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A MAGYAR TUDOMÁNYOS AKADÉMIA
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STATISTICAL INVESTIGATIONS OF SUNSPOTS BY A NEW METHOD

by

L. DEZSŐ

Summary: In this paper we classify evolutionary phases of sunspots, by variations in the umbral and penumbral areas of single spots and spot groups. Various distributions were studied for an entire solar cycle using Greenwich observational data, and the steps of development and decay classified. From these studies it is readily apparent that certain characteristic features of sunspots are readily recognizable and, thus, our method can be widely applied to classification of sunspots in general. Further, these preliminary investigations show that similar studies of sunspots can be profitably made in even greater detail. We investigated mainly relationships among evolutionary phases of sunspots and between umbra and penumbra and reached, among others, the following conclusions: Spots belonging to the same group show a close similarity in pattern of evolution, while those of different groups show only a relatively weak connection, if any. The development and the decay of sunspots cannot be considered as a simple, single event, in which the decay simply reverses the pattern of development. Our most important conclusion probably is that, while the penumbrae in general follow the course of the umbrae, their evolution is marked by a certain degree of inertia -i. e., the changes in the evolutionary phases of the penumbrae very often show a time-lag relative to the umbrae.* Using Greenwich data to study the distribution of penumbral spots -i. e., those recorded as having only a penumbral area-, over the solar disk, we came to the following conclusions: In reality, there are no such things as penumbral spots; at most only the initial and final phases in the life of sunspots can be called „penumbral". We explain the „physical" foreshortening effect as due to the presence of non-transparent faculae of at least 1500 km in height surrounding the sunspots.

Л. ДЕЖЁ:

СТАТИСТИЧЕСКИЕ ИССЛЕДНОВАННИЯ СОЛНЕЧНЫХ ПЯТЕН НОВЫМ МЕТОДОМ

Резюме: На основании изменений площадей тени и полутени может быть определена фаза развития как отдельного солнечного пятна, так и группы пятен. Для исследования различных распределений были использованы Гринвичские наблюдательные данные, охватывающие целый 11-летний цикл, разделенные по фазам развития. При этом легко обнаруживаются определенные характерные свойства солнечных пятен при отдельном рассмотрении наблюдательных данных, относящихся к моментам их развития и спада. Это оправдывает применение нового метода. Более того, отсюда следует, что и в дальнейших исследованиях наблюдения солнечных пятен необходимо обрабатывать аналогичным образом. Мы рассматривали, преимущественно, связь между пятнами одной группы и между площадями тени и полутени. При этом, в частности, было установлено, что пятна в группе развиваются в тесной связи друг с другом, в то время как у пятен разных групп связь если и есть, то очень слабая. Развитие пятна не происходит изолированно от осталных пятен группы. Наиболее важным результатом является то, что площадь полутени с некоторым запозданием повторяет кривую изменения со временем площади тени.* Исследуя распределение по солнечному диску полутеневых пятен (т. е.

пятен без тени) можно сделать вывод, что такие пятна представляют собою, в большинстве случаев, начальный или конечный период жизни обычных пятен. При этом можно сделать предположение о наличии эффекта видимого уменьшения площади в результате существования непрозрачного факела, окружающего пятно, на высоте по райней мере 1500 км.

DEZSŐ LORÁNT:

