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SYMPOSIUM ON WEAK INTERACTIONS

BALATONVILÁGOS

31 May — 5 June, 1966

Organized by the Roland Eötvös Physical Society
Supported by the Hungarian Academy of Sciences

The final manuscript was put into production on 15th September 1966
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OPENING ADDRESS

By

K. Nagy

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Ladies and Gentlemen,

it is an honour for me to welcome you on behalf of the Mathematics and Physics Division of the Hungarian Academy of Sciences and also of the Roland Eötvös Physical Society, to this conference on weak interactions.

As time goes by the Conferences on Physics at Lake Balaton are growing old enough to have traditions. Several years ago a much smaller group of physicists convened here, at Balatonvilágos, and later at Balatonföldvár, to discuss topics in elementary particle physics, which were then of interest. The last conference took place at Keszthely, two years ago. It already prided itself upon greater publicity. Among its participants one found well known physicists from distant countries and a noticeable reaction to it from abroad could be felt. All these conferences have been very important in improving our connections with the physicists of foreign countries. Let me sincerely hope that the same will hold for this conference on weak interactions. Nowadays active work in the natural sciences, especially in physics, is inconceivable without exchanging ideas during personal contact in the atmosphere of vivid, "open air" conferences. It is far from my intention to exaggerate the significance of these conferences at Lake Balaton, or their role in the promotion of physics. But they *are* significant to those doing physics in Hungary, in enabling them to meet eminent persons in the various fields covering their interests.

Physics in Hungary has its past, present and future. The name of Roland Eötvös, his investigations as to the equivalence of gravitational and inertial mass, are known everywhere. We are witnesses to a revival of interest toward this problem, evoked by recent measurements of Dicke. But "real action" to enlarge the basis for research work in Hungary was taken only during the last 10 or 15 years. We now possess good young physicists and they are engaged in up to date problems of modern physics. We do not undertake great things, yet by fitting in, from time to time, small bricks in a rather modest way, we still may contribute to what can be called the majestic building of modern physics.

Nobody, indeed, should be ashamed of being overwhelmed by the riches of the microcosmos, revealed by the rapid progress of the experimental techniques of recent years. We seek to bring to light laws and symmetries in order to find our way through the jungle of microscopic objects. We are led by the conviction that the world seen by a wider vision is much simpler than the fraction of it we are acquainted with at present. To make use of a foggy simile: we see perhaps the peaks emering from the clouds, without seeing the ridges, which connect them according to deeply established rules. We often meet the unexpected during our search for the ridges: such was the case in recognizing the parity-violation, or more recently, in the experimental discovery of *CP*-violation. Now we have the problem of the existence of the quarks, and whatever the outcome of the experiments it will be a surprise. There is an incentive in those surprises; they give wings to research work and, ultimately, bring us closer to reality. Personally, I tend to believe that the concept of a small number of elementary particles will prove realistic and the variety of phenomena we are aware of are produced by the various interactions.

In its title our conference bears the name of "Weak Interactions". But I hope this conference will be a place for a peculiar mutation of weak interactions into strong ones — as far as scientific cooperation is concerned.

That is predominant in my mind, when I open this conference and wish you good work and useful discussions.

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SESSION 1. WEAK INTERACTIONS

POSSIBLE TESTS FOR THE VIOLATION OF PARITY AND TIME REVERSAL IN NUCLEI

By

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The possible tests for the existence of weak nucleon-nucleon force and the violation of parity and time reversal in atomic nuclei are discussed.

1. Parity

In the present review the tests for the existence and properties of weak interaction acting between nucleons in atomic nuclei are discussed. It may seem at first sight that the detection of such an extremely weak and fine force cannot be possible in the background of the strong interactions between the nucleons in nuclei. In fact, the relevant parameter that estimates the relative strength of weak to strong force is

$$F = \frac{\text{weak force potential}}{\text{nuclear potential}} = \frac{G' MR_0}{\hbar} \approx 10^{-7} A^{1/3}\,, \tag{1}$$

where G is a constant appearing in the weak correction to nuclear potential (see formula (4) below). It is of the order of the weak coupling constant G. In equ. (1) R_0 denotes nuclear radius.

The weak effects of this size would never be detectable if the invariance properties of the weak force were not different from those of the strong interaction. The violence of parity invariance (P-violence) makes ever possible the investigations of such effects which would vanish if strong force were present as the only interaction between the nucleons.

The weak nucleon-nucleon force arrives from the self-interacting nucleon current. In the $V-A$ current-current theory [1] it has the form:

$$L_{self} = \sqrt{8}\, G \left(\bar{n}\, \gamma_n \frac{1+i\gamma_5}{2} p\right)\left(\bar{p}\, \gamma_n \frac{1+i\gamma_5}{2} n\right) \tag{2}$$

in the self-evident notation. Expanding this expression in powers of $1/c$ we get a contact P conserving interaction in zero order, while the first order term produces a pseudoscalar interaction containing expressions like $(\vec{\sigma}_n - \vec{\sigma}_p)$ $(\vec{p}_n - \vec{p}_p)$ tending to align the nucleonic spin in the direction of their motion. The expressions have been derived by BLIN—STOYLE [2] and MICHEL [3] and are of the form:

$$v_{np}^{(p)} = -\frac{G\sqrt{8}}{8MC}\left((\vec{\sigma}_p - \vec{\sigma}_n)\{\vec{p}_p - \vec{p}_n, \delta(\vec{x}_n - \vec{x}_p)\} - \right.$$
$$\left. - i\,\vec{\sigma}_p \times \vec{\sigma}_n\left[\vec{p}_p - \vec{p}_n, \delta(\vec{x}_n - \vec{x}_p)\right]\right). \tag{3}$$

The simple averaging procedure applied to the nuclear model consisting of a nuclear core plus one outside nucleon leads [3] to the contribution to the nuclear average potential

$$u^{(p)} = G'\,\vec{\sigma}\cdot\vec{p}\,, \tag{4}$$

where G is proportional to the weak coupling constant G. Here again, the potential tends to align nucleon spin along its linear momentum. The derivation of this formula is over-simplified, as it completely neglects the radial dependence. Expressions of a different form are also possible in principle (see [4]).

As a result of the weak, P-violating corrections to the nuclear potential we shall have to deal with nuclear states which are no longer exactly characterized by the parity quantum number. The wave functions will then contain the opposite-parity admixtures;

$$\psi = \psi_{\text{reg}} + F\sum \alpha\,\psi_{\text{irreg}}\,, \tag{5}$$

where α depends on the magnitude of nuclear matrix elements of $u^{(P)}/F$, while ψ_{reg}, denotes the regular part of the wave function (i.e. the eigenstate of a nuclear Hamiltonian without the correction term $u^{(P)}$). The other components, ψ_{irreg}, have the opposite parity with respect to ψ_{reg}.

Now, owing to the existence of that "irregular" admixture the well-known parity selection rules for electromagnetic transition will no longer hold exactly. If the lowest regular transition is for example $M1$ then a trace of simultaneous $E1$ multipolarity should also be seen.

There are two immediate consequences of the existence of such irregular transitions. In the case when the nuclear source emitting the radiation is *unpolarized* we should expect the outgoing gamma-ray to be slightly circularly polarized as the only consequence of P-violation. In other words, the number of right-handed photons N_+ is slightly different from that of the left-handed ones N_- and therefore the polarization

$$P = \frac{N_+ - N_-}{N_+ + N_-}$$

is different from zero.

If, on the other hand, *polarized* sources are used then the circular polarization of outgoing photons occurs anyway and is not a test for P-violation. However, in this case the existence of the pseudoscalar part $u^{(P)}$ (see equ. (4)) causes a slight asymmetry in the angular distribution of the outgoing photons with respect to the polarization. In other words, the expression for angular distribution $W(\theta)$ contains a term with $\cos\theta$:

$$W(\theta) = 1 + F\beta\cos\theta\,, \tag{6}$$

where angle θ is measured from the polarization direction, while β again depends on nuclear matrix elements.

As the expected sizes of all the effects are very small owing to the presence of factor F in formulae (5) or (6) one has to look for those suitable cases where the nuclear conditions strongly favour the irregular part of transition with respect to the regular one. In other words parameters α and β from equs. (5) and (6) have to be as large as possible. For example if the regular transition in the nucleus is forbidden for some nuclear reasons (such as approximate l-selection rule, for instance) then we may expect that the relative importance of the irregular transition is much higher. Following this line BOEHM and KANKELEIT [5] used the Ta^{181} nucleus as a test for the detection of circular polarization of the 482 keV gamma radiation. The regular $M1$ $(+\,E2)$ transition is accompanied by the irregular $E1$. As the regular $M1$ is approximately 10^5 times forbidden in this case the irregular $E1$ is relatively stronger and the measured polarization is

$$P = (-2 \pm 0{,}4)\cdot 10^{-4}\,.$$

The method of a polarized source was used in the experiment of ABOV, KRUPCHICKIJ, ORATOWSKIJ [6]. They bombarded a Cd^{113} target with polarized, thermal neutrons. A highly excited $1+$ state of a Cd^{114} nucleus is then produced, and the source is polarized. The $1+ \to 0+$ regular transition is then of the $M1$ multipolarity. The negative-parity admixtures in the $1+$ state cause a slight $E1$ transition and then the asymmetry in the angular distribution results as shown by equ. (6).

In this case of a very high excited state obtained by a thermal neutron capture there are many $1-$ states nearby, which can be mixed with the $1+$. Then the factor β from equ. (6) is quite appreciable and the measured asymmetry coefficient is:

$$\beta F = (-3{,}7 \pm 0{,}9)\cdot 10^{-4}\,.$$

This experiment has been repeated for Cd^{114} and some other nuclei by ABRAHAMS et al. [7].

Another method for testing the violation of P-invariance has been used by G. SCHARFF—GOLDHABER and MCKEOWN [8]. They have investigated a 58 keV gamma line of the transition 1143 keV (8—) 1085 keV (8+). Here the nuclear selection rule for the regular $E1$ transition (K — selection rule with $\Delta K = 8$) gives a hindrance factor of the order 10^{16}. Such a high degree of forbiddenness makes possible a direct comparison of the regular $E1$ transition with the irregular $M1$ component. The experimental determination of the conversion coefficients leads to the following suggested mixture:

$$90\%\ E1 + 10\%\ M1 .$$

Unfortunately, the result may be ambiguous as the penetration effects in the electron conversion process offer an alternative explanation for discrepancy between the experimental and calculated conversion coefficients.

The above nuclear experiments seem to be important not merely because they test the violation of P-invariance. After all, this fact has been known for a long time from beta-decay experiments. However, the above experiments on circular polarization or gamma-ray asymmetry are the first observations of a self-interaction in the current-current theory of weak interaction which is then confirmed very reasonably in this way. On the other hand the experiments give for the first time a measurement of the weak process in the first order of the coupling constant. Then, not only the magnitude but also the sign of the interaction can be determined. For this purpose, as well as for the quantitative determination of the weak coupling constant G all the steps of a sequence have to be investigated carefully:

$$\left\{\begin{array}{l}\text{weak}\\ \text{coupling}\\ \text{with a}\\ \text{constant } G\end{array}\right\} \underset{(1)}{\rightarrow} \left\{\begin{array}{l}\text{non-relativistic}\\ \text{two-body weak}\\ \text{force between}\\ \text{the nucleons}\end{array}\right\} \underset{(2)}{\rightarrow} \left\{\begin{array}{l}\text{average}\\ \text{nuclear}\\ \text{weak}\\ \text{potential}\end{array}\right\} \underset{(3)}{\rightarrow} \left\{\begin{array}{l}\text{experimental}\\ \text{effects in}\\ \text{nuclei (circular}\\ \text{polarisation, asymmetry)}\end{array}\right\}.$$

Steps (1) and (2) were examined by BLIN—STOYLE [2] and MICHEL [3] while step (3) was first estimated by BLIN—STOYLE [2], MICHEL [3]. Then WAHLBORN [9] has done more detailed calculation employing the present knowledge of nuclear structure and models. His calculation could be easily repeated for any particular case (see for example [10]). The calculations are unfortunately very involved. However, the sign of G determined in this way is compatible with the possibility of the existence of the intermediate boson. The opposite sign of G would have ruled out that hypothesis.

2. Time-reversal

The problem of testing invariance with respect to time-reversal T has become especially interesting after the discovery of a $K_2^0 \rightarrow 2\pi$ decay which violates CP and therefore also T by the CPT-theorem. The question whether this decay is a real proof for the existence of interactions which are odd under time-reversal does not seem to be settled yet. Many speculations have been introduced in order to explain the possibility of such interactions. We shall not attempt here a discussion of the various approaches to the problem. Assuming that the experimental consequences in the nucleus are essentially the same we shall pick out one of the suggested theories and try to investigate its consequences.

Let us take, for example, the suggestion of ZWEIG—ZACHARIASEN [12] that the weak-interaction Lagrangian contains not only V and A terms as in the usual theory but also $P - S$ and $T - \bar{T}(\bar{T}$ — pseudotensor) currents of baryons as well. Now, in the ordinary beta decay or in any other lepton decay, the current of leptons (which is assumed to have only the $V - A$ component) automatically picks up the $V - A$ part of the baryon current. Then the T-violating terms can occur only in non-leptonic processes and can also contribute to the nucleon-nucleon force. The same terms are also odd with respect to the parity transformation P. For reasons of symmetry (see [12]) the T-violating part of the Lagrangian which comes from $P - S$ or $T - \bar{T}$ interference should be expected to be of a much lower strength compared with the usual weak interaction. In the ZWEIG—ZACHARIASEN theory the corresponding factor is of the order of 40% (and this follows from the square of the sine of Cabibbo angle). Then, the resulting magnitude for the ratio $K_2^0 \to 2\pi / K_2^0 \to 3\pi$ comes with the right order of magnitude as compared with experiments [11].

As in the case of the usual weak Lagrangian (see equ. (2)) we may try to determine the nonrelativistic nucleon-nucleon interaction resulting from [12] as an example of a P- and T-violating theory. The result is:

$$v_{12}^{(P,T)} = \frac{G^{(T)} \sqrt{8}}{4MC} \left(i (\vec{\sigma}_2 - \vec{\sigma}_1) [\vec{p}_2 - \vec{p}_1, \delta(\vec{x}_2 - \vec{x}_1)] - \right. \\ \left. - (\vec{\sigma}_1 \times \vec{\sigma}_2) \{\vec{p}_1 - \vec{p}_2, \delta(\vec{x}_2 - \vec{x}_1)\} \right), \tag{7}$$

where $G^{(T)}$ is of the order of 4% G. The averaging procedure similar to that of MICHEL [3] performed in this case gives:

$$u^{(P,T)} = G'^{(T)} \vec{r} \cdot \vec{\sigma}, \tag{8}$$

which is the analogue of equ. (4) with $G'^{(T)}$ 4% G'.

The very small size of the expected interaction makes experimental tests extremely difficult, if not hopeless, at present time. Let us try, however, to consider the consequences of the T-violation imposed by such an interaction.

In order to better realize what the effects connected with T-violation in the nuclear system are let us recall the well-known theorems which follow from the assumption of the T-invariance of the system. They are:

1. The system containing the odd number of fermions exhibits an energy spectrum with at least doubly degenerate eigenstates (Kramers degeneracy).

2. The reality of matrix elements. In the case of electromagnetic transitions in nuclei we may make all the matrix elements real. For a standard phase-convention this is achieved by multiplying the electric transition operators by i^{λ} and the magnetic ones by $i^{\lambda+1}$ (λ-multipolarity).

3. The static odd-λ electric moments and even magnetic ones vanish. This follows from 2.

Now, if T-invariance is violated in the nucleus we may expect that none of the above theorems will be valid exactly. As for the splitting of the Kramers degeneracy in nuclei it turns out, that the principle of rotational invariance (or at least reflection symmetry in the deformed nuclei) still keeps the corresponding pair of energy eigenstates degenerate. Therefore, there is no Kramers splitting in nuclei.

Let us now turn to Theorem 2. We may expect the relative phase to be measurable in a whole variety of electromagnetic processes. As an example let us take 482 keV transition in Ta^{181}. A measurement of photon linear momentum $\vec{k}$ together with its linear polarization ε can be performed as a function of the asimuthal angle φ in the plane perpendicular to $\vec{k}$. If the initial target is aligned (with the degree of orientation even) the number of polarized photons is given by

$$W(\varphi) = 1 + 0.222 \cos 2\varphi + 1.5 \cdot 10^{-5} \cdot \sin 2\varphi \, .$$

Here, the last term corresponds to the irregular (P- and T-odd) part of the interaction. The resulting rotation of a polarization pattern would be very hard to detect because of the very small expected size of the effect.

Perhaps the best possibility for testing the simultaneous P- and T-violation is offered by the corrections to Theorem 3 if it is not valid exactly. The electric-dipole moment of a neutron for example should have the value of the order of

$$e \cdot r \cdot F \cdot f \, ,$$

where e is the elementary charge, r is the neutron dimension, F is the relative strength of weak interaction, while f determines the relative strength of $v^{(P,T)}$ compared with $v^{(P)}$ (see equs. (4), (8)). Taking $r = 10^{-13}$ cm, $F = 10^{-7}$; $f = 4\%$ we get for the neutron electric dipole moment a value of the 10^{-21} cm $\cdot$ e. Perhaps the measurement of a quantity of this order is not completely hopeless (the measurement of SMITH, PURCELL and RAMSEY [13] give 10^{-20} cm $\cdot$ e as an upper limit) and we may expect that tests of this type may be actually performed in the near future.

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ВОЗМОЖНЫЕ ПРОВЕРКИ НАРУШЕНИЯ *P*- И *T*-ИНВАРИАНТНОСТИ В ЯДРАХ

З. ШИМАНЬСКИЙ

Резюме

Обсуждены возможные проверки существования слабой нуклон-нуклонной силы и нарушения *P*- и *T*-инвариантности.

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RATIO C_A/C_V AND TIME REVERSAL INVARIANCE IN NEUTRON BETA DECAY

By

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The existing data on the asymmetry coefficients of the various correlations in the beta decay of free neutrons are applied to determine the A/V ratio and the relative phase A, V. It is shown that by using the above data the error of the generally accepted A, V value can be decreased by 25%.

1. Introduction

The neutron beta decay

$$n \to p + e + \bar{\nu}$$

is known to be described by the Universal-Fermi-Interaction-V,A-Theory [1, 2] i.e. by a Hamiltonian of the form

$$H = \frac{C_V}{\sqrt{2}} \langle e \,|\, \gamma_\mu (1 + \gamma_5) \,|\, \nu \rangle \langle p \,|\, \gamma_\mu (1 + C_A/C_V \, \gamma_5) \,|\, n \rangle + h.\,c.,$$

where C_V and C_A are the vector and axial vector coupling constants respectively. The ratio C_A/C_V can be written as

$$C_A/C_V = \alpha e^{i\varphi},$$

where α and φ are real constants.

If time reversal invariance holds in neutron beta decay, the ratio C_A/C_V must be a real number [3] which means that the only possible values for φ are either 0 or π. It is known that experimentally $\varphi = \pi$ [4]. However, after the discovery of the CP violating decay $K_2^0 \to 2\pi$ [5], it is interesting to rediscuss the experimental situation to see what information is actually available as far as time reversal invariance in neutron beta decay is concerned.

The value of α has recently been calculated on certain theoretical grounds [6]. This number is of fundamental importance in a number of questions in the field of weak interactions and its most accurate determination is clearly desirable.

Both α and φ can be determined from the existing data on the decay of free polarized neutrons. Further information on α can be obtained from a knowledge of the neutron lifetime and the study of $0^+ \to 0^+$ transitions.

It is the purpose of this paper to use all the experimental information at present available in order to obtain the best consistent determinations of α and φ.

2. Free neutron data

The values of the asymmetry coefficients a, A, B and D in the correlations

$$1 + a \frac{\hat{p}_e \cdot \hat{p}_\nu}{p_e \cdot p_\nu}, \tag{1}$$

$$1 + A \frac{\hat{J} \cdot \hat{p}_e}{J p_e}, \tag{2}$$

$$1 + B \frac{\hat{J} \cdot \hat{p}_\nu}{J p_\nu}, \tag{3}$$

$$1 + D \frac{\hat{J}}{J} \cdot \frac{\hat{p}_e \times \hat{p}_\nu}{p_e p_\nu}, \tag{4}$$

(p_e = electron momentum, $\hat{p}_\nu$ = neutrino momentum, $\hat{J}$ = neutron spin), measured in the beta decay of the free neutron, can be used to determine the best values of the quantities α and φ.

Assuming that only V and A terms are present, the expressions for the four considered coefficients are given by [3]

$$a = \frac{1 - \alpha^2}{1 + 3\alpha^2}, \tag{5}$$

$$A = \frac{-2\alpha^2 - 2\alpha \cos\varphi}{1 + 3\alpha^2}, \tag{6}$$

$$B = \frac{2\alpha^2 - 2\alpha \cos\varphi}{1 + 3\alpha^2}, \tag{7}$$

$$D = \frac{2\alpha \sin\varphi}{1 + 3\alpha^2}. \tag{8}$$

The available experimental determinations of the asymmetry coefficients are listed in Table I.

Table I

Experimental result	Reference
$a = \quad 0{,}07 \pm 0{,}12$	[7]
$a = -0{,}06 \pm 0{,}13$	[8]
$a = -0{,}12 \pm 0{,}04$	[9]
$A = -0{,}114 \pm 0{,}019$	[4]
$A = -0{,}09 \pm 0{,}05$	[10]
$B = \quad 0{,}88 \pm 0{,}15$	[4]
$B = \quad 0{,}96 \pm 0{,}40$	[11]
$D = \quad 0{,}04 \pm 0{,}05$	[4]
$D = -0{,}14 \pm 0{,}20$	[11]

The weighted averages of the results quoted in Table I are listed, together with the corresponding χ^2 values, in Table II.

Table II

Weighted average	χ^2
$\bar{a} = -0{,}098 \pm 0{,}036$	2,35
$\bar{A} = -0{,}111 \pm 0{,}018$	0,20
$\bar{B} = \quad 0{,}89 \pm 0{,}14$	0,03
$\bar{D} = \quad 0{,}029 \pm 0{,}048$	0,76

The χ^2 values obtained indicate that the various measurements of the same parameter are consistent among them.

The best determinations for α and φ and their errors can be obtained from the function $\chi^2(\alpha, \varphi)$ constructed by using formulae (5), (6), (7) and (8) and the results of Table I.

More precisely

$$\left.\begin{aligned} \chi^2(\alpha,\varphi) = \frac{(a(\alpha)+0{,}098)^2}{(0{,}036)^2} + \frac{(A(\alpha,\varphi)+0{,}111)^2}{(0{,}018)^2} + \\ + \frac{(B(\alpha,\varphi)-0{,}89)^2}{(0{,}14)^2} + \frac{(D(\alpha,\varphi)-0{,}029)^2}{(0{,}048)^2}\,. \end{aligned}\right\} \tag{9}$$

The best values α^* and φ^* are those for which $\chi^2(\alpha, \varphi)$ reaches the minimum value.

Fig. 1 shows the plot of the function $\chi^2(\alpha, \varphi^*(\alpha))$, where, for each α, $\varphi^*(\alpha)$ is the value of φ which minimizes $\chi^2(\alpha,\varphi)$. This curve does not show any appreciable deviation from the curve $\chi^2(\alpha, \varphi^*)$.

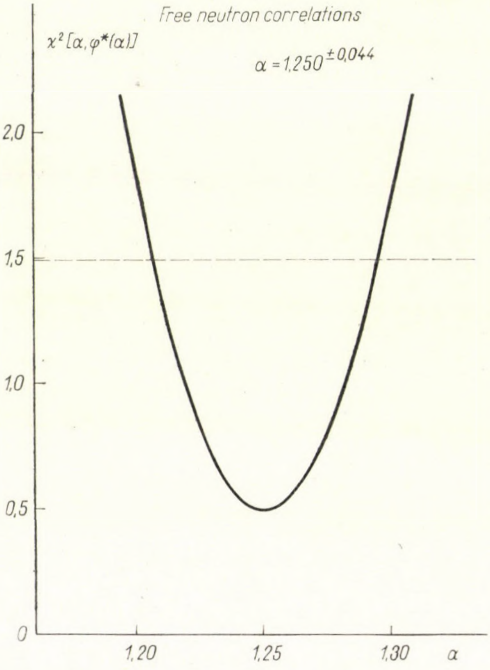

Fig. 1

A line drawn across the curve at $\chi^2(\alpha^*, \varphi^*) + 1$ encloses a 68% confidence interval (1 standard deviation). From the curve of Fig. 1, α is determined to be

$$\alpha = 1{,}250 \pm 0{,}044 \, .$$

Similarly, Fig. 2 shows the plot of the function $\chi^2(\alpha^*(\varphi), \varphi)$ where, for each φ, $\alpha^*(\varphi)$ is the value of α which minimizes $\chi^2(\alpha, \varphi)$. The dotted curve represents the functions $\chi^2(\alpha^*, \varphi)$. With the same procedure used for the determination of α, the best value for φ is

$$\varphi = 176{,}1^\circ \pm 6{,}4^\circ \, .$$

The minimum value of $\chi^2(\alpha, \varphi)$ is found to be

$$\chi^2(a^*, \varphi^*) = 0{,}49$$

which indicates that the correlation data are very well fitted by formulae (1), (2), (3) and (4).

The results obtained should be compared with the best determinations at present available in the literature [4]

$$\alpha = 1{,}25 \pm 0{,}05 \quad \text{(with the assumption } \varphi = 180°)$$

$$\varphi = 175° \pm 10° \quad \text{(with the assumption } \alpha = 1{,}25).$$

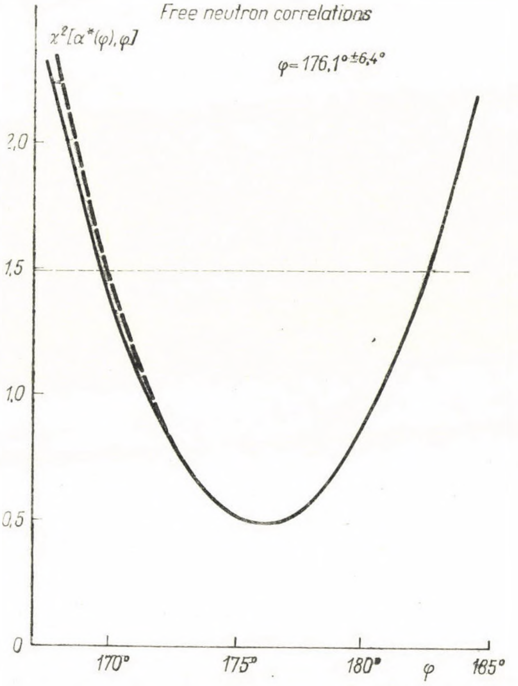

Fig. 2

3. $0^+ \to 0^+$ transition data

An independent determination of α can be derived from the relation [1]

$$\frac{ft\,(0^+ \to 0^+)}{ft\,(n)} = \frac{1}{2} + \frac{3}{2}\,\alpha^2 . \tag{10}$$

From the values

$$ft(n) = (1213{,}4 \pm 35)\ \text{sec} , \tag{11}$$

$$ft(O^{14}) = (3127{,}3 \pm 77)\ \text{sec}$$

it can be calculated that [12]

$$\alpha = 1{,}180 \pm 0{,}028 .$$

The errors in α and in $ft(O^{14})$ are assumed to take into account all statistical and systematical uncertainties. The largest contribution to the error in α in any case arises from the error in the measurement of the neutron lifetime [13].

The accuracy claimed for this determination of α is somewhat better than that obtained from the free neutron data. However it has to be pointed out that:

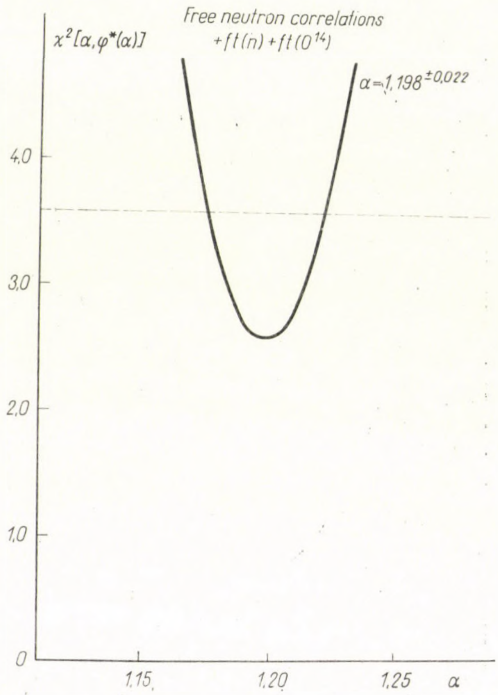

Fig. 3

a) The measurement of the neutron lifetime has never been repeated and actually some criticisms of this result have been raised [2]. The determination of α from the free neutron data comes, instead, from several measurement, all self-consistent.

b) The value of α obtained in Section 2 comes from more direct experimental observations and does not depend on any calculation involving nuclear physics and with very good approximation is not affected by radiative corrections [14].

At any rate, the two values of α are compatible and there is no a priori reason why all the available information should not be used in the determination of α and φ.

Equations (10) and (11) give a fifth term to be added in equation (9).

With this new definition of $\chi^2(\alpha, \varphi)$ one obtains the two functions plotted in Figures 3 and 4, in complete analogy to what has been shown in Figures 1 and 2.

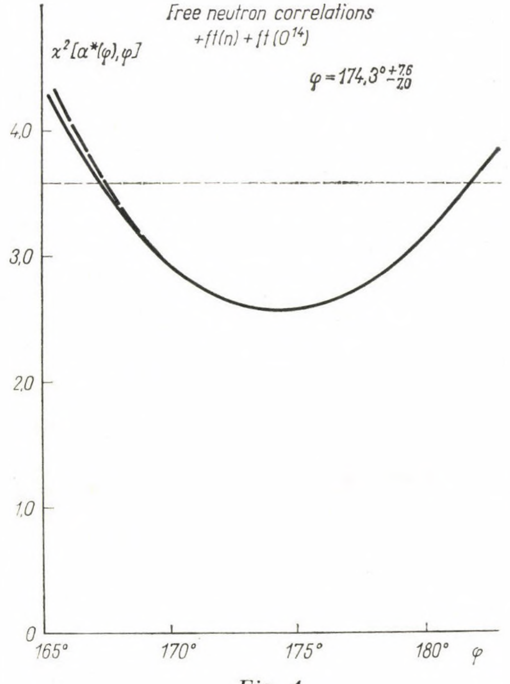

Fig. 4

The final result obtained by using the method discussed in Section 2, is

$$\alpha = 1{,}198 \pm 0{,}022\ ,$$

$$\varphi = 174{,}3^\circ \begin{array}{l} +7{,}6^\circ \\ -7{,}0^\circ \end{array}.$$

The minimum value of $\chi^2(\alpha, \varphi)$ is found to be

$$\chi^2(\alpha^*, \varphi^*) = 2{,}58$$

which indicates that the new information is, in fact, compatible with the correlation data.

4. Conclusion

To summarize it has been shown that:

a) For each correlation, the existing measurements of the asymmetry coefficient are consistent among them.

b) The experimental data relative to the four considered correlations are well fitted by the V, A theory ($\chi^2(\alpha^*, \varphi^*) = 0{,}49$).

c) The free neutron data give

$$\alpha = 1{,}250 \pm 0{,}044\,, \qquad \varphi = 176{,}1^\circ \pm 6{,}4^\circ\,.$$

d) The free neutron data are compatible with the value

$$\alpha = 1{,}180 \pm 0{,}028$$

derived from the O^{14} and neutron *ft* values ($\chi^2(\alpha^*, \varphi^*) = 2{,}58$).

e) The best determination of α and φ obtained by combining the free neutron correlation data with the O^{14} and neutron *ft* values is

$$\alpha = 1{,}198 \pm 0{,}022\,, \qquad \varphi = 174{,}3^\circ \begin{array}{l} 7{,}6^\circ \\ -7{,}0^\circ \end{array}.$$

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ОТНОШЕНИЕ C_A/C_V И T-ИНВАРИАНТНОСТИ В БЕТА-РАСПАДЕ НЕЙТРОНА

ДЖ. КОНФОРТО

Резюме

Существующие данные относительно коэффициентов асимметрии разных корреляций в бета-распаде свободного нейтрона используются для определения отношения A/V и относительной фазы A, V. Показано, что, учитывая эти данные, ошибка общепринятого значения A/V может быть уменьшена на 25%.

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THE VALUE OF THE FIERZ TERM AND THE RECENT DATA ON THE ε/β^+ RATIO IN FORBIDDEN TRANSITIONS

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In connection with the finite value of the Fierz term derived recently from an ε/β^+ ratio measurement, the new data on ε/β^+ are analysed in forbidden transitions.

Recently, Williams carried out measurements for the branching ratio of electron capture to positron emission in the decay of Na^{22} with much greater accuracy than in earlier studies [1]. The $0{,}1041 \pm 0{,}00098$ experimental value determined by him is less than the $0{,}1135 \pm 0{,}002$ theoretical value. From this datum for ε/β^+ he derived a value of the Fierz term $b = -(2{,}5 \pm 0{,}6)\%$ which definitely deviates from zero.

If the theory of two-component neutrinos holds then it demands the Fierz term to be identically zero even if the beta interaction has not a pure $V-A$ character. In this way the Williams' result would be very important from the view-point of both theories but a difficulty here is that the transition of Na^{22} to the 1275 keV excited state of Ne^{22} where the measurement was carried out is a l-forbidden transition in all probabilities.

My present short lecture is to analyse the recent measurements on ε/β^+ for forbidden transitions in order to examine whether the deviation obtained by Williams is explicable by the l-forbiddenness on the basis of the experimental data available.

A comparison between the experimental and theoretical values of ε/β^+ for the known three l-forbidden transitions was made [2] just at the time of the publication of [1]. It was stated that there exists a deviation from the permitted value at most in the case of Zn^{65} but according to the last measurement of Taylor and Merritt [3] this does not hold. In spite of these facts we can say nothing certain about the expectable deviation in l-forbidden cases because the number of such cases are very few and the relevant measurements are not sufficiently accurate in comparison with Williams' measurement.

However, the question arises, whether some general trend exists for ε/β^+ with the increase of the forbiddenness. According to the theory [4, 5] in general, the ratio either agrees with the permitted value or is higher for forbidden transitions. Experiments have supported this hypothesis until now [6, 7] and where deviation was indicated it was in a positive direction. As it is well known e.g., for the $2^- \to 2^+$ transitions, these experimental data were consistently 20—50% higher than the theoretical permitted values [8].

However, during the last year, some new data on the ε/β^+ for first forbidden transitions were published (Table I) which confused the previously relatively clear picture. These data are partly in contradiction with each other, too (cf. Table I).

Table I

Recently measured branching ratios ε_K/β^+ in first forbidden transitions

Nuclide	Transitions	$\Delta I, \Delta\pi$	$E^{max}_{\beta+}$ End-point energy (keV)	Allowed theoretical value of ε_K/β^+*	Experimental value of ε_K/β^+	Authors, year	Ref.
$^{145}_{63}Eu$	$5/2^+ \to 3/2^-$	1, yes	800	47	100 ± 20	AVOTINA et al. 1965	[10]
					120	ZHELEV, MUSIOL 1965	[11]
$^{145}_{63}Eu$	$5/2^+ \to 7/2^-$	1, yes	1740	3,4	≤ 5	AVOTINA et al. 1965	[10]
					3,4	ZHELEV, MUSIOL, 1965	[11]
$^{147}_{63}Eu$	$5/2^+ \to 3/2^-$	1, yes	433	407	160	AVOTINA et al. 1965	[9]
$^{147}_{63}Eu$	$5/2^+ \to 5/2^-$	0, yes	509	265	170	AVOTINA et al. 1965	[9]
$^{161}_{68}Er$	$3/2^- \to 1/2^+$	1, yes	820	64	400 ± 200	GROMOW et al. 1965	[12]

* From the ZYRYANOVA tabulation [14] by linear interpolation. The calculations of ZYRYANOVA differ from those of ZWEIFEL [15] in that in the earlier the finite nuclear size also is taken into account.

A measurement for Eu^{147}, not included in the Table, was also performed. MCNULTY et al. [13] stated that for the two transitions indicated in Table I, as well as for the transition to the Sm^{147} ground state $(5/2^+ \to 7/2^-)$ together, the experimental ε/β^+ ratio is definitely higher than the theoretical one.

On the basis of the existing experimental and theoretical results we can draw the following conclusions. To make clear the regularities for the electron capture — positron emission branching, it would be very useful to perform more accurate measurements for the ε/β^+ ratio for forbidden transitions. Accurate measurements for pure Fermi and *G-T* allowed transitions would be of great help in checking the theory of two-component neutrinos. If in such experiments the Fierz term has a non-zero value it is in contradiction with the theory of two-component neutrinos and at the same time the experiments can give information on the *S* or *T* admixture to the *V—A* interaction.

Finally, it is to be noted that the new data included in the Table have a very important bearing on the theory of branching between electron capture and positive beta-decay [6]. Namely, for the non-unique first forbidden transitions where the mentioned deviation exists between theory and experiment, experimental data have been available till now only for transitions of $2^- \to 2^+$, $\Delta I = 0$, yes type. Unfortunately, the accuracy of recent data on transitions with $\Delta I = 1$, yes (cf. Table I) is very poor.

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ЗНАЧЕНИЕ ИНТЕРФЕРЕНЦИОННОГО ЧЛЕНА ФИРЦА И НОВЫЕ ДАННЫЕ ПО ОТНОШЕНИЮ ε/β^+ В ЗАПРЕЩЕННЫХ ПЕРЕХОДАХ

Д. БЕРЕНИ

Резюме

Обсужденые новые данные по отношению ε/β^+ в запрещенных переходах, в связи с конечным значением интерференционного члена Фирца, полученным недавно из измерения отношения ε/β^+.

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NUCLEAR PARAMETERS FROM THE MEASUREMENTS OF THE β-DECAY SPECTRUM*

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A general formula is given for the shape factor of the energy spectrum of the β-decays forbidden in arbitrary high order. The method used allows us to take into account configuration mixing, too. New equations obtained from the measurements of the shape factor provide a possibility to learn experimentally about new matrix elements characteristic for the nuclei participating in the transition. The numerical discussion of the decay $Cl^{36} \rightarrow S^{36}$ is investigated as an example.

1. Introduction

In order to obtain information concerning the nature of weak interactions the theoretical analysis of the experimental data obtained from β-spectroscopy was of great importance in recent decades. After 1957 β-spectroscopy provided a new possibility to nuclear physics; after having learned the fundamental form of weak interactions it has become possible to collect useful information concerning nuclear structure from the observed properties of β-transitions.

All the results obtained by many authors, e.g. FIERZ [1], KONOPINSKI, UHLENBECK [2], ROSE, OSBORN [3, 4], MARSHAK [5] contain approximations and they give exact formulae only for second forbidden transitions at most. These formulae contain the reduced matrix elements as unknown nuclear parameters.

In the case of transitions forbidden in higher order, the calculation is rather complicated by the increasing number of unknown parameters. Thus, the analysis of the experimental data in terms of nuclear parameters is rather difficult.

On the basis of the calculations carried out by KONOPINSKI and UHLENBECK, and taking into account the fact that all the four particles participating in the transition are in energy, parity and angular momentum eigenstates, i.e. their eigenfunctions have the form

$$\psi_{\varkappa_f}(\mathfrak{r}) = \begin{pmatrix} F_{\varkappa_f}(r) & f_{\varkappa_f M_f}(\hat{r}) \\ iF'_{\varkappa_f}(r) & f'_{\varkappa_f M_f}(\hat{r}) \end{pmatrix} \tag{1}$$

* Part of this work was undertaken at the Institute for Theoretical Physics, Roland Eötvös University, Budapest.

MONTVAY [6] proposed a new method to calculate the transition probabilities of β-decays and electron capture (in Eq. (1) $\varkappa_f$ is the spin-orbital quantum number, $\hat{r} = \frac{\mathfrak{r}}{r}$, $f_{\varkappa_f M_f}$, $f'_{\varkappa_f M_f}$ are known angular functions. $F_{\varkappa_f}$ and $F'_{\varkappa_f}$ are known exactly only for leptons). The results of this paper [6] refer to transitions forbidden in arbitrary order but only for nuclei with one nucleon outside the closed shell. Later on he completed these results [7] for the case of nuclei with more nucleons outside the closed shell. For the sake of clarity the main results of [7] are given in Appendix 1. Moreover, he succeeded in reducing the unknown matrix elements to radial matrix elements of the nucleons.

Let us introduce the notation for the radial matrix elements

$$\begin{aligned} R_1(\varkappa_u, \varkappa_v) &= \int_0^R dr\, r^2\, U_{\varkappa_u}(r)\, V_{\varkappa_v}(r)\,, \\ R_2(\varkappa_u, \varkappa_v) &= \int_0^R dr\, r^2\, U'_{\varkappa_u}(r)\, V_{\varkappa_v}(r)\,, \\ R_3(\varkappa_u, \varkappa_v) &= \int_0^R dr\, r^2\, U_{\varkappa_u}(r)\, V'_{\varkappa_v}(r)\,, \\ R_4(\varkappa_u, \varkappa_v) &= \int_0^R dr\, r^2\, U'_{\varkappa_u}(r)\, V'_{\varkappa_v}(r)\,, \end{aligned} \tag{2}$$

where $\varkappa_u(\varkappa_v)$ stands for the spin orbital quantum number of the final (initial) nucleon state. $U(r)$ $(V(r))$ are the radial parts of the "large" components of the wave function of the final (initial) nucleons. The primed letters denote those of the "small" components and R is the radius of the nucleus.

The transition probability of β-decay can be expressed in terms of these quantities (cf. Appendix 1) as

$$\begin{aligned} \Gamma_\beta = \sum_{i,j=1}^{4} \Bigg[& \sum_{\substack{uv\ \mathrm{uv} \\ j_3}}^{even} r_{ij}(uv \mid \mathrm{uv})\, R_i(\varkappa_u \varkappa_v)\, R_j(\varkappa_\mathrm{u} \varkappa_\mathrm{v}) + \\ & + \sum_{\substack{u\ \mathrm{uv} \\ j_3}}^{odd} r'_{ij}(uv \mid \mathrm{uv})\, R_i(\varkappa_u \varkappa_v)\, R_j(\varkappa_\mathrm{u} \varkappa_\mathrm{v}) \Bigg]. \end{aligned} \tag{3}$$

The quantities r_{ij}, r_{ij} are easily calculable numerical coefficients. They contain the known radial integrals of the leptonic wave functions as well as the angular integrals of all the four wave functions. The other notations used in Eq. (3) are explained in Appendix 1.

The transition probability of the electron capture has a form similar to Eq. (3), differing only in the numerical coefficients of the unknown parameters.

In Eq. (3) the unknown parameters are characterized by $\varkappa_u \varkappa_v$. This means that in the case of pure configuration we are confronted with only 10 nuclear parameters independent of the order of forbiddenness, and the formula is also suitable for describing transitions between nuclei with configuration mixing. (In the case of configuration mixing the number of the unknown parameters becomes inconveniently large.)

Hence taken the values of $\Gamma_{\beta+}$, and the branching ratios K/β^+, L/K from the experiments, we are able to construct equations for them containing the nuclear parameters; these equations, however, will not be sufficient for calculating uniquely even the ten parameters occurring in the case of pure configuration. Therefore, we have to look for further equations regarding other measurements of β-spectroscopy.

In the present paper a general formula for the shape factor of the energy spectrum of β-decay is given (Sec. 2) and the second non-unique forbidden transition $Cl^{36} \to S^{36}$ is discussed as an example (Sec. 3).

2. The spectrum of the β-decay

The energy spectrum of the electron (positron) emitted by β-decay may be written in the following form:

$$P(\varepsilon)\, d\varepsilon = \frac{G^2}{2\pi^3}\, \varepsilon\, p q^2\, F(\varepsilon, \pm Z)\, C_\beta(\varepsilon, \varepsilon_0 Z)\, d\varepsilon\,, \tag{4}$$

$F(\varepsilon, \pm Z)$ is the Fermi function [8], $C_\beta(\varepsilon, \varepsilon_0, Z)$ is, by definition, the shape factor which depends on the energy of the electron only in the case of forbidden transitions. $p = \sqrt{\varepsilon^2 - 1}$, $q = \varepsilon_0 - \varepsilon$ and ε_0 is the energy difference between the initial and final nuclei. (In all our calculations and formulae the units $\hbar = m_e = c = 1$ are used except when specified.)

By a simple modification of Eq. (A2) one can also obtain the explicit form of the shape factor for transitions forbidden in arbitrary high order. The main steps of this procedure are as follows.

In the literature, the energy functions $\mathcal{N}^{(q)}$ and $\mathcal{E}$ (radial parts of the wave function of the neutrino and electron, respectively) are normed for a sphere with unit radius while our functions N and E in Eq. (A2) are normed for energy. The transition to that notation is given in the following expressions:

$$N_{\varkappa_n} = \frac{\mathcal{N}^{(q)}_{\varkappa n}}{\sqrt{\pi}} = \frac{q}{\sqrt{\pi}}\, \mathcal{N}_{\varkappa_n}\,, \tag{5a}$$

$$E_\varkappa = \sqrt{\frac{\varepsilon}{p\pi}}\, \mathcal{E}_{\varkappa_e}\,. \tag{5b}$$

On the surface of the nucleus we can approximate (5a) with the formula (cf. e.g. [6]):

$$\mathcal{N}_{\varkappa_n}(\mathrm{R}) = (q\,\mathrm{R})^{l_n} \frac{1}{(2l_n+1)!!} + \sigma\,(q^{l_n+1}\,\mathrm{R}^{l_n+2})\,. \tag{5c}$$

The tabulated functions in [8] are M_ν, L_ν, N_ν and the connection between them and the function $\mathcal{E}$ is given by

$$\mathcal{E}^2_{\varkappa_e} + \mathcal{E}'^2_{-\varkappa_e} = \begin{cases} F_0\,2p^2\,R^{2\nu+2}\,M_\nu & \text{if} \quad \varkappa_e > 0 \\ F_0\,2p^2\,R^{2\nu}\,L_\nu & \text{if} \quad \varkappa_e < 0 \end{cases} \tag{6}$$

and

$$\mathcal{E}_{\varkappa_e}\mathcal{E}'_{-\varkappa_e} - \mathcal{E}'_{-\varkappa_e}\,\mathcal{E}_{\varkappa_e} = \begin{cases} \;\;F_0\,2p^2\,R^{2\nu+1}\,N_\nu & \text{if} \quad \varkappa_e > 0 \\ -F_0\,2p^2\,R^{2\nu+1}\,N_\nu & \text{if} \quad \varkappa_e < 0 \end{cases} \tag{7}$$

where $\nu = |\,\varkappa_e\,| - 1$, R is the radius of the nucleus and

$$F_0\,(\varepsilon, Z) = \frac{\mathcal{E}^2_{-1} + \mathcal{E}'^2_1}{p^2\,(1 + \sqrt{1 - \alpha^2 Z^2})} \tag{8}$$

is the zero-order approximation of the Fermi function.

Introducing the notation

$$e^{(2)}_{\varkappa e} = \begin{cases} R^{2\nu+2}\,M_\nu & \text{or} \\ R^{2\nu}\,N_\nu & \text{or} \\ R^{2\nu+1}\,L_\nu \end{cases} \tag{9}$$

the new form of the energy integrands in Eq. (A2) will be

$$\frac{2}{\pi^2}\,d\varepsilon\varepsilon\,pq^2\,F(\varepsilon, \pm Z)\,\mathcal{N}^2_{\varkappa_n}\,e^{(2)}_{\varkappa e}. \tag{10}$$

Now the energy integral in Eq. (A2) can be replaced by the integrands in Eq. (9) and we can split off the factors independent of the variables inside the bracket notation. This procedure enables us to write the shape factor

$C_\beta\,(\varepsilon, \varepsilon_0, Z)$ as:

$$C_{\beta^+}(\varepsilon, \varepsilon_0, Z) = \{J_f\} \left\{ \sum_{\substack{uv\,\mathrm{uv}\\ j_3}}^{\mathrm{even}} \left[\left[\left\{ \begin{array}{l} \sum\limits_{\varkappa_n \nu} \mathcal{N}^2_{\varkappa_n} \left[\begin{array}{l} \mathrm{R}^{2\nu+2} M_\nu \\ \mathrm{R}^{2\nu} L_\nu \end{array} \right. \\ R_1(u,v) + R_4(u,v) \,|\, R_1(\mathrm{u},\mathrm{v}) + R_4(\mathrm{u},\mathrm{v}) \\ J_r(j_3) \,|\, J_r(j_3) + J_i(j_3) \,|\, J_i(j_3) \\ 2\{j_3\}^2 [\lambda < j_3 > + (1-\lambda)(j_3) \,|\, \lambda < j_3 > + \\ \qquad + (1-\lambda)(j_3)] \end{array} \right. + \right.\right.\right.$$

$$+ \left\{ \begin{array}{l} \sum\limits_{\varkappa_n \nu} \mathcal{N}^2_{\varkappa_n} \left[\begin{array}{l} \mathrm{R}^{2\nu} L_\nu \\ \mathrm{R}^{2\nu+2} M_\nu \end{array} \right. \\ R_3(u,v) - R_2(u,v) \,|\, R_3(\mathrm{u},\mathrm{v}) - R_2(\mathrm{u},\mathrm{v}) \\ J_r(j_3) \,|\, J_r(j_3) + J_i(j_3) \,|\, J_i(j_3) \\ 2\{j_3\}^2 [- <j_3> + (1-\lambda)(j_3) \,|\, - <j_3> + (1-\lambda)(j_3)] \end{array} \right. +$$

$$\left. + \left\{ \begin{array}{l} \sum\limits_{\varkappa_n \nu} \mathcal{N}^2_{\varkappa_n} \left[\begin{array}{l} \quad \mathrm{R}^{2\nu+1} N_\nu \\ - \mathrm{R}^{2\nu+1} N_\nu \end{array} \right. \\ R_1(u,v) + R_4(u,v) \,|\, R_3(\mathrm{u},\mathrm{v}) - R_2(\mathrm{u},\mathrm{v}) \\ J_r(j_3) \,|\, J_r(j_3) + J_i(j_3) \,|\, J_i(j_3) \\ 4\{j_3\}^2 [\lambda <j_3> + (1-\lambda)(j_3) \,|\, - <j_3> + (1-\lambda)(j_3)] \end{array} \right. \right] +$$

$$+ \sum_{\substack{uv\,\mathrm{uv}\\ j_3}}^{\mathrm{odd}} \left[\left\{ \begin{array}{l} \sum\limits_{\varkappa_n \nu} \mathcal{N}^2_{\varkappa_n} \left[\begin{array}{l} \quad \mathrm{R}^{2\nu+1} N_\nu \\ - \mathrm{R}^{2\nu-1} N_\nu \end{array} \right. \\ R_1(u,v) + R_4(u,v) \,|\, R_1(\mathrm{u},\mathrm{v}) + R_4(\mathrm{u},\mathrm{v}) \\ J_i(j_3) \,|\, J_r(j_3) - J_r(j_3) \,|\, J_i(j_3) \\ 2\{j_3\}^2 [\lambda<j_3> + (1-\lambda)(j_3) \,|\, \lambda<j_3> + (1-\lambda)(j_3)] \end{array} \right. + \right.$$

$$+ \left\{ \begin{array}{l} \sum\limits_{\varkappa_n \nu} \mathcal{N}^2_{\varkappa_n} \left[\begin{array}{l} \quad \mathrm{R}^{2\nu+1} N_\nu \\ - \mathrm{R}^{2\nu+1} N_\nu \end{array} \right. \\ R_3(u,v) - R_2(u,v) \,|\, R_3(\mathrm{u},\mathrm{v}) - R_2(\mathrm{u},\mathrm{v}) \\ J_i(j_3) \,|\, J_r(j_3) - J_r(j_3) \,|\, J_i(j_3) \\ 2\{j_3\}^2 [- <j_3> + (1-\lambda)(j_3) \,|\, - <j_3> + (1-\lambda)(j_3)] \end{array} \right. +$$

$$\left.\left. + \left\{ \begin{array}{l} \sum\limits_{\varkappa_n \nu} \mathcal{N}^2_{\varkappa_n} \left[\begin{array}{l} \mathrm{R}^{2\nu+2} M_\nu \\ \mathrm{R}^{2\nu} L_\nu \end{array} \right. \\ R_1(u,v) + R_4(u,v) \,|\, R_3(\mathrm{u},\mathrm{v}) - R_2(\mathrm{u},\mathrm{v}) \\ J_i(j_3) \,|\, J_r(j_3) - J_r(j_3 \,|\, J_i(j_3) \\ 4\{j_3\}^2 [\lambda <j_3> + (1-\lambda)(j_3) \,|\, - <j_3> + (1-\lambda)(j_3)] \end{array} \right. \right] \right\}. \tag{11}$$

By changing the order of lines in Eq. (11) we point out that one has to determine the coefficients of functions M_ν, L_ν, N_ν. From the angular momentum selection rules discussed in [6] it follows that the functions M_ν, L_ν, N_ν have to be in the relative order to each other just written in Eq. (11), i.e. if it happens that we have to put M_ν (L_ν) in the first term, in the other terms $L_\nu, N_\nu (M_\nu, -N_\nu)$ are to be written. For the sake of brevity in the argumentum of the nuclear radial matrix elements u and v are written instead of $\varkappa_u, \varkappa_v$. J_i and J_r are the imaginary and real parts of the quantity defined in Eq. (A4), J_f is the angular momentum of the final nucleus, $\lambda = G_A/G_V$. $\sum\limits_{\substack{uv\,\mathrm{uv}\\ j_3}}^{\text{even}}, \sum\limits_{\substack{uv\,\mathrm{uv}\\ j_3}}^{\text{odd}}$ denote that one has to summarize over the terms where $l_u + l_v = l_\mathrm{u} + l_\mathrm{v} +$ an even number $l_u + l_v = l_\mathrm{u} + l_\mathrm{v} +$ an odd number, respectively. The $\langle j_3 \rangle$ and (j_3) are the angular integrals written explicitly in Eqs. (A3).

The coefficients of the functions M_ν, L_ν, N_ν can easily be calculated, i.e. the values of M_ν, L_ν, N_ν are tabulated as functions of the electron energy [8], so there is nothing to do but to take Eq. (11) at as many given values of the electron energy as there are functions M_ν, L_ν, N_ν contained in it, or more appropriately, to fit the theoretical curve to the experimental points as closely as possible, e.g. with the least square method.

Thus, we obtain a system of equations in which the unknown quantities are the coefficients of the functions M_ν, L_ν, N_ν. By solving this system of equations and knowing how its results depend on our parameters one obtains further equations which can be solved uniquely. The number of these equations depends on the order of forbiddenness as can be seen from Eq. (8).

3. The decay $Cl^{36} \to S^{36}$

The method discussed above should be illustrated with the decay $Cl^{36} \to S^{36}$. It is a $J_i = 2 \to J_f = 0$ $\Delta\Pi$ no transition, i.e. it is second, non-unique forbidden (cf. [9]). There are five different measurements (partly carried out, partly under way) namely the ft-value of the β^+-decay [9], the ratios K/β^+ [9, 10], L/K [10], the energy spectrum of the positrons and that of the X-quanta emitted by the internal bremsstrahlung [11].

In the course of this calculation we neglect configuration mixing. The reasons for this are:

1. The nucleus Cl^{36} can be described with a state vector which is the superposition of 7 different configurations (cf. [12, 13]). In this case the transition $Cl^{36} \to S^{36}$ can be realized through two different transitions $1d_{3/2} \to 1d_{3/2}$ and $2s_{1/2} \to 1d_{3/2}$. Considering these two possibilities the number of unknown parameters would be 36, i.e. far more than the number of data obtainable from the measurements.

2. In [12] and [13] it was shown that in the state vector of Cl^{36} the weight of the state $(1d_{3/2})^4\,(2s_{1/2})^4$ is dominant (about 90%), i.e. the mixing is weak. There is no reason to take the mixing into account and by so doing to lose the possibility of obtaining information, at least for the dominant matrix elements.

Hence we calculate only the most probable transition $1d_{3/2} \rightarrow 1d_{3/2}$ of the decay $Cl^{36} \rightarrow S^{36}$ using the general formulae (3) and (11).

a) *The value* $ft_{\beta+}$

Putting the data of the individual nuclei into Eq. (A4) it can be seen that the real part of $J(j_3)$ vanishes. Therefore, in Eq. (3) the terms occurring under the symbol $\sum\limits_{\substack{uv\ \mathrm{uv} \\ j_3}}^{\mathrm{odd}}$ vanish. Further, as a consequence of having neglected the configuration mixing $uvne = \mathrm{uvne}$ there is no need to summarize over the initial and final nucleon states in Eq. (11).

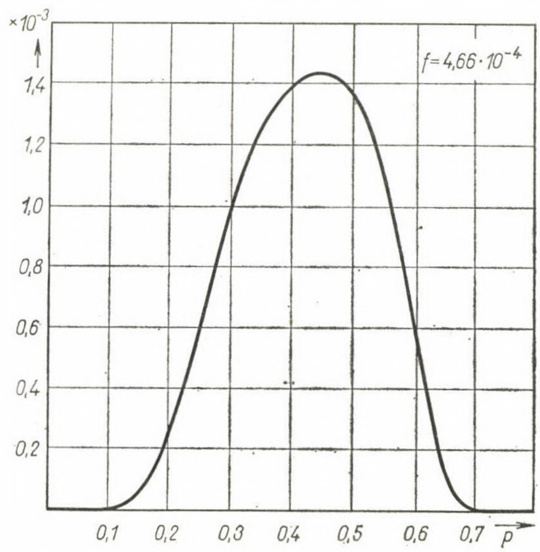

Fig. 1

The general formula of the *ft*-value expressed by the shape factor is

$$ft_{\beta^+} = \frac{2\pi^3 \log 2}{G^2 \langle C_\beta \rangle} \tag{12}$$

with the notation

$$\langle C_\beta \rangle = \frac{1}{f} \int\limits_1^{\varepsilon_0} d\varepsilon \varepsilon \, pq^2 \, F(\varepsilon, \pm Z)\, C_\beta(\varepsilon, \varepsilon_0, Z), \tag{13}$$

where $f = \int_1^{\varepsilon_0} d\varepsilon \varepsilon p q^2 F(\varepsilon, \pm Z)$. In the case of this transition $\varepsilon_0 = 1{,}224$, f is calculated by integrating the curve plotted in Fig. 1 graphically.

By computing with the help of Eqs. (11), (12), (13) and introducing the notation $C = 2\pi^3 \log 2/[G^2 \cdot 2\{j_f\}^2 \cdot \{j_3\}^2 J_i^2(j_3)]$ the following formula is obtained:

$$ft_{\beta^+} = C \left[\sum_{i,j=1}^{4} A_{ij} R_i(\varkappa_u \varkappa_v) R_j(\varkappa_u \varkappa_v) \right]^{-1}. \tag{14}$$

In the case of the most probable transition $1d_{3/2} \to 1d_{3/2}$

$$\varkappa_u = \varkappa_v = \begin{cases} 2 & \text{if } i,j = 1,2, \\ -2 & \text{if } i,j = 3,4, \end{cases}$$

$j_3 = 2$ and $J_i(dd\,2) = \dfrac{2\sqrt{3}}{3\sqrt{5}}$.

The coefficients of the unknown parameters in the case of pure configuration are symmetrical in suffixes i and j, and their explicit forms are:

$$\begin{aligned}
A_{11} &= f^{-1}[J(M_0)\, a_{11}(11) + J(M_1)\, a_{11}(02)], \\
A_{12} &= f^{-1}[J(N_0)\, c_{12}(11) + J(N_1)\, c_{12}(02)], \\
A_{13} &= -f^{-1}[J(N_0)\, c_{13}(11) + J(N_1)\, c_{13}(02)], \\
A_{14} &= f^{-1}[J(M_0)\, a_{14}(11) + J(N_1)\, a_{14}(02)], \\
A_{22} &= f^{-1}[J(L_0)\, b_{22}(11) + J(L_1)\, b_{22}(02)], \\
A_{23} &= -f^{-1}[J(L_0)\, b_{33}(11) + J(L_1)\, b_{23}(02)], \\
A_{24} &= f^{-1}[J(N_0)\, c_{24}(11) + J(N_1)\, c_{24}(02)], \\
A_{33} &= f^{-1}[J(L_0)\, b_{23}(11) + J(L_1)\, b_{33}(02)], \\
A_{34} &= -f^{-1}[J(N_0)\, c_{34}(11) + J(N_1)\, c_{34}(02)], \\
A_{44} &= f^{-1}[J(M_0)\, a_{44}(11) + J(N_1)\, a_{44}(02)],
\end{aligned} \tag{15}$$

with the notation $J(M_0), \ldots$ for the integrals of Eq. (10). The integrands are plotted in Figs. 2, 3, 4 where the values of the integrals are also shown. The

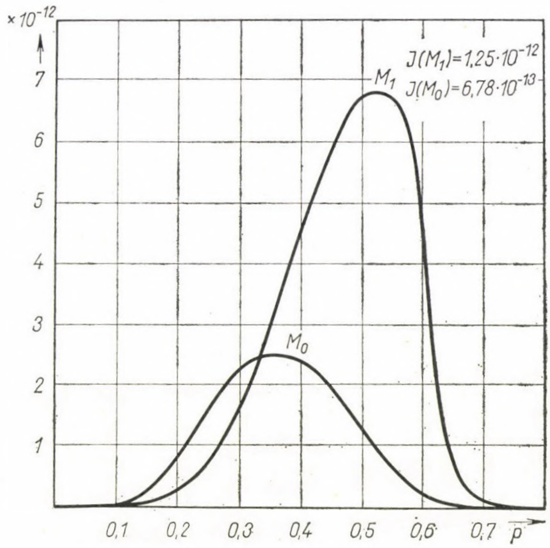

Fig. 2

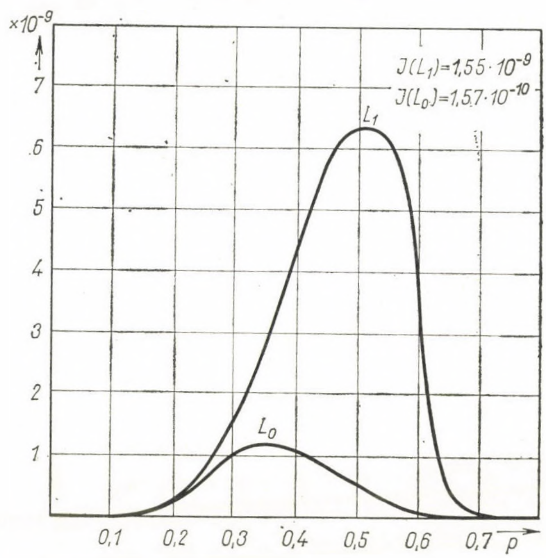

Fig. 3

explicit form of the quantities a_{ij}, b_{ij}, c_{ij} can be obtained from the bottom line of the first, second, or third term of Eq. (11), respectively. The suffices are the same as those of the products $R_i R_j$. The numbers in their argument indicate the quantum numbers (l_n, $|\varkappa_e|$). The numerical values of the quantities a_{ij}, b_{ij}, c_{ij}, A_{ij} and those of the angular integrals $< >$, ( ) are given in Appendix 2.

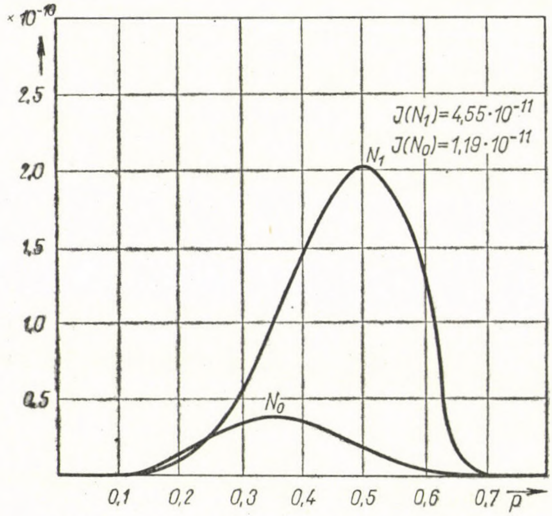

Fig. 4

b) *The branching ratios K/β^+, L/K*

With the simplifications occurring because we calculate only the most probable transition instead of Eq. (3) we get

$$\Gamma_{\beta^+} = \sum_{i,j=1}^{4} r_{ij} (\varkappa_u \varkappa_v \mid \varkappa_u \varkappa_v) R_i (\varkappa_u \varkappa_v) R_j (\varkappa_u \varkappa_v). \tag{16}$$

In [7] it is shown that the expressions of Γ_K, Γ_L can be brought to a similar form, differing only in their numerical coefficients. That is, in order to get the expression of the transition probability it is necessary only to replace the wave function of the electron moving in the Coulomb field of the nucleus by the wave function of the bounded one in Eq. (A2). Instead of summation over the infinite values of $\varkappa_n$ and $\varkappa_e$ one has only to sum over $\varkappa_n$ with the auxiliary condition that the value of $\varkappa_e$ is fixed. Having calculated the coefficients r_{ij}^K or r_{ij}^L following the procedure given above, the ratios K/β^+, L/K can be calculated.

c) *The shape factor of the positron spectrum*

The following form of the shape factor can be obtained from Eq. (11):

$$C_\beta (\varepsilon, \varepsilon_0, Z) = A\{(\varepsilon_0 - \varepsilon) [a_{M_0} M_0 (\varepsilon) + a_{N0} N_0 (\varepsilon) + a_{L_0} L_0 (\varepsilon)] + + a_{M_1} M_1 (\varepsilon) + a_{N_1} N_1 (\varepsilon) + a_{L_1} L_1 (\varepsilon)\}, \tag{17}$$

where ε is the energy of the positron, $A = 2\{j_3\}^2 \{j_f\}^2 J_i^2 (j_3)$, the functions $M_0, M_1, \ldots$ are tabulated in [8]. Their unknown coefficients can be written:

$$
\begin{aligned}
a_{M_0} &= \mathrm{R}^3 \,[a_{11}(11) R_1^2 + 2a_{14}(11) R_1 R_4 + a_{44}(11) R_4^2],\\
a_{M_1} &= \mathrm{R} \,[a_{11}(02) R_1^2 + 2a_{14}(02) R_1 R_4 + a_{44}(02) R_4^2],\\
a_{N_0} &= \mathrm{R}^2 \,[c_{12}(11) R_1 R_2 + c_{24}(11) R_2 R_4 - c_{13}(11) R_1 R_3 - c_{34}(11) R_3 R_4],\\
a_{N_1} &= \mathrm{R}^3 \,[c_{12}(02) R_1 R_2 + c_{24}(02) R_2 R_4 - c_{13}(02) R_1 R_3 - c_{34}(02) R_3 R_4],\\
a_{L_0} &= \mathrm{R}^2 \,[b_{22}(11) R_2^2 + 2b_{23}(11) R_2 R_3 + b_{33}(11) R_3^2],\\
a_{L_1} &= \mathrm{R}^4 \,[b_{22}(02) R_2^2 + 2b_{23}(02) R_2 R_3 + b_{33}(02) R_3^2].
\end{aligned} \tag{18}
$$

R is the radius of the nucleus and the coefficients of the parameters are known from Eq. (14).

Numerical values for the quantities $a_{M_0}, \ldots$ can be obtained by fitting Eq. (17) to the measured spectrum thus obtaining six new equations for the parameters. Together with Eq. (15) and the equations obtained for the branching ratios we already have nine equations. One further equation is required to identify all ten nuclear parameters uniquely. This is supplied from the evaluation of the internal bremsstrahlung accompanying the decay $Cl^{36} \to S^{36}$.

The experimental results for $ft_{\beta+}$ and for the branching ratios so far known are insufficient to determine all the unknown parameters. As an approximation, however, we can neglect those parameters containing more than one "small" component part, i.e. we assume that the products $R_i R_j$ vanish if $i > 1, j > 2$. This approximation is correct if the transition between the "large" components is not forbidden higher then the transition between the "large" component $\to$ "small" component, or vice versa. (This is exactly the case in the transition $Cl^{36} \to S^{36}$.)

Hence, by using Eqs. (14) and (16) as well as the data of [9, 10] a system of equations containing only three unknown parameters is obtained. Its solution gives:

$$
|R_1| = 2{,}61 \qquad |R_2| = 8{,}42 \cdot 10^{-2} \qquad |R_3| = 6{,}09 \cdot 10^{-2}.
$$

(For the calculation the wave functions of the electron given in [15] were used.)

The above results are unsatisfactory considering their order of magnitude. But the ratios $|R_2| / |R_1|$, $|R_3| / |R_1|$ are in good agreement with the assumption that the matrix elements containing one "small" component are at least two orders of magnitude smaller than the matrix elements containing only "large" components.

We hope that the discrepancy mentioned above will vanish if we are able to calculate the complete system of equations.

4. Conclusions

β-spectroscopy provides one possible check on the shell model of nuclei. However, the number of unknown parameters is, in general, high compared with the experimental data.

In the present paper we have calculated the shape factor of the energy spectrum of β-decay. From its explicit form equations can be derived containing the unknown parameters (in our case the products of the unknown radial matrix elements of the nucleons). The number of these equations depends on the order of forbiddenness.

Thus, we may hope that for certain transitions of nuclei with configuration mixing forbidden in high order it will be possible with the help of our equations and the formulae obtained by MONTVAY to calculate the numerical values of the unknown mixing parameters from the experimental data uniquely.

The calculation of the second non-unique forbidden transition $Cl^{36} \rightarrow S^{36}$ is given as a simple example. Only the most probable initial and final configuration transition was taken into account. The four measurements ($ft_{\beta+}$, $C_{\beta+}$, K/β^{+}, L/K) gave us nine equations for our ten parameters. The lacking equation (or more) may be expected from the measurements of the energy spectrum of the internal bremsstrahlung. We hope that finding these equations we shall be able to give experimentally ten new parameters characterizing the structure of nuclei Cl^{36}, S^{36}.

This will support information concerning the small components of the DIRAC wave functions of the nucleons which, in general, are neglected in current nuclear model theories.

The author would like to thank Prof. P. GOMBÁS for his kind interest in this work. Thanks are also due to Dr. D. BERÉNYI, Dr. I. LOVAS, Prof. G. MARX and Dr. I. MONTVAY for their valuable help and advice.

Appendix 1

Transition probability of the β-decay

Let us suppose that N nucleons are outside the closed shell, then the N-particle function describing them (using the isotopic formalism) has the following form

$$\psi_i^N(\mathfrak{r}_1, \ldots, \mathfrak{r}_N) = \sum_{\alpha j T j_v l_v} \left[\varphi_{jMTM_T}^{N-1}(\alpha \mid \mathfrak{r}_1, \ldots, \mathfrak{r}_{N-1})\, \psi_v^{\tau_v}(\mathfrak{r})\right]_{T_i M_{Ti}}^{j_i M_i} y(\alpha j T j_v l_v). \quad \text{(A1)}$$

The suffices i and v refer to the initial state. Replacing these by f and u, from Eq. (A1) we obtain the corresponding equation for the final nucleus. In Eq. (A1) stands for the wave function of the decaying nucleon with the

z-component of the isotopic spin τ_v . $\mathfrak{r}_1, \ldots, \mathfrak{r}_{N-1}$ are the coordinates of the other $N-1$ nucleons in the $(N-1)$-particle wave function $\Phi^{N-1}_{jMTM_T}$ with the resulting angular momentum (j, M) and isotopic spin (T, M_T). α denotes further quantum number specifying Φ^{N-1}. The numerical coefficients $y(\alpha jT\, j_v\, l_v)$ can be obtained from the coefficients of fractional parentage. The values of resulting angular momentum (j_i, M_i) and isotopic spin (T_i, M_{T_i}) correspond to the initial and final N-particle wave function in Eq. (A1). This is expressed by the notation $[\ldots]^{j_iM_i}_{T_iM_{Ti}}$, i.e.

$$[\Phi^{N-1}_{jMTM_T}\, \psi^{\tau_v}]^{j_iM_i}_{T_iM_{Ti}} = \sum_{\substack{M+M_v=M_i \\ M_T+\tau_v=M_{Ti}}} (jM\, j_v\, M_v \,|\, j_i\, M_i) \cdot$$

$$\cdot\, \Phi^{N-1}_{jMTM_T}\, \psi^{\tau}_{v_v} \left(TM_T\, \frac{1}{2}\, \tau_v \,|\, T_i\, M_{T_i}\right).$$

Based on the wave function of Eq. (A1) the transition probability per unit time is:

$$\Gamma_{\beta^+} = \frac{G^2}{4\pi}\{j_f\}^2 \left\{ \sum_{\substack{uv\; uv \\ l_3}}^{\text{even}} \left[\left\{ \begin{matrix} R_1(u,v) + R_4(u,v) \,|\, R_1(\mathrm{u,v}) + R_4(\mathrm{u,v}) \\ J_r(j_3)\,|\,J_r(j_3) + J_i(j_3)\,|\,J_i(j_3) \\ \sum_{\varkappa_n \varkappa_e} \int d\,\varepsilon\,(NE\,|\,NE + NE'\,|\,NE') \\ 2\{j_3\}^2[\lambda < j_3 > + (1-\lambda)(j_0 > |\, \lambda < j_3 > + \\ + (1-\lambda)(j_3] \end{matrix} \right. \right. \right. +$$

$$+ \left\{ \begin{matrix} R_3(u,v) - R_2(u,v) \,|\, R_3(\mathrm{u,v}) - R_2(\mathrm{u,v}) \\ J_r(j_3)\,|\,J_r(j_3) + J_i(j_3)\,|\,J_i(j_3) \\ \sum_{\varkappa_n \varkappa_e} \int d\,\varepsilon\,(NE\,|\,NE + NE'\,|\,NE') \\ 2\{j_3\}^2[- < j_3 > + (1-\lambda)(j_3)\,|- < j_3 > + (1-\lambda)(j_3)] \end{matrix} \right. + \qquad \text{(A2)}$$

$$+ \left. \left\{ \begin{matrix} R_1(u,v) + R_4(u,v) \,|\, R_3(\mathrm{u,v}) - R_2(\mathrm{u,v}) \\ J_r(j_3)\,|\,J_r(j_3) + J_i(j_3)\,|\,J_i(j_3) \\ \sum_{\varkappa_n \varkappa_e} \int d\varepsilon\,(NE\,|\,NE' - NE'\,|\,NE) \\ 4\{j_3\}^2[\lambda < j_3 > + (1-\lambda)(j_3)\,|- < j_3 > + (1-\lambda)(j_3)] \end{matrix} \right. \right] +$$

$$+\sum_{\substack{u,v\,\mathrm{u,v}\\ j_3}}^{\mathrm{odd}}\left\{\begin{array}{l} R_1(u,v)+R_4(u,v)\,|\,R_1(\mathrm{u,v})+R_4(\mathrm{u,v})\\ J_i(j_3)\,|\,J_r(j_3)-J_r(j_3)\,|\,J_i(j_3)\\ \sum_{\varkappa_n\,\varkappa_e}\int d\,\varepsilon(NE\,|\,NE'-NE'\,|\,NE)\\ 2\{j_3\}^2[\lambda<j_3>+(1-\lambda)(j_3)\,|\,\lambda<j_3>+(1-\lambda)(j_3)] \end{array}\right. +$$

$$+\left\{\begin{array}{l} R_3(u,v)-R_2(u,v)\,|\,R_3(\mathrm{u,v})-R_2(\mathrm{u,v})\\ J_i(j_3)\,|\,J_r(j_3)-J_r(j_3)\,|\,J_i(j_3)\\ \sum_{\varkappa_n\,\varkappa_e}\int d\,\varepsilon NE\,|\,NE'-NE'\,|\,NE\\ 2\{j_3)^2[-<j_3>+(1-\lambda)(j_3)\,|\,-<j_3>+(1-\lambda)(j_3)] \end{array}\right. +$$

$$+\left\{\begin{array}{l} R_1(u,v)+R_4(u,v)\,|\,R_3(\mathrm{u,v})-R_2(\mathrm{u,v})\\ J_i(j_3)\,|\,J_r(j_3)-J_r(j_3)\,|\,J_i(j_3)\\ \sum_{\varkappa_n\,\varkappa_e}\int d\,\varepsilon(NE\,|\,NE+NE'\,|\,NE')\\ 4\{j_3\}^2[\lambda<j_3>+(1-\lambda)(j_3)\,|\,-<j_3>+(1-\lambda)(j_3)] \end{array}\right.\Bigg]\Bigg) .$$

The bracket notation used in Eq. (A2) is familiar from Ref. [6] N, N', E, E' are the radial parts of the wave functions of the neutrino and electron respectively with their values valid on the surface of the nucleus (cf. Ref. [6]). G is the coupling constant of the weak interaction, ε is the energy of the electron. Further notations in Eq. (A2) are, with the abbreviation $\{x\}=\sqrt{2x+1}$:

$$<j_3> \;=\; <uvne\,j_3> \;=\sum_{a,\,a'} <uvne\,j_3\,a'\,a>\{a'\}^2,$$
$$(j_3)=(uvne\,j_3)$$

$$<uvne\,j_3> \;= j(uvne)\begin{pmatrix} l_u\,l_v\,a'\\ 0\;0\;0\end{pmatrix}\begin{pmatrix} l_n\,l_e\,a'\\ 0\;0\;0\end{pmatrix}\begin{Bmatrix} j_u\,l_u\,\frac{1}{2}\\ l_n\,j_n\,a\end{Bmatrix}\begin{Bmatrix} j_v\,l_v\,\frac{1}{2}\\ l_e\,j_e\,a\end{Bmatrix}\begin{Bmatrix} l_u\,l_v\,a'\\ l_e\,l_n\,a\end{Bmatrix}\begin{Bmatrix} j_u\,j_v\,j_3\\ j_e\,j_n\,a\end{Bmatrix},$$

$$(uvne\,j_3)=-\tfrac{1}{2}j\,(uvne)\begin{pmatrix} l_u\,l_v\,j_3\\ 0\;0\;0\end{pmatrix}\begin{pmatrix} l_n\,l_e\,j_3\\ 0\;0\;0\end{pmatrix}\begin{Bmatrix} j_n\,j_e\,j_3\\ l_e\,l_n\,\frac{1}{2}\end{Bmatrix}\begin{Bmatrix} j_u\,j_v\,j_3\\ l_v\,l_u\,\frac{1}{2}\end{Bmatrix}, \qquad \text{(A3)}$$

$$j(uvne)=\{j_v\}\{l_v\}\{j_u\}\{l_u\}\{j_e\}\{l_e\}\{j_n\}\{l_n\}$$

a and a' denote auxiliary quantities occurring in the calculation of the angular integrals (cf. App. of Ref. [6]).

$$J(j_3) = J(\varkappa_u \varkappa_v j_3) = y(uv)(-1)^j \begin{Bmatrix} j_u & j_v & j_3 \\ j_i & j_f & j \end{Bmatrix} \tag{A4}$$

and

$$y(uv) = \sum_{\alpha T M_T} y^*(\alpha j T j_u l_u)\, y(\alpha j T j_v l_v)\,(T M_T \tfrac{1}{2} \tau_u \mid T_f M_{T_f})(T M_T \tfrac{1}{2} \tau_v \mid T_i M_{T_i})$$

(cf. the notation after Eq. (A1)).

Appendix 2

Numerical results

The angular integrals $\langle \varkappa_u \varkappa_v l_n \varkappa_e \rangle$, $(\varkappa_u \varkappa_v l_n \varkappa_e)$ are calculated in the case of the transitions $1d_{3/2} \to 1d_{3/2}$, $2s_{1/2} \to 1d_{3/2}$ for β-decay as well as for K-, L^{I}-, L^{II}-, L^{III}-captures. For the calculation, the formulae (A3) and the tables of [14] were used. The results are given in Table I, the omitted values are zeros resulting from the selection rules (cf. [6]), while accidental zeros are tabulated.

The quantities a_{ij}, b_{ij}, c_{ij} are calculated only for the transition $1\,d_{3/2} \to 1\,d_{3/2}$. The results are given below:

$$a_{11}(11) = 7{,}18 \cdot 10^{-3}, \quad a_{14}(11) = 1{,}88 \cdot 10^{-3}, \quad a_{44}(11) = 4{,}76 \cdot 10^{-4},$$

$$a_{11}(02) = 4{,}76 \cdot 10^{-4}, \quad a_{14}(02) = 1{,}88 \cdot 10^{-3}, \quad a_{44}(02) = 7{,}18 \cdot 10^{-3},$$

$$b_{22}(11) = 6{,}40 \cdot 10^{-3}, \quad b_{23}(11) = 1{,}07 \cdot 10^{-2}, \quad b_{33}(11) = 1{,}78 \cdot 10^{-2},$$

$$b_{22}(02) = 1{,}78 \cdot 10^{-2}, \quad b_{23}(02) = 1{,}07 \cdot 10^{-2}, \quad b_{33}(02) = 6{,}40 \cdot 10^{-3},$$

$$c_{11}(11) = 6{,}84 \cdot 10^{-3}, \quad c_{13}(11) = 1{,}14 \cdot 10^{-2},$$

$$c_{11}(02) = 3{,}00 \cdot 10^{-3}, \quad c_{13}(02) = 1{,}79 \cdot 10^{-3},$$

$$c_{24}(11) = 1{,}79 \cdot 10^{-3}, \quad c_{34}(11) = 3{,}00 \cdot 10^{-3},$$

$$c_{24}(02) = 1{,}14 \cdot 10^{-2}, \quad c_{34}(02) = 6{,}84 \cdot 10^{-3}.$$

Table I

	$(\varkappa_u \varkappa_v l_n \varkappa_e)$				$\langle\varkappa_u\varkappa_v l_n\varkappa_e\rangle$							
$(l_n, \varkappa_e)$ / $(\varkappa_u, \varkappa_v)$	(0,2)	(1,1)	(2, −1)	(1, −2)	(0,2)	(1,1)	(2, −1)	(1, −2)	(0, −2)	(1, −1)	(2,1)	(1,2)
(2,2)	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{25}$	$\frac{2}{15}$	$\frac{2}{25}$	$\frac{2}{15}$				
(2, —1)	$-\frac{2}{5}$	$-\frac{2}{5}$	$-\frac{2}{5}$	$-\frac{2}{5}$	$\frac{1}{25}$	$\frac{1}{15}$	-1	$-\frac{1}{9}$				
(—2, —2)	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{15}$	$\frac{2}{25}$	$\frac{2}{15}$	$\frac{2}{15}$				
(—2,1)	$-\frac{2}{5}$	$-\frac{2}{5}$	$-\frac{2}{5}$	$-\frac{2}{5}$	$-\frac{1}{3}$	$-\frac{1}{9}$	$\frac{1}{15}$	$\frac{56}{225}$				
(—2,2)									$-\frac{2}{15}$	$-\frac{2}{25}$	$-\frac{6}{25}$	$-\frac{2}{25}$
(—2, —1)									$\frac{1}{3}$	1	$-\frac{1}{15}$	$-\frac{1}{9}$
(2, —2)									$-\frac{2}{25}$	$-\frac{2}{15}$	$-\frac{2}{25}$	$-\frac{6}{25}$
(2,1)									$-\frac{1}{25}$	$-\frac{1}{15}$	0	$-\frac{1}{3}$

By introducing the notation $r_{ij} = f A_{ij}$ and using the values of the radial integrals of the leptonic wave functions written in Figs. 2, 3, 4, we obtained the results:

$$r_{11} = 5{,}79 \cdot 10^{-15},\ r_{12} = -2{,}25 \cdot 10^{-13},\ r_{13} = 2{,}08 \cdot 10^{-13},\ r_{14} = 8{,}82 \cdot 10^{-15},$$
$$r_{22} = 3{,}05 \cdot 10^{-11},\ r_{23} = -2{,}03 \cdot 10^{-11},\ r_{24} = -6{,}12 \cdot 10^{-13},$$
$$r_{33} = 1{,}53 \cdot 10^{-11},\ r_{34} = 3{,}87 \cdot 10^{-13},$$
$$r_{44} = 1{,}38 \cdot 10^{-14}.$$

The quantities

$$r_{11}^K = 1{,}07 \cdot 10^{-14}, \quad r_{12}^K = -5{,}51 \cdot 10^{-12}, \quad r_{13}^K = 9{,}18 \cdot 10^{-12},$$
$$r_{11}^L = 2{,}50 \cdot 10^{-15}, \quad r_{12}^L = -3{,}52 \cdot 10^{-13}, \quad r_{13}^L = 6{,}06 \cdot 10^{-13}$$

were used for the calculation of the radial matrix elements R_1, R_2, R_3 .

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ЯДЕРНЫЕ ПАРАМЕТРЫ ИЗ ИЗМЕРЕНИЙ СПЕКТРА β-РАСПАДА

Е. БАЛАЖ

Резюме

Дается общая формула для формфакторов энергетического спектра β-распадов, запрещенных в любом порядке. С помощью употребляемого метода можно учесть и конфигурационное смешивание. Новые уравнения, полученные на основе измерений формфакторов, дают возможность для экспериментального изучения новых матричных элементов, характерных для ядер, участвующих в переходе, как например, в подробно рассмотренном случае распада $Cl^{36} \rightarrow S^{36}$.

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WEAK INTERACTIONS AND UNITARY SYMMETRY BREAKING

By

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A model for leptonic decays is considered using unitary symmetry. For the calculation of matrix elements a special method of unitary symmetry-breaking is applied taking into account all corrections due to baryon mass differences.

For the treatment of leptonic decays a model is considered using SU (3) symmetric expressions for the weak current of the hadrons and for the strong interaction between baryons and mesons. Computing the matrix elements the unitary symmetry is broken by putting the experimental masses in the propagators, i.e. in the contraction of two baryon field operators. So all corrections due to the breaking of unitary symmetry by mass differences of the baryons are included.

