns (6) and (7) present the cumulated values of these two series of multiplied rates. Columns (8) and (9) show the same calculated by using the data of their percentage distribution (included in columns (4) and (5) (see Figure 3). The figures in column (9) are smaller than those in column (8) and in the case of the female population they approach the upper limit 100 percent slower than in the case of the males. This slow convergence is also linked with a lower mortality level, i.e. with a longer life expectancy at birth of females.

Columns (10) and (11) in Table 5 show the distance between the cumulated general age-specific mortality rates and their sums. If we denote this distance by v_a, we may calculate it by using the formula $v_a = \left(\sum_{x=0}^{\omega} n_n m_x - \sum_{x=0}^{x=a} n_n m_x \right) / \sum_{x=0}^{\omega} n_n m_x$. The data in columns (6) and (7), and in columns (8) and (9) are both appropriate for realising this calculation. The cumulated values of the general age-specific mortality rates may be reproduced by using the *Baule–Mitscherlich* saturation function: $\sum_{x=0}^{x=a} {}_n m_x = \sum_{x=0}^{\omega} {}_n m_x (1 - v_a)$, or $100(1 - v_a)$, where v_a denotes the distance in question. The values of the v_a are bigger at all ages in the case of females than in the case of males, which is also due to the lower mortality level, i.e. to the higher life expectancy of females at birth. In the case of the male population the values of v_{50} = (5.578520 - 0.136170)/5.578520 = (100 - 2.440970)/100 = 0.975590, and the value of multiplied rates cumulated from the age 0 to 50 = 5.578520 (1 - 0.975590), or in percentages 100(1 - 0.975590) = 2.440970.

Figure 2. Distribution by age groups of the sum of general age-specific mortality rates for Hungarian males and females, 1966

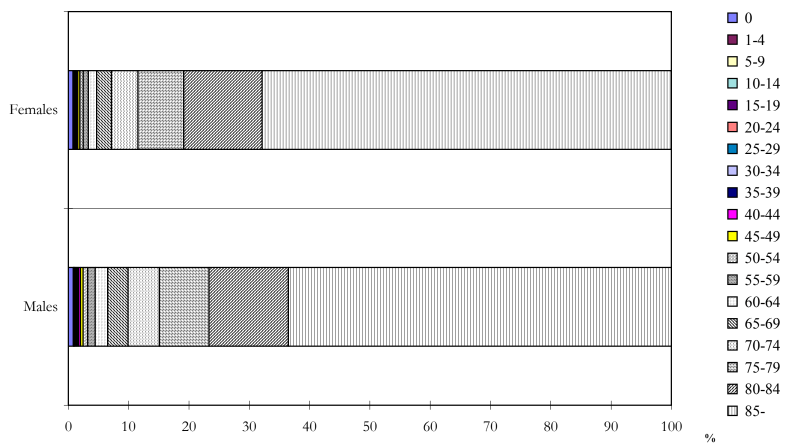
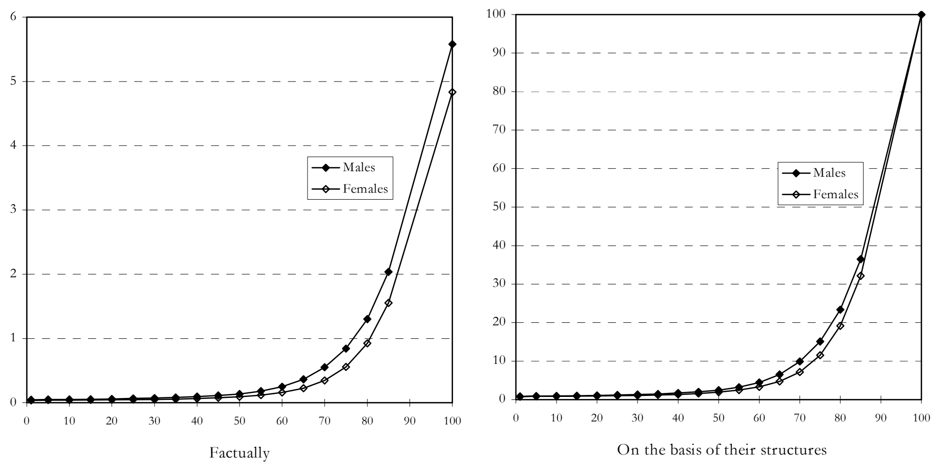