NAPFOLTOK STATISZTIKAI VIZSGÁLATAI ÚJ MÓDSZER ALAPJÁN

Összefoglalás: Ebben a dolgozatban fejlődési fázisokat különböztetünk meg napfoltokra vonatkozólag az umbra és penumbra területek változásaiból mind egyedi foltok, mind foltcsoportok esetében. Greenwichi észlelési adatok és a kifejlődés- és visszafejlődésre bevezetett fokozatok felhasználásával egy teljes napciklus különféle eloszlásait tanulmányoztuk. Ezen vizsgálatokból azonnal kitűnik, hogy a napfoltok bizonyos sajátos tulajdonságai könnyen felismerhetők és így módszerünket általában messzemenően alkalmazni lehet a napfoltok klasszifikációjánál. Látszik továbbá ezen bevezető kutatásokból, hogy hasonló napfoltvizsgálatokat érdemlegesen lehet végezni még nagyobb részletességgel is. Főleg a napfolt fejlődési fázisok, valamint az umbra és penumbra kapcsolatait kutattuk és többek között a következő eredményekre jutottunk: Azonos foltcsoporthoz tartozó foltok fejlődésének módja hű hasonlóságot mutat, míg a különböző foltcsoportoké csupán viszonylag gyenge kapcsolatról tanúskodik, ha van ilyen egyáltalán. Nem lehet a kifejlődést és visszafejlődést a napfoltoknál egyszerűen egyféle eseményként felfogni, amelynél a visszafejlődéskor a kifejlődési folyamat pusztán ellentétes irányúra fordul. Legfontosabb megállapításunk talán az, hogy miközben a penumbrák általában követik az umbrák menetét fejlődésüket bizonyos fokú tehetetlenség jellemzi, azaz a penumbrák fejlődési fázisainak megváltozásai igen gyakran megkésnek az umbrákéhoz képest.* Greenwichi adatok alapján a penumbrális foltok napkorongon való eloszlását tanulmányozva — azaz olyan foltokkal kapcsolatban melyeknél a közlések szerint csupán penumbrális területet tapasztaltak — az alábbi következtetésekre jutottunk: A valóságban olyan képződmények, mint penumbrális foltok nincsenek; legfeljebb a napfoltok életének kezdeti és végső szakaszát lehet „penumbrálisnak" nevezni. A „fizikai foreshortening effektus" pedig a foltokat körülövező legalább 1500 km magas átlátszatlan fáklyák jelenlétének tulajdonítható.

§ 1. Introduction

Since the ultimate cause of sunspot formation is still entirely unknown and the average period or „cycle" of sunspot activity is considered to be 11 or, rather, 22 years, in studying spots we must use, wherever possible, observational materials for a period at least this long. In addition, sunspots (whether we study them individually or as collective units in groups) are such complex phenomena — both qualitatively and quantitatively — that we may still hope to discover additional laws governing their formation by use of simple statistical methods.

We have a great number of measurements from observations made on a broad scale over many decades for statistical investigation. Among them is the outstanding observational material contained in the well-known „*Greenwich Photo-Heliographic Results.*" Statistical studies can be made only with large collections of homogeneous data; spot observations made with a single instrument under identical circumstances over a fairly long period of time yield substantial data which can be selected to fulfill the requirements of homogeneity. An examination of the literature shows that use and statistical evaluation of published sunspot data, mainly of area and heliographic positions (as well as magnetic data), are far from being exhausted.

Some years ago, on the basis of these considerations, we began statistical investigations which we to the present time have based mainly on Greenwich observational materials for the years from 1922 to 1934. There are many reasons why we selected the observations of this period for our first intensive studies. They include two successive sunspot minima and the complete solar cycle in between (*no. 16*). There is substantially more observational material of different kinds for this period than for preceding decades. The sunspot cycle of 1922—1934 is the first for which we have both the complete *Mt. Wilson* magnetic observations and the *Meudon „Cartes Synoptique . . . Solaire"*. Also during this period there are observations of limb prominences (published by *Arcetri* Observatory under the auspices of the International Astronomical Union) which may be regarded as homogeneous.