In the octet version of unitary symmetry the weak current has ten constituents permitting the ansatz

$$J_\mu = \sum_{r=V,A} G_L^{(r)} J_\mu^{(r)}(L) + G_F^{(r)}(0)\, j_\mu^{(r)}(F) + G_D^{(r)}(0)\, S_\mu^{(r)}(F) + + G_F^{(r)}(1)\, S_\mu^{(r)}(F) + G_D^{(r)}(1)\, S_\mu^{(r)}(D)\,, \tag{1}$$

where $J_\mu(L)$ is the leptonic part, j_μ is the strangeness-conserving part and S_μ is the strangeness-changing part, both of F- and D-type. In the limit of exact SU (3) symmetry the currents of the strongly interacting particles are

$$j_\mu(F) = Tr\,(\bar{B}\Gamma_\mu\,[B, \lambda_1 + i\lambda_2]) + \ldots \qquad j_\mu(D) = Tr\,(\bar{B}\,\Gamma_\mu\,\{B, \lambda_1 + i\lambda_2\}) + \ldots$$

$$S_\mu(F) = Tr\,(\bar{B}\,\Gamma_\mu\,[B, \lambda_4 + i\lambda_5]) + \ldots \quad S_\mu(D) = Tr\,(\bar{B}\,\Gamma_\mu\,\{B, \lambda_4 + i\lambda_5\}) + \ldots$$

where $\bar{B}$ and B are the 3×3 baryon matrices and the λ_i are given by GELL-MANN [1], Γ_μ is γ_μ for $r = V$ and $\gamma_\mu\gamma_5$ for $r = A$. The points indicate contributions of other hadrons. The ten G's are the corresponding unrenormalized coupling constants, $G(0)$ is involved in semileptonic processes with $\Delta S = 0$ and $G(1)$ in those with $|\Delta S| = 1$. The G's are determined by making the following assumptions for the weak current:

a) The contributions of the baryons and leptons have the same "weight"

$$\sum_r G_L^{(r)2} = \sum_r G_F^{(r)}(0)^2 + G_D^{(r)}(0)^2 + G_F^{(r)}(1)^2 + G_D^{(r)}(1)^2 . \tag{3}$$

b) The leptonic coupling constants are equal because of the absence of renormalization effects in μ-decay

$$G_L^V = G_L^A = 1 . \tag{4}$$

c) The CVC-theorem excludes the D-parts of the vector currents

$$G_D^V(0) = G_D^V(1) = 0 . \tag{5}$$

d) For the axial vector coupling constants the D/F-ratios are equal for $\Delta S = 0$ and $|\Delta S| = 1$ processes

$$G_D^A(0) = \xi\, G_F^A(0) , \qquad G_D^A(1) = \xi\, G_F^A(1) , \tag{6}$$

and it is further assumed that the D/F-ratio is the same both for strong and weak interactions

$$\xi_{\text{weak}} = \xi_{\text{strong}} . \tag{7}$$

e) The ratio of the vector coupling constants for processes with $\Delta S = 0$ and $|\Delta S| = 1$ is that of the corresponding axial vector coupling constants in the case of F-coupling

$$G_F^V(0)/G_F^V(1) = G_F^A(0)/G_F^A(1) . \tag{8}$$

These are 8 conditions for the 10 coupling constants providing for the weak current the expression

$$\begin{aligned} J_\mu = J_\mu^V(L) + J_\mu^A(L) + \cos\vartheta \Bigg[(2)^{\frac{1}{2}} \cos\alpha\, j_\mu^V(F) + \left(\frac{2}{1+\xi^2}\right)^{\frac{1}{2}} \sin\alpha\, j_\mu^A(F) + \\ + \left(\frac{2}{1+\xi^2}\right)^{\frac{1}{2}} \xi \sin\alpha\, j_\mu^A(D) \Bigg] + \sin\vartheta \Bigg[(2)^{\frac{1}{2}} \cos\alpha\, S_\mu^V(F) + \\ + \left(\frac{2}{1+\xi^2}\right)^{\frac{1}{2}} \sin\alpha\, S_\mu^A(F) + \left(\frac{2}{1+\xi^2}\right)^{\frac{1}{2}} \xi \sin\alpha\, S_\mu^A(D) \Bigg], \end{aligned} \tag{9}$$

where ϑ and α are arbitrary constants, ξ is the D/F-ratio. The parameters ϑ and α are determined by the $\pi_{\mu 2}$ and $K_{\mu 2}$ decays involving only axial vector matrix elements of the form

$$\langle 0|\, G_F^A(0)\, j_\mu^A(F) + G_D^A(0)\, j_\mu^A(D)\, |\pi\rangle = F_\pi q_\mu , \tag{10}$$

$$\langle 0|\, G_F^A(1)\, S_\mu^A(1) + G_D^A(1)\, S_\mu^A(D)\, |K\rangle = F_K q'_\mu , \tag{11}$$

where q_μ and q'_μ are the four-momenta of the π and K, F_π and F_K are certain constants (Their ratio is exactly cotan θ, where θ is the Cabibbo angle)

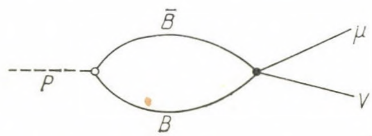

Fig. 1. The first order diagram for the considered decays

The diagram of Figure 1 is computed by making the following assumptions:

1) The strong baryon — meson interaction is given by

$$H_{\rm I} = g_F\, Tr\,(\bar{B}\,\gamma_5\,[B,P]) + g_D\, Tr\,(\bar{B}\,\gamma_5\,\{B,P\}) . \tag{12}$$

2) Assumption (7) means

$$\xi = \frac{G_D^A(0)}{G_F^A(0)} = \frac{G_D^A(1)}{G_F^A(1)} = \frac{g_D}{g_F} . \tag{13}$$

3) The contraction of two baryon field operators is given by

$$\bar{B}_i^{\cdot}(y)\, B_j^{\cdot}(x) = \frac{1}{2} S_F(x-y, Mi)\, \delta_{ij} - \frac{1}{4}\, \varepsilon_{ij}\, [S_F(x-y, M_4) - $$

$$- S_F(x-y, M_5)] \tag{14}$$

with

$$M_1 = M_2 = M_3 = M_\Sigma,\;\; M_4 = M_6 = M_N,\;\; M_5 = M_7 = M_\Xi,\;\; M_8 = M_\Lambda \tag{15}$$

$\varepsilon_{44} = \varepsilon_{66} = 1, \;\; \varepsilon_{55} = \varepsilon_{77} = -1, \;\; \varepsilon_{45} = -\varepsilon_{54} = -1, \;\; \varepsilon_{67} = -\varepsilon_{76} = -1$

$\varepsilon_{ij} = 0$ otherwise.

The result is

$$\frac{F_\pi}{F_K} = \mathrm{ctg}\vartheta \frac{\xi^2\left[\frac{2}{3}(M_\Sigma + M_\Lambda) + M_\Xi + M_N\right] + 2\xi\,[M_\Xi - M_N] + M_\Xi + 4M_\Sigma + M_N}{\xi^2\left[\frac{3}{2}M_\Sigma + \frac{1}{2}M_\Lambda + \frac{5}{6}(M_\Xi + M_N)\right] - \xi\,[M_\Xi - M_N] + \frac{3}{2}[M_\Sigma + M_\Lambda + M_\Xi + M_N]}\,, \tag{16}$$

Summing up all $\bar{B}\,B$ bubble diagrams [3] an additional correction is obtained which is within 1%.

Taking as the other conditions the experimental axial vector renormalization of the nuclear β-decay and the ratio between β- and μ-decay coupling constants the parameters are determined to

$$\vartheta \sim 18°, \quad \alpha \sim 42°, \quad \xi \sim 1{,}5 \tag{17}$$

providing for the coupling constants the values

$$G_F^V(0) \sim 0{,}98, \quad G_F^A(0) \sim 0{,}49, \quad G_D^A(0) \sim 0{,}74\,. \tag{18}$$

These results are now compared with the investigation of 400 000 Σ-decays done by Willis, Courant, Filthuth et al [2] using Cabibbo's theory. In their solution A they obtained for the parameters "F" and "D" the values 0,437 and 0,742 while our extrapolation to these quantities gives the values 0,50 and 0,75 indicating an agreement which is better than expected.

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СЛАБЫЕ ВЗАИМОДЕЙСТВИЯ И НАРУШЕНИЕ УНИТАРНОЙ СИММЕТРИИ

П. МЕБИУС

Резюме

Рассматривается модель для лептонных распадов в рамках унитарной симметрии. Для вычисления матричных элементов применяется специальный метод нарушения унитарной симметрии, учитывающий все поправки, связанные с расщеплением масс барионов.

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THE HOT MODEL OF THE UNIVERSE AND THE ELEMENTARY PARTICLES

By

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The recent radioastronomical observations strongly support the hot model of the Universe. The implications of this model are discussed, and, in particular, an estimation is given for the number density of the quarks.

1. The observational basis

The observations made by radioastronomers during the last year have shown that the hot model of the Universe is the correct one.

These observations revealed the existence of a Planck spectrum of black body equilibrium radiation with the temperature $3^\circ \pm 0{,}3^\circ$ K, isotropic in all directions and filling all space. This radiation is superimposed on the light of the stars mostly in the visible part of the spectrum, and of nonthermal radio-sources prevailing at $\lambda > 50$ cm.

The measurements were made by radiomethods at $\lambda = 20$ cm, 7,3 cm, 3 cm and 0,25 cm. The last point belongs to $\hbar\omega/kT \cong 2$. It is independently checked by the measurement of population ratio of two nearby states of the molecule CN. The bolometric and colour temperatures at different wavelengths coincide and give $3^\circ \pm 0{,}3^\circ$.

At 7 cm the blackbody intensity is some 10^2 more, at 0,25 some 10^5 more than the calculated intensity due to known sources (stars, radiosources, quasistellar sources).

The integrated density of electromagnetic radiation of the blackbody spectrum at 3° K is $6 \cdot 10^{-13}$ erg/cm^3. It exceeds by a factor of 30 or 50 the mean density of radiation of known sources.

The mean matter density in the Universe is still little known. The density of matter in the form of galaxies is said to be $5 \cdot 10^{-31}$ g/cm^2 including visible stars, gas, dust and all other forms of invisible matter. This density is evaluated by the dynamics of galaxies as gravitationally bound systems. Cosmological evidence does not exclude a mean density of $2 \cdot 10^{-29}$ g/cm^3. The greater part of it in this case should be in the form of a highly ionized highly transparent intergalactic plasma at $10^5 - 10^6$ °K, heated by cosmic rays. Of course this plasma is not in equilibrium with the 3° radiation, but is cooling very slowly.

Neither the radiation from stars or other discrete sources, nor the radiation of the hypothetical plasma can give the 3° blackbody radiation.

2. The hot model

The only possible past state of the Universe, compatible with the observed situation is the so-called hot model first put forward by GEORGE GAMOW in 1947.

The cosmological theory of A. A. FRIEDMAN (1922—24) which incorporates the Hubble red shift and expansion give the following picture: there was a moment $t = 0$ (10^{10} years ago) when the density was infinite; from this moment begins the general expansion. The expansion is isotropic and uniform: all directions in space are alike.

The mean density of baryons decreases like R^{-3}, where R is the linear scale. The density of quanta decreases in the same way like R^{-3}. But at the same time the wavelength of quanta λ increases like R, so their energy $E_\gamma = \hbar c/\lambda$ decreases like R^{-1}. The overall energy density of quanta decreases like

$$\varepsilon_\gamma = n_\gamma E_\gamma = R^{-4} = \sigma T^4 .$$

This is perfectly in accord with the law of adiabatic expansion. The entropy per baryon is given by $S = \frac{4}{3}\frac{\sigma T^3}{n_b}$, so $S = \text{const}$ corresponds to $\varepsilon_\gamma \sim T^4 \sim n_b^{4/3}$.

The dimensionless S divided by Boltzmann k (entropy per baryon) today is of the order of $10^8 - 10^9$. This means that the number of quanta per baryon is of the same order of magnitude. Then for the distant past, when the energy of quanta was far greater, we obtain the picture of a quite uniform plasma with overwhelming number and energy of quanta. The electrons and ions constituting "ordinary matter", from which stars and planets are built, were a small minority at this stage ($t < 10^5$ years). Only at a later stage the temperature of the radiation drops in the course of expansion, the electrons and ions go into neutral atoms, gravitational instability gathers them together in clusters, from which galaxies and stars are formed — in short, the present period of astronomical evolution begins. We shall not follow these questions further.

3. Nuclear reactions and the primordial composition

Let us return to the very beginning, with very high temperatures. The energy density as a function of time is given by the equations of mechanics quite independently of the composition of plasma:

$$\varrho = \frac{\varepsilon}{C^2} = \frac{3}{32\pi G t^2} = \frac{5 \cdot 10^5}{t^2} \; [\varrho] = \frac{g}{cm^3}, \quad [t] = \text{sec.}$$

The only assumption is that of isotropic and uniform expansion. At very high density the time of the equilibrium adjustment is much smaller than the time of considerable diminution of the density.

So to the first approximation the expansion goes through a succession of equilibrium states. The equilibrium at some moment for example when $t = 10^{-3}$ sec, $T = 30$ MeV, does not depend on previous states. So one can foresee that at this moment there are no measurable quantities of antibaryons, excited states (resonances) and mesons.

There are quanta, electrons, positrons, neutrinos and antineutrinos of both sorts in commensurable quantities. Contrary to this, the number of baryons is small ($10^{-8} - 10^{-9}$ of the mentioned particles), the equilibrium $e^+ + n \rightleftarrows p + \nu_e$, $e^- + p \rightleftarrows n + \nu_e$ is rapidly established so that $n/p = e^{-\Delta m \cdot C^2/KT} \cong 1$.

During the expansion at $T \cong 1$ MeV, $t \cong 1$ sec, the reaction rate is no longer great enough. The composition 16% N, 84% P is "quenched" i.e. is not much altered during subsequent expansion by the mentioned reactions. By subsequent nuclear reactions of the type

$$N + P = D + \gamma; \quad P + D = He^3 + \gamma;$$

$$D + D = He^3 + N; \quad D + D = T + P; \quad T + P = He^4 + \gamma;$$

$$He^3 + N = T + P, \quad T + D = He^4 + N; \quad He^3 + D = He^4 + D$$

one should obtain 70% H and 30% He^4 (by weight), with $10^{-4} \div 10^{-5}$ of D and He^3. Some astrophysicists claim that observations confirm this composition, but there are rumours of old stars with a smaller helium content.

The investigation of helium content is very difficult, owing to its high potential of ionization and excitation from ground state. At least the observations do not disprove the hot model, and the evidence from the 3° radiation is of overwhelming importance.

4. "Relict" thermal neutrinos and their detection

Near the moment when $N \rightleftarrows P$ equilibrium is quenched, the reaction $e^+ + e^- \rightleftarrows \nu_e + \bar{\nu}_e$ also becomes slow. It has no immediate consequences because the temperature of independent ν_e, $\bar{\nu}_e$ and e^+, $e^-\gamma$ drops in the same tempo when e^+, e^- are relativistic, at $T > 0{,}5\,\mathrm{MaV}$. But later when $e^\pm$ disappears at $T < \mathrm{m} \cdot c^2$, their energy is pumped into γ, the drop of γ-temperature is retarded, compared with the drop of ν-temperature. As a result now $T_{\nu\tilde{\nu}} = \left(\frac{4}{\pi}\right)^{1/3}$ T_γ. Today, when the $T_\gamma = 3°$, the theory predicts that there shall be an equilibrium distribution of $\nu_e\bar{\nu}_e$ and also of $\nu_\mu\bar{\nu}_\mu$ corresponding both to the temperature $T_\gamma \equiv 2°$ K.* Their energy density is of the order of $1{,}5 \cdot 10^{-13}$ erg/cm^3 $= 0{,}1$ eV/cm^3, the mean energy is E $5 \cdot 10^{-4}$ eV, the number density $n \sim 200$ 1/cm^3.

The experimental investigation of these neutrinos is of the utmost importance as a direct proof of the very basis of modern cosmology. But the task is immensely difficult. The various methods for investigating cosmic neutrinos is reviewed in the report of G. Marx at the Balaton school. So we make only a few remarks.

The energy flux of solar neutrinos is estimated to be 5% of the total energy flux (10^6 erg/cm^2 sec) *i.e.* $5 \cdot 10^4$ erg/cm^2 sec; it gives the energy density $5 \cdot 10^4\, c^{-1} = 10^{-6}$ erg/cm^3. But the energy of solar neutrinos is of the order of 2 MeV so their number density at the Earth is $n \sim 1$ cm^{-3}. As is well known, there are feasible projects for detecting solar neutrions.

The main difficulty in our case arises as a result of the very small energy of cosmological ("relict") neutrinos.

It has been proposed to determine the effects near the endpoint E_0 of β spectrum in tritium decay. At $E \cong E_0 - kT$, where $T = 2°$, $kT = 2 \cdot 10^{-4}$ eV, the electron number shall be half as small compared with the normal theory (Kurie plot). But it appears now a group of electrons with energy higher than E_0 from $\nu + T = \mathrm{He}^3 + e^-$. Their $\bar{E} - E_0 \cong 6\,kT = 10^{-3}$ their number is the same as the number of missing electrons from the other side of E_0, $\Delta N = 3\left(\frac{T}{E_0}\right)^3 N$ $\sim 3 \cdot 10^{-24}\, N$ for tritium-helium decay where N is the number of normal electrons.

One should speculate on what effect relict neutrinos have on cosmic rays or artificially accelerated particles. The relict neutrinos single out the frame of reference in which the nearby galaxies are at rest; only in this frame they are isotropic and with $\tilde{E} \sim 10^{-3}$ eV. In the frame of a relativistic particle with $E_p = \delta \cdot M_p C^2$; $\delta = (1-\beta^2)^{-1/2}$ the $\nu\bar{\nu}$ appear as a gegenstream, with $E \sim \delta\; 8 \cdot 10^{-3}$ eV, with the density of the order of $n = n_0\, \delta/2$ and effective angular spread $\bar{\Theta} \sim \delta^{-1}$. The increase of density is compensated by the

* During the stay at Balaton, an interesting remark was made in a discussion with Gershtein: the mass of ν_μ is known only to be smaller than 2 MeV by particle physics.

But cosmological evidence perhaps could put a much more stringent limit, $m\ (\nu_\mu) <$ $> me/w_0 = 5$ KeV, because in the opposite case the density of the rest mass of $\nu_\mu\, \bar{\nu}_\mu$ would be too great, more than the allowed $2 = 4 \cdot 10^{-29}$.

relativistic time dilatation, when one calculates the number of interactions per unit length of path in the laboratory frame. The non-specific scatterings on neutrinos are far smaller than the similar scattering of the particle on thermal electromagnetic quanta. For the specific reaction $P + \nu = N + e^+$, one must have P with $E = 10^{18}$ eV in laboratory frame, and still the equilibrium neutron content is only 10^{18} of the protons, if only interaction with relict neutrinos is considered. The effect of the smaller number of more energetic stellar neutrinos is overwhelming.

Perhaps the collective coherent effects are more promising: for a neutrino with $E \sim 10^{-3}$ eV, the wavelength $\lambda \cong 0{,}1$ cm, all the electrons of a macroscopic volume scatter in the same phase, and the ν interaction with the nuclei is of second order and does not compensate the electrons. These effects are best described by the notion of the refractive index of ordinary matter for neutrinos.

Let us start from the interaction Hamiltonian, $H_{\text{int}} = g\overline{\psi}_e O\psi_e \overline{\psi}_\nu O\psi_\nu$ (after Fierz transformation) single out $O = \gamma_4$ and sum the contribution of all electrons $\overline{\overline{\psi}_e \gamma_4 \psi_e} = n_e$ where n_e the electron density, cm^{-3}. So the energy of a neutrino with given impulse $p = E_0/c$ is altered in matter by the amount $\Delta E = g n_e$. It corresponds to the index of refraction η such that:

$$\Delta\eta = \eta - 1 = \frac{\Delta E}{E_0} = \frac{g\, n_e}{E_0} \cong 10^{-9} \text{ for } n_e = 6 \cdot 10^{24}\, (gold) \text{ and } E_0 = 10^{-3}\text{eV}.$$

The sign of $(\eta - 1)$ is opposite for ν and $\bar{\nu}$. Perhaps one should search for the low energy excitations by neutrinos in a solid cooled down to $T \ll 1°$ K. For phonon excitation it is the momentum of neutrinos which is limiting more than their energy.

Finally, on a uniformly moving macroscopic particle of $r \sim \lambda \sim 0{,}1$ cm a force of the type of friction is expected.

$$F = -\varepsilon_\nu r^2 \frac{v}{c} \quad (\Delta n)^2,$$

so that the inverse time of deceleration of the order

$$\frac{1}{\tau} = \left|\frac{F}{m}\right| = 3 \cdot 10^{-42}\,\text{sec}^{-1}$$

$\left(\frac{1}{\tau} \sim \varrho\right.$, because $\Delta n \sim \varrho$, as it must be for coherent $\left.\text{effects}\right)$. The same time τ characterizes the onset of Brownian motion, corresponding to 2°, of the particle due solely to the interaction with relict neutrinos. Obviously, these effects are quite unobservable.

5. Quarks in the hot model

Let us assume that quarks exist as normal heavy (m) particles and antiparticles with one sort $\left(\text{for example } q \text{ with } z = +\frac{2}{3} e \text{ with } \bar{q} \text{ with } -\frac{2}{3} e\right)$ stable against weak interaction. At the very beginning, when $kT \gg mc^2$ they were as numerous as any other particles. During cooling down their equilibrium concentration drops:

$$n_q = n_{\bar{q}} \cong \left(\frac{kT}{mc^2}\right)^{3/2} \left(\frac{mc}{\hbar}\right)^3 e^{-mc^2/kT}.$$

But then comes a moment when the reaction rate of the establishment of equilibrium is too small, the remaining q and $\bar{q}$ are "quenched". Their concentration is further diminished only by the general expansion, the ratio of q/λ, q/ν or q/P, N (short for $n_q/n_\lambda \ldots$) tends to a finite limit. The quarks are strongly interacting and the small rate of the reactions leading to the disappearance of quarks is due to the fact that one must have always two q or q and $\bar{q}$ for the reaction $q + q = B + \bar{q}$ (B for baryons) or $q + \bar{q} =$ energy. So every *one* q or $\bar{q}$ is stable, but on the other side, triple encounters $3\,q = B +$ $+$ energy, $3\,\bar{q} = B +$ energy are not necessary.

The inverse time of bimolecular reaction $\frac{1}{\tau} = \sigma v n_q$, where σ is cross-section, v velocity, n_q concentration. This tends to zero with $n_q \to 0$.

So there exists an n_q which makes $\frac{1}{\tau} \leq \frac{1}{t_1}$, t_1 being the hydrodynamic time of expansion. The corresponding n_q/P is conserved afterwards (up to a numerical coefficient) because during the expansion

$$\frac{d\,(n_q/P)}{dt} = -\,\sigma v\,(n_q/P)^2\,P$$

and the $\int P dt = \int_{t_1}^{\infty} t^{3/2}\, dt$ does not diverge on the $t = \infty$ side. What is important is that the condition of quenching singles out a definite $n_q = n_1$.

The equilibrium n_q depends exponentially on the mass of the quark. But if $n_q = n_1$ is defined, it means that the moment of quenching t_1 and the characteristic temperature T_1 adjust themselves to the mass M. So the resulting n_1 does not depend exponentially on M. The hydrodynamic time $t_1 \sim 1/\sqrt{G}$ because $\varrho \sim -1/Gt^2$, $t = \sqrt{G/\varrho}$. By the quenching condition

$$n_1 \sim \frac{1}{t_1} \sim \sqrt{G}.$$

If $\sigma \sim \left(\frac{\hbar}{mc}\right)^2 \frac{c}{v}$ (by the order of magnitude), then by dimension argument

$$\frac{n_1}{N_1} = \sqrt{\frac{Gm^2}{\hbar c^2}} \cong 10^{-19},$$

where N_1 is the number density of all other kinds of particles ($\gamma, \nu, \ldots$) at the moment of quenching.

But the number density of baryons remaining after cooling is also a small part of all kinds of particles, of the order of 10^{-9}.

So the model predicts that primordial matter contains some 10^{-10} quarks per baryon! Of course we have lost here logarithmic factors of the type $(\ln Gm^2/\hbar c)^{-1} \cong 0{,}01$ but the result is still impressive, because, for example, the ratio of gold to hydrogen is now of the order of 10^{-12}.

The result about quarks quenching is due to Okun, Pikelner and the author. One must acknowledge that at the time of their work, the hot model had not been proved correct by radioastronomers, so in the original paper there are two extreme figures — for the hot and for the cold model. Now the higher one (for the hot model) should be taken.

The burning out of quarks in stars is not very great (see the original papers). The last work by Domokos and independently done by Feinberg and al. has shown that the quark production in cosmic ray encounters is small. So the search for relict cosmological quarks appears to be more appealing — if quarks exist at all, of course!

6. The charge—quasi-symmetry of the Universe

There are no signs of charge symmetry in the contemporary state of the Universe. The attempts at a theory of charge symmetrical Universe with spontaneous division of matter and antimatter are artificial and not in accordance with general cosmological theory.

In my opinion this asymmetry of *state* (prevalence of "matter" over antimatter) does not in any way contradict the known symmetry of *properties* of particles and antiparticles.

But in the hot model of the Universe there is an enigma: early at $T \sim M_p c^2$, there were antibaryons, so that approximately

$$B/\bar{B} = 1 + 10^{-8}.$$

The early state was *almost* charge symmetrical, but the small departure (of the order of 10^{-8}) from full symmetry is of the utmost importance for the present state. Such a situation seems very strange.

Perhaps more appealing is the assumption that there was a previous history at $t < 0$, before the singular state of $t = 0$, $\varrho = \infty$, $T = \infty$. One could assume that at $t < 0$, there was no charge symmetry just as now. Normal matter prevailed. By some nuclear reactions and other processes the matter was heated. During the implosion at $t < 0$ the pairs $B, \bar{B}$ were born quite naturally, the excess of B over $\bar{B}$ remaining. This excess also remains when at $t = 0$ the implosion at $t < 0$ is reversed to the expansion at $t > 0$.

Such a cosmological theory has recently been elaborated.

The difficult point is of course near $t = 0$ where at high densities general relativity is intimately tied with quantum mechanics.

It is the beautiful work of Roland Eötvös which is the basis of Albert Einstein's general relativity and through this of modern cosmology.

It is a great honour to make this report to a conference held by the Roland Eötvös Society. But it is also a tribute to the memory of the great Hungarian Physicist.

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ГОРЯЧАЯ МОДЕЛЬ ВСЕЛЕННОЙ И ЭЛЕМЕНТАРНЫЕ ЧАСТИЦЫ

Я. Б. ЗЕЛЬДОВИЧ

Резюме

Новые радиоастрономические наблюдения дали поддержку горячей модели Вселенной. Обсуждены следствия этой модели, и, в частности, дается оценка плотности числа кварков.

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NEUTRINOS, GRAVITY AND COSMOLOGY

By

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The observation of the soft cosmic neutrino and neutretto background, predicted by different cosmological theories, may be observed efficiently only with the help of reactions having no energy threshold. A new method is suggested making use of local gravitational fields.

The mean free path of neutrinos and neutrettos amounts to light years in condensed matter, and 10^{30} light years in the cosmic gas filling the Universe. This has the consequence that the neutrino radiation of extragalactic origin is affected neither by time nor distance, so it contains vital information concerning the Universe as a whole. It was PONTECORVO and SMORODINSKY who first emphasized the cosmological importance of the observation of the cosmic neutrino background [1].

The various cosmological theories predict different neutrino and neutretto fluxes. For example in the steady state theory, where neutrons are created locally at rest, from their decay a flux of intensity $I = 10^6\ \tilde{\nu}/\mathrm{cm}^2\mathrm{s}$ is expected, corresponding to an average mass density $\varrho = 10^{-31}$ g cm^{-3}. The energy spectrum has its maximum evidently at 0,78 MeV, but as a consequence of the Hubble shift on average 43% of the energy is lost, so the mean neutrino energy is well below 0,5 MeV. The different Hot Universe theories predict higher or lower densities and different types of neutrinos. For example, ZELDOVICH assumed a thermal radiation, corresponding to $T = 2$°K neutrino temperature, indicated by the observed thermal electromagnetic radiation of $T = 3$°K in the long wave length radio region [2]. This means $\varrho = 10^{-35}$g cm^{-3} both for ν_e, $\tilde{\nu}_e$ and for ν_μ, $\tilde{\nu}_\mu$ if the rest masses are vanishing. In the case of a neutretto rest mass of $m(\nu_\mu) \approx m(e)$ from $T = 2$ °K we should have a neutretto mass density $\varrho = 5 \cdot 10^{-30}$ g cm^{-3}. A decision among the various theories would be made possible by observing the soft neutrinos.

The classical method of neutrino detection is given by the induced β decay:

$$\left.\begin{matrix} \nu_e \\ \tilde{\nu}_e \\ \nu_\mu \\ \tilde{\nu}_\mu \end{matrix}\right\} + A^Z + E \rightarrow A^{Z+1} + \left\{\begin{matrix} e^- \\ e^+ \\ \mu^- \\ \mu^+ \end{matrix}\right. . \qquad (1)$$

Here the emitted charged leptons can be counted. The charged leptons are massive, so the reactions have a certain $E_{\min}$ energy threshold, being of the order of MeV for neutrinos, 100 MeV for neutrettos. As mentioned above, the energy of the extragalactic neutrinos has been degraded far below these thresholds because of the Hubble shift, so this classical method may be useful in the case of nearby sources (for atmospheric and solar neutrinos, where the energy E *is covered by the kinetic energy of the incoming neutrinos)*, but not in cosmological investigations. In the latter case the excess energy E must be supplied by the other partner of the reaction.

S. WEINBERG [3] suggested the use of radioactive nuclei instead of stable ones as target particles. In this case E *is liberated from the binding energy of the nucleus.* Analyzing the observed spectrum of H^3 WEINBERG deduced the estimation

$$p_F < 10^3 \text{ eV}/c \text{ for } \nu_e ,$$

where p_F means the Fermi momentum of the neutrino gas, assumed to be degenerate. (This corresponds to a density limit $\varrho < 10^{-10}$ g cm^{-3}.)

The very same idea [4] applied to the reaction

$$\nu_\mu + \mu^+ \rightarrow e^+ + \nu_e$$

gives

$$p_F < 5 \text{ MeV/c} \quad \text{for } \nu_\mu .$$

Another idea, due originally to BERNSTEIN, FEINBERG and RUDERMAN [5] is *to support the energy E needed for the reaction* (1) *in the form of the kinetic energy of the target particles.* BERNSTEIN et al. argued that the mean free path of very energetic protons would be shortened by the presence of an intensive neutrino sea as a consequence of inelastic collision

$$\tilde{\nu} + p^{\text{energetic}} \rightarrow n + e^+ .$$

From evidence obtained by the Brookhaven machine we have $p_F < 10^5$ eV both for neutrinos and neutrettos. If we knew that the most energetic cosmic ray protons came from outside the Galaxy, we should be able to push the upper limit down to

$$p_F < 1 \text{ eV},$$

corresponding to $\varrho < 10^{-22}$ g cm^{-3}, somewhat approaching the density values of cosmological interest. The origin of cosmic radiation is, however, not completely clarified, so a direct search for the energetic neutrinos, produced from soft ones by collision has been suggested [6].

$$\tilde{\nu}^{\text{soft}} + p^{\text{energetic}} \rightarrow n^{\text{energetic}} + e\,(\mu)\,; \quad n^{\text{energetic}} \rightarrow p + e^- + \tilde{\nu}^{\text{energetic}}.$$

This mechanism is able to transform an indetectable soft neutretto into an energetic neutrino. The recent South African experiments of REINES and his co-workers [7] have put an upper limit on the neutretto flux in the 100 MeV region. Comparing this value with the intensity of the cosmic proton radiation, we have reached the estimation $p_F < 10^3$ eV. The accuracy of this direct method may be improved by the improvement of the neutrino experiments working in the MeV—GeV domain in the coming years.

The most promising development in the observational neutrino cosmology would be afforded by the use of a thresholdless interaction instead of the classical weak transition (1). In [8] the possible importance of local gravitational fields has been emphasised. Let us now risk a more quantitative estimation.

In weak gravity approximation the metric tensor can be given as

$$g_{ii} = 1 + \frac{2}{c^2}\varphi(\mathbf{x})\,, \qquad g_{00} = 1 - \frac{2}{c^2}\varphi(\mathbf{x})\,, \qquad g_{\mu\nu} = 0 \text{ for } \mu \neq \nu\,.$$

Here $\varphi(\mathbf{x})$ is the absolute value of the local Newtonian potential, produced e.g. by cluster of stars or galaxies. $\varphi(\mathbf{x}) \rightarrow 0$ at large distances. What will be the equilibrium distribution of a neutrino gas in this geometry? As a consequence of the vanishing rest-mass of neutrinos, we have for the neutrino wave number

$$k_\mu k_\nu g^{\mu\nu} = 0\,, \qquad \text{i.e. } |\mathbf{k}| = \left[1 + \frac{2}{c^2}\varphi(\mathbf{x})\right] k_0\,. \tag{2}$$

Now, the number of neutrinos in a three dimensional volume V is given by

$$N = \int_V f(k_0)\,\frac{dx^1\,dx^2\,dx^3\,dk_1\,dk_2\,dk_3}{(2\pi)^3}, \tag{3}$$

where, e.g., in the case of complete degeneracy

$$f(k_0) = \begin{cases} 1 \text{ for } k_0 < k_F\,, \\ 0 \text{ for } k_0 > k_F\,. \end{cases}$$

By making use of the connection (2), we integrate the expression (3) with respect to k, with the result

$$dN = \frac{4\pi}{(2\pi)^3}\,k_F^4\left(1 + \frac{6}{c^2}\varphi(\mathbf{x})\right) dx^1\,dx^2\,dx^3\,.$$

(Terms of higher order in φ/c^2 are neglected.) This shows that in the vicinity of gravitating bodies the number density of neutrinos is increased, the relative excess of neutrino density being proportional to φ/c^2. Evidently the same is true for energy and mass density. The neutrino energy present in a given volume V amounts to

$$E = \int_V p^0 f(k_0) \frac{dx^1\, dx^2\, dx^3\, dk_1\, dk_2\, dk_3}{(2\pi)^3} ,$$

where the neutrino energy p^0 is given by

$$p^0 = g^{00} k_0 ,$$

so, finally

$$dE = \frac{4\pi}{(2\pi)^3} k_F^4 \left(1 + \frac{8}{c^2} \varphi(\mathbf{x})\right) dx^1\, dx^2\, dx^3 ,$$

which corresponds to a mass distribution

$$\varrho(\mathbf{x}) = \varrho_{00} \left[1 + \frac{6}{c^2} \varphi(\mathbf{x})\right] + O\left(\frac{\varphi^2}{c^4}\right) .$$

ϱ_{00} is the mass density of the unperturbed degenerate neutrino sea at large distances from the gravitating body. The mass density excess due to gravitational polarisation of nearby masses is given by

$$\delta\varrho(\mathbf{x}) = \frac{6}{c^2} \varphi(\mathbf{x})\, \varrho_\infty$$

in first approximation in the constant of gravity k. This mass excess amplifies the gravitational stability of the cluster. Let the mass of the cluster of stars or galaxies be M, its radius R, the stellar mass density ϱ_* approximated by a constant, so the potential φ_* can be taken in Newtonian approximation,

$$\varphi_*(r) = -\frac{\varkappa M}{R} \cdot \begin{cases} \xi^{-1} & \text{for } \xi > 1 , \\ \frac{3}{2} - \frac{1}{2}\xi^2 & \text{for } \xi < 1 , \end{cases}$$

where $\xi = r/R$. The neutrino density excess in first approximation turns out to be

$$\delta\varrho_\nu(r) = \frac{6}{c^2} \varrho_\infty \varphi_*(r)$$

and its potential

$$\varphi_\nu(\mathrm{x}) = \frac{24\pi\,\varkappa^2 MR}{c^2}\,\varrho_\infty \left(\frac{D}{R} - \frac{3}{9} - \frac{1}{4}\xi^2 + \frac{1}{4}\xi^4\right) \quad \text{for} \quad \xi < 1\,.$$

Here D is the upper limit of radial integration, something like the main separation of the clusters. The over all potential energy of the cluster is evidently given by the formula

$$E_{\mathrm{pot}} = \frac{1}{2}\int (\varrho_* + \delta\,\varrho_\nu)\,(\varphi_* + \varphi_\nu)\,d^3 x\,,$$

which can be evaluated, let us say, up to second order in the constant of gravity, $\varkappa/c^2$. From the point of view of the stability of the cluster, the following expression is of importance:

$$\frac{dE_{\mathrm{pot}}}{dR} = \frac{3}{5}\,\frac{\varkappa\,M^2}{R^2}\left[1 + 125\pi\frac{\varkappa}{c^2}\,\varrho_\infty\,R^2\right].$$

We see that the effective stabilizing mass is given by

$$M_{\mathrm{eff}} = M\left[1 + 62{,}5\pi\,\frac{\varkappa}{c^2}\,\varrho_\infty\,R^2 + 0\left(\frac{\varkappa^2}{c^4}\right)\right].$$

E.g. the effective mass is doubled by the presence of the neutrino sea if the overall neutrino density is of the order

$$\varrho_\infty \sim \frac{c^2}{62{,}5\,\varkappa\,R^2}\,.$$

It can be seen that the role of gravitational polarisation(the gravitational feedback coming from cosmic neutrinos) is completely negligible in the case of stars, even for galaxies, but for a giant cluster of galaxies a moderate neutrino density may result in a considerable stabilizing effect. Let us put in as characteristic value $R = 10^8$ light years; for such a galaxy cluster a background density $\varrho_\infty = 10^{-26}$ g cm^{-3} may be of importance. Owing to the pioneering works of AMBARTSUMIAN and DE VANCOULEURS, the empirical study of stability problems is progressing, with very promising preliminary results. So the observation of invisible mass densities in space may be possible by taking the effects of gravitational polarisation into account.

If we put R the radius of the Universe, $R \approx 10^{10}$ light years, we obtain $\varrho \approx 10^{-30}$ g cm^{-3} as the critical neutrino density which may show itself in the gravitational behaviour of the Metagalaxy. In fact, in the framework of relativistic cosmologies, by assuming a vanishing cosmological constant* and by assuming homogeneity and isotropy, PONTECORVO and SMORODINSKY concluded that an average mass density higher than $2 \cdot 10^{-29}$ g cm^{-3} is hardly possible according to the astronomical evidence [1].

In conclusion, it can be stated that a cosmic neutrino density higher than the optically observed mass density, may show itself by its gravitational effects, if sufficiently sophisticated astronomical methods are used.

If, however, the actual neutrino density turns out to be lower than the optical recognisable mass density or the electromagnetic energy density the gravitational method cannot be applied. One has to go back to the selective atom-physical methods. To-day one cannot say how it would be possible to increase the atom-physical detection sensitivity by ten or twenty orders of magnitude. As can be seen the number of ideas is growing fast in time in this romantic field of astrophysics, but still more sophisticated concepts are needed if we are interested in hearing the background music of the Big Dawn of our Universe, which is hi-fi recorded in the cosmic neutrino fluxes.

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НЕЙТРИНО, ТЯГОТЕНИЕ И КОСМОЛОГИЯ

Г. МАРКС

Резюме

Фон мягких космических нейтрино и нейтретто, предсказываемый разными космологическими теориями, может быть эффективно наблюден только при помощи реакций, не имеющих энергетического порога. Предложен новый метод, связанный с локальными гравитационными полями.

* It should be noted, however, that the vacuum expectation value of the energy momentum tensor operator may give rise just to a term $< 0 \mid T_{\mu\nu} \mid 0 > = \lambda g_{\mu\nu}$ because of the Lorentz symmetry of the vacuum state.

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MEASUREMENT OF THE TIME-REVERSAL PARAMETER IN THE DECAY OF THE Λ PARTICLE

By

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Abstract

A high sensitivity experiment to measure the time-reversal parameter in lambda decay in progress at the CERN proton synchrotron is described. Preliminary results are presented.

ИЗМЕРЕНИЕ ПАРАМЕТРА ОБРАЩЕНИЯ ВРЕМЕНИ В РАСПАДЕ Λ-ЧАСТИЦЫ

ДЖ. КОНФОРТО

Резюме

Описывается высокочувствительный эксперимент по измерению параметра обращения времени на протонном синкротроне CERN. Даны предварительные результаты.

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ON THE LEPTONIC DECAYS OF HADRONS

By

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Abstract

The experimental data on leptonic decays of baryons have been reexamined in the light of a two-angle Cabibbo theory, where the two angles, Θ_V and Θ_A are characteristic of the vector and axial vector baryon currents, respectively. With certain assumptions about the energy dependence of the form factors in the vector (K_{e3}) and axial vector ($K_{\mu 2}$, $\pi_{\mu 2}$) decays of mesons, it can be shown that the angles Θ_V and Θ_A derived from baryon decays are compatible with the corresponding angles derived from meson decays. There is no discrepancy between the information from hyperon and meson decays and the information from the superallowed nuclear beta decays (O^{14}, Cl^{34}, . . .). From a fit of all data on leptonic decays of hadrons we obtain the values

$$\Theta_V = 0{,}212 \pm 0{,}004, \quad \Theta_A = 0{,}268 \pm 0{,}001, \quad \alpha = 0{,}665 \pm 0{,}018\,,$$

where the parameter α defines the relative content of D coupling in the baryonic axial vector current.

О ЛЕПТОННЫХ РАСПАДАХ АДРОНОВ

Н. БРЕНЕ, Ц. КРОНШТРЕМ, Л. ВЕЙЕ и М. РУУС

Резюме

Пересмотрены экспериментальные данные по лептонным распадам барионов с точки зрения двухугловой теории Кабиббо, в которой два угла Θ_V и Θ_A характеризуют соответственно векторный и аксильный ток барионов. При помощи некоторых предположений относительно зависимости формфакторов от энергии в векторных (K_{e3}) и аксиальвекторных ($K_{\mu 2}$, $\pi_{\mu 2}$) распадах мезонов можно показать, что Θ_V и Θ_A, полученные из барионных распадов, сравнимы с соответственными углами в мезонных распадах. Нет противоречий между информациями о гиперонных и мезонных распадах и данными о сверхразрешенных ядерных β-распадах (O^{14}, Cl^{34},. .). Из всех данных касающихся лептонных распадов адронов полученные значения

$$\Theta_V = 0{,}212 \pm 0{,}004, \; \Theta_A = 0{,}268 \pm 0{,}001, \; \alpha = 0{,}665 \pm 0{,}018,$$

где параметр α определяет относительную величину D-связи в барионном аксиальном токе.

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CONSISTENT THEORY OF WEAK INTERACTIONS

By

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Abstract

An S-operator theory of weak interactions will be discussed which fulfills all the requirements of consistency including unitarity. In an expansion to first order in G the results are identical to those obtained in first order perturbation theory from standard Lagrangian Formulation of weak interactions, but higher order corrections can be calculated. They are finite apart from a single infinite parameter which can be absorbed in a coupling-constant renormalization. The complete renormalization of the theory will be discussed.

НЕПРОТИВОРЕЧИВАЯ ТЕОРИЯ СЛАБЫХ ВЗАИМОДЕЙСТВИЙ

Х. ПИТЧМАНН

Резюме

Рассматривается S-операторная теория слабых взаимодействий, удовлетворяющая всем основным требованиям, включая унитарность. В разложении по G первый порядок дает аналогичные результаты с первым порядком в теории возмущений с обычным Лагранжевым формализмом слабых взаимодействий, но поправки более высокого попрядка могут быть рассчитаны. Они конечны кроме одного бесконечного параметра, который можно устранить с помощью перенормировки константы связи. Полная перенормировка теории будет изучена.

Acta Physica Academiae Scientiarum Hungaricae, Tomus 22 (1—4), p. 68 (1967)

TIME REVERSAL INVARIANCE IN BETA DECAY

By

F. JANAUCH

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Abstract

The time reversal invariance in beta decay is discussed.

T — ИНВАРИАНТНОСТЬ В БЕТА-РАСПАДЕ

Ф. ЯНАУХ

Резюме

Обсуждена *T* — инвариантность в бета-распаде.

Acta Physica Academiae Scientiarum Hungaricae, Tomus 22 (1—4), p. 69 (1967)

RADIATIVE DECAYS OF BARYONS

By

B. N. VALUEV

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Abstract*

The results of calculation for weak electromagnetic processes $B_1 \to B_2 + e^- + e^+$, $B_1 \to B_2 + \gamma$ will be given. (B_1, B_2 denote baryons with spin 1/2). Calculations were carried out in a pole approximation (one-photon approximation). In this case all quantities are expressed in terms of four form-factors.

The expressions obtained are used for the discussion of possible tests of *CP* conservation in such processes as $\Sigma^+ \to p + e^- + e^+$, $\Xi^- \to \Sigma^- + e^- + e^+$, etc.

РАДИАЦИОННЫЕ РАСПАДЫ БАРИОНОВ

Б. Н. ВАЛУЕВ

Резюме**

В докладе будут представлены результаты вычислений для слабо-электромагнитных процессов $B_1 \to B_2 + e^- + e^+$, $B_1 \to B_2 \to \gamma$, где B_1, B_2 — барионы со спином 1/2. Вычисления проведены в полюсном (однофотонном) приближении. В этом случае все величины выражаются через четыре формфактора. На основании полученных выражений обсуждаются возможые способны проверки *CP* сохранения в таких процессах как $\Sigma^+ \to p + e^- + e^+$, $\Xi^- - \Sigma^- + e^- + e^+$ и т. д.

* For details see preprint JINR, P-2823, Dubna, 1966.
** Подробности можно найти в препринте ОИЯИ, P-2823 Дубна, 1966.

Acta Physica Academiae Scientiarum Hungaricae, Tomus 22 (1—4) p. 70 (1967)

ON THE FORM FACTORS IN THE K_{l3} DECAY

By

N. Smiernitzky

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Abstract

The recent experimental results on the form factors in the K_{l3} decay are discussed.

О ФОРМФАКТОРАХ В РАСПАДАХ K_{l3}

Н. СМЕРНИЦКИЙ

Резюме

Обсуждены последние экспериментальные результаты по формфакторам в K_{l3} распадах.

Acta Physica Academiae Scientiarum Hungaricae, Tomus 22 (1—4), pp. 71—76 (1967)

SESSION 2. *CP* VIOLATION

CP VIOLATION AND $\pi^+\pi^-$ INTERFERENCE IN NEUTRAL *K* DECAY

By

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and

SLAC, STANFORD UNIVERSITY, STANFORD, CALIFORNIA, USA

The elementary facts concerning the decay of neutral K mesons and the proposed CP violating interactions are briefly summarized. The recent interference experiments on the decay mode for a coherent mixture of K_l and K_S are discussed.

Two years ago, a group of experimental physicists [1] at Princeton discovered the $\pi^+\pi^-$ decay mode of the long lived neutral K meson, with a branching ratio $R(K \to \pi^+ \pi^-)/(K \to \text{all charged}) \sim 2\times 10^{-3}$. The fundamental importance of this result is that it indicated that CP invariance, the combined symmetry of particle anti-particle conjugation and parity, is violated by some interactions in nature. As we shall see later on, we are still quite in the dark regarding the properties of this CP violating interaction. Actually several attemps have been made to save CP invariance by invoking additional fields or particles, or possible non-linear modifications of quantum mechanics. However, recent experiments have essentially disproved these hypotheses; the talk of M. ROOS will cover some of these points. Since you have received a copy of the excellent review article of G. MARX on the topic of the lectures today [2], we shall concentrate this discussion mainly on the recent beautiful experiments at CERN on the interference between the $\pi^+\pi^-$ decay mode of the short and of the long lived neutral K mesons [3], and attempt to relate the results to the fundamental questions concerning CP violation. We shall also remark on some experiments which are now in progress to resolve some of the ambiguities which we face at present.

To begin with, let me remind you of some of the elementary facts concerning the decay of neutral K mesons if there are CP non-conserving interactions. Assuming TCP invariance (on which we shall comment at the end), the short and the long lived states of the K mesons can be written in the form

$$\begin{aligned} K_S &= pK_0 + q\bar{K}_0 , \\ K_l &= pK_0 - q\bar{K}_0 . \end{aligned} \tag{1}$$

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These are the eigenstates of a non-hermitian operator with matrix elements

$$(K_i\,|\,H\,|\,K_j) + \left(K_i\,|T^+ \frac{1}{(H_0 - m_K + i\,\varepsilon)} T\,|\,K_j\right); \quad K_i = K_0, \bar{K}_0 \tag{2}$$

and complex eigenvalues $m_s - \dfrac{i}{2\tau_s}$ and $m_l - \dfrac{i}{2\tau_l}$ respectively, where m is the mass and τ is the lifetime of the K mesons; experimentally $|\Delta m| \equiv |m_l - m_s| \cong \dfrac{1}{2\tau_s}$, $\tau_l \sim 600\tau_s$ and $\tau_s \sim 10^{-10}$ sec. In Eq. 2, the matrix elements $(n\,|\,T\,|\,K_i)$ give the transition amplitude for $K_i \to n$, while the off diagonal matrix elements $(K_i\,|\,H\,|\,K_j)$ account for the possibility of a $\Delta S = 2$ CP violating transition.

For the ratios of the amplitudes of K_l and K_s going to the same 2π state, one obtains the results

$$\eta_{+-} \equiv \frac{(\pi^+\pi^-\,|\,T\,|\,K_l)}{(\pi^+\pi^-\,|\,T\,|\,K_s)} = \varepsilon_0 + \frac{1}{\sqrt{2}}\,\varepsilon_2\,,$$

$$\eta_{00} \equiv \frac{(\pi^0\pi^0\,|\,T\,|\,K_l)}{(\pi^0\pi^0\,|\,T\,|\,K_s)} = \varepsilon_0 - \sqrt{2}\,\varepsilon_2\,, \tag{3}$$

where

$$\varepsilon_0 = \frac{p-q}{p+q} \quad \text{and} \quad \varepsilon_2 = i\ \mathrm{Im}\ \frac{(A_2\,e^{-i\delta_2})}{|A_0|}\,e^{i(\delta_2-\delta_0)}\,.$$

The amplitude $A_I = (2\pi, I\,|T\,|\,K_0)$ is the $K_0 \to 2\pi$ transition amplitude in the isospin state I, where we have chosen the phase of K_0 relative to the $I = 0$, 2π state in such a way that $A_0 = |A_0|\,e^{i\delta_0}$. We can also express ε_0 directly in terms of the off diagonal absorptive part $\Gamma_{0,\bar{0}}$ and the dispersive part $M_{0,\bar{0}}$ of Eq. 2

$$\varepsilon_0 = \frac{i\,\mathrm{Im}\,M_{0\bar{0}} + \mathrm{Im}\,\Gamma_{0\bar{0}}}{(m_s - m_l) - \dfrac{i}{2}\left(\dfrac{1}{\tau_s} + \dfrac{1}{\tau_l}\right)}, \tag{4}$$

where

$$\Gamma_{0,\bar{0}} = \sum_n (n\,|\,T\,|\,K_0)^*\,(n\,|\,T\,|\,\bar{K}_0)\,\pi\,\delta(\varepsilon_n - m_K).$$

Note that with the phase convention described above, the $I = 0$ 2π state does not contribute to $\mathrm{Im}\,\Gamma_{0,\bar{0}}$.

Now let me briefly summarize the proposed *CP* violating interactions according to their strangeness changing properties [4] and indicate their relation to the amplitudes which enter in neutral $K \to 2\pi$ decays. The three possibilities that need to be considered are given in Table I.

Table I

ΔS	Interaction
2	superweak
1	weak
0	medium strong or electromagnetic

WOLFENSTEIN [5] has suggested that the *CP* violating interaction allows $\Delta S = 2$ transitions, in which case it should be much weaker than the *CP* conserving weak interactions. We can then ignore the second term in Eq. 2, so that

$$M_{ij} = (K_i | H | K_j) \text{ and } \Gamma_{ij} = 0 .$$

Hence, in this case

$$\varepsilon_0 = -\frac{i\,\mathrm{Im}\,(K_0 | H | \bar{K}_0)}{\Delta m + \frac{i}{2}\left(\frac{1}{\tau_s} - \frac{1}{\tau_l}\right)}\,,\quad \Delta m = m_l - m_s$$

and $\varepsilon_2 = 0$. We see immediately that the branching ratio

$$R\left(\frac{K_l \to \pi^+ \pi^-}{K_s \to \pi^+ \pi^-}\right) \equiv |\eta_{+-}|^2 = \varepsilon_0|^2$$

determines $|\mathrm{Im}\,(K_0 | H_w | \bar{K}_0)|$, and from the experiment of CHRISTENSON et al. [1] we have

$$|fm\,(K_0 | H | \bar{K}_0)| \simeq 2 \times 10^3 \left|\Delta m + \frac{i}{2\tau_s}\right| \sim 2 \cdot 4 \times 10^3 \Delta m .$$

Note that in this case the phase φ_0 of ε_0 is determined to be

$$\tan^{-1} \frac{2\tau_s\,\Delta m}{(1 - \tau_s/\tau_l)} \sim \pm 45°$$

and the branching ratio

$$R_0\left(\frac{K_l \to 2\pi^0}{K_l \to \pi^+ \pi^-}\right) = R\left(\frac{K_s \to 2\pi^0}{K_s \to \pi^+ \pi^-}\right).$$

According to the $|\Delta I| = \frac{1}{2}$ rule $R_0 = \frac{1}{2}$ which is satisfied experimentally. It should be added that with present techniques, the only observable effects of this superweak interaction appear in the neutral K decay.

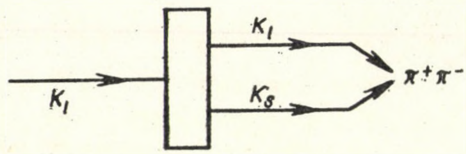

Fig. 1. Coherent superposition K_S and K_l obtained by passing a beam K_l through a slab of matter

If the CP violating interaction allows $\Delta S = 1$ transitions then no definite predictions can be made about φ_0 and R_0 unless further assumptions are made. In particular, if we suppose that this interaction also satisfies the $\Delta I = \frac{1}{2}$ rule, we have $R_0 = \frac{1}{2}$. Furthermore, if the $\Delta S = \Delta Q$ rule is satisfied, the same phase φ_0 as in the case of $\Delta S = 2$ interactions follows, since in this case the leptonic modes cannot contribute to $\Gamma_{0,\bar{0}}$, and we would then expect $\mathrm{Im}\,\Gamma_{0,\bar{0}} \ll \mathrm{Im}\,M_{0,0}$, unless there is an anomalously large CP violation in the 3π decay amplitudes. On the other hand, one should then expect to find observable consequences of CP violations in decays other than that of the neutral mesons [6].

Finally, there is the possibility that the CP violating interaction satisfies the $\Delta S = 0$ rule [4, 7]. In this case this interaction is of the same order as electromagnetic interaction (since its effect in K decays occur through second order processes) and no definite predictions seem possible about either φ_0 or R_0 (however, see PRENTKI's report in [2]). The consequences of CP violation in electromagnetic interactions will be discussed by M. JACOB in the next lecture.

Now let us turn to some of the recent interference experiments on the $\pi^+ \pi^-$ decay mode for a coherent mixture of K_l and K_S. At CERN, two experiments [3] have been carried out in which a long-lived K beam is incident on a slab of matter which then generates a coherent mixture of K_s and K_l (see Fig. 1). By measuring the $\pi^+ \pi^-$ intensity from the decay of the coherent K beam as a function of the distance x from the slab, the relative phase φ_0 of K_s and K_l into $\pi^+ \pi^-$ may be obtained. The amplitude for the $\pi^+ \pi^-$ state at $t = \frac{x}{v}$, where v is the velocity of the incident K meson is proportional to

$$\eta_{+-} e^{-i\left(m_l - \frac{i}{2\tau_l}\right)t} + r e^{-i\left(m_s - \frac{i}{2\tau_s}\right)t}, \tag{5}$$

where η_{+-} is given by Eq. 3 and r is the complex K_s regeneration amplitude

$$r = \frac{\pi(f - \bar{f})N}{m_K\left[\Delta m + \frac{i}{2}\left(\frac{1}{\tau_s} - \frac{1}{\tau_l}\right)\right]} (1 - e^{-i\left[\Delta m + \frac{i}{2}\left(\frac{1}{\tau_s} - \frac{1}{\tau_l}\right)\right]\frac{d m_K}{p_K}}, \tag{6}$$

where $f(\bar{f})$ is the forward K^0, $(\bar{K}^0)$ nuclear scattering amplitude and N is the density of the regenerator of thickness d.

The $\pi^+ \pi^-$ intensity is proportional to

$$|\eta_{+-}|^2 e^{-t/\tau_l} + |r|^2 e^{-t/\tau_s} + 2|\eta_{+-}||r| e^{-\frac{t}{2}\left(\frac{1}{\tau_s} + \frac{1}{\tau_l}\right)} \times \cos(\Delta m t + \varphi_{+-} - \varphi_r),$$

where φ_{+-} and φ_r are the phases of η_{+-} and r respectively. It is clear that this experiment can only determine the relative phase $\varphi_{+-} - \varphi_r$, and that to obtain φ_r the phase of the difference $(f - \bar{f})$ between the K^0 and $\bar{K}^0$ forward amplitudes has to be known. The recent fits obtained at CERN for $\varphi \equiv \varphi_{+-} - \arg i(f - \bar{f})$ and the value $\Delta m \tau_s$ are given in Table II.

Table II

	C. Steinberger et al.	M. Bott. Bodenhausen et al.
$\varphi \equiv \varphi_{+-} - \arg i(f - \bar{f})$	$80{,}4° \pm 10°$	$87{,}5° \pm 4{,}9°$
$\lvert\Delta m\rvert\, \tau_S$	$0{,}445 \pm 0{,}34$	$0{,}48 \pm 0{,}2$

It should be noted that the experiment of C. Alff-Steinberger et al. is carried out with a copper regenerator and a K beam of $2{,}5 \pm$ BeV, while M. Bott-Bodenhausen et al. use carbon regenerators and a K beam energy of $4{,}5 \pm 2$ BeV/c.

We cannot arrive at any definite conclusions regarding φ_{+-} until Arg $i(f - \bar{f})$ is determined. The remarkable agreement between the phase φ for these two recent CERN experiments carried out at different K beam energies and with different targets is consistent with the nuclear model calculations indicating that f and $\bar{f}$ are mainly imaginary and therefore do not contribute much to φ. In that case, the superweak model is ruled out. However, we can expect in the near future to have a determination of Arg $i(f-\bar{f})$ by measuring interference in the leptonic decays of a coherent mixture of K_s and K_l, and also by a determination of φ_{+-} measuring the $\pi^+\pi^-$ intensity relative to the distance from the production region, where the strong interaction conservation of strangeness determines the mixture of K_s and K_l at $t=0$.

I like to end this discussion with a brief comment. It is straightforward to carry out this phenomenological analysis without introducing the restriction of *TCP* invariance. In this case ε_0 depends on an additional parameter proportional to the difference between the diagonal elements of Eq. 2, and on the difference $(A_0 - \bar{A}_0)$ and the phase of ε_2 is no longer determined by the difference $(\delta_2 - \delta_0)$ *S*-wave $\pi\pi$ phase shifts. It will be interesting to bear in mind this possibility for the future.

Note added in proof: Recent experiments have ruled out the superweak model of *CP* violation. The branching ratio for $K_1 \to 2\pi^0$ has been measured by two groups giving $|\eta_{00}| = 4\cdot 9 \pm 0\cdot 5 \times 10^{-3}$, J. Cronin et al., Phys. Rev. Letters **18**, 25, 1967, and $|\eta_{00}| = 4\cdot 3^{+1\cdot 1}_{-0\cdot 8} \times$ $\times 10^{-3}$, J. Gaillard et al., Phys. Rev. Letters **18**, 20, 1967. Furthermore, recent total K^+ and K^- cross section measurements in Cu by Cool et al., Brookhaven (to be published), lead to the conclusion [see C. Rubbia and J. Steinberger (Phys. Letters to be published)] that the angle arg $i(f\text{-}\bar{f})$ is at most a few degrees.

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НАРУШЕНИЕ *CP* И $\pi^+\pi^-$-ИНТЕРФЕРЕНЦИЯ В НЕЙТРАЛЬНОМ *K*-РАСПАДЕ

М. НАУЭНБЕРГ

Резюме

Дается краткий обзор элементарных фактов по распаду нейтральных *K*-мезонов, и предложенных взаимодействий, нарушающих *CP*. Обсуждается последний эксперимент по интерференции в $\pi^+\pi^-$-распаде когерентной смеси K_l и K_s.

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THE STATUS OF *CP* INVARIANCE IN $K^0 \to \pi^+ \pi^-$ DECAYS

By

M. Roos

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The remaining possibilities to explain the long-lived $\pi^+ \pi^-$ decays of K^0 mesons by *CP* invariant theories are examined in the light of two recent regeneration experiments at CERN.

Among the possible explanations to the observation [1] of long-lived decays of K^0 mesons into $\pi^+ \pi^-$, some conjectures have retained strict *CP* invariance. Most of these conjectures have already been proved wrong. For the remaining *CP* invariant theories [2—4] feasible experiments have been proposed [5] as decisive tests of *CP* invariance. In this talk, we are going to report on two recent regeneration experiments [6, 7] at CERN which shed some light on this question.

For the purpose of this talk it is not necessary to distinguish between the different *CP* invariant theories [2—4]; we will just call them *CP* theories in contrast to $\overline{CP}$ theories which violate *CP* invariance. As remarked before [4], [5], *CP* theories and $\overline{CP}$ theories predict interference terms of different magnitude in regeneration experiments, when the K_2^0 beam is an incoherent mixture of K^0 and $\overline{K}^0$ mesons. Let the beam contain K^0 and $\overline{K}^0$ mesons in the ratio $Z/1 - Z$ (at the source), then the interference term in *CP* theories will be proportional to

$$(2Z - 1)\,, \tag{1}$$

whereas no such proportionality occurs in $\overline{CP}$ theories. Instead, $\overline{CP}$ theories correspond to taking the factor (1) equal to 1.

Since it is very difficult to obtain information of the value of Z, we have suggested [5] that the interference term be studied as a function of the measurable ratio K^+/K^-. This ratio is known to vary strongly with beam angle and momentum, and although the value of Z cannot be derived from the value of K^+/K^-, one would expect Z to vary detectably if K^+/K^- varies strongly.

Both CERN experiments [6], [7] have, however, been carried out at fixed beam energy and angle. Thus neither experiment has studied the variation of the interference term as a function of Z. Unfortunately, both experiments also claim identical K^+/K^- ratios, so that no information on the Z dependence can be obtained from combining the experiments.

The only useful information then comes from a best fit determination of the factor (1). The factor takes the values 1,20 ± 0,14 (a 56% likelihood solution) [6], and 1,08 ± 0,12 (a 98,5% likelihood solution) [7], respectively. The likelihood for $\overline{CP}$ theories ($2Z - 1 = 1$) is, in the two cases, 42% [6] and 98% [7], respectively.

If the ratios K^+/K^- and $K^0/\overline{K}^0$ are equal, the value of Z in these experiments is $\approx 3/4$. With this value, the best solution for CP theories has the likelihoods 0,3% [6] and 0,5% [7], respectively. If Z is underestimated even by a small amount, however, the best fit solutions of CP theories get acceptable likelihoods, although much worse than $\overline{CP}$ theories. To demonstrate the sensitivity on the true value of Z we tabulate below the likelihoods for CP theories as a function of Z.

Table

Z	Likelihoods in %	
	Ref. [6]	Ref. [7]
0,75	0,3	0,5
0,80	1,5	5
0,85	6	34
0,90	17	82
1,00	42	98

From this we conclude that CP violation is very much more likely than CP invariance, although the CP theories have not been ruled out in an entirely unambiguous way.

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ВОЗМОЖНОСТЬ CP-ИНВАРИАНТНОСТИ В РАСПАДЕ $K^0 \to \pi^+\pi^-$

М. РУУС

Резюме

Исследованы оставшиеся возможности для объяснения $\pi^+\pi^-$-распада долгоживущих K^0-мезонов с помощью CP-инвариантных теорий, в свете двух новых регенерационных экспериментов, проведенных в CERN.

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POSSIBLE *C* VIOLATION IN ELECTROMAGNETIC PROCESSES

By

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The various ways in which *C* non-invariance could appear in electromagnetic interactions are reviewed and the pertinent experimental consequences are analyzed with a detailed discussion of a few typical cases.

It is stressed that, even though *C* invariance might be badly broken, all experiments one can think of, at present, in order to observe its possible consequences, appear as searches for a measurable effect of at most a few per cent. This is due to various reasons which are indicated.

I. Introduction

During the two years which have now passed since the first announcement of an apparent *CP* violation in neutral *K* decay [1], the only solid piece of information which has become available with respect to this apparently puzzling fact is that the long lived *K* zero actually decays into both three pion and two pion states, thus violating invariance under *CP* [2]. The ingenious proposals [3] which have been put forward to explain the observed effect with an external interaction through which a K_2 could turn into a K_1 have been ruled out by experiment [4, 2]. At present, one is left with the ratio of two decay amplitudes corresponding respectively to the K_L and K_S $\pi^+ \pi^-$ decays. This ratio is known in absolute value

$$\left| \frac{A\,(K_L \to \pi^+ \pi^-)}{A\,(K_S \to \pi^+ \pi^-)} \right| = (1{,}85 \pm 0{,}12)\, 10^{-3}\,, \tag{1}$$

and its phase will be known soon [5, 2].

There is some reluctance to admit that the weak interactions, which otherwise have shown so far a perfect invariance under the combined operations of *P* and *C*, should also include *CP* violating terms [7]. Although this possibility should be kept in mind, pending better knowledge of weak reactions with large momentum transfer, another attitude is to put the blame on other interactions thus preserving an exact *CP* invariance for the proper weak interactions. One possibility is to postulate a *CP* violating but extremely weak interaction

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[8] which could manifest itself appreciably only in the $K_1^0 K_2^0$ transition, owing to the smallness of the mass difference [2]. Another possibility, which does not require the introduction of an hitherto unknown type of interaction, is to consider *CP* violation, as expressed by (1), as the result of the perturbation of basically *CP* invariant weak interaction by *C* violating electromagnetic [9, 10] or even medium strong [11] interactions. The possibility that *C* invariance could be violated by medium strong interactions is now hard to reconcile with recent experimental information on anti-proton anihilation [12] as well as several other results pertaining to Time-reversal Invariance in strong interactions [9]. On the other hand, a possible violation of *C* invariance in electromagnetic interactions is at present still compatible with all known experimental results [9] and could furthermore provide an intuitive explanation for the smallness of the K_L decay amplitude into two pions. At first guess, this value is α/π which is quite compatible with (1). The various possibilities so far mentioned have been presented and discussed in parallel by PRENTKI at the Oxford Conference [13]. Each hypothesis leads to several consequences beside the two pion decay of the K_L which each deserve proper theoretical and experimental attention. The purpose of this paper is more modest. It is to present the various consequences of a possible *C* violation in Electromagnetic Interactions together with the limitations already imposed by the experiments which have been done and are at present being done to test this possibility. Even though all that follows is still at the conjecture level, it is the precise meaning of an invariance property which is at stake, and the intense experimental activity which is at present proceeding along these lines makes the whole subject worthy of frequent discussions. As analysed in detail by LEE in a remarkable series of papers [14] the selection rules which could be associated with *C* violation in Electromagnetic processes already lead to many different predictions. We shall first present the different theoretical possibilities together with some of their implications. We shall then try to estimate the effects expected in the decay of mesons which provide apparently handy tests. A discussion of each currently usable case would be outside the scope of this paper and we shall put our emphasis on the η and X^0 decays which have been most extensively studied so far. We shall then finally consider the expected effects in experiments involving high energy photons [15]. Both types of experiments by no means provide an exhaustive list. On the contrary this omits many reactions with which similar tests could be conducted. We prefer here to focus on these two types of reactions, which are a priori quite typical, rather than to give survey of the many reactions available. Many more reactions are presented in references [9, 14, 15]. Some of these are listed in an Appendix. In the whole discussion, we shall assume *CPT* invariance. In particular, any violation of *CP* invariance will correspond to a violation of *T* invariance and we shall consider tests of both.

II. *C* non-invariant electromagnetic interactions

It may appear puzzling that *C* invariance still remains a question when it seems so deeply associated with Electromagnetic interactions which are comparatively extremely well understood. As emphasized, however, by BERNSTEIN, FEINBERG and LEE [9], if *C* invariance is a basic property of Standard Electrodynamics, its relevance whenever hadronic currents are introduced, still remains an hypothesis. It has often been taken for granted so far, without involving any contradiction, but this may be due merely to the fact that it often results anyway from other conditions such as Lorentz invariance, parity invariance and gauge invariance.

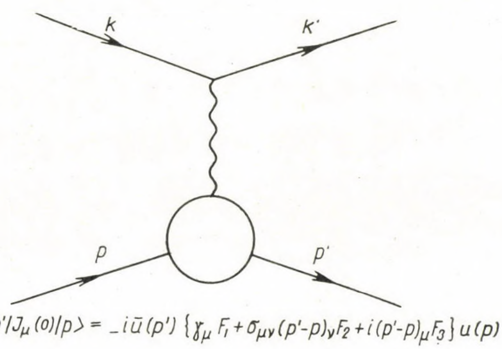

Fig. 1. Electron-baryon scattering

If one considers, for instance, electron-baryon scattering in lowest order in electromagnetic interactions, as shown in Figure 1, the whole amplitude can be explicitely written down except for the matrix element of the electromagnetic current operator J_μ between the one baryon initial and final states

$$\langle B(p^+)|J_\mu(0)|A(p)\rangle . \tag{2}$$

If both baryons are spin 1/2 particles, (2) is expressed in terms of three independent functions of the momentum transfer squared $t = -(p'-p)^2$ as [16]

$$\begin{aligned}\langle B(p')|J_\mu(0)|A(p)\rangle = -i\bar{u}_B(p')\{\gamma_\mu F_1(t) + \sigma_{\mu\nu}(p'-p)_\nu F_2(t) + \\ + i(p'-p)_\mu F_3(t)\}u_A(p).\end{aligned} \tag{3}$$

This is the most general expression when Lorentz invariance and parity invariance are required. If *T* (or *C*) invariance holds all terms should transform properly under *T* and this implies that F_1, F_2 and F_3, as defined in (3), should all be real. A relative phase, from which polarization effects would result even in lowest order, would show a failure of *T* (and *C*) invariance. Nevertheless, all detailed experiments so far conducted involve only nucleons for the *A* and *B* particles. The hermiticity of the current alone then implies that F_1 and F_2 should be real. F_3 should then be pure imaginary but it is not possible to test its presence since it does not contribute to the scattering amplitude as follows from the conservation of the electron current. In effect, F_3 should be zero also from the conservation of the hadronic current. This condition reads in general:

$$\langle B(p') | \partial_\mu J_\mu | A(p) \rangle = - i\, \bar{u}_B(p') \{(M_A - M_B) F_1(t) + t F_3(t)\} u_A(p) = 0$$

or

$$t F_3(t) = (M_B - M_A) F_1(t)$$

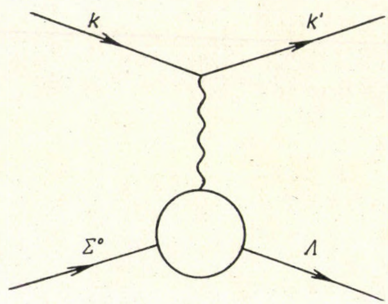

Fig. 2. $\Sigma^\circ \to \Lambda^\circ + e^+ + e^-$ decay

with $F_3(t) = 0$ if the baryons A and B are identical particles. So, present detailed information on electron proton scattering does not give any hint whether or not C invariance is satisfied. As explained in detail in reference [9] such an argument can be extended to most experiments so far analyzed. With different baryons, the same constraints no longer hold and tests are possible. The experiments, however, are much more difficult. A possible test could be provided by the $\Sigma^0 \to \Lambda^0 + e^+ + e^-$ decay as shown in Figure 2 [9] which is formally very close to the one just mentioned. It involves the matrix element:

$$\langle \Lambda(p') | J_\mu(0) | \Sigma(p) \rangle = - i\, \bar{u}_\Lambda(p') \left\{ \gamma_\mu \frac{t F_3(t)}{M_\Sigma - M_\Lambda} + \right. \\ \left. + \sigma_{\mu\nu} (p' - p)_\nu F_2(t) \right\} u_\Sigma(p)\,, \tag{5}$$

where F_2 and F_3 are new form factors, proper to this reaction, but which have been defined according to (3) and (4). We have not included a term $F_3(p' - p)_\mu$ since it does not contribute to the decay amplitude. A phase difference between the two form factors would violate T invariance and would result in a polarization of the Λ normal to the decay plane, allowed by parity invariance but forbidden by T invariance for a lowest order decay amplitude [18]. The normal polarization obtained with unpolarized Σ, is then proportional, beside known factors, to $t\ Im\ \{F_3^*(t)F_2(t)\}/\ |\ F_2(t)\ |^2$ at least for small t. In order to find a large effect, it is tempting to take large momentum transfer decays (large angle pairs) but these decays are infrequent. The process in analyzed in great detail in reference [9] and generalized to arbitrary hadronic states for A and B in reference [15]. The present limit is: 0,06 $\pm$ 0,03, inconclusive.

Even with a large violation to start with the measurable effect is a priori highly reduced with the imposed kinematics.

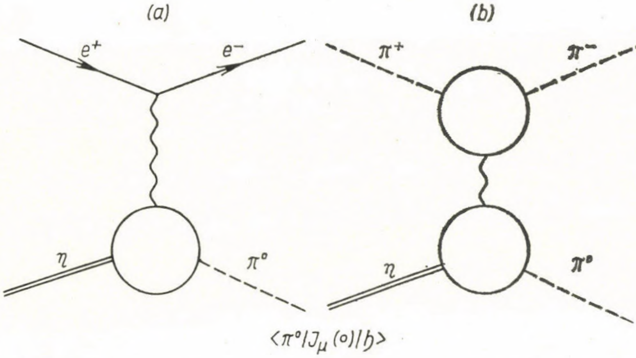

Fig. 3a. $\eta \to \pi^0 + e^+ + e^-$ decay *3b.* $\eta \to \pi^0\ \pi^-\ \pi^+$ decay with C violation (model)

This brief discussion illustrates two general points, the second of which will come up many times here. The first is that there is so far no direct proof of C invariance in electromagnetic processes involving hadrons, the second is that most C violation effects which could be expected turn out to be quite small, and this for various reasons, even if C violation would be large, that is C conserving and C violating coupling terms would be comparable in magnitude.

A similar argument can be carried for spinless particles, as shown in Figure 3. The most general expression for the matrix element of the Electromagnetic current operator now reads

$$\sqrt{4\,\omega_{p'}\,\omega_p}\,\langle b\,(p')\,|\,J_\mu\,(0)\,|\,a\,(p)\rangle = f_1\,(t)\,(p+p')_\mu + f_2\,(t)\,(p'-p)_\mu\,, \tag{6}$$

where current conservation requires

$$(m_b^2 - m_a^2)\,f_1\,(t) = t f_2\,(t)\,. \tag{7}$$

Only $f_1\,(t)$ is present when both a and b particles are identical. $f_2\,(t)$ does not contribute to the scattering amplitude as follows from the conservation of the electron current. If J_μ has non-zero matrix elements between two eigenstates of C with the same eigenvalue (η and π^0 say) C invariance will be violated.

Different particles are needed. The matrix element is otherwise zero since a photon cannot couple to a pair of identical spinless mesons.

As imposed by (7) this C-violating term should, however, by zero at zero momentum transfer squared. Following [9], f_1 is approximated as t/m^2, where $<r^2> = -6\ m^{-2}$ may be defined as a charge transition radius. If this characteristic mass is of the order of the vector meson mass, as seems to be the case in most meson decays [19] here again the kinematic limitation on t, will also quench the effect a priori.

So far we have introduced possible C (and T) invariance on a pure phenomenological basis, building up obvious C-violating amplitudes. Lee has, however, highly clarified the various ways, in which a violation of C-invariance could be brought into the electromagnetic interactions of the hadrons [14] and we now turn to this classification.

III. Classification of C non-invariant interactions

The successes of standard electrodynamics, a theory which is explicitly C-invariant, clearly warrants C invariance for the electromagnetic interactions involving charged leptons, whereas for the hadrons, for which no detailed theory is at hand, C-non-invariant terms could be constructed with no known reason to be ruled out. One should, however, specify clearly what is meant by C for the leptons and for the hadrons. In classical Electrodynamics and also for the electron and the positron, or μ^- and μ^+ as well, which are the only particles involved in quantum electrodynamics, C is basically associated with a change in the sign of the charge and the electromagnetic current accordingly changes sign under C. Following Lee [4] such an operation will from now on be denoted by C_γ. Charge conjugation, as defined for the hadrons, should change a particle into its antiparticle because it is under this operation that strong interactions are known to be invariant. It might be the same operation as C_γ but could be different, and will be denoted by C_{St}. In other words, whether particle—anti particle exchange always implies the change of the charge into its opposite is still an open question. If the electromagnetic current is odd under C_γ it could be either odd (as assumed so far) or also contain an even part under C_{St}. C_γ and C_{St} would then be non-compatible as invariance properties and C_{St} invariance, a property of strong interactions, would be violated by electromagnetic interactions. Parity invariance being a common property of strong and electromagnetic processes, the same parity operator P is introduced for both and, according to the CPT theorem, two time reversal operators T_γ and T_{St} are required in order that the Hamiltonian (strong and electromagnetic) commutes with the operator $C_\gamma PT_\gamma = C_{St} PT_{St}$. The two pion and three pion decay modes of the K meson are eigenstates of PC_{St} with eigenvalues $+1$ and

-1. At the same time the K_1^0 and K_2^0 are defined as eigenstates of PC_{St} with eigenvalues $+1$ and -1. PC invariance for weak interactions is then specified as PC_{St} [20] invariance (for the leptons C_γ and C_{St} are identical) and apparent violations may result from electromagnetic effects. In general, they could be expected in weak interactions whenever hadrons are involved.

The electromagnetic current J_μ (odd under C_γ) is then separated into two terms respectively odd I_μ and even K_μ under C_{St}. The presence of K_μ will lead to C_{St} non-invariance in electromagnetic interactions. Its presence might be associated with selection rules according to which its possible effect should be classified. As pointed out by LEE, the main classification, however, has to do with the charge Q_K associated with it. The charge is defined as

$$Q_I = i \int I_4(x)\, d_3 x\,, \qquad\qquad Q_K = i \int K_4(x)\, d_3 x\,,$$

and Q_K as well as Q_I might not be zero. The K charge as the current density is even under C_{St}. If Q_K is not zero, and as shown in detail by LEE [14], there should then exist at least one charged state (which can be chosen as an eigenstate of H_{St}) which is an eigenstate of C_{St}, $|\, a^+ >$. Through PCT, there would then also be another state $|\, a^- >$ (not related to it by C_{St}). The existence of any such state necessitates C non-invariance. C_{St} and C_γ invariance are then clearly incompatible. Q_J and Q_K are shown to be two commuting operators which are separately conserved by the strong and electromagnetic intractions [14]. It follows that all particles known at present must have a zero K-charge for C_{St} invariance to hold in strong processes as it does. Nevertheless, particles of the a^+ (and a^-) type with a non-zero K charge could exist which would be stable except for weak interactions [21]. These particles could be produced in pairs. Their non-observation in the data so far analyzed would put a lower limit of over 1 Bev for their mass [22]. These strongly interacting particles should be associated to SU_3 multiplets. If a is a SU_3 scalar, the pertinent part of K_μ will transform like a scalar but, as stressed by LEE [14], it is also possible that K_μ might transform like a scalar even though the a might be associated with higher multiplets (some a would then be neutral).

This possibility of C non-invariance thus corresponds to a $\Delta T = 0$ Isotopic Spin selection rule. Many reactions which could be considered as direct proofs of C non-invariance [9] would in effect not provide a test under this scheme. In particular, the Φ (or ω), $\varrho\gamma$ decay is forbidden. The $\eta \to \pi^0$ $e^+ e^-$ decay, which we shall consider in detail later on would also be forbidden. Similarly no C violation would be found in $\Sigma^0 \to \Lambda\, e^+ e^-$ decay nor in N^+ (3;3) (and any N^+ with $T \neq 1/2$) electroproduction on a polarized target, which would be one of the easiest practical tests to do with high energy electrons (Section V.). As discussed in particular examples in the next Section, C violation effects when allowed, would then often show a rather well defined pattern.

Asymmetry in the $\eta \to \pi^+ \pi^- \pi^0$ decay might be hard to detect but asymmetry in the $\eta \to \pi^+ \pi^- \gamma$ could appear.

On the other hand, Q_K might be zero (no particle such as the a's would then exist). K_μ would not be limited to any particular multiplet and its matrix elements could be, as previously done, written on purely phenomenological grounds. As listed in references [9] and [14], many reactions can then show directly the effect of such C non-invariant terms. If K_μ as I_μ is assumed to transform as a member of an Octet, many matrix elements are zero in perfect SU_3 symmetry and should be reduced by an order of magnitude.

Defined with only the known particle, that is without a's, Electromagnetic interactions, introduced in a minimal way, with ∂_μ replaced by $\partial_\mu - ieA_\mu$ everywhere in the strong interaction Lagrangian, may seem at first sight to imply C_{St} invariance. In effect, one does not know whether such a Lagrangian approach is correct to derive definite conclusions from it, but anyway, as shown by LEE [14], C non-invariant terms could be introduced in a minimal way starting from at least two neutral vector meson fields coupled to the Electromagnetic field. The C violating terms then come out as a magnetic moment coupling between the two fields.

IV. Possible C invariance violation in η^0 and X^0 decays

As particular examples of the various reactions so far mentioned, we now consider how C non-invariant terms would appear in η and X decays [9, 14, 23]. Such decays are, a priori, the simplest reaction in which to look for the effects of C violating electromagnetic interactions. The widths (especially the η width) are small and electromagnetic terms should manifest themselves in a conspicuous way either directly or as interference effects. Much information is already available on the η [24].

A striking effect would be the decay mode $\eta \to \pi^0 + e^+ + e^-$ as shown in Figure 3. The hadron vertex is obtained from the expression already written (6). The decay rate is then easily calculated as

$$\Gamma_1 = \frac{f^2}{4\pi} \frac{e^2}{4\pi} \frac{2M}{\pi m_4} \int \left\{ 4\omega_1 \omega_2 - 2M(\omega_1 + \omega_2) + (M^2 - \mu^2) \right\} d\omega_1 \, d\omega_2 \,, \tag{8}$$

where ω_1 and ω_2 are the electron and positron energies. M and μ are the η and π masses. We have defined $f = m^2 f_1(t)/t$ $(t = 0)$ and neglect the variation of $f_1 \cdot m^{-1}$ defining a radius (it is tempting [19] to take $m = \varrho$ meson mass). This decay mode has been now thoroughly explored [24] and one knows that

$$\Gamma_1 \leq 0{,}007\, \Gamma_{\gamma\gamma} \,, \qquad (\Gamma_{\gamma\gamma} = 1{,}54 \Gamma_{\pi^+\pi^-\pi^0}) \,,$$

where $\Gamma_{\gamma\gamma}$ is the rate of the $\eta \to 2\gamma$ decay mode. $\Gamma_{\gamma\gamma}$ can be related through SU_3 symmetry to the π^0 decay rate, i.e. $\Gamma_{\gamma\gamma} = 127$ ev. This gives the following higher limit for f^2,

$$\frac{f^2}{4\pi} \leq \left(\frac{m}{M}\right)^4 \times 5{,}10^{-5}\,. \tag{9}$$

This may sound deceptively small but it should be stressed that the reaction considered corresponds to a contribution from K with $\Delta T = 1$ which could be

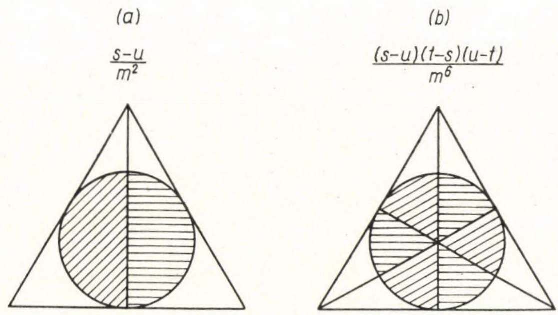

Fig. 4a. Dalitz plot for η decay $T = 1{,}2$ interference
4b. Dalitz plot for η decay $T = 0{,}1$ interference, final state $\pi\pi$ interaction increases the density toward the low π° kinetic energy

forbidden if a $\Delta T = 0$ selection rule holds. Also, as pointed out by CABIBBO [25] the process is forbidden if K_μ, as I_μ transforms under SU_3 as a member of an octet. The zero value of the diagonal matrix elements then implies a zero value for the $\eta\pi$ off diagonal elements. SU_3 symmetry could then decrease the expected effect by an order of magnitude. FEINBERG [26] has made a detailed analysis, assuming SU_3 matrix elements and $X-\eta$ mixing. The expected branching ratio $\Gamma \to (\eta \;\; \pi^0 e^+ e^-)/\Gamma(\eta \to \gamma\gamma)$ is of the order of 1%.

We now turn to the three pion decay mode of the η. This is known to be an electromagnetic decay. If both the K and I parts of the current are involved, the final state will not be in general an eigenstate of C_{St} as the initial η state, i.e. there will be an asymmetry between the π^+ and π^- energy distributions. This would arise from an interference between the basically constant. S wave amplitude associated to the C conserving electromagnetic decay of the η ($T = 1$ final state) and a C violating amplitude (with $T = 0$ or $T = 2$ final states). As follows from PCT invariance, the two decay matrix elements are relatively imaginary. Interference can occur only through final state interactions, here mainly the large S wave, $T = 0$ phase shift [26]. The observed asymmetry will reflect the Isotopic spin character of the interaction. If no particular selection rule holds, beside what is known for the I component, the asymmetry would result mainly from an S, P interference in the $\pi^+ \pi^-$ state.

This corresponds to a left right asymmetry on the η Dalitz plot (Figure 4a). It is then tempting to assume that the coupling between the P wave two pion system and the $\eta\pi^0$ system would occur through a simple IK interaction [23] (that is a one photon exhange as shown in figure 3b). The C violating term would then correspond to $T = 2$. The advantage of this model is that the asymmetry can be calculated in terms of the coupling constant f previously introduced (8). To do this, we write the dominant S wave rate as:

$$|\Gamma_0 = \frac{F_0^2}{(4\pi)^3 M} \int d\omega_1 \, d\omega_2 \tag{10}$$

and the left right asymmetry (assuming dominance of the C conserving mode) is found to be

$$\Delta = \frac{4 feM}{F_0 m^2} \frac{\int |\omega_1 - \omega_2| \, d\omega_1 \, d\omega_2}{\int d\omega_1 \, d\omega_2} \tag{11}$$

or

$$R = \frac{\Gamma_1}{\Gamma_0} = K \Delta^2$$

using (8), (10) and (11) with $K \sim 70$ independent of the range m^{-1}.

Therefore, the present lower limit of 1,1% on Γ_1 would limit the asymmetry to $\sim$1,3% which would be extremely hard to detect. Furthermore we have assumed maximum interference when a reasonable treatment including the $\pi\pi$ phase shift would reduce the asymmetry by a factor 2 at least. Nevertheless, the model is probably too restrictive though practical, and the non-observation of the $\eta \to \pi^0 e^+ e^-$ decay mode should not discourage a detailed check of the asymmetry in the three pion mode.

If the $\Delta T = 0$ rule operates for K the C violating term should correspond to $\Delta T = 0$. At the same time the $\eta \to e^+ e^-$ decay mode is excluded. In such a case the antisymmetry of the isotopic spin state implies a totally antisymmetric amplitude, namely $G(s - u)(u - t)(t - s)$, where s, t and u are the centre of mass energies squared of each pair of π mesons. G is a symmetric function of s, t and u, i.e. a constant with a good approximation. The asymmetry now would change from one sextant to the next, as shown in Figure 4b. If this is the case, one would expect an important quenching of the C violating amplitude as a result of centrifugal barrier effects. Whenever we introduce an explicit dependence on the energy of one of the pions, ω_1 say, we should actually introduce a factor ω/m where m is the inverse decay range. With m of the order of the vector meson mass, we are led to expect strong centrifugal barrier quenching whenever the matrix element introduced is complicated. The C violating term should be intrinsically very large [23] in order to bring a detectable asymmetry effect. Taking the range as an inverse ϱ mass, one obtains an average quenching factor for the asymmetry of 10^3 as opposed to a $\Delta I = 2$ transition.

Asymmetry could also occur in the $\pi^+ \pi^- \gamma$ decay mode between the P wave associated with the C invariant term and the D wave associated with the C non-invariant one (we limit the decay amplitude to the lowest possible waves). This should occur even with a unitary singlet current. This has been discussed in detail by BARRETT and TRUONG [28]. The point is that there is no large expected phase difference between the P and D wave at the energies available (both are expected to be rather small). Even though the violation might be strong to start with (that is the same amount of P and D wave), the

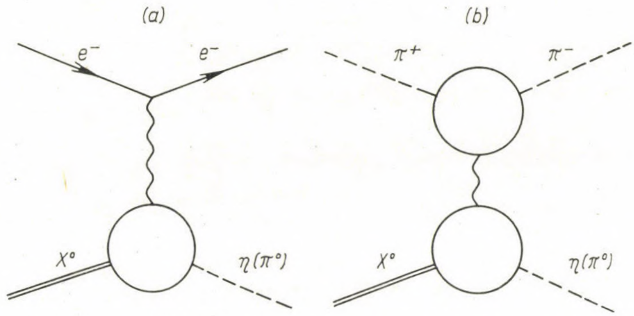

Fig. 5a. $X^\circ \to \pi(\eta) + e^+ + e^-$ *5b.* $X^\circ \to \pi(\eta) + \pi^+ + \pi^-$ (model)

interference cannot build up. As shown in [28] the maximum asymmetry would come out as 1,1% while the large D wave would contribute a rather large $\pi^0 \pi^0 \gamma$ branching ratio $\Gamma(\eta \to \pi^0 \pi^0 \gamma)/\Gamma(\eta \to \pi^+ \pi^- \gamma) \sim 0{,}25$ which might then perhaps be the easiest to detect, though this would be very hard against the $3\pi^0$ background. Present experimental information [29] based on 33 events gives a 0 asymmetry (with an error of 17%).