Figure 3. Cumulated values of general age-specific mortality rates of Hungarian males and females factually and on the basis of the age structures of their sums, 1966



Columns (12) and (13) in Table 5 show those multipliers of the cumulated values of the multiplied age-specific mortality rates $_nm_x^{(F)}$ and $_nm_x^{(N)}$ which assure that they reach their total sum. If we denote these multipliers by s_a, the formula used for their calculation may be written as follows: $s_a = \left(\sum_{x=0}^{\omega} {}_nm_x - \sum_{x=0}^{x=a} {}_nm_x \right) / \sum_{x=0}^{x=a} {}_nm_x$. Their values may be calculated by using the data of columns (6) and (7) or columns (8) and (9). In the case of the male population the value of

$$s_{50} = (5.578520 — 0.136170)/0.136170 = (100 — 2.440970)/2.440970,$$

because

$$5.578520 — 0.136170 = 39.967320 \cdot 0.136170 = 5.442350$$

and

$$5.442350/39.967320 = 0.136170,$$

and if we use the data of the percentage distribution:

$$100 — 2.440970 = 39.967320 \cdot 2.440970 = 97.559030$$

and

$$97.559030/39.967320 = 2.440970.$$

The calculated values of general age-specific mortality rates may be reproduced by using the well-known logistic function

$$\sum_{x=0}^{x=a} {}_nm_x = \sum_{x=0}^{\omega} {}_nm_x \left(\frac{1}{1+s_a} \right) = \sum_{x=0}^{\omega} {}_nm_x (1+s_a)^{-1} \quad \text{or} \quad 100 \left(\frac{1}{1+s_a} \right) = 100(1+s_a)^{-1}.$$

The sum of multiplied rates cumulated from age 0 to 50 equals $5.578850\left(1/(1+39.967320)\right) =$ $= 5.578850(1+39.967320)^{-1} = 0.136170$, or expressed in percentages $100\left(1/(1+39.967320)\right) =$ $= 100\,(1+39.967320)^{-1} = 2.440970$. The values of this multipliers s_a are higher in the case of the female population than in the case of the male population. This property of the general age-specific mortality rates is also linked with the lower mortality level, i.e. higher life expectancy at birth of females.

On the basis of the example we have just analysed, it is possible to state that when comparing two series of general age-specific mortality rates (and age- and cause-specific mortality rates) it is not sufficient to consider only their differences and their ratios, but, especially if we want to understand their roles in creating differences in mortality levels, it is necessary to consider the differences in their rates of descent and ascent, in their relative magnitudes, in their concavities, curvatures, difference quotients (or derivatives, if possible), the differences in the age structure of their sums, in the speed and acceleration of the convergence of their cumulated values to their sums, as well as the distances of their cumulated values from their sums, the differences of the multipliers which – in the different age groups – assure that they reach their sums by their cumulated values. These neglected properties of general age-specific mortality rates are related to their properties – which have already been considered many times –, i.e. to their differences and their ratios. We may formulate the following hypothesis: bigger differences between the corresponding values of the rates in question and their higher ratios involve bigger differences in their up until now neglected properties as well.

The same is true of the age- and cause-specific mortality rates with a few exceptions concerning mainly their absence in a few cases at some ages, the nature of their concavity and curvature, etc. which must be analysed in the case of each cause of death separately. It is very important to understand that they also have a double nature as well: they determine, with the number and age structure of those exposed to the risk of dying, the number and the age structure of the victims of given causes of death. In the case of the life tables by causes of death, the sum of the victims of different causes of death is equal to the radix of the life table $\left(l_0 = \sum_x \sum_i {}_n d_{i,x} \right)$ and thus it is easy to calculate the structure of the deceased in the life table by causes of death and the mean ages of victims of different causes of death. The mean age of all the deceased, as it has already been mentioned, is equal to the weighted arithmetic mean of the mean ages of victims of different causes of death.