§ 2. The basic idea of our method

It is not certain by any means that the photospheric spot areas are physically the most characteristic features of sunspots, but they are the only ones which can be utilized practically for our purposes. Certainly *we may describe the life-history of spots by means of the variations of their areas with time.*

It is striking from graphs that, in the case of large areas, not only the curves of the changes in area of whole spot groups, but those of single spots as well, are rarely composed exclusively of monotonically rising and declining segments. In many cases, even the curves for the period immediately following the birth of the spot are different, although, usually a significant process of increase and decrease is taking place in most cases during these first days. Often *in important spot groups*, as well as in fairly large *single spots*, whether they stand alone or in a group, *the curve describing the variation in area* has more than one steep-rising segment. (Examples are shown in Figs. 1a and 1b.) The *secondary maxima and minima* on these curves — that is, *the fluctuations — are mainly real* and do not originate from measuring errors. The Greenwich observations convince us of this in many ways. Fluctuations in values of the Greenwich area measurements are, in general, larger than any possible random error would admit and may be regarded as fully proved (indirectly) by the statistical investigations discussed in this paper.

As a preliminary quantitative example, we can mention some approximate data from an appropriate statistical sample for the years 1925—1930. These are graphs showing the curves of fairly strong average decreases in area of the umbrae of some significant p and f spots of bipolar spot groups on abscissae of 5—10 days length. Only one-fourth of these curves fell monotonically over their entire length. The same percentage showed more than one jump upward disturbing the monotony of the curve. The most significant fact is that one-fourth of the downward curves show at least one peak which exceeds the probable error in measurement [1] by several times — i. e., there must have been a true increase in area for some period of time.

Accordingly, *the area growth during the lifetime of a sunspot, and consequently of a spot group, is not a single, smooth process,* if the area attains any considerable size. *The course of the increase and decrease* — or as we may also call it, *development and decay — generally changes* more than once, and sometimes *several times*. The rapidity with which the area changes, the duration and the subsequent variations of the development and decay of spot groups are evidently both characteristic and important. In other words, the length and the steepness of the rising and falling segments of the curves of area growth, their different heights and the numbers of maxima and minima along them are of primary interest to us.

We can get a better understanding of the essential nature of the observable increase in spot area, and the circumstances and direct causes leading to it, in the first place from observations of the moments of development. These observations are likely to be very important to solar physics, since we have little possibility of applying inductive methods to the investigation of sunspot problems without considering their development and decay and the transitional periods between. The various phenomena observable in the atmosphere of the sun, which show some kinds of correlation with one another and with general levels of spot activity, probably have an even more direct connection with the individual spots or rather spot groups. Evidently, it will be to our advantage in studying these relations to show both the direction and the rate of change in the area of sunspots. As an illustration, it has been known for a long time that increase in the frequency of chromospheric flares appearing in the region of a spot group accompanies its average growth in area, in general.

Taking into consideration in our sunspot investigations all the factors described briefly above, *we have tried to account as far as possible for all factors creating the run of the curves of area change*. Practically, this means that we tried from the observational data to *distinguish the ascending and descending segments*, as well as all the peaks (*maxima*) and troughs (*minima*) of the curves -that is, to separate them into *four evolutional phases*. In short, *we considered the problems of sunspots from an evolutional point of view*. The results which we give in detail in *these Publications* prove that this method is both justified and useful.[1]

[1]The author has previously delivered three brief lectures on the first preliminary results of these invetigations: in the *Crimea* (September 1955), in *Budapest* (August 1956; it is mentioned on p.3 of the *Mitt. Sternw. Budapest*, Nr. 42) and in *Ondrejov* (November 1957). For a Russian summary of the Crimean lecture see the *IZVESTIA of the Crimean Astroph. Obs. 16*, p.208 (1956).

§ 3. Definitions and notations of classifications

We classified each observation of a single spot or spot group on the basis of a curve of variation in area, showing the ascending, descending, maximum and minimum phases. Since the data at our disposal for statistical investigation are generally based only on a daily single solar observation, we have had to use the spot measurements for at least three successive days. Our classification by four evolutionary phases is shown in Figures 1a and 1b.