The same argument can be applied to the $K_L \to \pi^+ \pi^- \gamma$ decay. Even though parity is not conserved, an asymmetry between the π^+ and π^- energy distribution would still be a proof of CP violation due to C violation in the Electromagnetic part of the decay interaction.

It is to be feared that centrifugal barrier effects may, in many cases, hide actual C violating terms. A striking example is the $\pi^0 \to 3\gamma$ decay where with a decay range of the order of the inverse mass of the ϱ, which seems to fit properly the 2γ mode [19] one would expect a quenching factor as large as 10^{-7}. The present limit for $\Gamma\, 3\gamma/\Gamma\, 2\gamma$, is 5×10^{-6}.

We now turn to X^0 decay. Several decay modes could provide evidence for C non-invariance. Among the many matrix elements of interest, we shall consider first $\langle \pi^0 \mid K_\mu \mid X \rangle$ and $\langle \eta \mid K_\mu \mid X \rangle$. As shown in Figure 5, they would correspond to $X \to \pi(\eta) + e^+ + e^-$ and $X \to \pi(\eta) + \pi^+ + \pi^-$. Estimation of C violating effects may proceed as follows. The ratio of the $\pi^0 e^+ e^-$ and 3π decay rates is readily obtained (the 3π decay assumed to proceed according to our model). One finds that the ϱ enhancement more than compensates for phase space. Namely

$$\frac{\Gamma(X \to \pi^0 e^+ e^-)}{\Gamma(X \to \pi^0 \pi^+ \pi^-)} \sim \frac{1}{5}.$$

The 3π decay rate has a contribution (second order electromagnetic) from C conserving terms but without ϱ enhancement. Assuming C violating terms to be present with a similar strength, we have neglected it.

There is at present, an upper limit for the $\pi^+ \pi^-$ (neutrals) branching ratio of the X [30]: $5 \pm 4\%$. We would then expect a branching ratio for the $\pi^0 e^+ e^-$ mode of at most 1%. The $\eta e^+ e^-$ decay mode implies a phase space reduction by a factor 50. With an $X\eta$, electromagnetic transition interference between the C conserving (strong interaction) mode and the C violating electromagnetic mode would yield an asymmetry on the Dalitz plot. As for the η, the same model relates this asymmetry Δ to the $\eta e^+ e^-$ branching ratio through the same relation. The pertinent factor K is now found to be about 30. This would assume complete interference and the asymmetry should in effect be still more reduced.

If K_μ transforms as a unitary singlet both matrix elements are zero except for $X—\eta$ mixing. If K_μ transforms like I_μ, the $\eta \to \pi e^+ e^-$, $X \to \pi e^+ e^-$ and $X \to e^+ e^-$ rates are all related [26] with a $X—\eta$ mixing angle of 0,18 [31]. The branching ratio of the $X \to \pi e^+ e^-$ mode should then be of the order of a few per cent, the $\eta e^+ e^-$ mode should contribute about 0,1%. Experimentally, the respective upper limits are 1,3% and 1,1% [32].

As for the η the most interesting test should here also be afforded by the $\pi^+ \pi^- \gamma$ mode, which has a branching ratio of 20% [32]. The C conserving contribution corresponds mainly to P wave, while the C violating interaction would correspond mainly to D wave. The transition is allowed even for a K current which would transform as a SU_3 scalar. The centrifugal barrier effect should play only a rather small role owing to the high mass of the X, and the ϱ resonance together with a still small D wave phase shift should provide an important interference. For all these reasons the Dalitz plot of the $X \to \pi^+ \pi^- \gamma$ decay should provide one of the most favourable pieces of data for seeking possible C violation. A detailed estimate has been made by BARRETT and TRUONG [28]. With maximum violation, i.e. equivalent P and D contributions, the expected asymmetry soars to 18% as follows from the large P wave phase shift. Such a large D wave contribution is, nevertheless, not too reasonable and a more likely figure of 10% would leave only a 4% $\pi^0 \pi^0 \gamma/$

$/\pi^+ \pi^- \gamma$ (due to the D wave contribution) branching ratio [28]. The present experimental situation [32] based on 152 events (including 86 in the ϱ peak) is compatible with no asymmetry but the statistical error is such that a 10% or less asymmetry cannot be ruled out. Further data on this decay mode are highly needed [33].

V. Experiment involving high energy electron scattering

We have seen that if, in principle, C invariance could be easily tested in meson decays, tests of T invariance are more readily available with fermions. One should then test directly T invariance in electromagnetic interactions involving fermions, C and T invariance being related by the CPT theorem. Many such possible tests have been proposed and analyzed in great detail by CHRIST and LEE [15]. Several of the reactions considered are formally related to the $\Sigma^0 \to \Lambda^0 e^+ e^-$ decay, already mentioned. The Σ and Λ may be replaced by two different hadronic states which can be one or more particle states. Both electron scattering and pair production are considered. In many such reactions, one should simply look for a T non-invariant term in the reaction amplitude. Since the reaction is described up to a very good approximation by a one photon exchange, it should then correspond to a T non-invariant term in the Hamiltonian [34]. This is the case for a contribution of the type $\langle\vec{\sigma}\rangle\vec{k}\times\vec{k}'$ to the cross section which was previously mentioned [18]. Such terms, as well as others [15] should be searched for in many electromagnetic interactions involving hadrons. The remaining question is to select that which would be, a priori, the most favourable.

Instead of giving a review, we shall single out one reaction, namely N^+ electroproduction on a polarized target and analyze its main features which are quite typical. This might be a feasible experiment at present [35]. We refer to the work of CHRIST and LEE [15] for a more detailed discussion as well a review of related possible experiments. The reaction studied is presented in Figure 6. N^+ stands for any nucleon isobar. For obvious practical reasons, production of the N^+ (3,3) at 1240 Mev could be most easily studied. Nevertheless, there are some indications that, owing to the particular relative value of the form factors involved, the polarization effect sought might not be large in any case [36]. Furthermore, no effect would be observed if C invariance is violated unless the current K_μ transforms like an isoscalar [14]. For these reasons, it is desirable to push the experiment to higher isobars and especially to the N^+ (1,3) at 1520 Mev which is also clearly produced in electroproduction [37].

The principle of the experiment is as follows: electrons are scattered off a polarized target and their energy spectrum at a fixed angle is analyzed. The

N^+ peak, due to protons, will stand above the background provided by the target compound nuclei as a whole, and its magnitude should change with inversion of the polarization if T (and C) invariance were violated. At present, polarized targets provide a 60% polarization, but with only a 3% proton content [35]. The background is therefore enormous. Nevertheless a rapid improvement of targets is expected.

The $N^+ N\gamma$ vertex, with a virtual photon (Figure 6) involves 3 form factors which are conveniently related to the matrix elements of the electromagnetic current between different helicity states of the initial and final

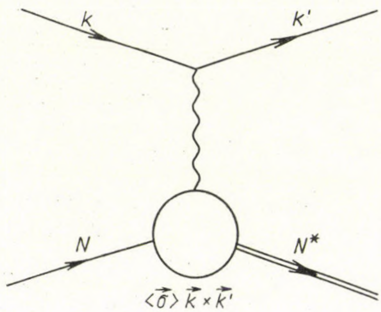

Fig. 6. $e + N \to e + N^+$ reaction

baryon [38]. We follow here the analysis of CHRIST and LEE [15], i.e. we write the differential cross-section in the laboratory system defining the z axis by the isobar momentum p and the y axis by the normal to the electron scattering plane $\vec{k}\times\vec{k}'$. 3 independent matrix elements are thereby introduced [15].

$$F_\pm = \mp \frac{1}{2}\left\langle N^+, \ \frac{1}{2} \pm 1 \,|\, (J_x(0) \pm i J_y(0)) \,|\, N, \ \frac{1}{2}\right\rangle,$$
$$F_z = \left\langle N^+, \ \frac{1}{2} \,|\, J_z(0) \,|\, N, \frac{1}{2}\right\rangle. \tag{12}$$

All other matrix elements are given by parity invariance [39]. The matrix elements of the time component of the current are obtained from current conservation, namely

$$p\left\langle N^+, \frac{1}{2} \,|\, J_z(0) \,|\, N, \frac{1}{2}\right\rangle = -i\left(\sqrt{p^2+M^2} - m\right)\left\langle N^+, \frac{1}{2} \,|\, J_4(0) \,|\, N, \frac{1}{2}\right\rangle, \tag{13}$$

where M and m stand for the isobar and nucleon mass.

If time reversal invariance holds all F's are relatively real in the scattering physical region and a zero polarization effect results [34]. On the other hand, a contribution to the cross-section proportional to the polarization normal to the production plane involves the imaginary part of the quantities $F_i^+ F_j$. This is easily seen since with our choice of axis, it is proportional to the non-symmetric part of the density matrix describing the polarization of the initial nucleon.

In order to be more specific, we write the isobar production differential cross-section in the laboratory system as

$$\frac{d\sigma}{d\cos\Theta} = \frac{2\pi\,\alpha^2\,k'}{2k\,m^2\,q^2}\,\frac{M^2+m^2+q^2}{1+\frac{2k}{m}\sin^2\frac{\Theta}{2}}\left\{2R_1 + R_2\,\mathrm{cotan}^2\frac{\Theta}{2} + \right.$$
$$\left. + p\,\frac{k^2-k'^2}{m^2}\,R_3\,\mathrm{cotan}\frac{\Theta}{2}\right\}, \tag{14}$$

where P is the average proton polarization (expectation value of the spin component) normal to the selected reaction plane. q^2 is the momentum transfer squared, $q^2 = 4kk'\sin^2\frac{\Theta}{2}$, θ is the scattering angle. The electron mass has been neglected.

Relation (14) is derived from the work of CHRIST and LEE [15] when their general relations are specialized to a particular isobar. R_1, R_2 and R_3 have been defined according to the 3 form factors that they have chosen (12). Namely

$$R_1 = |F_+|^2 + |F_-|^2,$$

$$R_2 = \frac{q^2}{p^2}\left(|F_+|^2 + |F_-|^2 + \frac{4m^2\,q^2}{(M^2+m^2+q^2)^2}\,|F_z|^2\right), \tag{15}$$

$$R_3 = 2\eta\,Im\,\{F_-^*\,F_z\}\,\frac{4m^4\,q^2}{p^2\,(M^2-m^2+q^2)^2},$$

where p, $p^2 = \frac{M^2}{m^2}\left(q^2 + \frac{(M^2-m^2-q^2)^2}{4M^2}\right)$, is the isobar momentum, and where η is the relative parity between the nucleon and the isobar time a factor $+(-1)$ if their spin difference is even (odd) $\eta = -1$ for the $N^+(33)$ and $+1$ for the N^+ (1,3).

(14) together with (15) follows from the definition of the F's when the cross-section associated with the graph of Figure 6 is calculated. The most readily measurable quantity is the asymmetry obtained with up and down

polarization. It reads [15]

$$\Delta = \frac{(k^2 - k'^2) R_3 \cotan \frac{\theta}{2}}{m^2 \left(2R_1 + R_2 \cotan^2 \frac{\theta}{2}\right)} . \tag{16}$$

We now turn to an estimate of this expected asymmetry assuming a maximal violation, that is $Im \{F_z^+ F_-\} \sim | F_z F_- |$ as seen in (14) and (15). The observation of the electron direction only yields two independent functions [41] which correspond to transverse and longitudinal photon polarization. Present results [37] clearly show that isobars are conspicuously produced, but the analysis of the data still leaves a great uncertainty in the knowledge of the F functions. For the N^+ (3,3), $| F_z |^2$ is compatible with zero and should not exceed 20% of $(| F_+ |^2 + | F_- |^2)$, [41]. The small value of F_z could be a result of higher symmetries [43]. A maximal effect, i.e. $Im\ F_z^* F_- \sim 0{,}5\ | F_- |^2 \sim \sim 0{,}5\ | F_+ |^2$, with 800 Mev incident electrons and a scattering angle of 60° ($q^2 \sim 0{,}26$ (Bev/c)^2) would yield an asymmetry $\Delta \sim 30\%$ as follows from the relation of CHRIST and LEE.

The asymmetry is zero in the forward direction, increases with momentum transfer to return to zero in the backward direction. The counting rate, however decreases sharply with q^2.

A detailed and refined analysis [36] concludes with a maximum asymmetry of the order of 25%. This may look favourable but, as we have already mentioned, this has to be observed on a 2% contribution above the background due to the still low hydrogen content of the targets.

As mentioned already, the probable smallness of F_z relative to F_- for the N^+ (3,3), together with the possible $\Delta T = 0$ selection rule, strongly indicates that the experiment, if feasible, should be extended to the N^+ (1,3). The pertinent value of F_z could be much more favourable [38] and a $\Delta T = 0$ rule would not forbid it.

Many other reactions have been proposed and analyzed in great detail including Compton scattering [44]. However they all seem difficult at present, as in the case for the test of reciprocity relations in photon reactions [15], or else are expected to yield small effects.

As a general conclusion, if there is yet no direct proof of C invariance in Electromagnetic processes involving hadrons, C non-invariance effects appear to be almost always limited to small contributions for various and independent reasons. This, however, should in no way discourage us from probing the profound meaning of this invariance principle.

It is a pleasure to thank G. SHAPIRO from Saclay and A. ROUSSET from l'École Polytechnique for discussions on the experimental aspects of the different tests at present being performed or planned. I also thank G. BOUCHIAT for a discussion.

Appendix

Several meson decays which could provide tests for *C* invariance in electromagnetic processes are listed

Reaction	forbidden for SU_3 singlet	forbidden for SU_3 octet	Expected quenching as opposed to the dominant mode or *C* conserving one. It is due to centrifugal barrier effects or the weakness of α
$\pi^\circ \rightarrow 3\gamma$			α, very strong
$\eta \rightarrow \pi^\circ e^+ e^-$	×	×	
$\eta \rightarrow \eta^+ \pi^- \pi^\circ$ (asymmetry)	×	×	strong (anyway) average (model)
$\eta \rightarrow \pi^+ \pi^- \gamma$ (asymmetry)			average but small *P* wave phase shift
$X \rightarrow \pi^+ \pi^- \eta$ (asymmetry)			α (Average)
$X \rightarrow \pi^\circ(\eta_0)\, e^+ e^-$	×		α^2
$X \rightarrow \varrho\pi$	$x(\varrho - \gamma)$		α^2
$X \rightarrow \pi^+ \pi^- \gamma$ (asymmetry)			
$K \rightarrow \pi^+ \pi^- \gamma$ (asymmetry)			average but small *P* wave phase shift
$\Phi \rightarrow \varrho + \gamma$	×		α
$\Phi \rightarrow \omega + \gamma$			α
$\omega \rightarrow \varrho + \gamma$	×		α
$\Phi \rightarrow \pi^+ \pi^- \gamma$ (asymmetry)	×		small branching ratio
$\omega \rightarrow \pi^+ \pi^- \gamma$ (asymmetry)	×		small branching ratio
$p \rightarrow \pi^+ \pi^- \gamma$ (asymmetry)			small branching ratio
$\omega \rightarrow \pi^+ \pi^- \pi^\circ$ (asymmetry)	×		α
$\Phi \rightarrow \pi^+ \pi^- \pi^\circ$ (asymmetry)	×		α

As a still possible search for *C* violation, it should be mentioned that:

— a more precise measurement of the neutron electric dipole moment would also be extremely interesting. Estimate with *C* violation [26] gives values as large as 10^{-19} excm when the present experimental limit obtained by RAMSEY and his collaborators is 10^{-20}. With a precision pushed higher, it could easily reveal a *C* violating effect if present.

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16. We use hermitian y matrices, each momentum k reads $(\vec{k}, i\omega_k)$.
17. Transformation under T of bilinear covariants are given in many text-books. See for instance J. Schweber, An Introduction to Relativistic Quantum Field Theory p. 236.
18. A term of the form $\langle\vec{\sigma}\rangle \cdot \vec{k}\times\vec{k}'$ in the decay amplitude is odd under time reversal. For a lowest order contribution (we consider electromagnetic interactions), this implies the same term in a phenomenological Hamiltonian, which would not be then invariant under time reversal. T invariance would be violated.
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39. Explicitely, one has:

$$\langle -\lambda' \,|\, J_z(0) \,|\, -\lambda \rangle = \eta (-1)^{s-\frac{1}{2}} (-1)^{\lambda-\lambda'} \langle \lambda' \,|\, J_z(0) \,|\, \lambda \rangle$$

$$\langle -\lambda' \,|\, \big(J_x(0) \pm iJ_y(0)\big) \,|\, -\lambda \rangle = \eta (-1)^{s-\frac{1}{2}} (-1)^{\lambda-\lambda'} \langle \lambda' \,|\, \big(J_x(0) \mp iJ_y(0)\big) \,|\, \lambda \rangle ,$$

where η is the relative parity of the nucleon and the isobar and s the isobar spin.

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ВОЗМОЖНОЕ НАРУШЕНИЕ *С*-ЧЕТНОСТИ В ЭЛЕКТРОМАГНИТНЫХ ПРОЦЕССАХ

М. ЖАКОБ

Резюме

Рассмотрены разные возможные способы несохранения *С*-четности в электромагнитных взаимодействиях, и дается анализ экспериментальных следствий с подробным обсуждением нескольких типичных случаев.

Подчеркивается, что хотя *С*-инвариантность может быть сильно нарушена, все эксперименты, которые в данный момент могут быть предложены, являются поисками измеримых эффектов порядка нескольких процентов. Это обстоятельство связано с разными причинами, которые кратко обсуждаются.

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REMARK ON THE $\pi^0 \to 3\gamma$ DECAY

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A theoretical upper limit on the rate of the *C* violating decay is given.

One of the discussed possibilities of (electromagnetic) *C* violation is the possible $\pi^0 \to 3\gamma$ decay [1]. Our aim was to find the simplest $\pi^0 - 3\gamma$ vertices and to estimate the expected branching ratio

$$R = \frac{\Gamma(\pi^0 \to 3\gamma)}{\Gamma(\pi^0 \to 2\gamma)}$$

for the case of an electromagnetic (or strong) *C* violation. Lorentz symmetry, *P* conservation, Bose statistics and gauge invariance for photons are assumed. It has been shown that all the vertices with six or less derivates vanish for $0^- \to 3\gamma$ (and also for $0^+ \to 3\gamma$) transition. Independent non-vanishing vertices with seven derivates are

$$\frac{e^3}{m^7} \partial_\varrho \partial_\delta \pi^0 \partial_\varrho F_{\alpha\beta} \partial_\gamma F_{\alpha\beta} F^D_{\gamma\delta}, \tag{1}$$

$$\frac{e^3}{m^7} \partial_\gamma \partial_\nu \pi^0 \partial_\mu F^D_{\alpha\beta} F_{\beta\gamma} \partial_\alpha F_{\mu\nu}, \tag{2}$$

$$\frac{e^3}{m^7} \partial_\nu \pi^0 \partial_\mu F^D_{\alpha\beta} F_{\beta\gamma} \partial_\alpha \partial_\gamma F_{\mu\nu}, \tag{3}$$

$$\frac{e^3}{m^7} \partial_\alpha \partial_\delta \pi^0 \partial_\varrho F^D_{\alpha\beta} F_{\beta\gamma} \partial_\varrho F_{\gamma\delta}, \tag{4}$$

$$\frac{e^3}{m^7} \partial_\delta \pi^\circ \partial_\varrho F_{\alpha\beta} \partial_\alpha F^D_{\beta\gamma} \partial_\varrho F_{\gamma\delta}, \tag{5}$$

$$\frac{e^3}{m^7} \partial_\delta \pi^\circ \partial_\varrho F_{\alpha\beta} \partial_\alpha F_{\beta\gamma} \partial_\varrho F^D_{\gamma\delta}. \tag{6}$$

Here $F_{\mu\nu} = \partial_\nu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor,

$$F^D_{\mu\nu} = \varepsilon_{\mu\nu\sigma\tau} F_{\sigma\tau} \tag{7}$$

the corresponding pseudotensor, m^{-1} is a constant of the dimension of length (the so-called decay length). There are several other independent vertices where the antisymmetric pseudotensor $\varepsilon_{\mu\nu\sigma\tau}$ and the field tensor $F_{\alpha\beta}$ are not contracted twice, so the vertex cannot be expressed in terms of π^0, $F_{\alpha\beta}$, $F^D_{\alpha\beta}$ only.

One of the simplest cases seems to be the vertex (1) which was quoted already by S. BARSHAY [2]. For a pion at rest in three dimensional language it can be rewritten in the following form:

$$\begin{aligned}\frac{e^3 m_\pi^2}{m^7} \pi^\circ \, [(\mathbf{H}_2 \mathbf{k}_1) \{[\mathbf{E}_3 \mathbf{H}_1 \mathbf{k}_2] + [\mathbf{H}_3 \mathbf{E}_1 \mathbf{k}_2]\} + \\ + (\mathbf{H}_3 \mathbf{k}_2) \{[\mathbf{E}_1 \mathbf{H}_2 \mathbf{k}_3] + [\mathbf{H}_1 \mathbf{E}_2 \mathbf{k}_3]\} + \\ + (\mathbf{H}_1 \mathbf{k}_3) \{[\mathbf{E}_2 \mathbf{H}_3 \mathbf{k}_1] + [\mathbf{H}_2 \mathbf{E}_3 \mathbf{k}_1]\} \,.\end{aligned} \tag{1a}$$

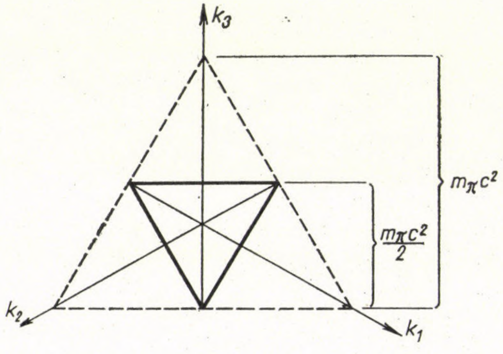

Fig. 1

The decay rate corresponding to the vertex (1a) can be expressed in a straightforward way. By making use of the δ functions several integrals can be evaluated directly; finally two integrals survive (with respect to the two photon energies k_1 and k_2). The integration domain is the central solid triangle of the Dalitz-diagram (Fig. 1). The numerical evaluation of this integral results in the estimation

$$\Gamma(\pi^0 \to 3\gamma) = 2 \cdot 10^9 \text{ s}^{-1}$$

if the decay length m^{-1} is put equal to the pion Compton wavelength. This corresponds to a branching ratio

$$R = 3.10^{-7} \,.$$

This numerical value may be considered to be an upper limit. A more realistic result can be obtained even in the case of maximum electromagnetic C violation by putting the ϱ meson Compton wavelength as decay length $(m = m_\varrho)$:

$$R \sim 10^{-12} \,.$$

The present best experimental upper limits are higher by two (or seven) orders of magnitude than this result. So we may conclude by saying that the observation of the C violating $\pi^0 \to 3\gamma$ decay may be considered to be rather difficult technically even for a maximum electromagnetic C violation.

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ЗАМЕЧАНИЕ О РАСПАДЕ $\pi^0 \to 3\gamma$

Л. ГАЛЬФИ и Г. МАРКС

Резюме

Дается теоретический верхний предел для распада с нарушением C-инвариантности.

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POSSIBLE *CP*-VIOLATION AND INTERFERENCE PHENOMENA IN DECAYS OF K_L^0 AND K_S^0 MESONS

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Abstract

Interference experiments in non-leptonic and leptonic modes of K_S^0 and K_L^0 are considered in connection with possible *CP*-violation. The interference effects in the $K_{S,L}^0 \to \pi^+ \pi^-$ system, which occur in the case of *CP*-violation, are investigated. It is shown that these effects are very pronounced under certain conditions. Their study makes it possible to solve finally the problems concerning the existence of the $K_L^0 \to \pi^+ \pi^-$ decay and the models of *CP*-violation. A comparison is made between the expected effects and recent experimental data. The influence of *CP*-violation upon leptonic decays of K_S^0 is analysed.

Some new interference effects which can be observed in experiments with $K^0 \bar{K}^0$ pair production are considered.

ВОЗМОЖНОЕ НАРУШЕНИЕ *CP* И ИНТЕРФЕРЕНЦИОННЫЕ ЯВЛЕНИЯ В РАСПАДАХ K_L^0 И K_S^0 МЕЗОНОВ

Э. О. ОКОНОВ

Резюме

Рассматриваются интерференционные эксперименты с лептонными и нелептонными распадами K_S^0 и K_L в связи с возможным нарушением *CP*-инвариантности. Исследованы интерференционные эффекты в системе $K_{S,L}^0 \to \pi^+\pi^-$, которые являются следствиями нарушения *CP*. Показано, что при определенных условиях эти эффекты весьма значительны. Их изучение дает возможность окончательно решить вопрос о существовании $K_L^0 \to \pi^+\pi^-$ распада и о моделях нарушения *CP*. Полученные экспериментальные данные сравниваются с ожидаемыми теоретическими эффектами. Проведен анализ влияния нарушения *CP* на лептонный распад K_S^0.

Рассмотрены некоторые новые интерференционные эффекты, которые могут быть наблюдены в экспериментах с рождением K^0K^0.

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EXPERIMENTAL INVESTIGATON OF $K_L^0 \to \pi\mu\nu$ AND $K_L^0 \to \pi^+\pi^-\pi^0$ DECAYS

By

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Abstract

An experiment on the charged K_L^0 decays has been carried out on the synchrotron Saturne in Saclay with a double magnetic spectrometer and spark chambers. 300 000 photos containing 80 000 K_L^0 decays have been obtained. The identification of the π, μ and e has been carried out by investigating their interactions in spark chambers. We present here preliminary results concerning 3000 $K_L^0 \to \pi^\pm \mu^\mp \nu$ and 1000 $K_L^0 \to \pi^+ \pi^- \pi^0$ decays. We have investigated the form factors of the $\pi\mu\nu$ mode and their possible energy dependence. The analysis of the π^0 spectrum in the $\pi^+ \pi^- \pi^0$ mode is carried out in terms of Weinberg's linear matrix element and of σ resonance as suggested by L. Brown.

ЭКСПЕРИМЕНТАЛЬНОЕ ИССЛЕДОВАНИЕ РАСПАДОВ $K_L^0 \to \pi\mu\nu$ и $K_L^0 \to \pi^+\pi^-\pi^0$

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Резюме

Проведен эксперимент по заряженный K_L^0 распадам на синхротроне Сатурн в Саклэ с двойным магнитным спектрометром и искровыми камерами. Получены 300 000 снимков, из которых 80 000 содержат K_L^0 распады. Идентификация π, μ и e проведена путем исследования их взаимодействия в искровых камерах. Мы сообщим здесь некоторые предварительные результаты на основе 3000 $._L \to \pi^\pm\mu\pm\nu$ и 1000 $K_L \to \pi^-\pi^+\pi^0$ распадов. Мы исследовали формфакторы $K^0 \to \pi^\pm\ \mu^\mp\ \nu$ распадов и их зависимость от энергии. Анализ спектра π^0 в распаде $K_L^0 \pm \pi^-\pi^+\pi^0$ сделан в терминах линейного матричного элемента Вейнберга и σ-резонанса, предложенного Л. Брауном.

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SESSION 3. QUARK MODELS

THE NON-RELATIVISTIC QUARK MODEL

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The classification scheme and decay properties of hadron states are investigated in the framework of the non relativistic quark model. The main successes and the unsolved difficulties of the model are presented indicating the future work which could be done to understand and exploit the non-relativistic quark ideas.

1. Introduction

If, at some time in the development of physics, one had known only states of pions or states with integer isospin (think of having "simply" switched off the weak interactions !) and one had observed isospin conservation one might have asked whether states with half integer isospin exist; one would have thus discovered the neutron and the proton, the lowest representation of SU_2.

Our present situation with SU_3 might be similar to the hypothetical situation described above. We do know the 8 and 10 representations of SU_3, but not the lowest one, that of dimension 3. Do particles corresponding to such a representation, the "quarks", exist? [1]

If this is so one can of course imagine that all the particles of the higher representations of SU_3 the 8 and 10 representations are really built with quarks, in the same way as FERMI and YANG built the pion through a nucleon and an antinucleon.

This is of course the basic idea of the quark model of elementary particles, an idea of which I will try to show the usefulness and the difficulties in what I am going to say; it must be emphasized that this approach may well be wrong but since more elaborate approaches may be wrong too, it seems useful to try to investigate its consequences.

To be more specific the main assumptions of the model are [2, 3]:

1. Quarks of only one kind do exist $\left(\text{with charges respectively } \frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$. As is well known also other models are possible with different kinds of

quarks; in these models one avoids fractionary charges, but they do not have the direct connection with SU_3 described above; we therefore will not consider these models further in this lecture.

2. The mass of the quark is rather large ($M \gtrsim 10$ GeV); we shall come back later to this point.

3. A description of the particles in terms of a fixed number of quarks and a fixed number of antiquarks is a good description. By this we mean that an approximation in which say a proton is described in terms of the coordinates of the three quarks which it contains is a good description; in the same sense in which a description in which He^3 is described in terms of the coordinates of the three nucleons is a good description, in spite of the fact that pions, that is exchange currents, are certainly present in the He^3 nucleus. In our case presumably the exchange currents are larger than in nuclei; so that it is by no means trivial to understand the validity of this assumption [4].

A simplifying feature which, if true, might also be useful in understanding assumption 3. above is now the following: contrarily to the FERMI—YANG description of the pion, it is not impossible, in the present model, that in spite of the high binding energy, the motion of the quarks inside an elementary particle is non-relativistic [2].

Consider, in fact, as an example, a pion described as a quark-antiquark system. The quark and antiquark will exchange mesons and we will assume that the force through which they are bound to form the meson is due to such an exchange. If the mesons which provide the binding force are, say, vector mesons, the range of this force will be of the order of $(5m_\pi)^{-1}$. Let us therefore represent the interaction between quark and antiquark as a potential well of radius $a = (5m_\pi)^{-1}$, and of such huge depth, of course, that the lowest bound level has the mass of the pion. In this potential well the relative velocity v of the quark and antiquark is non-relativistic. The relative momentum p is in fact, by the HEISENBERG principle only determined by the radius of the potential

$$p \cong 5m_\pi$$

and therefore $\frac{v}{c} = \frac{p}{M} = \frac{5m_\pi}{M}$; if M, the quark mass, is $\simeq 10$ GeV, one has $\frac{v}{c} \ll 1$.

It is obvious that the above argument will hold for any potential similar in shape to the square well, like, e.g. for a superposition of two Yukawians not having an attractive singularity at the origin. On the other hand, as has already been remarked [2], it does not hold for a pure $\frac{1}{r^n}$ potential (Virial

Theorem) nor for a pure YUKAWA potential, on account of its singularity. This last point can be explicitly checked analytically by making use of a HULTHÉN potential.

Since it is not unlikely that the realistic potential between two quarks (or a quark and an antiquark) may have a repulsive core of sufficient radius to make the above argument valid we will not insist further, for the moment, on the shape of the potential, and, for estimates of order of magnitude we shall use in what follows a square well potential. Indeed, assuming a square well potential of radius $(5m_\pi)^{-1}$ the distance between the two lowest s and p states of a quark and an antiquark can give us some information on the mass M of the quark; the lowest s, p, d, levels in an infinite square well potential of radius $(5m_\pi)^{-1}$ are given by $\alpha \frac{25m_\pi^2}{M}$ where $\alpha = (3{,}14)^2$, $(4{,}5)^2$, $(5{,}76)^2$, for s, p and d states, respectively. The distance between the first p and s states is $\cong 250 m_\pi \frac{m_\pi}{M}$. Now, experimentally, the distance of the average mass of the even parity mesons from the odd parity mesons is $\cong 500$ MeV. Therefore we obtain $M = 10$ GeV. Of course this figure depends strongly (quadratically) on the assumed range of the force and it becomes 20 GeV for a range of $(7m_\pi)^{-1}$ or 5 GeV for a range of $(3{,}5m_\pi)^{-1}$. In the following, as already stated, we will assume $M = 10$ GeV, when necessary for estimates of orders of magnitude.

2. A list of results and problems

We will list now several results of the model which can be considered rather succesful. They are:

1. The possibility of an interpretation of the values of the parameters in the GELL-MANN—OKUBO mass formula.

2. The magnetic moments and the $VP\gamma$ radiative decays.

3. The $N^* \to N + \gamma$ transitions, where N^* is an excited baryon state; in particular the absence of the $E2$ transition $N_{33}^* \to N + \gamma$.

4. The connection between BBP and VPP vertices, where B is a baryon, P a pseudoscalar and V a vector meson (in particular the connection between the $\varrho\pi\pi$ and $N_{33}^* N\pi$ widths).

5. The JOHNSON—TREIMAN—LIPKIN—SCHECK relations.

One can ask how typical the above list of results is of the non-relativistic quark model. The general answer to this question is that most of the above results can also be derived in different ways, but only at the expense of making some specific assumption in each case [5]. It appears to me that the quark model is, so far, the only way in which the results above can be derived in the simplest way and with a minimum number of general assumptions.

There are also, of course, things for which, at present, only tentative answers can be given in some cases.

1. Consider the quark-quark binding; as we have said this is produced by the exchange of meson states; assume, for a moment, that the vector mesons dominate this potential and that the effect of the other mesons is that of producing a convenient repulsive core. Of course we do not know if this is true but this schematization enables us to put the problem. In principle the $q\bar{q}V$ coupling constant can be determined from the $N\bar{N}V$ vertex. The question is now: is this coupling constant large enough for the $q\bar{q}$ bound state to have the mass it must have? This is a kind of self-consistency problem, or bootstrap problem as one prefers to call it. Of course, a similar problem can be put for the baryons and here also the question arises of which is the potential which binds the baryons.

2. The saturation problem: how can one explain the saturation of the baryons at three quarks? Why don't we have more binding when the number of quarks increases?

3. Why are the magnetic moments of the quarks so highly anomalous as we shall see they are?

4. How can one proceed to treat the dynamics when the kinematics is relativistic? Think for instance of the $\omega \to \pi^0 + \gamma$ decay or $\varrho \to \pi + \pi$ decay where the outgoing pions move at relativistic speeds.

5. Do quarks exist? and how can one get an estimate of their production cross section?

6. Taking the charge of the n quark as -1, why is the electron charge 3? Let us now take the above lists of questions and try to discuss them briefly.

3. The baryon wave functions

To start this discussion it is necessary to examine the internal structure of the baryons. To construct the baryon wave functions let us only assume that all the spin $\frac{3}{2}$ of the decuplet baryons is intrinsic spin, with no appreciable orbital angular momentum contribution. In other words, at least dominantly, $L = 0$ for the decuplet baryons. If this is so the spin and unitary spin wave functions of the decuplet baryons must be completely symmetric; and by the Pauli principle, the space part of the wave function must be completely antisymmetric. Calling $W^{(10)}$ the spin and unitary spin wave functions of the 10 particles ($i = 1 \ldots 10$) of the decuplet and $X(r_1, r_2, r_3)$ the space part, we therefore can write for the decuplet wave functions

$$D_i^{(10)} = X(r_1, r_2, r_3)\, W_i^{(10)}, \qquad (1)$$

where $X(r_1, r_2, r_3)$ is an antisymmetric $L = 0$ wave function and $W_i^{(10)}$ are the spin-unitary spin symmetric wave functions.

Coming now to the wave functions of the octet baryons we may argue as follows: if the dynamics is such that low lying levels are characterized by antisymmetric $L = 0$ space wave functions we should use these space wave functions to construct all the states which the Pauli principle allows; therefore not only the (decuplet) states with spin $\frac{3}{2}$ but also the (octet) states with spin $\frac{1}{2}$. The spin-unitary spin wave function with spin $\frac{1}{2}$ has (for a proton, say) the structure $\alpha_1(\beta_2 \alpha_3 - \alpha_3 \beta_2)$ and if this wave function has to be multiplied by the antisymmetrical spatial wave function $X(r_1, r_2, r_3)$, this spin-unitary spin wave function has to be symmetricized. Therefore the spin unitary spin wave function of a proton has the form

$$W_p^{(8)} = N\,S\,\alpha_1 (\alpha_2 \beta_3 - \alpha_3 \beta_2)\, p_1\, p_2\, n_3 ,$$

where N is a normalization constant and S is a symmetrization operator; and in a similar way we can construct all the baryon wave spin-unitary spin wave functions $W_i^{(8)}$. We can finally write for the octet baryon wave functions

$$B_i^{(8)} = X(r_1, r_2, r_3)\, W_i^{(8)} , \qquad (i = 1 \ldots 8) , \tag{2}$$

These results are of course the same as those which one obtains from SU_6; but we want to emphasize that the motivation here is different [2]: just as, in nuclear physics, the fact that the deuteron or α particle wave functions are dominantly S wave functions is not a proof of a spin independence of the Hamiltonian (we know, indeed, that tensor forces, for instance, are rather strong): here the fact that L is dominantly zero is not a proof that the Hamiltonian is invariant under SU_6. Things, in other words, may go, for particular configurations as if SU_6 were valid, even if the Hamiltonian is far from being spin-independent. It has been just to emphasize this point that we have given this detailed presentation of the baryon wave functions.

Of course the question arises why the lowest state of a three quark system has an antisymmetrical $L = 0$ wave function. There are two separate questions here:

1. why antisymmetric?
2. why $L = 0$?

If the quark forces are dominantly MAIORANA exchange forces, that is attractive in antisymmetric states, the fact that the wave function is antisymmetric can be understood. Of course, loosely speaking, the kinetic energy is higher with an antisymmetric wave function than with a symmetric one (that is why in nuclear physics low lying states possess the maximum possible symmetric space wave functions) but in the non-relativistic quark model the kinetic energy is supposed in any case to be small and if the forces are dominantly MAIORANA exchange the three quarks might well repel in a spatially symmetric state.

One should ask at this point: why MAIORANA forces? but the problem of the forces among quarks is not yet at a stage in which this question can be answered.

Now we ask the second question: even assuming that the forces are such that the lovest state is spatially antisymmetric, why do we have $L = 0$?

Here a remark of THIRRING [6] is important. THIRRING has, indeed, put this question to himself and has made it plausible that, at least in a particular model, it should not be an $L = 0$, but instead an $L = 1$ state, the lowest spatially antisymmetric state. The model considered by THIRRING consists of three particles which are performing harmonic oscillations around the three vertices of an equilateral triangle of side R. THIRRING has shown that both in the limit $R = 0$ and $R = \infty$ the lowest $L = 1$ antisymmetrical wave function, which turns out to be proportional to [7]

$$r_1 \times r_2 + r_3 \times r_1 + r_2 \times r_3 \tag{3}$$

is lower in energy than the lowest $L = 0$ antisymmetrical wave function proportional to

$$(r_{12}^2 - r_{13}^2)(r_{21}^2 - r_{23}^2)(r_{31}^2 - r_{32}^2) \,. \qquad (4$$

In our opinion this is a real problem (incidentally it should be interesting to solve THIRRING's model for intermediate values of R), but it is not an unsolvable problem. Indeed even if THIRRING's conclusion is valid for intermediate values of R, the tensor forces might well raise the $L = 1$ state over the $L = 0$ state. Of course they would, at the same time, introduce a mixing among states with different L (for instance the state with $L = 1$ might be mixed, for $J = \frac{1}{2}$, with that of $L = 0$ through the intermediary of some state with $L = 2$) but this might be small as it is in the deuteron problem; the 3% deviation of the ratio of the proton and neutron magnetic moments from the value $\frac{3}{2}$ can be interpreted in several ways but may perhaps indicate some admixture. What THIRRING's problem shows is that relatively low lying baryon states with $L = 1$ and parity + should exist. We shall come back to this point in some detail later (Section 5).

4. The magnetic moments of baryons and the $V \to P + \gamma\, M1$ transitions

With the wave functions of baryons discussed above one can calculate the magnetic moments of baryons, in terms of quark magnetic moments. To show this we first remark that, as is easily seen, SU_3 implies that the magnetic moments of the quarks are proportional to their charges

$$\vec{\mu}_i = \mu \frac{e_i}{e} \vec{\sigma}_i \qquad i = 1,\ 2,\ 3 \tag{5}$$

through the same proportionality constant μ. On calculating the expectation value of the magnetic moment operator $\Sigma_i \vec{\mu}_i$ in the neutron and proton state we obtain for the magnetic moments $\mathfrak{M}_N$ and $\mathfrak{M}_P$ of neutron and proton:

$$\mathfrak{M}_N = - \frac{2}{3} \mu\,, \tag{6}$$

$$\mathfrak{M}_P = \mu. \tag{7}$$

Therefore not only do we get the famous ratio $-\frac{3}{2}$ among the proton and neutron magnetic moments, but we also obtain $\mu = \mathfrak{M}_P = 2{,}79 \frac{e}{2\, M_p C}$ i.e. we have the values of the magnetic moments of the quarks. With a knowledge of the magnetic moments of the quarks we can calculate [8], in the same way as in nuclear physics, several pure magnetic dipole transitions of interest; in particular those corresponding to the radiative decays of vector mesons into pseudoscalar mesons: $V \to P + \gamma$. The results are given in Table I; as far as we know, the $\omega \to \pi + \gamma$ transition is in beautiful agreement with the experiment.

There is only one comment which we want to make here on this kind of calculation, referring for any other details to the original paper [8]. For this purpose let us concentrate on a particular transition, for instance $\varrho^+ \to \pi^+ + \gamma$. What one calculates is the matrix element of the magnetic moment operator between a ϱ^+ state, described by the wave function $\alpha_1 \alpha_{\bar{2}} f(r)$ and a π^+ state described by the wave function $\frac{1}{\sqrt{2}} (\alpha_1 \beta_{\bar{2}} - \beta_1 \alpha_{\bar{2}}) f(r)$. Because the space wave function $f(r)$ of ϱ and π is assumed to be the same, the matrix element $\langle \varrho \mid \pi \rangle$ in question is entirely known; we have

$$|<\varrho \mid \pi>|^2_{av} = \frac{2}{27} \mathfrak{M}_p^2 \frac{k}{2}\,,$$

where average means average on the ϱ polarization.

Table I

Process	Γ_γ (MeV)	Γ_γ/Γ
$\omega \to \pi^\circ + \gamma$	1,17	1,2 10^{-1}
$\omega \to \eta + \gamma$	6,4 10^{-3}	6,8 10^{-4}
$\varrho \to \pi + \gamma$	1,2 10^{-1}	1,1 10^{-3}
$\varrho^\circ \to \eta + \gamma$	4,4 10^{-2}	4,15 10^{-4}
$K^{*+} \to K^+ + \gamma$	7 10^{-2}	1,4 10^{-5}
$K^{*\circ} \to K^\circ + \gamma$	2,8 10^{-1}	1,6 10^{-3}
$\varphi \to \eta + \gamma$	3,04 10^{-1}	10^{-1}

Column 1: the process.
Column 2: the $V \to P + \gamma$ width.
Column 3: the branching ratio.
The wave functions assumed for φ, ω, and η in giving the figures of this Table are:

$$\varphi = D_3^3, \quad \omega = \frac{1}{\sqrt{2}}(D_1^1 + D_2^2), \quad \eta = \frac{1}{\sqrt{6}}(D_1^1 + D_2^2 - 2D_3^3).$$

A discussion of this point is given in ref. [8].
The $X \to \varrho\gamma$, $X \to \omega\gamma$ and $\varphi \to X\gamma$ decays are calculated in ref. [13] by the same method with the results: $\Gamma(X \to \varrho\gamma) = 155$ KeV, $\Gamma(X \to \omega\gamma) = 18$ KeV, $\Gamma(\varphi \to X\gamma) = 1$ KeV.

Now if the transitions were non-relativistic we should simply multiply this matrix element square by the (non-relativistic) phase space to obtain the rate of the $\varrho \to \pi + \gamma$ transition. However, the pion is relativistic. We must therefore interpret the matrix calculated above as the limit of the relativistic matrix element when the pion mass and the ϱ mass are equal. Now what is the form of the relativistic matrix element? There is only one invariant interaction leading to a transtion $V \to P + \gamma$. It is

$$f \varepsilon^{\alpha\beta\gamma\delta}\, \partial_\alpha A_\beta \partial_\gamma V_\delta P\,. \tag{8}$$

The matrix element of this interaction contains the mass of the ϱ, M_ϱ and the mass of the pion m_π only in the form $\sqrt{\frac{M_\varrho}{m_\pi}}$; the non relativistic limit of this factor is unique, being 1 whether we make $M_\varrho \to m_\pi$ or vice versa. This is a fortunate situation which allows us to relate uniquely f^2 in (8) to the non-relativistic matrix element $|\langle \varrho \mid \pi \rangle|^2$ calculated above. One can then proceed to the calculation of the rate by the relativistic interaction (8). The results of Table I have been obtained in this way.

We have insisted a little on this point because it clearly shows the necessity of improving the techniques used in such a way as to take relativistic kinematics into account not only in the phase space. For instance, in the $V \to P + P$ problem, which we shall consider in a moment, the transition to the non relativistic limit analogous to the one performed above for the $V \to P + \gamma$ problem leads to certain ambiguities; these can be avoided only by using a relativistic description of the wave functions, i.e. knowing how a wave function which is written, say, as $\alpha_1 \alpha_{\bar{2}} f(r)$ in the rest system must be written when we are moving at relativistic speeds with respect to the rest system. It is this problem which was listed (problem 4) in the second section and which we must leave open in this lecture.

We shall only state [16] that the BARGMAN—WIGNER description is, for two free particles both having the same momentum, a kinematic transformation of the required kind; but it is still unclear to me how to perform the transformation when the two particles are bound.

5. Electromagnetic transitions between baryon states

It was remarked by BEG, LEE and PAIS [9] that the transition magnetic moment $N_{\overrightarrow{33(M1)}} N + \gamma$ is related to the magnetic moment of the nucleon, by SU_6. Of course the same result holds in the non-relativistic quark model. The agreement between the experimental data and the prediction appears to be wrong by a factor $\simeq$1,6 in the matrix element but the points to consider in this comparison are many and we refer to the paper of DALITZ and SUTHERLAND [10] for a discussion of this problem.

An additional prediction of the model [11] is that the quadrupole transition

$$N^*_{\overrightarrow{33E2}} N + \gamma$$

is forbidden.

This is consistent with the present data. I will show very briefly how this prediction is obtained because this helps me in treating the transitions to the $L = 1$ baryonic excited states with positive parity mentioned in Section 3.

The $E2$ electromagnetic transition operator consists of two parts: the first part is independent of the quark spin, the second part (usually neglected in low energy nuclear spectroscopy) contains the spin, but is proportional also to a space vector. When we insert the $E2$ operator between the $B^{(10)}$ and $B^{(8)}$ wave functions described in Section 3 the first part gives a vanishing matrix element because of the orthogonality of the spin-unitary spin functions, and the second part is also zero because we are dealing with an $L = 0$ to $L = 0$ state transition.

Now we want to show that if, as discussed in Section 3, we assume that relatively low lying $L = 1$ baryonic excited states of positive parity exist an argument similar to that above shows that their excitation or decay either electromagnetically or through single pseudoscalar mesons is very small. Since this argument might provide a new interesting test of the model, we will describe it in some detail.

Let us, therefore, assume, as discussed in section 3, in connection with the THIRRING argument, that low lying baryonic excited states do exist having $L = 1$, positive parity and belonging either to an 8 or to a 10 representation of SU_3. Let us call $B(1+, a)$ the wave function of one of these states where 1 refers to the orbital angular momentum, $+$ is the parity and a reminds us that the space part of their wave function is antisymmetric. These states may have total angular momentum $J = \frac{1}{2}$ (twice) or $\frac{3}{2}$ or $\frac{5}{2}$ but a good part of the argument to be given holds for any one of these values of J so that we need not specify at this point the value of J with which we are dealing.

Now the matrix element for the transition from a state of the normal baryonic octet (say a proton) $B^{(8)}$ into one of these states $B(1+, a)$ is in general a sum of terms each of which is proportional to a factor of the type:

$$\langle X_{L=0}(r_1, r_2, r_3) \,|\, T \,|\, X^M_{L=1}(r_1, r_2, r_3)\rangle \,. \tag{9}$$

Here $X_{L=0}(r_1, r_2, r_3)$ is the space wave function of the normal octet $(L = 0)$ and $X^M_{L=1}(r_1, r_2, r_3)$ is the space wave function of the excited baryon, with $L = 1$ (and z component M); T is the operator inducing the electromagnetic or pionic transition which we wish to consider.

We now examine separately

a) an electromagnetic transition $(B(1+, a) \to B^{(8)} + \gamma)$;
b) a pseudoscalar meson transition $(B(1+, a) \to B^{(8)} + P)$.

The electromagnetic operator T_{em} has the general form

$$T_{em} = \sum_i \left[\frac{e_i}{M} p_i \cdot \varepsilon_i \exp i\, k.r_i + i\, \mathfrak{M}_P \frac{e_i}{l} \,\sigma_i\, (k \times \varepsilon) \exp i\, kr_i \right] . \tag{10}$$

Here, the first term is due to the current and the second to the magnetic moments of the quarks. In the second term $\mathfrak{M}_P$ is the proton magnetic moment (we recall that $\mathfrak{M}_P \frac{e_i}{e}$ is the magnetic moment of the i-th quark). Now, when inserted in (9), the second term does not contribute (because the matrix element $\langle L = 0^+ \,|\, \exp ikr \,|\, L = 1^+\rangle$ vanishes). The first term provides a contribution which in the long wave length approximation is given, by the matrix element of the orbital magnetic moment operator $i \frac{e_i \hbar}{2Mc} L \cdot k \times \varepsilon$. If the mass of the quark inside the baryon is its real mass (we are saying "if" because we are thinking now for instance of the argument proposed by TAVKHELIDZE [12] as a possible explanation of the anomalous quark magnetic moment — see, however, our point of view on this point in Section 10) the contribution of the first term is extremely small owing to the large mass of the quark, and the transition is practically forbidden.

Note in addition that the transition due to the first term in (10) to the decuplet $L = 1^+$ excited baryons $\left(s = \frac{3}{2}\right)$ is in any case forbidden because of the orthogonality of the spin—unitary spin functions.

It should be finally remarked that in the above argument concerning the electromagnetic transitions, we have not considered the exchange current contribution to the electromagnetic transition operator. This is admittedly an omission but it is perhaps better to wait for some data before considering this problem.

We now consider a $B(1+, a) \to B^{(8)} + P$ transition. In the static limit which should be appropriate in view of our non-relativistic dynamics, the coupling of quarks with pseudoscalar mesons P is of course proportional to

$$T_{P_S} \propto \sigma_i \cdot \nabla P(x_i) = \sigma_i \cdot k \exp i k r_i . \tag{11}$$

This coupling has the same form as the second term in T_{em} (10) and the same argument used there shows that the transition $B(1+, a) \to B^{(8)} + P$ is forbidden.

If the above arguments are correct the detection of the low lying $L = 1^+$ baryonic states considered here should be very difficult. They should not appear as peaks in pion scattering or in pion photoproduction. They should not decay in $B^{(8)}$ + one P meson or in $B^{(10)}$ + one P meson. They might perhaps be produced in nucleon collisions, but even there they would be presumably very inhibited.

Of course, the situation described above holds if these states are pure $L = 1^+$ states and do not contain an admixture of wave functions, corresponding to other values of L; and if the octet baryons are pure $L = 0$ states with no admixture of other L, too. If, as we have suggested, the $L = 1^+$ states are pushed above the normal $L = 0+$ states by tensor forces they contain presumably admixtures of states with different L. In such case the essential parameter determining the observability of these states as resonances in pion reactions or photoproduction reactions will be the degree of admixture.

We close this discussion with the following remark: if we have concentrated our attention here on the $L = 1+$ states it has certainly not been because in the literature there are many positive parity states [13]; on the contrary very few positive parity states are known; it is not impossible, but by no means clear that the resonance suggested by BELLETTINI et al. at 1450 MeV [14] is one of these 1+ parity states; the situation appears, however, really too obscure to say more than this. On the other hand, there is a great variety of — parity states on which we shall not enter here. They have been discussed, under certain specific assumptions concerning the wave functions, by DALITZ [13] and, as far as their photoproduction is concerned, by MOORHOUSE [15].

6. On the connection between *BBP* and *VPP* vertices [16]

In the quark model the $\varrho\pi\pi$ decay can be seen as a transition from a $^3S_1$ quark antiquark state (ϱ) into a $^1S_0$ quark antiquark state π with the emission of a pion. In other words a quark in the ϱ flips its spin at the same time emitting a pion. Of course, in the same way we can consider any other *VPP* process, say $\varphi \to K + \bar{K}$ or $K^* \to K + \pi$. Let us concentrate on the $\varrho^0\,\pi^+\,\pi^-$ decay, for definiteness. Once a quark-quark pion interaction has been written the calculation parallels very closely that of the $\varrho \to \pi + \gamma$ process sketched in Section 4. The $qq\pi$ interaction, in the static limit has necessarily the form

$$H_{q\pi} = \sqrt{2}\,\frac{f_q}{\mu}\,(\tau^-\sigma\cdot\nabla\,\Phi(x)) \quad + \text{h. c.}, \tag{12}$$

where f_q is the $qq\pi$ coupling constant to be determined, and μ is the numerical value of the pion mass.

It is clear that it is possible to express (16), approximately, the pion nucleon coupling constant $\left(\frac{f^2}{4\pi} = 0{,}08\right)$ in terms of f_q. One thus gets the (approximate) relation

$$f_q = \frac{3}{5} f\,. \tag{13}$$

With this knowledge of f_q one can calculate two quantities:

a) the $N^*_{33} \to N + \pi$ width: the calculation is straightforward and the result is:

$$\text{Rate}\,(N^*_{33} \to N + \pi) = \frac{48}{25}\,\frac{f^2}{4\pi}\,\frac{p^3}{\mu^2}\,\frac{M_p}{M^*} \cong 80\,\text{MeV}$$

to be compared with an experimental value $\cong$100 MeV.

b) the $\varrho \to \pi + \pi$ decay width: this calculation needs a little more attention. To present it in the simplest way let us first write the most general form for the $\varrho \to \pi\pi$ relativistic matrix element (in the rest system of the ϱ). For a ϱ^0 with spin up it is [17]

$$\text{M. E.}^{\uparrow}_{\text{rel}} = \frac{2g\,(p_x - ip_y)}{\sqrt{2}}\,\frac{1}{\sqrt{8\,M_\varrho\,\omega_\pi^2}}\,, \tag{14}$$

where g is a phenomenological coupling constant; the observed value of the $\varrho\pi\pi$ width is obtained with

$$\frac{g^2}{4\pi} \approx 2\,; \tag{15}$$

p in (14) is the momentum of one pion. The other symbols are obvious. Now if we calculate the same matrix element with the model we get

$$\text{M. E.}_{\text{model}} = \frac{4f_q}{\mu} \frac{p_x - ip_y}{\sqrt{2}} \frac{1}{\sqrt{2\omega_\pi}} . \tag{16}$$

Putting $\omega_p = \frac{M_\varrho}{2}$ in both (14) and (16) and equating (14) with (16) we get for $g^2/4\pi$ the expression:

$$\frac{g^2}{4\pi} = \frac{9}{25} \cdot 8 \left(\frac{M_\varrho}{\mu} \right)^2 \frac{f^2}{4\pi} . \tag{17}$$

Here we find the ambiguity which was already mentioned in Section 4.; Eq. (17) is valid only at the non-relativistic limit. But do we obtain the non-relativistic limit by putting $M_\varrho = 280$ MeV (twice the pion mass) in (17) or putting $M_\varrho = 750$ MeV (its real value) and imagining to increase the pion mass to 375 MeV? Note that, contrary to the case of the $V \to P + \gamma$ decays, these two cases are different, because the value of μ in (17) is fixed; it is the value (140 MeV) which appears in the non-relativistic limit of the pion—nucleon interaction. In view of the above ambiguity we can only say that the value of $g^2/4\pi$ which we obtain is in between 1 and 7,5. It is satisfactory that the correct value of 2 is comprised in this range.

In spite of the ambiguity just mentioned we consider this result rather significant; it is essentially the same result of GURSEY, PAIS and RADICATI [18] (these authors also have some ambiguity when they use a "central" mass μ_{00} for the mesons) but it is obtained without assuming [18, 19] that the "relativistic completed" meson SU_6 tensor must globally couple to the baryons and without assuming as these Authors do that the vector coupling of the ϱ to the isospin current is universal.

We realize of course that a number of problems arise when one wishes to improve this calculation; one problem has already been mentioned; it is a correct treatment of the wave functions of particles moving at relativistic velocities; the other is how to avoid treating (in the $\varrho \to \pi\pi$ dceay) one pion as a field and the other as a quark-antiquark system.

7. The Johnson—Treiman—Lipkin—Scheck relations

As is well known LIPKIN and SCHECK have recently shown [20] that the JOHNSON—TREIMAN relations [21] together with a set of many other relations between cross sections can be derived by simple considerations according to the ideas typical of the model. The general idea is that the forward scattering amplitude for, say, πN scattering may be written simply as the sum of the forward scattering amplitudes of the quarks or antiquarks which the pion and nucleon contain. By reducing the forward amplitudes of a certain number of physically accessible reactions to a sum of quark-quark or quark-$\overline{\text{quark}}$ forward amplitudes and using in some cases SU_3 to relate these quark-quark amplitudes one obtains a very interesting set of relations which are listed and compared with the experimental data in the paper of LIPKIN and SCHECK. The agreement with the experimental data appears to be fair and in some cases good. What has still to be justified in much more detail is the additive assumption of the forward amplitudes. This is probably connected with the transparency in the individual quark-quark collisions as pointed out by KOKKEDEE and VAN HOVE [22]. But these quarks are extremely off shell and while some kind of impulse approximation should be valid, it is doubtful whether two quarks, bound say in a pion moving with momentum p can be described when this pion collides with something else, as two free particles with momentum $\frac{p}{2}$ each and with an energy $\sqrt{\left(\frac{p}{2}\right)^2 + \left(\frac{m}{2}\right)^2}$ where m is the mass of the pion; i.e. as two free particles of mass $\frac{m}{2}$.

As already stated in Section 4 (see also [16]) such an assumption, if possible, would enable us to take into account the relativistic kinematics of bound states by a BARGMAN—WIGNER description and would, therefore, simplify things very much. We cannot, at the moment assert that such an assumption is impossible but in our opinion it is a very important problem to understand if and when it is possible. We do not think that the high value of the magnetic moment of the quark (in quarkic magnetons) is necessarily related to assuming a small effective mass (in the above sense) of strongly bound quarks.

8. The problem of the masses

To conclude this survey of the main results of the quark model and before passing to problems which are in a sense open, we should speak of the question of the masses. We shall confine ourselves to a few comments here because DALITZ has discussed this problem in some detail [13] and we may refer to his discussion.

The success of the GELL-MANN—OKUBO mass formula does not mean that the mass problem is understood. Indeed one has to understand:

1) the reason for this success;

2) the values of the parameters which appear in the GELL-MANN—OKUBO mass formula.

In the quark model, since ZWEIG [1], the principal reason for the mass splittings inside a multiplet is attributed to the mass difference among quarks; the mass M' of the λ quark 3 is assumed to be heavier than the common value of the mass M of the quarks 1 and 2. This fact alone explains a good part of the success of the GELL-MANN—OKUBO mass formula; indeed it explains the fact that the dominant SU_3 violating term in the Hamiltonian is of the form T_3^3. In addition a perturbative calculation of this mass difference term is equivalent to an exact calculation since this term commutes, *in the non relativistic model,* with the SU_3 invariant part of the Hamiltonian.

However the mass difference effect is not the only effect leading to splittings inside a multiplet as is obvious

a) from the $\Sigma - \Lambda$ mass difference;

b) from the fact that when one examines the values of the mass differences obtained for the various multiplets there are some differences among the values obtained.

These differences are to be explained as an effect of SU_3 violating potentials, as we shall discuss in a moment. To see this let us temporarily assume that the difference of mass among quarks is the only effect responsible for the mass splittings. One obtains for this mass difference $\Delta = M' - M$ the following values

1) Baryon $\frac{1}{2}+$ octet $\Delta = 190$ MeV;
2) Baryon $\frac{3}{2}+$ decuplet $\Delta = 146$ MeV;
3) Vector mesons (with linear mass formula) $\Delta = 130$ MeV;
4) 2^+ mesons (with linear mass formula) $\Delta = 80$ MeV.

I have not listed the excited baryon states because the situation appears to me so far too confused; and I have not listed the pseudoscalar mesons because their Δ depends very much on whether we use the linear or quadratic mass formula, being in the first case $\simeq 350$ MeV, in the second $\simeq 120$ MeV. We entirely agree with SOCOLOW and MACFARLANE [23] that this question of the linear or quadratic mass formula for the mesons has to be settled experimentally. I should also like to be more sure than I am that the X° at 960 MeV is effectively the ninth pseudoscalar meson. I am somewhat puzzled by the large separation of this unitary singlet from the other mesons.

The question, at this point, is to understand the reason for the differences among the values of Δ listed above; these values, though having the same order of magnitude are somewhat different and our purpose is to understand these differences. We shall concentrate our attention, from now on, on the baryons and our purpose will be to understand, following DALITZ [13] (see also KUO and RADICATI [24]), the difference between the 190 MeV value of

Δ obtained for the baryon octet and the 146 MeV value obtained for the baryon decuplet.

As we said the mass difference effect among quarks, though the largest effect, is not the only one responsible for the mass splittings inside a multiplet. Also the potential between two λ quarks i and k (call it $U_3\,(ik)$) may be somewhat different from the potential between a λ quark and an n or p (call it $U_2(ik)$) and this in turn may be different from the potential (call it $U_1(ik)$) between two quarks n or p. We have assumed, in the above, two body forces only among quarks.

It is now convenient to write the Hamiltonian of a baryon using the formalism of unitary spin in the same way as one would write the Hamiltonian of a three body system using the formalism of isotopic spin [2]. In the same way as one would introduce there the projection operators for proton and neutron, respectively $\frac{1 \pm \tau_3}{2}$, one can introduce here the projection operators for proton, neutron and λ quarks; they are, respectively

$$\frac{2 + 3\tau_3 + \lambda_8}{6}, \quad \frac{2 - 3\tau_3 + \lambda_8}{6} \quad \text{and} \quad \frac{2 - 2\lambda_8}{6},$$

where λ_8 is the diagonal matrix with elements 1, 1, -2.

In this notation the Hamiltonian can be written [2]

$$H = H_0 + H_1, \tag{18}$$

where H_0 is an SU_3 invariant Hamiltonian, which we do not need to write down explicitly, and H_1 is the part which violates SU_3. It is convenient to divide H_1 into two parts $H_1 = H_1^U + H_1^T$. Here

$$H_1^T = (M - M')\,Y + \sum_1^3{}_i \left(\frac{p_i^2}{2M} - \frac{p_i^2}{2M'}\right)\lambda_8^i \tag{19}$$

describes the effect of the mass difference among the quarks, proportional to the hypercharge $Y = \frac{1}{3}\sum \lambda_8^i$ and, therefore, commuting with the remaining Hamiltonian; and a second kinetic term which is rather small ($\simeq$ 5 MeV) in the non-relativistic model.

The second of part H_1, that is H_1^U, describes the effect of the SU_3 violating potential energy. It can be written, in the notation already introduced,

$$H_1^U = \sum_{i,k} W_{\text{linear}}\,(ik)\,(\lambda_8^{(i)} + \lambda_8^{(k)}) + \sum_{i,k} W_{\text{quad}}\,(ik)\,\lambda_8^{(i)}\,\lambda_8^{(k)}, \tag{20}$$

where

$$W_l \propto 2\,U_1 - U_2 - U_3 \tag{21}$$

and

$$W_q \propto U_1 - 2\,U_2 + U_3\,. \tag{22}$$

In order that no quadratic term (of the kind, in tensor notation T^{33}_{33}) appears in the mass formula it is necessary that $W_q \cong 0$ i.e.

$$U_2 \cong \frac{U_1 + U_3}{2} \tag{23}$$

a condition already conjectured by ZWEIG [1]. The problem of the splittings is then determined by $(M - M')\ Y$, by the second term in H_1^T and by the first term in H_1^U.

The second term in H_1^T can be omitted for simplicity: the ensuing discussion is not affected by its presence. We can therefore confine ourselves to calculate the expectation value of

$$W = \sum_{i,k} W_l\,(ik)(\lambda_8^{(i)} + \lambda_8^{(k)})$$

for the various states indicated as α of the decuplet and octet baryons. Now, if we take our decuplet and octet baryon wave functions which consist of the product of a purely antisymmetric space part (the same for the octet as for the decuplet) and a symmetric spin unitary spin part and if we assume that $W_l(ik)$ in (20) is independent of the spin, the expectation value $< W >_\alpha$ is simply proportional for all the states in question

$$\langle W \rangle_\alpha = \langle W \rangle\, Y_\alpha\,, \tag{24}$$

where Y_α is the hypercharge of the state α under consideration and $< W >$ is a constant.

Therefore, for both the decuplet and octet states the part of the mass formula which determines the splittings would be

$$[(M - M') + \langle W \rangle]\, Y_\alpha\,. \tag{25}$$

This means that we should obtain, both for the decuplet and the octet a splitting proportional simply to the hypercharge, the coefficient $(M - M') + \langle W \rangle$ being the same for octet and decuplet. In particular one would have no $\Sigma - \Lambda$ mass difference. A way to obtain, in terms of central two body forces, such a mass difference is to assume that the expression $W(ik)$ in (20) has a part which is spin dependent, i.e. that it is different when two quarks are in a singlet or in a triplet state [25].

In this case the expectation value $\langle W\rangle_\alpha$ is given by an expression of this kind:

$$\langle W\rangle_\alpha = \langle W_1\rangle Y_\alpha + \langle W_2\rangle\langle \sum_{i,k} \sigma_i\, \sigma_k\, (\lambda_8^{(i)} + \lambda_8^{(k)})\rangle_\alpha . \tag{26}$$

This expression contains only two constants $\langle W_1\rangle$ and $\langle W_2\rangle$; therefore, on adding the term $(M - M')\,Y$, the splittings both in the decuplet and in the octet are determined by

$$((M' - M) + \langle W_1\rangle)\, Y_\alpha + \langle W_2\rangle\langle \sum_{i,k} \sigma_i \cdot \sigma_k\, (\lambda_8^{(i)} + \lambda_8^{(k)})\rangle_\alpha . \tag{27}$$

Now the expression $\langle \sum_{i\,k} \sigma_i \cdot \sigma_k\, (\lambda_g^{(i)} + \lambda_g^{(k)})\rangle$ can be evaluated as

$$C + \left[- Y + 2\left\{I\,(I+1) - \frac{1}{4}\,Y^2\right\}\right]$$

with a different C but the same coefficients of Y and of $\{\quad\}$ both for the decuplet and the octet. It follows that the mass splitting formula becomes

$$((M - M') + \langle W_1\rangle - \langle W_2\rangle)\; Y_\alpha + 2\,\langle W_2\rangle \left(T_\alpha\,(T_\alpha + 1) - \frac{Y_\alpha^2}{4}\right) \tag{28}$$

that is, more concisely, as already discussed by DALITZ [13], and using now his notation:

$$a\,Y_\alpha + b\left(T_\alpha\,(T_\alpha + 1) - \frac{Y_\alpha^2}{4}\right) \tag{29}$$

with a and b being the same for decuplet and octet. Now the octet masses determine $a = -190$ MeV as already said and $b = 37$ MeV. For the decuplet formula (29) reduces to

$$\left(a + \frac{3}{2}\,b\right) Y_\alpha$$

and we have $a + \frac{3}{2} b = 135$ MeV to be compared with the already mentioned experimental value of -146 MeV. As noted by DALITZ the agreement is not perfect but rather good since we have omitted Coulomb effects and effects of order T_{33}^{33} arising from small violations of (23); we note also that possible tensor forces effects and consequent $L \neq 0$ admixtures may affect the situation.

It must be noted finally that the starting point of this discussion has been the mass difference between the quarks. It is now evident, however, that all one can know is $M - M' + \langle W_1 \rangle$ and that $M' - M$ though presumably larger than W_1, cannot be known with precision without additional arguments.

9. The problem of saturation

We have listed in Section 3 problems to which only a tentative solution can be given; in this Section and in the next one we shall consider only part of these problems; for instance we shall not consider the first problem in the list of Section 3 since we have not yet a sufficient knowledge of the qq or $q\bar{q}$ potentials; in this Section we shall concentrate our attention on the saturation problem [26]. The question is, of course, why 3 quarks are so strongly bound, to form, say, a proton, and four or more quarks, if bound at all, give rise to objects which are certainly much heavier than the proton. Indeed, assuming that there are only two body forces between quarks and neglecting, as one can do as a first approximation in the non-relativistic model, the average kinetic energy with respect to the average potential energy, the average potential energy of a pair of quarks in the proton is

$$|\bar{V}| \approx M - \frac{1}{3} M_P , \tag{30}$$

where M is mass of the quark and M_P the mass of the proton. One sees from (30) that a biquark would have a mass much larger than that of the proton

$$M_2 = 2M - \left(M - \frac{1}{3} M_P\right) = M + \frac{1}{3} M_P ; \tag{31}$$

in fact, M_2 has a value near to the mass of one quark. But a system of 4 quarks, for instance, should have a mass (remember that there are 6 pairs of quarks in a 4 quark object)

$$M_4 = 4M - 6|\bar{V}| \tag{32}$$

and therefore, on using (30),

$$M_4 = -2M + 2M_P$$

a negative value for mass!

This obviously absurd conclusion shows that there is something wrong in this argument. In fact, it is easy to realize that if a 3 quark system is so strongly bound as to have formed a proton which is a very light object in the quark mass scale, the dynamics of a 4 quark object cannot be treated non-relativistically. To see this let us consider the 4 quark object from the following point of view: as a quark which is bound to a three quark subsystem. More specifically let us consider a particular model of binding (one would say, in the variational language a particular trial wave function) in which we have already our three quark system forming a proton and we are trying to bind the fourth quark to it. In this case the mass formula (32) is more conveniently written

$$M_4 = M_P + M - 3\,|\overline{V}| \tag{33}$$

since the fourth quark can interact with all the three quarks of the proton.

It is now sufficient to remark that the proton mass is of the same order of magnitude as the inverse range of the forces holding the proton to the fourth quark to conclude that the relative motion of the fourth quark with respect to the proton is necessarily relativistic. Non-relativistic concepts cannot, therefore, be applied and we should interrupt our discussion at this point noting only that this conclusion is independent of the particular model of binding considered above.

However, we may try to go a little further assuming that we can continue to discuss on the basis of a non-relativistic formula like (33). Since what we have just learnt is that the quark motion inside a four quark system cannot be treated non-relativistically, we have no justification for substituting in (33) for $|\overline{V}|$ the non relativistic value (30) obtained from the binding of the proton. It may well happen that the potential between two quarks, which is strongly attractive when they move non-relativistically depends on the relative velocity and becomes much less attractive or even repulsive for relativistic velocities. Along these lines a possible explanation of the saturation problem can be given; for more details we refer to a published note on the subject [26], noting here that the above argument only shows that a consistent situation in which saturation is reached in the way it is (at three quarks) *can* be generated in a quark model; it is a different question and becomes a kind of consistency problem to show that saturation *must* effectively be reached at *three* quarks.

10. The problem of magnetic moments

Another problem which must be understood is that of the magnetic moments of quarks. It is easy to show that in *non violated* SU_3 these magnetic moments are proportional to the charge of the quarks. In fact the electromagnetic current is the T_1^1 component of an irreducible SU_3 tensor and the magnetic moments of the quarks are therefore proportional to $\langle q_i \mid T_1^1 \mid q_i \rangle$, where q_i

$(i = 1, 2, 3)$ are the three quark states. Observe now that T_1^1 belongs to the representation 8 and that the product $3 \times 3^*$ contains, of course, 8 only once. Therefore, the three expectation values $\langle q_i \mid T_1^1 \mid q_i \rangle$ can all be expressed in terms of a single reduced matrix element; the proportionality constants can be determined by simply taking $T_1^1 = Q$ (the charge); hence they are 2/3, $-1/3$, $-1/3$.

Note the difference with the SU_2 case (isospin) where the electromagnetic current does not transform according to a different representation of the isospin group, but is the sum of a scalar plus the third component of a vector in isospin space.

The real problem with the magnetic moments of the quarks is why they are so large. The Landé g factor of the quarks is

$$g = 5{,}58 \frac{M}{M_P},$$

where M is the quark mass and M_P the proton mass. The larger the quark mass, the more anomalous is the magnetic moment. To explain this large g factor the attitude is often that the quarks, when strongly bound as in the proton have a small effective mass, and it is this small effective mass, call it M^*, which determines the magnetic moment of the quark inside a proton as $\frac{e\hbar}{2M^* c}$. For instance, if $M^* = \frac{M_P}{3}$ one would have a g factor of a bound quark equal to $6 \frac{M}{M_P}$.

According to this point of view the magnetic moment of a free quark should be very small, of the order $\frac{e\hbar}{2Mc}$ and the large anomaly is produced in the binding. We do not share, at the moment, this point of view. We observe that if a very heavy particle such as the quark is coupled strongly to much lighter particles such as mesons the magnetic moment is essentially determined by the meson cloud. It is entirely unnatural to measure this magnetic moment in Bohr magnetons of the heavy particle; one can do so, of course, but one must not be amazed when one finds very anomalous values for the magnetic moment. These value turn out to be "anomalous" simply because measured in "anomalously" small units.

In other words, according to our point of view [27], the order of magnitude of the magnetic moment of a free quark might be calculated, for instance, in a fixed source theory, in a way similar to that used some time ago for the nucleon (compare e.g. [28]). It is not extraordinary then that, since the coupling of the pion (just to consider one of the mesons) to the the quark has the same order of magnitude as the coupling of the pion to the nucleon (compare equation [13]), also the magnetic moment of the quark and the nucleon turn out to have the same order of magnitude. Indeed, assuming that this kind of explanation is correct, we may reverse it to say that the couplings of the quarks to the mesons are not generally expected to have a substantially different order of magnitude from the couplings of the nucleons to the mesons. If this is so, the fact that the binding energy in the baryons, for instance, is so much higher than that of nucleons in nuclei might be due to the fact that the potential between two quarks, like that between two nucleons, is very deep at small distances (but always with such a repulsive core as to allow non-relativistic motion) and that quarks being much heavier than nucleons can remain practically at rest at small distances, with a negligible zero point motion.

(There is no need to stress at this point that all these are only very qualitative and tentative suggestions.)

11. Final questions

All the questions posed in the introductory Sections have been considered more or less at length, except for the last two:

a) why, normalizing the charge of the n quark as -1, the electron charge is 3? and

b) do quarks exist? and how can one get an estimate of their production cross section?

The fact that no answer at the moment can be given to question *a)* is a brutal way of recognizing that we have no idea of the connection between strong and weak interactions; this is not typical of the quark model, but this question remains and becomes in a sense more acute with the introduction of the quarks. Why are leptons not composed of quarks? Or does some kind of leptonic quark exist too? We do not know, although it is difficult to conceive weakly interacting leptonic quarks which are strongly bound. If one were to be optimistic at all costs one might even say that the existence of baryonic quarks but not of leptonic quarks can explain the conservation of nucleonic number. In fact, a quark, having fractionary charge, cannot decay into a lepton $+ \gamma$ or into leptons and, therefore, a proton is stable if it is assumed that the total decay amplitude of a proton is the sum of the amplitudes from the three quarks.

But this is a too optimistic way of seeing the situation; the real status of which is a complete lack of understanding of the relation between leptons and strongly interacting particles.

Coming now to the second question: do quarks exist? it is obvious that the model which we have described implies the real existence of quarks. It is therefore very important to find them, either in cosmic rays from experiments with future accelerators or in terrestrial matter [30]. The problem is complicated by an absolute lack of knowledge of the cross section for quark production, of which it is extremely difficult to give a reliable estimate. The only published estimate which we know of is a statistical estimate by DOMOKOS and FULTON [29] which gives an exceedingly small value of the cross section for quark production in pp collisions at energies where the pion production cross section in the same reaction is $\cong 40$ mb ($\sigma_q \cong 10^{-7}\ \mu b$ for $M = 9$ GeV).

How reliable a statistical estimate is when it gives rise to such small numbers we do not know, since some form of direct reaction (to use the language of nuclear reactions) can well be more important. In any case it is essential to try to have theoretical estimates of these quark production cross sections (the simplest process for this purpose is perhaps $p + \bar{p} \to q + \bar{q}$) in order to be able to understand the meaning of the experiments performed or to be made in future. These experiments are in progress at various laboratories with the different methods listed above, but their survey would lead us too far away and it seems appropriate to end this report here [31].

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НЕРЕЛЯТИВИСТСКАЯ МОДЕЛЬ КВАРКОВ

Г. МОРПУРГО

Резюме

Исследованы схема классификации и распадные свойства адронов в рамках статической модели кварков. Изложены главные успехи и нерешенные проблемы модели, указывая задачи, которые должны быть решены для лучшего понимания и использования нерелятивистских идей.

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RECENT RESULTS FROM THE QUARK MODEL

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The basic ideas and methods of the quark model of the elementary particles, and the main results of the model, including the new applications to high energy scattering are summarized and partly discussed in detail. Also the quark models of weak interactions are briefly commented.

I. Introduction

In the past few months, quite a number of interesting calculations have been done under more or less direct inspiration of a simple quark model of strongly interacting particles. To my knowledge, the theory of weak interactions has least benefited from this work. Thus, the only justification for including a lecture on quark models in a conference on weak interactions is the hope that a description of some of the recent applications of the quark model elsewhere in particle physics will lead someone in the audience to a useful idea about the weak interactions. Most of this talk will concern applications outside weak interactions; only a few remarks about quark models of weak interactions will be found at the end.

I assume that this audience is familiar with the $SU(3)$ group and its representations, and is aware of the kinds of problems in particle physics to which it has been applied.

A part of this lecture will be based on still unpublished material, and I want to make sure that proper credit is given. I had the pleasure of visiting the Weizmann Institute of Science, Rehovoth, Israel, two months ago, and I was greatly stimulated by discussions with Professor HARRY LIPKIN, Dr. HECTOR RUBINSTEIN and Dr. FLORIAN SCHECK. I am taking the liberty of presenting some of their ideas and calculations here, trusting that the work will be correctly attributed to them.

I make no claims to being aware of all of the published literature on quark models, much less of the unpublished literature. Furthermore, of the articles known to me, those discussed below are a sample chosen with con-

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siderable arbitrariness. I hope no one whose work is not mentioned will feel offended.

This lecture was written at CERN, where I have enjoyed the hospitality and the inspiring presence of L. VAN HOVE. I have also benefited from helpful conversations at CERN with N. CABIBBO, J. HARTE, J. J. J. KOKKEDEE, Y. NE'EMAN and J. PRENTKI.

II. History of the quark model

A basic idea of the quark model is that the known strongly interacting particles can be usefully viewed as composite particles. This idea can be traced at least as far back as FERMI's and YANG's suggestion [1] that the π mesons might be deeply bound states of a nucleon and an antinucleon. SAKATA [2] extended this idea to incorporate hypercharge, describing all mesons as deeply bound states of the nucleon, the lambda, and their antiparticles, and his model led to the first appearance of SU (3) in particle physics. The FERMI—SAKATA idea has led in two directions, down two different roads. One direction led to CHEW's idea of "nuclear democracy" where all of the strongly interacting particles are viewed as composites of one another [3]. This road will not be explored here. The other direction also led to theories which equalize the roles of mesons and baryons, but in a different sense. It led to theories in which the known baryons and mesons are all made of *something else*.

The farthest outpost along this second road today is the *literal* "quark model", according to which entirely new physical objects, called "quarks" by GELL-MANN, are supposed to have an independent existence in nature, and the baryons and mesons are understood to be bound states of these quarks. People have set out to look for these quarks, and they have not proved easy to find. The search continues. We shall not have to go quite this far down this road, however, for we shall never need to assume that quarks have an independent existence. In particular, we shall not need to introduce a free quark mass, because we shall almost completely avoid any discussion of the dynamics from which bound states of the quarks arise. Needless to say, eventually quark dynamics will have to be faced more directly.

The first step down the road from the SAKATA model to the quark model was taken by GELL-MANN [4] and NE'EMAN [5], who were seeking an explanation for the existence of eight spin 1/2 baryons. If the eight pseudoscalar mesons were bound states of a triplet of particles and a triplet of antiparticles, perhaps the eight baryons were also. (In the baryon case, of course, the two particle triplets would have to be distinct.) These triplets might have only a mathematical existence; in any event, they did not have to be identified with any known particles.

The next step down this road came with the identification of one $SU(3)$ multiplet which was not an octet: the decuplet of spin 3/2 states. This identification was made secure by the discovery of the Ω^-, the tenth member, at just the predicted mass [6]. For our story, the importance of the identification of this decuplet lies in the fact that it suggested a different model for the baryon *octet.* A decuplet cannot be made from a triplet and an antitriplet. However, both an octet and a decuplet can be made from three of the same kind of quarks. Gell-Mann [7] and Zweig [8] proposed such a model, in which there is only one triplet of quarks, with spin 1/2 and baryon number 1/3, and the mesons are quark-antiquark bound states while the known octet and decuplet are both bound states of three quarks.

This point of view achieved major confirmation in 1964, with the $SU(6)$ theory [9]. In particular the grouping of the baryon octet with the decuplet was confirmed by: i) the availability of the *56* representation which exactly accommodated these states, and ii) the derivation of the ratio of the static magnetic moments of the neutron and proton [10]. The latter calculation we shall repeat in detail below.

There are other quark models besides the Gell-Mann—Zweig one-triplet model. In particular, nothing we have said so far (though some of what we shall say subsequently) requires the mesons and the baryons to be made of the same kind of quark-stuff. There are a number of other models which begin with a triplet and a singlet or two triplets. To my knowledge, such models do not readily lead to the correct proton-neutron magnetic moment ratio. Though this is probably not sufficient reason to dismiss these models, we shall not mention them further.

III. Baryon and meson "wave functions" in terms of quarks

In order to do most quark model calculations, we must know something about the wave functions of the baryons and mesons in terms of quarks. We assume that these wave functions *factor* into a part containing the internal quark quantum numbers and quark spins, and another part dependent on the quark positions (and perhaps velocities). Particles belonging to the same $SU(6)$ supermultiplet are assumed to have the same spatial wave functions, in general not needed here.

Most of the states which have been successfully classified in the $SU(3)$ theory have also been classified in the $SU(6)$ theory. There is a 56-plet of baryons (the fully symmetric three-quark representation) and a 35-plet of mesons, made of a quark and an antiquark. In both cases, the quarks are assumed to be in a total orbital angular momentum zero state, which is another way of saying that the spin of the composite particle is accounted for entirely by the spins of the quarks.

The properties of the fundamental quark triplet are given in the Table below:

Quark	I	I_z	Charge	Hypercharge
p	1/2	1/2	2/3	1/3
n	1/2	—1/2	—1/3	1/3
λ	0	0	—1/3	—2/3

All quarks have a spin of one-half. The two charge states will be denoted by (+) for $S_z = +1/2$ and (—) for $S_z = -1/2$. The charge, hypercharge, and I_z assignments of the antiparticles are obtained by a change of sign. The third integral values of charge and hypercharge are required to keep the triplet representation *centered* (meaning that the average charge and hypercharge equal zero). This, in turn, is required if the supermultiplet constructed out of three quarks is also to be centered (which the physical 56 happens to be). Note that the notation p, n, λ, is intended to remind us that the quarks have the same values of I and I_z as the physical baryons with the same names. We shall try to use the capital letters P, N, Λ, when we refer to the physical baryons.

We can arrive at the baryon wave functions in more than one way. A simple way is the following: to construct the proton state with $S_z = +1/2$, we need to use two p quarks and one n quark, two spinning up and one down. We obtain a state of spin 1/2 by first combining two quarks to make spin zero, and then attaching the third:

$$[p(+)n(-) - p(-)n(+)]\, p(+) \,.$$

The state which belongs to the 56 is then obtained by fully symmetrizing, that is, adding together the six permutations of each term above. We get

$$\begin{aligned}|P(+)\rangle = (18)^{-1/2}\,[&2\,p(+)\,p(+)\,n(-) + 2\,p(+)\,n(-)\,p(+) + \\ &+ 2\,n(-)\,p(+)\,p(+) - p(+)\,p(-)\,n(+) - \\ &- p(-)\,n(+)\,p(+) - n(+)\,p(+)\,p(-) - \\ &- p(-)\,p(+)\,n(+) - p(+)\,n(+)\,p(-) - \\ &- n(+)\,p(-)\,p(+)]\,,\end{aligned}$$

where the radial $(18)^{-1/2}$ assures that normalized quark states lead to normalized physical states.

We obtain the remaining states of the spin 1/2 octet as follows:

a) the $S_z = -1/2$ states are obtained by a spin reflection, interchanging $(+)$ and $(-)$;

b) the neutron states are obtained by a charge reflection, interchanging p and n;

c) the Σ^+ states obtained from the proton and the Ξ° from the neutron by a U spin reflection, interchanging n and λ.

d) the Σ^- and Ξ^- states are obtained by a charge reflection of the Σ^+ and Ξ^0 states;

e) the Σ^0 and Λ states are constructed by beginning with the combination (for $S_z = +1/2$)

$$[p(+)\lambda(-) - p(-)\lambda(+)]n(+) \pm [n(+)\lambda(-) - n(-)\lambda(+)]p(+) ,$$

where the upper sign gives $I = 1$ and the lower sign gives $I = 0$. Symmetrizing and normalizing we get

$$| \Sigma^\circ(+) > = (36)^{-1/2} [2\, p(+)n(+)\lambda(-) - p(+)n(-)\lambda(+) - \\ - p(-)n(+)\lambda(+) + \text{perms.}] ,$$

$$| \Lambda (+) > = (12)^{-1/2}[p(+)n(-)\lambda(+) - p(-)n(+)\lambda(+) + \text{perms.}] ,$$

where "perms" means: add the five additional permutations of each term.

In a similar way, but with less work, we may construct the forty spin 3/2 decuplet states. For example, the Δ^{++} state with $S_z = +1/2$ has the quark representation:

$$(3)^{-1/2}[p(+)p(+)p(-) + p(+)p(-)\, p(+) + p(-)p(+)p(+)] .$$

The other states are constructed in a similar manner.

The pseudoscalar and vector meson wave functions are made from one quark and one antiquark. For many applications, one has to be careful to distinguish ordinary spin and W-spin (a crucial minus sign creeps in when antiquarks are involved [11].) For our purposes, we only need the quark *content* of the charged mesons and neutral K mesons:

$$\pi^+,\ \varrho^+ \ : (p, \bar{n}) ,$$

$$\pi^-,\ \varrho^- \ : (n, \bar{p}) ,$$

$$K^+,\ K^{*+} : (p, \bar{\lambda}) ,$$

$$K^\circ,\ K^{*\circ} : (n, \bar{\lambda}) ,$$

$$\bar{K}^\circ,\ \bar{K}^{*\circ} : (\lambda, \bar{n}) ,$$

$$K^-,\ K^{*-} : (\lambda, \bar{p}) .$$

Many other strongly interacting states are known. What is to be done with them? An honest answer is: we do not know. We probaly shall not be able to talk about their quark content until we at least identify the SU (3) representation to which they belong. But at this time only one more SU (3) representation has been identified that people generally believe in. This is the octet and singlet of spin 2^+ mesons [12]. Two distinct quark pictures for these mesons are: i) states made of two quarks and two antiquarks, with no orbital angular momentum between the quarks. (The corresponding $SU(6)$ multiplet can have a dimension of either 189 or 405. The 405 requires twenty-seven more 2^+ mesons. Both 189 and 405 require many more axial vector and scalar mesons.) ii) states made of one quark and one antiquark, with one unit of orbital angular momentum and one unit of spin coupling to give total spin 2. (Three units of orbital angular momentum are also possible.) It is not known if either of these pictures is correct, and even though the experimental situation is somewhat uncertain, it appears that all of these pictures share a common difficulty: they all give a ratio of the strength of the spin 2 coupling to a vector and a pseudoscalar meson to the strength of the spin 2 meson coupling to two psedoscalar mesons, which is too small by a factor of two [13].