In case of the period life-tables by causes of death, we may ask whether the number and age structure of those exposed to the risk of dying are really separate immediate determinants of the number and age structure of the deceased or are also determined by general age-specific mortality rates which are the sums of cause- and age-specific mortality rates. Demographers know that one of the possibilities of calculating the probabilities of surviving from birth to age a, if $l_0 = 1$, is the use of cumulated (or integrated) values of general age-specific mortality rates $l_a / l_0 = {}_a p_0 = exp\left(- \sum_{x=0}^{a} n_n m_x \right)$ and $l_a/l_0 = {}_a p_0 = {}_a p_0^{(1)} \cdot {}_a p_0^{(2)} \cdot {}_a p_0^{(3)} \ldots$, where ${}_a p_0^{(1)}, {}_a p_0^{(2)}, {}_a p_0^{(3)}$ denote the corresponding probabilities of surviving from birth to the age a by causes of death denoted here by (1), (2), (3) etc. The number and age structure of exposed to risk of dying may easily be calculated by using simply the general age-specific or age- and cause-specific mortality rates and the total number of exposed to risk of dying in a period life table. If $l_0 = 1$, it is equal to the life expectancy at birth i.e. to the mean age of all the deceased in the life table

$$e_0^0 = \sum_{x=0}^{\omega} {}_n L_x = T_0 = \sum_{x=0}^{\omega} x d_x / \sum_{x=0}^{\omega} d_x \; .$$

Another important property of the general age-specific and age- and cause-specific mortality rates therefore, is that their already known and up until now neglected properties determine the number and age composition of the deceased by specifying the number and age composition of those exposed to the risk of dying as well: a higher mean age of all the deceased in a period life-table is, among others, the result of a higher mean age of those exposed to the risk of dying and inversely: a lower mean age of all the deceased is, among others, the result of a younger age structure of those exposed to the risk of dying.

Let us consider after this introduction the method of decomposing the differences between the life expectancies at birth elaborated and used in the Demographic Research Institute of the HCSO.

Table 6

The mean ages at death and the proportions of victimes of different causes of death according to the abridged Hungarian life-tables by causes of death for 1966

Causes of death studied	Infectious and parasitic diseases	Neoplasms (all forms)	Diseases of the circulatory system	Cerebrovas-cular diseases	Diseases of the respiratory systems	Diseases of the digestive system	Chronic liver disease and cirrhosis of liver	Congenital anomalies	Certain conditions originating in the perinatal period	Injury and poisoning	All other causes of death	Averages
	001–139	140–239	390–429 440–459	430–438	460–519	520–570 572–579	571	740–759	760–779	800–999		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Male population												
Mean ages at death (years)	63.495	68.272	74.375	75.658	65.450	63.034	64.535	1.051	0.145	52.674	68.733	67.528
Proportion of deceased (%)	3.266	19.323	35.325	14.987	4.933	2.918	1.036	0.739	2.781	7.452	7.241	100.000
Female population												
Mean ages at death (years)	63.141	68.497	78.388	77.731	68.190	68.182	69.371	1.062	0.132	67.650	69.565	72.278
Proportion of deceased (%)	1.409	16.857	41.018	19.131	3.664	3.050	0.608	0.738	2.171	4.735	6.619	100.000

2. The method of decomposing the differences between the life expectancies at birth elaborated and used in the Demographic Research Institute of the HCSO

When using this method first we calculate the number of the deceased in each age group of the life table based on the causes of death studied by using the elements of distribution of the deceased relying on the causes of death in reality or the composition by causes of death of the general age-specific death rates, which are sums of age- and cause-specific death rates

It is natural that $\sum_{i} {}_n d_{i,x} = {}_n d_x$; $\sum_x \sum_i {}_n d_{i,x} = l_0$; $\sum_x \sum_i {}_n d_{i,x} / \sum_{x=0} {}_n d_x = 1$ where i denotes the causes of death ($i = 1, ..., 11$).