It is evident that the assignment of an observed spot to an evolutional phase is determined by the area values for the days preceding and following the observation -that is, we need to know whether the area curve for the moment of observation is rising or falling or shows minimum or maximum development. If there was no apparent change in the area for several days, which is a relatively rare occurrence, we used the nearest different measured value to determine the phase. Thus, *every classification is a qualitative statement about three successive different daily observations;* it shows the relation of the middle observation to the other two.

It was worthwile for several reasons to refine our system of classifications to include quantitative changes. For example, it is useful to dis tinguish whether the area was changing greatly from day to day. To do this, we introduced an average daily variation of the spot areas and considered anything over this to be a great change. This created four additional classifications within each of the four evolutionary phases. Figure 2 shows schematically these 4^2 possibilities for change in area.

We have followed the generally accepted system of classifying the area of sunspots numerically by using $10^{-6}\times$ (the area of the sun's hemisphere). On this scale, *ten* is already a *large change in classification of umbrae;* Figure 2 was plotted accordingly.

In Figures 1a and 1b, the symbols represent the qualitative changes in area; the *three-letter abbreviations show the quantitative change as well through the scheme shown in Figure 2.* Both these symbols and letters are used throughout this paper and in its tables and figures. The types of change shown in the left column of Figure 2 will be called „rapid"; those in the right column „slow" and those in the two middle columns „alternative". Generally, we found it necessary to distinguish only the rapid ones. We can speak collectively about the slow and alternative cases as „generally slow" ones. A line drawn over the three-letter symbol for slow changes in area (for example $\overline{\text{ASC}}$) indicates this generally slow category. Generally rapid variations are denoted similarly by a line over the symbol ($\overline{\text{A—S—C}}$). The patterns in the second and third columns of Figure 2 denote „earlier" and „later" rapid changes, respectively. When the rate of area change does not concern us (as shown is the concrete examples of Figures 1a and 1b) we use the form *Asc, Des,* etc. for the classification; the *three capital letters always indicate that we made quantitative distinctions* as well as qualitative ones.