However, even if we only know the quark content of two $SU(6)$ multiplets, these multiplets include most of the states we are usually interested in, and all of the states which are stable under the strong interactions, so let us proceed to applications.

IV. Group theoretic results: mass formulae and baryon magnetic moments

It is necessary to distinguish two varieties of results obtained from the quark model. First, there are a set of results which amount to finding a more picturesque language in which to describe a group theoretical calculation. These derivations have a heuristic value, for often the picturesque language will suggest further calculations. In particular, the language of quarks provides a simple and appealing way to discuss broken symmetry. Second, there are results which cannot be reduced to a group theoretical statement. Among these are relations between meson and baryon properties, since mesons and baryons belong to different supermultiplets, and relations which presuppose some definite quark dynamics. In this Section we shall describe the first kind of result, and in the next Section the second kind.

As a first example of a group theoretic calculation in quark language, let us consider *mass formulae* valid in the absence of electromagnetism. A simple quark-language statement would be "all baryon masses would be the same except that λ quarks are heavier than n and p quarks; the extra mass is proportional to the number of λ quarks". This statement is sufficient to "explain" the equal mass spacing in the decuplet. It corresponds to the statement in group theory language that the mass operator transforms like a 35 of $SU(6)$, since the mass operator may be written

$$M = M_0 + M_1 \left[a^+_{\lambda(+)}\, a_{\lambda(+)} + a^+_{\lambda(-)}\, a_{\lambda(-)}\right],$$

where a^+, a are the creation and destruction operators for quarks, which themselves transform like the basic 6 and 6* representations, so that bilinear forms transform like a 35.

The mass operator above does not account for the $\Sigma - \Lambda$ splitting or the splitting between octet and decuplet masses, and it is well known that we allow additional terms in the $SU(6)$ mass formula. Recently, Federman, Rubinstein, and Talmi found a picturesque language for these extra terms. Suppose that "mass differences are also caused by interactions between *pairs* of quarks, the pairing energy depends both on the kinds of quarks and whether they are in an $S = 0$ or an $S = 1$ state, and the net effect on the particle masses is the sum of the separate pairing energies". We can guess the result in advance, because pairing terms in the mass operator, a^+a^+aa, transform like the 405 of $SU(6)$ and the group theoretic calculation has been done before (that is, the Clebsch—Gordan coefficients have been tabulated). The most general mass operator for the 56 of $SU(6)$ which neglects electromagnetic effects has eight terms:

$$M = M(1,1) + M(35,8) + M(405,1) + M(405,8) +$$
$$+ M(405,27) + M(2695,8) + M(2695,27) + M(2695,64),$$

where the $SU(6)$ and $SU(3)$ representations are given in parentheses. In the language of the quark theory, the 405 terms correspond to pairing interactions and the 2695 terms correspond to the three-quark interactions. Neglecting the three-quark interactions gives three mass formulae.

$$\Xi^* - Y^* = \Xi - \Sigma, \tag{1. a}$$

$$\Omega - \Delta = 3(\Xi^* - Y^*), \tag{1. b}$$

$$(Y^* - \Delta) - (\Omega - \Xi^*) = (3\Lambda + \Sigma - 2\Xi - 2N). \tag{1. c}$$

The first is a famous $SU(6)$ result. The second is well known as the weaker form of the equal spacing law which still holds when the conventional octet $SU(3)$ breaking is treated to second order. The third mass formula equates two combinations of mass differences, each transforming like the $I = Y = 0$

member of the 27 of $SU(3)$. It is not often mentioned as a mass formula. Experimentally, both sides of (1. c) have the same sign. (It is well known that the Λ is too heavy and the Ω^- is too light to satisfy the GELL-MANN—OKUBO formula exactly.) Although one is comparing quantities which depend considerably on *electromagnetic* effects, and one is not sure what sort of average of charged states to insert in (1. c), it is probable that the right-hand side of (1. c) is about five times as large as the left-hand side.

The GELL-MANN—OKUBO formula (the vanishing of both sides of Eq. (1. c)) does not follow from the pairing assumption alone. FEDERMAN, RUBINSTEIN, and TALMI find that it *does* follow if one makes the additional assumption that all $S = 1$ pairing forces have the same strength. (In fact a weaker additional assumption is sufficient — see the Appendix.) RUBINSTEIN [15] notes that if one extends this assumption to include quark-antiquark pairing forces as well, one obtains

$$\Phi - K^* = K^* - \varrho = \Xi^* - Y^*, \tag{2}$$

(using meson *masses)*, which is quite well satisfied. This is an example of a quark model result of the second kind following from a quark model result of the first kind, for (2) has no obvious group theoretic origins.

The principal significance of the idea of pairing forces is that it provides a quasi-dynamical explanation for the observed mass spectrum. One had already noticed that in $SU(3)$ the "octet splitting" is larger than the „27 splitting" and that in $SU(6)$ also, the smaller the representation, the larger its contribution [16]. The $SU(3)$ situation had been explained by postulating a symmetry breaking interaction transforming like an octet; it was then difficult to explain why lowest order perturbation theory works so well. One can now speak instead of one-quark effects, pairing effects, and, eventually, three-quark effects, increasingly unimportant in determining the physical mass [17].

As an example of the use of the quark wave functions presented in the previous section, the pairing calculation is worked out in detail in the Appendix.

As a second example of a derivation of a group theoretic result from the quark model, consider the baryon magnetic moments [18]. The well-known results are "one-quark" results in the sense just explained above. One begins again with an additivity assumption: that the magnetic moment operator $\vec{M}$ is the sum of the magnetic moment operators for the separate quarks:

$$M_z = \sum_i \left[a^+_{i(+)}\, a_{i(+)} - a^+_{i(-)}\, a_{i(-)}\right] \mu_i\,,$$

where $i = p, n, \lambda$ and μ_i is the magnetic moment of the corresponding quark. Using the wave functions given above and taking expectation values, we find for the spin 1/2 baryon:

$$\mu_P = (4\mu_p - \mu_n)/3\,, \tag{3.a}$$

$$\mu_N = (4\mu_n - \mu_p)/3\,, \tag{3.b}$$

$$\mu_\Lambda = \mu_\lambda\,, \tag{3.c}$$

$$\mu_{\Sigma^+} = (4\mu_p - \mu_\lambda)/3\,, \tag{3.d}$$

$$\mu_{\Sigma^0} = (2\mu_p + \mu_n - \mu_\lambda)/3\,, \tag{3.e}$$

$$\mu_{\Sigma^0\Lambda} = (-\mu_p + \mu_n)/\sqrt{3}\,, \tag{3.f}$$

$$\mu_{\Sigma^-} = (4\mu_n - \mu_\lambda)/3\,, \tag{3.g}$$

$$\mu_{\Xi^0} = (4\mu_\lambda - \mu_p)/3\,, \tag{3.h}$$

$$\mu_{\Xi^0} = (4\mu_\lambda - \mu_n)/3\,. \tag{3.i}$$

The transition moment $\mu_{\Sigma^0\Lambda}$ governs the rate for $\Sigma^0 \to \Lambda + \gamma$, which is not well known experimentally. (It is interesting that it may be expressed in terms of μ_P and μ_N without further assumptions.) The magnetic moments of the Λ and Σ^+ are now known approximately. Making no assumptions about the relative magnitudes of the quark magnetic moments, one obtains from (3a)—(3d):

$$(3\mu_{\Sigma^+} + \mu_\Lambda)/4 = (4\mu_P + \mu_N)/5\,. \tag{4.a}$$

If we assume that the magnetic moments of the p and n quarks are proportional to their charges ($\mu_p = -2\mu_n$), we get the additional relation

$$\mu_P = -(3/2)\,\mu_N\,, \tag{4.b}$$

which is the celebrated $SU(6)$ result. Finally, if we also demand that the magnetic moment of the λ quark has the same constant of proportionality (so that $\mu_n = \mu_\lambda$) we get the $SU(3)$ predictions:

$$\mu_{\Sigma^+} = \mu_P \tag{4.c}$$

$$\mu_\Lambda = \frac{1}{2}\mu_N\,. \tag{4.d}$$

We should expect (4a) to hold better than (4c) or (4d), since in (4a) we are allowing for some of the medium strong symmetry breaking. The n

and λ quarks might have different magnetic moments because they have different masses. Experimentally

$$\mu_{\Sigma^+} = 4{,}3 \pm 1{,}5\,,$$

$$\mu_{\Lambda} = -0{,}69 \pm 0{,}13$$

in units of ***nucleon*** Bohr magnetons. Better experimental accuracy will be required before it will be possible to say whether (4.a) is indeed better satisfied than (4.c) or (4.d). If (4.b) turns out to be better satisfied than (4.a), this would indicate that the medium strong symmetry breaking is making itself felt elsewhere than in a difference between the λ and n magnetic moments.

We see here that even when discussing a "one-quark" effect like magnetic moments, the quark model provides a natural language in which to discuss symmetry breaking. Setting $\mu_n \neq \mu_\lambda$ is equivalent to allowing U spin violation. Accurate tests of U-spin conservation do not exist as yet, and it would be interesting to perform them.

It turns out that the electromagnetic decays of the decuplet resonances provide a sensitive test of U-spin conservation. The magnetic moments for decuplet-octet transitions are of course given in terms of the quark magnetic moments under the same assumptions. One finds

$$\mu_{\Delta^+ P} = (2\sqrt{2}/3)(\mu_p - \mu_n)\,, \tag{5.a}$$

$$\mu_{\Delta^0 N} = (2\sqrt{2}/3)(\mu_n - \mu_p)\,, \tag{5.b}$$

$$\mu_{Y^{*+}\Sigma^+} = (2\sqrt{2}/3)(\mu_p - \mu_\lambda)\,, \tag{5.c}$$

$$\mu_{Y^{*0}\Sigma^0} = (\sqrt{2}/3)(\mu_p + \mu_n - 2\mu_\lambda)\,, \tag{5.d}$$

$$\mu_{Y^{*0}\Lambda} = (\sqrt{6}/3)(\mu_p - \mu_n)\,, \tag{5.e}$$

$$\mu_{Y^{*-}\Sigma^-} = (2\sqrt{2}/3)(\mu_n - \mu_\lambda)\,, \tag{5.f}$$

$$\mu_{\Xi^{*0}\Xi^0} = (2\sqrt{2}/3)(\mu_\lambda - \mu_p)\,, \tag{5.g}$$

$$\mu_{\Xi^{*-}\Xi^-} = (2\sqrt{2}/3)(\mu_\lambda - \mu_n)\,, \tag{5.h}$$

where

$$\mu_{\Delta^+ P} = \langle \Delta^+, S_z = 1/2 \,|\, M_z \,|\, P, S_z = 1/2 \rangle\,, \text{ etc,}$$

We see that $\mu_n = \mu_\lambda$ forbids the decays $\Xi^{*-} \to \Xi^- + \gamma$ and $Y^{*-} \to \Sigma^- + \gamma$, which is the well-known U spin result. The calculated rates for $\Xi^{*0} + \gamma$ and $Y^{*+} \to \Sigma^+ + \gamma$ both turn out to be somewhat greater than 0,1 MeV. In this case, the electromagnetic decay rates may be measurable (the Ξ^* width is only 7 MeV) and the degree of suppression of the negatively charged resonance decays may be ascertained [19].

V. Non-group-theoretic results: high energy scattering

In addition to the group-theoretic results described in the previous Section, the quark model has yielded a number of results of a more dynamical kind. An example is the LIPKIN—SCHECK [20] and LEVIN—FRANKFURT [21] work on high energy scattering. This work, in my opinion, is the most interesting application of the quark model thus far and alone justifies further study of the quark model.

There now exists a large amount of accurate data on high energy total cross-sections, for a variety of incident particles with proton and with neutron (deuteron) targets. Assuming charge symmetry, ten independent processes are measured in the laboratory: K^+P, K^+N, K^-P, K^-N, π^+P, π^-P, PP, NP, $\bar{P}P$, $\bar{P}N$. Charge symmetry allows us to imagine that we always have a *proton* target and are scattering the then particles (K^+, K^0, K^-, $\bar{K}^0$, π^+, π^-, P, N, $\bar{P}$, $\bar{N}$). These fully equivalent processes can be discussed with a simplified notation: henceforth "K^0" will refer to K^0P and K^+N scattering, etc.

LIPKIN and SCHECK, and also LEVIN and FRANKFURT, made the extremely simple assumption that at high energies the meson-baryon and baryon-baryon elastic scattering amplitudes could be approximated by the sum of the elastic scattering amplitudes of the constituent quarks. In this approximation, one neglects the effects of quark spin and all details of the quark wave functions. Letting the quark and particle names stand for the quark-proton and particle-proton elastic scattering amplitudes, we may then write:

$$K^+ = p + \bar{\lambda}\,, \tag{6.a}$$

$$K^0 = n + \bar{\lambda}\,, \tag{6.b}$$

$$K^- = \lambda + \bar{p}\,, \tag{6.c}$$

$$\bar{K}^0 = \lambda + \bar{n}\,, \tag{6.d}$$

$$\pi^+ = p + \bar{n}\,, \tag{6.e}$$

$$\pi^- = n + \bar{p}\,, \tag{6.f}$$

$$P = p + p + n\,, \tag{6.g}$$

$$N = p + n + n\,, \tag{6,h}$$

$$\bar{P} = \bar{p} + \bar{p} + \bar{n}\,, \tag{6.i}$$

$$\bar{N} = \bar{p} + \bar{n} + \bar{n}\,. \tag{6.j}$$

This gives us four relations among the particle-proton elastic scattering amplitudes:

$$K^+ + \pi^- + K^0 = K^- + \pi^+ + \bar{K}^0, \tag{7.a}$$

$$K^+ - K^0 = P - N\,, \tag{7.b}$$

$$K^- - \bar{K}^0 = \bar{P} - \bar{N}\,, \tag{7.c}$$

$$3\,(\pi^+ + \pi^-) = P + N + \bar{P} + \bar{N}\,. \tag{7.d}$$

These relations *may* hold for all values of the momentum transfer at sufficiently high energies. They are most easily compared, however, in the forward direction, where the imaginary part of the elastic scattering amplitude is related to the total cross-section [22]. Equation (7.a), which involves only meson-baryon cross-section, is found to be satisfied to better than one percent for six out of seven experimental values of the incident meson momentum, between 6 and 18 GeV/c. [23]. It is not yet possible to make useful quantitative tests of equations (7.b) and (7.c), because experimental errors in the total baryon cross-section are of the order of the splittings themselves. However, it does appear that the baryon differences appearing in (7.b) and (7.c) are consistently larger than the meson differences; also the two experimental mass differences appearing in (7.c) show a tendency to have opposite signs, but the errors here are particularly large. The left-hand side of (7.d) is 150 $\pm$ 2 mb at 12 GeV/c, somewhat smaller than the right-hand side, which is 185 $\pm$ 5 mb at 12 GeV/c and 173 $\pm$ 10 mb at 18 GeV/c. That (7.b)—(7.d) are less well satisfied than (7.a) is attributed to differences in the quark properties inside baryons and mesons.

It is well to bear in mind that in testing relations between meson-baryon and baryon-baryon amplitudes, we need to compare the collisions at the same centre-of-mass energies of the *quark-quark* systems. It is reasonable to argue that since an incident baryon carries three quarks while an incident meson carries two quarks, the *quarks* in the two situations have the same energies when the baryon incident energy is roughly three-halves of the meson incident energy. It is interesting and probably significant that, for all relations of this kind derived from the quark model, the agreement with experiment is improved when baryon and meson momenta are chosen to be in a 3-to-2 ratio rather than in a 1-to-1 ratio [24].

Motivated by the success of these relations, LIPKIN [25] went on to examine whether any relations among the quark-quark amplitudes could be found. For this purpose, we must decompose the proton target into its constituent *n* and *p* quarks. There are then eleven quark-quark amplitudes, of which six are independent because we are insisting on charge reflection symmetry:

$$(pp) = nn) = p',$$
$$(pn) = n',$$
$$(p\lambda) = (n\lambda) = \lambda',$$
$$(p\bar{p}) = (n\bar{n}) = \bar{p}',$$
$$(p\bar{n}) = (n\bar{p}) = \bar{n}',$$
$$(p\bar{\lambda}) = (n\bar{\lambda}) = \bar{\lambda}',$$

where we adopt a compact notation at the right. In terms of these amplitudes, the physical amplitudes become:

$$K^+ = 2\,p' + n' + 3\,\bar{\lambda}', \tag{8.a}$$
$$K^0 = p' + 2\,n' + 3\,\bar{\lambda}', \tag{8.b}$$
$$\pi^+ = 2\,p' + n' + \bar{p}' + 2\,\bar{n}', \tag{8.c}$$
$$\pi^- = p' + 2\,n' + 2\,\bar{p}' + \bar{n}' \tag{8.d}$$
$$\bar{K}^0 = 3\,\lambda' + \bar{p}' + 2\,\bar{n}', \tag{8.e}$$
$$K^- = 3\,\lambda' + 2\,\bar{p}' + \bar{n}', \tag{8.f}$$
$$P = 5\,p' + 4\,n', \tag{8.g}$$
$$N = 4\,p' + 5\,n', \tag{8.h}$$
$$\bar{P} = 5\,\bar{p}' + 4\,\bar{n}', \tag{8.i}$$
$$\bar{N} = 4\,\bar{p}' + 5\,\bar{n}'. \tag{8.j}$$

Of course, Eqs. (7.a)—(7.d) are still obtained. But suppose we use the experimental values of the total cross-sections to solve for the total quark-quark cross-sections. We begin with the meson-baryon scattering, using the experimental total cross-sections at 14 GeV/c as a typical high energy value:

$$K^+ = 17{,}4 \pm 0{,}1 \text{ mb.}$$
$$K^0 = 17{,}5 \pm 0{,}4 \text{ mb.}$$
$$\pi^+ = 23{,}9 \pm 0{,}2 \text{ mb.}$$
$$\pi^- = 25{,}4 \pm 0{,}3 \text{ mb.}$$
$$\bar{K}^0 = 20{,}1 \pm 0{,}4 \text{ mb.}$$
$$K^- = 21{,}5 \pm 0{,}2 \text{ mb.}$$

[At this energy, Eq. (7.a) happens to be satisfied exactly by the central values.]

One finds that the six quark-quark amplitudes are now expressed in terms of a common parameter. The total quark-quark cross-sections, in millibarns, with errors of about $\pm$ 0,2 mb, are:

$$p' = 3{,}6 - \alpha\,, \tag{9.a}$$

$$n' = 3{,}7 - \alpha\,, \tag{9.b}$$

$$\lambda' = 2{,}4 - \alpha\,, \tag{9.c}$$

$$\bar{p}' = 5{,}3 + \alpha\,, \tag{9.d}$$

$$\bar{n}' = 3{,}8 + \alpha\,, \tag{9.e}$$

$$\bar{\lambda}' = 2{,}1 + \alpha\,, \tag{9.f}$$

where α cannot be determined unless we make use of the data on baryon-baryon scattering.

The equality of the $p - p$ and $p - n$ total cross-sections is seen to be independent of the parameter α. LIPKIN takes this equality to mean that one is in an asymptotic region where one can neglect the effects of charge exchange in quark-quark scattering. On the other hand, the $p - \bar{p}$ and $p - \bar{n}$ cross-sections are quite different, indicating that here charge exchange is still significant. LIPKIN supposes that the fact that the asymptotic region for quark-quark scattering is at a lower energy than for quark-antiquark scattering, is a reflection of the fact that annihilation channels are open in the latter case. We might recall that at these energies one has a substantial cross-section for nucleon-antinucleon annihilation.

We notice that *if α is near zero,* we have two additional approximate equalities:

$$n' \approx \bar{n}', \tag{10.a}$$

$$\lambda' \approx \bar{\lambda}'\,. \tag{10.b}$$

Dynamically, these equalities are a statement of the POMERANCHUK theorem for $n - p$ and $\lambda - p$ quark scattering. This is another phenomenon of the asymptotic region. The fact that we do not simultaneously obtain $p' = \bar{p}'$, (i.e., the equality of the $p - p$ and $p - \bar{p}$ total cross-sections) is attributed by LIPKIN to the importance of the annihilation channel with $I = 0$.

But is α near zero? We may express the baryon-baryon total cross-sections (in millibarns) in terms of α:

$$P = 32{,}8 - 9\alpha\,, \tag{11.a}$$

$$N = 32{,}9 - 9\alpha\,, \tag{11.b}$$

$$\bar{P} = 41{,}7 + 9\alpha\,, \tag{11.c}$$

$$\bar{N} = 40{,}2 + 9\alpha\,. \tag{11.d}$$

These cross-sections are uniformly 8 to 10 mb lower than the experimental values at 14 GeV/c, and the baryon-baryon total cross-sections are decreasing sufficiently slowly that the situation is not appreciably improved by using incident baryon momenta closer to 21 GeV/c. But the four cross-sections (11.a)—(11.d) are approximately correctly spaced. If we solve for α by fitting to the experimental *difference* between PP and $P\bar{P}$ total cross-sections at 14 GeV/c, (which is 11,6 $\pm$ 1,5 mb), we obtain $\alpha = 0{,}1$; if we extrapolate to 21 GeV/c, (which is somewhat beyond the highest well-measured antiproton data), we get $\alpha = 0{,}0$.

In LIPKIN's paper [25], the argument is reversed relative to that given above. He begins by supposing $p' = n'$, in which case he obtains

$$K^{+} = K^{0} \, . \tag{12}$$

Adding the assumptions $n' = \bar{n}'$, $\lambda' = \bar{\lambda}'$, he obtains

$$\pi^{+} - \pi^{-} = K^{0} - \bar{K}^{0} \, . \tag{13}$$

The equality (12) has always been a challenge to any theory, because these total cross-sections differ by less than 3% down to incident momenta below 6 GeV/c. Equations (7.a) and (13) are the JOHNSON—TREIMAN relations [26], which here follow from some plausible dynamics. [Actually, we see that $n' - \lambda' = \bar{n}' - \bar{\lambda}'$ is sufficient to derive (13).] [27].

Note that we have not used any $SU(3)$ predictions to relate the quark-quark amplitudes. These relations, $n' = \lambda'$ and $\bar{n}' = \bar{\lambda}'$, are in fact seen to be badly violated. We use $SU(3)$ only to arrive at the quark content of the physical states, and then make use of a plausible quark dynamics in which $SU(3)$ is broken in order to go further. In the end, ten physical cross-sections are given in terms of three quark-quark cross-section (p', $\bar{p}'$, λ').

Successful relations between baryon and meson properties provide positive evidence for the existence of a simple type of hadronic quark matter. There are several other applications of the quark model which also give evidence for a common quark-stuff out of which both baryons and mesons might be composed. Of these, I will mention here the successful calculation of the electromagnetic decay rate $\omega \to \pi^{0} + \gamma$ in terms of the nucleon magnetic moment by BECCHI and MORPURGO [28] and the analysis of proton-antiproton annihilation into mesons by RUBINSTEIN [29].

VI. Towards a quark model of weak interactions

Two sorts of hypotheses about quarks have continually appeared in our discussion of applications in the previous two Sections. One is the *additivity* hypothesis, which states that the amplitude for some interaction of the strongly interacting particles is the sum of the amplitudes for processes involving the constituent quarks. The second is the idea that single-quark processes should predominate over multiple quark processes, so that a reasonable perturbation theory is provided by ordering quark effects according to the number of quarks which participate. Can either of these hypotheses help to construct a quark theory of the weak interactions? We confine ourselves to a very brief discussion of this question.

The strongly interacting particles decay leptonically and non-leptonically. The decays into leptons are known to be described by a Hamiltonian density $J_\mu{}^+ j_\mu$, where j_μ is the lepton current and J_μ is the hadron current. J_μ is one of the most exhaustively studied objects in particle physics. One has relations between its baryon and meson parts via both the conserved vector current and partially conserved axial vector current hypotheses. One has relations between the strangeness changing and strangeness conserving parts via the Cabibbo theory. One knows how to calculate many of the renormalization effects via the $SU(3) \times SU(3)$ algebra formed from its vector and axial vector components (an algebra which, in turn, was suggested by the properties of the currents constructed from free quark fields).

The quark model accounts for some of the facts of the leptonic decays which have been built into the more sophisticated theory. If the leptonic decays are to involve only a single quark, then one is restricted to the processes

$$n \to p l^- \bar{\nu},$$

$$\lambda \to p l^- \bar{\nu}$$

and the corresponding antiparticle decays into positive leptons. Thus one automatically has a $|\Delta Y| \leq 1$ law and a $\Delta Y = \Delta Q$ law for the hadronic transitions. One guesses that these processes are universal and hence is less surprised than in the absence of a quark model when one finds a common ratio of strengths of strangeness changing and strangeness conserving interactions in both meson and baryon decays. (Without further assumptions, one might not expect the equality of these ratios for vector and axial vector transitions.) All of this and more is of course built into the Cabibbo theory.

When we turn to the non-leptonic decays, we have greater hope of getting new results from the quark model, largely because a satisfactory theory is still lacking. Two paramount regularities appear in these decays, the $|\Delta Y| \leq 1$ law, which appears to be rigorously satisfied (in view of the magnitude of the $K_L - K_S$ mass difference), and the $\Delta I = \frac{1}{2}$ law, which is approximately satisfied. (The decay $K^+ \to \pi^+ \pi^0$ provides a measure of its violation, and suggests violations of 5% in the amplitude.) The first law, $|\Delta Y| \leq 1$, follows from that has generally been regarded as the most likely non-leptonic interaction Hamiltonian, $J_\mu{}^+ J_\mu$, where J_μ is the same current as in the leptonic decays. The law is rigorous if the leptonic selection rules $|\Delta Y| \leq 1$ and $\Delta Y = \Delta Q$ are rigorous. However, the second law, $\Delta I = 1/2$, does not follow in any straightforward way from the current-current interaction.

The quark model provides one additional single-quark transition

$$\lambda \rightarrow n,$$

which does not appear in the leptonic transitions because the leptonic current is charged. And the quark model all but demands that this transition appears when it has an opportunity to do so. In non-leptonic decays, the $\lambda \rightarrow n$ transition is a one-quark effect, while the current-current interaction is a two-quark effect, $(\lambda p) \rightarrow (pn)$. From what we have already seen, we expect the single-quark transition to predominate.

We are led to the following picture of the non-leptonic decays. There are *two* mechanisms by which these decays can proceed, shown pictorially below:

Both interactions preserve the $|\Delta Y| \leq 1$ law, which we then expect to be rigorous. Only the first interaction satisfies a pure $\Delta I = \frac{1}{2}$ law, which we then expect to be broken by the second interaction. The second interaction should be weaker, so the $\Delta I = \frac{1}{2}$ law should be approximately valid.

According to this picture, the current-current interaction should correctly relate only the $\Delta I = \frac{3}{2}$ parts of the non-leptonic decays. The $\Delta I = \frac{1}{2}$ parts should be dominated by the single-quark transition. There should be *no* $\Delta I = \frac{5}{2}$ transitions to lowest order, a prediction which will eventually be tested by the K decays into two pions.

It is tempting to imagine that CP violation is introduced at the quark level by a difference in phase between the one-quark and two-quark terms in the weak Hamiltonian. This would require that no CP violation be seen in any leptonic interactions or in strangeness-conserving non-leptonic interactions. It would also predict that there would be no CP violation in $\Delta I = \frac{3}{2}$ non-leptonic decays, but how this could be verified is not clear since $\Delta I = \frac{1}{2}$ decays always accompany $\Delta I = \frac{3}{2}$ decays.

Let me conclude on a still more fanciful note. There appear to be two fundamental ways in which the quark content of hadronic matter can be changed. The first is quark-antiquark pair creation and destruction; these processes allow the formation of resonances and many-particle final states in high energy scattering. The second is *quark change*, by which we mean the three fundamental processes $p \to n$, $p \to \lambda$, $n \to \lambda$. Because of strict charge conservation, the first two processes only occur in the presence of leptons. But all three processes may have a comparable, slow rate, which gives the characteristic features of the weak interactions [30].

The usual weak interaction coupling constant, which is 10^{-5} when measured in units of the proton mass, is of order unity when measured in units of a particle whose mass is 300 GeV. Far down the road which the quark model has led us along, far beyond our farthest outposts today, might there lie a calculation of the weak interaction coupling constant in terms of quark parameters?

Appendix

The baryon self-energies

We want to work out a simple calculation in some detail in order to illustrate the use of quark wave functions. For this purpose, we express the self-energy of the baryon octet and decuplet states as a sum of single terms and pairing terms, as discussed in Section IV. The pairing term requires a sum over the energies of interaction of all pairs of quarks contained in these states.

The baryons are three-quark systems, and hence for each quark configuration there are three different pairing interactions which must be summed. The baryon wave functions are linear combinations of distinct quark configurations, and we must sum over these as well. Because the wave functions are symmetrized with respect to the three constituent quarks, it is sufficient to calculate the pairing energy due to the interaction of the first and second quark, for example, and then to multiply the result by three to obtain the total interaction.

We assume isotopic spin and ordinary spin conservation. Thus, it is necessary to rewrite the wave functions in such a way that the first and second quarks are in eigenstates of spin and isospin. For example, consider the Δ^{++} state with $S_z = 3/2$:

$$| \Delta^{++} > = [p(+)\, p(+)]\, p(+)\,.$$

Here the pairing energy is three times the $p(+)\, p(+)$ interaction energy. Using the notation of [14] we call this energy D_{nn}^{11}, where the first and second upper indices refer respectively to the spin and isospin of the two-quark system. Naturally, the pairing energy is the same for the other charge and spin states of Δ. We write:

$$E(\Delta) = 3\, m_n + 3\, D_{nn}^{11}, \tag{A.1}$$

where we include in the self energy $E(\Delta)$ both the one-quark contribution (a sum over constituent quark masses) and the pairing contribution.

For the proton and Λ states with $S_z = 1/2$, we rearrange the terms of the wave function given in Section III as follows:

$$| P(+) > = (18)^{-1/2} \{2[p(+)\,p(+)]n(-) +$$

$$+ \frac{1}{2}[p(+)n(-) + p(-)n(+) + n(+)p(-) + n(-)p(+)]p(+) +$$

$$+ \frac{3}{2}[p(+)n(-) - p(-)n(+) - n(+)p(-) + n(-)p(+)]p(+) -$$

$$- [p(+)p(-) + p(-)p(+)]n(+) - [p(+)n(+) + n(+)p(+)]p(-)\},$$

$$| \Lambda(+) > = (12)^{-1/2} \{[p(+)n(-) - p(-)n(+) + n(-)p(+) -$$

$$- n(+)p(-)]\lambda(+) + [\lambda(+)p(+)]n(-) - [\lambda(+)n(+)]p(-) -$$

$$- [n(+)\lambda(+)]p(-) + [p(+)\lambda(+)]n(-) +$$

$$+ \frac{1}{2}[n(-)\lambda(+) + n(+)\lambda(-)]p(+) + \frac{1}{2}\,[n(-)\lambda(+) -$$

$$- n(+)\lambda(-)]p(+) + \frac{1}{2}[\lambda(+)n(-) + \lambda(-)n(+)]p(+) +$$

$$+ \frac{1}{2}[\lambda(+)n(-) - \lambda(-)n(+)]p(+) - \tfrac{1}{2}[\lambda(+)p(-) +$$

$$+ \lambda(-)p(+)]n(+) + \frac{1}{2}[\lambda(+)p(-) - \lambda(-)p(+)]n(+) -$$

$$- \frac{1}{2}[p(-)\lambda(+) + p(+)\lambda(-)]n(+) + \frac{1}{2}[p(-)\lambda(+) -$$

$$- p(+)\lambda(-)]n(+)\,.$$

The expressions in brackets now have definite spin and isospin, so we may write down the self-energy by inspection:

$$E\,(P) = 3m_n + 3 \times \frac{1}{18}\,[4D^{11}_{nn} + D^{11}_{nn} + 9D^{00}_{nn} + 2D^{11}_{nn} + 2D^{11}_{nn}] =$$

$$= 3m_n + \frac{3}{2}\,D^{11}_{nn} + \frac{3}{2}\,D^{00}_{nn}\,, \qquad \text{(A.2)}$$

$$E\,(\Lambda) = 3m_n + m_\lambda + 3 \times \frac{1}{12}\,[4D^{00}_{nn} + \mathrm{u}D^{1\,1/2}_{n\lambda} + 2D^{1\,1/2}_{n\lambda} + 2D^{0\,1/2}_{n\lambda}] =$$

$$= 2m_n + m_\lambda + D^{00}_{nn} + \frac{3}{2}\,D^{1\,1/2}_{n\lambda} + \frac{1}{2}\,D^{0\,1/2}_{n\lambda}\,. \qquad \text{(A.3)}$$

The other self-energies are found in exactly the same way, and we simply state the results:

$$E(Y^*) = 2m_n + m_\lambda + D^{11}_{nn} + 2D^{1\,1/2}_{n\lambda}\,, \tag{A.4}$$

$$E(\Xi^*) = m_n + 2m_\lambda + 2D^{1\,1/2}_{n\lambda} + D^{10}_{\lambda\lambda}\,, \tag{A.5}$$

$$E(\Omega) = 3m_\lambda + 3D^{10}_{\lambda\lambda}\,, \tag{A.6}$$

$$E(\Sigma) = 2m_n + m_\lambda + D^{11}_{nn} + \frac{1}{2}D^{1\,1/2}_{n\lambda} + \frac{3}{2}D^{0\,1/2}_{n\lambda}\,, \tag{A.7}$$

$$E(\Xi) = m_n + 2m_\lambda + \frac{1}{2}D^{1\,1/2}_{n\lambda} + \frac{3}{2}D^{0\,1/2}_{n\lambda} + D^{10}_{\lambda\lambda}\,, \tag{A.8}$$

From equations (A.1) — (A.8), we may verify that we obtain equations (1.a) — (1.c) of section IV, and no other relations. If we also assume the following relation between $S = 1$ quark-quark interactions:

$$D^{11}_{nn} + D^{10}_{\lambda\lambda} = 2D^{1\,1/2}_{n\lambda}\,, \tag{A.9}$$

then we get the full equal-spacing law for the decuplet and the GELL-MANN—OKUBO formula for the octet. Equation (A.9) is a weaker assumption than the assumption that all quark-quark pairing energies are equal. Using the stronger assumption:

$$D^{11}_{nn} = D^{1\,1/2}_{n\lambda} = D^{10}_{\lambda\lambda} \tag{A.10}$$

does not lead to any further relations between the eight baryon masses. However, (A.10) suggests the further assumption (see [15]) that the $S = 1$ quark-quark *and* quark-antiquark pairing energies are equal. Under the additional very reasonable assumption that the Φ is a $\lambda\bar{\lambda}$ state, this yields an interesting relation between meson and baryon mass differences, namely equation (2) of Section IV.

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НОВЫЕ РЕЗУЛЬТАТЫ ИЗ МОДЕЛИ КВАРКОВ

Р. Х. СОКОЛОВ

Резюме

Излагаются основные идеи и методы кварковой модели элементарных частиц. Дается обзор, и частью подробное обсуждение главных результатов, включая новое применение модели к рассеянию при высоких энергиях. Кратко рассмотрены также кварковые модели слабых взаимодействий.

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THE BARYON MODEL AS A TETRAHEDRON OF QUARKS AND ONE ANTIQUARK

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A baryon model consisting of 4 quarks and one antiquark is proposed.

The idea of quarks has given a clear physical picture underlying the $SU(3)$ and $SU(6)$ symmetries. The lowest energetic states of mesons have the structure q, $\bar{q}$ in s-state with total spin 0 or 1. Concerning the baryons one usually assumes that they are built up from $3q$. The lowest energetic states (octet + decuplet = 56 in $SU(6)$) have wave functions totally symmetric in spin and unitary spin. From the Pauli principle then follows that the space wave function of three quarks must be totally antisymmetric. It is tempting to assume that there are attractive forces between q and $\bar{q}$, and repulsive ones between two quarks. However, in that case it cannot be understood, how three quarks can form a baryon; new, more or less artificial assumptions are necessary, like the hypothesis of attraction between two quarks at great distances and repulsion at small ones, or that concerning three-body forces [1,] [2].

We propose a baryon model consisting of 4 quarks and one antiquark [3]. The antibaryon gives a common attractive field, in which the quarks are moving. The lowest one-particle states in this field are an s-state and a three times degenerate p-wave. The quarks prefer to be on different energetic levels because of the repulsion between them. (The Pauli principle would not give this result at all, as the quarks have three charge states and two spin states. On the contrary, the Pauli principle leads to the considered spin-unitary spin structure only, if the dynamics give the desribed space wave function.) The configuration with $1q$ on the s-level corresponds to the meson, while the sp^3 configuration gives the structure $4q$, $\bar{q}$, i.e. the baryon. Thus one gets automatically $(4 - 1)\, q = 3q$ for the baryon, that means, the quark has $B = \frac{1}{3}$.

Our model allows an interesting geometric interpretation. Namely, from the s and p-functions one can build — like in organic chemistry — linear combinations, so called σ-functions, which have preferred axes directed to the vertices of a tetrahedron. So we may consider the baryon as a tetrahedron with quarks in each vertex, and the antiquark is moving symmetrically around them. The privileged character of the number 4 is connected with the fact that the tetrahedron is the simplest regular figure in our three-dimensional space. So we prefer PYTHAGORAS' hellenistic ideas instead of the eightfold way of BUDDHA.

The difficulties of the model are the following.

1. The quarks should be placed in the vertices of the tetrahedron with the maximal probability. Besides that, the wave function has to change sign (and not only go through the minimum) when in the process of interchanging two quarks the system passes through a configuration, in which all the four quarks are in the same plane. It is possible that for that purpose the repulsion between two quarks is not sufficient, and we have to consider different effective interaction potentials for the symmetric and antisymmetric cases [2].

2. As it was pointed out by R. SOCOLOW, the additivity principle for total cross sections at high energies [7, 8] gives in our model $\frac{2}{5}$ for the ratio of the meson-baryon cross section to the baryon-baryon or antibaryon-antibaryon cross section. The ratio $\frac{2}{3}$, which is given by the usual $3q$ baryon model, fits much better the experiment.

The best proof for the $4q$, $\bar{q}$ model would be the discovery of baryon resonances with electric charge $Q = +3$ or $Q = -2$ (and strangeness $S = 0$) or resonances with strangeness $S = +1$ or $S = -4$. Resonances with an unusual strangeness have not been discovered so far. The baryon resonance N^{+++} (1580 MeV) which is decaying into $p + \pi^+ + \pi^+$ is mentioned in the literature [5, 6], but it is not clear, whether it is a resonance or an interference effect of the π^+ waves in the simultaneous decays of Δ^{++} and ϱ^+.

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БАРИОН КАК ТЕТРАЭДР ИЗ КВАРКОВ И ОДИН АНТИКВАРК

Я. Б. ЗЕЛЬДОВИЧ и А. Д. САХАРОВ

Резюме

Рассматривается модель бариона, состоящего из 4 кварков и одного антикварка.

Acta Physica Academiae Scientiarum Hungaricae, Tomus 22 (1—4), pp. 153—157 (1967)

MASS FORMULAS FOR MESONS AND BARYONS IN THE QUARK MODEL

By

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A unified mass formula is given, linear both for mesons and baryons. The spin-spin interaction and the annihilation interaction of strange quark pairs is proposed to be much weaker than that of non-strange quark pairs.

For the charged pseudoscalar and vector mesons $(q, \bar{q})$ and for the 56-plet of baryons the following mass formulas of similar form are proposed:

$$m_M = a_0 + c \sum_{i=1,} |S_i| + c_0 (\vec{\mu}_1 \vec{\mu}_2) , \tag{1}$$

$$m_B = a_1 + b \sum_{i=1,2,3} |S_i| + c_1 \sum_{i,K=1,2,3} (\vec{\mu}_i \vec{\mu}_K) . \tag{2}$$

Note that the formula (1) is written for the mass (and not for the mass square) of the meson. Here S_i is the strangeness of the i-th quark or antiquark; thus the coefficient $b = 180$ MeV (the same in both equations (1) and (2)) gives the mass difference between the λ quark and the p and n quark. The coefficients c_0 and c_1 characterize the spin-spin interaction of quarks and so lead to the mass splitting between the vector mesons and pseudoscalar mesons, and the baryonoctet and decuplet respectively.

The experimental results predict $c_0 = 620$ MeV and $c_1 = 206$ MeV. The difference is understandable, if we take into account, that in a meson the q and $\bar{q}$ are in an s-state: $\psi(r_{12} = 0) \neq 0$ while in a baryon the wave function is totally antisymmetric, i.e. $\psi(r_{12}, r_{13}, r_{23})$ is equal to zero, if any $r_{ik} = 0$.

We assume, that the spin-spin interaction of the strange λ quark is weaker than that of the p, n quarks. Therefore we write

$$\vec{\mu}_i = \vec{\sigma}_i (1 - \alpha |S_i|) ,$$

where $\vec{\sigma}_i$ is the spin of the i-th quark. The best fit with the experiment is given by the value $\alpha = 0{,}42$ for both the mesons and baryons.

The difference between the spin-spin interaction of the λ and the p, n leads to the mass splitting of Σ^0 and Λ^0, having the same quark structure

(λ, p, n). Indeed, it can be seen, that for the Σ^0

$$< \vec{\sigma}_p \vec{\sigma}_n > = \frac{1}{4}, \qquad < \vec{\sigma}_p \vec{\sigma}_\lambda > = < \vec{\sigma}_n \vec{\sigma}_\lambda > = -\frac{1}{2}$$

and for Λ^0

$$< \vec{\sigma}_p \vec{\sigma}_n > = -\frac{3}{4}, \qquad < \vec{\sigma}_p \vec{\sigma}_\lambda > = < \vec{\sigma}_n \vec{\sigma}_\lambda > = 0.$$

In the case of baryons the only difference between the formula (2) and that of GELL-MANN—OKUBO is that the coefficient of $\langle \vec{\sigma}_\lambda \vec{\sigma}_\lambda \rangle$ is $(1 - \alpha)^2$ instead of $(1 - 2\alpha)$. The comparison of the theoretical and experimental results is given in the following Table for the values $a_0 = 608$, $a_1 = 1083$, $b = 180$, $c_0 = 620$, $c_1 = 206$, $\alpha = 0{,}42$:

Table I

particle	π	K	ϱ	K^*	ω	φ		
$\text{mass}_{\text{theory}}$ (MeV)	143	500	763	878	763	1020		
mass_{exp} (MeV)	137	494	750	890	780	1020		
particle	N	Λ	Σ	Ξ	Δ	Y^*	Ξ^*	Ω^-
$\text{mass}_{\text{theory}}$ (MeV)	928	1108	1195	1340	1238	1375	1520	1675
mass_{exp} (MeV)	939	1115	1193	1317	1238	1385	1530	1675

Let us consider now the neutral mesons, the quark structure of which is the superposition of $p\bar{p}$, $n\bar{n}$ and $\lambda\bar{\lambda}$. It is useful to recall the analogy of the well understood systems consisting of charged particles like positronium $e^+ e^-$, muonium $\mu^+ e^-$ and antimuonium $\mu^- e^+$ or $\mu^+ \mu^-$.

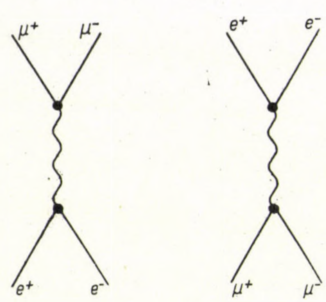

Fig. 1

In the system $\mu^+ e^-$ the Coulomb interaction between μ^+ and e^- plays the fundamental role (see Fig. 1). This diagram contains also the magnetic interaction of the spins of μ and e, which leads to the splitting of the $^1S_0$ and $^3S_1$ muonium states.

In the case of positronium we have to take into account the annihilation diagram of the $e^+ e^-$ pair (Fig. 2). It is well known that this process is possible only for the orthopositronium $^3S_1$, as the electromagnetic field is a vector field. The annihilation process gives a correction to the energy of the positronium, which is measured and is in excellent agreement with the theoretical result.

Fig. 2

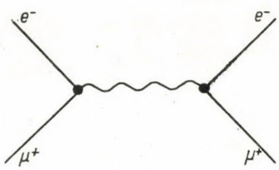

Fig. 3

Assuming the existence of $\mu^+ \mu^-$ pairs we have to consider diagrams like in Fig. 3.

We can conclude that the ortho $e^+ e^-$ and ortho $\mu^+ \mu^-$ are mixed in the superposition. (Here we neglect the fact that the mass difference of $\mu^+ \mu^-$ and $e^+ e^-$ is more than the binding energy of positronium.) On the other hand, para $\mu^+ \mu^-$ and para $e^+ e^-$ are not mixed, because in electrodynamics there are no diagrams with a pseudoscalar neutral intermediate state. $\left(\text{We do not consider diagrams higher than first order in } \frac{e^2}{\hbar c}\right)$.

On the contrary, for the neutral mesons i.e. for pairs

$$p\bar{p} = r_1', \qquad n\bar{n} = r_2, \qquad \lambda\bar{\lambda} = r_3 .$$

The experimental results show that in the case of vector mesons (the analogue of ortho-positronium, spin 1) there are no annihilation processes. Thus r_3, i.e. the φ meson is an eigenstate; this can be proved by the fact that φ decays mostly into K and $\bar{K}$.

The r_1 and r_2 states are degenerate, and a very weak interaction (which must not be taken into account for r_3 and r_{12}) is enough to split $(r_1 - r_2)/\sqrt{2} = \varrho^0$ and $(r_1 + r_2)/\sqrt{2} = \omega$. The mass splitting between ϱ^0 and ω is small. As in the system of ϱ^0, ω and φ there is no considerable annihilation, we give the masses of the neutral mesons in Table I together with the charged meson masses.

Let us consider now the pseudoscalar neutral mesons. The annihilation term in the Hamiltonian is

$$H_{ik} r_i^+ r_k ,$$

where r_i^+ is the production operator of the i-th pair; r_k is the annihilation operator of the k-th pair.

The other interaction terms in the Hamiltonian are of the type of H_{11}; in that case $r_1 = p\bar{p}$ goes into $r_1 = p\bar{p}$ again. Such a process is clearly possible for charged pairs as $p\bar{n}$, which are not contained in r_1, $r_2 r_3$, . Thus we can say, that the non-diagonal terms in H_{ik} are characteristic for annihilation processes.

The unitary symmetry group $SU(3)$ is in agreement only with such a Hamiltonian $H_{ik} = g$, which does not depend on any indices, e.g.

$$H = \begin{vmatrix} g & g & g \\ g & g & g \\ g & g & g \end{vmatrix} .$$

The usual interaction

$$H_{ik} = f \delta_{ik}$$

is allowed only, if there is such a term f in the energy of charged pairs like $p\bar{n}$ also. In that case we have an $SU(3)$ singlet:

$$x = \frac{r_1 + r_2 + r_3}{\sqrt{3}} , \qquad m = f + 3g$$

and two degenerate states

$$\pi^0 = \frac{r_1 - r_2}{\sqrt{2}} \qquad \text{and} \qquad \eta = \frac{r_1 - r_2}{\sqrt{6}} - \sqrt{\frac{2}{3}}\, r_3 , \qquad m = f .$$

As the annihilation interactions exist for pseudoscalar mesons (and not for vector mesons) they are in agreement not only with $SU(3)$ but with $SU(6)$ also.

Finally let us consider the $SU(3)$ symmetry breaking (isospin is conserved) caused by the mass difference between λ and p, n. We will assume, that r_3 is always multiplied by $(1 - \beta)$, so

$$H_{ik} = g \begin{vmatrix} 1 & 1 & (1-\beta) \\ 1 & 1 & (1-\beta) \\ (1-\beta) & (1-\beta) & (1-\beta)^2 \end{vmatrix} .$$

Considering the usual interaction, in which (in the absence of annihilation) the different mass and spin interaction of λ is taken into account (see (1)), we get a secular equation for the neutral pseudoscalar mesons. It gives

$$\pi^0 = \frac{r_1 - r_2}{\sqrt{2}} . \qquad m_{\pi^0} = m_{\pi^\pm} ,$$

$$\eta = 0{,}54\, r_1 + 0{,}54\, r_2 - 0{,}65\, r_3 ,$$

$$X = 0{,}46\, r_1 + 0{,}46\, r_2 + 0{,}76\, r_3 .$$

To fit the experimental values $m_\eta = 548$ MeV and $m_X = 958$ MeV we take

$$g = 580 \text{ MeV} , \qquad \beta = 0{,}75 .$$

That means that not only the spin-spin interaction, but especially the annihilation interaction of $r_3 = \lambda\bar{\lambda}$ is much weaker than that of the non-strange quark pairs. This result is in good agreement with the smaller cross section of λ scattering and smaller probability of strange particle production in the additivity theory of scattering (see the lecture of R. H. SOCOLOW at the Summer School at Lake Balaton).

Note that the sign of the annihilation interaction is the same as for the orthopositronium i.e. it is as if there would exist an intermediate pseudoscalar boson with zero mass. As such a boson does not seem to exist, we have to assume that the annihilation processes correspond to simple four-fermion diagrams.

Most of the ideas in this work were already mentioned by other authors. The list of references is given in [1].

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МАССОВЫЕ ФОРМУЛЫ ДЛЯ МЕЗОНОВ И БАРИОНОВ В КВАРКОВОЙ МОДЕЛИ

Я. Б. ЗЕЛЬДОВИЧ и А. Д. САХАРОВ

Резюме

Построена единая массовая формула, линейная в массах мезонов и барионов. Спин-спиновое и аннигиляционное взаимодействие с участием странных кварков предполагается ослабленным в одинаковой степени для мезонов и барионов.

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A RELATIVISTIC QUARK MODEL WITH APPLICATION TO MESON DECAY RATES

By

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The pseudoscalar and vector mesons are assumed to be quark-antiquark bound states. By using the BETHE—SALPETER equation, the wave functions of mesons are determined and, in turn, these wave functions are applied to calculate several meson decay rates. For the $\varrho \to 2\pi$ decay we get $\Gamma = 200$ MeV which is roughly in agreement with the experimental value.

At this Conference we have heard several reports on the successes and failures of the static quark model. I should like to report now on some of the consequences of a relativistic quark model of the hadrons. The work is motivated by the probable large binding energy of quarks in hadrons and the consequent difficulties of achieving non-relativistic quark motion in a dynamical model. We shall assume that the hadrons are described by appropriate bound state solutions to the BETHE—SALPETER equation with quarks taken as the constituent particles. Because of the difficulties associated with the three-body problem, our attention here will be restricted to the mesons which will be assumed to be quark, antiquark bound states.

The BETHE—SALPETER wave function equation [1] for a fermion-antifermion bound state reads

$$S_{F,1}^{-1}(P,p)\,\chi(P,p)\,S_{F,2}^{-1}(P,p) = \int d^4 q\, I(P,p,q)\,\chi(P,q)\,. \tag{1}$$

where χ is the Bethe—Salpeter wave function which is, in general, a 4×4 matrix, S_F is the quark propagator, and I is the interaction kernel.

The variables P and p, q refer to the sum and difference respectively of the quark four momenta. We shall assume the ladder approximation for the interaction kernel and take the binding mechanism to be that of meson exchange.

In order to simplify the dynamics and allow us to obtain exact solutions readily, we shall set the mass of the exchanged meson and the total centre-of-mass four momentum of the bound state equal to zero. This may be a reasonable approximation if the quark mass is sufficiently large, in which case our solutions may represent the first term of an expansion of the full wave function in powers of the parameter $M_{\text{meson}}/M_{\text{quark}}$. We refer the reader to [2] for a more complete discussion of this point. The solutions of Eq. (1) are still not uniquely determined and therefore we impose two additional conditions of the wave function.

First we assume a self-consistency or bootstrap condition which states that the exchanged meson is identical to the bound state. This condition implies that the coupling constant, g, which appears in the interaction kernel in the form (neglecting spin complications)

$$I(P, p, q) = \frac{i g^2}{(2\pi)^4} \frac{1}{(p-q)^2 - \mu^2} \tag{2}$$

is equal to the effective coupling constant for the bound state to annihilate into a quark, antiquark pair. Using the relation between the vertex function and the BETHE—SALPETER wave function, we can formulate this condition by writing

$$g = (2\pi)^2 S_{F,1}^{-1}(P, p)\, \chi(P, p)\, S_{F,2}^{-1}(P, p), \tag{3}$$

where the right-hand side is evaluated for momentum variables corresponding to on-shell quark and antiquark.

Secondly, we impose a normalization condition on the bound state wave function which has been derived by many authors [3] and reads

$$\begin{aligned} Tr \Bigg\{ \int d^4 p\, \bar{\chi}(P, p) \frac{\partial}{\partial P_\mu} \left[S_{F,1}^{-1}(P, p)\, S_{F,2}^{-1}(P, p) \right] \chi(P, p) \\ - \iint d^4 p\, d^4 q\, \bar{\chi}(P, p) \frac{\partial}{\partial p_\mu} \left[I(P, p, q) \right] \chi(P, q) \Bigg\} = 2 i P_\mu . \end{aligned} \tag{4}$$

We shall omit here the minor complications which arise in writing the multichannel generalizations of Eqs. (1), (2), (3), (4) appropriate to interacting SU_3 multiplets of particles and simply state the results; further details can be found in [2].

For the pseudoscalar mesons we obtain the wave function

$$\chi_{ps} = c_1 \gamma_5 F(3/2, 3/2; 3/2; 2; p^2/m_q^2), \tag{5}$$

where F is a hypergeometric function, and m_q is the quark mass, while for the vector mesons, with polarization vector ε^r, we obtain

$$\chi_V^r = c_2\, p \cdot \varepsilon^r \left(\frac{1}{p^2 (p^2 - m_q^2)} - \frac{\ln \left| \frac{p^2 - m_q^2}{m_q^2} \right|}{p^4} \right). \tag{6}$$

c_1 and c_2 are determined from Eq. (4). We shall now apply these wave functions to the calculation of several meson decay rates.

We consider first the $\varrho \to 2\pi$ decay rate, or, in general, the rate for $V \to PP$. The simplest dynamical mechanism to assume for this process is that shown in Fig. 1. The shaded blobs refer to the wave functions for the vector

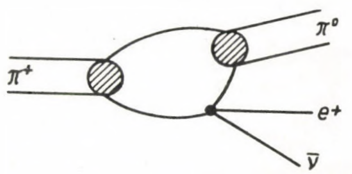

Fig. 1

and pseudoscalar particles and the three internal quark lines will be described by free particle propagators. The integration around the loop is convergent because of the fall-off of the wave functions at infinity in momentum space and the result, for the ϱ meson width, is

$$\Gamma_{\varrho\to 2\pi} = 200 + O\left(\frac{m_{\text{meson}}}{m_{\text{quark}}}\right) \qquad \text{MeV} . \tag{7}$$

The first term in this expression is independent of any adjustable parameter such as the quark mass or a cut-off constant and depends only on the ϱ and π masses and, of course, the dynamical assumptions discussed above. The rough agreement with the experimental value $\Gamma_\varrho \sim 125$ MeV is satisfying.

We can also calculate the decay rates $\pi^+ \to \mu\bar{\nu}$, $\pi^+ \to \pi^0 e\bar{\nu}$ in our model. We assume for the weak Hamiltonian an expression of the form

$$H_w = [G\,\bar{q}_n \cdot \gamma_\mu (1 - b\,\gamma_5)\, q_p + G'\,\bar{q}_\lambda\, \gamma_\mu (1 - b'\,\gamma_5)\, q_p]\, j^l_\mu, \tag{8}$$

where b and b' measure the axial vector renormalization of the $\Delta S = 0$ and $\Delta S = 1$ hadronic currents, respectively and j^l_μ is the leptonic current. The processes illustrated in Figs. 2 and 3 will be assumed to dominate the decay rates. It is most convenient to compare the calculated ratio $\pi^+ \to \pi^0\, e\, \bar{\nu}/\pi^+ \to \mu\nu$ with experiment. We then find agreement with the experimental ratio of 10^{-8} if the condition

$$m_q\, b^2 = 80 \text{ BeV} \tag{9}$$

is satisfied. The parameter b is expected to be of the order of magnitude of 1, and hence the quark mass is predicted to be of the order of magnitude of $100\times$

the nucleon mass. The calculation of other mesonic decay rates is in progress.

We wish to emphasize, in conclusion, that the description of the mesons that we have attempted here cannot be considered to be complete even if the exact solutions of Eq. (1) could be obtained with physical mass parameters.

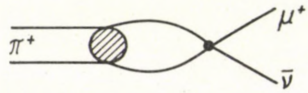

Fig. 2

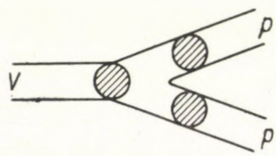

Fig. 3

The reason is that because of the singular nature of the meson exchange force and the assumed large quark mass, our wave functions describe highly condensed or localized mesons. Thus we are describing, at most, the meson core. If the "radius" of the physical meson is of the order of $1/M_{\text{meson}}$, it may be necessary to include $qq\bar{q}\bar{q}$ and perhaps more complicated contributions to the wave functions [4]. Nevertheless, it may still be valid to calculate processes involving internal pair production of quarks such as that illustrated in Fig. 1 if the production occurs primarily from the meson core. The processes illustrated in Figs. 2 and 3, however, would be expected to receive a significant contribution from the peripheral structure of the meson. It will be of interest to calculate non-leptonic hyperon decays in this model, since the simplest mechanism to assume for these processes is the pair production mechanism of Fig. 1.

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4. Another possibility for surmounting this difficulty and also to avoid additional unpleasant features of the marginally singular Bethe—Salpeter equation would be to introduce a core in the binding force via appropriate form factors in the exchanged meson vertices.

РЕЛЯТИВИСТСКАЯ МОДЕЛЬ КВАРКОВ И ЕЕ ПРИМЕНЕНИЕ К РАСПАДАМ МЕЗОНОВ

ДЖ. ХАРТ

Резюме

Предполагается, что псевдоскалярные и векторные мезоны являются связанными состояниями кварк-антикварк. С помощью уравнения Бете—Сольпитера определяются волновые функции мезонов, и потом эти волновые функции применяются для вычисления вероятностей некоторых мезонных распадов. Для распада $\varrho \to 2\pi$ получается $\Gamma = 200$ Мэв, это приближенно соответствует экспериментальному значению.

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ON THE CROSS SECTION OF QUARK PRODUCTION

By

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It is shown that in certain circumstances the cross section of quark production may be much larger than that predicted on the basis of the statistical model.

From papers of FAINBERG et al. and DOMOKOS and FULTON it follows that the cross-section of quark production is very small. On the basis of the statistical model they predict

$$\sigma \sim e^{-\frac{2M}{T}}, \tag{1}$$

where M is the quark mass, and T is constant.

If that is true, then the search for quarks with large mass will be hopeless even at cosmic energies.

I should like to make a short remark and emphasize that prediction (1) may be wrong.

In the statistical model a compound system is supposed to be produced. In such a system the statistical weight of mesonic states increases exponentially with the energy. Thus the cross-section of quark production becomes small.

A similar consideration may be applied for instance to elastic $p-p$ scattering at large angles. Indeed, HAGEDORN and others have obtained for the cross-section of elastic $p-p$ scattering the following formula:

$$\frac{d\sigma}{d\omega} \sim e^{-E/T}, \tag{2}$$

where $T = 0{,}15-0{,}16$ GeV, and E is the energy in the c. m. system.

This formula describes well the energy dependence of the cross-section at an angle near 90°. On the other hand, in the range from 30° to 80° the experimental data are described by OREAR's formula:

$$\frac{d\sigma}{d\Omega} \sim e^{-p \sin \Theta / T} \simeq e^{-\sqrt{-t}/T}, \tag{3}$$

where t is the momentum transfer.

When t decreases, we have diffraction scattering with a cross-section equal to

$$\frac{d\sigma}{d\omega} \sim e^{At} . \tag{4}$$

At an energy of 30 GeV the cross-section decreases by 10 orders.

It must be stressed that this dependence can be observed not only in elastic scattering, but in some inelastic processes too, e.g.

$$p + p \to D + \pi^+ ,$$
$$p + p \to N + p .$$

Suppose now that we have some reactions in which quarks can be produced, for instance:

$$\pi + p \to p + q + \tilde{q},$$
$$N + N \to N + N + q + \tilde{q},$$
$$N + \tilde{N} \to q + \tilde{q}$$

and that the energy is much larger than the mass of the quarks.

Then the momentum transfer is

$$\sqrt{-t} \sim (2Mq)^2/2E = \frac{2M_T^2 q^2}{E} .$$

At sufficiently large energies the momentum transfer can be small, and we shall be in OREAR's region. If OREAR's formula is valid, then the cross-section of quark production may be much larger than that predicted by the statistical model with the energy in the c. m. system.

ОБ ЭФФЕКТИВНОМ СЕЧЕНИИ РОЖДЕНИЯ КВАРКОВ

С. С. ГЕРШТЕЙН

Резюме

Показано, что при некоторых обстоятельствах эффективное сечение рождения кварков может быть намного больше того, что предсказывается статистической моделью.

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THE STATIC QUARK MODEL

By

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The radiative decay widths for pseudoscalar and vector mesons are calculated in the static quark model.

1. Introduction

The following review of the static quark model could perhaps be characterized as "an outsider's view" on this subject because the author will not show any particular eagerness in defending some of the rather drastic assumptions entering the model. Rather, aim is made at showing the coherent picture of many different physical processes and experimental facts which this model provides with only very few adjustable parameters.

Since this is a conference on weak interactions, it will not be assumed that everybody is familiar with the concept of quarks; a short-hand introduction has to suffice, however, to leave ample time for results and predictions.

The idea that mesons could be actually bound states of a fermion—antifermion system is rather old and goes back to E. Fermi and C. N. Yang [1]. Later on, one tried to reduce the number of "fundamental particles" even further; in the Sakata model, it was assumed that all elementary particles are formed as bound states of only 3 fermions, the 2 nucleons and the Λ-hyperon which can then be grouped in a triplet

$$S = \begin{pmatrix} P \\ N \\ \Lambda \end{pmatrix} . \tag{1}$$

The SU_3-symmetric version of the Sakata model [2] was eventually ruled out by experiments. (It predicts, for example, odd $\Sigma - \Lambda$ parity.) But its formal beauty inspired people not to discard it altogether.

The 2 diagonal generators of SU_3 will be defined by

$$H_1 = \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}, \qquad H_2 = \frac{1}{6} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} . \tag{2}$$

In the SU_3-symmetric version of the SAKATA model, they are related to the 3rd component of isospin and to hypercharge by

$$I_3 = \sqrt{3}\, H_1 , \qquad Y = 2(H_2 + h/3) , \tag{3}$$

where h is some quantum number equal to zero for the representations *8*, *10*, *27*, ... which we now know to be relevant for the classification of elementary particles. However, $h = 1$ for the triplet representation of SU_3. The charge is then given by the generalized GELL-MANN—NIJISHIMA formula

$$Q = \sqrt{3}\, H_1 + H_2 + h/3 . \tag{4}$$

Just as hypercharge has to be added to I_3 to keep the charge integer also for the fundamental representation of SU_2, one here has to add the new quantum number h. In nature there is, however, no indication of such a quantum number. This lead GELL-MANN and ZWEIG [2] to define $h \equiv 0$ by force and hence to arrive at a new triplet, called "quarks" or "aces",

$$q = \begin{pmatrix} p \\ n \\ \lambda \end{pmatrix} \tag{5}$$

with quantum numbers shown in Table I.

Table I

Quantum numbers of the fundamental particles

	Q	Y	N	S	I	I_3
p	2/3	1/3	1/3	0	1/2	1/2
n	—1/3	1/3	1/3	0	1/2	—1/2
λ	—1/3	—2/3	1/3	—1	0	0

All bosons are assumed to be bound states in the quark anti-quark system, whereas fermions should be made up of 3 quarks.

The static quark model goes one step further and assumes that the motion of the quarks in the potential that binds them together is non-relativistic. They are assumed to be bound in S-states. The present lower limit on the mass of the quarks is around 5 GeV, so that the binding energy per particle is of the same order as the rest energy. It follows that the potential can not satisfy the virial theorem, otherwise the average kinetic energy would be of the same order and the motion would be relativistic. Discussions of these problems are found in the literature [4, 5].

The question immediately arises why only quark-antiquark and 3-quark bound states are found in nature. According to rough arguments, given by MORPURGO [4], 2-quark bound states for instance could in fact exist, but their mass would be of the same order as the quark mass.

2. "Classical" predictions of the static quark model

The extremely simplified model described in the introduction turns out to be rather powerful in its predictions. Most of them coincide with predictions, earlier derived from SU_6. This is not at all surprising; if SU_6 has any range of application at all, the static quark model is the most natural candidate. All one has to assume is that the binding forces are approximately independent of spin and unitary spin. Since there are 6 different quark states, 3 with spin up and 3 with spin down, one predicts 36 different boson states which group into 35 + 1 according to the representations of SU_6. Likewise, one has 56 baryon states. The decomposition of these states in terms of quark states is given in [6]. Here, we only list a few characteristic examples in an obvious notation

$$\begin{aligned} &\varrho^+\uparrow = \underset{\bar{n}}{\uparrow}\,\underset{p}{\uparrow}\,, \qquad \pi^+ = \frac{1}{\sqrt{2}}\left(\underset{\bar{n}}{\uparrow}\,\underset{p}{\downarrow} - \underset{\bar{n}}{\downarrow}\,\underset{p}{\uparrow}\right), \\ &K^{*+}\uparrow = \underset{\bar{\lambda}}{\uparrow}\,\underset{p}{\uparrow}\,, \qquad K^+ = \frac{1}{\sqrt{2}}\left(\underset{\bar{\lambda}}{\uparrow}\,\underset{p}{\downarrow} - \underset{\bar{\lambda}}{\downarrow}\,\underset{p}{\uparrow}\right), \\ &P\uparrow = \frac{1}{\sqrt{18}}\left(2\underset{p}{\uparrow}\,\underset{n}{\downarrow}\,\underset{p}{\uparrow} + 2\underset{p}{\uparrow}\,\underset{p}{\uparrow}\,\underset{n}{\downarrow} + 2\underset{n}{\downarrow}\,\underset{p}{\uparrow}\,\underset{p}{\uparrow} - \underset{p}{\uparrow}\,\underset{p}{\downarrow}\,\underset{n}{\uparrow} - \underset{p}{\uparrow}\,\underset{n}{\uparrow}\,\underset{p}{\downarrow} - \right. \\ &\qquad \left. - \underset{p}{\downarrow}\,\underset{n}{\uparrow}\,\underset{p}{\uparrow} - \underset{n}{\uparrow}\,\underset{p}{\downarrow}\,\underset{p}{\uparrow} - \underset{n}{\uparrow}\,\underset{p}{\uparrow}\,\underset{p}{\downarrow} - \underset{p}{\downarrow}\,\underset{p}{\uparrow}\,\underset{n}{\uparrow}\right). \end{aligned} \tag{6}$$

The neutron state with spin up is obtained from the negative of $P\uparrow$ by interchanging $p \leftrightarrow n$. In a more closed form, the nucleon states can be written as (suppressing space-dependence)

$$\begin{aligned} P\uparrow &= \frac{1}{\sqrt{18}} S\,\alpha_1(\alpha_2\beta_3 - \alpha_3\beta_2)\,p_1(p_2 n_3 - p_3 n_2), \\ N\uparrow &= \frac{1}{\sqrt{18}} S\,\alpha_1(\alpha_3\beta_2 - \alpha_2\beta_3)\,n_1(n_2 p_3 - n_3 p_2), \end{aligned} \tag{7}$$

where

$$\sigma_3\,\alpha = \alpha\,, \qquad \sigma_3\,\beta = -\,\beta \tag{8}$$

and S is an operator symmetrizing with respect to the indices (1, 2, 3).

When the λ-quark is assumed to have a slightly higher mass, $\omega - \Phi$ mixing is predicted with a mixing angle arctan $(1/\sqrt{2}) \approx 35{,}3°$. Since η and x^0 belong to different representations of SU_6, their spatial overlap integral should vanish and no $\eta - x^0$ mixing is predicted. Both these predictions come close to experimental findings. From the decay of $\Phi \to \varrho + \pi$ [7], the $\omega - \Phi$ mixing angle is obtained to be about 40° and the masses of pseudoscalar mesons allow for an estimate of the $\eta - x^0$ mixing angle of about 10°.

Nucleon magnetic moments can be obtained in the static quark model by taking matrix elements of

$$\sum_i \mu Q_i (\sigma_3)_i \tag{9}$$

between the nucleon states (7). The sum goes over all quarks contained in the states and μQ_i is the magnetic moment of quarks with charge Q_i. Two relations are obtained [6]:

$$\mu_P = \mu \,, \tag{10}$$

$$\mu_N = -\frac{2}{3}\mu \,.$$

Elimination of μ yields the famous SU_6 result

$$\mu_P/\mu_N = -3/2 \tag{11}$$

which compares favourably with the experimental value $-1{,}46$. The result that equates the total proton magnetic moment to the quark magnetic moment is a bit surprising. Because of the large quark mass, a quark magneton is a small unit and to obtain eqs. (10), the quarks have to have an appreciable anomalous magnetic moment. The situation is improved when one takes into account that in their potential the quarks move with an effective mass that is much lower than their free mass [8].

The *axial vector coupling constant* for β-decay is obtained in the same way. One computes

$$(P \mid \sum_i (\sigma_3)_i (\tau^+)_i \mid N) = 5/3 \tag{12}$$

to obtain the SU_6-result

$$g_A/g_V = -5/3 \tag{13}$$

This value is a little bit too large. But one knows from calculations of B. W. LEE [9] that the same result is obtained from current-algebra without assuming SU_6 but an approximation where one only uses the 56-representation in the intermediate state. On the other hand, the excellent calculations of ADLER and WEISBERGER [10] have shown that the value of g_A/g_V is too high if one only takes into account the (3,3) resonance and neglects higher resonances; all this is consistent and seems to point out that one should not expect better agreement in eq. (13) from the static quark model.

3. Electromagnetic decays

It will not be possible here to give an account of all aspects of the static quark model. Electromagnetic boson decays shall be taken as a representative example [11].

There are essentially 2 kinds of electromagnetic decays: Lepton-pair decays of vector mesons and radiative decays. To investigate the first kind, one has to write down the most general form of the $V-\gamma$ vertex from invariance reasons

$$<V|j_\mu(x)|0> = \frac{1}{\sqrt{2p^\circ \Omega}} e f_{V\varphi}\, \varepsilon_\mu\, e^{ipx}. \tag{14}$$

To compute the constant $f_{V\gamma}$ from the quark model, one has to compare (14) with matrix-elements of the quark current

$$j_\mu \propto \frac{2}{3} \bar{p}\, \Gamma_\mu P - \frac{1}{3} \bar{n}\, \Gamma_\mu n - \frac{1}{3} \bar{\lambda}\, \Gamma_\mu \lambda\,, \tag{15}$$

where

$$\Gamma_\mu = e\gamma_\mu + i\,\mu'\,\sigma_{\mu\nu}\, q^\nu \tag{16}$$

and μ' is the anomalous magnetic moment of the quark in its bound state. It can therefore be obtained from the quark "effective mass" $m_V/2$ [8] by

$$\mu' = \mu_p = \frac{e}{m_V}\,. \tag{17}$$

The first result one can derive from eqs. (14) and (15) is

$$f_{\varrho\gamma} : f_{\omega_i\gamma} : f_{\varphi_i\gamma} = 3 : 1 : -\sqrt{2}\, c\,, \tag{18}$$

where ω_i and Φ_i refer to the "ideally mixed" particles with mixing angle arctan $(1/\sqrt{2})$. With $c = 1$, this is the SU_3-result which has to be obtained as SU_3 is, of course, built into the model. However, symmetry-breaking effects are included in a natural way because in eq. (17), the physical masses are inserted. This leads to $c = 0{,}58$ due to the different mass of the Φ. Comparing dimensionless constants, one has

$$\frac{f_{\omega_i\gamma}}{m_\omega^2} = \frac{f_{\varphi_i\gamma}}{m_\varphi^2} \cdot 1{,}7. \tag{19}$$

In principle, eq. (19) can be checked against measurements of the isoscalar nucleon form factors; these measurements are still somewhat uncertain, but they agree qualitatively [12].

A parameterless computation of $f_{V\gamma}$ is unfortunately impossible, but these constants can be related to some sort of radius r of the vector mesons. This radius depends, of course, on the precise form of the wave function, but a good compromise leads to

$$f_{\omega;\gamma} = \sqrt{\frac{2m_\omega}{9r^3}} \cdot (1 + \mu'_\omega\, m_\omega). \tag{20}$$

The bracket in (20) contributes a correction due to the anomalous magnetic moment of 0,8. The width for lepton-pair decay is related to $f_{V\gamma}$ by the standard formula [13]

$$\Gamma_{V\to l^+l^-} = m_v \cdot \frac{4\pi\,\alpha^2}{3} \cdot \left(\frac{f_{V\gamma}}{m_v^2}\right)^2 + 0\left(\frac{m_l^4}{m_v^4}\right). \tag{21}$$

Experimental data and the resulting coupling constants are shown in Table II.

Table II

Vector meson decays into lepton-pairs

V	$l^+ l^-$	$\Gamma_{V\to l^+l^-}/\Gamma_V$	Ref.	$f_{V\gamma}/m_V^2$
ω	$e^+ e^-$	2.10^{-4}	[14]	1/8,2
ω	$e^+ e^-$	1.10^{-4}	[7]	1/11,7
ϱ°	$e^+ e^-$	$0{,}65 \cdot 10^{-4}$	[7]	3/13,6
ϱ°	$\mu^+ \mu^-$	$0{,}3 \cdot 10^{-4}$	[15]	3/20

Relying on [15] one obtains from eq. (20) $r = 0{,}97 \cdot 10^{-13}$ cm, a rather reasonable value.

Turning to radiative decays, it should be pointed out that the $PV\gamma$ vertex can be obtained without adjustable parameters. Its relativistic form is

$$L_{VP\gamma} = 2\mu_{VP}\,(P\varepsilon^{\mu\nu\sigma\lambda}\,V_{\sigma,\lambda})_{,\nu}\,A_\mu, \tag{22}$$

where the transition moment μ_{VP} is defined by

$$\mu_{VP} = (P\,|\,\mu_z\,|\,V_j j_z = 0). \tag{23}$$

Its values as obtained from the static quark model are listed in Table III.

Table III

Transition moments in units of proton moments

	η	X°	π°
ϱ	$-1/\sqrt{3}$	$\sqrt{2}/\sqrt{3}$	1/3
ω_i	$-1/3\sqrt{3}$	$\sqrt{2}/3\sqrt{3}$	1
Φ_i	$-2\sqrt{2}/3\sqrt{3}$	$-2/3\sqrt{3}$	0

From Table III one easily obtains decay widths for $V \to P + \gamma$ and $P \to V + \gamma$. In particular [6, 11].

$$L(\omega \to \pi^\circ + \gamma) = 1{,}2 \text{ Mev}$$

in absolute agreement with experiment [7].

Turning to 2γ decays of measons now, one observes that in a nonrelativistic picture the emission of the first γ should just straighten out the two quark spins and then the quark—antiquark system can transform into the second γ. The decay widths are again obtained from a standard formula [16]

$$L_{P\to 2\gamma} = \alpha \cdot \frac{\mu_{VP}^2}{4} \cdot \left(\frac{f_{V\gamma}}{m_V^2}\right)^2 \cdot m_P^3 \,. \tag{25}$$

In this way, one obtains

$$\Gamma(\pi^\circ \to 2\gamma) = 37 \text{ ev} \tag{26}$$

which is by about a factor 4 larger than the experimental value [17]. It is interesting to observe that the value of eq. (26) can be pushed down if the radius of the pion is abnormally large. This is consistent with calculations of J. Kuti [18], who computed this decay in analogy with positronium decay and also found an abnormally large pion radius. A recent measurement [19] yielded an electromagnetic r. m. s. radius of the pion

$$r_\pi = (1{,}8 \pm 0{,}8) \times 10^{-13} \text{ cm.} \tag{27}$$

A calculation of the $\eta \to 2\gamma$ decay along the same lines gives

$$\Gamma_\eta/\Gamma_\pi = 46. \tag{28}$$

In a different model, in which the pseudoscalar meson first goes into 2 virtual vector mesons which then convert into γ-s, H. Faier [20] obtains for the same ratio a value of 6. This value is pushed to about 12 when $\eta - X^\circ$ mixing is added to his calculations, still leaving a discrepancy of a factor 4. This may just show that the above-mentioned approximation schemes are not too reliable.*

There is, however, yet another support for the assumption of a large pion radius. An analysis entirely similar to the above but with j_λ replaced by the axial vector current of weak interactions allows a computation of meson radii from the decays $P \to \mu + \nu_\mu$. One obtains

$$r_K = 1{,}1 \times 10^{-13} \text{ cm}$$

$$r_\pi = 1{,}75 \times 10^{-13} \text{ cm}. \tag{29}$$

in consistency with the above assumptions.

4. Conclusion

Our rough survey of some of the predictions of the static quark model were intended to show that there is some internal consistency in the model, in spite of the fact that the assumptions are sometimes extremely crude. It is therefore desirable to find some better understanding of this fact. The static quark model will probably not be enough for this undertaking and some extensions are desirable. About relativistic extensions we are going to hear at this conference and I think we are all eager to watch the development in this field.

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СТАТИЧЕСКАЯ МОДЕЛЬ КВАРКОВ

Х. ПИТЧМАНН

Резюме

Вычислены ширины радиационных распадов псевдоскалярных и векторных мезонов в статической модели кварков.

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MESON DECAYS IN THE STATIC QUARK MODEL II

By

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Abstract

The radiative decay $\pi^- \to \bar{e} + \widetilde{\nu} + \gamma$ is investigated in the framework of the static quark model. After having the separation of the internal bremsstrahlung one is left with two form factors to characterize the structure radiation which is extremely sensitive to the internal dynamics of the decaying hadron state. As the static quark model is a possible hadron structure we are challenged to analyse the structure radiation just to see what are the consequences of the drastic assumptions concerning the fixed particle number and slow internal motion of quarks inside the hadron state. According the *CVC* hypothesis the vector form factor is connected with the decay ratio $\pi^0 \to 2\gamma$ and the ratio of the vector and axialvector form factor is calculated explicitly. Moreover, considering the leptonic decay $\pi^- \to \mu^- + \widetilde{\nu}$ we have some information on the spatial extension of the bound state wave function and, therefore, the form factors could be calculated absolutely.

РАСПАДЫ МЕЗОНОВ В СТАТИЧЕСКОЙ МОДЕЛИ КВАРКОВ II.

Ю. КУТИ

Резюме

Рассматривается радиационный распад $\pi^- \to e^- + \tilde{\nu} + \gamma$ в статической модели кварков. Отделив внутреннее тормозное излучение, остается два формфактора, характерезующие структурное излучение, которое очень чувствительно к внутренней динамике распадающегося адронного состояния. Так как статическая модель кварков является возможной адронной структурой, следует изучать структурное излучение, чтобы увидеть, к каким следствиям ведет очень сильное предположение фиксированного числа частиц и медленного внутреннего движения кварков внутри адронного состояния. Согласно гипотезе о сохранении векторного тока, векторный формфактор связан с распадом $\pi^\circ \to 2\gamma$, и соотношение векторного и аксиальвекторного формфакторов вычислено явным образом. Более того, из лептонного распада $\pi^- \to \eta^- + \tilde{\nu}$ мы имеем некоторую информацию относительно пространственных размеров волновой функции связанного состояния, и поэтому можно точно определить формфакторы.

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SESSION 4. HIGHER SYMMETRIES

RELATIVISTIC $SU(6)$ SYMMETRIES WITH INFINITE MULTIPLETS

By

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The group $P \times SL(6, C)$ and its physical interpretation is discussed and the predictions which can be derived from the model are surveyed. In conclusion it is stressed that the theories with infinite multiplets of the type studied here encounter great difficulties connected with the violation of crossing symmetry.

1. Introduction

In static $SU(6)$ theory the spin operators are identified with the generators of the spin group $SU(2)$,

$$S_k = \frac{1}{2}\sigma_k, \quad k = 1, 2, 3.$$

Rotations of the rotational subgroup of the homogeneous Lorentz group transform these operators as a three-vector operator

$$[M_k, S_l] = i\,\varepsilon_{klm}\, S_m.$$

In relativistic formulations of $SU(6)$ symmetry the spin operators must necessarily transform as a tensor operator under the transformation of the homogeneous Lorentz group. Characteristic for the abstract group approach is the additional assumption that the spin operators should form a Lie algebra. This requirement rules out such operators as Wigner's spin operators W_μ.

The most natural choice for the spin algebra is the algebra of the well-known group $SL(2, C)$ (see [1]). Its commutation relations are

$$[s_{\mu\nu}, s_{\varkappa\lambda}] = -i\,(g_{\mu\varkappa}\, s_{\nu\lambda} + g_{\nu\lambda}\, s_{\mu\lambda} - g_{\mu\lambda}\, s_{\nu\varkappa} - g_{\nu\varkappa}\, s_{\mu\lambda}).$$

Let us first neglect the translations completely.

If we couple these generators of $SL(2, C)$ to the unitary symmetry operators λ_i by simply extending the compact part of $SL(2, C)$, $SU(2)$ to the complex domain, we obtain the group $SL(6, C)$. This group possesses the generators

$$s^t_{\mu\nu;i},\ s^s_i,\ s^p_i,$$

representing tensor operators and scalar (pseudoscalar) operators under transformations of the homogeneous Lorentz group.

Let us now assume that the dynamics is not only Lorentz invariant, but that the spin algebra also defines a symmetry.

Since we want to extend the static $SU(6)$ symmetry it is very plausible to assume that the compact subgroup $SU(6)$ of this $SL(6, C)$ group does not change the state of motion if it is applied to a particle at rest, but transforms simply states belonging to irreducible representations of $SU(6)$ into another. We take these representations to be idealized physical $SU(6)$ multiplets with degenerate mass. The algebra of $SU(6)$ is closed under transformations of the rotational part of the homogeneous Lorentz group by construction.

Having fixed the meaning of the compact part of the $SL(6, C)$ group our further arguments are straightforward. Let us assume that we have a kinematical situation with all particles moving in the direction of one spatial direction, say the third axis, with arbitrary velocities. We can create these states by applying finite transformations generated by the element M_{03} of the Lorentz algebra. Those generators of $SU(6)$ which commute with M_{03} will still operate between states of the multiplet without changing the state of motion. They form a subalgebra of $SU(6)$ as can be seen from the commutation relations. This subalgebra can be used to deduce restrictions on the S-matrix elements taken between states which are eigenstates of the momentum operator and for which the momenta are collinear in one Lorentz frame.

This subalgebra is the well-known collinear algebra spanned by

$$\frac{1}{2}(s_i^s \pm s_{12,i}^t), \qquad i = 1,2,..8, \qquad \text{and} \qquad s_{0,12}^t.$$

The group is $S(U(3) \otimes U(3))$ or $SU(3) \otimes SU(3) \otimes U(1)_{\text{helicity}}$.

For the derivation of this collinear symmetry it is therefore not necessary to know the detailed group structure. In particular, we can overlook the problems of the translation subgroup. Instead we need some heuristic arguments on particles, transformations to rest or to a definite state of motion, which have to be justified in complete group models.

There are now two essentially different groups which permit the spin-algebra to possess these transformation properties. In the first case we can assume that the $SL(2, C)$ group contained in $SL(6, C)$ coincides with the homogeneous Lorentz group. Such an ansatz leads to theories with more than four momentum components. In the second case the homogeneous Lorentz group and the $SL(2, C)$ group of $SL(6, C)$ are regarded as independent. We then obtain a total group of the structure

$$(L \times T_4) \times SL(6, C) = P \times SL(6, C), \; (SL(6, C) \text{ in invariant subgroup}),$$

as the simplest example. Models of such structure have been proposed as physically interesting $SU(6)$ generalizations by FRONSDAL, see [2].

The group L, the homogeneous Lorentz group, can be split into two parts which are both isomorphic to the homogeneous Lorentz group

$$L = L_0 + L_r, \quad M_{\mu\nu} = L_{\mu\nu} + s_{\mu\nu},$$

such that L_r is a subgroup of $SL(6, C)$. In this way we obtain another possibility of writing the group, namely as the direct product

$$(L_0 \times T_4) \otimes SL(6, C).$$

Indeed, it is trivial to check that the generators $L_{\mu\nu}$ and $s_{\mu\nu}$ commute.

2. The physical interpretation of the group $(L \times T_4) \times SL(6, C)$

If we write the group in the form

$$(L_0 \times T_4) \otimes SL(6, C), \qquad L_0 \times T_4 = P',$$

the problem of finding representations for this group is reduced to the construction of tensor products of unitary representations for the group $L_0 \times T_4$, which has the structure of the inhomogeneous Lorentz group, and of representations for the group $SL(6, C)$ which we require also to be unitary. Whereas the representations of the inhomogeneous Lorentz group are well-known to every physicist the representations of the group $SL(6, C)$ are less popular. They are all infinite dimensional. The representations of the group $L_0 \times T_4$ bring in the mass M, which is common to the complete multiplet, and a spin S', denoted orbital spin.

On the other hand the representations of $SL(6, C)$ can be reduced to an infinite direct sum of unitary representations of the compact subgroup $SU(6)$ of $SL\ (6, C)$.

In the physical picture we want to deal with state functions which depend on the momentum p_μ and belong to moving particles which form $SU(6)$ multiples with degenerate mass. Mass splitting can be introduced by additive terms in the mass operator which break the symmetry and are handled with perturbation theory. The connection between these state functions and the states belonging to the tensor product representations is quite simple in principle. For simplicity we now assume $S' = 0$ (see below).

Let τ characterize the $SL(6, C)$ representation, ν a given $SU(6)$ multiplet and ω a state of this multiplet. Then we may denote a physical state function by

$$| \nu, \omega, p > .$$

By a rotation free Lorentz transformation we bring this state to rest, the $SU(6)$ state ω is unchanged

$$| \nu, \omega, 0 > = \Lambda(p)^{-1} | \nu, \omega, p > .$$

We assume that we know by any principle the representation τ of $SL(6, C)$ which contains the $SU(6)$ representation ν as a representation of the compact subgroup. We define the connection then as

$$| \nu, \omega, 0 > = | S' = 0, 0 > | \tau, \nu, \omega > .$$

We apply on both sides the "booster" $\Lambda(p)$ but note that on the right-hand side we can split this operator into the commuting product

$$\Lambda(p) = \Lambda_0 (p) \Sigma(p)$$

with $\Lambda_0(p) \in L_0$, $\Sigma(p) \in L_r \subset SL(6, C)$.

In this fashion we obtain

$$\begin{aligned} | \nu, \omega, p > &= \Lambda(p) | \nu, \omega, 0 > \\ &= | S' = 0, p > \cdot \Sigma(p) | \tau, \nu, \omega > . \end{aligned}$$

In this manner we have fixed the interpretation of the model:

(α) If we decompose states at rest of representations for $P' \otimes SL(6, C)$ into $SU(6)$ representations, these representations are thought to be the physical $SU(6)$ multiplets. This identification is permitted because elements belonging to the compact subgroup $SU(6)$ of $SL(6, C)$transform one state ω of $SU(6)$ multiplet at rest into another state of the same multiplet ν.

The elements which lie outside the compact part $SU(6)$, as e.g. the booster $\Sigma(p)$ induced by the pure Lorentz transformation $\Lambda(p)$, connect different $SU(6)$ representations contained in the $SL(6, C)$ representation. Indeed, we may write

$$\Sigma (p) | \tau, \nu, \omega > = \sum_{\nu', \omega'} C (\nu', \omega', \nu, \omega, p) | \tau, \nu', \omega' > .$$

A multiplet in motion contributes therefore to different representations of the compact subgroup $SU(6)$.