The structure by the causes of death of the deceased in a life table is different from that of the deceased based on the causes of death in reality mainly because of the differences in the age structure of the real and the stationary life-table populations.

The differences between the life expectancies at age x can be calculated by using two methods. If we are interested only in calculating the differences between life expectancies at birth, the easiest way is perhaps first to calculate directly the mean age of victims of different causes of death, then those of all causes of death.

The mean age of death in different age groups may be calculated by using the formula

$$\bar{x}^{(M)} = x + \frac{{}_n L_x^{(M)} - n l_{x+n}^{(M)}}{{}_n d_x^{(M)}}$$

in the case of males and

$$\bar{x}^{(F)} = x + \frac{{}_n L_x^{(F)} - n l_{x+n}^{(F)}}{{}_n d_x^{(F)}}$$

in the case of females.

The mean age at death of all victims and those of different causes of death may be calculated by using the formulae in the case of males:

$$e_0^{0(M)} = \frac{\sum_{x=0}^{\omega} \bar{x}^{(M)} {}_n d_x^{(M)}}{\sum_{x=0}^{\omega} {}_n d_x^{(M)}} \quad \text{and} \quad e_{i,0}^{0(M)} \approx \frac{\sum_{x=0}^{\omega} \bar{x}^{(M)} {}_n d_{i,x}^{(M)}}{\sum_{x=0}^{\omega} {}_n d_{i,x}^{(M)}}$$

and in the case of females the formulae:

$$e_0^{0(F)} = \frac{\sum_{x=0}^{\omega} \bar{x}^{(F)} {}_n d_x^{(F)}}{\sum_{x=0}^{\omega} {}_n d_x^{(F)}} \quad \text{and} \quad e_{i,0}^{0(F)} \approx \frac{\sum_{x=0}^{\omega} \bar{x}^{(F)} {}_n d_{i,x}^{(F)}}{\sum_{x=0}^{\omega} {}_n d_{i,x}^{(F)}} .$$

If we divide the number of years the deceased lived by their mean ages, we obtain their numbers in the corresponding life table:

$$\frac{\sum_{x=0}^{\omega} \bar{x}^{(M)}\, {}_n d_{i,x}^{(M)}}{e_{i,0}^{0(M)}} \approx \sum_{x=0}^{\omega} {}_n d_{i,x}^{(M)} \quad \text{and} \quad \frac{\sum_{x=0}^{\omega} \bar{x}^{(M)}\, {}_n d_x^{(M)}}{e_0^{0(M)}} = \sum_{x=0}^{\omega} {}_n d_x^{(M)}$$

in the case of males and

$$\frac{\sum_{x=0}^{\omega} \bar{x}^{(F)}\, {}_n d_{i,x}^{(F)}}{e_{i,0}^{0(F)}} \approx \sum_{x=0}^{\omega} {}_n d_{i,x}^{(F)} \quad \text{and} \quad \frac{\sum_{x=0}^{\omega} \bar{x}^{(F)}\, {}_n d_x^{(F)}}{e_0^{0(F)}} = \sum_{x=0}^{\omega} {}_n d_x^{(F)}$$

in the case of females.

The most important question remains the same as what it was before: how to transform the differences between the age-specific mortality rates into differences between life expectancies at birth?