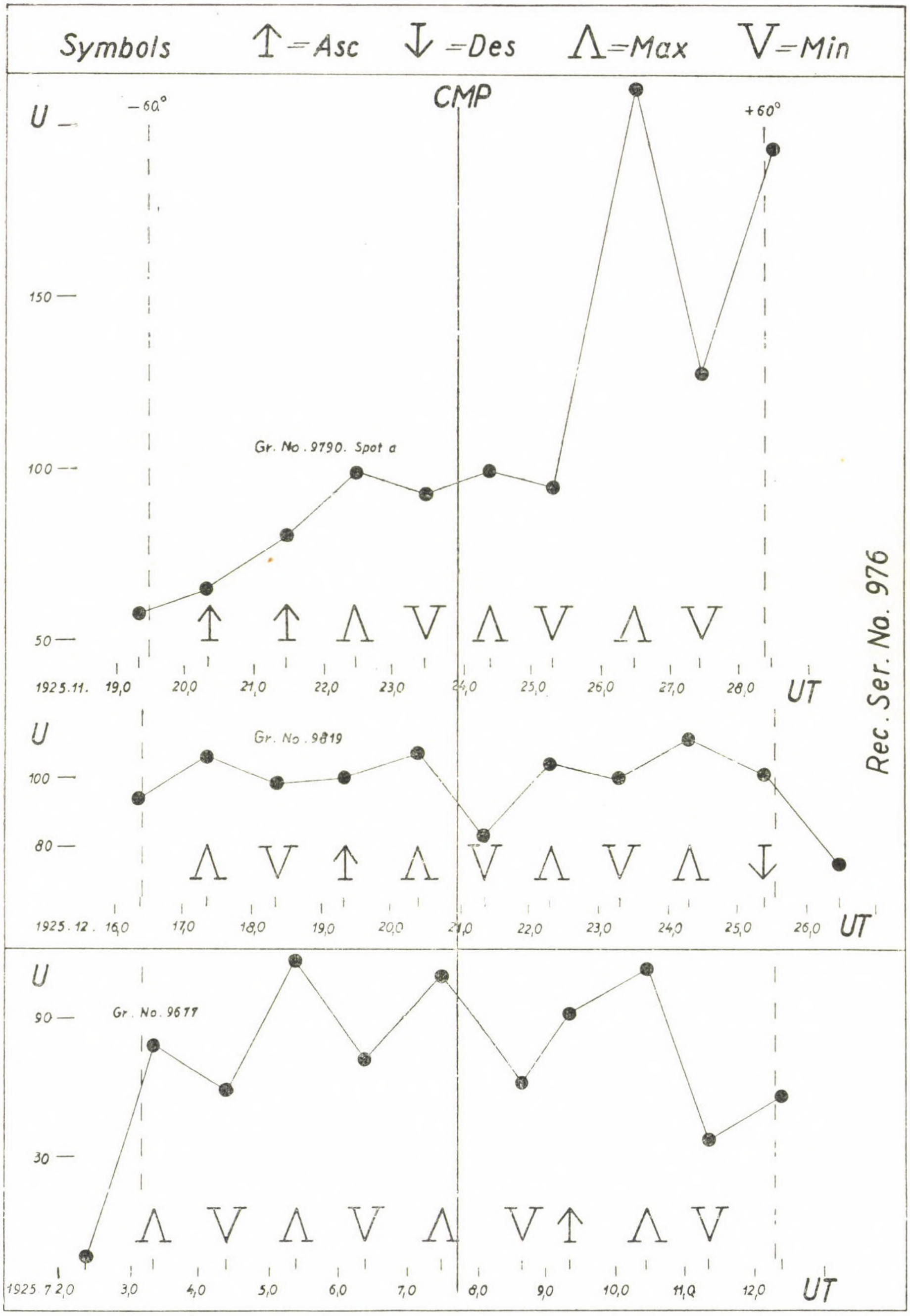

Fig. 1a. *Classification of sunspot observations by evolutional phase.* (Further examples in Fig. 1b.)

The curves represent the variations of umbral area (U). The upper and didmle curves refer to a single long-lived spot. The upper curve shows the evolution of the p spot of a large recurrent group during its first observational period. When it again came onto the solar disk, it was alone. The middle curve shows this period (there was a third appearance, also). The lower curve and those of Fig. 1b show spot groups composed of several spots.