Thus we have recognized that all the praemissa needed for the collinear subgroup symmetry are satisfied. The collinear subgroup is a good symmetry group for these infinite multiplet models, in the following sense:

If we assume that a given set of physical $SU(6)$ multiplets ω_i are contained in $SL(6, C)$ representations τ_i and these representations τ_i are known to couple to at least one invariant form, the predictions of the collinear subgroup consist in identities between $SU(3)$ amplitudes which do not all vanish identically. (The same statement can be made for $SU(6,6)$ models etc.) This result was first stated in [3]. The fact that one physical multiplet in motion belongs to different representations of the compact subgroup is the reason for the non-conservation of spin or orbital angular momentum separated. The decays

$$\varrho \rightarrow \pi + \pi$$

$$N^* \rightarrow N + \pi$$

are really allowed in principle in such models.

The condition $S' = 0$ is made above for reasons of convenience. In a search for representations which are to fit the experiments one has to account also for representations with $S' \neq 0$. In such a case the spin of a particle at rest consists of contributions from both the groups P' and $SL(6, C)$. Let us look at a 35-plet. For $S' = 0$ we have the content

$$\{S' = 0{,}35\} = [8{,}1] \oplus [8{,}3] \oplus [1{,}3],$$

whereas for $S' = 1$ we obtain

$$\{ S' = 1, 35 \} = [8, 3] \oplus [8{,}5 \oplus 3 \oplus 1] \oplus [1{,}5 \oplus 3 \oplus 1] .$$

Since this bears some resemblance with the 1-excitation of elementary particles (see [4]) we may adopt this notion for theories $S' \neq 0$. On the other hand, we may denote the excitation implied by the non-compactness of $SL(6, C)$ "relativistic $SU(6)$" excitation. The idea that non-compact groups could serve to generate an infinite sequence of multiplets with or without nontrivial (that means non-perturbative) mass relations, goes back to Barut [5].

3. Parity

The choice between all those representations which contain a given $SU(6)$ multiplet and which can therefore be considered to be of physical interest can be restricted by the requirement that the parity operator must be definable as a unitary operator on the representation space. For non-unitary representations of $SL(2, C)$ it is known that parity reflections force us to double the representation space. Indeed, a representation (j_1, j_2) goes over into (j_2, j_1) under parity and both representations are, in general, not equivalent.

The requirement that the representation should not be doubled has the following physical origin. The compact subgroup $SU(6)$ of $SL(6, C)$ possesses only generators of positive parity, the generators of boosters $\Sigma(p)$, on the other hand, are negative parity operators (they contain the velocity). If we take these properties together, we can formulate the following statement: If $D(S)$ is a representation of elements S of $SL(6, C)$ parity acts as

$$D(S) = PD'(S)P^{-1},$$

where $D'(S) = D(S^{-1+})$. If and only if D and D' are equivalent, P can be defined as a unitary operator in the representation space (see [2] and [6]). If D and D' are not equivalent P maps an $SU(6)$ multiplet of the representation D on the same multiplet of the representation D'. Each $SU(6)$ multiplet would appear twice. We know, however, that such parity doublets do not exist in nature.

Investigating the consequence of the postulate that D and D' should be equivalent we get relations for the Casimir invariants of the $SL(6, C)$ representations which in the finite example correspond to $j_1 = j_2$.

4. Physical implications of the model, I

The physical predictions of the model are principally of two kinds. Either they involve from each $SL(6, C)$ representation only one $SU(6)$ multiplet or they concern different $SU(6)$ multiplets from one representation. Let us first study the first kind.

We know that collinear symmetry is valid in the group model we are investigating. Are the predictions which are beyond those of the collinear subgroup? We shall see that this can be decided only if particular representations of $SL(6, C)$ are considered. It depends on the number of invariant forms which can be built out of the representations. Two extreme cases are: the meson-baryon vertex with maximal possible degeneracy, we obtain only one vertex. With this same degeneracy of the mesons we obtain an infinite set of invariants for the meson-meson-meson coupling. We expect, therefore, new results for the meson-baryon vertex only.

The collinear subgroup gives some identities for strong formfactors of the baryon octet, namely:

$$a_m^D : a_m^F : a_m^S = 3 : 2 : 1,$$

and

$$a^F = -2/3\, a_c^F - 5/9\, a_c^D,$$

$$a^D = -a_c^F + 2/3\, a_c^D.$$

a_c and a_m are the formfactors of charge and magnetic moment (Sachs) type for the vector mesons, "a" are the formfactors (vector-coupling) for the pseudoscalar mesons. In addition static $SU(6)$ gives the result

$$a^D = 0 \quad \text{at} \quad \mu^2 = 4M^2.$$

The well-known result

$$a^D/a^F = 3/2 \quad \text{at} \quad \mu^2 = 0$$

is not (!) a consequence of the collinear subgroup or static $SU(6)$, but can be derived only if certain properties of analytic continuation of the representations are satisfied. These are satisfied for the inhomogeneous $SL(6, C)$ group with 72 translations (the analytic behaviour is analogous as for the inhomogeneous Lorentz group) but are different for the group now under discussion. Therefore, we expect different results in the present case. Other interesting results are the ratios

$$\frac{a_m^F}{a_c^F} \quad \text{at} \quad \mu^2 = 0 \text{ or in general.}$$

FRONSDAL and WHITE have shown that the ratio a^D/a^F depends on the representation of $SL(6, C)$ used for the mesons. For a particular one they obtain [7]

$$a^D/a^F = 9/5$$

and as a consequence

$$a_c^D/a_c^F = -3/25, \quad \mu^2 = 0.$$

It is, therefore, possible to enlarge the value of the D/F ratio for the pseudoscalar mesons without changing the D/F ratio for the vector mesons too much from the value zero.

Since for this vertex only one invariant form can be found it could be suggested that the electromagnetic current is similarly determined up to one factor, which would imply a fixed value for the magnetic moment. However, this is not true.

Indeed, we can immediately find two contributions to the current. First we have a convection current. Since the translations commute with $SL(6, C)$ we can build a fourvector out of a unitary representation of $SL(6, C)$ and its conjugate, by first forming the $SL(6, C)$ scalar product and then multiplying by the momentum. We treat the $SL(6, C)$ states like scalars! Such a convection current implies a pure electric current for the $SU(6)$ multiplets, the total magnetic moment is zero, see [8]. However, we can also define the magnetization current as the divergence of the expectation value for the magnetization operator

$$s^t_{\mu\nu,3} + 3^{-1/2}\, s^t_{\mu\nu,8}$$

taken between states of the infinite multiplet. So far nobody has calculated such an expression, the result for a physical multiplet as the 56-plet would certainly be of interest. In any case the magnetic moment of the nucleons is not fixed since the convection and magnetization give completely independent contributions.

5. Physical implications, II

The other type of predictions which can be derived from such a model relates different $SU(6)$ multiplets contained in one infinite representation of $SL(6, C)$. Such predictions make, however, sense only if such multiplets are known to exist in nature. This raises the fundamental problem whether there is any hint in the data of particles and resonances which favors the assignment of such objects to infinite multiplets. Today, this question must still be answered in the negative.

Indeed, for the mesonic resonances we know that the 35^--plet must be followed by a 405^+-plet. The 405^+ contains

$$405 = [27 \oplus 8 \oplus 1, 5] \oplus [27 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10}, 3] \oplus$$
$$\oplus [27 \oplus 8 \oplus 1, 1] .$$

No members of the 27-plets are definitely known. In the case of the 189 the [27, 5] and [27, 3] are missing, but this supermultiplet lies in a certain sense far off the 35-plet if it is contained at all in the infinite sequence.

It might well be that the $SL(6, C)$ group is too big a group to generate the infinite multiplets and that $SL(2, C) \otimes SU(3)$ serves better for this purpose. In addition such a smaller group is technically simpler to handle and gives definite predictions which might also have relevance for more general models involving infinite multiplets.

Some results are known: (see [9])

(1) If we consider a ladder of octets containing the spins $S = k_0 + n$, $n = 0, 1, 2, \ldots$, parity $(-1)^S$, all the decay states of this multiplet consisting of two pseudoscalar mesons can be shown to have signature $(-1)k_0$. If the vector resonances belong to such a ladder they must necessarily have $k_0 = 1$. On the other hand the resonance K^{**} (1405) fits into a similar octet ladder only if $k_0 = 0$ or 2, since it also decays into two pseudoscalar mesons. This implies, therefore, that the known 1^- and 2^+ octets belong to different ladders. No example is known, even for this smaller group, where two $SU(3)$ multiplets could be assigned to one ladder. In terms of our $SL(6, C)$ ladders we can interpret this result in the way that the 1^- mesons lie in a ladder of $SL(2, C)$ starting in the 35^--plet and going through the 405^+ whereas the 2^+ resonances lie in a $SL(2, C)$ ladder starting in a 189^+ multiplet.

(2) To gain some insight we assume that a hypothetical 0^+ meson and some 2^+ lie together in an infinite ladder and decay into two pseudoscalar mesons. The ratio for the corresponding partial widths can be computed. The width of the 0^+ turns out to be a factor of 1000 bigger, somewhat depending on the masses of both particles (we have broken symmetry !). If the mass of the 2^+ is of the order of 10 GeV, 0^+ mass equal 1 GeV, this ratio becomes of the order one. The origin of this result is formally so simple that we may very likely draw the conclusion that higher rungs of a ladder are very weakly coupled to a channel which is already open to a lower rung of the same ladder. This result makes it unprobable that these higher rungs are observable at present and the principal question for the existence of ladders remains open.

6. Concluding remarks

One of the aims of studying these mathematically non-trivial models which generalize $SU(6)$ symmetry in a relativistic manner was to learn something about the contradiction of $SU(6)$ symmetries with the unitarity of the S-matrix. We note that collinear symmetry and static $SU(6)$ symmetry give no answer to the unitarity problem. They tell us only that a complete set of states must contain at least one $SU(6)$ multiplet. From the model $SL(6, C)\times T_{72}$ we deduce that one multiplet is in general not enough to achieve completeness; there is a leakage of probability, formally into the remaining 68 dimensions. On the other hand the infinite multiplet model assures us that an infinite number of $SU(6)$ multiplets suffices to yield completeness. The natural question of the physicist is then how the higher $SU(6)$ multiplets in the completeness sum are weighed, if we consider an infinite multiplet model as a good physical symmetry. The answer is contained in the statement made earlier: A given channel is mainly coupled to the lowest $SU(6)$ multiplet possible. The same multiplet must consequently give the main contribution to the unitarity sum in this channel. In other words: Violation of unitarity is not severe if only one $SU(6)$ multiplet is taken into account in the sum. This is the meaning of the notion of "saturation" known from the current algebra approach in this complementary approach based on abstract groups. The conflict with unitarity can be considered as the characteristic difficulty of the model based on the inhomogeneous $SL(6, C)$ group. The characteristic defect of the theories with infinite multiplets of the type studied here is connected with crossing symmetry. In familiar field theories particles and antiparticles are dealt with simultaneously in fields and field equations. In our models this is not the case. Fields can be defined. They contain only particles similar to fields of the WEYL type and the field equations are absolutely trivial: they are all equal to the KLEIN—GORDON equation.

A typical effect of the violation of crossing symmetry can be observed in the following fact. In usual field theory the Lagrangian

$$i \int d^{\nu}x \, \tilde{\psi}(x)\gamma_5 \, \psi(x) \, \varphi(x)$$

describes the coupling of the pion to the nucleon current as well as the annihilation of a pion into a virtual nucleon pair. In momentum space the pion is either coupled to the nucleon spinors via the matrix

$$\sigma_k \, (p' - p)^k$$

or the identity matrix. Both matrices are hermitian. This property can be understood to be a consequence of the reality of the Lagrangian in x-space.

Examples are known where hermiticity in one channel implies non-hermiticity in another channel if we apply the infinite multiplet models. The result of the symmetry depends on whether we make the vertex hermitian in one channel and continue then or continue first and add the conjugate later.

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РЕЛЯТИВИСТСКИЕ СИММЕТРИИ $SU(6)$ С БЕСКОНЕЧНЫМИ МУЛЬТИПЛЕТАМИ

В. РЮЛЬ

Резюме

Обсуждается группа $P \times SL$ (6, C) и ее физическая интерпретация. Резюмируются предсказания, которые могут быть получены из модели. В заключении подчеркивается, что теории с бесконечными мультиплетами типа изучаемого в данной работе, встречают большие трудности в связи с нарушением кроссингсимметрии.

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NON-COMPACT SYMMETRY GROUPS, UNITARY S-MATRIX AND QUANTUM FIELD THEORY

By

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We study the non-compact symmetry groups and their connections with field theory. We prove explicitly that in the symmetry with a non-compact group there exists no contradiction with the unitarity condition of the S-matrix. We have suggested a method of describing the symmetry with a non-compact group by means of the apparatus of the local quantum field theory. Within our method the field operators satisfy the normal commutation relations, and the S-matrix is crossing-symmetrical.

1. Introduction

In a series of papers [1—8] the symmetry groups $SL(6, C)$ and $\widetilde{U}(12)$ have been suggested as the relativistic extensions of the $SU(6)$ symmetry group, the $SL(2, C)$ subgroup of these relativistic symmetry groups being identified with the homogeneous Lorentz group. In contrast to the isotopic invariance and the unitary symmetry, no ideal case with exact symmetry exists for these new groups: these symmetries are intrinsically broken. More concretely, it is impossible to write down the wave equations even for the free particles or the Lagrangian for the free fields in such a manner that they are invariant under the $SL(6, C)$ or the $\widetilde{U}(12)$ groups if we do not consider the 4-dimensional momenta of the particles as the components of some many-dimensional tensors of the corresponding groups: 36-dimensional for the $SL(6, C)$ group and 143-dimensional for the $\widetilde{U}(12)$ group. Because of the breaking of the symmetry by the wave equations the matrix elements of the scattering processes are not strictly invariant under the $SL(6, C)$ or $\widetilde{U}(12)$ groups. If we require the exact symmetry of the S-matrix, then this requirement should lead to a contradiction with the unitary condition even in the one-particle approximation, i.e. in the pole approximation [9, 10]. In order to take into account the intrinsic breaking of the symmetry we have suggested a spurion method [5]. According to this method the matrix elements of the scattering processes contain explicitly particle momenta (irregular coupling in [11]) and therefore could be invariant under the $SL(6, C)$ or $\widetilde{U}(12)$ groups only if we introduce many-dimensional momenta (a very similar method was also suggested in [11, 12]). In some special cases the scattering amplitudes obtained within this formalism do not contradict the unitarity condition [12, 13] but in the general case the above mentioned spurion method does not remove the contradiction with the unitarity condition [14]. It is possible to show that this contradiction arises because of the following cause: exact symmetry requires the existence of the many-dimensional momenta but, in fact, particle momenta have only 4 components. In other words, the intrinsic breaking of the symmetry is closely connected with the breakdown of the unitarity of the S-matrix. In order to construct a relativistic $SU(6)$ symmetry theory with a unitarity S-matrix we have to introduce a symmetry group G such that the 4-dimensional momenta form it in irreducible representations; the group G containing the homogeneous Lorentz group and the $SU(6)$ group as its subgroups. One of the symmetry groups of this kind has been suggested by MICHEL [15] and by BUDINI and FRONSDAL [16]. It was studied in a series of papers by FRONSDAL, WHITE et al. [17], DELBOURGO, SALAM and STRATHDEE [19, 20] and RUHL [21]. This symmetry group G is the semi-direct product of the homogeneous Lorentz group and the internal symmetry group S

$$G = L \cdot S, \tag{1}$$

S being the $SL(6, c)$ or the $U(6,6)$ group. If we also consider the translations, then we have the group

$$G_p = P \cdot S, \tag{2}$$

where P is the Poincaré group, generators of the translation group and of the symmetry group S being commuted.

In the present paper we study the structure of the unitary S-matrix in such a symmetry. We also consider the connection between the symmetry theory with a non-compact group, and the local quantum field theory.

2. The symmetry group

Before studying the group G and the classification of elementary particles in the symmetry with this group, we first give some physical arguments which show that the introduction of the symmetry group G in the form of Eq. (1) is natural or even necessary. Let us formulate the conditions of the group G. It must contain the homogeneous Lorentz group L and the internal symmetry group S as subgroups, S and L not being commuted because we want to find such an internal symmetry group S that its irreducible representations contain particles with different spins. Further, the group S must be relativistically invariant in the following sense: in any Lorentz transformation $\lambda \in L$ every element σ of S goes into another element of this group, e.g. if $\sigma \in S$ then $\lambda \sigma \lambda^{-1} \in S$. In addition to this condition, we require that the irreducible representations of the group G exist which can be identified with the 4-momenta of the particles. We show that in this case the homogeneous Lorentz group L must be isomorphic to the factor-group G/S of the group G with respect to the invariant subgroup S. Remember that by definition a representation of the group G is a homomorphism of this group to a group R of the linear transformations in some vector space. If we denote by N the kernel of this homomorphism then the group R is isomorphic to the factor group G/N of the group G with respect to the invariant subgroup N. On the other hand, the group of linear transformations of the 4-momenta p_μ which conserve the square p^2 is the homogeneous Lorentz group L. Thus the 4-momenta of the particles form an irreducible representation of the group G only in the case when the Lorentz group L is isomorphic to the factor-group G/N of G with respect to some invariant subgroup N. By definition N is the set of all elements of G which go into the identity transformation of the momenta p_μ in the homomorphism $G \to L$. On the other hand, in all the transformations of the internal symmetry group S particle momenta do not change. Therefore, the group S in general must be a subgroup of some invariant subgroup N. If S is not identical with N then instead of S we can choose N as the internal symmetry group. Thus, the internal symmetry group S can be chosen in such a manner that the homogeneous Lorentz group L is isomorphic to the factor-group G/S

$$L \sim G/S .$$

It is natural to suppose L and S have only one common element, namely, the identity transformation, i.e. there does not exist any nontrivial Lorentz transformation which may be considered at the same time as a transformation from the symmetry group S. In this case the group G is the semidirect product of the invariant subgroup S and the factor group L .

We suppose that the group S is a simple group. Then it must contain some subgroup S_L isomorphic to L . This property of S follows from the fact that the Lorentz group L is a group of automorphisms of S and from the theorem according to which every automorphism of a semisimple group is an inner automorphism. More concretely, to every Lorentz transformation λ there corresponds such a transformation s from the group S that for any $\sigma \in S$

$$\lambda \sigma \lambda^{-1} = s \sigma s^{-1},$$

the mapping $\lambda \to s$ being a homomorphism. Hence the group L is a simple one, then the mapping $\lambda \to s$ is an isomorphic. We suppose that the internal symmetry group S is the $SL(6, C)$ group or the $U(6,6)$ group. In this way we obtain a symmetry theory which is a relativistic extension of the $SU(6)$ symmetry.

In studying the classification of the elementary particles, i.e. in studying the one-particle states it is sufficient to consider only the internal symmetry group S. Hence, S is a non-compact group, we have infinite multiplets if we use the unitary representations of this group. The idea of using the unitary representations of the non-compact groups to classify elementary particles was also suggested in the paper by DOTHAN, GELL-MANN and NE'EMAN [22].

We denote the generators of L by $I_{\mu\nu}$, the generators of the subgroup S_L isomorphic to L by $s_{\mu\nu}$. For the corresponding infinitesimal operators of the representations of the group G we use the same notations. We put

$$I_{\mu\nu} = I'_{\mu\nu} + s_{\mu\nu}. \tag{3}$$

It is not difficult to show that $I'_{\mu\nu}$ commute with all generators of S and form the Lie algebra of a group L' isomorphic to L. Thus the group G may be considered as the direct product of the internal symmetry group S and some group L' isomorphic to L:

$$G = L' \otimes S\,, \tag{4}$$

L' together with the translation group forms some group P' isomorphic to the Poincaré group P, and the group G_p is the direct product of P' and S

$$G_p = P' \otimes S\,. \tag{5}$$

3. Particle classification

We introduce some notations. The maximal compact subgroup of S which contains the $SU(2)$ subgroup with generators s_{ij}, $i, j = 1, 2, 3$, is denoted by S_0. Consider the unitary representations of S. We denote by ξ the sets of all parameters characterizing these representations. Each such representation splits into a direct sum of the irreducible (finite-dimensional) unitary representations of S_0 which are characterized by the sets of parameters j. The sets of the eigenvalues of the diagonal infinitesimal operators of S_0 are denoted by ν. The basis vectors of the irreducible unitary representations of S may be represented as $\xi j\nu\rangle$. In the following we call this basis the caconical basis corresponding to the reduction $S \supset S_0$. On the other hand, each irreducible representation of P' is characterized by two numbers: $m = \sqrt{-p^2}$ and s', $s' = 0, \frac{1}{2}, 1 \ldots$. The basis vectors of such a representation are denoted by $|\,p\, s' >$. Hence, as G_p is the direct product of S and P, the unitary irreducible representations of G_p, i.e. the Hilbert spaces of the state vectors of particles may be considered as direct products of the representations of S and P'.

Consider rest particles and denote their momenta by $\hat{p} : \hat{p}_i = 0, \hat{p}_4 = im$. State vectors of these particles are of the form

$$\hat{p}s', \xi j \nu> = |\hat{p}s'> \otimes |\xi j \nu>. \tag{6}$$

In the pure Lorentz transformation $\lambda_{p \leftarrow \hat{p}}$ in which the momentum $\hat{p}$ goes into p the state vectors (6) are transformed in the following manner

$$|ps', \xi j \nu> \to U(\lambda_{p \leftarrow \hat{p}})(\hat{p}s', \xi j \nu> = U'(\lambda_{p \to \hat{p}})|\hat{p}s'> \otimes U^S(\lambda_{p \leftarrow \hat{p}})|\xi j \nu>, \tag{7}$$

where

$$\begin{aligned} U(\lambda) &= e^{iI_{\mu\nu}\omega_{\mu\nu}(\lambda)}, \\ U'(\lambda) &= e^{iI'_{\mu\nu}\omega_{\mu\nu}(\lambda)}, \\ U^S(\lambda) &= e^{is_{\mu\nu}\omega_{\mu\nu}(\lambda)}, \end{aligned} \tag{8}$$

and $\omega_{\mu\nu}(\lambda)$ are the parameters of the Lorentz transformation λ. The first factor on the right-hand side of Eq. (7) is the basis vector $|ps'>$, and we denote the second factor by $|\xi \tilde{j} \tilde{\nu}>$. Instead of the generators s_α of S (some of s_α are identical to $s_{\mu\nu}$) we introduce new operators depending on p:

$$s_\alpha(p) = U^S(\lambda_{p \leftarrow \hat{p}})\, s_\alpha\, U^S(\lambda_{p \leftarrow \hat{p}})^{-1}. \tag{9}$$

These operators may also be considered as the generators of the group S. They satisfy the same commutation relations as the generators s_α. It is not difficult to show that in the basis $|\xi \tilde{j} \tilde{\nu}>$ the operators $s_\alpha(p)$ have exactly the same matrix elements as s_α have in the basis $|\xi j \nu>$. This means that if $|\xi j \nu>$ is the canonical basis of a representation of the group S with generators s_α then $|\xi \tilde{j} \tilde{\nu}>$ is the canonical basis of the same representation of S but with new generators $s_\alpha(p)$. In particular, the sets of parameters $\tilde{j}$ characterize the irreducible representations of the maximal compact subgroup $S_0(p)$ containing the $SU(2)_p$ group with generators

$$s_{ij}(p) = U^S(\lambda_{p \leftarrow \hat{p}})\, s_{ij}\, U^S(\lambda_{p \leftarrow \hat{p}})^{-1}. \tag{9'}$$

In other words, particles with different momenta are classified according to the representations of different maximal compact subgroups of S.

The state vectors on the right-hand side of Eq. (7) will be denoted by $|ps'\, \xi \tilde{j} \tilde{\nu}>$. We consider only the case $s' = 0$, and denote the state vectors by $|p\, \xi \tilde{j} \tilde{\nu}>$. In the Lorentz transformations $\lambda_{q \leftarrow p}$ $(q^2 = p^2)$ they are transformed in the following manner

$$U(\lambda_{g \leftarrow p})|p\, \xi \tilde{j} \tilde{\nu}> = R_{\nu\nu'}|p\, \xi \tilde{j} \tilde{\nu}'>, \tag{10}$$

where $R_{\nu\nu'}$ are the matrix elements of the Wigner rotation operators.

In order to classify the elementary particles we use the canonical basis corresponding to the reduction $S \supset S_0(p)$, i.e. depending on p. However, in studying the structure of the S-matrix it is convenient to use for all momenta the canonical basis corresponding to the same reduction $S \supset S_0$. In other words, we must introduce a new basis $| p\, \xi\, j\, \nu >$ in which the generators s_α have the same matrix elements as they have in the basis for rest particles $| \hat{p}\, \xi\, j\, \nu >$. It is not difficult to show that this new basis is related to the physical one in the following manner:

$$|p\,\xi\,\tilde{j}\,\tilde{\nu}> = U^S(\lambda_{p\leftarrow\hat{p}})\,|p\,\xi\,j\,\nu>, \tag{11}$$

this transformation not being a Lorentz one. This relation can be rewritten explicitly in the following manner

$$|p\,\xi\,\tilde{j}\,\tilde{\nu}> = d^{\xi}_{j\nu,j'\nu'}\,(\lambda_{p\leftarrow\hat{p}})\,|p\,\xi\,j'\,\nu'>. \tag{12}$$

We remember that the state vectors $| p\, \xi \tilde{j} \tilde{\nu} >$ and $| p\, \xi\, j\, \nu >$ are of the form of Eq. (6). Therefore, we have

$$|\xi\,\tilde{j}\,\tilde{\nu}> = d^{\xi}_{j\nu,j'\nu'}\,(\lambda_{p\leftarrow\hat{p}})\,|\xi\,j'\,\nu'>, \tag{13}$$

which is the special case of a more general formula

$$U^S(\lambda)\,|\xi\,j\,\nu> = d^{\xi}_{j\nu,j'\nu'}\,(\lambda)\,|\xi\,j'\,\nu'>, \tag{14}$$

where λ is any Lorentz transformation. From the group properties and the unitarity of $U^S(\lambda)$ if follows that the functions $d^{\xi}_{j'\nu,j\nu}(\lambda)$ satisfy the relations

$$d^{\xi}_{j_1\nu_1,j_2\nu_2}(\lambda_1)\,d^{\xi}_{j_2\nu_2,j_3\nu_3}(\lambda_2) = d^{\xi}_{j_1\nu_1,j_3\nu_3}(\lambda_1\,\lambda_2), \tag{15}$$

$$d^{\xi}_{j_1\nu_1,j\nu}(\lambda)\,d^{\xi}_{j_2\nu_2,j\nu}(\lambda) = d^{\xi}_{j\nu,j_1\nu_1}(\lambda)\,d^{\xi}_{j\nu,j_2\nu_2}(\lambda) = \delta_{j_1j_2}\,\delta_{\nu_1\nu_2}, \tag{16}$$

the summation over the continuous parameters on the left-hand sides Eqs. (15) and (16) being the integration.

In conclusion, we note that in the space inversion some irreducible unitary representations of S go to themselves. However, representations exist which go to other representations non-equivalent to themselves. For these last representations the space inversion invariance leads to the degeneracy in parity.

4. S-matrix

We now study the structure of the S-matrix. We show explicitly that in the symmetry with a non-compact group no contradiction with the unitarity condition exists. For simplicity we consider in detail the elastic scattering of a singlet on the particles from some multiplet of the group S. The

4-momenta of the singlet before and after the scattering will be denoted by q_1 and q_2, the 4-momenta and other quantum numbers of the second particle before and after scattering will be denoted by $p_1, \xi_1, \tilde{j}_1, \tilde{\nu}_1$ and $p_2, \xi_2, \tilde{j}_2, \tilde{\nu}_2$, respectively. Matrix element of the T-Matrix, $S = 1 + i\,T$, is of the form

$$<q_2, p_2\,\xi_2 \tilde{j}_2 \tilde{\nu}_2 \,|\, T \,|\, q_1, p_1\,\xi_1 \tilde{j}_1 \tilde{\nu}_1> = i\,(2\pi)^4\,\delta^4\,(p_1 + q_1 - p_2 - q_2)\,\delta_{\xi_2\xi_1}$$
$$T^{\xi}\,(q_2, p_2 j_2 \nu_2 \,|\, q_1 p_1 j_1 \nu_1), \xi = \xi_1 = \xi_2. \tag{17}$$

Going from the physical basis $|\,p\,\xi \tilde{j} \tilde{\nu}>$ to the non-physical one $|\,p\,\xi\,j\,\nu>$ which is the canonical basis corresponding to the reduction $S \supset S_0$, i.e. making the transformation (12), we can show that the amplitudes $T^{\xi}\,(q_2, p_2 \tilde{j}_2 \tilde{\nu}_2 \,|$ $q_1, p_1 \tilde{j}_1 \tilde{\nu}_1)$ have the following structure

$$T^{\xi}\,(q_2, p_2 \tilde{j}_2 \tilde{\nu}_2 \,|\, q_1, p_1 \tilde{j}_1 \tilde{\nu}_1) = d^{\xi}_{j_2\nu_2, j\nu}\,(\lambda_{p_2 \leftarrow \hat{p}_2})\,\cdot$$
$$\cdot\, d^{\xi}_{j_1\nu_1, j\nu}\,(\lambda_{p_2 \leftarrow \hat{p}_2})\,F^{\xi}\,(s,t), \tag{18}$$

where $F^{\xi}(s, t)$ is a scalar function of s and t. This means that in the given case the structure of the scattering amplitude is completely determined by the functions $d^{\xi}_{j_1\nu_1, j_2\nu_2}\,(\lambda)$. In the general case it is completely determined by the functions $d^{\xi}_{j_1\nu_1, j_2\nu_2}\,(\lambda)$ and the CLEBSH—GORDAN coefficient of the group S. We denote the antihermitian part of the scattering amplitude by $A^{\xi}(q_2\,p_2\,\tilde{j}_2\,\nu_2 \,|\, q_1\,p_1\,\tilde{j}_1\,\tilde{\nu}_1)$. From Eqs. (17) and (18) it follows that

$$A^{\xi}\,(q_2, p_2 \tilde{j}_2\,\tilde{\nu}_2 \,|\, q_1, p_1 \tilde{j}_1 \tilde{\nu}_1) = d^{\xi}_{j_2\nu_2, j\nu}\,(\lambda_{p_2 \leftarrow \hat{p}_2})\,\cdot$$
$$\cdot\, d^{\xi}_{j_1\nu_1, j\nu}\,(\lambda_{q_1 \leftarrow \hat{q}_1})\,Im\,F^{\xi}\,(s, t). \tag{19}$$

Consider now the unitarity condition in the two-particle approximation. We put the expressions (17) and (18) into the right-band side of the relation

$$\frac{1}{i} <f|\,T - T^{+}\,|\,i => \sum_{n} <f|\,T\,|\,n><n\,|\,T^{+}\,|\,i>, \tag{20}$$

where $\sum_n$ denotes the summation over the infinite number of all intermediate states of the system consisting of the singlet and the particle in the given multiplet. Using Eq. (16) we get:

$$A^{\xi}\,(q_2, p_2 \tilde{j}_2\,\tilde{\nu}_2 \,|\, q_1, p_1 \tilde{j}_1\,\tilde{\nu}_1) = d^{\xi}_{j_2\nu_2, j\nu}\,(\lambda_{p\,\leftarrow \hat{p}_2})$$
$$d^{\xi}_{j_1\nu_1, j\nu}\,(\lambda_{p_1 \leftarrow \hat{q}_1})\,\frac{1}{8\pi^2}\int \frac{d^{3\prime}_{p}}{2'_{p_0}}\,\frac{d^{3\prime}_{q}}{2'_{q_0}}\,\delta^4\,(p' + q' - p_1 - q_1)\,F^{\xi}\,(s, t_2)\,F^{\xi}\,(s, t_1) \tag{21}$$

Thus, after the summation over the infinite number of all particles from the given multiplet we get for the antihermitian part $A^{\xi}(q_2, p_2\ \tilde{j}_2\ \tilde{\nu}_2\,|\,q_1, p_1\ \tilde{j}_1\ \tilde{\nu}_1)$ the same structure which we suppose. In the given case the unitarity condition leads only to an integral equation for $F_{\xi}(s, t)$ which is similar to the integral equation for the scalar particle elastic scattering in the two-particle approximation.

Consider now the scattering of two arbitrary particles. We denote by $C^{\xi j \nu n}_{\xi_1 j_1 \nu_1 \xi_2 j_2 \nu_2}$ the Clebsh—Gordan coefficients for the group S

$$|\,\xi_1 j_1 \nu_1 > \otimes\, |\,\xi_2 j_2 \nu_2 > = \sum_{\xi j \nu n} C^{\xi j \nu n}_{\xi_1 j_1 \nu_1\, \xi_2 j_2 \nu_2}\, |\,\xi\, j\, \nu\rangle_n . \tag{22}$$

In principle, the product of two irreducible representations of the group S can contain some irreducible representation many times. In order to distinguish between these different, but equivalent representations, we use the index n. These equivalent representations can be chosen in such a manner that the vectors from the representations with different n are orthogonal. We can show that the scattering amplitude is of the form

$$\begin{aligned} T\,(q_2\ \xi_2\ \tilde{j}_2\ \tilde{\nu}_2, p_2\ \eta_2\ \tilde{i}_2\ \tilde{\mu}_2\,|\,q_1\ \xi_1\ \tilde{j}_1\ \tilde{\nu}_1, p_1\ \mu_1\ \tilde{i}_1\ \tilde{\mu}_1) = d^{\xi_2}_{j_2 \nu_2, j'' \nu''}\,(\lambda_{q_2 \leftarrow \hat{p}_2})\\ d^{\eta_2}_{i_2 \mu_2, i'' \mu''}(\lambda_{p_2 \leftarrow \hat{p}_2})\, d^{\xi_1}_{j_1 \nu_1, j' \nu'}\,(\lambda_{q_1 \leftarrow \hat{q}_1})\, d^{\eta_1}_{i_1 \mu_1, i' \mu'}\,(\lambda_{p_1 \leftarrow \hat{p}_1})\\ \sum_{\zeta k \varkappa n_i} C^{\zeta k \varkappa n_2}_{\xi_2 j'' \nu'' \eta_2 i'' \mu} \cdot C^{\zeta k \varkappa n_1}_{\xi_1 j' \nu' \eta_1 i' \mu'}\, F^{\zeta}_{n_2 n_1}\,(s, t), \end{aligned} \tag{23}$$

where $F^{\xi}_{n_2 n_1}(s, t)$ is an infinite number of scalar functions of s and t. Note that the C, P and T-invariances lead to some relations between these scalar functions.

Consider now the unitarity condition in the two-particle approximation. We put the expressions in the form of Eq. (23) into the right-hand side of Eq. (20) and sum over all intermediate two-particle states. Using Eq. (16) for the functions $d^{\xi}_{j_1 \nu_1, j_2 \nu_2}(\lambda)$ and the following properties of the Clebsh—Gordan coefficient

$$\sum_{\xi_i j_i \nu_i} C^{\xi j\, \nu_1\, \nu n}_{\xi_1 j_1 \xi_2 j_2 \nu_2}\, C^{\xi' j' \nu' n'}_{\xi_1 j_1 \nu_1 \xi_2 j_2 \nu_2} = \delta_{\xi \xi'}\, \delta_{j j'}\, \delta_{\nu \nu'}\, \delta_{n n'}, \tag{24}$$

$$\sum_{\xi j \nu n} C^{\xi j n \nu}_{\xi_1 j_1 \nu_1 \xi_2 j_2 \nu_2}\, C^{\xi j \nu n}_{\xi'_1 j'_1 \nu'_1 \xi'_2 j'_2 \nu'_2} = \delta_{\xi_i \xi'_i}\, \delta_{j_i j'_i}\, \delta_{\nu_i \nu'_i} \tag{25}$$

we can show that the antihermitian part obtained after the summation over all two-particle intermediate states has the same structure as the amplitude (23), and the unitarity condition leads only to a system of integral equations for scalar functions $F^{\xi}_{n_2 n_1}(s, t)$. Thus in the two-particle approximation there exists no contradiction with the unitarity condition. This takes place owing to the fact that the summation over the intermediate states is an operation invariant under the symmetry group. Therefore, in any many-particle approximation after the summation over all intermediate states we always obtain expressions invariant under the symmetry group. We can show explicitly that in any many-particle approximation the symmetry and the unitarity of the S-matrix are compatible.

5. Connections with field theory

We now study the possibilities of describing the symmetry with the group G of the form (1) by means of the apparatus of the local quantum field theory. We show that for the particles in the infinite multiplets of the group G we can introduce the corresponding quantized fields in such a manner that for the field operators we have the normal commutation relations, and the scattering amplitudes satisfy the crossing symmetry condition.

For simplicity we consider first the case $S = SL(2, C)$ and we omit the index ξ. The creation and destruction operators for a particle in the state $| p\, j\, \nu \rangle$ will be denoted by $a^+(p\, j\, \nu)$ and $a(p\, j\, \nu)$, respectively. Consider the set of operators $a(p\, j\, \nu)$ with given $j(\nu = -j, -j + 1, \ldots j - 1, j)$. Using the method suggested by WEINBERG [23] and developed by FELDMAN and MATHEWS [24], we construct the corresponding field operators which form the non-unitary spinor representations of the homogeneous Lorentz group. Let $\varphi_{a_1 \ldots a_n}\,(p\, j\, \nu)$ be the wave functions of particles with spin $j, n = 2\,j$, and with spin projection ν. These wave functions form the representation $(j, 0)$ of the homogeneous Lorentz group. From these functions and the momentum $(p)_a^b$ we form other representations $\left(j - \frac{1}{2}, \frac{1}{2}\right)$, $(j - 1, 1) \ldots (0, j)$:

$$\varphi_{\dot{a}_1 a_2 \ldots a_n}(pj\,\nu) = \left(-\frac{ip}{m}\right)_{\dot{a}_1}^{b} \varphi_{b a_2 \ldots n}(pj\,\nu)$$

$$\varphi_{\dot{a}_1 \dot{a}_2 \ldots \dot{a}_n}(pj\,\nu) = \left(\frac{-ip}{m}\right)_{\dot{a}_n}^{b} \varphi_{\dot{a}_1 \ldots \dot{a}_{n-1} b}(pj\,\nu). \tag{26}$$

All these representations are the components of the BARGMANN—WIGNER wave function $U_{\alpha_1 \ldots \alpha_n}\,(p\, j\, \nu)$, $\alpha_i = a_i$ or $\dot{a}_i$; which satisfies the wave equation

$$(ip + m)_{\alpha_i}^{\beta_i} u_{\alpha_1 \ldots \alpha_{i-1} \beta_i \alpha_{i+1} \ldots \alpha_n}(pj\,\nu) = 0. \tag{27}$$

We put

$$A_{\alpha_1\ldots\alpha_n}(p) = \sum_\nu U_{\alpha_1\ldots\alpha_n}(pj\nu)\, a(pj\nu). \tag{28}$$

The operators $A_{\alpha_1\ldots\alpha_n}(p)$ are transformed in a Lorentz transformation as components of a spinor of rank n

$$U(\lambda)\, A_{\alpha_1\ldots\alpha_n}(p)\, U(\lambda)^{-1} = S^{\beta_1\ldots\beta_n}_{\alpha_1\ldots\alpha_n} A_{p_1\ldots p_n}(p'), \tag{29}$$

as has been shown by FELDMAN and MATHEWS [24].

Consider now some transformation X from the group S. We have

$$Xa(pj\nu)\, X^{-1} = x_{j\nu,\, j'\nu'}(p)\, a(pj'\nu'), \tag{30}$$

and therefore

$$XA_{\alpha_1\ldots\alpha_n}(p)\, X^{-1} = \sum_m X^{\beta_1\ldots\beta_m}_{\alpha_1\ldots\alpha_n}(p)\, A_{p_1\ldots p_m}(p), \tag{31}$$

where

$$X^{\beta_1\ldots\beta_m}_{\alpha_1\ldots\alpha_n}(p) = \sum_{\nu,\nu'} U_{\alpha_1\ldots\alpha_n}(pj\nu)\, x_{j\nu,j'\nu'}(p)\, \overline{U}_{\beta_1\ldots\beta_m}(pj'\nu'),$$

$$n = 2j, m = 2j'. \tag{32}$$

Note that if X does not depend on p then $X^{\beta_1\ldots\beta m}_{\alpha_1\ldots\alpha_n}(p)$ and $x_{j\nu,\, j'\nu'}(p)$ depend on p. On the other hand, we can choose the p-dependence of X in such a manner that the matrix elements $x_{j\nu,\, j'\nu'}$ in (30) does not depend on p.

The transition from the canonical basis corresponding to the reduction $S \supset S_0(p)$ to the canonical basis corresponding to the reduction $S \supset S_0$, i.e. the transformation of the form of Eq. (11) is a particular case of the transformation (30). In this transition

$$A_{\alpha_1\ldots\alpha_n}(p) \to A'_{\alpha_1\ldots\alpha_n}(p) = \sum_m D^{\beta_1\ldots\beta_m}_{\alpha_1\ldots\alpha_n}(p)\, A_{\beta_1\ldots\beta_m}(p). \tag{33}$$

In constructing the matrix elements of S-matrix it is convenient to use the non-physical operators $A'_{\alpha_1\ldots\alpha_n}(p)$, because they are transformed in a simple way in the transformations of the group S. Namely, if $X \in S$ does not depend on p, then

$$XA'_{\alpha_1\ldots\alpha_n}(p)X^{-1} = \sum_m X^{\beta_1\ldots\beta_m}_{\alpha_1\ldots\alpha_n}(p)\, A'_{\beta_1\ldots\beta_m}(p), \tag{34}$$

where

$$X^{\beta_1\ldots\beta_m}_{\alpha_1\ldots\alpha_n}(p) = \sum_{\nu,\nu'} U_{\alpha_1\ldots\alpha_n}(pj\nu)\, \varkappa_{j\nu,j'\nu'}\, \overline{U}^{\beta_1\ldots\beta_m}(pj'\nu'), \tag{35}$$

$x_{j\nu,\, j'\nu'}$ being p-independent.

The operators $A_{\alpha_1 \ldots \alpha_n}(p)$ are the Fourier transforms of the positive frequency parts $\psi^{(+)}_{\alpha_1 \ldots \alpha_n}(x)$ of the field operators

$$\psi^{(+)}_{\alpha_1 \ldots \alpha_n}(x) = \frac{1}{(2\pi)^{3/2}} \int e^{ipx} A_{\alpha_1 \ldots \alpha_n}(p)\, \delta(p^2 + m^2)\, \theta(p^\circ)\, d^4_p. \tag{36}$$

Making the transition to the canonical basis corresponding to the reduction $S \supset S_0$ we obtain new non-physical operators

$$\psi^{(+)}_{\alpha_1 \ldots \alpha_n} \to \psi'^{(+)}_{\alpha_1 \ldots \alpha_n}(x) = \sum_m D^{\beta_1 \ldots \beta_m}_{\alpha_1 \ldots \alpha_n}\left(-i \frac{\partial}{\partial x}\right) \psi^{(+)}_{\beta_1 \ldots \beta_m}(x). \tag{37}$$

To study the negative frequency parts $\psi^{(-)}_{\alpha_1 \ldots \alpha_n}(x)$ of the field operators we introduce the creation and destruction operators for the antiparticle $b^+(p\, j\, \nu)$ and $b(p\, j\, \nu)$, respectively, and we put

$$B_{\alpha_1 \ldots \alpha_n}(p) = \sum_\nu V_{\alpha_1 \ldots \alpha_n}(pj\nu)\, b^+(pj\nu). \tag{38}$$

The negative frequency parts of the field operators are

$$\psi^{(-)}_{\alpha_1 \ldots \alpha_n}(x) \frac{1}{(2\pi)^{3/2}} \int e^{-ipx} B_{\alpha_1 \ldots \alpha_n}(p)\, \delta(p^2 + m^2)\, \theta(p^\circ)\, d''_p. \tag{39}$$

The field operators equal

$$\psi_{\alpha_1 \ldots \alpha_n}(x) = \psi^{(+)}_{\alpha_1 \ldots \alpha_n}(x) + \psi^{(-)}_{\alpha_1 \ldots \alpha_n}(x). \tag{40}$$

The transition from the canonical basis corresponding to the reduction $S \supset S_0(p)$ to the canonical basis corresponding to the reduction $S \supset S_0$ for the field operators is of the four of Eq. (37).

$$\psi_{\alpha_1 \ldots \alpha_n}(x) \to \psi'_{\alpha_1 \ldots \alpha_n}(x) = \sum_m D^{\beta_1 \ldots \beta_m}_{\alpha_1 \ldots \alpha_n}\left(-i \frac{\partial}{\partial x}\right) \psi_{\beta_1 \ldots \beta_m}(x). \tag{41}$$

In this transformation the operators $B_{\alpha_1 \ldots \alpha_n}(p)$ are transformed in the following manner

$$B_{\alpha_1 \ldots \alpha_n}(p) \to B'_{\alpha_1 \ldots \alpha_n}(p) = \sum_m D^{\beta_1 \ldots \beta_m}_{\alpha_1 \ldots \alpha_n}(-p)\, B_{\beta_1 \ldots \beta_m}(p). \tag{42}$$

The matrix elements of scattering processes and also the vertex parts contain explicitly the non-physical operators $A'_{\alpha_1 \ldots \alpha_n}$ and $B'_{\alpha_1 \ldots \alpha_n}$ and their conjugate. These operators are transformed in a simple manner in the transformations from group S and from them we can form immediately the invariants of G. Let the matrix element of some process with the destruction of a particle described by the field $\psi_{\alpha_1\ldots\alpha_n}(x)$ be

$$\begin{aligned} M_1 &= <f|\ldots A'_{\alpha_1\ldots\alpha_n}(p)|i> \\ &= \ldots:\ldots D^{\beta_1\ldots\beta_m}_{\alpha_1\ldots\alpha_n}(p)\, U_{\beta_1\ldots\beta_m}(p). \end{aligned} \tag{43}$$

Then the matrix element of the corresponding process with the creation of an antiparticle is of the form

$$\begin{aligned} M_2 &= <f|\ldots B'_{\alpha_1\ldots\alpha_n}(p)|i> \\ &= \ldots\ldots\ldots D^{\beta_1\ldots\beta_m}_{\alpha_1\ldots\alpha_n}(-p)\, V_{\beta_1\ldots\beta_m}(p). \end{aligned} \tag{44}$$

The matrix elements (43) and (44) are connected by the normal crossing symmetry relations (the substitution law of LOY).

The destruction and creation operators satisfy the normal commutation relations

$$\begin{gathered} \delta(p^2+m^2)\,\delta(p'^2+m^2)\,\theta(p'^\circ)\,[a(pj\nu), a^+(p'j'\nu')]_\pm = \\ \delta(p^2+m^2)\,\delta(p'^2+m^2)\,\theta(p^\circ)\,\theta(p'^\circ)\,[b^+(pj\nu), b(p'j'\nu')]_\pm = \\ \delta(p^2+m^2)\,\delta(p-p')\,\theta(p^\circ)\,\delta_{jj'}\,\delta_{\nu\nu'}. \end{gathered} \tag{45}$$

From these relations we get in the x-representation

$$[\psi_{\alpha_1\alpha_2\ldots\alpha_n}(x), \bar\psi^{\beta_1\ldots\beta_n}(y)]_\pm = \frac{1}{n!}\sum_{P(\alpha_1\ldots\alpha_n)} (\hat\partial+m)^{\beta_1}_{\alpha_1}\ldots(\hat\partial+m)^{\beta_n}_{\alpha_n}\,\Delta(x-y). \tag{46}$$

It is not difficult to see that the commutation relations of the form of Eqs. (45) and (46) are invariant under the group G. The field operators $\psi_{\alpha_1\ldots\alpha_n}(x)$ satisfy the BARGMANN—WIGNER equation which is also invariant under the group G.

The results obtained may be generalized to the case $S = GL(6, C)$ for example. In this case instead of the BARGMANN—WIGNER [8] wave functions we use the broken $U(12)$ wave functions $U^{\beta_1\ldots\beta_m}_{\alpha_1\ldots\alpha_n}(p\,j\,\nu)$, the upper and the lower indices being symmetrized according to some YOUNG table. Field operators are

$$\begin{aligned} \psi^{\beta_1\ldots\beta_m}_{\alpha_1\ldots\alpha_n}(x) = \frac{1}{(2\pi)^{3/2}} \int [&e^{ipx}\, U^{\beta_1\ldots\beta_m}_{\alpha_1\ldots\alpha_n}(pj\nu)\, a(pj\nu) \\ &+ e^{-ipx}\, V^{\beta_1\ldots\beta_m}_{\alpha_1\ldots\alpha_n}(pj\nu)\, b^+(pj\nu)]. \\ &\delta(p^2+m^2)\,\theta(p^\circ)\,d^4p. \end{aligned} \tag{47}$$

In this case, to each irreducible unitary representation of the group $S = GL(6, C)$ we introduce an infinite set of irreducible finite dimensional representations of the broken $\widetilde{U}(12)$ group. The transformation of the form (42) for this case is

$$\psi^{\beta_1 \ldots \beta_m}_{\alpha_1 \ldots \alpha_n}(x) \to \psi'^{\beta_1 \ldots \beta_m}_{\alpha_1 \ldots \alpha_n}(v) = \sum_{p,q} D^{\beta_1 \ldots \beta_m, \delta_1 \ldots \delta_q}_{\alpha_1 \ldots \alpha_n, \gamma_1 \ldots \gamma_p}\left(-i \frac{\partial}{\partial x}\right) \psi^{\gamma_1 \ldots \gamma_p}_{\delta_1 \ldots \delta_q}(x). \tag{48}$$

The new operators $\psi'^{\beta_1 \ldots \beta_m}_{\alpha_1 \ldots \alpha_n}(x)$ have the following transformation properties under the group S

$$X \psi'^{\beta_1 \ldots \beta_m}_{\alpha_1 \ldots \alpha_n} X^{-1} = \sum_{p,q} X^{\beta_1 \ldots \beta_m, \delta_1 \ldots \delta q}_{\alpha_1 \ldots \alpha_n, \gamma_1 \ldots \gamma p} \left(-i \frac{\partial}{\partial x}\right) \psi'^{\gamma_1 \ldots \gamma_p}_{\delta_1 \ldots \delta_q}(x), \tag{49}$$

where

$$X^{\beta_1 \ldots \beta_m, \delta_1 \ldots \delta_q}_{\alpha_1 \ldots \alpha_n, \gamma_1 \ldots \gamma_p}(p) = \sum_{v,v'} U^{\beta_1 \ldots \beta_m}_{\alpha_1 \ldots \alpha_n}(pj\nu)\, x_{j\nu, j'\nu'}\, \overline{U}^{\delta_1 \ldots \delta_q}_{\gamma_1 \ldots \gamma_p}(pj' \nu'), \tag{50}$$

$x_{j\nu, j'\nu'}$ being p-independent if X is p-independent.

We can show that the wave equations for the field operators are invariant under the transformation of the form of Eq. (49), i.e. under the group S. The field operators satisfy the following commutation relations

$$[\psi^{\beta_1 \ldots \beta_m}_{\alpha_1 \ldots \alpha_n}(x), \overline{\psi}^{\alpha'_2 \ldots \alpha'_n}_{\beta'_1 \ldots \beta'_m}(y)]_{\pm} = \frac{1}{n!} \frac{1}{m!} \sum_{P(\alpha_i, \beta_i)} (\hat{\partial} + m)^{\alpha'_1}_{\alpha_1} \ldots (\hat{\partial} + m)^{\alpha'_n}_{\alpha_n} (-\hat{\partial} + m)^{\beta_1}_{\beta'_1} \ldots (-\hat{\partial} + m)^{\beta_m}_{\beta'_m} \Delta(x-y), \tag{51}$$

which are also invariant under the group G. Here the $+$ sign is used for fermions, and the $-$ sign is for bosons.

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НЕКОМПАКТНЫЕ ГРУППЫ СИММЕТРИИ, УНИТАРНАЯ *S*-МАТРИЦА И КВАНТОВАЯ ТЕОРИЯ ПОЛЯ

НГУЕН ВАН ХЬЕУ

Резюме

Изучаются некомпактные группы симметрии и их связь с теорией поля. Явно доказано, что в симметрии с некомпактной группой нет никакого противоречия с унитарностью *S*-матрицы. Предлагается метод для описания симметрии с некомпактной группой с помощью аппарата локальной квантовой теории поля. В нашем методе операторы полей удовлетворяют нормальным перестановочным соотношениям, и сохраняется кроссинг-симметрия *S*-матрицы.

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ON THE THEORY OF UNITARY REPRESENTATIONS OF THE *SL*(2*C*) GROUP

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The irreducible unitary representation of the noncompact $SL(2, C)$ is discussed by a method based on the use of the homogeneous functions. The matrix elements of the finite transformations are calculated explicitly. The method is very convenient for applications in physics and the results of GELFAND and NAIMARK are obtained in a very simple way.

§ 1. Introduction

The possibility of using the unitary representations of non-compact groups to classify the elementary particles has been discussed in a series of papers (BARUT, BUDINI and FRONSDAL [1], DOTHAN, GELL-MANN and NE'EMAN [2], FRONSDAL [3], DELBOURGO, SALAM and STRATHDEE [4], RUHL [5], MICHEL [6], TODOROV [7] and NGUYEN VAN HIEU [8]). It has been shown that in the symmetry theory with the group G

$$G = P \boxed{\sim} S, \quad S \supset SL(2, C)$$

which is the semi-direct product of the Poincaré group P and some interna symmetry non-compact group S containing some $SL(2, C)$ subgroup, no contradiction with the unitarity condition for S-matrix exists [8], and in this theory we can introduce field operators in such a manner that the free field operators obey the normal commutation or anticommutation relations (with the normal connection between spins and statistics).

Before studying the experimental consequences of this new symmetry theory we must solve some mathematical problems:

1. To study the irreducible unitary representations of the internal symmetry non-compact group S and the splitting of these representations into the direct sum of the irreducible finite-dimensional representations of the maximal compact subgroup of S.

2. For the unitary representations to calculate the matrix elements of the finite transformations of the group S which correspond to the Lorentz transformations.

These problems have been considered in all the above-mentioned papers, where some partial results have been obtained, but none of which has been solved finally.

In the present work we study the irreducible unitary representation of the $SL(2, C)$ group and calculate the matrix elements of the finite transformations for these representations. The method developed here can be generalized to study the groups $SL(n, C)$ and $SU(p, q)$. We note that the theory of unitary representations of these groups was developed in the work of GELFAND and NAIMARK [9]. The GELFAND—NAIMARK theory is a rigorous one mathematically. However, the method used by GELFAND and NAIMARK is not convenient for physical applications. Here we apply another method based on the use of the homogeneous functions to realize the irreducible unitary representations of non-compact groups. This method is very convenient for applications to physics. The possibility of using the homogeneous functions to study the representations of non-compact groups has been discussed in references [10, 3, 5, 11]. With this method we can obtain the GELFAND—NAIMARK results in a very simple manner.

§ 2. Unitary representations of the $SL(2, C)$ group

$SL(2, C)$ is the group of all 2×2 complex matrices with determinant equal to 1. We now realize the representations of this group in the Hilbert space of the functions $f(z_1, z_2)$ depending on two complex variables z_1 and z_2. For every matrix $g \in SL(2, C)$ we define a corresponding operator T_g in the given Hilbert space of functions $f(z_1, z_2)$:

$$g \rightarrow T_g$$

$$T_g f(z_1, z_2) = f(z_1', z_2')\,; \;\; z_a' = z_b\, g_{ba} \;\; (a, b = 1, 2)\,. \tag{2.1}$$

It is not difficult to prove that

$$T_{g_1} T_{g_2} = T_{g_1 g_2}\,.$$

Therefore the correspondence $g \rightarrow T_g$ is a representation of the group $SL(2, C)$. We now define the scalar product in the Hilbert space of functions $f(z_1, z_2)$:

$$<f_1, f_2> = \left(\frac{i}{2}\right)^2 \int f_1(z_1, z_2)\, \bar{f}_2(z_1, z_2) \cdot dz_1\, d\bar{z}_1 \cdot dz_2\, d\bar{z}_2\,. \tag{2.2}$$

In this case our Hilbert space consists of all square integrable functions of two complex variables. It is not difficult to prove that with this scalar product

(2.2) the operators T_g are unitary:

$$< T_g f_1, T_g f_2 > = < f_1, f_2 > .$$

Thus we obtain a unitary representation of the $SL(2, C)$ group, which is still not irreducible. In order to get the irreducible representations we use the homogeneous functions. A function $f(z_1, z_2)$ is called a homogeneous function of degree (λ_1, λ_2), where λ_1 and λ_2 are complex numbers, if for any complex number $\sigma \neq 0$ we have

$$f(\sigma z_1, \sigma z_2) = \sigma^{\lambda_1} \bar{\sigma}^{\lambda_2} f(z_1, z_2) . \tag{2.3}$$

This definition makes sense only if the difference $\lambda_1 - \lambda_2$ is an integer. From this definition and from (2.1) it follows that if $f(z_1, z_2)$ is a homogeneous function of degree (λ_1, λ_2) then $T_g f(z_1, z_2)$ is also a homogeneous function with the same degree. Thus the Hilbert space D_λ of homogeneous functions of some degree $\lambda = (\lambda_1, \lambda_2)$ realizes a representation of the group $SL(2, C)$.

For the homogeneous functions we cannot use the scalar product defined as in (2.2). Indeed, each function $f(z_1, z_2)$ from D_λ is determined uniquely by a corresponding function of one variable $f(z) \equiv f(z, 1)$, since

$$f(z_1, z_2) = z_2^{\lambda_1} \bar{z}_2^{\lambda_2} f\left(\frac{z_1}{z_2}, 1\right) = z_2^{\lambda_1} \bar{z}_2^{\lambda_2} f\left(\frac{z_1}{z_2}\right) \tag{2.4}$$

and the integral on the right-hand side of (2.2) can be written in the form of the product of two independent integrals

$$\int f_1(z_1, z_2) \overline{f_2(z_1, z_2)} \cdot dz_1 \, d\bar{z}_1 \cdot dz_2 \, d\bar{z}_2 =$$

$$= \int | z_2^{\lambda_1} \bar{z}_2^{\lambda_2} |^2 f_1\left(\frac{z_1}{z_2}\right) \overline{f_2\left(\frac{z_1}{z_2}\right)} \cdot | z_2 |^2 \cdot d\left(\frac{z_1}{z_2}\right) d\overline{\left(\frac{z_1}{z_2}\right)} \cdot dz_2 \, d\bar{z}_2 =$$

$$= \int f_1(z) \overline{f_2(z)} \, dz \, d\bar{z} \cdot \int | z_2^{\lambda_1+1} \bar{z}_2^{\lambda_2} |^2 \, dz_2 \, d\bar{z}_2 .$$

It is not difficult to show that the second integral tends to infinity.

Thus, in the Hilbert space D_λ we must define the scalar product in another manner. Since the homogeneous functions $f(z_1, z_2)$ effectively depend on only one of the variables we must use the complex path integral instead of the complex surface integral to define the scalar product. That is, we define the scalar product in the following manner:

$$< f_1, f_2 > = \frac{i}{2} \int f_1(z_1, z_2) \overline{f_2(z_1, z_2)} \, d\omega_z , \tag{2.5}$$

where $d\omega_z$ is some measure invariant under the transformations

$$z_a \to z'_a = z_b\, g_{ba}\,, \quad \det g = 1\,.$$

It is easy to see that such a measure can be of the form

$$d\omega_z = (z_2\, dz_1 - z_1\, dz_2)\,(\bar z_2\, d\bar z_1 - \bar z_1\, d\bar z_2)\,. \tag{2.6}$$

As in the case of (2.2) it follows from the invariance of the measure that all the operators T_g are unitary with respect to the scalar product (2.5), and the representation of the $SL(2, C)$ group in the Hilbert space D_λ is a unitary one.

The norm of an element $f(z_1, z_2) \in D_\lambda$ is determined according to (2.5):

$$\|f\|^2 \equiv <f,f> = \frac{i}{2}\int |f(z_1, z_2)|^2\, d\omega_z. \tag{2.7}$$

Inserting the integral in the right-hand side of (2.7) $z_a = \sigma z'_a$ and using (2.3) we get

$$\|f\|^2 = |\,\sigma^{\lambda_1+\bar\lambda_2+2}\,|^2 \cdot \|f\|^2 \tag{2.8}$$

for any complex $\sigma \neq 0$. This relation shows that λ'_1 and λ'_2 must satisfy the equation

$$\lambda_1 + \bar\lambda_2 + 2 = 0 \tag{2.9}$$

the solution of which is

$$\lambda_1 = \nu_0 + \frac{i\varrho}{2} - 1\,,$$

$$\lambda_2 = -\,\nu_0 + \frac{i\varrho}{2} - 1\,,$$

where ν_0 and ϱ are real numbers. Since $\lambda_1 - \lambda_2 = 2\,\nu_0$ must be an integer number, then ν_0 is an integer or half-integer number. Thus we have obtained the unitary representations of the $SL(2, C)$ group in the Hilbert space D_λ of homogeneous functions of degrees $\lambda = \left(\nu_0 + \frac{i\varrho}{2} - 1,\ -\nu_0 + \frac{i\varrho}{2} - 1\right)$, where ν_0 is any integer or half-integer number, and ϱ is any real number. These representations will be denoted by $\mathfrak{S}_{\nu_0\varrho}$. They are irreducible [10] and form the so-called principal series. Together with this principal series there also exists the supplementary one [9, 10, 12] which can be considered in a similar manner. However, we shall not study this series here.

Finally, we note that from (2.6) and (2.9) it follows that for the representations of the principal series the scalar product defined by (2.5) is identical to the scalar product introduced by GELFAND and NAIMARK

$$<f_1,f_2> = \frac{i}{2}\int f_1(z)\,\overline{f_2(z)}\, dz\, d\bar z\,. \tag{2.10}$$

§ 3. Equivalent representations. Splitting the unitary representations of the $SL(2, C)$ group into direct sums of the representations of the $SU(2)$ subgroup

Let $\mathfrak{S}_{\nu_0\varrho}$ (with operators T_g) and $\mathfrak{S}_{\nu_0'\varrho'}$ (with operators T_g') be two irreducible unitary representations of the group $SL(2, C)$ which are realized in the Hilbert spaces of homogeneous functions:

$$D_{\lambda=\left(\nu_0+\frac{i\varrho}{2}-1,\,-\nu_0+\frac{i\varrho}{2}-1\right)} \quad \text{and} \quad D_{\lambda'=\left(\nu_0'+\frac{i\varrho'}{2}-1,\,-\nu_0'+\frac{i\varrho'}{2}-1\right)},$$

respectively. We now find the conditions for the equivalence of these two representations. The representations $\mathfrak{S}_{\nu_0\varrho}$ and $\mathfrak{S}_{\nu_0'\varrho'}$ are called equivalent if there exists an operator A which realizes a one-to-one mapping D_λ onto $D_{\lambda'}$, such that

$$T_g' A = A T_g \tag{3.1}$$

for any $g \in SL(2, C)$. From this definition we see immediately that if $\lambda = \lambda'$ (i.e. $\nu_0 = \nu_0'$, $\varrho = \varrho'$) then $\mathfrak{S}_{\nu_0\varrho}$ and $\mathfrak{S}_{\nu_0'\varrho'}$ are equivalent. This case is trivial and of no interest, because here D_λ and $D_{\lambda'}$ coincide, $A = 1$.

Let $f(\xi_1, \xi_2) \in D_\lambda$, $f'(\eta_1, \eta_2) \in D_{\lambda'}$. We represent the operator A in the form of an integral transformation with some kernel K:

$$f'(\eta_1, \eta_2) = Af(\xi_1, \xi_2) \equiv \frac{i}{2}\int K(\eta_1, \eta_2; \xi_1, \xi_2)\, f(\xi_1, \xi_2)\, d\omega_\xi. \tag{3.2}$$

Then the condition of equivalence (3.1) can be rewritten explicitly in the form

$$T_g' f'(\eta_1, \eta_2) = \frac{i}{2}\int K(\eta_1, \eta_2; \xi_1, \xi_2)\, T_g f(\xi_1, \xi_2)\, d\omega_\xi. \tag{3.3}$$

Using Eq. (2.1) we can rewrite the last equation in the form:

$$f'(\eta_1', \eta_2') = \frac{i}{2}\int K(\eta_1, \eta_2; \xi_1, \xi_2)\, f(\xi_1' \xi_2')\, d\omega_{\xi'}, \tag{3.4}$$

where $\eta_a' = \eta_b g_{ba}$, $\xi_a' = \xi_b' g_{ba}$. Comparing Eqs. (3.2) and (3.4) we get:

$$K(\eta_1', \eta_2';\ \xi_1', \xi_2') = K(\eta_1, \eta_2;\ \xi_1, \xi_2).$$

Thus, the kernel K must be an invariant function of ξ_a and η_a. As is well known in the theory of spinor representations of the group $SL(2, C)$ from the variables ξ_a and η_a we can form the following invariant

$$\xi_1\eta_2 - \xi_2\eta_1 = \text{inv}.$$

The kernel K must be a function of this invariant combination $(\xi_1\eta_2 - \xi_2\eta_1)$. It must also be a homogeneous function since $f'(\eta_1, \eta_2)$ is homogeneous. Let K be a homogeneous function on two variables (ξ_1, ξ_2) of degree (μ_1, μ_2). Then putting $\xi_a = \sigma\xi'_a$ into (3.2) we get:

$$\int K(\eta_1, \eta_2; \xi_1, \xi_2) f(\xi_1, \xi_2)\, d\omega_\xi =$$
$$= (\sigma)^{\mu_1+\lambda_1+2} (\overline{\sigma})^{\mu_2+\lambda_2+2} \int K(\eta_1, \eta_2; \xi'_1, \xi'_2) f(\xi'_1, \xi'_2) d\omega_{\xi'}$$

for any complex $\sigma \neq 0$. Therefore, we must have

$$\mu_1 = -\lambda_1 - 2 = -\nu_0 - \frac{i\varrho}{2} - 1, \qquad \mu_2 = -\lambda_2 - 2 = +\nu_0 + \frac{i\varrho}{2} - 1. \tag{3.5}$$

Thus, $K(\eta_1, \eta_2; \xi_1, \xi_2)$ must be a homogeneous function on (ξ_1, ξ_2) of degr e $\left(-\nu_0 - \frac{i\varrho}{2} - 1, \nu_0 - \frac{i\varrho}{2} - 1\right)$. On the other hand, the kernel $K(\eta_1, \eta_2; \xi_1, \xi_2)$ is a function of the combination $(\xi_1\eta_2 - \xi_2\eta_1)$ and, therefore, it is also a homogeneous function on (η_1, η_2) of the same degree $\left(-\nu_0 - \frac{i\varrho}{2} - 1, \nu_0 - \frac{i\varrho}{2} - 1\right)$. This means that if $\nu'_0 = -\nu_0$, $\varrho' = -\varrho$ then the representations $\mathfrak{S}_{\nu_0\varrho}$ and $\mathfrak{S}_{\nu'_0\varrho'}$ are equivalent.

Consider now the splitting of irreducible unitary representations of the group $SL(2, C)$ into direct sums of the irreducible representations of the maximal compact subgroup $SU(2)$. In the theory of spinor representations of the group $SL(2, C)$ we know that if z_a is transformed as a spinor φ_a then $\overline{z}_a$ is transformed as a spinor $\varphi^{\dot{a}}$ and the sum $z_a\overline{z}_a$ is transformed as the sum $\varphi_a\varphi^{\dot{a}}$ i.e. is an invariant of the $SU(2)$ subgroup. In the following we denote $\overline{z}_a$ by $\overline{z}^{\dot{a}}$ or $z^{\dot{a}}$ for convenience.

For clarity we now illustrate our method by some simple examples. The general case will be considered in the following section. Consider first the representation $\mathfrak{S}_{0\varrho}$. This representation is realized in the Hilbert space $D_{\left(\frac{i\varrho}{2}-1, \frac{i\varrho}{2}-1\right)}$ of homogeneous functions $f(z_1, z_2)$ of degree $\left(\frac{i\varrho}{2} - 1, \frac{i\varrho}{2} - 1\right)$. One of these functions is

$$f_{00}(z_1, z_2) \sim (z_1\overline{z}^1 + z_2\overline{z}^2)^{\frac{i\varrho}{2} - 1}. \tag{3.6}$$

As has been noted, this function is invariant under the $SU(2)$ subgroup and therefore characterizes the spin-zero state. In order to obtain the functions corresponding to the states with non-zero spin we must construct them in such a manner that they contain some factors z_a and $\bar{z}^b$ without summation. It is not difficult to see that for the spin 1 states we have the following functions:

$$f_{11}(z_1, z_2) \sim (z_1 \bar{z}^1 + z_2 \bar{z}^2)^{\frac{i\varrho}{2} - 2} z_1 \bar{z}^2 \quad \text{for } j = 1,\ m = 1, \tag{3.7}$$

$$f_{10}(z_1, z_2) \sim (z_1 \bar{z}^1 + z_2 \bar{z}^2)^{\frac{i\varrho}{2} - 2} (z_2 \bar{z}^2 - z_1 \bar{z}^1) \quad \text{for } j = 1,\ m = 0, \tag{3.8}$$

$$f_{1,-1}(z_1, z_2) \sim (z_1 \bar{z}^1 + z_2 \bar{z}^2)^{\frac{i\varrho}{2} - 2} z_2 \bar{z}^1 \quad \text{for } j = 1, m = -1. \tag{3.9}$$

(j and m denote the spin and its projection on the $z -$ axis.)

Consider now the representations $\mathfrak{S}_{\nu_0\varrho}$ ($\nu_0 \neq 0$). Since the representations $\mathfrak{S}_{\nu_0\varrho}$ and $\mathfrak{S}_{-\nu_0, -\varrho}$ are equivalent then we can assume that $\nu_0 > 0$. We choose the basis elements of the representations $\mathfrak{S}_{\nu_0\varrho}$ in the form of the products of the quantity $(z_1\bar{z}^1 + z_2\bar{z}^2)^{\frac{i\varrho}{2} - 2}$ $(n \geqslant \nu_0 + 1)$ and some factors z_a and $\bar{z}_b$ without summation. The products with the minimal number of free factors z_a and $\bar{z}^b$ are of the form

$$f(z_1, z_2) \sim (z_1 \bar{z}^1 + z_2 \bar{z}^2)^{\frac{i\varrho}{2} - 1 - \nu_0} z_{a_1} z_{a_2} \ldots z_{a_{2\nu_0}}: \tag{3.10}$$

These functions describe the states with spin $j_0 = \nu_0$. The other functions correspond to the states with spins $j = \nu_0 + 1, \nu_0 + 2, \ldots$ Thus, the representation $\mathfrak{S}_{\nu_0\varrho}$ splits into the direct sum of irreducible finite-dimensional representations of the $SU(2)$ subgroup. Each of them is contained in given representation $\mathfrak{S}_{\nu_0\varrho}$ once and describes a state with definite spin $j = j_0 + + n \ldots, n = 0, 1, 2, \ldots$.

§ 4. Matrix elements of finite transformations

As was noted in the Introduction, in studying the structure of the vertex parts and the scattering amplitudes we must use the matrix elements of the finite transformations of the group S and in particular of the group $SL(2, C)$. Note that this problem was first considered in the paper by DOLGHINOV and TOPTYGHIN [13] for the case with $\nu_0 = 0$. These authors chose the analytic continuations of the 4-dimensional spherical functions as the basis functions. (see [13, 14]). Our method is based on the results obtained in § 2.

The matrix element $D^{\nu_0\varrho}_{jm;j'm'}(g)$ corresponding to the representation $g \to U_g$ is defined in the following manner:

$$U_g \,|\, \nu_0 \varrho; jm\rangle = \sum_{j'm'} D^{\nu_0\varrho}_{jm, j'm'}(g) \,|\, \nu_0 \varrho; j' m'\rangle , \tag{4.1}$$

where $|\nu_0\varrho; jm>$ is the canonical basis of representation $\mathfrak{S}_{\nu_0\varrho}$; j and m are the spin and its projection on the z — axis. Generalizing the results obtained (see (3.6) — (3.10)) we determine first the canonical basis in the space of homogeneous functions $D_{\lambda=\left(\nu_0+\frac{i\varrho}{2}-1,\, -\nu_0+\frac{i\varrho}{2}-1\right)}$ in the following manner:

$$|\nu_0\,\varrho; jj> \to f_{jj}^{\nu_0\varrho}(z_1, z_2) = c_{jj}\,(z_1\bar{z}^1 + z_2\bar{z}^2)^{\frac{i\varrho}{2}-1-j}(z_1)^{j+\nu_0}(\bar{z}^2)^{j-\nu_0}, \tag{4.2}$$

where C_{jj} are the normalization constants. Using (2.10) we get:

$$c_{jj} = \left\{\frac{i}{2}\int |f_{jj}^{\nu_0\varrho}(z)|^2 dz\,d\bar{z}\right\}^{-1/2} = \frac{1}{\sqrt{\pi}}\left\{\frac{(2j+1)!}{(j+\nu_0)!\,(j-\nu_0)!}\right\}^{1/2}. \tag{4.3}$$

From $|\nu_0\varrho; jj>$ we can find $|\nu_0\varrho; jm>$:

$$|\nu_0\,\varrho\,; jm> \to f_{jm}^{\nu_0\varrho}(z_1, z_2) = N_{jm}\,(I^-)^{j-m} f_{jj}^{\nu_0\varrho}(z_1, z_2)\,, \tag{4.4}$$

where

$$N_{jm} = \left\{\frac{(j+m)!}{(2j)!\,(j-m)!}\right\}^{1/2}, \tag{4.5}$$

$$I^- = z_2\frac{\partial}{\partial z_1} - \bar{z}^1\frac{\partial}{\partial \bar{z}^2}\,. \tag{4.6}$$

From (4.2) — (4.6) we get the final expression for $f_{jm}^{\nu_0\varrho}(z_1, z_2)$:

$$f_{jm}^{\nu_0\varrho}(z_1, z_2) = \frac{1}{\sqrt{\pi}}\{(2j+1)\,(j+m)!\,(j-m)!\,(j+\nu_0)!\,(j-\nu_0)!\}^{1/2}\cdot$$

$$\cdot\,(z_1\bar{z}^1 + z_2\bar{z}^2)^{\frac{i\varrho}{2}-1-j}\sum_d(-1)^d\cdot \tag{4.7}$$

$$\cdot\,\frac{1}{d!\,(j-m-d)!\,(\nu_0+m+d)!(j-\nu_0-d)!}(z_1)^{\nu_0+m+d}(z_2)^{j-m-d}(\bar{z}^1)^d(\bar{z}^2)^{j-\nu_0-d}.$$

Having the explicit expression for the canonical basis $|\nu_0\varrho; jm>$ we can find the matrix elements $D_{jm;\,j'm'}^{\nu_0\varrho}(g)$. It is well known that every matrix g can be represented in the form

$$g = u_1\,\varepsilon\,u_2,$$

where u_1 and u_2 are unitary unimodulary matrices which correspond to the space rotation;

$$\varepsilon = \begin{pmatrix}\varepsilon^{-1} & 0\\ 0 & \varepsilon\end{pmatrix}\quad (\varepsilon\text{—real number})$$

and corresponds to a pure Lorentz transformation in the plane (x_3, x_4). Thus, without losing generality we can consider only the matrix element $D^{\nu_0\varrho}_{jm;\,j'm'}(\varepsilon)$

From (4.1) and from the orthonormalization relations

$$\frac{i}{2}\int f^{\nu_0\varrho}_{jm}(z)\,\overline{f^{\nu_0\varrho}_{j'm'}}(z)\,dz\,d\bar{z} = \delta_{jj'}\cdot\delta_{mm'}$$

we have

$$D^{\nu_0\varrho}_{jm;\,j'm'}(\varepsilon) = \frac{i}{2}\int T_\varepsilon f^{\nu_0\varrho}_{jm}(z)\cdot\overline{f^{\nu_0\varrho}_{j'm'}}(z)\cdot dz\,d\bar{z}. \tag{4.8}$$

From (2.1) and (4.7) we get:

$$D^{\nu_0\varrho}_{jm;\,j'm'}(\varepsilon) =$$

$$= \delta_{mm'}\frac{1}{\pi}\{(2j+1)(2j'+1)(j+m)!\,(j-m)!\,(j+\nu_0)!\,\cdot$$

$$\cdot\,(j-\nu_0)!\,(j'+m)!\,(j'-m)!\,(j'+\nu_0)!\,(j'-\nu_0)!\}^{1/2}\cdot$$

$$\cdot\sum_{d,d'}(-1)^{d+d'}\frac{1}{d!\,d'!\,(j-m-d)!\,(j'-m-d')!\,(\nu_0+m+d)!\cdot(\nu_0+m+d')!\,(j-\nu_0-d)!\,(j'-\nu_0-d')!}\cdot \tag{4.9}$$

$$\cdot\,\varepsilon^{-2\left(2d+m+\nu_0+1-\frac{i\varrho}{2}\right)}\cdot\frac{i}{2}\int dz\,d\bar{z}\,|z|^{2(d+d'+m+\nu_0)}\,\cdot$$

$$\cdot\,(1+|z|^2)^{-\frac{i\varrho}{2}-1-j'}(1+\varepsilon^{-4}|z|^2)^{\frac{i\varrho}{2}-1-j},$$

where d and d' can take any integer number which does not entail each factor under the factorial becoming a negative number. By putting $z = \sqrt{v}\,e^{i\varphi}$ $(0 \leqslant v < \infty;\ 0 \leqslant \varphi \leqslant 2\pi)$ the integral in (4.9) can be rewritten in the form

$$I = \pi\int_0^\infty dv\cdot v^{d+d'+m+\nu_0}(1+v)^{-\frac{i\varrho}{2}-1-j'}(1+\varepsilon^{-4}v)^{\frac{i\varrho}{2}-1-j} =$$

$$= \pi\cdot\varepsilon^{4(d+d'+m+\nu_0+1)}\frac{(d+d'+m+\nu_0)!\,(j+j'-d-d'-m-\nu_0)!}{(j+j'+1)!}\,\cdot$$

$$\cdot\,F\left(j'+1+\frac{i\varrho}{2},\,d+d'+m+\nu_0+1;\,j+j'+2;\,1-\varepsilon^4\right), \tag{4.10}$$

where $F(\alpha, \beta; \gamma; z)$ is a hypergeometric function. By setting (4.10) into (4.9) we get the final result:

$$D^{\nu_0\varrho}_{jm;j'm'}(\varepsilon) =$$
$$= \frac{\delta_{mm'}}{(j+j'+1)!}\{(2j+1)(2j'+1)(j+m)!(j-m)!(j+\nu_0)!\cdot$$
$$\cdot (j-\nu_0)!(j'+m)!(j'-m)!(j'+\nu_0)!(j'-\nu_0)!\}^{1/2}\sum_{d,d'}(-1)^{d+d'}\cdot$$
$$\cdot\frac{(d+d'+m+\nu_0)!(j+j'-d-d'-m-\nu_0)!}{d!d'!(j-m-d)!(j'-m-d')!(\nu_0+m+d)!(\nu_0+m+d')!\cdot(j-\nu_0-d)!(j'-\nu_0-d')!}\cdot \tag{4.11}$$
$$\cdot\varepsilon^{2\left(2d'+m+\nu_0+1+\frac{i\varrho}{2}\right)}F\left(j'+1+\frac{i\varrho}{2}, d+d'+m+\nu_0+1; j+j'+2; 1-\varepsilon^4\right)\cdot$$

This result exactly coincides with that obtained earlier by the authors in another way [15].

Now we note some simple properties of $D^{\nu_0\varrho}_{jm\;j'm'}(\varepsilon)$.

1. Putting in (4.11) $\varepsilon = 1$ and taking into account $F(\alpha, \beta; \gamma; 0) = 1$ we have:

$$D^{\nu_0\varrho}_{jm;j'm'}(1) = \delta_{jj'}\,\delta_{mm'}. \tag{4.12}$$

This is a trivial relation. It means that we deal here with the identity transformation.

2. Making a permutation of jm and $j'm'$ in (4.11) and using the properties of the homogeneous functions

$$F(\alpha, \beta; \gamma; z) = (1-z)^{-\beta}F\left(\gamma-\alpha, \beta; \gamma; \frac{z}{z-1}\right) = $$
$$= (1-z)^{-\alpha}F\left(\alpha, \gamma-\beta; \gamma; \frac{z}{z-1}\right), \tag{4.13}$$

we obtain

$$D^{\nu_0\varrho}_{j'm';jm}(\varepsilon) = \overline{D^{\nu_0\varrho}_{jm;j'm'}}(\varepsilon^{-1}). \tag{4.14}$$

3. Making the permutation $m \to -m$, $m' \to -m'$ in (4.14), putting $j-\nu_0-d \equiv d_1$, $j'-\nu_0-d' \equiv d_1'$ and using (4.13) we can prove that

$$D^{\nu_0\varrho}_{jm;j'm'}(\varepsilon) = (-1)^{j+j'-2\nu_0}D_{j,-m,j',-m'}(\varepsilon^{-1}). \tag{4.15}$$

4. From (4.12), (4.14) and from group properties of $D_{jm;\,j'm'}$ we get:

$$\sum_{jm} D^{\nu_0\varrho}_{jm,j'm'}(\varepsilon)\,\overline{D^{\nu_0\varrho}_{jm;j''m''}}(\varepsilon) = \delta_{j'j''}\cdot\delta_{m'm''}.$$

This last relation means the unitarity condition of the representation.

§ 5. Generalized tensors

From the canonical basis which was given in (4.7) we go to another basis called generalized tensors of the $SU(2, C)$ group. They are constructed in the following manner:

$$f^{b_1 b_2 \ldots b_{j-\nu_0}}_{a_1 a_2 \ldots a_{j+\nu_0}}(z_1, z_2) = (z_c \bar{z}^c)^{\frac{i\varrho}{2} - 1 - j} \Phi^{b_1 b_2 \ldots b_{j-\nu_0}}_{a_1 a_2 \ldots a_{j+\nu_0}}, \quad (a, b = 1,2), \tag{5.1}$$

where

$$\Phi^{b_1 b_2 \ldots b_t}_{a_1 a_2 \ldots a_{t+k}} = \sum_{s=0}^{t} \alpha(t, s, k) (z_c \bar{z}^c)^s S \delta^{b_1}_{a_1} \delta^{b_2}_{a_2} \ldots \delta^{b_s}_{a_s} z_{a_{s+1}} \ldots z_{a_{t+k}} \bar{z}^{b_{s+1}} \bar{z}^{b_t} \tag{5.2}$$

$$\alpha(t, s, k) = (-1)^s \frac{t!\,(t+k)!\,(2t+k-s)!}{s!\,(t-s)!\,(t+k-s)!\,(2t+k)!}. \tag{5.3}$$

S denotes the symmetrization on the upper and lower indices a and b separately:

$$S(T^{b_1 b_2 \ldots b_j}_{a_1 a_2 \ldots a_i}) = \frac{1}{i!\,j!} \sum_{P(a\ b)} T^{b_1 b_2 \ldots b_j}_{a_1 a_2 \ldots a_i}.$$

($\sum_{P(a\ b)}$ stands for summation over all permutations *af* d and all permutations of b).

The tensors $\Phi^{b_1 b_2 \ldots}_{a_1 a_2 \ldots}$ are symmetrical in upper and in lower indices and are traceless with respect to contraction of any upper index with any lower index. They are irreducible under the $SU(2)$ subgroup. Putting in (5.1), e.g., $a_1 = a_2 = \ldots = a_{j+\nu_0} = 1$, $b_1 = b_2 = \ldots = b_{j-\nu_0} = 2$ and using (5.1), (5.3) and (4.2) we get:

$$f^{\overbrace{22\ldots2}^{j-\nu_0 \text{ times}}}_{\underbrace{11\ldots1}_{j+\nu_0 \text{ times}}} = \sqrt{\pi} \left\{ \frac{(j+\nu_0)!\,(j-\nu_0)!}{(2j+1)!} \right\}^{1/2} f^{\nu_0 \varrho}_{jj}.$$

The inverse expansion is

$$z_{a_1} z_{a_2} \ldots z_{a_{t+k}} \bar{z}^{b_1} \bar{z}^{b_2} \ldots \bar{z}^{b_t} = \sum_{s=0}^{t} \beta(t, s, k) (z_c \bar{z}^c)^{t-s} S \Phi^{b_1 b_2 \ldots b_s}_{a_1 a_2 \ldots a_{s+k}} \delta^{b_{s+1}}_{a_{s+k+1}} \delta^{b_t}_{a_{t+k}}, \tag{5.4}$$

where

$$\beta(t, s, k) = \frac{t!\,(t+k)!\,(2s+k+1)!}{s!\,(s+k)!\,(t-s)!\,(t+k+s+1)!}. \tag{5.5}$$

Under the transformation g the tensor $f^{b_1 b_2 \ldots b_{j-\nu_0}}_{a_1 a_2 \ldots a_{j+\nu_0}}$ is transformed as

$$f^{b_1 b_2 \ldots b_{j-\nu_0}}_{a_1 a_2 \ldots a_{j+\nu_0}}(z_1, z_2) \to T_g f^{b_1 b_2 \ldots b_{j-\nu_0}}_{a_1 a_2 \ldots a_{j+\nu_0}}(z_1, z_2) =$$

$$= \sum_{j'} D^{b_1 b_2 \ldots b_{j-\nu_0};\ c_1 c_2 \ldots c_{j'+\nu_0}}_{a_1 a_2 \ldots a_{j+0};\ d_1 d_2 \ldots d_{j'-\nu_0}}(g) f^{d_1 d_2 \ldots d_{j'-\nu_0}}_{c_1 c_2 \ldots c_{j'+\nu_0}}(z_1, z_2), \tag{5.6}$$

where the matrix elements $D^{b_1 b_2 \ldots ;\, c_1 c_2 \ldots}_{a_1 a_2 \ldots ;\, d_1 d_2 \ldots}(g)$ are the generalization of $D_{jm;\, j'm'}(g)$. The explicit expressions of these matrix elements will be given in the next section.