If we assume as before that $l_0 = 1$, it is clear that

$$l_x = exp\left[-M_x\right] = exp\left[-\int_0^x \mu_x d\,x\right] = exp\left[ln_x\, p_0\right] = exp\left[ln\, l_x\right]$$

where ${}_x p_0 = l_x/l_0 = exp[-M_x]$, i.e. the probability of surviving from birth till the exact age x and the number of survivors in the life table with $l_0 = 1$; μ_x denotes the value of the definite integral of the force of mortality within the limits of age groups, i.e. approximately the value of the age-specific mortality rate denoted generally m_x (or ${}_n m_x$) in life tables ${}_n m_x = (-ln_n\, p_x)/n$;

$$M_x = \int_0^x \mu_x dx = \sum_{x=0}^{x} {}_1 m_x = \sum_{x=0}^{x} n_x m_x = -ln_x\, p_0 = -ln\left(l_x / l_0\right)$$

If we consider the additivity of μ_x or m_x subdivided by causes of death, i.e. the fact that

$$\mu_{1,x} + \mu_{2,x} + \ldots = \sum_i \mu_{i,x} = \mu_x\,,$$

or

$${}_n m_{1,x} + {}_n m_{2,x} + \ldots = \sum_i {}_n m_{i,x} = {}_n m_x\,,$$

then we may write

$$e_0^{0(F)} - e_0^{0(M)} = \int_0^{\infty} \left\{exp\left[-M_x^{(F)}\right] - exp\left[-M_x^{(M)}\right]\right\} dx =$$

$$= \int_0^\infty \left\{ exp\left[-(M_{1,x}^{(F)} + M_{2,x}^{(F)} + \ldots)\right] - exp\left[-(M_{1,x}^{(M)} + M_{2,x}^{(M)} + \ldots)\right] \right\} dx =$$

$$= \int_0^\infty \left\{ \left[exp\,(-M_{1,x}^{(F)})\, exp\,(-M_{2,x}^{(F)})\ldots\right] - \left[exp\,(-M_{1,x}^{(M)})\, exp\,(-M_{2,x}^{(M)}\ldots\right] \right\} dx =$$

$$= \int_0^\infty \left\{ exp\left[-\int_0^x \mu_x^{(F)}\, dx\right] - exp\left[-\int_0^x \mu_x^{(M)}\, dx\right] \right\} dx =$$

$$= \int_0^\infty \left\{ exp\left[-(\int_0^x \mu_{1,x}^{(F)}\, dx + \int_0^x \mu_{2,x}^{(F)}\, dx + \ldots)\right] - exp\left[-(\int_0^x \mu_{1,x}^{(M)}\, dx + \int_0^x \mu_{2,x}^{(M)}\, dx + \ldots)\right] \right\} dx =$$

$$= \int_0^\infty \left\{ \left[exp\,(-\int_0^x \mu_{1,x}^{(F)}\, dx)\, exp\,(-\int_0^x \mu_{2,x}^{(F)}\, dx)\ldots\right] - \left[exp\,(-\int_0^x \mu_{1,x}^{(M)}\, dx)\, exp\,(-\int_0^x \mu_{2,x}^{(M)}\, dx)\ldots\right] \right\} dx =$$

$$= \int_0^\infty \left\{ exp\left[ln\, {}_x p_0^{(F)}\right] - exp\left[ln\, {}_x p_0^{(M)}\right] \right\} dx =$$

$$= \int_0^\infty \left\{ exp\left[ln\, {}_x p_{1,0}^{(F)}\right] - exp\left[ln\, {}_x p_{1,0}^{(M)}\right] \right\} dx + \int_0^\infty \left\{ exp\left[ln\, {}_x p_{2,0}^{(F)}\right] - exp\left[ln\, {}_x p_{2,0}^{(M)}\right] \right\} dx + \ldots =$$

$$= \int_0^\infty \left[l_x^{(F)} - l_x^{(M)}\right] dx = \int_0^\infty \left[l_{1,x}^{(F)} - l_{1,x}^{(M)}\right] dx + \int_0^\infty \left[l_{2,x}^{(F)} - l_{2,x}^{(M)}\right] dx + \ldots =$$

$$= \sum_{x=0}^{\omega} \left[L_x^{(F)} - L_x^{(M)}\right] = \sum_{x=0}^{\omega} \left[L_{1,x}^{(F)} - L_{1,x}^{(M)}\right] + \sum_{x=0}^{\omega} \left[L_{2,x}^{(F)} - L_{2,x}^{(M)}\right] + \ldots = \sum_{x=0}^{\omega} \sum_i \left[L_{i,x}^{(F)} - L_{i,x}^{(M)}\right] =$$