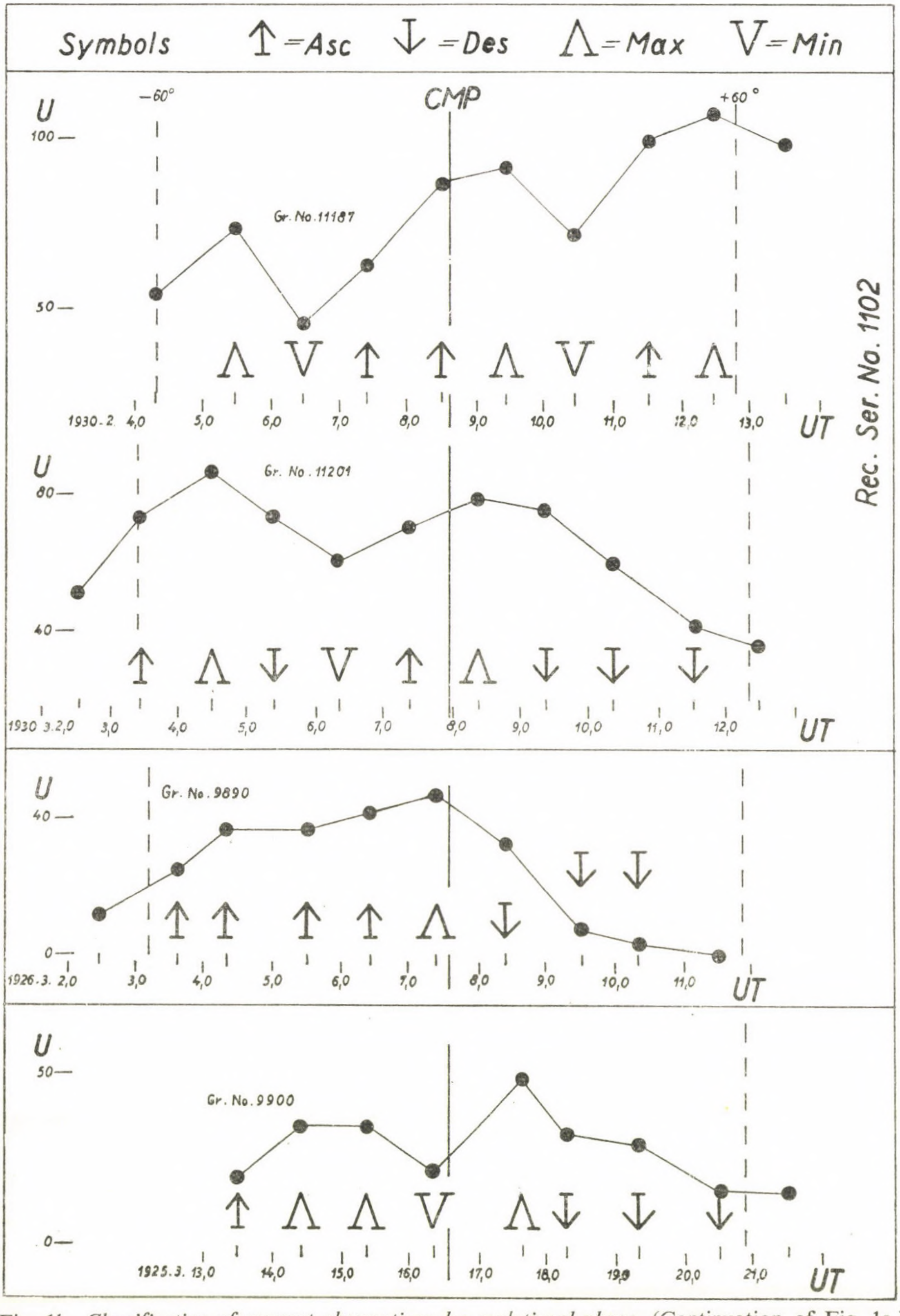

Fig. 1b. *Classification of sunspot observations by evolutional phase.* (Continuation of Fig. 1a.)

The dots on the curves, representing variations in area of the umbra, are the published data of Greenwich measurements The values of the ordinates are expressed in millionths of the solar hemisphere. The lines marked CMP and the dotted lines show the moments of central meridian passage and that of the 60° limits, reckoned from central meridian and expressed in heliographic longitude differences. The groups are given their Greenwich serial numbers.

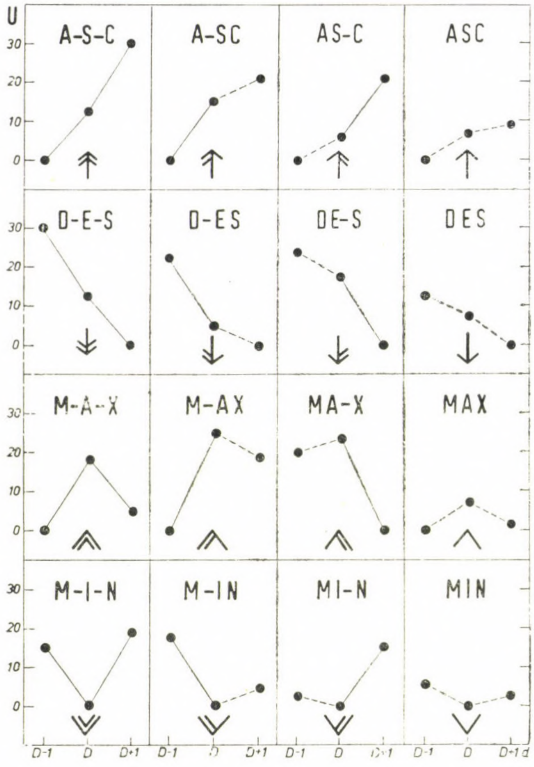

Fig. 2. *Schematic illustration of the classifications used to distinguish large and small area changes.*