Any homogeneous function $\varphi(z_1, z_2)$ from the space $D_{\left(\nu_0 + \frac{i\varrho}{2} - 1, -\nu_0 + \frac{i\varrho}{2} - 1\right)}$ can be represented in the form:

$$\varphi(z_1, z_2) = \sum_{j'} \varphi^{c_1 c_2 \ldots c_{j'+\nu_0}}_{d_1 d_2 \ldots d_{j'-\nu_0}} f^{d_1 d_2 \ldots d_{j'-\nu_0}}_{c_1 c_2 \ldots c_{j'+\nu_0}}(z_1, z_2) . \tag{5.7}$$

The components $\varphi^{c_1 c_2 \ldots}_{d_1 d_2 \ldots}$ are also symmetrical in upper and lower indices and traceless. These quantities will be called generalized tensors. Under g function $\varphi(z_1, z_2)$ transforms into $T_g \varphi(z_1, z_2)$ which can also be represented in the form of (5.7):

$$T_g \varphi(z_1, z_2) = \sum_{j} T_g \varphi^{a_1 a_2 \ldots a_{j+\nu_0}}_{b_1 b_2 \ldots b_{j-\nu_0}} \cdot f^{b_1 b_2 \ldots b_{j-\nu_0}}_{a_1 a_1 \ldots a_{j+\nu_0}}(z_1, z_2) . \tag{5.8}$$

On the other hand, from (5.6) and (5.7) it follows that

$$T_g \varphi(z_1, z_2) = \sum_{j'j} \varphi^{c_1 c_2 \ldots c_{j'+\nu_0}}_{d_1 d_2 \ldots d_{j'-\nu_0}} D^{d_1 d_2 \ldots d_{j'-\nu_0};\, a_1 a_2 \ldots a_{j+\nu_0}}_{c_1 c_2 \ldots c_{j'+\nu_0};\, b_1 b_2 \ldots b_{j-\nu_0}}(g) \cdot f^{b_1 b_2 \ldots b_{j-\nu_0}}_{a_1 a_2 \ldots a_{j+\nu_0}} . \tag{5.9}$$

Comparing (5.8) we find immediately the transformation law for tensors $\varphi^{a_1 a_2 \ldots}_{b_1 b_2 \ldots}$:

$$\varphi^{a_1 a_2 \ldots a_{j+\nu_0}}_{b_1 b_2 \ldots b_{j-\nu_0}} \to T_g \varphi^{a_1 a_2 \ldots a_{j+\nu_0}}_{b_1 b_2 \ldots b_{j-\nu_0}} = \sum_{j'} D^{d_1 d_2 \ldots d_{j'-\nu_0};\, a_1 a_2 \ldots a_{j+\nu_0}}_{c_1 c_2 \ldots c_{j'+\nu_0};\, b_1 b_2 \ldots b_{j-\nu_0}}(g)\, \varphi^{c_1 c_2 \ldots c_{j'+\nu_0}}_{d_1 d_2 \ldots d_{j'-\nu_0}} . \tag{5.10}$$

§ 6. Matrix elements for generalized tensors

Now we determine the matrix elements $D^{b_1 b_2 \ldots ,\, c_1 c_2 \ldots}_{a_1 a_2 \ldots ,\, d_1 d_2 \ldots}(g)$ defined in (56) At first we rewrite (5.1) and (5.2) in the form

$$f^{b_1 b_2 \ldots b_{j-\nu_0}}_{a_1 a_2 \ldots a_{j+\nu_0}}(z_1, z_2) = \sum_{s=0}^{j-\nu_0} \alpha(j-\nu_0, s, 2\nu_0)\, (z_c \bar{z}^c)^{\frac{i\varrho}{2} - 1 - j + s} \cdot$$

$$\cdot S \delta^{b_1}_{a_1} \delta^{b_2}_{a_2} \ldots \delta^{b_s}_{a_s} z_{a_{s+1}} \ldots z_{a_{j+\nu_0}} \bar{z}^{b_{s+1}} \ldots \bar{z}^{b_{j-\nu_0}} . \tag{6.1}$$

Since under the transformation g

$$z_a \to z'_a = z_b\, g_{ba} \equiv z_b\, g^b_a ,$$

$$\bar{z}^a \to \bar{z}'^a = \bar{z}^b\, \bar{g}_{ba} \equiv \bar{z}^b\, \overset{+}{g}{}^a_b ,$$

$$f(z_1, z_2) \to T_g f(z_1, z_2) = f(z'_1, z'_2) ,$$

then the tensor $f_{a_1a_2\ldots}^{b_1b_2\ldots}$ is transformed in the following manner:

$$f_{a_1a_2\ldots a_{j+\nu_0}}^{b_1b_2\ldots b_{j-\nu_0}} \to T_g f_{a_1a_2\ldots a_{j+\nu_0}}^{b_1b_2\ldots b_{j-\nu_0}} =$$

$$= \sum_{s=0}^{j-\nu_0} \alpha(j-\nu_0, s, 2\nu_0) \left\{z_p \bar{z}^q (g \overset{+}{g})_q^p\right\}^{\frac{i\varrho}{2}-1-j+s} \cdot \tag{6.2}$$

$$\cdot z_{c_{s+1}} z_{c_{s+2}} \ldots z_{c_{j+\nu_0}} \bar{z}^{d_{s+1}} \bar{z}^{d_{s+2}} \ldots \bar{z}^{d_{j-\nu_0}} \cdot$$

$$\cdot \underset{(a;b)}{S} \delta_{a_1}^{b_1} \delta_{a_2}^{b_2} \ldots \delta_{a_s}^{b_s} g_{a_{s+1}}^{c_{s+1}} g_{a_{s+2}}^{c_{s+2}} \ldots g_{a_{j+\nu_0}}^{c_{j+\nu_0}} \overset{+}{g}{}_{d_{s+1}}^{b_{s+1}} \overset{+}{g}{}_{d_{s+2}}^{b_{s+2}} \ldots \overset{+}{g}{}_{d_{j-\nu_0}}^{b_{j-\nu_0}} \cdot$$

We represent the 2×2 matrix $g\overset{+}{g}$ in terms of the Pauli matrices:

$$g\overset{+}{g} = \alpha_0 \sigma_0 + \widehat{\alpha} = \alpha_0 (1 + \widehat{\beta}); \; \widehat{\beta} \equiv \frac{1}{\alpha_0} \widehat{\alpha} = \frac{1}{\alpha_0} \vec{\alpha} \cdot \vec{\sigma}. \tag{6.3}$$

Putting this expression for $g\overset{+}{g}$ into $\{z_p \bar{z}^q (g\overset{+}{g})_q^p\}^{\frac{i\varrho}{2}-1-j+s}$ and performing some elementary expansion

$$\left\{z_p \bar{z}^q (g \overset{+}{g})_q^p\right\}^{\frac{i\varrho}{2}-1-j+s} = \alpha_0^{\frac{i\varrho}{2}-1-j+s} \left\{z_c \bar{z}^c + z_p \widehat{\beta}\,{}_q^p \bar{z}^q\right\}^{\frac{i\varrho}{2}-1-j+s} = \tag{6.4}$$

$$= \alpha_0^{\frac{i\varrho}{2}-1-j+s} \sum_{k=0}^{\infty} \frac{\Gamma\left(\frac{i\varrho}{2} - j + s\right)}{k!\,\Gamma\left(\frac{i\varrho}{2} - j + s - k\right)} (z_c \bar{z}^c)^{\frac{i\varrho}{2}-1-j+s-k} (z_p \widehat{\beta}\,{}_q^p \bar{z}^q)^k .$$

we can rewrite (6.2) in the form

$$T_g f_{a_1a_2\ldots a_{j+\nu_0}}^{b_1b_2\ldots b_{j-\nu_0}} = \sum_{s=0}^{j-\nu_0} \sum_{k=0}^{\infty} \alpha(j-\nu_0, s, 2\nu_0) \frac{\Gamma\left(\frac{i\varrho}{2} - j + s\right)}{k!\,\Gamma\left(\frac{i\varrho}{2} - j + s - k\right)} \cdot$$

$$\cdot \alpha_0^{\frac{i\varrho}{2}-1-j+s} (z_c \bar{z}^c)^{\frac{i\varrho}{2}-1-j+\;-k} . \tag{6.5}$$

$$\cdot \widehat{\beta}\,{}_{q_1}^{p_1} \widehat{\beta}\,{}_{q_2}^{p_2} \ldots \widehat{\beta}\,{}_{q_k}^{p_k} z_{p_1} z_{p_2} \ldots z_{p_k} z_{p_{k+1}} z_{p_{k+2}} \ldots z^{p_{k+j+\nu_0-s}} \bar{z}^{q_1} \bar{z}^{q_2} \ldots \bar{z}^{q_k} \bar{z}^{q_{k+1}} \bar{z}^{q_{k+2}} \ldots \bar{z}^{q_{k+j-\nu_0-s}} \cdot$$

$$\cdot \underset{(a,b)}{S} \delta_{a_1}^{b_1} \delta_{a_2}^{b_2} \ldots \delta_{a_s}^{b_s} g_{a_{s+1}}^{p_{k+1}} g_{a_{s+2}}^{p_{k+2}} \ldots g_{a_{j+\nu_0}}^{p_{k+j+\nu_0-s}} \overset{+}{g}{}_{q_{k+1}}^{b_{s+1}} \overset{+}{g}{}_{q_{k+2}}^{b_{s+2}} \ldots \overset{+}{g}{}_{q_{k+j-\nu_0-s}}^{b_{j-\nu_0}} \cdot$$

Here for convenience we have made some changes in the notations of the summation indices:

$$\begin{array}{ll} c_{s+1} \to p_{k+1} & d_{s+1} \to q_{k+1} \\ c_{s+2} \to p_{k+2} & d_{s+2} \to q_{k+2} \\ \ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots \\ c_{j+\nu_0} \to p_{k+j+\nu_0-s} & d_{j-\nu_0} \to q_{k+j-\nu_0-s} \end{array}$$

Now we express the product $z_{p_1} z_{p_2} \ldots z_{p_{k+j+\nu_0-s}} \bar{z}^{q_1} \bar{z}^{q_2} \ldots \bar{z}^{q_{k+j-\nu_0-s}}$ in terms of $\Phi^{q_1 q_2 \ldots}_{p_1 p_2 \ldots}$ by means of (5.4) and we put these expressions into (6.5). We get:

$$T_g f^{b_1 b_2 \ldots b_{j-\nu_0}}_{a_1 a_2 \ldots a_{j+\nu_0}} =$$

$$= \sum_{s=0}^{j-\nu_0} \sum_{k=0}^{\infty} \sum_{r=0}^{k+j-\nu_0-s} \alpha(j-\nu_0, s, 2\nu_0) \cdot \beta(k+j-\nu_0-s, r, 2\nu_0) \cdot$$

$$\cdot \frac{\Gamma\left(\frac{i\varrho}{2}-j+s\right)}{k!\,\Gamma\left(\frac{i\varrho}{2}-j+s-k\right)} \alpha_0^{\frac{i\varrho}{2}-1-j+s} \cdot (z_c \bar{z}^c)^{\frac{i\varrho}{2}-1-\nu_0-r} \hat{\beta}^{p_1}_{q_1} \hat{\beta}^{p_2}_{q_2} \ldots \hat{\beta}^{p_k}_{q_k} \cdot \tag{6.6}$$

$$\cdot \underset{(a;b)}{S}\, \delta^{b_1}_{a_2} \delta^{b_2}_{a_2} \ldots \delta^{b_s}_{a_s} g^{p_{k+1}}_{a_{s+1}} g^{p_{k+2}}_{a_{s+2}} \ldots g^{p_{k+j+\nu_0-s}}_{a_{j+\nu_0}} \overset{+}{g}{}^{b_{s+1}}_{q_{k+1}} \overset{+}{g}{}^{b_{s+2}}_{q_{k+2}} \ldots \overset{+}{g}{}^{b_{j-\nu_0}}_{q_{k+j-\nu_0-s}} \cdot$$

$$\cdot \underset{(a;b)}{S}\, \Phi^{q_1 q_2 \ldots q_r}_{p_1 p_2 \ldots p_{r+2\nu_0}} \delta^{q_{r+1}}_{p_{r+2\nu_0+1}} \delta^{q_{r+2}}_{p_{r+2\nu_0+2}} \ldots \delta^{q_{k+j-\nu_0-s}}_{p_{k+j+\nu_0-s}} .$$

Note that the product

$$\left\{\hat{\beta}^{p_1}_{q_1} \ldots \hat{\beta}^{p_k}_{q_k} g^{p_{k+1}}_{a_{s+1}} g^{p_{k+2}}_{a_{s+2}} \ldots g^{p_{k+j+\nu_0-s}}_{a_{j+\nu_0}} \overset{+}{g}{}^{b_{s+1}}_{q_{k+1}} \overset{+}{g}{}^{b_{s+2}}_{q_{k+2}} \ldots \overset{+}{g}{}^{b_{j-\nu}}_{q_{k+j-\nu_0-s}}\right\} \cdot$$

$$\cdot \sum_{P(p,q)} \left\{\Phi^{q_1 q_2 \ldots q_r}_{p_1 p_2 \ldots p_{r+2\nu_0}} \delta^{q_{r+1}}_{p_{r+2\nu_0+1}} \delta^{q_{r+2}}_{p_{r+2\nu_0+2}} \ldots \delta^{q_{k+j-\nu_0-s}}_{p_{k+j+\nu_0-s}}\right\}$$

can also be represented in the following manner

$$\Phi^{q_1 q_2 \ldots q_r}_{p_1 p_2 \ldots p_{r+2\nu_0}} \delta^{q_{r+1}}_{p_{r+2\nu_0+1}} \delta^{q_{r+2}}_{p_{r+2\nu_0+2}} \ldots \delta^{q_{k+j-\nu_0-s}}_{p_{k+j+\nu_0-s}} \cdot$$

$$\cdot \sum_{P(p,q)} \hat{\beta}^{p_1}_{q_1} \ldots \hat{\beta}^{p_k}_{q_k} g^{p_{k+1}}_{a_{s+1}} g^{p_{k+2}}_{a_{s+2}} \ldots g^{p_{k+j+\nu_0-s}}_{a_{j+\nu_0}} \overset{+}{g}{}^{b_{s+1}}_{q_{k+1}} \overset{+}{g}{}^{b_{s+2}}_{q_{k+2}} \ldots \overset{+}{g}{}^{b_{j-\nu_0}}_{q_{k+j-\nu_0-s}} .$$

Therefore, we can rewrite (6.6) in a more convenient form

$$T_g f^{b_1 b_2 \ldots b_{j-\nu_0}}_{a_1 a_2 \ldots a_{j+\nu_0}} =$$

$$= \sum_{s=0}^{j-\nu_0} \sum_{k=0}^{\infty} \sum_{j'=\nu_0}^{k+j-s} (z_c \bar{z}^c)^{\frac{i\varrho}{2}-1-j'} \Phi^{q_1 q_2 \ldots q_{j'-\nu_0}}_{p_1 p_2 \ldots p_{j'+\nu_0}} \cdot \alpha_0^{\frac{i\varrho}{2}-1-j+s} .$$

$$\cdot \alpha(j-\nu_0, s, 2\nu_0)\ \beta(k+j-\nu_0-s, j'-\nu_0, 2\nu_0) \frac{\Gamma\left(\frac{i\varrho}{2}-j+s\right)}{k!\,\Gamma\left(\frac{i\varrho}{2}-j+s-k\right)} \cdot \tag{6.7}$$

$$\cdot \delta^{q_{j'-\nu_0+1}}_{p_{j'+\nu_0+1}} \delta^{q_{j'-\nu_0+2}}_{p_{j'+\nu_0+2}} \ldots \delta^{q_{k+j-\nu_0-s}}_{p_{k+j+\nu_0-s}} \frac{1}{(j-\nu_0)!\,(j+\nu_0)!\,(k+j-\nu_0-s)!\,(k+j+\nu_0-s)!} \cdot$$

$$\sum_{P(a;b;p;q)} \hat{\beta}^{p_1}_{q_1} \ldots \hat{\beta}^{p_k}_{q_k} \delta^{b_1}_{a_1} \ldots \delta^{b_s}_{a_s} g^{p_{k+1}}_{a_{s+1}} \ldots g^{p_{k+j+\nu_0-s}}_{a_{j+\nu_0}} \overset{+}{g}{}^{b_{s+1}}_{q_{k+1}} \ldots \overset{+}{g}{}^{b_{j-\nu_0}}_{q_{k+j-\nu_0-s}} ,$$

where we put $r + \nu_0 \equiv j'$. From (6.7) and from definitions (5.1) and (5.6) we get the following expression for $D^{b_1 b_2 \ldots,\, c_1 c_2 \ldots}_{a_1 a_2 \ldots,\, d_1 d_2 \ldots}(g)$:

$$D^{b_1 b_2 \ldots b_{j-\nu_0};\, c_1 c_2 \ldots c_{j'+\nu_0}}_{a_1 a_2 \ldots a_{j+\nu_0};\, d_1 d_2 \ldots d_{j'-\nu_0}}(g) =$$

$$= \sum_{k=\max(0,\, j'-j)}^{\infty} \sum_{s=0}^{\min(j-\nu_0,\, j+k-j')} \alpha_0^{\frac{i\varrho}{2}-1-j+} \alpha(j-\nu_0, s, 2\nu_0) \cdot$$

$$\cdot \beta(k+j-\nu_0-s, j'-\nu_0, 2\nu_0) \frac{\Gamma\left(\frac{i\varrho}{2}-j+s\right)}{k!\,\Gamma\left(\frac{i\varrho}{2}-j+s-k\right)} \cdot \tag{6.8}$$

$$\cdot \frac{1}{(j-\nu_0)!\,(j+\nu_0)!\,(k+j-\nu_0-s)!\,(k+j+\nu_0-s)!} \delta^{d_{j'-\nu_0+1}}_{c_{j'+\nu_0+1}} \delta^{d_{j'-\nu_0+2}}_{c_{j'+\nu_0+2}} \ldots \delta^{d_{k+j-\nu_0-s}}_{c_{k+j+\nu_0-s}} \cdot$$

$$\cdot \sum_{(a;\, b;\, c;\, d)} \hat\beta^{c_1}_{d_1} \ldots \hat\beta^{c_k}_{d_k} \delta^{b_1}_{a_1} \ldots \delta^{b_s}_{a_s} g^{c_{k+1}}_{a_{s+1}} \ldots g^{c_{k+j+\nu_0-s}}_{a_{j+\nu_0}} \overset{+}{g}{}^{b_{s+1}}_{d_{k+1}} \ldots \overset{+}{g}{}^{b_{j-\nu_0}}_{d_{k+j-\nu_0-s}} \cdot$$

Putting here the explicit expressions (5.3) and (5.5) for α and β we obtain the final result:

$$D^{b_1 b_2 \ldots b_{j-\nu_0}\, c_1 c_2 \ldots c_{j'+\nu_0}}_{a_1 a_2 \ldots a_{j+\nu_0}\, d_1 d_2 \ldots d_{j'-\nu_0}}(g) =$$

$$= \sum_{k=\max(0,\, j'-j)}^{\infty} \sum_{0=s}^{\min(j-\nu_0,\, j+k-j')} (-1)^s \frac{(2j-s)!\,(2j'+1)!}{\begin{gathered}s!\,k!(2j)!\,(j'-\nu_0)!\,(j'+\nu_0)!\,(j-\nu_0-s)!\\ (j+\nu_0-s)!\,(k+j-j'-s)!\,(k+j+j'-s+1)!\end{gathered}} \cdot$$

$$\cdot \frac{\Gamma\left(\frac{i\varrho}{2}-j+s\right)}{\Gamma\left(\frac{i\varrho}{2}-j+s-k\right)} \alpha_0^{\frac{i\varrho}{2}-1-j+s} \delta^{d_{j'-\nu_0+1}}_{c_{j'+\nu_0+1}} \delta^{d_{j'-\nu_0+2}}_{c_{j'+\nu_0+2}} \ldots \delta^{d_{k+j-\nu_0-s}}_{c_{k+j+\nu_0-s}} \cdot \tag{6.9}$$

$$\cdot \sum_{P(a;\, b,\, c,\, d)} \hat\beta^{c_1}_{d_1} \ldots \hat\beta^{c_k}_{d_k} \delta^{b_1}_{a_1} \ldots \delta^{b_s}_{a_s} g^{c_{k+1}}_{a_{s+1}} \ldots g^{c_{k+j+\nu_0-s}}_{a_{j+\nu_0}} \overset{+}{g}{}^{b_{s+1}}_{d_{k+1}} \ldots \overset{+}{g}{}^{b_{j-\nu_0}}_{d_{k+j-\nu_0-s}} \cdot$$

Consider now some particular cases:

1. If g is a pure Lorentz transformation in the (x_3, x_4) plane, i.e. $g = \varepsilon = \begin{pmatrix} \varepsilon^{-1} & 0 \\ 0 & \varepsilon \end{pmatrix}$, then

$$g\overset{+}{g} = \begin{pmatrix} \varepsilon^{-2} & 0 \\ 0 & \varepsilon^2 \end{pmatrix} = \frac{1}{2}(\varepsilon^{-2}+\varepsilon^2)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2}(\varepsilon^{-2}-\varepsilon^2)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and therefore

$$\alpha_0 = \frac{1}{2}(\varepsilon^{-2}+\varepsilon^2);\ \hat\beta = \frac{\varepsilon^{-2}-\varepsilon^2}{\varepsilon^{-2}+\varepsilon^2}\sigma_3. \tag{6.10}$$

Putting (6.10) into (6.9) we get:

$$D^{b_1 b_2 \ldots b_{j-\nu_0};\, c_1 b_2 \ldots c_{j'+\nu_0}}_{a_1 a_2 \ldots a_{j+\nu_0};\, d_1 d_2 \ldots d_{j'-\nu_0}}(\varepsilon) =$$

$$= \sum_{k=\max(0,\, j'-j)}^{\infty} \; \sum_{s=0}^{\min(j-\nu_0,\, j+k-j')} \; \sum_{m=0}^{\infty} (-1)^{s+m} \cdot$$

$$\cdot \frac{(2j-s)!\,(2j'+1)!}{m!\, s!\, k!\,(2j)!\,(j'-\nu_0)!\,(j'+\nu_0)!\,(j-\nu_0-s)!\,(j+\nu_0-s)!\,(k+j-j'-s)!\,(k+j+j'-s+1)!} \cdot \tag{6.11}$$

$$\cdot \frac{1}{2^{k+m}} \cdot \frac{\Gamma\left(\frac{i\varrho}{2} - j + s\right)}{\Gamma\left(\frac{i\varrho}{2} - j + s - k - m\right)} \varepsilon^{2\left(j-s+1-\frac{i\varrho}{2}\right)} (1-\varepsilon^4)^{m+k} .$$

$$\cdot\, \delta^{d_{j'-\nu_0-1}}_{c_{j'+\nu_0+1}} \delta^{d_{j'-\nu_0+2}}_{c_{j'+\nu_0+2}} \cdots \delta^{d_{k+j-\nu_0-s}}_{c_{k+j+\nu_0-s}} \cdot$$

$$\cdot \sum_{P(a,\, b;\, c;\, d)} (\sigma_3)^{c_1}_{d_1} \cdots (\sigma_3)^{c_k}_{d_k} \delta^{b_1}_{a_1} \cdots \delta^{b_s}_{a_s} \varepsilon^{c_{k+1}}_{a_{s+1}} \cdots \varepsilon^{c_{k+j+\nu_0-s}}_{a_{j+\nu_0}} \varepsilon^{c_{s+1}}_{d_{k+1}} \cdots \varepsilon^{b_{j+\nu_0}}_{d_{k+j-\nu_0-s}} .$$

If we put $j = j'$ in Eq. (6.11) and

$$a_1 = a_2 = \ldots = a_{j+\nu_0} = c_1 = c_2 = \ldots = c_{j+\nu_0} = 1 ,$$
$$b_1 = b_2 = \ldots = b_{j-\nu_0} = d_1 = d_2 = \ldots = d_{j-\nu_0} = 2 ,$$

then we have

$$D^{\nu_0 \varrho}_{jj,\, jj}(\varepsilon) = \overset{j-\nu_0 \text{ times}}{\overbrace{22\ldots2;}} \underset{j+\nu_0 \text{ times}}{\underbrace{11\ldots1;}} \;\overset{j+\nu_0 \text{ times}}{\overbrace{11\ldots1}} \underset{j-\nu_0 \text{ times}}{\underbrace{22\ldots2}} (\varepsilon) = \varepsilon^{2\left(j-\nu_0+1-\frac{i\varrho}{2}\right)} .$$

$$\cdot F\left(j+1-\frac{i\varrho}{2},\, j-\nu_0+1;\; 2j+2;\; 1-\varepsilon^4\right) .$$

This result can be obtained also from (4.11).

2. If g is a Lorentz transformation from the rest frame of a particle with mass m_0 to the frame in which this particle has momentum $p_\mu = (\vec{p}, iE)$ then

$$g = \frac{E + m_0 - \vec{\sigma}\vec{p}}{\sqrt{2m_0(E+m_0)}} ; \; g\overset{+}{g} = \frac{E - \vec{\sigma}\vec{p}}{m_0}$$

and therefore

$$\alpha_0 = \frac{E}{m_0} ; \; \hat{\beta} = -\frac{1}{E}\vec{\sigma}\vec{p} .$$

3. In the case $j = 0$ ($\nu_0 = 0$) formula (6.9) becomes

$$D^{;c_1c_2\ldots c_{j'}}_{;d_1d_2\ldots d_{j'}}(g) = \alpha_0^{\frac{i\varrho}{2}-1} \sum_{k=j'}^{\infty} \frac{(2j'+1)!}{(j'!)^2(k-j')!\,(k+j'+1)!} \cdot$$

$$\cdot \frac{\Gamma\left(\frac{i\varrho}{2}\right)}{\Gamma\left(\frac{i\varrho}{2}-k\right)} \delta^{d_{j'+1}}_{c_{j'+1}} \delta^{d_{j'+2}}_{c_{j'+2}} \ldots \delta^{d_k}_{c_k} \sum_{P(c)} \hat{\beta}^{c_1}_{d_1}\, \hat{\beta}^{c_2}_{d_2} \ldots \hat{\beta}^{c_k}_{d_k} \,.$$

§ 7. Most degenerate principal series of $SL(n, C)$

For the most degenerate principal series of $SL(n, C)$ we can immediately use the method developed in §§ 5,6, for the group $SL(2, C)$ with some slight changes. Thus, instead of Eq. (5.1) we have

$$f^{b_1b_2\ldots b_{j-\nu_0}}_{a_1a_2\ldots a_{j+\nu_0}}(z_1, \ldots, z_n) = (z_c \bar{z}^c)^{\frac{i\varrho}{2}-\frac{n}{2}-j}\, \Phi^{b_1b_2\ldots b_{j-\nu_0}}_{a_1a_2\ldots a_{j+\nu_0}}, \quad (a, b = 1, 2, \ldots, n)$$

where j, ν_0 are also integer or half-integer, however, j still does not denote the spin. The formulae (5.2) and (5.4) remain if instead of the expressions (5.3) and (5.5) for α and β we take

$$\alpha(t, s, k) = (-1)^s \frac{t!\,(t+k)!\,(2t+k+n-2-s)!}{s!\,(t-s)!\,(t+k-s)!\,(2t+k+n-2)!}\,,$$

$$\beta(t, s, k) = \frac{t!\,(t+k)!\,(2s+k+n-1)!}{s!\,(s+k)!\,(t-s)!\,(t+k+n+s-1)!}\,.$$

In order to obtain the formula for matrix elements

$$D^{b_1b_2\ldots b_{j-\nu_0};\,c_1c_2\ldots c_{j'+\nu_0}}_{a_1a_2\ldots a_{j+\nu_0};\,d_1d_2\ldots d_{j'-\nu_0}}(g)$$

corresponding to the transformation $z_a \to z'_a = z_b\, g_{ba}$ we make the analogous procedure as in § 6. The only difference is that we must now expand gg^+ in terms of the matrices generators λ_i of the subgroup $SU(n)$. The result is:

$$D^{b_1b_2\ldots b_{j-\nu_0};\,c_1c_2\ldots c_{j'+\nu_0}}_{a_1a_2\ldots a_{j+\nu_0};\,d_1d_2\ldots d_{j'-\nu_0}}(g) =$$

$$= \sum_{k=\max(0,\, j'-j)}^{\infty} \; \sum_{=0}^{\min(j-\nu_0,\; j+k-j')} (-1)^s \cdot$$

$$\cdot \frac{(2j+n-2-s)!\,(2j'+n-1)!}{s!\,k!\,(j-\nu_0-s)!\,(j+\nu_0-s)!\,(2j+n-2)!\,(j'-\nu_0)!\,(j'+\nu_0)! \;\; !(k+j-j'-s)!\,(k+j+j'-s+n-1)!}\,.$$

$$\cdot \frac{\Gamma\left(\frac{i\varrho}{2} - \frac{n}{2} - j + s + 1\right)}{\Gamma\left(\frac{i\varrho}{2} - \frac{n}{2} - j + s + 1 - k\right)} \alpha_0^{\frac{i\varrho}{2} - \frac{n}{2} - j + s} \cdot \delta^{d_{j'-\nu_0+1}}_{c_{j'+\nu_0+1}} \delta^{d_{j'-\nu_0+2}}_{c_{j'+\nu_0+2}} \dots \delta^{d_{k+j-\nu_0-s}}_{c_{k+j+\nu_0-s}} \cdot$$

$$\cdot \sum_{P(a,\, b;\, c;\, d)} \hat{\beta}^{c_1}_{d_1} \hat{\beta}^{c_2}_{d_2} \dots \hat{\beta}^{c_k}_{d_k} \delta^{b_1}_{a_1} \delta^{b_2}_{a_2} \dots \delta^{b_s}_{a_s} g^{c_{k+1}}_{a_{s+1}} \dots g^{c_{k+j+\nu_0-s}}_{a_{j+\nu_0}} \overset{+}{g}^{b_{s+1}}_{d_{k+1}} \dots \overset{+}{g}^{b_{j-\nu_0}}_{d_{k+j-\nu_0-s}},$$

where

$$g\overset{+}{g} = \alpha_0 I + \sum_{i=1}^{n^2-1} \alpha_i \lambda_i = \alpha_0 (1 + \hat{\beta}); \quad \hat{\beta} \equiv \frac{1}{\alpha_0} \sum_{i=1}^{n^2-1} \alpha_i \lambda_i .$$

§ 8. Space reflection for the group $SL(2, C)$

Now we identify the group $SL(2, C)$ with the homogeneous proper Lorentz group and consider the space reflection P. The following relations exist between P and the generators of the $SL(2, C)$ group:

$$\begin{aligned} PM^i_j P^{-1} &= M^i_j , \\ PN^i_j P^{-1} &= -N^i_j , \\ P^2 &= 1 , \end{aligned} \tag{8,1}$$

where M^i_j and N^i_j are compact and non-compact generators, respectively.

As is known, the commutation relations for M^i_j and N^i_j are of the form:

$$\begin{aligned} [M^i_j, M^k_l] &= \delta^i_l M^k_j - \delta^k_j M^i_l , \\ [N^i_j, N^k_l] &= -\delta^i_l M^k_j + \delta^k_j M^i_l , \\ [M^i_j, N^k_l] &= \delta^i_l N^k_j - \delta^k_j N^i_l . \end{aligned} \tag{8.2}$$

It is easy to see that in the space of homogeneous functions with canonical basis (4.7) these generators are[1]:

$$\begin{aligned} M^i_j &= Z_j \frac{\partial}{\partial Z_i} - \bar{Z}^i \frac{\partial}{\partial \bar{Z}^j} - \frac{1}{2} \delta^i_j \left(Z_a \frac{\partial}{\partial Z_a} - \bar{Z}^a \frac{\partial}{\partial \bar{Z}^a} \right), \\ iN^i_j &= Z_j \frac{\partial}{\partial Z_i} + \bar{Z}^i \frac{\partial}{\partial \bar{Z}^j} - \frac{1}{2} \delta^i_j \left(Z_a \frac{\partial}{\partial Z_a} + \bar{Z}^a \frac{\partial}{\partial \bar{Z}^a} \right). \end{aligned} \tag{8.3}$$

[1] The correspondence between our M^i_j, N^i_j and H, F in reference [12] is the following:
$M^1_2 = H_-$, $M^2_1 = H_+$, $M^1_1 = -M^2_2 = H_3$,
$N^1_2 = F_-$, $N^2_1 = F_+$, $N^1_1 = -N^2_2 = F_3$.

From eqs. (8.1) and (8.3) it follows that the operator P acts in such a way that

$$Z_a \to \varepsilon_{ab} \bar{Z}^b ,$$
$$\bar{Z}^a \to Z_b \varepsilon^{ba} ,$$
$$\varepsilon_{ab} = \varepsilon^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

and therefore

$$Pf(Z_1, Z_2; \bar{Z}^1, \bar{Z}^2) = f(\bar{Z}^2, -\bar{Z}^1; -Z_2, Z_1) . \tag{8.4}$$

From eqs. (8.4) and (8.5) after some simple calculations we get:

$$Pf_{jm}^{\nu_0 \varrho}(Z_1, Z_2) = (-1)^{\frac{i\varrho}{2} - 1 - j} f_{jm}^{-\nu_0 \varrho}(Z_1, Z_2) . \tag{8.5}$$

Thus, under P the basis elements of representation $\mathfrak{S}_{\nu_0 \varrho}$ transform into the basis elements of $\mathfrak{S}_{-\nu\,\varrho}$ which in its turn is equivalent to $\mathfrak{S}_{\nu_0,\,-\varrho}$. This means that under P only the space $D_{\left(\frac{i\varrho}{2} - 1,\, \frac{i\varrho}{2} - 1\right)}$ $(\nu_0 = 0)$ or the space $D_{(\nu_0 - 1,\, -\nu_0 - 1)}$ $(\varrho = 0)$ transforms into itself. Moreover, it is seen from (8.5) that parities of the basis vectors f_{jm} differ from one another by a factor $(-1)^j$.

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К ТЕОРИИ УНИТАРНЫХ ПРЕДСТАВЛЕНИЙ ГРУППЫ SL (2, C)

ДАО ВОНГ ДЫК и НГУЕН ВАН ХЬЕУ

Резюме

В работе рассматривается неприводимое унитарное представление некомпактной группы SL (2, C) методом, основанным на применении однородных функций. Матричные элементы конечных преобразований вычисляются точно. Метод является очень подходящим для применений в физике и результаты Гельфанда и Наймарка получены очень простым путём.

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DECAYS OF THE POSITIVE PARITY MESONS IN $SU(6)_W$

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$SU(6)_W$ invariant amplitudes are given for the decays of the $\underline{189}$ and $\underline{405}$ meson multiplets into two pseudoscalar mesons and into one vector meson and one pseudoscalar meson. The decay widths and branching ratios of the 2^+ mesons are calculated; the results obtained for $\underline{189}$ are closer to the experimental data.

In a previous paper [1] we tried to classify the positive parity meson resonances according to the $\underline{189}$ and $\underline{405}$ representations of $SU(6)$. The classification was based on simple mass formulae [2—4]. We have concluded that $\underline{405}$ seems to be a more suitable choice, in so far as it allows a great many of the observed resonances to be identified. The existence of most of the reported positive parity resonant states [5] is, however, rather questionable [6], the 2^+ being the only well established multiplet; therefore, the classification based on mass formulae alone is not convincing enough. The additional information which is needed can be provided by the investigation of decay widths and branching ratios.

In order to obtain relations for the decay amplitudes in $\underline{189}$ and $\underline{405}$, we use the $SU(6)_W$ formalism [7]. The $SU(6)_S$ states are linear combinations of $SU(6)_W$ states from various $SU(6)_W$ representations, occurring in the $U(6) \otimes U(6) \supset SU(6)$ decomposition

$$(\underline{15}, \underline{15}^*) = \underline{189} + \underline{35} + \underline{1}$$

in the case of $\underline{189}$, and

$$(\underline{21}, \underline{21}^*) = \underline{405} + \underline{35} + \underline{1}$$

in the case of $\underline{405}$. The $SU(6)_S - SU(6)_W$ mixing for $\underline{189}$ and $\underline{405}$ is given in Tables I and II, respectively. (For the definition of $\underline{8}_F$ and $\underline{8}_D$ see [1].) In Table III we give the $SU(6)_W$ invariant amplitudes for all possible decays into two pseudoscalar mesons and into one vector meson and one pseudoscalar meson. (The notations are as follows: $n_a^{m,\mu}$ denotes the $SU(3)$ state $|\, n, a>$, with $2J + 1 = m$, $J_z = \mu$, where J is the spin; $C_{ab}^{27,c}$ etc. are the CG-coefficients for $\underline{8} \otimes \underline{8} = \underline{27} + \underline{10} + \underline{10}^* + \underline{8}^F + \underline{8}^D + 1$; $\delta_a = (-1)^{I_{za}+1/2 \cdot Y_a}$.) The results for the 2^+ mesons are the same as in [8]. The 27^5 mesons in $\underline{405}$ can decay only into two vector mesons or into more than two particles.

The fact that the decays of particles with different spin go with different orbital momenta gives rise to ambiguities when making a quantitative comparison with the experimental data. For this reason we shall discuss only the decays of the 2^+ mesons. According to [1] the 2^+ states which can decay into the two-body states studied here are those given in the following Table:

Mass (MeV)		Mixing	
		$\underline{189}$	$\underline{405}$
K^*	1405	$\|8>$	$1/\sqrt{5}\,(\sqrt{2}\,\|27> - \sqrt{3}\|8>)$
$K^{*\prime}$	1650	—	$1/\sqrt{5}\,(\sqrt{3}\,\|27> + \sqrt{2}\|8>)$
A_2	1324	$\|8>$	$1/\sqrt{5}\,(\|27> - 2\|8>)$
A_2'	1580	—	$1/\sqrt{5}\,(2\|7> + 8>)$
f	1253	$1/\sqrt{3}\,(\|8> + \sqrt{2}\,\|1>)$	$1/\sqrt{10}\,(27> - 2\|8> + \sqrt{5}\|1>)$
f'	1500	$1/\sqrt{3}\,(\sqrt{2}\,\|8> - \|1>)$	$1/\sqrt{15}\,(3\|27>\| - 8> - \sqrt{5}\|1>)$
f''	1670	—	$1/\sqrt{30}\,(3\|27> + 4\|8> + \sqrt{5}\|1>)$

The decays can be described by two parameters both in $\underline{189}$ and $\underline{405}$ (A, B and A', B', respectively). By fitting the width of A_2 and f we get the results summarized in Table IV. The decay rates are corrected by a phase space factor k^5, where k is the momentum of the decay products in the c.m. system.

The widths of the unobserved resonances in $\underline{405}$ are very large. The results for K^* and A_2 are well known from earlier works (see e.g. [8]), and they do not distinguish between $\underline{189}$ and $\underline{405}$. The theoretical branching ratios for the f meson are approximately the same for both representations and are in accordance with experiment. Thus, there remains only one point where the predictions of the two representations differ: the decay of f'. And in this case the $\underline{189}$ seems to be preferred, contrary to the considerations based on mass relations. Now, the arguments given in favour of $\underline{405}$ have been connected with the possibility of identifying the 1^+ mesons and $S^0(700)$. Recently the experimental evidence for these resonances is very bad [6]: the $A_1(1072)$ and $B(1220)$ are almost surely kinematic effects, while $K^*_{3/2}(1270)$, $K\pi\pi(1320)$ and $S^0(700)$ have never been well-established resonant states. On the other hand, the well established $C(1215)$, $D(1286)$ and $E(1410)$ can be assigned to $\underline{189}$ as well as to $\underline{405}$. Clearly, much more experimental data are needed in order to be able to decide which of the two representations is correct, or whether such classification schemes make any sense at all.

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Table I

$SU(6)_S - SU(6)_W$ mixing for <u>189</u>

$SU(6)_S$	$\|SU(3), SU(2); J_z\rangle$	$\|SU(6_w, SU(3), SU(2); J_z\rangle$
189	$\|8,5;2\rangle$	$\|8,5;2\rangle$
	$\|1,5;2\rangle$	$\|1,5;2\rangle$
	$\|8,5;1\rangle$	$1/2(\sqrt{3}\,\|189,8_D,3;1\rangle - \|35,8,3;1\rangle)$
	$\|1,5;1\rangle$	$-\|35,1,3;1\rangle$
	$\|8,5;0\rangle$	$1/3(-\|189,8,5;0\rangle - \sqrt{5}\,\|189,8,1;0\rangle + \sqrt{3}\,\|35,8,1;0\rangle)$
	$\|1,5;0\rangle$	$1/3\sqrt{5}(-\sqrt{5}\,\|189,1,5;0\rangle - 4\,\|189,1,1;0\rangle +$ $+ 2\sqrt{6}\,\|1,1,1;0\rangle)$
	$\|10,3;1\rangle$	$\|189,10,3;1\rangle$
	$\|10^*3;1\rangle$	$-\|189,10^*3;1\rangle$
	$\|8_F,3;1\rangle$	$-1/2(\|189,8_D,3;1\rangle + \sqrt{3}\,\|35,8,3;1\rangle)$
	$\|8_D,3;1\rangle$	$1/2(\sqrt{3}\,\|189,8,5;1\rangle - \|189,8_F,3;1\rangle)$
	$\|10,3;0\rangle$	$-\|189,10,3;0\rangle$
	$\|10,^*3;0\rangle$	$-\|189,10^*3;0\rangle$
	$\|8_F,3;0\rangle$	$-\|189,8_F,3;0\rangle$
	$\|8_D,3;0\rangle$	$1/2\,(\|189,8_D,3;0\rangle - \sqrt{3}\,\|35,8,3;0\rangle)$
	$\|27,1;0\rangle$	$-\|189,27,1;0\rangle$
	$\|8,1;0\rangle$	$-1/6(2\sqrt{5}\,\|189,8,5;0\rangle + \|189, 8,1;0\rangle + \sqrt{15}\,\|35,8,1;0\rangle)$
	$\|1,1;0\rangle$	$-1/15\,(4\sqrt{5}\,\|189,1,5;0\rangle + 7\,\|189,1,1;0\rangle +$ $+ 4\sqrt{6}\,\|1,1,1;0\rangle)$
35	$\|8,3;1\rangle$	$-1/2\,(\|189,8,5;1\rangle + \sqrt{3}\,\|189,8_F,3;1\rangle)$
	$\|8,3;0\rangle$	$-1/2\,(\sqrt{3}\,\|189,8_D,3;0\rangle + \|35,8,3;0\rangle)$
	$\|8,1;0\rangle$	$1/2\,\sqrt{3}(2\,\|189,8,5;0\rangle - \sqrt{5}\,\|189,8,1;0\rangle -$ $- \sqrt{3}\,\|35,8,1;0\rangle)$
	$\|1,3;1\rangle$	$-\|189,1,5;1\rangle$
	$\|1,3;0\rangle$	$\|35,1,3;0\rangle$
1	$\|1,1;0\rangle$	$1/5\sqrt{3}\,(2\sqrt{10}\,\|189,1,5;0\rangle - 4\sqrt{2}\,\|189,1,1;0\rangle -$ $- \sqrt{3}\,\|1,1,1;0\rangle$

Table II

$SU(6)_S - SU(6)_W$ mixing for 405

$SU(6)_S$	$\mid SU(3), SU(2); J_z >$	$\mid SU(6)_W, SU(3), SU(2); J_z >$
405	$\mid 27,5;2>$	$\mid 27,5;2>$
	$\mid 8,5;2>$	$\mid 8,5;2>$
	$\mid 1,5;2>$	$\mid 1,5;2>$
	$\mid 27,5;1>$	$-\mid 405,27,3;1>$
	$\mid 8,5;1>$	$-1/2\sqrt{6}(3\mid 405,8_D,3;1> + \sqrt{15}\mid 35,8,3;1>)$
	$\mid 1,5;1>$	$-\mid 35,1,3;1>$
	$\mid 27,5;0>$	$1/3(-\mid 405,27,5;0> + 2\sqrt{2}\mid 405,27,1;0>)$
	$\mid 8,5;0>$	$1/3\sqrt{2}(-\sqrt{2}\mid 405,8,5;0> + \mid 405,8,1;0> + \sqrt{15}\mid 35,8,1;0>)$
	$\mid 1,5;0>$	$1/3\sqrt{7}(-\sqrt{7}\mid 405,1,5;0> + 2\sqrt{2}\mid 405,1,1;0> + 4\sqrt{3}\mid 1,1,1;0>)$
	$\mid 27,3;1>$	$-\mid 405,27,5;0>$
	$\mid 10,3;1>$	$-\mid 405,10,3;1>$
	$\mid 10,{*}3;1>$	$\mid 405,10,{*}3;1>$
	$\mid 8_F,3;1>$	$1/2\sqrt{2}(-\sqrt{5}\mid 405,8_D,3;1> + \sqrt{3}\mid 35,8,3;1>)$
	$\mid 8_D,3;1>$	$-1/2\sqrt{2}(\sqrt{3}\mid 405,8,5;1> + \sqrt{5}\mid 405,8_F,3;1>)$
	$\mid 27,3;0>$	$\mid 405,27,3;0>$
	$\mid 10,3;0>$	$-\mid 405,10,3;0>$
	$\mid 10,{*}3;0>$	$-\mid 405,10,{*}3;0>$
	$\mid 8_F,3;0>$	$-\mid 405,8_F,3;0>$
	$\mid 8_D,3;0>$	$1/4(-\mid 405,8_D,3;0> + \sqrt{15}\mid 35,8,3;0>)$
	$\mid 27,1;0>$	$1/3(2\sqrt{2}\mid 405,27,5;0> + \mid 405,27,1;0>)$
	$\mid 8,1;0>$	$1/12(2\sqrt{2}\mid 405,8,5;0> - 11\mid 405,8,1;0> + \sqrt{15}\mid 35,8,1;0>)$
	$\mid 1,1;0>$	$1/21(2\sqrt{14}\mid 405,1,5;0> - 17\mid 405,1,1;0> + 4\sqrt{6}\mid 1,1,1;0>)$
35	$\mid 8,3;1>$	$1/2\sqrt{2}(-\sqrt{5}\mid 405,8,5;1> + \sqrt{3}\mid 405,8_F,3;1>)$
	$\mid 8,3;0>$	$1/4(\sqrt{15}\mid 405,8_D,3;0> + \mid 35,8,3;0>)$
	$\mid 8,1;0>$	$1/4\sqrt{3}(2\sqrt{10}\mid 405,8,5;0> + \sqrt{5}\mid 405,8,1;0> + \sqrt{3}\mid 35,8,1;0>)$
	$\mid 1,3;1>$	$-\mid 405,1,5;1>$
	$\mid 1,3;0>$	$\mid 35,1,3;0>$
1	$\mid 1,1;0>$	$1/7\sqrt{3}(4\sqrt{7}\mid 405,1,5;0> + 4\sqrt{2}\mid 405,1,1;0> + \sqrt{3}\mid 1,1,1;0>)$

Table III

Decay amplitudes*

Decay	189	405
$8_c^{5,1} \to 8_a^{3,1}\, 8_b^1$	$-\sqrt{3}\, AC_{ab}^{Fc}$	$\sqrt{15}\, A'C_{ab}^{Fc}$
$8_c^{5,0} \to 8_a^1\, 8_b^1$	$\sqrt{5}/3\, AC_{ab}^{Dc}$	$-5/3\, A'C_{ab}^{Dc}$
$1^{5,0} \to 8_a^1\, 8_{a*}^1$	$-1/\sqrt{2}\, B\, \delta a$	$B'\, \delta a$
$27_c^{3,1} \to 8_a^{3,1}\, 8_b^1$	—	$\sqrt{2}\, C'C_{ab}^{27,c}$
$27_c^{3,0} \to 8_a^{3,0}\, 8_b^1$	—	$\sqrt{2}\, C'C_{ab}^{27,c}$
$10_c^{3,1} \to 8_a^{3,1}\, 8_b^1$	$CC_{ab}^{10,c}$	$C'C_{ab}^{10,c}$
$10_c^{3,0} \to 8_a^{3,0}\, 8_b^1$	$CC_{ab}^{10,c}$	$C'C_{ab}^{10,c}$
$10_c^{*3,1} \to 8_a^{3,1}\, 8_b^1$	$-CC_{ab}^{10*,c}$	$C'C_{ab}^{10*,c}$
$10_c^{*3,0} \to 8_a^{3,0}\, 8_b^1$	$-CC_{ab}^{10*,c}$	$C'C_{ab}^{10*,c}$
$8_{Fc}^{3,1} \to 8_a^{3,1}\, 8_b^1$	$(2C-3A)\, C_{ab}^{Fc}$	$(2C'-2A')\, C_{ab}^{Fc}$
$8_{Fc}^{3,0} \to 8_a^{3,0}\, 8_b^1$	$-2CC_{ab}^{Fc}$	$-2C'C_{ab}^{Fc}$
$8_{Dc}^{3,1} \to 8_a^{3,1}\, 8_b^1$	$\sqrt{5}\, CC_{ab}^{Dc}$	$1/\sqrt{2}\, C'C_{ab}^{Dc}$
$\to 8_c^1\, 1^{3,1}$	$-1/\sqrt{2}\, C$	$\sqrt{5}/2\, C'$
$8_{Dc}^{3,0} \to 8_a^{3,0}\, 8_b^1$	$\sqrt{5}/C - A/C_{ab}^{Dc}$	$1/\sqrt{2}\, (C'-5A')\, C_{ab}^{Dc}$
$\to 8_c^1\, 1^{3,0}$	D	D'
$27_c^1 \to 8_a^1\, 8_b^1$	$1/2\, CC_{ab}^{27,c}$	$1/2\, \sqrt{3}\, C'C_{ab}^{27,c}$
$8_c^1 \to 8_a^1\, 8_b^1$	$1/6\, (9C-5A)\, C_{ab}^{Dc}$	$1/6\, \sqrt{2}\, (9C'-5A')\, C_{ab}^{Dc}$
$1^1 \to 8_a^1\, 8_{a*}^1$	$1/4\, \sqrt{10}\, (3C+8B)\, \delta a$	$1/2\, \sqrt{14}\, (3C'+8B')\, \delta a$

* For the notations see the text.

Table IV

Particle	Γ (MeV)			Decay	Branching ratio (%)		
	189	405	Exp.		189	405	Exp.
K^*	112	84	95 ± 11	$K^*\pi$	15,5	15,5	~ 50
				ϱK	4	4	
				ωK	1	1	
				$K\pi$	77	77	~ 50
				$K\eta$	2,5	2,5	
$K^{*\prime}$	—	212		$K^*\pi$	—	22,5	
				ϱK		14,5	
				$K^*\eta$		3,5	
				φK		0,7	
				ωK		0,5	
				$K\pi$		55,5	
				$K\eta$		2,8	
A_2	90	90	90 ± 10	$\varrho\pi$	64,5	64,5	91
				$K\bar{K}$	12,5	12,5	5,5
				$\eta\pi$	23	23	3,6
A_2'	—	113		K^*K	—	3	
				$\varrho\pi$		68	
				$K\bar{K}$		13,5	
				$\eta\pi$		15,5	
f	118	118	118 ± 16	$\pi\pi$	98	98,8	large
				$K\bar{K}$	2	1	< 4
				$\eta\eta$	$2 \cdot 10^{-2}$	0,2	
f'	82	49	80	K^*K	8	2	~ 40
				$\pi\pi$	10,5	28,5	
				$K\bar{K}$	62	57	~ 60
				$\eta\eta$	19,5	12,5	
f''	—	235		K^*K	—	33	
				$\pi\pi$		16	
				$K\bar{K}$		36	
				$\eta\eta$		15	

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РАСПАДЫ МЕЗОНОВ С ПОЛОЖИТЕЛЬНОЙ ЧЕТНОСТЬЮ В СХЕМЕ $SU(6)_W$

И. МОНТВАИ и Т. НАДЬ

Резюме

Даны $SU(6)_W$ — инвариантные амплитуды для распадов $\underline{189}$- и $\underline{405}$-мультиплетов в два псевдоскалярных мезона и в один векторный мезон и один псевдоскалярный мезон Вычислены ширины распада 2^+ мезонов; результаты, полученные для $\underline{189}$ лучше согласуются с экспериментальныыми данными.

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A FIELD THEORETICAL MODEL WITH $SL(6, C)$ INVARIANT INTERACTION

By

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Abstract

A nonlinear spinor equation is considered, which is form-invariant under the transformations of the SO_6-group. The components of the field operator are the trion fields introduced by BACRY, NUYTS and VAN HOVE. The formal interaction Lagrangian is characterized by $V-A$ coupling and $SL(6,C)$ invariance.

МОДЕЛЬ ТЕОРИИ ПОЛЯ С $SL(6, C)$-ИНВАРИАНТНЫМИ ВЗАИМОДЕЙСТВИЯМИ

К. ЛАДАНИ

Резюме

Рассматривается нелинейное спинорное уравнение, инвариантное относительно группы преобразований SO_6. Компонентами оператора поля явгяются триплетные поля, введенные Бакри, Нюитсом и Ван-Ховом. Формальный Лагранжиан взаимодействия охарактеризован $V-A$ связью и $SL(6, C)$ инвариантностью.

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SYMMETRIES OF SCATTERING INCLUDING MASS-SPLITTING

By

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Symmetries of scattering including mass splitting are discussed.

It is known that the usual perturbative treatment of symmetry breaking, when applied to the symmetries of scattering, affords no practical possibility to include unambiguously the effect of mass-splitting. This fact causes serious difficulties whenever the relations among scattering amplitudes have to be compared with experiment: one does not know for what values of kinematic variables the amplitudes of different processes have to be compared. The comparison for the same Q-value usually made is an ad hoc assumption devoid of any theoretical foundation.

I should like to present here briefly an attempt to give a new and more accurate meaning to the notion of symmetries of scattering which would not encounter such difficulties. Let me begin with the formulation of my basic assumptions.

Let $F^{(1)}, \ldots$ be a set of operators commuting with the scattering operator with the following properties:

1. All one-particle states can be divided into subspaces (multiplets) which are closed and irreducible with respect to $F^{(i)}$

$$F^{(i)} | a > = \sum | a' > .$$

2. The space of two-particle states built up by taking one-particle from one multiplet and the second from another is again closed under $F^{(i)}$

$$F^{(i)} | a, b > = \sum | a', b' > .$$

3. $F^{(i)}$ change the conserved observables Q and Y by $0, \pm 1, \pm 2, \ldots$

$$\Delta Q, \Delta Y = 0, \pm 1, \pm 2, \ldots$$

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4. $F^{(i)}$ are rotationally invariant.

$$[F^{(i)}, L_{jk}]_- = 0 .$$

It is not explicitly assumed that $F^{(i)}$ form an algebra but for the moment let us imagine that they do. Consider various realizations of this algebra in terms of normal products of creation and annihilation operators of in- and/or out-going particles i.e. in terms of the sum of operators like

$$\begin{aligned} &\int d^3 k_1\, d^3 k_2 f(\vec{k}_1 \vec{k}_2)\, a_\alpha^+ (\vec{k}_1)\, a_\beta (\vec{k}_2) + \\ &+ \int d^3 k_1 \ldots d^3 k_4 f(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4)\, a_\gamma^+ (\vec{k}_1)\, a_\delta^+ (\vec{k}_2)\, a_\lambda (\vec{k}_3)\, a_\varrho (\vec{k}_4) + \ldots . \end{aligned} \tag{1}$$

The assumptions 1—4. have then to be considered as constraints which indicate what realizations express the symmetry of the S-matrix.

Usually one considers only realizations which are bilinear in the field operators. This corresponds to the explicitly made assumption that two-particle states transform like members of the space which is a direct product of one-particle representation spaces

$$F^{(i)} | 1 > \otimes | 2 > = (F^{(i)} | 1 >) \otimes | 2 > + | 1 > \otimes (F^{(i)} | 2 >). \tag{2}$$

In our approach it is necessary to go beyond this assumption whenever we want to include inelastic processes. This means that more general realizations must be used and the WIGNER—ECKHART theorem can no longer be applied.

Nevertheless, a rather simple general method exists in which it is possible, on the basis of assumptions 1—4, to obtain well-defined relations among the scattering amplitudes whenever the classification of one-particle states is given.

I cannot demonstrate this method here but only present some typical results.

As a simple example we consider spinless quarks and antiquarks forming two-multiplets in the sense of assumption 1. We shall label them by Greek indices 1, 2, 3 corresponding to p, n, λ and assume $m_1 < m_2 < m_3$.

When only $Q—Q$ scattering is considered the situation is very simple because here only elastic processes can occur. The result is then

$$a_l^{\alpha\beta} (p^2) = a_l^{\alpha\gamma} (p^2) ; \quad \alpha, \beta, \gamma = 1, 2, 3 \tag{3}$$

where $a_l^{\alpha\beta}(p^2)$ is a partial wave for the elastic scatering

$$\alpha + \beta \rightarrow \alpha + \beta$$

as a function of the c.m. momentum squared. For $\alpha \equiv \beta$ this equality holds, of course, for even partial waves only. In this case of purely elastic scattering relations satisfying (2) can still be used.

More complicated and interesting results are obtained for quark-antiquark scattering. In this case we have a degeneracy of states $|\alpha, \bar{\alpha}>$ with respect to I_3 and Y and therefore inelastic processes like

$$\alpha + \bar{\alpha} \rightarrow \beta + \bar{\beta}$$

can take place. Now inclusion of two-particle terms (see assump. 2) is necessary to get nontrivial scattering. Let us denote by $\varphi_l^{\alpha\bar{\beta}}(p^2)$ the amplitudes of elastic process

$$\alpha + \bar{\beta} \rightarrow \alpha + \bar{\beta} \qquad (\alpha \neq \beta)$$

as a function of c.m. momentum squared, and by $f_l^{\alpha\beta}(s)$ amplitudes of

$$\alpha + \bar{\alpha} \rightarrow \beta + \bar{\beta}$$

as a function of c.m. energy squared in the corresponding channel. Then

$$\varphi_l{}^{1\bar{2}}(p^2) = \varphi_l{}^{1\bar{3}}(p^2) = \varphi_l{}^{2\bar{3}}(p^2) = \ldots \qquad \equiv \varphi_l(p^2)$$

and

$$\det\left[B(s) - 2\Delta_{\alpha 1}^{1/2}\,\varphi(\Delta_{\alpha 1}) \otimes 1\right] = 0,$$

$$\alpha = 1, 2, 3, \qquad s > 4m_3^2$$

where $B(s)$ is a 3×3 matrix

$$B_{\alpha\beta}(s) = f^{\alpha\beta}(s)\,(s - 4m_\alpha^2)^{1/2},$$

where

$$\Delta_{\alpha 1} = p^2 + m_1^2 - m_\alpha^2,$$

$$s = 4(p^2 + m_1^2)\,.$$

Finally let me make two comments:

1. Besides relations among scattering amplitudes, relations among certain matrix elements of $F^{(i)}$ appear. This means that if $F^{(i)}$ have to form an algebra this algebra cannot be arbitrary. It is not even clear whether $F^{(i)}$ can form an algebra at all. Moreover, the present method does not exclude the possibility that our basic assumptions are incompatible with nontrivial scattering, as only some of their consequences have been derived.

2. The main question, of course is, whether this approach and its consequences have anything to do with physics at all. Up to now I cannot give a definite answer to that question, but it is possible to examine the following

point of view: it is clear that such a symmetry of the S-matrix cannot include electromagnetic forces of infinite range. This is seen from

$$\sigma_{NN} = \sigma_{PP} .$$

On the other hand, it cannot be excluded a priori that the breakdown of higher symmetries, including U-spin etc., can be provided by the inclusion of mass-differences in the way proposed here. However, this needs to be directly verified. It is possible because the present method is general and can be applied equally well to such cases as meson-baryon or baryon-baryon scattering. However, this needs the inclusion of spin which slightly changes the method of derivations. This work is now in progress.

СИММЕТРИИ РАССЕЯНИЯ, ВКЛЮЧАЯ РАСЩЕПЛЕНИЕ МАСС

Й. ШТЕРН

Резюме

Обсуждены симметрии рассеяния, включая расщепление масс.

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SYMMETRIES OF QUARK-ANTIQUARK SCATTERING AND MESON NONETS MASS-SPLITTING

By

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Linear mass formulae for meson nonets are obtained. The derivation is based on equalities among quark-antiquark partial wave scattering amplitudes obtained in the preceding paper, and on assumptions concerning its analytical properties. The resulting formulae are in very good agreement with experimental mass-values of the 1^- and 2^+ meson nonets, but fail completely for pseudoscalar mesons.

1. The idea of quarks [1] as basic elements of all particles is rather attractive. Its applicability goes beyond the $SU(3)$ or $SU(6)$ content of the original proposition, as is well seen in the recent literature [2]. However, all dynamical calculations in quark models are made within a non-relativistic limit, what seems to be rather naive, to say at least. In this paper we shall try to exploit some results of the preceding talk by STERN [3] about the quark-antiquark scattering amplitude to obtain certain properties of the one-particle meson states.

As in the original proposition [1], we take mesons to be bound states of a quark-antiquark system. We shall show that from relations among the $Q-\bar{Q}$ scattering amplitudes and a usual assumption about the analytical properties of the amplitude we can obtain ordering of mesons into multiplets and non-trivial mass formulae, which are very well satisfied for 1^- and 2^+ meson nonets, but fail for pseudoscalar mesons.

2. We denote by 1, 2, 3 the three quarks p, n, λ with $m_1 < m_2 < m_3$. The antiquarks are then $\bar{1}$, $\bar{2}$, $\bar{3}$. The partial wave scattering amplitude for the process [4]

$$\alpha + \bar{\beta} \rightarrow \alpha + \bar{\beta}\,; \qquad (\alpha \neq \alpha\,; \alpha, \beta = 1,\ 2,\ 3)$$

(elastic scattering of a quark α and an antiquark $\bar{\beta}$) is $\varphi_l^{\alpha\bar{\beta}}(p^2)$, where p^2 is the c.m. three momentum of the pair $(\alpha, \bar{\beta})$. The amplitude for the process

$$\alpha + \bar{\alpha} \rightarrow \beta + \bar{\beta}$$

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is $f_l^{\alpha\beta}(s)$; here s is the total c.m. energy squared. These amplitudes are normalized in such a way that the differential cross-section is

$$\frac{d\sigma^{\alpha\bar{\beta}}}{d\Omega} = \sum_e (2l+1)\,\varphi_l^{\alpha\bar{\beta}}$$

and

$$\frac{d\sigma^{\alpha\beta}}{d\Omega} = \sum_e (2l+1) f_l^{\alpha\beta}\,.$$

Here we must make a remark about the partial wave decomposition. In [3] quarks were taken as spinless particles for simplicity, and l was the orbital momentum of the partial wave. However, this is inessential for us, because we expect that the same relations as those derived in [3] for partial waves will be true for the helicity amplitudes with definite values of spin and parity J^P in the case of quarks with spin. Further on we shall not write the index l, and shall consider all amplitudes to have definite values of J^P.

In [3] it is shown that the amplitudes φ and f fulfill the following relations

$$\varphi^{\alpha\bar{\beta}}(p^2) \equiv \varphi(p^2) \tag{1}$$

(all are equal) and

$$\det\left[B(s) - 2\Delta_{\alpha_1}^{1/2}\,\varphi(\Delta_{\alpha_1})\right] = 0\,,$$

$$\Delta_{\alpha_1} = \frac{1}{4}s - m_\alpha^2\,, \quad s > 4m_3^2\,, \tag{2}$$

where $B(s)$ is a 3×3 matrix with the elements

$$B^{\alpha\beta} = (s - 4m_\alpha^2)^{1/2} f^{\alpha\beta}(s)\,. \tag{3}$$

Eq. (2) is a cubic relation between the amplitudes φ and f. It can be easily linearized when looked upon as a secular equation for three eigenvalues $2\Delta_{\alpha_1}^{1/2}\,\varphi(\Delta_{\alpha_1})$. One gets

$$Sp\,B(s) = 2\sum_{\alpha=1}^{3} \Delta_{\alpha_1}^{1/2}\,\varphi(\Delta_{\alpha_1})\,. \tag{4}$$

Two other conditions (one quadratic and one cubic) can be obtained also but they have no value for us at present.

3. Relations (1) and (4) are valid in their respective physical regions. Instead of making any artificial assumption about the interaction of a quark-antiquark system we suppose that our scattering amplitudes have the usual analytical properties in the total c.m. energy squared. This is the only dynamic assumption we make. Mesons as quark-antiquark bound states will manifest

themselves as poles in the corresponding amplitudes (with specified J^P). From analyticity we conclude that (1) and (4) can be extended to the non-physical region and the pole terms can be compared.

Let us start with (1). The relevant pole term corresponds to a "charged" meson ($|I_3| + |Y| > 0$). We write e.g.

$$\varphi^{3\bar{2}}(p^2) = \frac{(g^{3\bar{2}})^2}{s_{32}(p^2) - m_{32}^2} + \text{no pole terms}\,, \tag{5}$$

where

$$s_{32}(p^2) = [(p^2 + m_3^2)^{1/2} + (p^2 + m_2^2)^{1/2}]^2$$

and m_{32} is the position of the pole. As a function of p^2, φ has a pole on the first Rieman sheet of both square roots whenever

$$|m_3^2 - m_2^2| < m_{32}^2$$

is true. At the position of the poles we have

$$m_{32} = (p_0^2 + m_3^2)^{1/2} + (p_0^2 + m_2^2)^{1/2} \tag{6}$$

Now it is easy to see from (1) that all $\varphi^{\alpha\bar{\beta}}(p^2)$ must have a pole at the same value of $p^2 = p_0^2$:

$$\varphi^{\alpha\bar{\beta}}(p^2) = \frac{(g^{\alpha\bar{\beta}})^2}{s_{\alpha\beta}(p^2) - m_{\alpha\beta}^2} + \text{no pole terms}, \tag{7}$$

where

$$s_{\alpha\beta}(p^2) = [(p^2 + m_\alpha^2)^{1/2} + (p^2 + m_\beta^2)^{1/2}]^2.$$

This means

$$m_{\alpha\beta} = (p_0^2 + m_\alpha^2)^{1/2} + (p_0^2 + m_\beta^2)^{1/2}. \qquad \alpha \neq \beta \tag{8}$$

This can be done for every value of J^P. Therefore, it is possible to identify uniquely $m_{\alpha\beta}$ ($\alpha \neq \beta$) with masses of "charged" mesons with the same J^P and Y, I_3 values of the corresponding channel (see Table I).

Table I

J^P	$1\bar{2}$	$1\bar{3}$	$2\bar{3}$
0^-	π^+	K^+	K°
1^-	ϱ^+	K^{+*}	$K^{\circ*}$
2^-	A_2^+	K^{+**}	$K^{\circ**}$

Eliminating p_0^2 from (8) we get

$$m_\alpha^2 - m_\beta^2 = m_{\alpha\beta}(m_{\alpha\gamma} - m_{\beta\gamma}) \quad (\alpha \neq \beta \neq \gamma \neq \alpha) \tag{9}$$

A similar procedure can be followed with (4). The r. h. s. of (4) has three different poles, according to the previous discussion, which must appear at the l. h. s. too. Their position in the s-variable we denote by $m_{\alpha\alpha}^2$ and choose $m_{11} < m_{22} < m_{33}$. We write, therefore,

$$f^{\alpha\beta}(s) = \sum_{\gamma=1}^{3} \frac{\eta_\gamma^\alpha \eta_\gamma^\beta}{s - m_{\gamma\gamma}^2} + \dots . \tag{10}$$

Here again $m_{\gamma\gamma}$ must have the meaning of masses of bound states — (absoutely) neutral mesons — in the $(\gamma\bar{\gamma})$ system. Comparing positions of the poles we get

$$m_{\alpha\alpha} = 2\,(p_0^2 + m_\alpha^2)^{1/2}, \tag{11}$$

where p_0^2 is the same as in (8). Formulae (8) and (11) can therefore be written in a common form

$$m_{\alpha\beta} = (p_0^2 + m_\alpha^2)^{1/2} + (p_0^2 + m_\beta^2)^{1/2}. \tag{12}$$

We have now to find the correspondence between bound states $(\gamma\bar{\gamma})$ and the real neutral mesons. It can be chosen according to the increasing masses of the mesons with given J^P. The results of the assignment are in Table II. We remark that this cannot be done for pseudoscalar mesons.

Table II

J^P	$1\bar{1}$	$2\bar{2}$	$3\bar{3}$
0^-	π°	?	?
1^-	ϱ°	ω	Φ
2^+	f	A_2°	f'

From (12) we obtain three linear relations among the meson masses (with given J^P)

$$2m_{\alpha\beta} = m_{\alpha\alpha} + m_{\beta\beta} \tag{13}$$

and two quadratic ones (with (9)).

$$4\,(m_\alpha^2 - m_\beta^2) = m_{\alpha\alpha}^2 - m_{\beta\beta}^2 = 4m_{\alpha\beta}(m_{\alpha\gamma} - m_{\beta\gamma})\,. \tag{14}$$

Formulae (13) are extremely well satisfied for 1^- and 2^+ meson nonets. If one calculates e.g. (using [5]) Φ and f' masses, one gets

$$M(\varphi) = (1017 \pm 6)\,\text{MeV}$$

and

$$M(f') = (1520 \pm 20)\,\text{MeV}.$$

One can estimate also the electromagnetic mass difference for K^*-mesons:

$$M(K^{*0}) - M(K^{*+}) = \frac{1}{2}[M(\omega) - M(\varrho^0)] = (8 \pm 3)\,\text{MeV}.$$

From (14) relations among masses of members of different multiplets follow. In this way a remarkably good value for the A_2-meson is obtained:

$$M(A_2) = (1324 \pm 11)\,\text{MeV}.$$

Besides formulae (13) and (14) it is also possible to write down several equalities among the coupling constants $(g^{\alpha\bar{\beta}})^2$ and $(\eta_{\gamma}^{\alpha})^2$ by comparing residues of the poles of φ and f. These relations can be of some value for quark models by giving information about different physical quark-antiquark-meson coupling constants.

4. In our opinion the important point of our result is not the relations (13) but the method by which they were obtained. It seems that it can be generalized to more complicated cases.

We made no use of any kind of perturbation method in our calculation. From this point of view the results obtained are exact. Relation (13) is exactly the "relativistic" version of the additivity rule for hadron masses frequently met in non-relativistic hadron models.

One point is rather interesting. Formulae identical to (13) were obtained by FORMÁNEK [6] in a completely different way. He found that an infinite dimensional algebra exists, connecting in a nontrivial way $SU(3)$ with the whole Poincaré algebra. Among other things, it has representations suitable for classification of nine particles with different masses. The relevant mass formula — also obtained without symmetry breaking — is equivalent to (13).

Both these approaches are different, but nevertheless have something in common. Namely, they are free from any kind of approximation, and use no symmetry breaking. This suggests that there is a deeper connection between the "spectrum generating" infinite dimensional symmetry scheme and unbroken symmetry of scattering proposed in [3].

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СИММЕТРИИ РАССЕЯНИЯ КВАРКОВ-АНТИКВАРКОВ И МАССОВОЕ РАСЩЕПЛЕНИЕ МЕЗОННЫХ НОНЕТОВ

Й. ШТЕРН и Й. ВАНЧУРА

Резюме

Выведены линейные массовые формулы для мезонных нонетов. Вывод основывается на приравнении амплитуд парциальных волн в рассеянии кварков-антикварков, полученных в предыдущей работе; далее на предположениях, касающихся их аналитических свойств. Результирующие формули очень хорошо согласуются с экспериментальными данными для массовых значений для мезонных нонетов 1^- и 2^+, но этого согласия нет у псевдоскалярных мезонов.

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SESSION 5. CURRENT ALGEBRAS

DISPERSIVE SUM RULES FROM CURRENT ALGEBRA

By

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One of the most successful and popular trends in our present approach to Elementary Particle Physics seems to be, beside group theory, the method based on the so called "current algebra". In these lectures we shall try to recall and summarize the main physical ideas and the most important results of this new approach as well as our present attitude toward still unsettled problems.

As the subject is a rapidly developing one, our exposition will suffer from the imperfections of the theory, which is still at its heuristic stage. Namely, it is incomplete and the formulation is not the most elegant one, but, on the other hand, it offers many reasons for genuine enthusiasm, which we shall try to make evident. Thus, we apologize from the beginning for the personal plan of exposition and for the numerous interesting contributions which will not be mentioned explicitly.

I. General considerations

1. One of the most powerful tools with which to investigate Elementary Particle Physics is group theory. We all know the very satisfactory classification of elementary particles based on $SU(3)$ and $SU(6)$ and the other outstanding results obtained with this approach. However, we find some difficulties in comparing group theoretical results with reality. The reason is that in nature all the proposed symmetries are more or less badly broken so that an estimate of the corrections due to symmetry-breaking interactions is necessary. This is particularly relevant in the case of groups like $SU(3)\times SU(3)$ where the symmetry limit is far from reality. Thus, to achieve a complete understanding of the role of group theory we need a scheme where, on the one hand, the group theoretical results are reproduced but, on the other hand, a control on the "corrections" is maintained. To deal with these problems we shall profit by the methods of quantum field theory and dispersion theory through the "current algebra approach".

The study of weak phenomena has revealed the appealing feature of an analogy between electromagnetic and weak interactions, in the sense that both are represented as interactions of currents. This similarity is sharpened if we assume that, like electromagnetic current, the weak (vector and axial vector) hadronic currents can exhibit a conservation, at least partially. After a set of currents $J_\mu^{(\alpha)}(x)$, which can be considered divergenceless in a suitable symmetry limit

$$D_\alpha(x) = \partial_\mu J_\mu^{(\alpha)}(x) = 0$$

we can define a set of charges

$$Q_\alpha(t) = \int J_0^{(\alpha)}(\vec{x}, t)\, d\vec{x} \tag{1}$$

which are approximate constants of motion

$$[Q_\alpha, H_0] = 0.$$

(H_0 is part of the total Hamiltonian H). The next step, i.e. the bridge between physical hadronic currents* and symmetry operators of the theory, is based on the fundamental suggestion by GELL-MANN [1] of identifying the physical charges with the generators (aside from a coupling constant) of a symmetry group, which is explicitly proposed to be $SU(3)$ (for vector currents) or $SU(3)\times SU(3)$ (for vector and axial vector currents). In so doing the equal time commutation relations between charges and currents are taken to generate the algebra of the corresponding group i.e.

$$\begin{gathered}[Q_\alpha, Q_\beta] = c_{\alpha\beta\gamma}\, Q_\gamma\,, \\ [Q_\alpha(t)\,,\, j_\mu^{(\beta)}(\bar{x}, t)] = c_{\alpha\beta\gamma}\; j_\mu^{(\gamma)}(\bar{x}, t)\,.\end{gathered} \tag{2}$$

where the $c_{\alpha\beta\gamma}$ are the structure constants of the algebra. After this we can look at the group as underlying the structure of baryons and mesons e.g. in the unitary symmetry scheme. Anyway, the existence of the algebra and the hypothesis that the related group is a symmetry group are independent things. Thus we can still exploit the group (through its algebra) without assuming invariance under it. In other words the commutation relations reflect the existence of the symmetry group for the hadrons but they are supposed to be *exact* and not to be affected by the presence in the total Hamiltonian of a symmetry-breaking part. Thus, the commutation relations can be the tool we are looking for.

2. One important problem is, therefore, to translate these *exact* commutation relations into exact relations among observable quantities i.e., into sum rules.

* We shall consider here the first oder in electromagnetic and weak interactions.

The simplest way [2] to do this is to take the matrix element of an equal time (e. t.) commutator between one-particle states and to use completeness

$$[Q_\alpha , M_\beta] = h_{\alpha\beta\gamma} \; M_\gamma , \tag{3}$$

where Q_α is a generator and M_β any tensor operator with well defined transformation properties (which determine the constant $h_{\alpha\beta\gamma}$). Then

$$\begin{aligned} h_{\alpha\beta\gamma} < a_1 | M_\gamma | a_2 > &= < a_1 | [Q_\alpha , M_\beta] | a_2 > = \\ = \sum_n < a_1 | Q_\alpha | n > < n | M_\beta | a_2 > &- \sum_n < a_1 | M_\beta | n > < n | Q_\alpha | a_2 > \end{aligned} \tag{4}$$

and this is a sum rule.

Now in the limit of exact symmetry we can classify particles as belonging to a given irreducible representation of the group, so that once having fixed the irreducible representation of $| a_1 >$, $| a_2 >$ we can separate in the sum the diagonal and non diagonal contributions. By diagonal we mean the contribution from one-particle states of the same multiplet as $| a_1 >$, $| a_2 >$; non diagonal otherwise. The fundamental point is that if Q_α is approximately conserved, the ratio of the non diagonal to the diagonal terms is of the order of the symmetry breaking λ. In fact, if $m_a \neq m_\nu$, i.e. $| a >$, $| \nu >$ belong to different multiplets, then

$$\begin{aligned} <a | Q_\alpha | \nu> &= \frac{<a | [Q_\alpha , H] | \nu>}{E_\nu - E_a} = \lambda \, \frac{<a | [Q_{\alpha 0} H_1] | \nu>}{E_\nu - E_a} = \\ &= O(\lambda) ; \quad H = H_0 + \lambda H_1 . \end{aligned} \tag{5}$$

From this it is clear that when $\lambda \to 0$, the sum rule contains only diagonal contributions which reproduce the group theoretical result. Then, as a consequence of switching off the symmetry breaking part of H, non-diagonal matrix elements come into the sum and give rise to $O(\lambda)$ corrections. The reason is that a set of states, transforming in the symmetry limit as an irreducible representation of the group, now contains mixtures of other representations since the physical states are eigenstates of the total Hamiltonian, containing both symmetry preserving and symmetry violating parts.

From this it also follows that the one-particle term is modified and we must include a "renormalization factor"

$$< a_1 | Q_\alpha | a > = \delta (\vec{p}_1 - \vec{p}) \, C (a, \alpha, a_1) \, r^{(\alpha)}_{aa_1} (p) \tag{6}$$

and the deviation of $r^{(\alpha)}(p)$ from unity is a measure of the symmetry breaking, again due to non-diagonal terms.

In fact, we can find a sum rule for $r^{(\alpha)}(p)$. From the e. t. commutator between opposite elements

$$[Q_\alpha, Q_{-\alpha}] = C(\alpha, i)\, Q_i \,; \quad Q_{-\alpha} = Q_\alpha^+ , \tag{7}$$

where Q_i is a diagonal generator (Y or I_3 in $SU(3)$), we obtain, by selecting the one particle term

$$| r^{(\alpha)}(p) |^2 + \delta r^{(\alpha)}(p) = 1 . \tag{8}$$

We notice that the 1 factor on the r. h. s., which takes into account the non-renormalization of the diagonal generators, helps, together with the non linear relation (8), to "fix the scale" for the renormalized coupling constants [1]. $\delta\, r^{(\alpha)}$ represents the many particle contribution

$$\begin{aligned} \delta r^{(\alpha)}(p) &= \sum_{n \neq a} | < a | Q_\alpha | n > |^2 - \text{crossed term} = \\ &= \sum_{n \neq a} \lambda^2 \cdot \frac{|< a\, [Q_\alpha, H_1]\, n >|^2}{(E_n - E_a)^2} - \text{c. t.} = O(\lambda^2) . \end{aligned} \tag{9}$$

This is the most general form of the ADEMOLLO—GATTO [3] theorem which shows that the deviation of $r^{(\alpha)}$ from the symmetric limit 1 is of the second order in λ. Moreover we have squared denominators which improve the convergence of the sum rule. Of course the distinction discussed here has a practical value as long as the symmetry breaking parameter "λ" is small enough to justify the denomination of corrections for the many particle contribution (as for $SU(3)$ or $SU(2)$ violation).

3. Let us now discuss a very peculiar point which arises when we take into account the dependence of the sum rule on the external momenta $\vec{p}_1, \vec{p}_2$ of $| a_1 >$, $| a_2 >$ (one common momentum p if M of Eq. (3) is an integrated operator). In this way we get a continuous set of sum rules. Of course the total sum rule is independent of "p" (in the simplest case of the sum rule 8) but different choices can lead to different splittings between lowest order and higher order terms, namely they can affect the rapidity of convergence of the sum rule. From this point of view a possible criterion of choice is the study of the relative size of the corrections and "as best sum rule" we can consider the one corresponding to the "p" value for which $\delta r^{(\alpha)}$ is as small as possible. As can be shown on the basis of some models, this occurs for $| \vec{p} | \to \infty$. In the text, of course, we assume that it is possible to do this inside the sum.

The existence of a continuous set of sum rules is due to the fact that the separation between one-particle and many-particle contributions depends on the reference frame. The fact that covariance is not automatic can be easily understood by considering that we started from an equal time commutator and in so doing we defined a reference frame. The choice $p \to \infty$ corresponds to specifying in that frame the state of motion of the external particles.

To appreciate further the choice $p \to \infty$ let us consider a typical corrective term

$$< a(p) \,|\, \lambda [Q_\alpha, H_1] \,|\, n(k) > = i \int d\vec{x} < a(p) \,|\, D_\alpha(x) \,|\, n(k) > = \\ = (2\pi)^3 i \; \delta(\vec{p} - \vec{k}) < a(p) \,|\, D_\alpha \,|\, n(k) > = (2\pi)^3 i \; \delta(\vec{p} - \vec{k}) \, F_\alpha(q^2) \tag{10}$$

and

$$q^2 = (p - k)^2 = (\sqrt{p^2 + m_a^2} - \sqrt{p^2 + m_n^2})^2 > 0. \tag{9}$$

In the limit $p \to \infty$, $q^2 \to 0$, independent of the mass of the intermediate state and, for instance, we find the familiar definition of renormalization ratio as the limit of zero momentum transfer of the form factor, i.e. $r^{(\alpha)} \equiv r^{(\alpha)}(\infty)$. The fact of avoiding form factors in the time-like region is important to have some confidence in the approximation of keeping only a few resonant states in the total sum. In fact if p is finite, q^2 is time-like and varies with m_n, so that non-resonant states also can give important contributions provided that q^2 for a suitable m_n, crosses a peak of the form factor $F_\alpha(q^2)$.

Finally, in the limit $p \to \infty$ it is possible to give to the sum rule an explicit covariant form, completely equivalent to a fixed masses dispersion relation. To this end let us start from the commutator (3) assuming for simplicity M_β to be a scalar operator* and a_1, a_2 spinless particles of equal masses:

$$< a_1 \,|\, [Q_\alpha, M_\beta] \,|\, a_2 > = h_{\alpha\beta\gamma} < a_1 \,|\, M_\gamma \,|\, a_2 > = h_{\alpha\beta\gamma} \; F_\gamma(\Delta^2) \,, \tag{11}$$

where we introduce the quantities

$$P = \frac{p_1 + p_2}{2} \,, \qquad \Delta = p_1 - p_2 \,. \tag{12}$$

$$\Delta^2 \leq 0$$

Selecting the one-particle term, we have:

$$h_{\alpha\beta\varphi} F_\gamma(\Delta^2) = r^{(\alpha)}_{a_1 a}(p_1) \, F_\beta(p_1) + \sum_{n \neq a} (2\pi)^3 i \; \delta(\vec{p}_1 - \vec{p}_n) \cdot \\ \cdot \frac{<a_1 \,|\, D_\alpha \,|\, n> <n \,|\, M_\beta \,|\, a_2>}{E_n - E_1} - \text{crossed term} \tag{13}$$

and the corrective term can be written, after introduction of an auxiliary four vector $q \equiv (q_0, \vec{q} = 0)$, as

* This example has only an indicative value. The use of sum rules when a scalar operator is involved should require a more complete discussion (see later the case of the mass formula).

$$\int \frac{dq_0}{q_0} \sum_{n \neq a} (2\pi)^3 i\delta^{(4)} (p_1 + q - p_n) < a_1 | D_\alpha | n >$$
$$< n | M_\beta | a_2 > - \text{ c. t.} = \int \frac{dq_0}{q_0} \varrho^{(\alpha\beta)} (q_0, p_1, p_2) . \tag{14}$$

As $\varrho^{(\alpha\beta)}$ is a scalar, it is actually a function of the independent invariants

$$\varrho^{(\alpha\beta)} = \varrho^{(\alpha\beta)} (q^2, k^2, \Delta^2, \nu),$$
$$\nu = q \cdot P = q_0 P_0, \qquad q^2 = q_0^2 = \frac{\nu^2}{P_0^2}, \tag{15}$$
$$k^2 = (\Delta + q)^2 = \Delta^2 + q^2 + 2\Delta \cdot q.$$

We see explicitly the dependence on the reference frame, since different choices of P_0 correspond to different paths of integration in the plane q^2, k^2, Δ^2, ν.

Now we perform the limit p_1, $p_2 \to \infty$ i.e., $P \to \infty$, taking $\vec{\Delta} = \vec{p}_1 - \vec{p}_2$ fixed and going to the limit of infinite momentum along a direction (z say) orthogonal to $\vec{\Delta}$. We have that the path of integration becomes the $q^2 = 0$ $k^2 = \Delta^2$ one (Δ^2 fixed) and it is easy to verify that in so doing the sum rule takes the form of a dispersion relation at fixed Δ^2 (spacelike) and fixed external masses

$$h_{\alpha\beta\gamma} F_\gamma (\Delta^2) = r^{(\alpha)}_{a_1 a} \, F_\beta (\Delta^2) + \int \frac{d\nu'}{\nu'} \varrho^{(\alpha\beta)} (\nu', \Delta^2, q^2 = 0 , \quad \Delta \cdot q = 0). \tag{16}$$

This relation can be visualized as corresponding to the pseudoprocess of Fig. 1.:

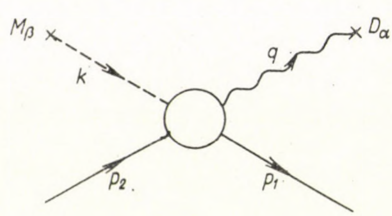


Fig. 1

Of course, it is clear that the implicit hypothesis that the exchange of the limit $P \to \infty$ and of the summation over infinite states is harmless, amounts to assume the convergence of the dispersion integral (16).

A more direct derivation of this representation for the e. t. commutator and some applications have been discussed in [4]. Let us only stress the fact that the corrections can be expressed in terms of the *local* operator $D_\alpha(x)$ which is usually taken to be a "gentle" one, i.e. with matrix elements dominated by low lying states.

The choice $P \to \infty$ is fundamental but we have a price to pay for it: while the situation $\vec{p} = 0$ does not allow the excitation of higher angular momenta, this is possible when $|\vec{p}| \to \infty$.*

4. Up to now we have discussed sum rules derived from e. t. commutators of the form $[Q, Q]$ or $[Q, J]$ i.e. an integrated operator is present at least once (this has the consequence that at least one "external mass" is zero, $q^2 = 0$). The experimental verification of these sum rules represents a test of the validity of the assumed commutation relations, *i.e., a test of the algebra.* In fact to deduce these commutation relations we begin with a well defined field theoretical model (usually the quark model) and then we abstract the result, assuming its general validity. In other words we renounce explicit field-theoretical expressions of the currents of hadrons and in order to characterize them we exploit their equal time commutation relations.

In this spirit it is natural to look at the more specific commutation relations between non-integrated quantities i.e. between current densities components like

$$[j_0^{(\alpha)}(\bar{x}, t), j_\mu^{(\beta)}(\bar{y}, t)] = c_{\alpha\beta\gamma} j_\mu^{(\gamma)}(\bar{x}, t) \; \delta(\bar{x} - \bar{y}) + \ldots . \tag{17}$$

It is clear that commutators where one integrated quantity is involved are more restrictive because possible gradient terms present in (17) disappear in (2). Moreover, in this case the link with the underlying symmetry group is lost and diagonal and non-diagonal terms cannot be separated.

The consideration of commutators between densities introduces the complication of possible additional gradient terms whose presence was originally indicated by SCHWINGER [5] in the case of electric current densities, when space components are involved. We have no complete theory for them, but at least in some models [6] it is possible to show that, for space labels, they have definite symmetry properties so that they can be presumably eliminated by considering properly symmetrized commutators. Analogously for both time components there is, for instance from perturbation theory, the indication that there are no gradient terms.

Anyway, let us start from the commutator, where we assume the absence of Schwinger terms

$$[j_0^{(\alpha)}(\bar{x}, t), j_0^{(\beta)}(\bar{y}, t)] = c_{\alpha\beta\gamma} j_0^{(\gamma)}(\bar{x}, t) \; \delta(\bar{x} - \bar{y}) . \tag{18}$$

We consider its matrix element between $|a_1>$, $|a_2>$, we multiply it by $e^{i\vec{q}_1 \vec{x}}$, $e^{-i\vec{q}_2 \cdot \vec{y}}$, we integrate over $\bar{x}$ and $\bar{y}$ and after insertion of a complete set of intermediate states, we find

* The last (but not the least) advantage by the choice $P \to \infty$ is the possibility of neglecting disconnected graph contributions. In fact they correspond to states of infinite mass, which on the basis of the assumed convergence are taken to be negligible.

$$c_{\alpha\beta\gamma}\, F_\gamma\,(\Delta^2) = \sum_n (2\pi)^3\, \delta\,(\bar{p}_1 + \bar{q}_1 - \bar{p}_n) < a_1 \,|\, j_0^{(\alpha)} \,|\, n > \cdot < n \,|\, j_0^{(\beta)} \,|\, a_2 > - \text{ c. t.} \quad (19)$$

From the previous discussion we can guess the result we shall obtain after performing the limit $P \to \infty$, taking Δ^2 fixed. In this case, we have no energy denominators and the external "masses" of the particles will be fixed at the values $q_{1,2}^2 = -\bar{q}_{1,2}^2$. Thus the sum rule will be of the form*

$$c_{\alpha\beta\gamma}\, F^{(\gamma)}\,(\Delta^2) = \int d\nu'\, a^{(\alpha\beta)}\,(\nu', \Delta^2, q_1^2, q_2^2)\,. \quad (20)$$

where "$a^{(\alpha\beta)}$" is a suitable amplitude.