$$= T_0^{(F)} - T_0^{(M)} = \left[T_{1,0}^{(F)} - T_{1,0}^{(M)}\right] + \left[T_{2,0}^{(F)} - T_{2,0}^{(M)}\right] + \ldots = \sum_i \left[T_{i,0}^{(F)} - T_{i,0}^{(M)}\right] =$$

$$= \sum_{x=0}^{\omega} \left[x\, d_x^{(F)} - x\, d_x^{(M)}\right] = \sum_{x=0}^{\omega} \left[x\, d_{1,x}^{(F)} - x\, d_{1,x}^{(M)}\right] + \sum_{x=0}^{\omega} \left[x\, d_{2,x}^{(F)} - x\, d_{2,x}^{(M)}\right] + \ldots = \sum_{x=0}^{\omega} \sum_i \left[x\, d_{i,x}^{(F)} - x\, d_{i,x}^{(M)}\right],$$

where 1, 2, ... etc. are the different causes of death, denoted previously by i.

John H. Pollard has shown that

$$e_0^{0\,(F)} - e_0^{0\,(M)} = \int_0^\infty \left\{ exp\left[-M_x^{(F)}\right] - exp\left[-M_x^{(M)}\right] \right\} dx =$$

$$= \int_0^\infty \left\{ exp\left[-M_x^{(M)} - M_x^{(F)}\right] - 1 \right\} {}_0 p_x^{(M)}\, dx =$$

$$= \int_0^\infty \left[\frac{{}_x p_0^{(F)}}{{}_x p_0^{(M)}} - 1 \right] {}_0 p_x^{(M)}\, dx =$$

$$\int_0^\infty \left[{}_x p_0^{(F)} - {}_x p_0^{(M)} \right] dx = \int_0^\infty \left[l_x^{(F)} - l_x^{(M)} \right] dx = \text{etc.}$$

His demonstration is undoubtedly correct, but the result we obtain by using his final formula is very different from our results.

If we are interested in calculating the differences between the life expectancies at higher ages too, we cumulate from the highest ages the values of ${}_n d_{i,x}$ for obtaining the numbers of survivors as future victims of different causes of death ($l_{i,x}$). It is obvious that $\sum_i l_{i,x} = l_x$ and the sum of the elements of the structures of survivors as future victims of different causes of death is equal to 1 at each exact age.

The next step is the calculation of the total stationary population and the stationary subpopulation in the life table by causes of death (${}_nL_{i,x}$). It is natural that

$$\sum_i {}_nL_{i,x} = {}_nL_x \ .$$

For the age intervals 0 to 1, 1 to 4 and 5 to 9, the calculation can be done by using the following well-known formulae:

$${}_1L_0 = (0{,}07 + 1{,}7M_0)d_0 + l_1, \text{ where } M_0 \text{ is the mortality rate for 0 year of age},$$

$${}_4L_1 = 1{,}5{}_4d_1 + 4l_5, \quad \text{and} \quad {}_5L_5 = 2{,}5{}_5d_5 + 5l_{10} \ .$$

For the following five-year age intervals (until the age of 85) we have the formula:

$${}_nL_x = \frac{65}{24}(l_x + l_{x+5}) - \frac{5}{24}(l_{x-5} + l_{x+10})$$

For the last (open ended interval) the result may be obtained by using the following formula:

$${}_\infty L_{85} = l_{85} \cdot e_{85}^0 = l_{85}(1/{}_\infty M_{85}) = l_{85}/{}_\infty M_{85} \ ,$$

where ${}_\infty M_{85}$ is the mortality rate for 85 years of age and above.