5. To derive this sum rule in a more elegant way, we prefer to follow the approach given by FUBINI [7], where explicit use is made of dispersion relation techniques.

We consider the two quantities

$$\begin{aligned} T_{\mu\nu}^{\alpha\beta} &= i \int dx\, e^{iq_1\cdot x}\, \Theta\,(x_0) < p_1 \,|\, [J_\mu^{(\alpha)}\,(x)\,,\; J_\nu^{(\beta)}\,(0)] \,|\, p_2 > \\ t_{\mu\nu}^{\alpha\beta} &= \frac{1}{2} \int dx\, e^{iq_1\cdot x} < p_1 \,|\, [J_\mu^{(\alpha)}(x)_1 J_\nu^{(\beta)}(0)] | p_2 > \end{aligned} \quad (21)$$

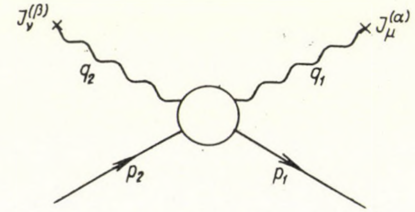


Fig. 2

which are in some way related to the scattering process of Fig. 2 (p_1, p_2 represent spinless particles). We introduce the kinematical variables ($p_1 + q_1 = p_2 + q_2$)

$$\begin{aligned} P &= \frac{p_1 + p_2}{2}\,, \qquad \Delta = p_1 - p_2\,, \\ \nu &= P \cdot q_1\,, \qquad t = (p_1 - p_2)^2 = \Delta^2. \end{aligned} \quad (22)$$

On invariance grounds we can develop $T_{\mu\nu}^{\alpha\beta}$** and $t_{\mu\nu}^{\alpha\beta}$ in elementary invariants,

* It can be shown that this procedure introduces a limitation of the form $\sqrt{\bar{q}^2} + \sqrt{\bar{q}_2^2} \geq \sqrt{\Delta^2}$ and analogous ones obtained by circular permutation.

** In this context we can recognize the SCHWINGER terms from another (equivalent) point of view. The time ordered product is not a completely covariant quantity in the sense that it does not transform as a second order tensor. In fact, owing to the incomplete definition at the origin, we can add a certain number of derivatives of $\delta(x - y)$ which result in a polynominal in q. This means that the true covariant amplitude given by the decomposition (23) and $T_{\mu\nu}$ defined in (21) differ just by the SCHWINGER terms. It is clear that these terms can affect the asymptotic behaviour and therefore the non-subtraction philosophy of Eq. (25).

namely

$$\begin{aligned} T_{\mu\nu} &= A_1 P_\mu P_\nu + A_2 q_{1\mu} P_\nu + A_3 q_{2\mu} P_\nu + B_1 P_\mu q_{1\nu} + \ldots \\ t_{\mu\nu} &= a_1 P_\mu P_\nu + a_2 q_{1\mu} P_\nu + a_3 q_{2\mu} P_\nu + b_1 P_\mu q_{1\nu} + \ldots \end{aligned} \tag{23}$$

where the scalar functions $A_1 \ldots, a_1$ depend on ν, t, q_1^2, q_2^2. The mathematical relation between $T_{\mu\nu}$ and $t_{\mu\nu}$ can be expressed as

$$T_{\mu\nu} = H\, t_{\mu\nu}\,, \tag{24}$$

where H means "Hilbert transform" with respect to the variable ν and (24) is understood to hold among the components of $T_{\mu\nu}$ and $t_{\mu\nu}$, developed in the set of invariants (23). In other words, we assume unsubtracted dispersion relations, at fixed t, for the A functions:

$$A_i(\nu, t) = \frac{1}{\pi} \int \frac{d\nu'}{\nu' - \nu}\, a\, i(\nu', t)\,, \tag{25}$$

To exploit the e. t. commutator we apply $q_{1\mu}$ to $T_{\mu\nu}$:

$$\begin{aligned} q_{1\mu} T_{\mu\nu} &= i \int (-i\, \partial_\mu e^{iq_1 \cdot x})\, dx\, \Theta(x_0) < p_1 \,|\, [J_\mu^{(\alpha)}(x), J_\nu^{(\beta)}(0)] \,|\, p_2 > \\ &= - \int \Theta(x_0)\, e^{iq_1 \cdot x} dx < p_1 \,|\, [D_\alpha(x), J_\nu^{(\beta)}(0)] \,|\, p_2 > - \\ &- \int \delta(x_0)\, e^{iq_1 \cdot x}\, dx < p_1 \,|\, [J_0^{(\alpha)}(x), J_\nu^{(\beta)}(0)] \,|\, p_2 > = id - c_{\alpha\beta\gamma} < p_1 \,|\, J_\nu^{(\gamma)} \,|\, p_2 >, \end{aligned} \tag{26}$$

where an integration by parts has been performed, and we have used the commutation relation

$$[j_0^{(\alpha)}(x), j_\nu^{(\beta)}(0)]_{x_0=0} = c_{\alpha\beta\gamma}\, j_\nu^{(\gamma)}(0)\, \delta(\vec{x})\,. \tag{27}$$

We can summarize the result of this operation as

$$q_{1\mu} T_{u\nu} - H\, q_{1\mu} t_{\mu\nu} = (q_{1\mu} H - H\, q_{1\mu})\, t_{\mu\nu} = c_{\alpha\beta\gamma} < p_1 \,|\, J_\nu^{(\gamma)} \,|\, p_2 >. \tag{26'}$$

Putting:

$$\begin{aligned} D_\nu^{\alpha\beta} &= - \int \Theta(x_0)\, e^{iq_1 \cdot x}\, dx < p_1 \,|\, [D_\alpha(x), J_\nu^{(\beta)}(0)] \,|\, p_2 > = \\ &= D_1 P_\mu + D_2\, q_{1\mu} + D_3\, q_{2\mu} \end{aligned} \tag{28}$$

$$d_\nu^{\alpha\beta} = \frac{i}{2} \int e^{iq_1 \cdot x}\, dx < p_1 \,|\, [D_\alpha(x), J_\nu^{(\beta)}(0)] \,|\, p_2 > = d_1 P_\mu + d_2\, q_{1\mu} + d_3\, q_{2\mu}$$

i.e.

$$D_\nu = H\, d_\nu \tag{29}$$

and

$$<p_1 | J_\nu^{(\gamma)} | p_2> = 2P_\nu F_1^{(\gamma)}(t) + \Delta_\nu F_2^{(\gamma)}(t) \tag{30}$$
$$(F_2(t) = 0 \text{ if } J_\nu^{(\gamma)} \text{ is conserved})$$

we obtain from (26), comparing the P_ν coefficient

$$\begin{aligned} \nu A_1 + q_1^2 \cdot Q A_2 + q_1 \cdot q_2 \Delta A_3 - D_1 &= -2c_{\alpha\beta\gamma} F_1^{(\gamma)}(t), \\ \nu a_1 + q_1^2 \cdot Q a_2 + q_1 \cdot q_2 \Delta a_3 - d_1 &= 0 \end{aligned} \tag{31}$$

and from (26')

$$\frac{1}{2\pi} \int d\nu' \, a_1^{(\alpha\beta)}(\nu', t, q_1^2, q_2^2) = c_{\alpha\beta\gamma} F_1^{(\gamma)}(t), \tag{32}$$

where according to its definition (23), "a_1" is related to the imaginary part of the scattering amplitude for the process of Fig. 2. In the same way we can get two other sum rules by comparing the $q_{1\nu}$ and $q_{2\nu}$ coefficients.*

The sum rule (31) is the most general consequence we can deduce from the assumed e. t. commutator (27). There are some comments to be made about this result.

a) First of all our sum rule has nothing to do with symmetry breaking because only currents, and not charges, are involved. As a consequence we have not such "gentle" operators, as divergences, to deal with. Anyway, if we want to get back to the sum rules of the kind (16) it is sufficient to consider the limit $q_1^2 = q_1 \cdot q_2 = 0$, $q_2^2 = t$ so that from (31)

$$a_1 = \frac{d_1}{\nu}$$

and Eq. (32) becomes

$$\frac{1}{2\pi} \int d_1(\nu', t, q_1^2 = 0, \; q_2^2 = t) \frac{d\nu'}{\nu'} = c_{\alpha\beta\gamma} F_1^{(\gamma)}(t) \tag{32'}$$

completely analogous to (16).

The second remark concerns the dependence of the sum rule on q_1^2, q_2^2, t. We know that it can be obtained for q_1^2, q_2^2, t spacelike but can we find a "best sum rule" in this case also, namely a configuration corresponding to the most rapid convergence of the sum rule? Or, more modestly, does the sum rule converge for any value of the above momenta? We have no complete answer to this question [8] and we shall limit ourselves to a few considerations.

* For these sum rules the presence of SCHWINGER terms is by no means excluded.

b) To better visualize the situation let me consider an explicit case, namely the commutator between isospin currents $(+, -) \to (3)$. After selection of the one pion contribution we get

$$F_\pi(t) = F_+(q_1^2)\, F_+(q_2^2) + \frac{1}{2\pi}\int \varrho^{(+\,-)}(\nu', t, q_1^2, q_2^2)\, d\nu'. \tag{33}$$

As long as the t dependence is concerned, the right hand side of the previous equation is an analytic function in t with given poles and cuts. How can the left hand side reproduce these singularities? For instance, if we are concerned with the ϱ-meson pole the simplest possibility is to add a sort of subtraction constant $\sim (m_\varrho^2 - t)^{-1}$, assuming the good convergence of the integral, or, alternatively, to deduce the pole in t from the blowing up to the integral i.e. from the large ν behaviour of a_1. In fact in a Regge-pole model, the asymptotic behaviour of a_1 is

$$a_1(\nu, t) \sim \nu^{\alpha(t)-2}\, \varphi(t, q_1^2, q_2^2) \tag{34}$$

and

$$\int_{\nu_0}^{\infty} d\nu'\, a_1(\nu', t) \sim \frac{\varphi(t, q_1^2, q_2^2)\,(\nu_0)^{\alpha(t)-1}}{\alpha(t) - 1} \underset{t \to m_\varrho^2}{\sim} \frac{G^2}{t - m_\varrho^2}\,. \tag{35}$$

This means that in the timelike t region the dispersion integral can provide the "asymptotic tail" which guarantees the consistency of the sum rule, developing the right hand side "t" singularities.*

c) The last point concerns the q_1^2, q_2^2 dependence. As we remark from Eq. (32) or (33) the right-hand side is independent of q_1^2, q_2^2 so that we must presume that strong cancellations occur in the left-hand side integral. (Of course our sum rule rests on the assumed simplicity of the e. t. c.). To fully exploit this fact, we multiply (32) by $(q_1^2 - m_\alpha^2)(q_2^2 - m_\beta^2)$ m_α, m_β, being the massess of vector mesons (the ϱ-meson in ex. 32) with the same quantum numbers of $J_\mu^{(\alpha)}$, $J_\nu^{(\beta)}$. If we extrapolate to $q_1^2 \to m_\alpha^2$, $q_2^2 \to m_\beta^2$, the right-hand side becomes zero while the integrand of the left hand side now involves the quantity $\tilde{a}_1(\nu, t, m_\alpha^2, m_\beta^2)$ to be related to the imaginary part of the scattering amplitude $p_2 + \beta \to p_1 + \alpha$.

$$\int d\nu'\, \tilde{a}_1(\nu', t, m_\alpha^2, m_\beta^2) = 0\,. \tag{36}$$

This is a sum rule for a strong-interaction amplitude i.e. only strong interaction parameters are involved. Thus, starting from the region $q_{1,2}^2 \leq 0$ where weak and e. m. interactions are described through currents, we go to the $q_{1,2}^2 = m_{\alpha,\beta}^2 > 0$ region of strong interactions. The interesting point is that result (36) is completely independent of the form of the e. t. commutator (provided it

* The structure of $\varphi(t, q_1^2, q_2^2)$ should be rather complicated. We could guess for it a form
$\varphi(t, q_1^2, q_2^2) = \alpha(\alpha - 1)\, \varphi_1(t, q_1^2, q_2^2) + G^2 \varphi_2(t)$; $\varphi_2(m_\varrho^2) = 1$.

is a local operator) i.e. independent from current algebra. This strongly suggests that a sum rule like (36) can be directly derived from the theory of strong interactions. This will be discussed in the final part.

II. Applications

1. In the previous lecture we have seen a general method which enables us to obtain sum rules from current algebra. The general form of the sum rule is

$$\frac{1}{2\pi}\int d\nu' \, a(\nu', t, q_1^2, q_2^2) = F(t), \tag{1}$$

where an "on mass shell" form factor $F(t)$ is given as an integral over the imaginary part of an "off mass shell" scattering amplitude $a(\nu', t, q_1^2, q_2^2)$, since the q_i^2 should be considered as arbitrary "masses" associated with some object simulating particles fields.

In this lecture we shall, primarily, give an account of the most interesting sum rules which derive directly from current algebra and so involve weak interaction quantities. We shall obtain our results using the FUBINI dispersive method, by an appropriate definition of commutators and states. We shall point out that many of the results we shall illustrate have been obtained by the original authors using different methods, namely either following the FUBINI, FURLAN [2] suggestion of introducing completeness in the matrix elements of the equal time commutators of "charge" and currents in the "infinite momentum frame of reference", or applying the equivalent, but completely covariant approach proposed by FUBINI, FURLAN and ROSSETTI [4].

Let us begin by briefly recalling the main steps of the method, as has been explained in the previous lecture. One starts by considering the equal-time commutator between two currents (which are *local* operators) and then constructs the general quantities

$$T_{\mu\nu}^{(\alpha\beta)} = i\int dx\, e^{iq_1\cdot x}\,\Theta(x_0) < p_1 | [J_\mu^{(\alpha)}(x), J_\nu^{(\beta)}(0)] | p_2 >, \tag{2}$$

$$t_{\mu\nu}^{(\alpha\beta)} = \frac{1}{2}\int dx\, e^{iq_1\cdot x} < p_1 | [J_\mu^{(\alpha)}(x), J_\nu^{(\beta)}(0)] | p_2 >. \tag{3}$$

μ, ν are Lorentz indexes and α, β those of internal symmetry, and the states are, for the moment, scalar particles states. Introducing the kinematic variables

$$\begin{aligned} q_2 &= p_1 + q_1 - p_2\,; \qquad P = \frac{p_1 + p_2}{2}, \\ \Delta &= p_1 - p_2\,, \qquad \nu = P\cdot q_1\,, \qquad t = \Delta^2 \end{aligned} \tag{4}$$

one decomposes then, on invariance grounds, $T_{\mu\nu}$ and $t_{\mu\nu}$ in the same set of elementary invariants, namely

$$T_{\mu\nu} = A_1 P_\mu P_\nu + A_2 q_{1\mu} P_\nu + A_3 q_{2\mu} P_\nu + \ldots, \tag{5}$$

$$t_{\mu\nu} = a_1 P_\mu P_\nu + a_2 q_{1\mu} P_\nu + a_3 q_{2\mu} P_\nu + \ldots \tag{6}$$

the invariant functions $A, a, \ldots$ depending, of course, on ν, t, q_1^2, q_2^2. The A functions are then Hilbert transforms, with respect to the variable ν, of the corresponding a:

$$A_i(\nu, t) = Ha_i = \frac{1}{\pi}\int \frac{d\nu'}{\nu' - \nu} a_i\,(\nu', t)\,. \tag{7}$$

By taking the "divergences" $q_{1\mu} T_{\mu\nu}$ and $q_{1\mu} t_{\mu\nu}$, it can easily be shown that starting from Eqs. (2), (3) and after partial integration the following relation holds:

$$(q_{1\mu} H - H q_{1\mu}) + {}_{\mu\nu} = C_{\alpha\beta\gamma} < p_1 | j_\nu^{(\gamma)} | p_2 > . \tag{8}$$

To write (8) we have taken into account the equal time commutation relation

$$[j_0^{(\alpha)}(x), j_\nu^{(\beta)}(0)]_{x_0=0} = C_{\alpha\beta\gamma}\, j_\nu^{(\gamma)}(0)\,\delta(\vec{x})\,. \tag{9}$$

By observing that, on invariance grounds,*

$$< p_1 | j_\nu^{(\gamma)} | p_2 > = (p_1 + p_2)_\nu F_1^{(\gamma)}(t) + (p_1 - p_2)_\nu F_2^{(\gamma)}(t) \tag{10}$$

it is almost straightforward to derive from (8), using the decomposition (5) and (6), the fundamental equation

$$\frac{1}{2\pi}\int a_1^{(\alpha\beta)}(\nu, t, q_1^2, q_2^2)\, d\nu = C_{\alpha\beta\gamma} F_1^{(\gamma)}(t)\,. \tag{11}$$

To derive from (11) explicit sum rules, we shall, first of all, separate the one-particle term from the continuum of many particles contribution. This is easily done by writing the general one-particle matrix element of a current as

$$< p_1 | j_\mu^{(\alpha)} | m > = (p_1 + m)_\mu F_1^{(\alpha)}(q_1^2) - q_{1\mu} F_2^{(\alpha)}(q_1^2)\,; \tag{12}$$
$$q_1 = m - p_1.$$

* We assume here, in order to simplify the notation, an invariant normalization for the states, namely

$$< p_1 | p_2 > = (2\pi)^3\, 2E\delta(\vec{p}_1 - \vec{p}_2)\,.$$

From (3), introducing a complete set of physical intermediate states, and selecting the $P_\mu P_\nu$ coefficients, in the one-particle contribution, evaluated with the help of (12), it follows immediately that (11) can be written as:

$$F_1^{(\alpha)}(q_1^2)\, F_1^{(\beta)}(q_2^2) - (\alpha \leftrightarrow \beta) + \frac{1}{2\pi} \int_{\text{cont}} a_1(\nu)\, d\nu = C_{\alpha\beta\gamma} F_1^{(\gamma)}(t) . \tag{13}$$

In many of our applications we shall be concerned with the particularly simple kinematic configuration of $q_1^2 = q_2^2 = t = 0$. If this is the case, the evaluation of the continuum contribution can be put in the most suitable form by observing that from (3), by integrating twice by parts, it follows that

$$q_{1\mu}\, q_{2\nu}\, t_{\mu\nu}' = \frac{1}{2} \int e^{iq_1 \cdot x} < p_1 \,|\, [D_\alpha(x), D_\beta(0)] \,|\, p_2 > dx = w_{\alpha\beta}(\nu)\, ; \tag{14}$$

where w is a scalar function; on the other side from (4) one has simply

$$q_{1\mu}\, q_{2\nu}\, t_{\mu\nu} = \nu^2\, a(\nu, 0, 0, 0)$$

and no other contribution appears since $q_1^2 = q_2^2 = q_1 \cdot q_2 = 0$. Our sum rule then takes the form

$$\{F_1^{(\alpha)}(0)\, F_1^{(\beta)}(0) - (\alpha \to \beta)\} + \frac{1}{2\pi} \int_{\text{cont}} \frac{w^{(\alpha\beta)}(\nu)}{\nu}\, d\nu = C_{\alpha\beta\gamma} F_1^{(\gamma)}(0), \tag{15}$$

where w can now be interpreted as the absorptive part of the amplitude for the scattering of the zero mass "particles" $D_{\alpha,\beta}$ on the particles p_1 and p_2.

2. Besides current-current commutators, we shall consider another type of commutator in some applications, namely that between a current and a local scalar operator, say $M_\beta(x)$, β being an index of internal symmetry. We can treat this simpler case exactly in the same way as we derived the previous sum rule. We introduce the quantities

$$T_\mu^{(\alpha\beta)} = i \int e^{iq_1 \cdot x}\, dx\, \Theta(x_0) < p_1 \,|\, [j_\mu^{(\alpha)}(x), M_\beta(0)] \,|\, p_2 > , \tag{17}$$

$$t_\mu^{(\alpha\beta)} = \frac{1}{2} \int e^{iq_1 \cdot x}\, dx < p_1 \,|\, [j_\mu^{(\alpha)}(x), M_\beta(0)] \,|\, p_2 > \tag{18}$$

which we decompose as

$$T_\mu = AP_\mu + Bq_{1\mu} + C\, q_{2\mu}\, , \tag{19}$$

$$t_\mu = aP_\mu + bq_{1\mu} + c\, q_{2\mu}\, . \tag{20}$$

We then derive

$$(q_{1\mu} H - H q_{1\mu}) t_\mu = h_{\alpha\beta\gamma} < p_1 | M_\gamma | p_2 > \tag{21}$$

having taken into account the commutation relation

$$[j_0^{(\alpha)}(x), M_\beta(0)]_{x_0=0} = h_{\alpha\beta\gamma} M_\gamma(0) \, \delta(\bar{x}) . \tag{22}$$

The right-hand side of Eq. (21) is now a scalar function, say $R^{(\gamma)}$ and we obtain the sum rule

$$\frac{1}{2\pi} \int a^{(\alpha\beta)}(\nu, t, q_1^2, q_2^2) \, d\nu = h_{\alpha\beta\gamma} R^{(\gamma)}(t) . \tag{23}$$

Again the one-particle contribution is easily separated (by looking at the P_μ coefficient in the one-particle contribution to the expansion of t_μ through a complete set of intermediate states) and we can write our sum rule as:

$$\{F_1^{(\alpha)}(q_1^2) R^{(\beta)}(q_2^2) - (\alpha \longleftrightarrow \beta)\} + \frac{1}{2\pi} \int_{\text{cont}} a^{(\alpha\beta)}(\nu) \, d\nu = h_{\alpha\beta\gamma} R^{(\gamma)}(t) . \tag{24}$$

Also in this case, in the particular kinematic configuration $q_1^2 = q_2^2 = t = 0$, the continuum contribution can be more easily expressed by observing that

$$q_{1\mu} t_\mu = \frac{i}{2} \int e^{iq_1 \cdot x} <p_1 | [D_\alpha(x), M_\beta(0)] | p_2> dx = w_{\alpha\beta}(\nu) , \tag{25}$$

w being a scalar function which, looking at the decomposition (20), equals $\nu \, a$ in the considered limit. In this particular case the sum rule (24) can thus be written as

$$\{F_1^{(\alpha)}(0) R^{(\beta)}(0) - (\alpha \longleftrightarrow \beta)\} + \frac{1}{2\pi} \int \frac{w_{\alpha\beta}(\nu)}{\nu} \, d\nu = h_{\alpha\beta\gamma} R^{(\gamma)}(0). \tag{26}$$

As in the previous case w can be interpreted as the imaginary part of the scattering amplitude for the process $D_\alpha + p_1 \to M_\beta + p_2$ at the limit of zero masses for the "particles" D and M.

Up to now our considerations refer only to the simplest case where the external particles p_1, p_2 are spinless particles. If we want to generalize the method to higher spin external particles, the only complication which would arise would be the appearance of more and more terms in the general invariant decomposition of the kind (5), (6) or (19), (20). We shall not treat any higher spin case in its most general form, since particular kinematic configurations allow us to greatly simplify the deduction of the particular sum rules in which we are interested.

Let us point out only that if it is possible to average the spin of the external particles, exactly the same formulae hold as those we have written for the scalar case.

3. The general formalism we have discussed allows us to obtain explicit examples of sum rules, by only particularizing the commutator from which we start and the physical states between which it is sandwiched. As first examples we shall consider a set of sum rules which present two common features; first, the evaluation of the continuum contribution can be connected to the imaginary part of amplitudes for a physical process through the assumption of the validity of the PCAC (partial conservation of the axial vector currents) principle and thus calculated from experimental information on scattering processes; secondly, they must actually be considered as a sort of "low energy theorems" since they are deduced in the limit $q_1^2 = q_1 \cdot q_2 = \ldots = 0$.

As a first explicit application let us discuss in some detail the famous and elegant sum rule, obtained independently, one year ago, by S. L. ADLER [9] and W. I. WEISBERGER [10], for the axial vector coupling constant renormalization in neutron β decay. To this end we must consider the commutator between the two opposite axial vector currents having the same internal quantum numbers as the π^+ and the π^- which we shall call $A_\mu^{(+)}$ and $A_\mu^{(-)} = [A_\mu^{(+)}]^+$ and we take it between proton states. Putting $q_1^2 = q_2^2 = t = 0$ we have $p_1 = p_2$ and we can average over the spin of the external protons so that the general treatment can be applied. We shall exploit the commutation rule

$$[A_0^{(+)}(x),\ A_\nu^{(-)}(0)]_{x_0=0} = 2J_\nu^{(3)}(0)\,\delta(\vec{x}), \tag{27}$$

where $J_\nu^{(3)}$ is the isovector part of the electromagnetic current (which is not renormalized), whose matrix element between proton states is, after an average on the proton spin,

$$\overline{<p\,|\,J_\mu^{(3)}\,|\,p>} = \frac{1}{2}\,\overline{\bar{u}\,\{\gamma_\mu F_1^v(0)\}\,u} = p_\mu \tag{28}$$

the isovector form factor being normalized to

$$F_1^v(0) = 1\,. \tag{29}$$

Thus, one immediately has from (11) the sum rule

$$\frac{1}{2\pi}\int a\,(\nu, 0, 0, 0)\,d\nu = 1\,, \tag{30}$$

where "a" is the coefficient of the $P_\mu\,P_\nu = p_\mu\,p_\nu$ term in the decomposition of

$$t_{\mu\nu} = \frac{1}{2}\int e^{iq_1\cdot x} <p|\,[A_\mu^{(+)}(x),\ A_\nu^{(-)}(0)]\,|\,p>\,dx. \tag{31}$$

We remember that the general form of the matrix element of A_μ between a proton and a neutron state is

$$< p_1 | A_\mu^{(+)} | n > = i \bar{u}_1 \{ r_A \gamma_5 \gamma_\mu G(q_1^2) - q_{1\mu} \gamma_5 \beta(q_1^2) \} u_n , \qquad (32)$$

$$q_1 = n - p_1,$$

where r_A is the ratio between the renormalized axial vector coupling constant g_A and the bare one $g_{A^\circ} = g_V$:

$$r_A = g_A / g_V$$

and

$$G(0) = 1 . \qquad (33)$$

We can now introduce a complete set of intermediate states in (31) and easily extract the neutron contribution (which is the only single-particle contribution, since in the crossed term we should need a twice charged particle). We have thus (remember that an average on the external proton is understood)

$$t_{\mu\nu}^{(n)} = \frac{1}{2} (2\pi)^4 \Sigma \int \frac{d^3 n}{(2\pi)^3 2E} \delta(p + q - n) i^2 \bar{u} r_A \gamma_5 \gamma_\mu u_n \bar{u}_n$$

$$r_A \gamma_5 \gamma_\nu u = -\pi \delta \{ (p + q)^2 - m_n^2 \} \frac{1}{2} r^2{}_A \cdot$$

$$\cdot Tr \{ (\hat{p} + m_p) \gamma_5 \gamma_\mu (\hat{p} + \hat{q} + m_n) \gamma_5 \gamma_\nu \} =$$

$$= \frac{r_A^2}{4} \pi \delta \left(\nu - \frac{m_n^2 - m_p^2}{2} \right) 8 p_\mu p_\nu + \ldots = 2\pi r_A^2 \delta(\nu) p_\mu p_\nu \ldots$$

The one-particle contribution to the sum rule (30) is then simply the square of the axial vector renormalization ratio; since we are considering the case $q_1^2 = q_2^2 = t = 0$, we can now write (30) in a form similar to Eq. (15), namely

$$r_A^2 + \frac{1}{2\pi} \int_{\text{cont}} \frac{w(\nu)}{\nu^2} d\nu = 1 , \qquad (34)$$

where in this case the scalar function w is given by

$$w = \frac{1}{2} \int e^{i q_1 \cdot x} < p | [\bar{D}_+(x), \bar{D}_-(0)] | p > dx \; ; \; \bar{D} = \partial_\mu A_\mu . \qquad (35)$$

Two comments now arise: the first is that, looking at Eq. (34) and remembering the general discussion, the value 1 would be considered as the symmetric value of r_A in agreement with the fact that, in the symmetry limit, w would be automatically zero since the currents would be conserved and then the $\bar{D}$'s would be zero. In the case we are treating, however, we do not believe in this limit, since the underlying symmetry group is the $SU_2 \times SU_2$ group (or, more generally, the $SU_3 \times SU_3$ one) and we know such a symmetry to be very badly violated, being valid only in the limit of zero baryon masses. In this and analogous cases we can thus never disregard the higher contributions which are of fundamental importance. The second comment is concerned with the form of these higher contributions; we will indeed point out that (35) has the form of the elastic unitary condition where only squared amplitudes appear and they are related to the imaginary part of a forward scattering amplitude and then, through the optical theorem to total cross-sections. Some care should, however, be taken in handling experimental data due to the "soft" character of one of the incoming particles.

Our final task is now to transform the expression of the continuum contribution into a suitable form for practical evaluation. To do this in the simplest way we admit, as already mentioned, the validity of the PCAC principle, i.e. we assume the validitiy of the GELL-MANN, LEVY [11] proportionality relation between the divergences of the axial currents and the pion fields,

$$\bar{D}^{\pm} = C\varphi_{\pm} \tag{36}$$

and we admit that (36) still remains valid when extended to zero mass pions. The value of the constant C can be deduced in various ways; the simplest is to consider separately the matrix elements of the two sides of (36) between nucleon states; by remembering (32) we have for the l. h. s. matrix element

$$< p \,|\, \bar{D}_+ \,|\, n > \, = - \left\{ 2m\, r_A\, G(q^2) + q^2 \beta(q^2) \right\} \bar{u}_p \gamma_5 u_n \tag{37}$$

and from the r. h. s., one has

$$C < p \,|\, \varphi_+ \,|\, n > \, = C \frac{< p \,|\, j_+ \,|\, n >}{m_\pi^2 - q^2} = C\, \frac{\sqrt{2}\, g_{\pi N}\, G_{PN}(q^2)}{m_\pi^2 - q^2}\, \bar{u}_r \gamma_5 u_n\,. \tag{38}$$

$$G_{PN}(m_\pi^2) = 1\,.$$

By equating (37) and (38) and letting q^2 go to zero we derive

$$C = - \frac{\sqrt{2}\, m\, r_A\, m_\pi^2}{g_{\pi N}\, G_{PN}(0)}\,, \tag{39}$$

where $G_{PN}(0)$ takes into account the fact that we are working with zero mass pions.

Introducing (36) in (35) the spectral function w becomes

$$w = \frac{(2\pi)^4}{2}\left(\frac{C}{m_\pi^2}\right)^2 \sum_m \{ | < p_1 | j_+ | n > |^2 \delta(p+q-p_n) - | < p | j_- | n > |^2 \cdot \cdot \delta(p-q+p_n)\} \tag{40}$$

and it can then be easily expressed in terms of total $\pi^\pm$ proton cross-sections. Remembering that, with our normalization

$$(\text{flux}) \cdot \sigma^{\text{tot}} = \sum_n (2\pi)^4 | T_n |^2 \delta(P_I - P_n) \tag{41}$$

and

$$(\text{flux}) = 4 p_0 q_0 | V_r | ; \qquad V_r^2 = \{(pq)^2 - p^2 q^2)\}/p_0^2 q_0^2 \tag{42}$$

we have thus, at the $q^2 = 0$ point:

$$\sum_n (2\pi)^4 | T_n |^2 \delta^{(4)} = 4\nu\sigma^{\text{tot}}(\nu, q^2 = 0) = 4\nu [G_{in}(0)]^2 \sigma^{\text{tot}}(\nu), \tag{43}$$

where we have introduced an inelastic form factor to correct the physical cross-sections to zero pion mass.

Introducing (43) in (40) and making the value (39) for C explicit, we have

$$w = \frac{4m^2 r_A^2}{g_{\pi N}^2} \left| \frac{G_{in}(0)}{G_{PN}(0)} \right|^2 \nu \{\sigma_{\pi^- p}^{\text{tot}} - \sigma_{\pi^+ p}^{\text{tot}}\} \tag{44}$$

so that our sum rule (34) attains its final form

$$r_A^{-2} = 1 + \frac{2m^2}{g_{\pi N}^2} \frac{1}{\pi} \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} \{\sigma_{\pi^- p}^{\text{tot}} - \sigma_{\pi^+ p}^{\text{tot}}\}, \tag{45}$$

where we have made the approximation of taking $(G_{in}(0)/G_{PN}(0) = 1$ assuming the two extrapolated values to go in the same direction.

The sum rule for r_A in the form (45), which is that obtained by ADLER and WEISBERGER, presents, besides a truly elegant aspect, the possibility of easily evaluating the continuum contribution. This evaluation, based on the experimental data on pion-proton scattering has been carefully performed separately by ADLER and WEISBERGER, who respectively give the values of

$$(r_A) = 1{,}24 \quad \text{and} \quad (r_A) = 1{,}15^*$$

* The difference between the two results can be mainly ascribed to the different procedures in handling the zero mass pion limit. ADLER uses PCAC directly in the form (36), while WEISBERGER uses analytic continuation in q^2 by retaining only the pion pole. In so doing only physical quantities are involved, and the sum rule (45) becomes

$$r_A^{-2} = 1 + \frac{2m^2}{g^2_{\pi N}} \frac{1}{\pi} \int k(\nu) \frac{d\nu}{\nu^2} \{\sigma_{\pi^- p}^{\text{tot}} - \sigma_{\pi^+ p}^{\text{tot}}\}$$

where k is the momentum of the (physical) π in the laboratory system.

to be compared with the experimental one [12] of

$$(r_A)_{\rm exp} = 1{,}20 \pm 0{,}02\,.$$

We wish to emphasize that the convergence of the sum rule (i.e. the validity of the assumed unsubtracted dispersion relation) is ensured by the POMERANCHUK theorem which we could consider as the dynamical mechanism which minimizes the corrections.*

The result we have obtained, namely that the higher corrections to the axial vector renormalization for β decay can be expressed in terms of physical cross-sections, depends, of course, on the fact that the $\bar{D}$'s have the same quantum numbers as the pions. It is then clear that analogous results could be obtained by exploiting other commutators involving operators having the same quantum number as other physical particles as e.g. K mesons. Investigations of this kind have been actually undertaken by several authors [13], generally by means of the infinite momentum frame of reference method. By translating their language into our present one, we sketch, briefly, the investigation undertaken, for instance, by D. AMATI, C. BOUCHIAT and J NUYTS. One starts by considering the commutators

$$[A_\mu^{(k+)}(x), A_\nu^{(k-)}(0)] \qquad \text{and} \qquad [A_\mu^{(L+)}(x), A_\nu^{(L-)}]$$

where $A_\mu^{(k+)} = [A_\mu^{(k-)}]^+$ and $A_\mu^{L+} = [A_\mu^{(L-)}]^+$ are the axial strange currents with the same quantum numbers as the K^+ and K^0, respectively. Following exactly the same procedure as in the previous case one obtains two sum rules strictly analogous to (45), where total cross-sections of charged or neutra kaons on protons appear. Owing particularly to the lack of experimental data the numerical evaluations are not so good as in the ADLER—WEISBERGER case. The main result of the investigation by AMATI, BOUCHIAT and NUYTS is the derivation of the ratio between the symmetric (D) and the antisymmetric axial coupling (F) which we must consider in SU_3. The value obtained is $D/F = 2{,}7$ to be compared with the experimental one [14] of about 2.

4. The relations we have examined, deriving from the commutators of two axial vector currents, can be referred to as "scattering sum rules". The question arises spontaneously if there are analogous "photoproduction sum rules". The next example will, therefore, deal with such sum rules, which derive from the commutator of an axial vector current with the electromagnetic current. In particular we shall obtain two sum rules for the nucleon anomalous magnetic moments which have been derived by FUBINI, FURLAN

* Experimentally one can fit the high energy data with

$$\sigma_{\pi-p} - \sigma_{\pi+p} \underset{\nu\to\infty}{\sim} b\nu^{-0\cdot 7}.$$

and ROSSETTI [15], using the covariant approach. By translating again into our present language, we shall consider the two commutators

$$\left[A_\mu^{(3)}(x), J_\nu^{(3,8)}(0)\right], \tag{46}$$

(where we have adopted the $SU_3 \times SU_3$ notations to label the isovector axial current ($A_\mu^{(3)}$), the isovector ($J_\nu^{(3)}$) and the isoscalar ($J_\nu^{(8)}$) part of the electromagnetic current), taken between nucleon states. Following our general considerations we define

$$T_\mu = i\,\varepsilon_\nu \int < p_1 \left| \left[A_\mu^{(3)}(x), \; J_\nu^{(3,8)}(0)\right] \right| p_2 > e^{iq_1\cdot x}\,\Theta(x^0)\,dx\,, \tag{47}$$

$$_\mu = \frac{1}{2}\,\varepsilon_\nu \int < p_1 | \; A_\mu^{(3)}(x), J_\nu^{(3,8)}(0)\right] | p_2 > e^{iq_1\cdot x}\,dx\,, \tag{48}$$

where ε_ν is an arbitrary (for the moment) vector introdu..d for convenience Since, as one easily realizes, T_μ and t_μ are to be connected to photoproduction amplitudes, we shall look for their decomposition to that given by CHEW, GOLDBERGER, LOW and NAMBU [16] in their fundamental paper on photoproduction theory. We can, however, choose the particularly simple kinematic configuration $q_1^2 = q_2^2 = q_1 \cdot q_2 = 0$, and, furthermore, we can choose ε_ν as being orthogonal to the photon momentum q_2 and also to q_1: $q_1 \cdot \varepsilon = q_2 \cdot \varepsilon = 0$. In the C. G. L. N. decomposition only two terms survive in our limit, namely

$$M = \gamma_5\,(\gamma\cdot\varepsilon)\,(\gamma\cdot q_2), \qquad N = \gamma_5\,(\gamma\cdot\varepsilon)\left[\gamma\cdot q_1\,, \gamma\cdot q_2\right] \tag{49}$$

so that we can decompose T_μ and t_μ as

$$T_\mu = P_\mu(A_1\,M + B_1\,N) + q_{1\mu}(A_2\,M + B_2\,N) + \ldots, \tag{50}$$

$$t_\mu = P_\mu(a_1\,M + b_1\,N) + q_{1\mu}(a_2\,M + b_2\,N) + \ldots. \tag{51}$$

In this case we obtain a sum rule for the invariant function a_1, since we have

$$(q_{1\mu}\,H - H q_{1\mu})\,t_\mu = M\,\frac{1}{\pi}\int a_1(\nu)\,d\nu = 0 \tag{52}$$

having taken into account the commutation rules

$$A_0^{(3)}(x), J_\nu^{(3,8}\;(0)]_{x_0=0} = 0\,. \tag{53}$$

The sum rules we shall consider are then:

$$\int a_1^{(3,8)}(\,\nu)\,d\,\nu = 0 \tag{54}$$

To evaluate the one-particle contribution we define the following matrix elements

$$< p_1 | A_\mu^{(3)} | p_2 > = i \bar{u}_1 \{ \bar{r}_3 \gamma_5 \gamma_\mu G(q_1^2) - q_{1\mu} \gamma_5 \beta(q_1^2) \} u_2 \qquad (55)$$

$$< p_1 | J_\mu^{(3,8)} | p_2 > = \bar{u}_1 \frac{1}{2} \left\{ \gamma_\mu F_1^{V,s}(q_1^2) + \sigma_{\mu\nu} q_{1\nu} \frac{F_2^{V,s}}{2m}(q_1^2) \right\} u_2 . \qquad (56)$$

$$q_1 = p_2 - p_1 , \qquad \sigma_{\mu\nu} = \frac{1}{2} [\gamma_\mu, \gamma_\nu]_-$$

nd for the nucleon contribution we can then derive

$$\frac{1}{\pi} \int a_{\text{Nucleon}}^{(3,8)}(\nu)\, d\nu = - \frac{\bar{r}_3}{m} F_2^{V,s}(0) = - \frac{\bar{r}_3}{m} K_{V,S} \qquad (57)$$

$$K_v = K_P - K_N ; \qquad K_s = K_P + K_N .$$

Now, to express the continuum contribution we can follow an analogous procedure to that used to obtain Eq. (26). In this way we are finally led to the two sum rules

$$\bar{r}_3 K_{V,s} - \frac{2m}{\pi} \int \alpha^{(3,8)}(\nu) \frac{d\nu}{\nu} = 0 . \qquad (58)$$

where $\alpha^{(3,8)}$ are the coefficients of M in the continuum contribution to $q_{1\mu} t_\mu$, i.e. the α's can be deduced by selecting the coefficients of the M terms from the general quantities

$$A^{(3.8)} = \frac{1}{2} (2\pi)^4 \sum_{n \neq p} \{ < p_1 | \bar{D}_3 | n > < n | J^{(3,8)} \cdot \varepsilon | p_2 > \cdot \delta(p_1 + q_1 - p_n) - - < p_1 | J^{(3,8)} \cdot \varepsilon | n > < n | \bar{D}_3 | p_2 > \delta(p_1 - q_2 - p_n) \} . \qquad (59)$$

By neglecting the continuum contribution in (58), we obtain, as usual, the symmetry limit. In this case we find the (expected)* bad prevision $K_V = K_S = 0$.

However, we do not have to believe in it, since the axial currents are considered, and we need the continuum contributions. Through PCAC, these are expressed as dispersive integrals over the imaginary parts of π photoproduction amplitudes; in the present case they cannot, however, be directly connected to physical cross-sections.

* It is, in fact, well known that as a consequence of the γ_5 invariance the anomalous magnetic moment is zero for massless fermions.

For a first approximate discussion of the continuum part of Eq. (58) we shall assume the dominant contribution to come from the lowest lying resonance N_{33}, i.e. in an SU_3 scheme, from the decuplet states. This "decuplet dominance hypothesis" is suggested by two kinds of considerations. First of all, in most of the dispersion treatments the dominance of the 33 state has been assumed in good agreement with experiment; secondly SU_6 considerations seem to put the decuplet 3/2 states on somewhat privileged ground as members of the 56 supermultiplet. Under this decuplet dominance hypothesis the two sum rules (58) reduce to

$$\bar{r}_3 K_V = c_{33}, \tag{60}$$

$$\bar{r}_3 K_S = 0, \tag{61}$$

since, as a consequence of isospin conservation, only in the first case can the 33 resonance be excited. As from Eq. (60) $\bar{r}_3 \neq 0$, Eq. (61) tells us that, in this approximation, K_S still remains zero.

In order to study Eq. (60) together with the possible corrections to the decuplet model we need to use experimental information about the continuum distribution. Assuming PCAC, we can introduce in Eq. (58) the π^0 photoproduction amplitudes $A^{(3,8)}$

$$\alpha^{(3,8)} = \bar{r}_3 \frac{2m}{g_{\pi N}} Im\, A^{(3,8)}. \tag{62}$$

and obtain the two sum rules

$$K_V - \frac{4m^2}{g_{\pi N}} \frac{1}{\pi} \int Im\, A^{(V)}(\nu') \frac{d\nu'}{\nu'} = 0, \tag{63}$$

$$K_S - \frac{4m^2}{g_{\pi N}} \frac{1}{\pi} \int Im\, A^{(S)}(\nu') \frac{d\nu'}{\nu'} = 0. \tag{64}$$

Let us first study Eq. (64) under the hypothesis of a decuplet dominance. In order to have an estimate of these contributions we shall use the isobaric model of GOURDIN and SALIN [17] which gives a satisfactory description of the photoproduction process. In so doing we obtain

$$K_V = \frac{\lambda C}{g_{\pi N}} \frac{2}{3} \left(\frac{m}{m_\pi}\right)^2 \left(\frac{m}{M}\right) \left(1 + \frac{m}{M}\right), \tag{65}$$

where λ and C are phenomenological constants describing the $N \pi N_{33}$ and $N \gamma N_{33}$ vertices which have the values*

$$\lambda = 1{,}81\,; \qquad C = 0{,}345\,.$$

* These values are not exactly those given by GOURDIN and SALIN. For a discussion on this point see [15].

From (65) one obtains for K_V the value

$$K_V = 3.98\,, \text{ whereas } (K_V)_{\exp} = 3{,}70\,.$$

In the same approximation

$$K_S = 0\,, \text{ whereas } (K_S)_{\exp} = -0{,}12\,.$$

The calculated values are in reasonable agreement with the experimental data. We can observe at this point that with our approximation, we overestimate both K_V and K_S; the same thing happens in the ADLER—WEISBERGER case: if one takes only the 33 contribution, the value obtained is indeed $r_A = 1{,}44$; the contribution of the higher states then reduces the value of r_A approximately to the experimental one.

To estimate the role of higher states in modifying both values of K_V and K_S, we have considered the next resonant state, namely the $N_{13}(1515)$ which plays an interesting part in the ADLER—WEISBERGER relation. From the standpoint of experiment the N_{13} plays a much more relevant role in scattering than in photoproduction, so that we can expect that the rather good agreement given by the decuplet dominance model is not going to be spoiled. Indeed taking into account the 13 contribution, always with the help of the isobaric model, we obtain the values

$$K_V = 3{,}80; \quad K_S = 0{,}176\,.$$

These results show that the 13 contribution has the correct sign and order of magnitude to improve the agreement of both sum rules.

Finally we shall point out that the assumption of retaining only certain states has no absolute justification, but is an approximation depending on the kind of the sum rules studied.

The sum rules discussed for the nucleon anomalous magnetic moments were related to the π photoproduction amplitude. It is then rather obvious that by considering axial currents having the same quantum numbers as the K mesons, one can relate, in an analogous way, the anomalous magnetic moments of the hyperons to K photoproduction amplitude. For instance, by considering the commutator

$$[A_\mu^{(L+)}(x),\ J_u^{em}(0)]$$

taken between a Σ^+ and a proton states, one can obtain a sum rule for the anomalous magnetic moment K_{Σ^+} of the Σ^+. This sum rule has been analyzed by MATHUR and PANDIT [18] under the decuplet dominance assumption and the calculated value is

$$K_{\Sigma^+} = 3{,}6 \frac{e}{2m_p}, \text{ whereas } (K_{\Sigma^+})_{\text{exp.}} = (3{,}5 \pm 1{,}5)\, e/2m_p.$$

The same authors derive analogously also the Λ anomalous magnetic moment for which they obtain

$$K_\Lambda = -0{,}84 \frac{e}{2m_p}, \text{ whereas } (K_\Lambda)_{\text{exp}} = (-0{,}69 \pm 0{,}13)\, e/2m_p\,.$$

The agreement between the calculated values and experimental data is not bad even if many further approximations are necessary in these cases, such as that of connecting the coupling constants relative to the Y_1^* (1385) resonance to those of the N_{33} by a simple Clebsh—Gordan coefficient, owing to the lack of precise experimental information.

6. Up to now we have restricted ourselves to consider $q_1^2 = q_2^2 = t = 0$. If we now retain the restriction $q_1^2 = q_1 \cdot q_2 = 0$, but allow $q_2^2 = t \neq 0$, we can obtain sum rules not only for coupling constants, but even for form factors. As an example of this kind of sum rule, we shall mention the two sum rules that can be derived from the commutators

$$[A_\mu^{(-)}(x), J_\nu^{(3)}(0)] \text{ and } [A_\mu^{(3)}(x), J_\nu^{(3)}(0)] \tag{66}$$

taken between nucleon states. It is easily recognized that we are then led to dispersive integrals involving electroproduction amplitudes instead of photoproduction amplitudes. By treating the commutators (66) in the usual way, assuming PCAC and approximating the continuum contribution under the decuplet dominance assumption through the GOURDIN—SALIN isobaric model, the two sum rules can be written as

$$G(t) = F_1^v(t) - t \frac{\lambda' c'(t)}{r_A\, m_\pi^2 \sqrt{3}} \cdot \frac{M+m}{3M^2}, \tag{67}$$

$$0 = K_V F_2^V(t) + 4m^2 \frac{\lambda' c'(t)}{r_A\, m_\pi^2 \sqrt{3}} \cdot \frac{M+m}{3M^2}, \tag{68}$$

$$F_1^V(0) = F_2^V(0) = 1,$$

where λ' and c' are proportional to the $N \pi N_{33}$ and $N \gamma N_{33}$ coupling constants. Eqs. (67) (68) have been derived in a recent paper by FURLAN, JENGO and REMIDDI [20]. By combining these two equations one has

$$G(t) = F_1^V(t) - \frac{t}{4m^2} K_V F_2^V(t). \tag{69}$$

This equation allows the evaluation of $G(t)$ in terms of the well-known electromagnetic form factors and it should represent a good approximation for not too large "t" ($t/m^2 < 1$). In particular, looking at the slope at $t = 0$, we can obtain an indication of the mass of the axial vector meson. Assuming the simple polar form

$$G(t) = \frac{M_A^2}{M_A^2 - t} \tag{70}$$

and using the experimental fit for the vector form factors we obtain $M_A =$ $= 0{,}815$ BeV. If we take into account the effect of N_{13} resonance the resulting mass is $M_A = 0{,}765$ BeV. These figures have to be compared with the result from the CERN neutrino experiment $M_A \lesssim 0{,}7$ BeV.

7. A particular case of our general sum rule (13) which is remarkably interesting is that in which we assume the kinematical configuration $q_1^2 =$ $= q_2^2 = q^2$ and $t = 0$. Let us illustrate this with the following example. We consider the commutator between two opposite vector currents

$$[j_\mu^{(+)}(x), j_\nu^{(-)}(0)] \tag{71}$$

sandwiched between proton states. As $t = 0$ we can again average on the proton spin, so that all the general formulae which we have deduced at the beginning, apply. We can, thus, write a sum rule of the kind (11):

$$\frac{1}{2\pi} \int a(\nu, q^2, q^2, 0)\, d\nu = 1\,. \tag{72}$$

In order to separate the one-particle (neutron) contribution we write for the matrix element of $J_\mu^{(+)}$ between proton and neutron the usual expression

$$< p \,|\, J_\mu^{(+)} \,|n> = \bar{u}_p \left\{ \gamma_\mu F_1^V(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2^V(q^2)}{2m} \right\} u_n\,,$$

so that the neutron contribution to $t_{\mu\nu}$ is

$$t_{\mu\nu}^{(n)} = \frac{1}{2} (2\pi)^4 \frac{1}{2} \delta \left(\nu - \frac{m_n^2 - m_p^2}{2} \right) \frac{1}{2} Tr \left\{ (\hat{p} + m_p) \cdot \right.$$

$$\left. \cdot \left(\gamma_\mu F_1 - \sigma_{\mu\varrho} q_\varrho \frac{F_2}{2m} \right) (\hat{p} + q + m_n) \left(\gamma_\nu F_1 + \sigma_{\nu\sigma} q_\sigma \frac{F_2}{2m} \right) \right\} =$$

$$= \pi \delta(\nu) \frac{1}{4} Tr \left\{ \hat{p} \gamma_\mu \hat{p} \gamma_\nu F_1^2 - \hat{p} \sigma_{\mu\varrho} q_\varrho \hat{p} \ldots \right\} =$$

$$= 2\pi \left\{ p_\mu p_\nu F_1^2 + \frac{q^2}{4m^2} F_2^2 p_\mu p_\nu + \ldots \right\}$$

and (72) becomes

$$1 = \{F_1^V(q^2)\}^2 + \frac{q^2}{4m^2}\{F_2^V(q^2)\}^2 + \frac{1}{2\pi}\int_{\text{cont}} a(\nu, q^2)\, d\nu\,, \tag{73}$$

This sum rule has already been derived in one form or another by several authors [21]. The interest of (73) lies in the fact that it shows (almost) explicitly the dependence on q^2. At $q^2 = 0$ we have the trivial identity $F_1^V(0) = 1$ since it could be demonstrated that "a" is proportional to q^2.

On the contrary, if we take the derivative with respect to q^2 at $q^2 = 0$, we obtain:

$$2\left\{\frac{d}{dq^2} F_1^V(q^2)\right\}_{q^2=0} + \frac{[F_2^V(0)]^2}{4m^2} + \frac{1}{2\pi}\int_{\text{cont}}\left\{\frac{\partial}{\partial q^2} a(\nu, q^2)\right\}_{q^2=0} d\nu = 0\,. \tag{74}$$

As long as the many-particle contribution is concerned one could demonstrate, as has been shown by CABIBBO and RADICATI [19], that the sum rule can be finally written as

$$\frac{1}{3} \langle r_V^2 \rangle = \frac{\{F_2^V(0)\}^2}{4m^2} + \frac{1}{2\pi^2 \alpha}\int_{\nu_0}^{\infty} \frac{d\nu}{\nu}\,(2\sigma_{1/2}^V - \sigma_{3/2}^V)\,, \tag{75}$$

where $\langle r_V^2 \rangle$ is the mean square isovector charge radius and $\sigma_{1/2}^V$ and $\sigma_{3/2}^V$ are the total cross-sections for production of $I = 1/2$ and $I = 3/2$ states by "isovector" photons on protons. If we try to saturate the integral by a few low lying resonances the agreement of (75) with experiment is not exceptionally good, since the contribution of the 33 resonance is of the wrong sign. Roughly speaking, $F_2^2/4m^2$ is equal to $\langle r_V^2 \rangle$ and the contribution of the N_{33} to the integral is negative and of the order of magnitude of one half the anomalous magnetic moment term. However, above the 33 resonance, the cross-section is mostly $I = 1/2$ and there is a good chance that the sum rule is, in fact, satisfied. As far as we know, nobody has given a satisfactory evaluation of the continuum contribution to $\langle r_V^2 \rangle$. This sum rule is indeed on a different footing from the previous ones.

8. All the sum rules we have considered up to now involve axial charges, i.e. quantities which should be considered in the framework of the $SU_3 \times SU_3$ algebra.

There is a very simple reason for having so far disregarded the simpler SU_3 algebra built up from vector currents, which, moreover, corresponds to a symmetry which is much better observed in nature. When we have to deal with vector currents the evaluation of the continuum contribution to the sum rules is more unpleasant.

In the axial vector case the evaluation of the many-particle contribution is based, in most cases, on PCAC, i.e. on the fact that the pseudoscalar mesons have the same quantum numbers as the divergences $\bar{D}$. For vector currents we do not postulate a proportionality relation between current divergences and particle fields to connect the many-particle contribution to "physical amplitudes", since, up to now there is no experimental evidence on the existence of a (well established) fundamental multiplet of *scalar* particles. It is clear, however, that we could introduce scalar particles into the theory, either believing in their existence, or as a suitable tool to express our results in a more compact form, through scattering amplitudes and cross-sections; but such a procedure is, at least at the present moment, of rather academic interest. In so doing we should indeed introduce a lot of unknown masses and coupling constants which would be presumably very difficult to relate to known quantities.

To evaluate the many-particle contribution we shall thus, in this case, apply different methods, for instance, perturbative-like ones. As, however, SU_3 gives a much better symmetry than $SU_3 \times SU_3$ the only polar terms give, in general, a not too bad saturation of the sum rule, so that, even if the methods used to evaluate the further contributions are rather rough, the results are sufficiently good.

We shall now illustrate the sum rules coming from SU_3 algebra with a few examples.

First, we shall consider a sum rule for the renormalization ratio for the weak strangeness changing vector current, which was first derived by FURLAN, LANNOY, ROSSETTI and SEGRÉ [22], some time ago, using the infinite momentum frame method.

With the present method we start from the commutator

$$[J_\mu^{(K+)}(x),\ J_\nu^{(K-)}(0)], \tag{76}$$

$J_\mu^{(K+)} = [J_\mu^{(K-)}]^+$ being the vector current having the same quantum numbers as the K^+, taken between π^+ states. We can then choose the simple configuration of $q_1^2 = q_2^2 = t = 0$ and then we can immediately write a sum rule of the type (15). For the one-particle contribution we take into account that

$$<\pi^+ | J_\mu^{(K+)} | \bar{K}_0> = (p_\pi + p_K)_\mu F_1(q^2) - q_\mu F_2(q^2), \tag{77}$$

where

$$F_1(0) = r_{K\pi} \tag{78}$$

and, to write the right-hand side of Eq. (15) we remember the commutation relation

$$[j_0^{(K+)}(x),\ j_\nu^{(K-)}(0)]_{x_0=0} = \left\{ J_\nu^{(3)} + \frac{3}{2} J_\nu^{(\gamma)} \right\} \delta(\vec{x}). \tag{79}$$

Our sum rule then takes the form

$$r^2_{K\pi} + \frac{1}{2\pi} \int\limits_{\text{cont}} \frac{w(\nu)}{\nu^2} d\nu = 1 , \tag{80}$$

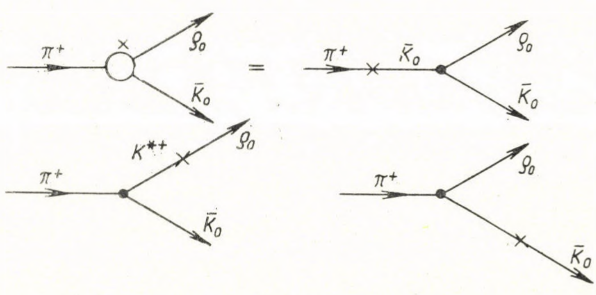
where, according to the definition (14)

$$\begin{aligned} w(\nu) = \frac{1}{2} (2\pi)^4 \Sigma_{n \neq \bar{K}^\circ} \{ | <\pi^+ | D_{K^+} | n >|^2 \delta (p_1 + q_1 - p_n) - \\ - | <\pi^+ | D_{K^-} | n > |^2 \delta (p_1 - q_2 - p_n) \} . \end{aligned} \tag{81}$$

To give an estimate of the many-particle contribution we performed a very simplified calculation taking into account only the lowest lying two-particle states treated in a perturbative way. To be more explicit we observe that in the first term in the right-hand side of Eq. (81) the states $| n >$ should be 1^+ with the same internal quantum numbers as the $\bar{K}_0$, whereas in the second term $| n >$ should have strangeness 1 and charge 2. The lowest two-particle states with these quantum numbers are composed of a pseudoscalar and a vector meson and these are the only states we have taken into account. We then had to deal e.g. with matrix elements of the type

$$< \pi^+ | D_{K^+} | \varrho_0 \bar{K}_0 > \tag{82}$$

which we treated in the polar approximation represented by the following graphs



(83)

Fig. 3

(the cross stands for D_{K^+}). In this way the matrix elements of the kind (82) are reduced to known quantities, i.e. to the matrix elements of the D between one-particle states and strong coupling constants. In so doing the result is still independent of the transformation properties of the breaking Hamiltonian and one obtains for the continuum contribution:

$$\frac{1}{2\pi} \int\limits_{\text{cont}} \frac{w(\nu)}{\nu^2} d\nu \simeq 0{,}07 \tag{84}$$

This number must not be taken too seriously, but only as an indication of the order of magnitude and sign of the renormalization effects.* They are quite small, as one can expect as a consequence of the ADEMOLLO and GATTO theorem, and, anyway, too small to change the present considerations about universality.

9. We shall now point out that if we are interested in the renormalization of the $\Delta S = \Delta Q$ vector current for the semileptonic decays of baryons, the same kind of procedure can be applied. In this case, however, the evaluation of the many-particle contribution can be rather simplified. Let us take as an example the renormalization ratio $r_{\Sigma N}$ for the process $\Sigma^- \to n e^- \tilde{\nu}_e$. We can start again from the commutator (1) taken between neutron states and assuming $q_1^2 = q_2^2 = t = 0$ we can average on the external neutron spin, so that the general formulae hold.

Thus, we reach the relation

$$r_{\Sigma N}^2 + \frac{1}{2\pi} \int\limits_{\text{cont}} \frac{w(\nu)}{\nu^2} d\nu = 1\,, \tag{85}$$

where now

$$\begin{aligned} w(\nu) = \frac{1}{2} (2\pi)^4 \Sigma_{\alpha \neq \Sigma^-} \{ | <n | D_{K^+} | \alpha> |^2 \delta (p_1 + q_1 - p_\alpha) - \\ - | <n | D_{K^-} | \alpha> |^2 \delta (p_1 - q_2 - p_\alpha) \}\,. \end{aligned} \tag{86}$$

In this case, in analogy with the procedure followed in many of the previous examples, the decuplet resonant states can be taken to simulate the continuum contribution, using an appropriate isobaric model.

The evaluation of the continuum contribution will thus depend on the value of an unknown parameter g^*, simulating the baryon-resonance $-D_{K+}$ coupling constant. If we believe in scalar mesons, g^* could be connected to the scalar meson-baryon-resonance coupling constant, invoking for instance a PCVC relation $D_K \sim \Phi_{K'}$. An estimate of the value of g^* can be made, with the same kind of approximation, on the ground of unitarity arguments applied to the pseudoprocess D_{K+} baryon scattering. This value has been determined by BOITI and REBBI in two different ways. The first [23] applies the method suggested by AMATI and FUBINI [24] for $\pi - N$ scattering to the D-baryon scattering. One considers the amplitude for the D-baryon scattering, projected in partial waves, taking into account only the contribution of intermediate baryon and decuple-resonant states. In the static model, one then requires

* We notice that the square of the term in which one breaks the symmetry on the vector line is logarithmically divergent. We have evaluated it by introducing a cut-off at $s = 3$ (baryon mass)^2. This contribution, however, is multiplied by a rather small Clebsch-Gordan coefficient and, in any case, is far from being the leading corrective term.

them to satisfy the high energy behaviour predicted by unitarity. Thus one obtains the value

$$g^{*2} = 4,9\,. \tag{87}$$

The second method relates the g^* coupling constant to the well known meson-baryon and meson-baryon-resonance coupling constants by applying analogous considerations to the "process"

$$D + B \rightarrow M + B\,. \tag{88}$$

The value so obtained is [25]

$$g^* = -\,2,2 \rightarrow (g^*)^2 = 4,83 \tag{89}$$

in excellent agreement with the previous one.

Thus, assuming for g^* the value (89) one easily derives, for the continuum contribution to the sum rule for $r^2_{\Sigma N}$:

$$\frac{1}{2\pi} \int\limits_{\text{cont}} \frac{w\,(\nu)}{\nu^2}\, d\nu \simeq 0,02 \tag{90}$$

so that

$$r^2_{\Sigma N} \simeq 0,98\,. \tag{91}$$

Again we see that the second order corrections, arising from the so-called medium strong interactions which break SU_3, are small.

10. As a final example we shall consider a new kind of sum rules, which, neglecting the many-particles contribution, coincide with the well-known Gell-Mann, Okubo SU_3 mass formulae.

We start by considering the commutator between a vector current and a vector divergence, both having the same quantum numbers or the K^+ meson:

$$[J_\mu^{(K+)}\,(x),\; D_{K^+}\,(0)] \tag{92}$$

and we take it between a K^+ and a K^- state, in order to derive a relation among meson masses. By choosing, as usual, $q_1^2 = q_2^2 = t = 0$, we can write the sum rule in a form of the kind (23), i.e.

$$\int a\,(\nu, 0, 0, 0)\, d\,\nu = 0 \tag{93}$$

since in the present case*

* In handling current-divergence commutators some care is necessary. From current algebra one cannot indeed derive a general commutation relation of the kind $[j^{(\alpha)}, D_\beta] = C_{\alpha\beta\gamma}\, D_\gamma$ since the D_α do not transform, under group operation as a basic octet. The validity of (94) is insured if the SU_3 breaking Hamiltonian transforms like the hypercharge as it is usually assumed. Eq. (94), however, is less restrictive and it holds also for more complicated Hamiltonians.

$$[j_0^{(K^+)}(x),\; D_{K^+}(0)]_{x_0=0} = 0. \tag{94}$$

The one particle contributions to Eq. (93) are easily evaluated by remembering that, following our convention

$$\left\langle K^+ \middle| J_\mu^{(K^+)} \middle| \begin{matrix}\pi^\circ \\ \eta\end{matrix} \right\rangle = \begin{pmatrix} 1/\sqrt{2} \\ \sqrt{3/2} \end{pmatrix} \{(p_K + p_{\pi,\eta})_\mu F_1(q^2) - q_\mu F_2(q^2)\} \tag{95}$$

and thus ($q^2 = 0$) $F_1(0) = r_{K\pi,\eta}$

$$\left\langle K^+ \middle| D_{K^+} \middle| \begin{matrix}\pi^\circ \\ \eta\end{matrix} \right\rangle = i \begin{pmatrix} 1/\sqrt{2} \\ \sqrt{3/2} \end{pmatrix} \begin{pmatrix} m_K^2 - & \begin{matrix} m_\pi^2 \\ m_\eta^2 \end{matrix} \end{pmatrix} r_{K\pi,\eta}\,. \tag{96}$$

The sum rule, written as the general form (26), becomes:

$$(m_K^2 - m_\pi^2)\, r_{K\pi}^2 + 3\,(m_K^2 - m_\eta^2)\, r_\eta^2 = \frac{1}{2\pi} \int_{\text{cont}} \frac{w(\nu)}{\nu}\, d\nu\,, \tag{97}$$

where, according to the general definition (25)

$$w(\nu) = \frac{1}{2}(2\pi)^4\, 2\Sigma_\mu < K^+ | D_{K^+} | n > < n | D_{K^+} | K^- > \delta(p + q - p_n) \tag{98}$$

(the factor 2 arises from crossing properties).

We remember now that

$$r_{K\pi,\eta}^2 = 1 + O(f^2)$$

if f is the strength of the breaking of the symmetry, and $m_K^2 - m_{\pi,\eta}^2 \sim O(f)$ by definition of f. Disregarding the third order corrections we can safely put $r_{K,\pi\eta} = 1$ and the sum rule (97) becomes:

$$4m_K^2 - 3m_\eta^2 - m_\pi^2 = \frac{1}{2\pi} \int_{\text{cont}} \frac{w(\nu)}{\nu}\, d\nu\,. \tag{99}$$

By looking at Eq. (98) one sees clearly that the right-hand side is $O(f^2)$ and (99 is actually the GELL-MANN—OKUBO mass formula for the pseudoscalar meson octet. The present method, however, presents at least in principle, a possibility of evaluating the continuum contribution, i.e. the $O(f^2)$ corrections. We can try to evaluate the right-hand side of Eq. (99) using the same approximation made in calculating $r_{K\pi}^2$. This calculation has actually been done by FURLAN, LANNOY, ROSSETTI and SEGRÉ [2]. In this case, however, due to the appearance of only one "denominator", the integral is divergent. A natural way to get rid of this difficulty would be to introduce a strong interaction form factor instead of the point-like coupling between the vector and the pseudoscalar mesons which we have taken in Eq. (83). However, in order to have a first indication, we have simply a cut-off on the integral, obtaining:

$$\frac{1}{2\pi}\int \frac{w(\nu)}{\nu}\,d\nu = \begin{cases} 0{,}55\, m_0^2 \\ 0{,}19\, m_0^2 \end{cases} \text{for} \quad \begin{cases} \Lambda = 2m_B^2 \\ \Lambda = m_B^2 \end{cases}$$

m_0 and m_B being the mean masses of pseudoscalar mesons and baryons. These values are to be compared to the experimental one of the left-hand side of Eq. (99):

$$(4m_K^2 - 3m_\eta^2 - m_\pi^2)_{\exp} = 0{,}36\, m_0^2\,.$$

Our calculation shows that even a very rough approximation gives the right sign for the correction and also a reasonable order of magnitude.

In what the mass formula for the baryon octet is concerned the decomposition of the basic quantities T_μ and t_μ is a little more complicated, since in this case we must take the commutator (94) between a proton and a Ξ^- state and we cannot average over the external particle spins.

We shall limit ourselves to writing explicitly the sum rule discussed in [4].

$$2m_P + 2m_\Xi - 3m_\Lambda - m_\Sigma = \frac{1}{2\pi}\int_{\text{cont}} \frac{w(\nu)}{\nu}\,d\nu\,, \tag{100}$$

where now $w(\nu)$ can be obtained from

$$w = \frac{1}{2}(2\pi)^4 \sum\nolimits_{n\neq\Lambda,\Sigma_0}\{ < p\,|\,D_{K^+}\,|n> < n\,|\,D_{K^+}\,|\,\Xi^- > \cdot \tag{101}$$

$$\cdot\;\delta\,(p_1 + q_1 - p_n)\text{-crossed term}\}$$

by decomposing it as

$$w = \bar{u}_p\,\{w_1(\nu, 0) + (\gamma\cdot q_1)\,w_2(\nu, 0)\}\,u_\Xi\,. \tag{102}$$

It is easy to see that all the $SU(3)$ results can be rederived in the current algebra framework. Moreover using some particular models, like the decuplet approximation, a first, rough* indication can be done for the size of the corrections. For instance, starting from the commutator

$$[j_\mu^{(L+)}(x), j_\nu^{(em)}(0)] \tag{103}$$

* We should remark that for the matrix element $N^* \to ND$ a $l = 2$ transition is required.

taken between Σ^+ and proton, one can derive a sum rule for the anomalous magnetic moment of the Σ^+ in the framework of broken SU_3 algebra. Assuming decuplet dominance one can obtain

$$K_{\Sigma^+} = 2.21\ e/2m_p,\ \text{whereas}\ (K_{\Sigma^+})_{\text{exp}} = (3{,}5 \pm 1{,}5)\ e/2m_p .$$

This value has been calculated in a recent paper [26] where, under the same assumptions, the magnetic moment of all the baryons are evaluated. For the magnetic moment, coming from a more complicated calculation, these authors give:

$$\mu_\Lambda = -\,0{,}80\ e/2m_p,\ \text{whereas}\ (\mu_1)_{\text{exp}} = (-\,0{,}69 \pm 0{,}13)\ e/2m_p .$$

Also in this case the calculated corrections to the symmetry limit are of the right sign and order of magnitude.

In conclusion we shall point out that in any studied case the agreement of the results obtained from current algebra dispersive sum rules and experimental data are good, and in some cases even spectacularly so. This shows, in our opinion at least, the soundness of the commutator dispersion approach to elementary particle physics.

III. Further developments: Strong interactions sum rules

1. We hope that in these two lectures we have been able to present a sufficiently impressive, although incomplete, description of the successes of the "current algebra" approach. There are some rather interesting results which we have not discussed, like the treatment of higher symmetries [27] or the specific application of our machinery to weak interactions[28].

Let us make a few remarks on the practical evaluation of the sum rules. In the previous lecture you have seen that, apart from the happy case of the ADLER—WEISBERGER relation, we are forced to approximate the contribution of the continuum with a few resonant states. Then we are faced with the twofold question of which states are to be consistently included and of the evaluation of their parameters, in some cases not directly deducible from experiments. As far as this latter point is concerned, we remind you that a relation between the coupling constants g_{B*BD} (unknown) and g_{BBD} can be found using simple unitarity considerations. [See the discussion on the $SU(3)$ corrections.] We have to keep this point in mind for what follows.

We want only to mention the first problem which arises when we try to exhaust the sum rule with a small number of states of discrete i.e. bound states or resonances. This set of particles has to saturate the algebra: taking the matrix element between two such states, a consistent result is obtained by limiting the intermediate contribution to states of the same set. This means that the low lying hadron states should be, with a good approximation, described as a mixture of a few irreducible representations. Therefore, we have to find what these representations are. This point of view and the related problems are discussed elsewhere [29].

2. Let us describe instead, for the second problem, a completely different approach [30] which starts from the very attractive suggestion we gained at the end of the first lecture. We considered there the extrapolation of the

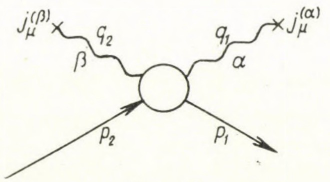


Fig. 4

momenta q_1^2, q_2^2 to the masses of physical particles with the same quantum numbers as the currents $j_\mu^{(\alpha)}, j_\mu^{(\beta)}$. (Thus if $j_\mu^{(\alpha)}$ is the isotopic current the physical particle is the $\varrho^{(\alpha)}$ meson.)

More precisely we notice that for $q_{1,2}^2 \sim m_{\alpha,\beta}^2$ our amplitude is dominated by the graph of Fig. 4.

Then we can write the exact relation

$$\lim_{q_1^2, q_2^2 \to m_\alpha^2, m^2\beta} (q_1^2 - m_\alpha^2)(q_2^2 - m_\beta^2)\, a(\nu, q_1^2, q_2^2, t) = \text{const. Im}\, A, \qquad (1)$$

where A is defined according to the decomposition of the $\beta(q_2) + p_2 \to \alpha\,(q_1) + p_1$ scattering amplitude

$$T_{\mu\nu}\, \varepsilon_\mu^{(\alpha)}\, \varepsilon_\nu^{(\beta)} = (P\cdot\varepsilon^{(\alpha)})\,(P\cdot\varepsilon^{(\beta)})\, A\,(\nu, t) + \dots . \qquad (2)$$

Moreover, the right-hand side of Eq. (31). I

$$\frac{1}{2\pi}\int d\,\nu\; a_1^{(\alpha\beta)}(\nu, q_1^2, q_2^2, t) = C_{\alpha\beta\gamma}\, F_1^{(\gamma)}(t)\,. \qquad (31)$$

is not singular in $q_{1,2}^2$ so that our sum rule becomes

$$\int d\,\nu\, \text{Im}\, A\,(\nu, t) = 0\,. \qquad (3)$$

As already stressed this result is independent of the explicit form of the e. t. commutator between current components. The only requirement we need is locality, i.e. the presence of δ function derivatives of finite order. Thus, we are in the pure strong interactions region, where there is no trace of weak currents. It is rather natural to look for the possibility of deriving Eq. (3) directly from the general properties we are accustomed to ascribe to strong interactions, namely analiticity (connected with locality), unitarity and asymptotic behaviour.

In fact, let us assume an unsubtracted dispersion relation, at fixed t, for the function $A(\nu)$:

$$A(\nu) = \frac{1}{\pi} \int d\nu' \frac{\operatorname{Im} A(\nu')}{\nu' - \nu - i\varepsilon} . \tag{4}$$

If $A(\nu)$ is subjected to the asymptotic bound

$$|A(\nu)| < \nu^{\beta} \quad \nu \to \infty , \qquad \beta < -1 , \tag{5}$$

we deduce at once from Eqs. (4), (5)

$$\int \operatorname{Im} A(\nu') \, d\nu' = 0. \tag{6}$$

The crux of the matter is the asymptotic condition (5) so that we must specify our present knowledge about the large ν behaviour of the scattering amplitude. Let us remind you that for spinless particles we have for the scattering amplitude the upper bound $\nu \ln^2 \nu$, the well known FROISSART limit [31]. However, if we consider, as we are doing, spin one particles we shall see that, at least for some amplitudes, unitarity requires more drastic limitations such that, for instance, condition (5) can be verified.

To appreciate better the working mechanism let us rederive the FROISSART bound in an heuristic way. As a consequence of unitarity and as the total cross-section is larger than the purely elastic one, we have, for large s,

$$\operatorname{Im} A(\nu, 0) > \text{const} \frac{1}{\nu} \int_{-\nu}^{0} |A(\nu, t)|^2 \, dt . \tag{7}$$

Now, at high energy the elastic scattering is practically concentrated around the diffraction peak and, assuming as a first, rough approximation a constant shape of the diffraction peak we obtain

$$|A(\nu, 0)|^2 < \text{const.}\, \nu \; \operatorname{Im} A(\nu, 0) , \tag{8}$$

namely

$$|A(\nu, 0)| < \text{const.}\, \nu . \tag{9}$$

Of course this is only an indicative derivation and to include logarithmic terms in the asymptotic bound we have to specify the variation of the diffraction peak (as in a Regge poles model).

Let us now consider a scattering process where spin 1 particles are involved e.g. $\pi\varrho$ scattering (neglecting for the moment isospin). The general form of the amplitude is:

$$T = A(P \cdot \varepsilon_1)(P \cdot \varepsilon_2) + B \frac{1}{2} [(P \cdot \varepsilon_1)(Q \cdot \varepsilon_2) + (Q \cdot \varepsilon_1)(P \cdot \varepsilon_2)] + C_1 (Q \cdot \varepsilon_1)(Q \cdot \varepsilon_2) + C_2 (\varepsilon_1 \cdot \varepsilon_2); Q = \frac{q_1 + q_2}{2}. \tag{10}$$

To sketch shortly the core of the proof let us assume the oversimplified situation where only forward scattering is considered $p_1 = p_2 = p, q_1 = q_2 = q$ and

$$T_{\mu\nu} \sim A p_\mu p_\nu + C_2 g_{\mu\nu} \quad (Q \cdot \varepsilon_1 = Q \cdot \varepsilon_2 = 0). \tag{11}$$

The sum over the polarizations of the intermediate ϱ implied by the unitarity condition, introduces an additional factor ν^2. In fact

$$\sum_r T_{\mu\lambda} T^+_{\lambda'\nu} \varepsilon_\lambda^{(r)} \varepsilon_\lambda^{(r)} = (A p_\mu p_\lambda + C_2 g_{\mu\lambda}) \cdot \left(- g_{\lambda\lambda'} + \frac{q_\lambda q_{\lambda'}}{M^2}\right)(A^* p_{\lambda'} p_\nu + C_2^* g_{\lambda'\nu}) \sim |A|^2 p_\mu p_\nu \left|\frac{\nu^2}{M^2}\right| + \dots \tag{12}$$

Thus relation (8) for the amplitude A now becomes

$$\nu^2 \mid A(\nu, 0) \mid^2 < \text{const.}\, \nu \, \mathrm{Im} A(\nu, 0) \tag{13}$$

i.e.

$$\mid A(\nu, 0) \mid < \text{const.}\, \nu^{-1}. \tag{14}$$

Of course a more complete derivation is possible using an orthogonal decomposition for T, and the following relations are derived, assuming a constant shape for the diffraction peak,

$$\mid A(\nu, 0) \mid < \text{const.}\, \nu^{-1}, \quad \mid B(\nu, 0) \mid < \text{const.} \tag{15}$$

$$\mid C_{1,2}(\nu, o) \mid < \text{const.}\, \nu.$$

The remarkable point is that we get an improved asymptotic behaviour for the different amplitudes, classified according to the power of P_μ they multiply. Now we can be more realistic, taking into account isospin and the general features of strong interactions at high energies, summarized by a Regge pole model with the empirical value of the trajectories in the small t region. On the basis of Eq. (15) we shall assume that if the behaviour of a given isospin amplitude for spinless particles is ν^{α} (corresponding to the exchange of "spin" α), the behaviours of A, B, $C_{1,2}$ are

$$A(s, 0) \sim \nu^{\alpha-2}, \quad B(s, 0) \sim \nu^{\alpha-1}, \quad C_{1,2} \sim \nu^{\alpha}. \tag{16}$$

Thus we get the asymptotic bound (5) for A if $\alpha < 1$ and for B if $\alpha < 0$.

To exemplify these considerations let us investigate in more detail the $\pi - \varrho$ case. We limit ourselves to the $t = 0$ situation and the isospin dependence leads to the decomposition of each amplitude according to the $I = 0, 1, 2$ values (in the t-channel)

$$T = P_0 T_0 + P_1 T_1 + P_2 T_2 \tag{17}$$

being the *Pi* projection operators on isospin eigenstates. At high energies T_1 is dominated by the exchange of the ϱ trajectory, and T_2 by double charge exchange. The candidate amplitudes to verify a sum rule (3) are:

$$A^{(1)}, \; A^{(2)}, \; B^{(1)}, \; B^{(2)}.$$

Experimentally $\alpha_{\varrho}(0) \simeq 0{,}5 < 1$, so that for $A^{(1)}$ we have

$$\int d\nu \; \mathrm{Im} \; A^{(1)}(\nu) = 0. \tag{18}$$

If we assume that for double charge exchange $\alpha_2(0) < 0$ we can still write

$$\int \mathrm{Im} \; A^{(2)}(\nu)\, d\nu = 0, \quad \int d\nu \; \mathrm{Im} \; B^{(2)}(\nu) = 0. \tag{19}$$

(Actually the sum rule $A^{(2)}$ turns out to be trivial by crossing.)**

Finally, on the basis of the assumed asymptotic behaviour and of the crossing properties, there is nothing against the additional sum rule

$$\int \nu \; \mathrm{Im} \; A^{(2)}(\nu)\, d\nu = 0. \tag{20}$$

To evaluate $\mathrm{Im} A^{(1,2)}$, $\mathrm{Im} B^{(2)}$ we assume tentatively that only the well known particles π, ω, Φ give an important contribution as intermediate states. We obtain from Eqs. (18), (19), (20) respectively***

** The crossing properties are $A(\nu) = (-1)^T \; A^*(-\nu)$; $B(\nu) = (-1)^T \; B^*(-\nu)$; $T = 0, 1, 2$.

*** The various coupling constants are defined by the interaction Lagrangians

$$g_{\varrho\pi\pi}\, \varepsilon_{ijk}\, \varrho_{\mu}^{i} \pi^{j} \; \partial_{\mu} \pi_{k},$$

$$g_{\varrho\pi\substack{\omega\\\varphi}}\, \varepsilon_{\alpha\beta\gamma\delta}\, \partial_{\alpha} \begin{pmatrix} \omega \\ \varphi \end{pmatrix}_{\beta} \partial_{\gamma}\, \varrho_{\delta}^{i}\, \pi^{i}.$$

$$4\ g^2_{\varrho\pi\pi} = m^2_\varrho\,(g^2_{\varrho\pi\omega} + g^2_{\varrho\pi\varphi}), \tag{18'}$$

$$4g^2_{\varrho\pi\pi} = (\nu_\omega + m^2_\varrho)\, g^2_{\varrho\pi\omega} + (\nu_\varphi + m^2_\varrho)\, g^2_{\varrho\pi\varphi}, \tag{19'}$$

$$2\ g^2_{\varrho\pi\pi} = \nu_\omega\, g^2_{\varrho\pi\omega} + \nu_\varphi\, g^2_{\varrho\pi\varphi}, \tag{20'}$$

where

$$\nu_{\omega,\,\varphi} = (m^2_{\omega,\varphi} - m^2_\varrho - m^2_\pi)\,. \tag{21}$$

The solution is the trivial one

$$g_{\varrho\pi\pi} = g_{\varrho\pi\omega} = g_{\varrho\pi\varphi}\,. \tag{22}$$

and it shows that the naive approximation of retaining only the above low lying single particle states is not completely reliable (at least in this problem).

3. In this context it is perhaps not useless to further recognize the similarity between results derived from current algebra (after extrapolation to the time-like region) and from unitary sum rules for partial wave amplitude strong interactions. We want in particular to mention the relation between the $\pi\, NN_{33}$ and $\pi\, NN$ coupling constants g^* and g. In the current algebra approach we start from the e.t. commutators [32]

$$[\bar I_3,\ A^{(3)}_\mu] = 0\,,\ \ [\bar I^+,\ A^{(-)}_\mu] = 2J^{(3)}_\mu\,. \tag{24}$$

We consider their matrix elements between nucleon states, and saturating the sum rule with the N and N_{33} intermediate states only it is possible after extrapolation to $q_2^2 = t = m_\pi^2$ to derive the result,

$$\frac{g^{*2}}{g^2} = \frac{9}{2}\,\frac{M^2\, m^2_\pi}{m\,(M+m)^2\,(2M-m)}\,, \tag{25}$$

(M, m, m_π are the N_{33}, N, π masses).

If we introduce the more familiar definitions for dispersion relations theorists [24]

$$\frac{g^2}{4\pi} = f^2\,\frac{4m^2}{m^2_\pi}\,, \qquad \frac{g^{*2}}{4\pi} = f^{*2}\, 3\,\frac{M}{m}\,. \tag{26}$$

Eq. (26) can be written

$$f^{*2} = \frac{6M\, m^2}{(M+m)^2\,(2M-m)}\, f^2 \simeq f^2\,, \tag{27}$$

while the experimental value is about $1{,}1\, f^2$.

Now let us consider the πN scattering amplitude in the $J = T = 3/2$ channel (p wave):

$$f_{33}(q) = \frac{e^{i\delta_{33}} \sin \delta_{33}}{q} \tag{28}$$

(q is the momentum in the c.m. system).

As a consequence of the unitarity condition for partial waves $f(q) \lesssim q^{-1}$ for large q. Moreover we have the threshold behaviour $f_{33} \sim q^2$ for small q. Thus we can introduce a new amplitude

$$F_{33}(q) = \frac{f_{33}(q)}{q^2}$$

such that

$$F_{33}(q) \lesssim q^{-3} \qquad q \to \infty$$

$$F_{33}(q) \sim \text{const.} \quad q \to 0 .$$

It is important to remark that both the large and small q behaviours must be taken into account to choose correctly the improved F_{33} (q) ampli ude.

Assuming an unsubtracted dispersion relation for $F_{33}(\omega)$ we have

$$F_{33}(\omega) = \frac{1}{\pi} \int\limits_{m_\pi}^{\infty} \frac{\operatorname{Im} F_{33}(\omega')}{\omega' - \omega} d\omega' + \frac{1}{\pi} \int\limits_{\text{left cut}} \frac{R_{33}(\omega')}{\omega' - \omega} d\omega' . \tag{29}$$

The unitarity limitation on the asymptotic behaviour of F_{33} enables us to derive from (29) the sum rule

$$\frac{1}{\pi} \int\limits_{m_\pi}^{\infty} d\omega' \operatorname{Im} F_{33}(\omega') + \frac{1}{\pi} \int\limits_{\text{left cut}}^{\infty} d\omega' R_{33}(\omega') = 0. \tag{30}$$

There are various contributions to the unphysical left cut. If we limit ourselves to the crossed N exchange, the second integral introduces the πN coupling constant f^2, while the first one is, in the limit of zero width, exactly f^{*2}. Thus a relation of the type (27) results. In particular, neglecting nucleon recoil, we find [24]

$$R_{33}(\omega) \simeq -\frac{4}{3} f^2 \delta(\omega) \tag{31}$$

and we obtain the static model result

$$f^{*2} = \frac{1}{\pi} \int \frac{\sin^2 \delta_{33}(\omega)}{q^2} d\omega \simeq \frac{4}{3} f^2 = 1.33 f^2 . \tag{32}$$

This example reminds you again that the asymptotic constraints due to unitarity introduce some restrictions on the bound states parameters [33]. In particular we can have strong self consistency requirements which lead to the so called "boot-strap" approach [34]. We believe that our sum rules, discussed at the beginning of this Section, represent the relativistic generalization of the bootstrap conditions, resulting in relations among masses and coupling constants of particles and resonances.

However let us stress the fact that while in the above approach you have to deal with the complicated singularities of the left cut, the superconvergence sum rules (18), (19), (20) give rise to simple, algebraic relations among the different parameters.

To conclude let us remind you two points: the first one concerns the crucial role played by the spin of the involved particles. If we increase the spin, we increase at the same time the number of coupling constants and the number of sum rules, i.e. of equations. This means that the more complicated (i. e. with more parameters) is the theory the higher is the number of constrains we have on it. It should be necessary to ascertain the relative rate of growth of these two sets or more directly to study the problem of the exact saturation of the superconvergent sum rules.

On the other hand equations like (18) or (25) show the possibility of a connection between coupling constants of different dimensions. This is important because different dimensionality of coupling constants corresponds to a different degree of singularity of the theory, so that we could hope to reduce more d vergent theories to less divergent ones. It is clear the superconvergent sum rules do not represent the complete solution to Quantum Field Theory, but rather they constitute an algebraic approximation to it in terms of many narrow resonances.

Really at present it is not given to know how far these speculations can go. We had better to wait for the next Balaton-meeting.

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ДИСПЕРСИОННЫЕ ПРАВИЛА СУММ ИЗ АЛГЕБРЫ ТОКОВ

Г. ФУРЛАН и К. РОССЕТТИ

Резюме

Одним из самых успешных и популярных направлений в настоящем подходе к физике элементарных частиц, кроме теории групп, является так называемая «алгебра токов». В этой работе дается обзор основных физических идей и самых важных результатов этого нового подхода, и обсуждается нынешняя позиция относительно пока открытых проблем.

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SOME APPLICATIONS OF CURRENT ALGEBRAS

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The relations of current algebras to strong interaction dynamics are studied by considering charge — current commutators between meson states and the vacuum.