The calculation of the stationary subpopulation by causes of may be obtained supposing that

$${}_nL_{i,x} = n\, l_{i,x+n} + {}_nd_{i,x} \frac{{}_nL_x - nl_{x+n}}{{}_nd_x} \ ,$$

instead of

$${}_nL_{i,x} = n\, l_{i,x+n} + {}_nd_{i,x} \frac{{}_nL_{i,x} - nl_{i,x+n}}{{}_nd_{i,x}} \ .$$

Obviously this is not true; the distribution of the victims of different causes of death, especially in five-year age intervals, may differ from that of victims of all causes of death. More precise results may be obtained if the distribution of the deceased by causes of death for single-year intervals is available.

The next step is to calculate the total after-life time of all survivors and of the survivors as future victims with different causes of death. It may be realized by cumulating the ${}_nL_x$ and ${}_nL_{i,x}$ values from the highest ages and so $\sum_i T_{i,x} = T_0$.

The life expectancies at age x of all survivors and survivors as future victims with different causes of death may be calculated by using the formulae:

$$e_x^0 = \frac{T_x}{l_x} \quad \text{and} \quad e_{i,x}^0 = \frac{T_{i,x}}{l_{i,x}} \ .$$

Table 7

Contribution of mortality based on causes of death to the differences between life expectancies at birth of Hungarian females and males

The causes of death studied	Calculated using the method of				Calculated using the method of			
	Pollard	Andreev	Pressat	DRI of the HCSO	Pollard	Andreev	Pressat	DRI of the HCSO
	data for 1966				data for 1994			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(9)	(9)
Infectious and parasitic diseases	0.341	0.342	0.341	-1.185	0.126	0.124	0.128	-0.014
Neoplasms(all forms)	0.640	0.677	0.647	-1.646	1.954	1.979	1.889	-1.339
Diseases of the circulatory system	1.068	1.056	1.056	5.881	2.503	2.518	2.457	8.616
Cerebrovascular diseases	0.169	0.180	0.175	3.532	0.658	0.665	0.641	2.678
Diseases of the respiratory system	0.324	0.339	0.333	-0.730	0.557	0.568	0.525	0.729
Diseases of the digestive system	0.138	0.134	0.136	0.240	0.199	0.193	0.204	0.229
Chronic liver disease and cirrhosis	0.101	0.099	0.101	-0.247	1.244	1.214	1.275	-1.479
Congenital anomalies	0.007	0.006	0.006	0.001	0.009	0.009	0.010	-0.051
Certain conditions originating in the perinatal period	0.450	0.427	0.440	-0.001	0.158	0.154	0.165	0.000
Injury and poisoning	1.282	1.227	1.267	-0.720	1.736	1.722	1.837	0.168
All other causes of death	0.234	0.267	0.252	-0.371	0.320	0.318	0.333	-0.073
Total	4.754	4.754	4.754	4.754	9.464	9.464	9.464	9.464

The life expectancy at birth ($e^0{}_0$), i.e. the mean age of all the deceased in the life table, as it has already been mentioned, is a weighted arithmetic mean of mean ages at the death of victims with different causes of death. If we denote the proportions of victims of different causes of death by f_i then

$$e_0^0 = \sum_i f_{i,0}\, e_{i,0}^0 \quad (\sum_i f_{i,0} = 1)\ .$$

The ‘mean of means’ nature of life expectancy at birth, or the mean age of all the deceased in the life table is sometimes presented by showing the balances with two hands. The weights hanging on both hands of balances are the numbers of the deceased in the life-tables due to different causes of death. Their sum is equal in our case to 100,000 (i.e. to the radix of the life-tables we use). The points of suspension of weights are the mean ages at the death of victims of corresponding causes of death. The sum of weights multiplied by the differences between the suspension points of weights and suspension point of balances is the same on both hands of the balances. The sign of these equal sums is nevertheless different and their algebraic sum is therefore equal to zero; corresponding to the concept of weighted arithmetic mean. The decomposition of the differences between the points of suspension of the balances means, in this case, the decomposition between the life expectancies at birth.

If we want to show not only the contribution of the different causes of death to the differences between the life expectancies at birth, but to present the contributions in question as the sums of ‘structural effects’ and ‘mortality level effects’ as well, we may use for this purpose the method of double standardization elaborated by *E.M. Kitagawa* (1955, 1964).