In applications of current algebras three assumptions are generally made: *a)* unsubtracted dispersion relations wherever possible, *b)* partial axial vector current conservation (PCAC), and frequently *c)* dominance of certain intermediate states in sum rules. It is generally believed that a knowledge of strong interaction dynamics, based on *a)*, will make assumptions *b)* and *c)* unnecessary and may even justify current algebras, but decisive evidence may be very far at present.

To learn about the relations of current algebras to strong interaction dynamics we study particularly simple cases: charge-current commutators between meson states and the vacuum. Here Lorentz-invariance restricts the spin of contributing intermediate states to zero or one, disconnected pairs are eliminated by the application of dispersion theory [1], the decomposition of the dispersion integral into independent kinematic forms provides a further selection.

When dealing with $SU_2 \times SU_2$ we can use the conservation of the vector currents. Between $< \pi^+ \mid$ and the vacuum $\mid 0 >$ we consider

$$< \pi^+ \mid [(j^V_{I_3}(0))_\mu , Q^A_{I+}] \mid 0 > = \lim_{q \to 0} F_\mu(p, q) = < \pi^+ \mid (j^A_{I+}(0))_\mu \mid 0 > \quad (1)$$

$$F_\mu(p, q) = \int \Theta(-x_0)\, e^{iqx} < \pi^+ \mid [(j^V_{I_3}(0))_\mu ,\ D^A_{I+}(x)] \mid 0 > dx^4 = \\ = A(\nu)\, p^\pi_\mu + B(\nu)\, q_\mu . \quad (2)$$

Following the general method [1] a dispersion relation for (2) in $\nu = (p^\pi \cdot q)/m^\pi$ with fixed $q^2 = 0$ is assumed.

$$f_\pi = A(0) = \frac{1}{\pi} \int \frac{\alpha(\nu')}{\nu'}\, d\nu' , \quad (3)$$

where the absorptive part $\alpha(\nu')$ is the coefficient of p_μ^π to be picked from

$$\begin{aligned} & i/2\,(2\pi)^4 \sum_n < \pi^+ \,|\, (j_{I_3}^V(0))_\mu \,|\, n > < n \,|\, D_{I+}^A(0) \,|\, 0 > \cdot \delta(p^n + q') \\ & - < \pi^+ \,|\, D_{I+}^A(0) \,|\, n > < n \,|\, (j_{I_3}^V(0))_\mu \,|\, 0 > \delta(p^\pi + q' - p^n). \end{aligned} \tag{4}$$

Only vector mesons contribute, we introduce a mass spectrum $\varrho^V(M)$ and rewrite without loss of generality

$$< \pi^+ \,|\, D_{I+}^A(0) \,|\, V(M) > = i(p^\pi - q^1)\,\varepsilon^V \frac{f_\pi\, m_\pi^2\, g_{V\pi\pi}(M)}{(m_\pi^2 - (q')^2)\, K_{V\pi}^\pi(q'^2)} \tag{5}$$

using a pion pole model and correcting for higher contributions by a form factor $K_{V\pi}^\pi(q'^2) \cdot K_{V\pi}^\pi(m_\pi^2) = 1$ and $K_{V\pi}^\pi(q'^2) \approx 1$ characterizes the range of pole dominance or approximate validity of PCAC. Relation (3) gives

$$f_\pi = - \int\limits_{(2m_\pi)} \frac{\varrho^V(M) f_\pi \cdot g_{V\pi\pi}(M) \cdot f_V(M)}{M^2 \cdot K_{V\pi}^\pi(0)}\, dM \tag{6}$$

to be compared with the dispersion integral for the isospin current form factor between π-mesons

$$1 = - \int\limits_{(2m_\pi)} \frac{\varrho^V(M)\, g_{V\pi\pi}(M) f_V(M)}{M^2}\, dM\,. \tag{7}$$

We derive $(K_{V\pi}^\pi(0))_{av.} = 1$, an average validity of PCAC as a consequence of current algebras and unsubtracted dispersion relations [2].

Similarly we can apply the commutator of an axial charge and an axial current between a ϱ^+ meson and the vacuum. With related notations the integral F_μ is now split into kinematic forms

$$F_\mu(p, q) = A(\nu)\,\varepsilon_\mu^\varrho + B(\nu)\,(\varepsilon^\varrho \cdot q)\, p_\mu^\varrho + C(\nu)\,(\varepsilon^\varrho \cdot q)\, q_\mu \tag{8}$$

with $A(0) = f_\varrho$ where $< \varrho^+ \,||\, (j_I^V + (0))_\mu \,|\, 0 > = f_\varrho\, \varepsilon_\mu^\varrho$. In calculating the absorptive part (4) we find that the pion intermediate state contributes only to B and C and that an isotriplet of axial vector meson states is necessary to saturate the sum rule for $A(0)$.

Explicitly we find:

$$f_\varrho = - \frac{1}{\sqrt{2}} \int \frac{f_\pi\, G_{A\varrho\pi}(M) \cdot f_A(M)\, \varrho^A(M)}{M^2 - M_\varrho^2}\, dM\,. \tag{9}$$

If we make the additional assumption of dominance by a single resonance corresponding to ϱ-meson universality we can apply the commutator of equ. (1) between $< A^+ |$ and $| 0 >$ to deduce $f_\varrho = - f_A$ and obtain a relation between mass and effective coupling constant

$$\sqrt{2} = G_{A\varrho\pi} \cdot f_\pi (M_A^2 - M_\varrho^2)^{-1}. \tag{10}$$

With A_1 (1070) the coupling constant comes out too large, but this may only show the limitations of the single resonance assumption, the postulate for axial vector mesons is maintained [3].

We can also make applications to SU_3 violations to eliminate ambiguities of scaling when approximating matrix elements by SU_3 Clebsch—Gordan coefficients.

So it is found that $f_\varrho = f_{K^*}$ up to second order SU_3 violations [4] and to the same accuracy $g_{K^*K^+\pi^-}/g_{\varrho^+\pi^+\pi^0} = (m_{K^*})^2/2\, m_\varrho^2$ for dimensionless coupling constants in vector meson decays.

For the pseudoscalar meson decay constants can be found:

$$\begin{aligned} f_K/f_\pi = & - \sqrt{2} \int \frac{\varrho^V(M) f_V(M) g_{VK\pi}(M)}{M^2} dM + \\ & + \sqrt{2} \int \frac{\varrho^S(M) f_S(M) g_{SK\pi}(M)}{M^2 - m_K^2} dM . \end{aligned} \tag{11}$$

from the commutator $[Q_I^A, (j_{K^+}^V)_\mu] = \frac{1}{2} (j_{K^+}^A)_\mu$, between $< K^+ |$ and $| 0 >$. Scalar mesons (with a spectrum $\varrho^S(M)$) may contribute because $(j_{K^+}^V)_\mu$ is not conserved. We compare (11) with dispersion integrals of form factors

$$< K^+ | (j_{K^+}^V)_\mu | \pi_0 > = \frac{1}{\sqrt{2}} (F^+(q^2)(p^K + p^\pi)_\mu + F^-(q^2))(p^K - p^\pi)_\mu), \tag{12}$$

$$\frac{1}{\sqrt{2}} F^+(q^2) = - \int \frac{\varrho^V(M) \cdot f_V(M) g_{VK\pi}(M)}{M^2 - q^2} dM, \tag{13}$$

$$\begin{aligned} \frac{1}{\sqrt{2}} F^-(q^2) = & (m_K^2 - m_\pi^2) \int \frac{\varrho^V(M) \cdot f_V(M) g_{VK\pi}(M)}{M^2 (M^2 - q^2)} dM + \\ & + \int \frac{\varrho^S(M) f_S(M) g_{SK\pi}(M)}{M^2 - q^2} dM . \end{aligned} \tag{14}$$

With $q^2 = m_K^2$ we rederive the relation of CALLAN and TREIMAN [5]

$$f_K/f_\pi = F^+(m_K^2)\left(1 - \frac{m_\pi^2}{(m_V^2)_{av.}}\right) + F^-(m_K^2) \tag{15}$$

with a small correction from the finite π-meson mass. If we assume dominance of vector mesons over scalar ones we derive

$$f_K/f_\pi = F^+(0)\,; \qquad F^-(0)/F^+(0) = \frac{m_\pi^2 - m_K^2}{(m_V^2)_{av.}}\,. \tag{16}$$

It should be noted that scalar meson contributions produce an increase of f_K/f_π and $F^-(0)/F^+(0)$ over the values of equ. (16), both in the observed direction. This supports equality of the fundamental CABIBBO angles for axial and vector currents [6] and attributes to SU_3 violations an enhancement of the rate $K^+ \to \mu + \nu$ relative to $\pi^+ \to \mu + \nu$ recently discussed [7].

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НЕКОТОРЫЕ ПРИМЕНЕНИЯ АЛГЕБРЫ ТОКОВ

Б. РЕННЕР

Резюме

Изучается отношение алгебры токов и динамики сильных взаимодействий, рассматривая коммутаторы зарядов и токов между мезонными состояниями и вакуумом.

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WEAK AND ELECTROMAGNETIC CURRENTS AND HADRON CLASSIFICATION

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We make a few remarks about the various possibilities of representation mixing. In connection with magnetic moment relations it is shown that assuming "partial conservation of the tensor currents" and a vector meson — axial vector meson mixing, excellent agreement with experiment cau be reached.

A rather wide justification has been given to the striking successes of $SU(6)$ theory in the frame of the algebra of currents [1]. It has been realized that the emergence of the $SU(6)$-symmetry results for baryons was linked to the saturation of the algebra of vector and axial vector currents by a few one-particle states, which build up a single $SU(6)$ representation, namely the *56*.

By allowing the saturation within more than one irreducible representation one gets results that "correct" the $SU(6)$ predictions, and one can look at such corrections in order to gain some insight into what these additional representations should be.

As L. MAIANI has shown [2], we see that the inclusion of a (20,1) $SU(6) \otimes \otimes 0(3)_L$ representation, which is supposed to describe the leakage of the axial current operators into negative parity baryon resonances, leads to the well verified relation

$$-\frac{G_A}{G_V} = \frac{1}{3}\,\frac{D+F}{D-F} \tag{1}$$

and also to a lowering of the $N^* - N$ axial current transition matrix element from its $SU(6)$ value, which is quite in agreement with the experiments.

In the first part of this talk I shall be concerned with the discussion of the possibility that the leakage of axial currents is directed towards representations different from the previous proposed (20,1); and the ultimate answer to such a question must of course rely on the comparison of the testable predictions, if any at all, with the experiments.

At the very budding of $SU(6)$ theory it was proposed that the negative parity baryon resonances could be arranged into a 70^- representation [3]. Let me recall that the physical particles content of this representation is [in the notation $(SU(3), J^P)$]:

$$(8, 1/2^-)\,,\ (8, 3/2^-)\,,\ (1, 1/2^-)\,,\ (10, 1/2^-)\,.$$

We see that in this set of states there can be arranged the quite well established γ-octet, the possible $1/2^-$ η-octet, and the singlet Y_0^* (1405) — whilst the $1/2^-$ decuplet is still very dubious.

With the very simple method of mixing representations, one can again quickly derive a relation between $\frac{G_A}{G_V}$ and $\frac{D}{F}$ [4], i.e.

$$-\frac{G_A}{G_V} = \frac{D + F}{3F - D}\,. \tag{2}$$

Owing to the fact that $-G_A/G_V$ must be smaller than its $SU(6)$ value $\frac{5}{3}$, because of the leakage of axial currents, we see immediately that $\frac{D}{F}$ must also be smaller than the 3/2 $SU(6)$ prediction, which is in conflict with the latest experimental figures [5].

One can also show [4] that Eq. (2) is also provided by adding to the (*56*, 0) the *20*$^-$ representation with negative parity, though I must say that such representation has never been seriously considered, as far as I know, to be a good candidate for arranging some of the known negative parity resonances.

We can go further and ask whether the spectrum generating non-compact groups which have recently been proposed [6], can naturally give some set of states, whose connection with the familiar baryon octet and decuplet through axial currents is able to lead to good predictions of the type of Eq. (1).

DOTHAN, GELL—MANN and NEEMAN [6] have proposed that the baryon states should be contained in a particular infinite dimensional unitary representation of the $U(6,6)$ group. This representation, when analyzed in terms of its $U(6)$ compact subgroup, leads to the following sequence or "tower":

$$56^+,\, 56^- \oplus 700^-,\ \ldots,$$

and one may consider the saturation of the chiral algebra within this huge set of states.

The saturation problem can be solved [7] and we finally get:

$$-\frac{G_A}{G_V} = \frac{D + F}{9F - 5D}\,. \tag{3}$$

Again one sees that if $-G_A/G_V < \frac{3}{2}$, as is experimentally the case, also $\frac{D}{F} < \frac{3}{2}$, which is in conflict with the experiments.

Thus it seems to us that this additional set of states does not lead to an acceptable picture of the baryons axial transitions.

Another proposal has been made by SALAM et al. [6] in the frame of the "L-excitation" scheme, whose orbital angular momentum pattern is provided by an infinite dimensional unitary representation of the group 0(3,1), thus giving rise to the $SU(6) \otimes 0(3)$ tower:

$$(56,0)^+, \quad (56,1)^-, \quad (56,2)^+, \quad \ldots .$$

The negative parity resonances are thus put into a (56,1), and one can again play the game of saturation with such states. The results are the following:

$$\frac{D}{F} = \frac{3}{2}, \quad -\frac{G_A}{G_V} < \frac{5}{3}, \tag{4}$$

which are really not bad, as $-\frac{G_A}{G_V}$ is reduced from the $\frac{5}{3}$ value, whilst the $\frac{D}{F}$ ratio remains $\frac{3}{2}$. However, this is not completely accurate, though it must be said that we are at the limits where $SU(3)$ breaking effects could play quite an important role.

Among the various other possibilities, we last mention that the $(70,1)^-$ multiplet discussed by DALITZ [8] leads to no testable prediction, owing essentially to the fact that the mixing scheme is complicated by the presence of two 1/2 octets.

As a conclusion of this brief review I have to say that we are quite confident that the mixing of the octet and decuplet with the resonances of the (20,1) multiplet contains a lot of physical truth, at least as far as low energy parameters of baryons are concerned.

In order to have another confirmation of this point of view, I shall now consider the problem of magnetic moments [9]. We know that the $SU(6)$ predictions are very good for the octet baryons $\left(\text{i.e. } \frac{\mu_p}{\mu_n} = -\frac{3}{2}\right)$, but are in disagreement with the experimental value of the $M(1)$ $N^* N \gamma$ transition [10], i.e.

$$(\text{Exp.}) = (1{,}28 \pm 0{,}05)\ (SU_6 \text{ theory}).$$

Thus $SU(6)$ has to be corrected, though not too much!

A natural frame for such corrections is the one we have discussed, and we can try again whether they happen to be in the right direction. Before discussing the results I would like to say a few words about the way the magnetic moments are to be calculated.

The frame in which $p \to \infty$, as has been discussed by DASHEN and GELL-MANN [11], selects upon the various "charges" which build up the $U(12)$ compact algebra, some algebras of "good charges", that is of operators with non-vanishing matrix elements between single particle states.

The $SU(6)_W$ algebra is one such, and also the $U(3) \otimes U(3)$ chiral algebra which coincides in this limit with the $U(3) \otimes U(3)$ collinear algebra. Only within these algebras can we expect to have saturation among single-particle states.

Let us now consider the electric dipole operator

$$D_1^i = \int d^3x\, x_1\, j_0^i(x) \qquad (i = 1, \ldots, 8)\,;$$

it has been shown [12] that its matrix elements between baryons are proportional to the anomalous magnetic moments. As D_1^i belong to a $(1,8) + (8,1)$ representation of the chiral $U(3) \otimes U(3)$ group with $|\Delta h| = 1$ (h is the helicity) we can use the Wigner—Eckart theorem to compute its matrix elements. By restricting ourselves to the $|h| = 1/2$ states we obtain a relation between the $M(1)\ N^*\ N\gamma$ transition μ^* and the magnetic moment of the neutron, namely

$$\mu^* = -\frac{\sqrt{2}}{\cos\theta}\,\mu_n \simeq \frac{1}{\cos\theta}\,(\mu^*)_{SU_6}, \tag{5}$$

where $(\mu^*)_{SU_6} = \frac{2\sqrt{2}}{3}\mu_P$ is the value obtained in $SU(6)$, and θ is the mixing angle as derived from the commutation relations of axial charges within the (56,0) and (20,1) states. By putting $-\frac{G_A}{G_V} = 1{,}18$, we get $\frac{1}{\cos\theta} \simeq 1{,}25$ to be compared with the factor 1,28 derived from an analysis of N^* photoproduction [13].

However, if we want to have consistency between this result and that which we get for the $h = 3/2 \to h = 1/2$ matrix elements we see that all anomalous magnetic moments and μ^* must vanish. A very unpleasant result!

In our opinion this model has the additional disadvantage of not reproducing the well known $SU(6)$ results in the no-mixing case, i.e. $\theta = 0$.

Thus, we are forced either in putting into doubt the commutation relations between D_1^i and the vector and axial charges, by invoking, for instance, the existence of the so called "SCHWINGER terms", or by thinking that the relevant magnetic moment operator is to be identified in a different way.

This second alternative rests mainly in the so called "Partial conservation of the tensor currents" [14], which by a type of GELL-MANN—LÉVY identity relates the divergence of the tensor current to the vector meson field, i.e.

$$\partial_\mu \, T^i_{\mu\nu} = K \Phi^i_\nu , \tag{6}$$

where i is a $SU(3)$ index.

By further assuming that the electromagnetic form factors are dominate d by such vector mesons, we can link the matrix elements of the tensor charges T^i_{02} and T^i_{01}, whose operator form in a quark model is given by

$$T^i_{0K} = i \int d^3x \, q^+ \lambda^i \gamma_K \, q \qquad (K = 1{,}2) \tag{7}$$

to the magnetic moments.

Actually one very delicate problem is to decide whether we have total magnetic moments or anomalous ones. An ultimate answer cannot at present be given, as has been emphasized in [14] and the choice can be based only on the final agreement with the experimental numbers.

We note at this point that the collinear $U(3) \otimes U(3)$ operators and the T^i_{0k} charges generate, in the quark model, a $SU(6)$ algebra in which F^i_{35} (the integrated third component of axial vector current) and T^i_{0k} have the commutation relations of the $\sigma_\alpha \lambda^i$ ($\alpha = 1, 2, 3$) $SU(6)$ generators. As we are actually saturating these commutation relations with $SU(6)$ multiplets, the matrix elements of T^i_{0k} have the same $SU(3)$ structure as those of F^i_{35} which coincide at $p \to \infty$ with the axial charges. Thus we have

$$\left(\frac{D}{F}\right)_{\text{magn.moments}} = \left(\frac{D}{F}\right)_{\text{axial}} \tag{8}$$

and we get the following predictions:

$$\mu_n = -2{,}05 , \qquad \mu^* = (1{,}16) \; (\mu^*)_{SU_6} \tag{9}$$

in nuclear magnetons.

These results are less accurate than those found for the axial matrix elements.

A possible improvement of the situation can be achieved if one realizes that the vector mesons in this framework can get mixed with the axial vector mesons with $C = -1$, which are contained in the (35,1) "Kinetic multiplet" [15] and that this mixing from the beginning has been neglected in the PCTC hypothesis, and in the extrapolation to zero vector meson mass which is entailed in it.

By adding this new term in the magnetic moment matrix elements (it corresponds to a $[(1,8)-(8,1)]_{L_z} = \pm 1$ tensor $U(3) \otimes U(3) \otimes U(1)_{L_z}$) we finally get only the relation:

$$\mu^* = \frac{\sqrt{2}}{\cos\theta}\left(\mu_p + \frac{1}{2}\mu_n\right) \simeq 1{,}26\ (\mu^*)_{SU_6} \tag{10}$$

in very good agreement with experiment.

To conclude this talk I should like to emphasize that a consistent picture of low energy parameters of baryons can be given in terms of the mixing scheme of the representations of the chiral algebra at $p \to \infty$, and that from this picture the previous proposed (20,1) "Kinetic Supermultiplet" [16] seems clearly to emerge as a tool for classifying the higher negative parity baryon resonances. I believe that much can still be said following this line of reasoning in order to classify also the other positive parity resonances, whose evidence is daily growing. Work is in progress in this direction.

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СЛАБЫЕ И ЭЛЕКТРОМАГНИТНЫЕ ТОКИ И КЛАССИФИКАЦИЯ АДРОНОВ

Г. ПРЕПАРАТА

Резюме

Обсуждаются разные возможности смешивания представлений. В связи с соотношениями между магнитными моментами показано, что предполагая «частичное сохранение «Тензорного тока» и смешивание векторных и аксиальных мезонов получается отличное согласие с опытом.

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MIXING EFFECTS IN BARYON SPECTROSCOPY

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The role of the $(20, L = 1)$ states in describing the low energy weak and electromagnetic parameters of the stable baryons and of the $3/2^+$ resonances is discussed.

In the past year two lines of research have focused the attention of many theoretical physicists. The first was the search for a group-theoretical classification for the negative parity baryon states like Y_0^* (1405), N^* (1518), Y_0^* (1519) and others, for which growing experimental evidence was being reported. The second was the exploitation of the physical content of quark-model commutation relations among weak and electromagnetic currents. It culminated in the discovery of the ADLER—WEISBERGER sum rule, which allowed the evaluation of the neutron beta-decay axial coupling constant.

I shall be mainly concerned with these two arguments, as well as the relations among them which have been worked out in a series of papers made in Florence by R. GATTO, G. PREPARATA and myself. Moreover, I shall consider only the quark model commutation rules of the integrated four components of vector and axial currents (generating the $U(3) \otimes U(3)$ chiral algebra) and of the integrated third components of the axial current (which, together with vector charges, generate the collinear $U(3) \otimes U(3)$), when saturated with single particle states at $p_3 \to \infty$.

It was rather early recognized [1] that symmetries like $SU(6)$ may stem out from a saturation of the commutation rules of certain algebras with few single-particle low lying states. In fact, as shown by BERGIA and LANNOY [2], and GERSTEIN [3], the saturation of the $U(3) \otimes U(3)$ chiral or collinear algebras, when restricted to the $1/2^+$ and $3/2^+$ octet and decuplet, leads directly to the static $SU(6)$ results.

Corrections to such results may arise from a violation of these algebras, when considered as symmetries of the Hamiltonian, in the sense that even if the implied commutation relations are to be retained, the physical octet and decuplet cannot be put together in a single irreducible representation, but become mixed, through the generators, to other low lying states. This is, of course, equivalent to saying that the saturation of the commutation relations with the $1/2^+$ octet and the $3/2^+$ decuplet fails, and other states have to be taken into account. As a matter of fact, this is just what is suggested by the ADLER—WEISBERGER sum rule [4] in which the contribution of the nearest negative parity intermediate states is far from being negligible.

These considerations also indicate a possible way to evaluate the matrix elements of axial charges to a better approximation than $SU(6)$, simply on a group theoretical ground without having to recur to dispersion-like treatments. One has to assume that only resonant states come into play (which amounts, for example, to approximate the continuum in the A. W. rule with few resonances) and try to saturate the chiral or collinear algebra with an enlarged set of $SU(3)$ multiplets, containing some negative parity multiplets besides the $1/2^+$ octet and the $3/2^+$ decuplet. The problem is thus reduced to a purely algebraic one. The output of such calculation are the renormalized axial strengths among the states under consideration, many of them being directly comparable with the experimental data.

As it has been formulated, this programme requires a somewhat detailed knowledge of the structure of the negative parity baryon states. For instance, a supermultiplet grouping of such particles as that indicated by the non-compact group approach [5] [6] would indicate which $SU(3)$ multiplets are to be put in the calculation together with the γ-octet.

On the other hand, it is clear that such an approach could allow tests of the various proposed classification schemes on the basis of their predictions for the axial charges matrix elements.

Other expected outputs are the matrix elements among the states of interest of operators, like magnetic moments, whose commutation rules with axial and vector charges are known. The saturation of such rules with the chosen set of states will provide these matrix elements. Note that his *does not imply* that these operators have unappreciable leakage outside this set.

Let me return now to the classification schemes for negative parity baryon states.

Using as a guide the well known quark model, schemes have been proposed, according to the different pictures of

i) bound states of three quarks with one unit of orbital angular momentum ($(56, L = 1)$ [6], $(20, L = 1)$ [7], $(70, L = 1)$ [8] of $SU(6) \otimes 0(3)$);

ii) three quarks plus a quark—antiquark pair (700^-) [5].

Another proposed possibility was the 70^- $SU(6)$ representation [9].

I shall not enter here into the details of all these models, apart from $(20, L = 1)$. I shall simply remark that in all the proposed supermultiplets embarrassing low-lying negative parity decuplets appear for which, at present, the experimental evidence is rather poor.

The $(20, L = 1)$ has the following content:

two nonets with $J^P = 1/2^-, 3/2^-$;

one singlet with $J^P = 5/2^-$.

Consistently with the mass formulae derived in the "kinetic supermultiplets" model [7], the $3/2^-$ nonet can be identified with the well known γ-octet plus a still lacking isosinglet predicted at 1670 ± 30 MeV. The $1/2^-$ octet can be identified with a possible η-octet containing the $N - \eta$ (1510), the $\Lambda - \eta$ (1660) threshold resonances plus a lacking isotriplet predicted at 1530 ± 40 MeV and a Ξ^* predicted at 1520 ± 60 MeV. The well known Y_0^* (1405) completes the $1/2^-$ nonet, whereas the $5/2^-$ singlet, predicted at 1760 ± 25 MeV is still missing.

It is worthwhile mentioning that one is also able to derive a quadratic mass formula (7) among the masses of known particles, which is experimentally well verified.

Having this supermultiplet at hand, we put its $SU(3)$ multiplets together with the $1/2^+$ octet and $3/2^+$ decuplet in the commutation rules of the chiral algebra, and looked for a consistent solution of the equations thus found [10, 11].

Under the requirement that it should provide, either for the stable octet and the decuplet, and for the other $SU(3)$ multiplets, the $SU(6)$ solution in the limit of no leakage between the two sets, we found only one solution, which, as for the $8\,1/2^+ - 8\,1/2^+$ and $8\,1/2^+ - 10\,3/2^+$ matrix elements, depends upon one single parameter. Its elimination leads to the relations:

$$-\frac{G_A}{G_V} = \frac{1}{3}\,\frac{D+F}{D-F}\,, \tag{1}$$

$$G^{*2} = -\frac{4}{9}\left(1 + 3\frac{G_A}{G_V}\right), \tag{2}$$

$\left(-\frac{G_A}{G_V}\right.$ is the neutron axial beta-decay coupling constant, D and F the independent axial strengths over the octet, G^* the axial $N^* - N$ strength.$\left.\right)$

Relations (1) and (2) are in good agreement with data: inserting $-\frac{G_A}{G_V} = 1{,}18$, one finds: $\alpha = D(D+F) = 0{,}69$; $G^* = 1{,}08$ to be compared with the experimental values: $\alpha = 0{,}67 \pm 0{,}03$, $G^* \simeq 1$.

Note that putting $-\frac{G_A}{G_V} = 5/3$ one recovers the $SU(6)$ predictions $D/F = 3/2$; $G^* = 4/3$.

Thus, it appears that the $(20, L = 1)$ provides a fairly good picture of the additional states needed to saturate our commutation rules. In addition it has to be said that the alternative classification schemes do not provide such good relations as eqs. (1) and (2) [13, 14] (on this argument see the report by G. PREPARATA [15]).

These latter calculations have been made by using an equivalent but much simpler method devised by H. HARARI [16] and, independently, by N. CABIBBO and H. RUEGG [14].

Let me briefly explain this method by which the actual solution of the complicated commutator equations is directly bypassed.

The essential point lies in the observation that a solution of the commutator equations will provide us with a reducible $U(3) \otimes U(3)$ representation. In the absence of leakage one would have, as irreducible components, the same representations appearing in the reduction of $(56, L = 0)$ and $(20, L = 1)$ of $SU(6) \otimes 0(3)$. Moreover, the physical 1/2 stable octet would be represented by the 1/2 octet contained inside the $(56, L = 0)$. One possible solution in the presence of leakage (and the only one which contains the latter situation as a limit) is that which reduces according to the same $U(3) \otimes U(3)$ components as before, the physical 1/2 stable octet being now a mixture of the octets contained in $(56, L = 0)$ and $(20, L = 1)$.

Owing to the absence of decuplets in $(20, L = 1)$ the physical decuplet cannot become mixed. In this situation one can write the $8-8$ and $8-10$ axial charges matrix elements in terms of the mixing angle, thus obtaining eqs. (1) and (2).

It has to be said that the requirement of having the $SU(6)$ limit also on the negative parity states is by no means obvious. If one omits this requirement [14], one finds another set of states giving the same eqs. (1) and (2). However, going further to the study of magnetic moments, one finds that again the $(20, L = 1)$ is successful in predicting a good relation among μ_p, μ_n, and the magnetic $N^* - N$ transition strength [15, 17, 18].

In conclusion, I should like to add a few remarks. We have seen that combining the $U(3) \otimes U(3)$ commutation relations with the hypotheiss of saturation with the states of $(56, L = 0)$ and $(20, L = 1)$, we found a consistent picture of the low energy weak and electromagnetic parameters of the stable baryons and of the $3/2^+$ resonances. Naturally, we are very far from having tested all the predictions contained in such a model, for which we have to wait for more accurate data of photo and neutrino production of negative parity resonances. Anyway the fact itself that the other classification schemes are not so successful even on the testable predictions gives us more confidence that we are on the right track towards finding the structure of the baryon levels.

Finally, the same appearance of the $(20, L = 1)$, after $(56, L = 0)$, is a puzzling feature of the theory. Perhaps this is an indication of the existence of a larger group than $SU(6) \otimes 0(3)$, with a single irreducible representation containing $(56, L = 0)$ together with $(20, L = 1)$, (as well as other possible $SU(6) \otimes 0(3)$ multiplets). This group, although badly violated as a symmetry group, could have the role of providing, with its irreducible representations the blocks which saturate the chiral, or collinear algebras.

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ЭФФЕКТЫ СМЕШИВАНИЯ В БАРИОННОЙ СПЕКТРОСКОПИИ

Л. МАЙАНИ

Резюме

Обсуждается роль состояний $(20, L = 1)$ в написании низкоэнергетических слабых электромагнитных параметров стабильных барионов и 3/2* резонансов.

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REMARKS ON THE WOLF $\pi\pi$ PHASE SHIFTS AND THE CALCULATION OF g_A/g_V RATIO FROM ADLER $\pi\pi$ SUM RULE

By

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The consistency of pion-pion phase shifts recently proposed by G. WOLF with dispersion relations is briefly mentioned. Using these phase shifts the g_A/g_V ratio is calculated from ADLER $\pi\pi$ sum rule. The calculated value is $g_A/g_V = 1{,}08$. Some possibilities of reducing the discrepancy between the calculated and experimental values are discussed.

Introduction

Pion-pion interaction plays an important role in the interpretation of many scattering and decay processes involving strongly interacting particles. Most of the experimental data are based on the application of the modified one pion exchange model to the reaction $\pi N \rightarrow \pi\pi N$. Recently, G. WOLF [1] has summarized the cross-section and asymmetry parameters data and on this basis he has proposed a plausible $\pi\pi$ phase-shift analysis [1]. In the first part of the present paper the consistency of these phase shifts with dispersion relations (*DR*) and with *DR* sum rules is briefly discussed. In the second part the ADLER $\pi\pi$ sum rule [2] for g_A/g_V ratio is calculated using WOLF $\pi\pi$ phase shifts. The third part contains a discussion of discrepancies between experimental and calculated values of g_A/g_V.

I. Remarks on the consistency of Wolf $\pi\pi$ phase shifts with *DR*

We shall first introduce the notation. The amplitude for elastic $\pi\pi$ scattering in the CMS system for pions in the definite isotropic spin state is written as

$$F^T = \frac{\omega}{q} \sum (2l+1)\, P_l(\cos\vartheta) \sin\delta_l^T \exp(i\,\delta_l^T), \qquad (1)$$

where q is the CMS pion momentum $\nu = q^2$, $\omega = \sqrt{\nu+1}$ and nuclear units $\hbar = c = m_\pi = 1$ are used. The elastic scattering amplitudes for pions in definite charge states are

$$F^{++,++} = F^2, \qquad F^{+-,+-} = \frac{1}{3} F^0 + \frac{1}{2} F^1 + \frac{1}{6} F^2,$$

$$F^{+0,+0} = \frac{1}{2} F^1 + \frac{1}{2} F^2, \qquad F^{00,00} = \frac{1}{3} F^0 + \frac{2}{3} F^2, \tag{2}$$

$$F^{00,+-} = -\frac{1}{3} F^0 + \frac{1}{3} F^2.$$

Respecting the identity of pions, we get the following expressions for elastic cross-sections

$$\sigma^{ij,kl} = (4 - 2\delta ij)\, \omega^{-2} \int |F^{ij,kl}|^2 \, d\Omega, \tag{3}$$

where $kl(ij)$ are the pion charges in the initial (final) state.

From eqs. (2) and (3) the optical theorem for elastic scattering amplitudes can be easily obtained:

$$\operatorname{Im} F(\nu) = \frac{\nu^{1/2} (\nu + 1)^{1/2}}{8\pi} \sigma^{\text{tot}}(\nu), \tag{4}$$

where $F(\nu)$ is some elastic scattering amplitude in eq. (2). The DR for forward scattering with two subtractions in the point $\nu = -1/2$ are [3]

$$F(-1/2) + (\nu + 1/2) \frac{d}{d\nu} F(-1/2) = Re\, F(\nu) -$$

$$- \frac{(2\nu + 1)^2}{8\pi^2} P \int_0^\infty \frac{[\nu' (\nu' + 1)]^{1/2}}{(2\nu' + 1)^2} \left[\frac{\sigma^{\text{tot}}(\nu')}{\nu' - \nu} + \frac{\sigma_c^{\text{tot}}(\nu')}{\nu' + \nu + 1} \right] d\nu', \tag{5}$$

where σ_c^{tot} is the total cross-section for the crossing symmetric reaction. In the paper [3] the consistency of Wolf $\pi\pi$ phase shifts [1] with DR [eq. (5)] was examined in the region $3 < \nu < 15$. The phase shifts data and cross-sections data which are inserted on the right hand side of eq. (5) are known [1] only in the range $\nu < 24$. The errors on the right hand side of eq. (5) which are due to the unknown cross-sections for $\nu > 24$ are relatively large but within these errors the consistency of Wolf $\pi\pi$ phase shifts with DR in the range $3 < \nu < 15$ is quite reasonable.

In principle, it is of course possible to calculate the right hand side of eq. (5) also in the range $\nu < 3$, but in this region the cross-sections, their derivatives and the experimental errors are very large. The principal value integrals are very sensitive to the derivatives of the integrand and the errors of the right hand side of eq. (5) in this region would be greater than the values of it.

The low energy behaviour of phase shifts is characterised by scattering lengths. In analogy to the πN scattering it is also possible to derive sum rules for $\pi\pi$ scattering lengths [5]. In order to have rapidly convergent integrals it is convenient to use only crossing symmetric amplitudes $F^{+0,+0}$ and $F^{00,00}$. The sum rule for these amplitudes has the form [5]

$$F(0) = F(-1/2) + \frac{1}{\pi}\int\limits_0^\infty \frac{d\nu' \, Im\, F(\nu')}{(2\nu'+1)\,\nu'\,(\nu'+1)} \, . \tag{6}$$

The values $F(0)$ are directly related to the scattering lengths, $F(-1/2)$ can be calculated from eq. (5) and integrals can be evaluated using phase shift data [1]. The scattering lengths calculated in this way are:

$$a_0 = -1{,}3 \pm 0{,}6 \, , \quad a_2 = 0{,}38 \pm 0{,}2 \, .$$

A similar value for the a_0 was obtained by a different method by H. ROTHE [6].

These values of scattering lengths are approximately the same in absolute value as WOLF's, but the sign is reversed. WOLF's $\pi\pi$ phase shift analysis [1] was based on the experimental data of the cross-sections $\sigma^{+0,+0}$, $\sigma^{++,++}$ and $\sigma^{+-,+-}$. In the low energy region where the $\pi\pi$ phase shift is small it is impossible to determine from these data the sign of the $\delta_0^{(0)}$ and $\delta_0^{(2)}$ phase shifts. It is probable that the signs of these phase shifts should be reversed in the low energy region. The cross-sections corresponding to original WOLF $\pi\pi$ phase shifts will be almost the same. Therefore, we shall use the original phase shifts in the calculation of the g_A/g_V ratio in the next section.

II. Adler pion-pion sum rule

Recently, the following sum rule which relates the g_A/g_V ratio to the $\pi\pi$ cross-sections was proposed by S. ADLER [2]

$$\frac{2}{(g_A/g_V)^2} = \frac{4M_N^2}{g_r^2\,K^{NN\pi}(0)^2}\,\frac{1}{2\pi}\int\limits_0^\infty \frac{d\nu}{\nu+3/4}\,[\sigma_0^{+-}(\nu) - \sigma_0^{++}(\nu)]\, , \tag{7}$$

where $\sigma_0^{+-}(\nu)$, $[\sigma_0^{++}(\nu)]$ is the total cross-section for scattering of a zero mass π^- $[\pi^+]$ meson on a physical π^+ meson, M_N is the nucleon mass, g_r is the rationalized, renormalized πN coupling constant and $K^{NN\pi}(0)$ is the pionic form factor of the nucleon.

If we neglect inelastic processes and partial waves with $l > 3$ and use the ADLER form for off mass shell corrections we can write the right hand side of eq. (7) as

$$R_0^{(0)} + R_2^{(0)} + R_1^{(1)} + R_3^{(1)} + R_0^{(2)} + R_2^{(2)},$$

where $R_l^{(T)}$ is a contribution from l, T partial wave. Using WOLF $\pi\pi$ phase shifts the values of $R_l^{(T)}$ terms are

$$R_0^{(0)} = 1{,}20\,, \qquad R_2^{(0)} = 0{,}114\,, \qquad R_1^{(1)} = 0{,}760\,, \qquad R_3^{(1)} = 0{,}072\,,$$
$$R_0^{(2)} = -0{,}385\,, \qquad R_2^{(2)} = -0{,}034\,.$$

Then $2 \Big/ \left(\dfrac{g_A}{g_V}\right)^2 = 1{,}73$ and $g_A/g_V = 1{,}08$.

III. Concluding remarks

There are various possibilities for avoiding the discrepancy between the experimental and calculated value. First of all it must be noted that the $R_1^{(1)}$ term calculated here is much larger than ADLER's estimate $R_1^{(1)} = 0{,}42$. This discrepancy is due to the different ϱ-meson width used. ADLER uses $\Gamma_\varrho = 105$ MeV, while WOLF $\pi\pi$ phase shift corresponds to $\Gamma_\varrho = 140$ MeV. A second possibility arises from the fact that the $R_0^{(0)}$ term which dominates the right hand side of eq. (7) is very sensitive to the small variations of a_0. For instance if we use $R_1^{(1)} = 0{,}76$ and take $|a_0| = 1{,}1$ in the region $0 < \nu < 0{,}5$ we get $g_A/g_V = 1{,}17$.

Acknowledgement

The original version of the author's short report at Balatonvilágos Conference on Weak Interactions contained only the discussion of low energy $\pi\pi$ phase shifts. After the report, Prof. M. JACOB suggested the calculation of g_A/g_V from the ADLER sum rule. The author is very indebted to Prof. JACOB for this proposal.

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ЗАМЕЧАНИЯ О $\pi\pi$ ФАЗОВОМ СДВИГЕ ВОЛЬФА И ВЫЧИСЛЕНИЕ ОТНОШЕНИЯ g_A/g_V ИЗ ПРАВИЛА СУММ АДЛЕРА

Я. ПИШУТ

Резюме

Кратко обсуждается совместимость $\pi\pi$ фазовых сдвигов, предложенных Г. Вольфом, с дисперсионными соотношениями. Используя эти фазовые сдвиги, вычисляется отношение g_A/g_V из правила сумм Адлера. Рассуждаются некоторые возможности для уменьшения расхождения между теоретическим и экспериментальным значениями.

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ELECTROMAGNETIC MASS DIFFERENCES AND CURRENT ALGEBRAS

By

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A sum-rule for the electromagnetic mass difference of elementary particles is derived using the methods of current algebras. An expression for the proton-neutron mass difference is given in terms of the proton-electron and neutron-electron cross-sections.

Current algebras have been applied to many physical problems in recent times [1]. By using this method it was possible in some cases to derive relations among different measurable quantities. In general, these relations were in satisfying agreement with experimental data. Several electromagnetic phenomena were examined using current commutation relations [2]. The success of these calculations encouraged us to apply this method to the calculation of electromagnetic mass differences. We shall use a $U(2)$ algebra (isotopic spin group extended by baryon gauge group) for deriving closed expressions for the mass difference of any particles belonging to the same isotopic spin multiplet. Our relations are exact if we assume as usual the convergence of a dispersion integral.

The $U(2)$ algebra consists of the generators T^+, T^-, T^3 and T^0, where $T^i = \int j_0^i(x) d^3x$. The generators T^3 and T^- are assumed to be exactly conserved. The conservation of T^+ and T^- is violated by electromagnetic interactions. The generators T^i satisfy the usual $U(2)$ commutation relations. We denote the Hamiltonian of the system by $H = H_s + H_{e.m.}$, where

$$[H_s, T^i] = 0, \; i = +, -, 3, 0 \tag{1}$$

$$H_{e.m} = e \int \left(j_\mu^0 (x) + j_\mu^3 (x) \right) A_\mu (x) \, d^3 x. \tag{2}$$

Using relations (1) and (2) we can obtain the following equation [3]:

$$\frac{dT^\pm}{dt} = i\,[H, T^+] = i\,[H_\gamma, T^\pm] = \pm\, ei \int j_\mu^\pm \, A_\mu \, d^3 x. \tag{3}$$

On the other hand

$$\frac{dT^\pm}{dt} = \int \frac{\partial j_0^\pm}{\partial t} \, d^3 x = \int \partial_\mu j_\mu^\pm \, d^3 x. \tag{4}$$

Comparing equations (3) and (4) we obtain

$$\int \partial_\mu j_\mu^\pm (x)\, d^3 x = \pm\, ie \int j_\mu^\pm A_\mu\, d^3 x. \tag{5}$$

In what follows we shall use eq. (5) without integration. Such a relation can be derived in a quark model for some types of interaction. We remark that our final formulae can be derived assuming only (5) as it stands and using a non-covariant method of derivation. Nevertheless, we shall follow the more elegant covariant treatment [4]. As usual we shall define a retarded function:

$$F^i_{ab}(\nu, q^2) = \int e^{iqx}\, \Theta(x_0)\, d^4 x < A_a(p_1) \,|\, [\partial_\mu j_\mu^-(x), eA_\mu j_\mu^i] \,|\, A_b(p_2) >, \tag{6}$$

where A_a and A_b are particles of the same isotopic multiplet, $\nu = \dfrac{q(p_1 + p_2)}{2}$.

For the sake of simplicity we shall use zero three-momentum initial and final states, $P_1 = (m_a, 0, 0, 0)$, $p_2 = (m_b, 0, 0, 0)$. i can be chosen as $+$, $-$, 3 or 0 . As stated the existence of a dispersion relation for F^i_{ab} without subtractions will be assumed:

$$F^i_{ab}(\nu, q^2) = \frac{1}{2\pi i}\int \frac{f^i_{ab}(\nu', q^2)}{\nu' - \nu - i\varepsilon}\, d\nu', \tag{7}$$

where

$$f^i_{ab}(\nu, q^2) = \int d^4 x e^{iqx} < A_a(p_1) \,|\, [\partial_\mu j_\mu^-(x), eA_\mu j_\mu^i] \,|\, A_b(p_2) >. \tag{8}$$

After integration by parts we obtain from (6)

$$F^i_{ab}(0,0) = < A_a(p_1) \,|\, eA_\mu(0)\, [j^i_\mu(0), T^-] \,|\, A_b(p_2) >. \tag{9}$$

Inserting a closed system of physical states inside the commutator of eq. (8) using (7) and (9) we arrive at the following expression (if we perform our calculations in second order of e it is enough to include in the sum over intermediate states, the corresponding state from the A multiplet and states containing one photon)

$$\begin{aligned} < A_a \,|\, eA_\mu [j^i_\mu, T^-] \,|\, A_b > &= - R^-_{ac} < A_c \,|\, e A_\mu j^i_\mu \,|\, A_b > \\ &+ R^-_{db} < A_a \,|\, eA_\mu j^i_\mu \,|\, A_d > + e^2 \int \frac{d\nu}{\nu} \sum_{\alpha, P'} \delta\left((q + p_1 - P')^2\right) \\ &\Theta(q_0 + p_{10} - P'_0) [< A_a \,|\, j_\mu^- \,|\, \alpha(P') > < \alpha(P') \,|\, j^i_\mu \,|\, A_b > \\ &+ < A_a \,|\, j^i_\mu \,|\, \alpha(P') > < \alpha(P') \,|\, j_\mu^- \,|\, A_b >]. \end{aligned} \tag{10}$$

The summation on the r.h.s. is understood over all possible hadron states. R^-_{ac} is essentially a Clebsch—Gordan coefficient, with $R^-_{ac} = (2\,\pi)^3 < A_a \,|\, j_0^- \,|\, A_b >$.

Eq. (10) is our final formula for the mass differences. For any types of isotopic multiplets one can choose i in such a way that the l.h.s. and the first terms of the r.h.s. can be expressed by mass differences. The sum on the r.h.s. can be given in terms of integrals over cross-sections, or may be approximated by a few low-lying levels.

The simplest application of eq. (10) is that for isotopic triplets or higher multiplets. For the sake of definiteness we apply it to triplets. If we choose $i = -$ the l.h.s. of eq. (10) vanishes. Using eq. (5) we obtain an expression for the following combination of masses:

$$\frac{m_+ + m_- - 2m_0}{2} = \frac{e^2 (2\pi)^3}{R^-_{0+} R^-_{-0}} \int \frac{d\nu}{\nu} \sum_{\alpha,P'} \delta((q + p_1 - P')^2) \cdot \tag{11}$$

$$\cdot \Theta(q_0 + p_{10} - P'_0) < A_- | j^-_\mu | \alpha(P') > < \alpha(P') j^-_\mu | A_+ > .$$

We shall apply eq. (11) in a separate paper [5] to calculate the mass difference of charged and neutral pions. In what follows we shall apply eq. (10) to doublets. We choose $i = 0$, so the l.h.s. again vanishes. On the other hand, having $a = d$ and $c = b$ in eq. (10) the first two terms on the right hand side (we have chosen the members of our doublet as positively charged and neutral particles) give

$$R_1 = -\frac{1}{\sqrt{2}} (< A_+ | e A_\mu j^0_\mu | A_+ > - < A_0 | e A_\mu j^0_\mu | A_0 >).$$

The isoscalar part of the photon source will not make any contribution to R_1, so

$$R_1 = -\frac{1}{\sqrt{2}} (< A_+ | e A_\mu j^3_\mu | A_+ > - < A_0 | e A_\mu j^3_\mu | A_0 >),$$

where now only the isoscalar part of the photon source contributes. Using isotopic spin invariance, R_1 can be written as

$$R_1 = - < A_0 | e A_\mu j^-_\mu | A_+ > = \frac{m_0 - m_+}{(2\pi)^3} \frac{1}{\sqrt{2}},$$

which gives

$$m_0 - m_+ = -2\sqrt{2}\,(2\pi)^3 e^2 \int \frac{d\nu}{\nu} \sum_{\alpha,P'} \delta((q + P - P')^2) \tag{12}$$

$$\Theta(q_0 + P_0 - P'_0) < A_0(p) | j^-_\mu | \alpha(P') > \cdot$$

$$\cdot < \alpha(P') | j^0_\mu | A_+(P) >,$$

(The mass difference of A_0 and A_+ gives fourth order effects on the r.h.s. and can be neglected). Eq. (12) is essentially the same as that obtained by CINI, FERRARI and GATTO [6] for the proton-neutron mass difference using field theory methods. The calculation of the one nucleon term does not give a correct result as is well known (we obtain $m_p - m_n = 0{,}6$ MeV). Because of the baryon current appearing in the sum rule of eq. (12) $N^*_{3/2,3/2}$ resonance makes no contribution. We may expect the N^* (1518) resonance to make an essential contribution. Unfortunately, the electromagnetic properties of this resonance are not yet well known. There is a further possibiliy that non-resonant many-particle terms may make essential contributions if they go through one meson exchange diagrams. Calculations of that type are in progress. We can express $m_p - m_n$ through the electron-proton, electron-neutron cross-sections in the following way[1]. Using isotopic invariance we can write

$$m_n - m_p = -(2\pi)^3 e^2 \int \frac{d\nu}{\nu} \sum_{\alpha,P'} \delta\left((q+P-P')^2\right) \Theta(q_0 + P_0 - P'_0)$$

$$[< p(P) | j_\mu^{e\cdot m} | \alpha(P') > < \alpha(P') | j_\mu^{e\cdot m} | p(P) > -$$

$$- < n(P) | j_\mu^{e\cdot m} | \alpha(P') > < \alpha(P') | j_\mu^{e\cdot m} | n(P) >],$$

where p and n denote proton and neutron, $j_\mu^{e\cdot m} = j_\mu^0 + j_\mu^3$.

We define $W_{\mu\nu}$ as [7]

$$W^p_{\mu\nu} = \frac{1}{2} \sum_{\substack{\text{initial}\\ \text{spin}}} \sum_{\alpha,P'} < p(P) | j_\mu^{e\cdot m} | \alpha(P') > \tag{13}$$

$$< \alpha(P') | j_\nu^{e\cdot m} | p(P) > \delta(q + P - P').$$

Following from Lorentz invariance and current conservation $W_{\mu\nu}$ can be expressed as a linear combination of two scalar functions

$$W_{\mu\nu} = W_1(q^2, qP) \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) +$$

$$+ W_2(q^2, qP) \frac{1}{m^2} \left(P_\mu - \frac{P_q}{q^2} q \right) \left(P_\nu - \frac{Pq}{q^2} q\nu \right).$$

[1] A little different sum-rule for the proton-neutron mass difference was obtained by COTTINGHAM [8].

W_1 and W_2 depend on two invariants: q^2 and qP. Comparing (12) and (13) we obtain for the mass difference

$$m_n - m_p = -(2\pi)^3 e^2 \int \frac{d(p_1 P)}{p_1 P} \int \frac{d^3 p_2}{2p_{20}} \Big\{ \big[W_1^P ((p_1 - p_2)^2, (p_1 - p_2) P) -$$

$$- W_1^n ((p_1 - p_2)^2, (p_1 - p_2) P, (p_1 - p_2) P) \big] 3 + \big[W_2^P ((p_1 - p_2)^2, (p_1 - p_2) P) -$$

$$- W_2^n ((p_1 - p_2)^2, (p_1 - p_2) P) \big] \left(1 - \frac{[(p_1 - p_2) P]^2}{m^2 (p_1 - p_2)^2} \right) \Big\}. \tag{14}$$

The total (in the sense, that we sum up over all possible hadron final states at a given final state electron momentum) differential cross-section for electron-nucleon scattering is given by the formula

$$\frac{d^2 \sigma_{eN}}{d\Omega_2 dp_2} = \frac{\alpha^2}{2\pi^2 q^4} \frac{|\underset{\sim}{p_2}|^2}{|\underset{\sim}{p_1}| p_{20}} \Big\{ q^2 W_1 (q^2, qP) + \tag{15}$$

$$+ \left[\frac{q^2}{2} + 2 \frac{E}{m} (Em - qP) \right] W_2 (q^2, qP) \Big\}.$$

In eqs. (14) and (15) $\alpha = \frac{e^2}{4\pi}$, q^2 is the invariant momentum transfer between the two leptons, $q = p_1 - p_2$, p_1 and p_2 are the four-momenta of the electrons in the initial and final states, respectively. E is the primary energy $E = p_{10}$. The components of p_1 and p_2 appearing in eq. (15) are taken in the laboratory system. The special combination of W_1 and W_2 appearing in eq. (14) may be expressed as a linear combination of *ep*, *en* cross-section and their derivatives with respect to E at constant q^2 and qP. By inserting this linear combination into eq. (14) we obtain after integration by parts

$$m_n - m_p = -\frac{(2\pi)^6}{\alpha} \int_0^\infty dE \int d\Omega_2 \int_0^a dp_2 g(E, p_2, \cos\vartheta)$$

$$\left(\frac{d^2 \sigma_{ep}}{d\Omega_2 d|\underset{\sim}{p_2}|} - \frac{d^2 \sigma_{en}}{d\Omega_2 d|\underset{\sim}{p_2}|} \right) - \frac{(2\pi)^5}{2\alpha} \int_0^\infty dE \int_0^b dp_2 \cdot \tag{16}$$

$$\cdot [2E + p_2 (1 - \cos\vartheta)]^2 \left(\frac{d^2 \sigma_{ep}}{d\Omega_2 d|\underset{\sim}{p_2}|} - \frac{d^2 \sigma_{en}}{d\Omega_2 d|\underset{\sim}{p_2}|} \right) \cos\vartheta = -1.$$

The quantities appearing in eq. (16) are defined as follows

$$a = \frac{EM}{M + E(1 - \cos\vartheta)}, \quad b = \frac{EM}{2E + M}$$

$$g(E, p_2, \cos\vartheta) = \frac{p_2(1 - \cos\vartheta)}{(E + p_2)^2}\left[(2E + p_2)Ep_2\cos\vartheta - 4E^3 - 5E^2p_2 - Ep_2^2 + p_2^3\right].$$

The present experimental data are still insufficient for the right hand side of eq. (16) to be calculated. However, it is not excluded that in the near future we shall have sufficient data about the electron spectrum in inelastic electron-nucleon scattering to evaluate these integrals.

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ЭЛЕКТРОМАГНИТНЫЕ РАЗНОСТИ МАСС И АЛГЕБРЫ ТОКОВ

П. ШУРАНИ

Резюме

Получено правило сумм для электромагнитных разностей масс элементарных частиц, используя метод алгебры токов. Дается выражение для разности масс протона и нейтрона через сечения рассеяния электрона на протонах и нейтронах.

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ON LOCALIZATION OF RELATIVISTIC MICROPARTICLES IN SPACE AND TIME

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The present day theory in a formal way permits one particle states which are localized in space with any degree of accuracy. This gives the possibility to formulate conditions of macroscopic causality directly for S-matrix.

The scattering matrix S for real "in" — and "out" — states should obey certain causality conditions. However, these conditions can be formulated only if "in" states are given in the form of localized wave packets instead of plane waves.

In this connection it is necessary to consider possibilities of construction of narrow wave packets for relativistic particles, which do not spread essentially during the time $T = \frac{R}{v}$ much longer than the collision time τ (here R is the distance between wave packets, and v their relative velocity). It is important to mention that when we say that the matrix S transforms the state given at $T = -\infty$ to the state $T = +\infty$ it is implied that we neglect the terms of the order τ^2/T^2 and retain the terms of the order τ/T.

Thus, we are looking for wave packets which satisfy the conditions:

$$R \gg \Delta \gg \lambda \tag{1}$$

(λ the typical wave length, Δ the dimension of wave packet, R the distance between them) and

$$|\Delta(T) - \Delta(-T)| \ll \Delta(-T)\,. \tag{1'}$$

The smaller the wave length λ the more precise are the conditions for the formulation of macroscopic causality for S matrix.

The fact is that in many papers devoted to the problem of relativistic particle localization it is asserted that a spinor particle cannot be exactly localized since the states of positive energy do not form a complete set of functions (see [1]). Therefore, the eigenfunction $\delta(x - x')$ of the coordinate operator $\hat{X}$ cannot be expanded in the eigenfunctions corresponding only to positive energy states.

The same is related to spinless particles obeying the Klein equation.

We shall show that if quadratically integrable wave packets are used instead of the δ-function then particles can be localized in positive energy states to any degree of accuracy.

A. First we consider the case of Dirac particles. Let us take a one-particle state, represented by a quadratically integrable wave function:

$$\psi(\vec{x}, t, \alpha) = \int C(\vec{P})\, U(\vec{P}, \alpha)\, e^{i(\vec{P}\vec{X} - Et)}\, d^3 P, \tag{2}$$

where $E = +\sqrt{m^2 + \vec{P}^2}$, $U(\vec{P}, \alpha)$ Dirac spinor, and

$$\int |C(\vec{P})|^2 d^3 P = 1, \tag{3}$$

$$S_P U^* U = 1. \tag{4}$$

Now we calculate the mean square value of a coordinate, for instance, of Z. Assuming that at $t = 0$ $\bar{Z} = 0$ we obtain after simple calculations:

$$\overline{\Delta Z^2} = \overline{Z^2} = \int \left| \frac{\partial C(\vec{P})}{\partial p_z} \right|^2 d^3 P + \int |C\ \vec{P})|^2\, S_p \left(\frac{\partial U^*}{\partial p_z} \frac{\partial U}{\partial p_z} \right) d^3 p\,. \tag{5}$$

The last term is characteristic of the relativistic case.

Now we represent $C(\vec{P})$ in the form:

$$C(\vec{P}) = f(\vec{\xi})\, 1/P_0^{3/2}\,, \qquad \vec{\xi} = \frac{\vec{P}}{P_0}, \tag{6}$$

where P_0 is the quantity describing the momentum dispersion in the considered state:

$$\overline{\Delta P_Z^2} \simeq P_0^2\,. \tag{7}$$

The first integral in eq. (5) gives:

$$I_1 = \int \left| \frac{\partial C}{\partial p_z} \right|^2 d^3 P = \frac{\alpha}{P_0^2}\,. \tag{8}$$

The second integral is

$$I_2 = 4\pi \int_0^\infty |f(\xi)|^2\, \xi^2\, d\xi\, M(\xi)\,, \tag{8'}$$

where

$$M(\xi) = \int S_p \left(\frac{\partial U^*}{\partial p_z} \frac{\partial U}{\partial p_z} \right) \frac{d\Omega}{4\pi} \tag{8''}$$

and $M(\xi)$ is equal:

$$M(\xi) = \begin{cases} \dfrac{1}{4m^2}\,; & \xi \ll m/P_0\,, \\[2ex] \dfrac{1}{4m^2}\,\dfrac{4}{3}\,\dfrac{m^2}{P_0^2}\,\dfrac{1}{\xi^2}\,; & \xi \gg \dfrac{m}{P_0}\,. \end{cases} \tag{9}$$

(See Appendix IA).

Therefore, we have that at $t = 0$

$$\overline{\Delta Z^2} = \alpha \frac{\hbar^2}{\Delta P_Z^2} + \beta^2 \frac{\hbar^2}{m^2 c^2}\,, \tag{10}$$

if $\Delta P_Z^2 \ll m^2c^2$.

For $\Delta P_Z^2 \gg m^2c^2$ we have

$$\overline{\Delta Z^2} = \alpha' \frac{\hbar^2}{\Delta P_Z^2}\,, \tag{10'}$$

where α, β, α' are of the order of unity. It is well seen that although in eq. (10) an additional term $\hbar^2/m^2c^2$ appears as if indicating that the Dirac particle cannot be localized more exactly than within $\Delta Z \sim \dfrac{\hbar}{mc}$ but, in fact, it is of no importance since at $\overline{\Delta P_Z^2} \to \infty$ eq. (10) transforms into eq. (10′).

Notice, however, that at $\Delta P_Z^2 \to \infty$ the considered state is not described by the function:

$$\Psi_{Z'}(Z) = \delta(Z - Z')\,, \tag{11}$$

since this function is not quadratically integrable but the considered state is described by quadratically integrable functions. This quadratically integrable function $\Psi_{Z'}, (Z, P_0)$ localized about $Z = Z'$ is related to the function (11) as follows:

$$Z\,\Psi_{Z'}(Z, P_0) = Z'\,\Psi_{Z'}(Z, P_0) + \Delta(Z - Z', P_0)\,, \tag{12}$$

$$\Delta(Z - Z', P_0) = \frac{1}{P_0^{1/2}} \left[(Z - Z')\,\Psi_{Z'}(Z, P_0)\,P_0^{1/2}\right], \tag{12'}$$

in this case

$$\Psi_{Z'}(Z, P_0)/P_0^{1/2} \to \delta(Z - Z')$$

at $P_0 \to \infty$. Therefore, if the function $\Psi_{Z'}(Z)$ is considered as an "ideal" eigenfunction of the operator of the coordinate Z then the function $\Psi_{Z'}(Z, P_0)$ approximates it so that $\Delta(Z - Z', P_0) \to 0$ at $P_0 \to \infty$ (see Appendix IB).

B. We now turn to the spinless particles, and consider again the one-particle state. The field $\varphi(x)$ may be represented in the form:

$$\varphi(x) = \int A(\vec{K})\, U_K(x)\, d^3 K, \qquad U_K = \frac{l^{iKx}}{\sqrt{\omega}}, \tag{13}$$

where $KX = \vec{K}\vec{X} - \omega t$, $\omega = +\sqrt{m^2 + K^2}$ (see Appendix IIA).

The density $\varrho(x)$ is

$$\varrho(x) = \frac{1}{2}[\Omega \varphi^* \varphi + \varphi^* \Omega \varphi], \qquad \Omega = +\sqrt{m^2 - \nabla^2} \tag{14}$$

and, generally speaking, is non-definite even for positive energy states $\omega = \sqrt{m^2 + K^2}$.

In this case it is also impossible to represent the δ function as a superposition of waves U_K with $\omega > 0$.

Now let us consider localized states with integrable density ϱ. We calculate the quantity $\overline{Z^2}$ at $t = 0$ under the condition:

$$\int \varrho(x)\, d^3 x = +1. \tag{15}$$

We have

$$\overline{\Delta Z^2} = \overline{Z^2} = \frac{1}{2}\int Z^2 (\Omega \varphi^* \varphi + \varphi^* \Omega \varphi)\, d^3 x. \tag{16}$$

After simple calculations we find that:

$$\overline{Z^2} = \int \left| \frac{\partial A}{\partial K_Z} \right|^2 d^3 K - \frac{1}{4}\int |A|^2 \frac{K_Z^2}{\omega^4}\, d^3 K. \tag{16'}$$

This expression is non-definite, therefore the density $\varrho(\vec{x}, 0)$ cannot be treated as a density of any probability.

It might be expected that such "anomalies" in the behaviour of $\varrho(\vec{x})$ arise only when the density $\varrho(\vec{x})$ is concentrated within $\Delta x \sim \frac{\hbar}{mc}$. But this is not the case: $\varrho(\vec{x})$ may assume negative values also when $\Delta x \sim \hbar/mc$ (see Appendix IIB and IIC). Taking A in the form

$$A(\vec{K}) = A(\omega) = f\left(\frac{\omega}{\omega_0}\right) 1/\omega_0^{3/2}, \qquad \omega/\omega_0 = \xi$$

we find

$$\overline{Z^2} = \frac{1}{\omega_0^2} \int_{\xi > m} \left[|f'|^2 - \frac{1}{4\xi^2} |f|^2 \right] \frac{\xi_Z^2}{\xi^2}\, d^3 \xi. \tag{16}$$

It is not difficult to choose a function f such that $\left[f'^2 - \frac{1}{4\xi^2}|f|^2\right] \geq 0$. Then it is seen that at $\omega_0 \to \infty$, $\overline{Z^2} = 0$ and we come to the state with a well localized density i.e. a density which at $t = 0$ is concentrated within an arbitrary small region $\Delta Z \sim \hbar/\omega_0 \to 0$ (see Appendix III).

Thus, as far as the possibility of localization is concerned, the situation is quite similar to that which takes place for the Dirac particle. However, the $\varrho(x, t)$ for the spinless particle might not be interpreted as the density of the probability to detect the particle near the point $\vec{x}$ at moment t.

The quantity $\varrho(\vec{x}, t)$ should be considered as a purely "field" quantity representing a spinless particle in space-time.

C. Now we consider the behaviour of relativistic wave packets in the course of time. All the states discussed above localized at $t = 0$ are spreading: the quantities $\overline{\Delta X^2}$, $\overline{\Delta Y^2}$, $\overline{\Delta Z^2}$ increase. However, this increase is such that under certain conditions it may be said that the relativistic packet is moving during a rather long time T conserving its characteristic size.

In other words, the change in the packet size during time T may be small compared with its initial size even for long time intervals. Here a long time interval implies an interval such that $R = cT \gg \Delta X, \Delta Y, \Delta Z$ where $\Delta X, \Delta Y, \Delta Z$ are taken at $t = 0$, C is the velocity of light.

It is easy to show that the packet width Δ_{11} measured in the direction parallel to the packet motion increases with t according to the law:

$$\Delta_{\rm II}^2(t) = \Delta_{\rm II}^2(0) + \frac{\lambda}{\Delta_{\rm II}^2(0)} \frac{m^4}{E^4} v^2 t^2 \tag{17}$$

and the width $\Delta_\perp$ measured in the direction perpendicular to the packet motion increases according to the law

$$\Delta_\perp^2(t) = \Delta_\perp^2(0) + \frac{\lambda}{\Delta_\perp^2(0)} v^2 t^2 = \Delta_\perp^2(0) + \frac{\hbar}{m^2 c^2} \frac{1}{\Delta_\perp^2(0)} \frac{m^2}{E^2} t^2 . \tag{17'}$$

Here λ is the particle wave length, $v \frac{\partial E}{\partial p}$ is the packet velocity, m is the particle rest mass, $\Delta^2(0)$ is the value of $\Delta^2(t)$ at $t = 0$ (see Appendix IV). From these equations it follows that

$$\left| \frac{\Delta^2(t) - \Delta^2(0)}{\Delta^2(t)} \right| \ll 1 \tag{18}$$

if

$$R = ct < \frac{\Delta^2(0)}{\lambda} . \tag{18'}$$

Now we come back to the conditions (1), (1') and combine them with the result (18'). We find the inequalities:

$$\Delta \frac{\Delta}{\lambda} > R \gg \Delta \gg \lambda \tag{19}$$

which can be realized for any t under the condition that $\lambda \to 0$ (i. e. $v \to c$).

This important condition of a possible long existence of a localized relativistic packet is exclusively the result of relativistic effects: delay of the clock in a moving frame of reference and increase of the particle mass with increasing velocity.

D. Summary:

The present day theory in a formal way (because there is no practical way to construct an arbitrary narrow split) permits one-particle states, which are localized in space with any degree of accuracy $\Delta \to 0$, and for the time intervals $T < \frac{\Delta^2}{\lambda c} \to \infty$ at $\lambda \to 0$. This gives the possibility to formulate conditions of macroscopic causality directly for the S-matrix, taken on the mass and energy surface.

Appendix I

A. The spinor $U_r(P,\alpha)$ can be written for $E > 0$ in the form (see [5]):

$$\begin{array}{ll} r = 1\,, & r = 2\,, \\ U(1) = N\,, & U(1) = 0\,, \\ U(2) = 0\,, & U(2) = N\,, \\ U(3) = \dfrac{m+E}{P_Z N}\,, & U(3) = \dfrac{\Pi^* N}{m+E}\,, \\ U(4) = \dfrac{m+E}{\Pi N}\,, & U(4) = -\dfrac{P_Z N}{m+E}\,, \\ N = \dfrac{1}{\sqrt{2}}\left(1+\dfrac{m}{E}\right)^{1/2} & \Pi = P_x + iP_y\,. \end{array} \tag{1}$$

Hence, it is seen that the traces of bilinear combinations for $r = 1$ and 2 are identical. A simple calculation gives

$$
\begin{aligned}
S_P\left(\frac{\partial U^*}{\partial P_Z}\frac{\partial U}{\partial P_Z}\right) &= \frac{1}{8}\left(1+\frac{m}{E}\right)^{-1}\frac{m^2 P_Z^2}{E^6} + \\
&+ \frac{1}{2}\left(1+\frac{m}{E}\right)^{-1}\frac{1}{E^2}\left\{1+\frac{P_Z^4}{\left(1+\frac{m}{E}\right)^2 E^4}+\right. \\
&+ \frac{1}{4}\frac{m^2 P_Z^4}{\left(1+\frac{m}{E}\right)^2 E^6} - \frac{2P_Z^2}{\left(1+\frac{m}{E}\right)E^2} - \frac{mP_Z^2}{\left(1+\frac{m}{E}\right)E^3} + \\
&+ \left.\frac{mP_Z^4}{\left(1+\frac{m}{E}\right)^2 E^5}\right\} + 1/2\,\frac{(E^2-m^2-P_Z^2)\,P_Z^2}{E^6}\left(1+\frac{m}{E}\right)^{-3}\times \\
&\times\left(1+1/2\,\frac{m}{E}\right)^2 \geq 0\,.
\end{aligned}
\tag{2}
$$

Noting that

$$\int P_Z^2\, d\,\Omega = \frac{4\pi}{3}\, p^2\,, \qquad \int P_Z^4\, d\,\Omega = \frac{4\pi}{5}\, P^4 \tag{3}$$

we find

$$M = \frac{1}{4\pi}\int S_P\left(\frac{\partial U^*}{\partial P_Z}\,\frac{\partial U}{\partial P_Z}\right) d\,\Omega = \begin{cases} \dfrac{1}{4m^2} & P \ll mc \\[2mm] \dfrac{1}{3}\,\dfrac{1}{P^2} & P \gg mc \end{cases} \tag{4}$$

or

$$M\left(\xi\,,\frac{m}{P_0}\right) = \begin{cases} \dfrac{1}{4m^2}\,; & \xi \ll \dfrac{m}{P_0} \\[2mm] \dfrac{1}{4m^2}\,\dfrac{4}{3}\,\dfrac{m^2}{P_0^2}\,\dfrac{1}{\xi}\,; & \xi \gg \dfrac{m}{P_0}\,. \end{cases} \tag{5}$$

The integral of $M(\xi)$ is of the form

$$I_2\left(\frac{m}{P_0}\right) = \int_0^\infty |f(\xi)|^2\,\xi^2\,d\,\xi\, M\left(\xi\,,\frac{m}{P_0}\right),$$

$$I_2\left(\frac{m}{P_0}\right) = \int_0^\infty |f(\xi)|^2\,\xi^2\,d\,\xi\, M\left(\xi\,,\frac{m}{P_0}\right). \tag{6}$$

This integral tends to zero at $\frac{m}{p_0} \to 0$ since the region where $M(\xi) \simeq \frac{1}{4m^2}$ reduces is $\frac{m}{p_0}$. At $\frac{m}{p_0} \to \infty$ it is finite and equal to $\frac{1}{4m^2}$.

B. Let us consider the connection between the wave function representing the state localized about the point $X = X'$ and the δ function. We denote this function by $\Psi_{Z'}(X, \alpha)$ where $\alpha \simeq \frac{1}{P_0}$. It can be of the form

$$\Psi_{X'}(X, a) \simeq \frac{e^{-\frac{(X-X')^2}{a^2}}}{\sqrt{a}}. \tag{7}$$

This function leads to $\overline{(X - X')^2} = a^2$ so that at $a \to 0$, $\overline{(X - X')^2} \to 0$ the function $\psi_{X'}(X, a)/\sqrt{a}$ has a limit $\delta(X - X')$ at $a \to 0$

$$\Psi_{X'}(X, a)/\sqrt{a} \underset{a \to 0}{\to} \Psi_{X'}(X) = \delta(X - X'). \tag{8}$$

Therefore

$$X \Psi_{X'}(X, a) = X' \Psi_{X'}(X, a) + \sqrt{a}\left[(X - X') \frac{\Psi_{X'}(X, a)}{\sqrt{a}}\right]. \tag{9}$$

The last term tends to zero at $a \to 0$ owing to (8) and the relation $(X - X') \times \times \delta(X - X') = 0$.

Appendix II

A. Usually the Fourier representation for the scalar field is written in the form

$$\varphi(X) = \int \frac{c(\vec{K})\, e^{iKX}}{2\omega} d^3 K = \int \frac{c(K)}{\sqrt{2\omega}} \int U_{\vec{K}}(X) d^3 K = \int A(\vec{K})\, U_{\vec{K}}(X)\, d^3_K; \tag{1}$$

$A(K)$, in contrast to $C(\vec{K})$ is not a scalar.

B. The fact that the quantity $\varrho(\vec{X}, t)$ is non-definite is seen from the following example. We put

$$\frac{c(\vec{K})}{\omega} = \frac{c_1 e^{-\frac{(\vec{K}-\vec{K}_1)^2}{2b^2}}}{\omega} + \frac{c_2 e^{-\frac{(\vec{K}-\vec{K}^2)^2}{2b^2}}}{\omega}. \tag{2}$$

We find $\varrho(\vec{X}, 0)$

$$\varphi(x) = c_1 \int \frac{e^{-\frac{(\vec{K}-\vec{K}_1)}{2b^2} + i\vec{K}\vec{X}}}{\omega} d^3K + c_2 \int \frac{e^{-\frac{(\vec{K}-\vec{K}_2)^2}{2b^2} + i\vec{K}\vec{X}}}{\omega} d^3K . \tag{3}$$

For simplicity we assume that $(\vec{K}_1 - \vec{K}_2) \gg b$. Further

$$\Omega\bar{\varphi} = c_1^* e^{-\frac{b^2X^2}{2} - i\vec{K}_1\vec{X}} + c_2^* e^{-\frac{b^2X^2}{2} - i\vec{K}_2\vec{X}} .$$

From here

$$\varrho(\vec{X}_1 0) = \frac{1}{2}(\Omega\bar{\varphi}\varphi + \bar{\varphi}\Omega\varphi) = e^{-\frac{b^2X^2}{2}} \left[\frac{|c_1|^2}{\omega_1} + \right.$$

$$\left. + \frac{|c_1 c_2|}{\omega_1} \cos(\Delta\vec{K}\vec{X} + \varphi) + \frac{|c_1 c_2|}{\omega_2} \cos(\Delta\vec{K}\vec{X} + \varphi) + \frac{|c_2|^2}{\omega_2}\right], \tag{4}$$

where $\varphi = \arg\frac{c_2}{c_1}$. Now we believe that $\omega_2 \gg \omega_1$, $|c_1|$, $|c_2|$ are comparable. Then

$$\varrho(\vec{X},0) = e^{-\frac{b^2X^2}{2}} \frac{|c_1|^2}{\omega_1} \left\{1 + \left|\frac{c_2}{c_1}\right| \cos(\Delta\vec{K}\cdot\vec{X} + \varphi)\right\}. \tag{5}$$

It is seen that if $\left(\frac{c_2}{c_1}\right) > 1$ then $\varrho(\vec{X}, 0)$ periodically changes the sign.

The density in this case is not too strongly localized (it was assumed that b is small) and in any case the quantity $\Delta X^2 = \frac{1}{b^2}$ is by no means connected with the Compton wave-length $\hbar/mc$.

C. Negative values of $\overline{\Delta_{\angle}^2}$. Now we turn to the case of one dimension. Eq. (34) reads now

$$\overline{\Delta Z^2} = \int_{-\infty}^{+\infty} \left[\left(\frac{\partial A}{\partial K}\right)^2 - 1/4\, A^2 \frac{K^2}{\omega^4}\right] dK.$$

We put $A = 1$ for $\omega = \sqrt{K^2 + 1} < \Omega \gg 1$

$$A = e^{-a2/2(\omega-\Omega)^2} \qquad \omega > \Omega$$

(the particle mass is taken to be unity). Going over to the integration over ω we get

$$1/2\,\overline{\Delta Z^2} = \int_{\Omega}^{\infty} e^{-a^2(\omega-\Omega)^2}\, a^4 (\omega - \Omega)^2 \left(1 - \frac{1}{\omega^2}\right)^{1/2} d\,\omega -$$

$$- \frac{1}{4}\int_{1}^{\Omega}\left(1 - \frac{1}{\omega^2}\right)^{1/2}\frac{d\,\omega}{\omega^2} - 1/4\int_{\Omega}^{\infty}\left(1 - \frac{m^2}{\omega^2}\right)^{1/2} e^{-a^2(\omega-\Omega)^2}\frac{d\,\omega}{\omega^2}\,. \tag{7}$$

It is sufficient to consider the first two integrals.

Assuming $a\,(\omega - \Omega) = \xi$ we find that the first integral will be of the order a and the second is simply calculated and for $\Omega \gg 1$ is $-1/4 \left(\frac{\pi}{4}\right) = -\frac{\pi}{16}$, the third integral is far smaller. At $a \ll \frac{\pi}{16}$, $\overline{\Delta Z^2} \simeq -\frac{\pi}{8}\frac{\hbar}{m^2 c^2}$. In the case of three dimensions, under the normalization condition (31) we have not succeeded in finding an example with $\overline{\Delta Z^2} < 0$.

Appendix III

Let us consider a relativistic packet described by the field $\varphi(\vec{X}, t)$

$$\varphi(\vec{X},t) = \int \frac{c\,(\vec{K})}{\omega}\, e^{i(\vec{K}\vec{X}-\omega t)}\, d^3 K\,, \tag{1}$$

$$i\,\frac{\partial\,\varphi^*\,(\vec{X}_1\,t)}{\partial\,t} = \int c^*\,(\vec{K})\; e^{-i\,(\vec{K}\vec{X}-\omega t)}\, d^3 K\,. \tag{2}$$

The density $\varrho(\vec{X}, t)$ is determined by the expression

$$\varrho\,(\vec{X}_1\,t) = \frac{i}{2}\left(\varphi\,\frac{\partial\,\varphi^*}{\partial\,t} - \varphi^*\,\frac{\partial\,\varphi}{\partial\,t}\right). \tag{3}$$

The localization will be strong if φ or $\frac{\partial\,\varphi}{\partial\,t}$ are strongly localized, We choose $(C(K)$ in the form

$$c(\vec{K}) = N e^{-\frac{(\vec{K}-\vec{K}_1)^2}{2b^2}}\,, \tag{4}$$

where $N = \frac{1}{\sqrt{b^3}}$. Then

$$\frac{\partial \varphi (X,t)}{\partial t} = - iN \int e^{-\frac{(K-K_1)^2}{2b^2} + iKX} d^3K \\ \equiv e^{i\vec{K}\vec{X}} \frac{e^{-\frac{X^2}{2a^2}}}{\sqrt{a}}; \tag{5}$$

where $a \sim \frac{1}{b}$. At $a \to 0$ this function is arbitrarily strongly localized about $x = 0$. The connection of such a function with the δ function was considered in Appendix IB.

Appendix IV

We calculate the spreading of a relativistic packet starting from its representation in the form (III. 1) and take $c(\vec{K})$ in the form (III. 4). If b is not too large then the field $\varphi(\vec{X}, t)$ can be represented in the form:

$$\varphi (X,t) = \frac{N}{\omega_1} e^{i(\vec{K}\vec{X} - \omega t)} I(\vec{X}, t), \tag{1}$$

where

$$I(\vec{X},t) = \int e^{-\frac{(\vec{K}-\vec{K}_1)}{2b^2} + i(K-K_1,X) - i(\omega-\omega_1)t} d^3 K. \tag{2}$$

For definiteness we put $\vec{K}_1 = (\vec{K}_{X_1}, 0, 0)$. Then

$$\omega - \omega_1 = \frac{K_X}{\omega_1} q_X + \frac{1}{2\omega_1}(q_X^2 + q_Y^2 + q_Z^2) - {}^1/_2 \frac{K_{X_1}^2}{\omega_1^3} q_X^2, \tag{3}$$

where $q = K - K_1$. A simple calculation yields

$$I(X_1 t) = A(t) e^{i\alpha(X_1 t) - \frac{(X - V_1 t)^2}{2\Delta_{11}^2(t)} - \frac{Y^2 + Z^2}{2\Delta_{\perp}^2(t)}}, \tag{4}$$

where A(t) is a slowly changing quantity, α is a real number and the quantities $\Delta_{11}^2(t)$ and $\Delta_{\perp}^2(t)$ are

$$\Delta_{11}^2(t) = \frac{1}{b^2} + \frac{b^2 m^4}{\omega^6} t^2, \tag{5}$$

$$\Delta_{\perp}^2(t) = \frac{1}{b^2} + \frac{b^2}{\omega^2} t^2. \tag{5'}$$

Putting $\frac{1}{b_2} = \Delta^2(0)$ these formulae can be rewritten in the form

$$\Delta_{11}^2(t) = \Delta^2(0) + \frac{\lambda^2}{\Delta^2(0)} \frac{m^4}{E^4} V^2 t^2 , \tag{6}$$

$$\Delta_{\perp}^2(t) = \Delta^2(0) + \frac{\lambda^2}{\Delta^2(0)} V^2 t^2 . \tag{6'}$$

Here $\lambda = \frac{\hbar}{P}$, P is the particle momentum, $V = \frac{P}{E}$ is its velocity. From the first formula it is seen that for $m = 0$ the wave packet does not spread in the longitudinal direction as is necessary for particles without rest mass (in this case there is no dispersion of the de Broglie waves). The formula for $\Delta_{\perp}^2(t)$ can also be derived from diffraction theory (see [6]). The increase of the beam width due to diffraction is determined by the multiplier

$$\sim e^{-\frac{a^2}{\lambda^2} \sin^2 \vartheta} , \tag{7}$$

where a is the diameter of the diaphragm. orifice, λ is the wave length, ϑ is the angle defining the beam width. The width $\varrho = R \sin \vartheta$, where $R = Vt$ is the distance to the diaphragm.

Therefore

$$\sim e^{-\frac{a^2}{\lambda^2} \sin^2 \vartheta} \simeq e^{-\frac{a^2}{\lambda^2} \frac{\varrho^2}{V^2 t^2}} \simeq e^{-\frac{\varrho^2}{\Delta^2 \varrho}} , \tag{8}$$

so that

$$\Delta_{\varrho}^2 = \frac{\lambda^2}{a^2} V^2 t^2 \tag{9}$$

according to eq. (6) for $\Delta_{\perp}^2$.

This formula can also be represented in the alternative form

$$\Delta_{\varrho}^2 = \frac{\Lambda_0^2}{a^2} \left(\frac{m}{E}\right)^2 c^2 t^2 , \tag{10}$$

where $\Lambda_0 = \frac{\hbar}{mc}$. In this formula the multiplier $\frac{m}{E}$ characterizing the delay of the clock is clearly seen.

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О ЛОКАЛИЗАЦИИ РЕЛЯТИВИСТСКИХ МИКРОЧАСТИЦ В ПРОСТРАНСТВЕ И ВРЕМЕНИ

Д. И. БЛОХИНЦЕВ

Резюме

Нынешняя теория формально позволяет одночастичные состояния, локализованные в пространстве произвольной точностью. Это дает возможность для сформулирования условия макроскопической причинности прямо для S-матрицы.

Acta Physica Academiae Scientiarum Hungaricae, Tomus 22 (1—4), p. 321 (1967)

CALCULATION OF THE $\pi^+ - \pi^0$ MASS DIFFERENCE

By

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Abstract

An attempt has been made to calculate the $\pi^+ - \pi^0$ mass difference $((m_+ - m_0)_{\exp} = +4{,}7$ Mev) on the basis of isospin current algebra. The contribution of the lowest $\pi^0\gamma$ intermediate state turned out to be $+1$ Mev, while that of the $\omega\gamma$ state equals $+8{,}8$ Mev. In the calculation the Hofstadter form-factors have been approximated by functions of the type $\Lambda^2 m_\varrho^2 (\Lambda^2 - t)^{-1} (m_\varrho^2 - t)^{-1}$ and for the $\omega\gamma$ case the form-factor has been normalized to the observed $\Gamma(\omega \to \pi^0 \gamma) \approx 1{,}26$ Mev decay rate. The result shows that the lowest $\pi^0\gamma$ state does not make a dominant contribution and that either much more convergent form-factors or non-negligible negative contributions to the mass-difference are needed to get the experimental result.

ВЫЧИСЛЕНИЕ РАЗНОСТИ МАСС π^+ И π^0 МЕЗОНОВ

А. ФРЕНКЕЛЬ, М. ПОШ, Г. ШУРАНИ и П. ШУРАНИ

Резюме

Оценена разность масс π^+ и π^0 мезонов $((m_+ - m_0)_{\text{эксп}} = 4{,}7$ Мэв) на основе алгебры токов изоспина. Вклад низшего промежуточного состояния $\pi^0\gamma$ составляет $+1$ Мэв, а вклад $\omega\gamma$ состояния равняется $+8{,}8$ Мэв. При расчете форм факторы Гофштадтера были представлены функциями типа $\Lambda^2 m_\varrho^2(\Lambda^2 - t)^{-1}(m_\varrho^2 - t)^{-1}$, и для случая $\omega\gamma$ состояния форм фактор был нормирован при помощи экспериментального значения $\Gamma(\omega \to \pi^0\gamma) \approx \approx 1{,}26$ Мэв. Результат показывает, что низшее состояние $\pi^0\gamma$ не дает доминирующего вклада и что для согласия с экспериментальным значением разности масс необходимы или форм факторы, сходящиеся намного сильнее использованных нами, или же нужны другие вклады со знаком минус.

Acta Physica Academiae Scientiarum Hungaricae, Tomus 22 (1—4), p. 322 (1967)

RESTRICTIONS ON THE CURRENTS AND SUM RULES

G. Pócsik

INSTITUTE OF THEORETICAL PHYSICS, ROLAND EÖTVÖS UNIVERSITY, BUDAPEST

Abstract

The general properties of a set of not necessarily conserved currents interacting with massive vector particles, the positivity and selected problems are examined.* From the positivity some restrictions on the currents are obtained. It is shown that all the conventional fermion currents, e.g. weak currents, contradict the positivity.

ОГРАНИЧЕНИЯ НА ТОКИ И ПРАВИЛА СУММ

Г. ПОЧИК

Резюме

Исследованы общие свойства ряда не обязательно сохраняющихся токов, взаимодействующих векторными частицами с массой, а также положительность и особые проблемы. Из положительности получены некоторые ограничения на токи. Показано, что обычные фермионные токи, т.е. слабые токи, противоречат положительности.

* G. Pócsik, Nuovo Cimento, **43A**, 541, 1966.

Acta Physica Academiae Scientiarum Hungaricae, Tomus 22 (1—4), p. 323 (1967)

ON THE STRUCTURE OF WEAK CURRENTS

By

F. CSIKOR and G. PÓCSIK

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Abstract

Restrictions on the currents arising from positivity are studied in both vector boson and conventional theories of weak interactions.* It is shown that the usual definition of the current does not fulfil these requirements. The correct expression for the current which satisfies all the prescriptions, is also given.

О СТРУКТУРЕ СЛАБЫХ ТОКОВ

Ф. ЧИКОР и Г. ПОЧИК

Резюме

Ограничения, навязаные условием положительности на токи, изучены и в теориях с векторными бозонами и в конвенциональных теориях слабых взаимодействий. Показано, что обычное определение тока не удовлетворяет этим требованиям. Дано правильное выражение тока, удовлетворяющее всем требованиям.

*F. CSIKOR and G. PÓCSIK, Nuovo Cimento, **42A**, 413, 1966.

Acta Physica Academiae Scientiarum Hungaricae, Tomus 22 (1—4), p. 324 (1967)

MODELS OF HIGH ENERGY SCATTERING

By

N. DOMBEY

RUTHERFORD HIGH-ENERGY LABORATORY, CHILTON NEAR DIDCOT, BERKS. ENGLAND

Abstract

Complex potential models of proton-proton high-energy large angle scattering are investigated. One finds that there are two essentially different kinds of potential which generate exponentially small scattering amplitudes.

МОДЕЛИ РАССЕЯНИЯ ПРИ ВЫСОКИХ ЭНЕРГИЯХ

Н. ДОМБЕИ

Резюме

Изучены комплексные модели потенциалов для протон-протонного рассеяния на большие углы при высоких энергиях. Найдено, что существуют два существенно разных сорта потенциалов, которые приводят к экспоненциально падающим амплитудам рассеяния.

INDEX

* Plenary lectures are distinguished by *. Talks represented only by abstracts will be published elsewhere.

SESSION 4. HIGHER SYMMETRIES

SESSION 5. CURRENT ALGEBRAS

1. СЕССИЯ, СЛАБЫЕ ВЗАИМОДЕЙСТВИЯ

2. СЕССИЯ. НАРУШЕНИЕ *CP*

3. СЕССИЯ. МОДЕЛИ КВАРКОВ

4. СЕССИЯ. ВЫСШИЕ СИММЕТРИИ

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