If we denote the weights when studying e.g. the differences between the life expectancies at birth of females and males by $f_{i,0}^{(F)}$ and $f_{i,0}^{(M)}$, and the life expectancies at birth of future victims with different causes by $e_{i,0}^{0(F)}$ and $e_{i,0}^{0(M)}$, then life expectancies at birth (exact 0 years of age) will be

$$e_0^{0(F)} = \sum_i e_{i,0}^{0(F)} f_{i,0}^{(F)} \quad \text{and} \quad e_0^{0(M)} = \sum_i e_{i,0}^{0(M)} f_{i,0}^{(M)}\ ,$$

and the difference between the expectancies at birth for females and males will be equal to

$$e_0^{0(F)} - e_0^{0(M)} = \sum_i [e_{i,0}^{0(F)} f_{i,0}^{(F)} - e_{i,0}^{0(M)} f_{i,0}^{(M)}]\ .$$

The contribution of mortality, based on different causes of death, to the differences between life expectancies at birth is very different from that calculated by using the methods of *Pollard* (1982, 1988), *Andreev* (1982), *Pressat* (1985, 1995) and *Arriaga* (1984). (See Table 7.) An explanation for the origin of these differences has been provided in two of the previous papers of *Valkovics* (1991, 1996).

In order to show the effect of the differences of the structures of the deceased based on causes of death and the effect of the differences of the mean ages in the death of victims with different causes of death in corresponding life-tables, we may use one of the following formulae:

$$e_0^{0(F)} - e_0^{0(M)} = \sum_i (f_{i,0}^{(F)} - f_{i,0}^{(M)})\, e_{i,0}^{0(M)} + \sum_i (e_{i,0}^{0(F)} - e_{i,0}^{0(M)})\, f_{i,0}^{(F)} =$$

$$= \sum_i (f_{i,0}^{(F)} - f_{i,0}^{(M)})\, e_{i,0}^{0(F)} + \sum_i (e_{i,0}^{0(F)} - e_{i,0}^{0(M)})\, f_{i,0}^{(M)} =$$

$$= \sum_i [f_{i,0}^{(F)} - f_{i,0}^{(M)}][0{,}5\,(e_{i,0}^{0(F)} + e_{i,0}^{0(M)})] + \sum_i [e_{i,0}^{0(F)} - e_{i,0}^{0(M)}][0{,}5\,(f_{i,0}^{(F)} + f_{i,0}^{(M)})]\ .$$

The first part of these formulae shows the impact of the differences of the structure of the deceased by causes in corresponding life-tables. The second part of these formulae shows the effect of the differences of mean ages of victims with different causes of death in corresponding life-tables.

We emphasise that double standardization method may only be used for decomposing the contributions of different causes of death to the differences between life expectancies at birth into 'structural effects' and ‘mortality level effects’.

When comparing two mortality structures by causes of death (in other words: two structures of the deceased by causes of death) we can see that the mortality structure which is more favourable from the point of view of the mortality level is the one where the proportion of causes of death killing their victims at older ages is higher. When we compare two sets of mean ages of victims of different causes of death, the set with higher mean ages is more favourable. A more favourable mortality structure and a more favourable set of mean ages at the death of victims with different causes of death result a higher life expectancy at birth, i.e. a lower mortality level and vice versa.

The observed mean ages at death and the mean ages at death in the life tables by causes of death of victims of different causes of death we use in our contribution are naturally not independent, they are influenced by the fact that each cause of death is acting in coexistence with all the other causes of death. I few special cases, when it is possible to calculate them in pure state, as every demographer knows it, the non-independent mean ages may be even very different from the independent mean ages.

If we consider the method elaborated and used in the Demographic Research Institute of the HCSO we must focus on the influence of rising or diminishing proportions of victims of a given cause of death in the life table death function on the diminishing or rising proportions of victims of other causes of death which contribute also the rise or decline of general mortality level as well.

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