Examples show the umbral areas (U) and their changes over a three-day period ($D-1$, D and $D+1$) with symbols and notations. (The dotted lines indicate small changes.)

§ 4. The classified observational material

All of the spot groups, and almost every single spot, observed on at least two days during the years 1922—1934 and recorded in the *Greenwich Photo—Heliographic Results* were classified by evolutional phase, if the spot or group (expressed in heliographic longitude difference, ΔL) was within the dictance $|L_{CM}| = 60^\circ$ of the sun's central meridian (CM) during at least one observation. The classifications were made individually both for umbral and penumbral areas (U and P respectively) and for all objects between $L_{CM} = \pm 60^\circ$. We considered in all about *50* thousand measurements of area and position. In the beginning we classified whole spot areas ($U+P$) also. Most observes today still talk about this heterogeneous sum of 2 different kinds of spot area, but we found that it has virtually no physical significance and disregarded it in further classifications.

Naturally, all our data of area were corrected for simple geometrical foreshortening, but they are still affected by the so-called „physical" foreshortening effect, which is still not completely understood and makes observations near the limb of the sun somewhat unreliable. On the basis of the investigations of ARCHENHOLD [2] and others, we established a limit of $|L_{CM}| = 60°$ in which observations could be relied upon. In practice, we sometimes had to consider measurements between 60.0 and 73.2° distance from central meridian, too, in order to classify objects observed between 46.7 and 59.9°. Evidently, the error in these cases is not important. Physical foreshortening enters only differentially into our classifications as a source of error; ΔL of $\pm 13.2°$ cannot change the foreshortening effect too much. In our classifications of rapid area changes (for example, A—S—C) we have already eliminated the problem of physical foreshortening completely with our choice of $|\Delta U| \geq 10$.

Our evolutional phase classifications are more significant when they come from observations a ken at constant intervals. The material is very good from this point of view, too. The majority of the photographic plates which were used in the measurements were exposed sometime between 0.30^d and 0.44^d U. T. The frequency curves of the moments of observation show only a slight skew towards the latter part of the day and the maxima and minima in both the winter and summer half-years are within the given interval. In the summer the observations were made during this period 76% of the time and in the winter 72%. Less than 3% of all observations occur outside the interval 0.25^d—0.64^d U. T.

According to the final Greenwich classifications, we used observations from 1919 spot groups and *14965* $U+P \neq 0$ observations.

There are 184 recurrent and 1735 single-passage groups among the 1919 used. (In the Greenwich classifications, 430 different group numbers were assigned the 184 recurrent groups.) These recurrent groups appeard on an average of 2.27 times between $\pm 60°$ from the central meridian. *11462* of the *14 695* observations were for whole spot groups. An average of six observations of all the spot groups falls within the $\pm 60°$ limit. In almost one-fourth (23.7%) of the cases we were certain that the observation showed a group which consisted of only a single spot. The remaining *3503* observations all show single spots, too, but they make up one or more „important" components of „principal" spot groups consisting of two or more spots (using Greenwich terminology). All of these belong to one of the 1919 groups; they are found in 315 groups overall. More than 1/3 of these 315 groups is recurrent and we have at least two × two simultaneous observations of over half of the 315 groups among the *3503* observations. We made an effort to take into consideration as far as possible only those details of spot groups that appeared to be single spots. As a result, we did not use all the separately measured and published data of spot group components. We immediately eliminated as a matter of course those details of groups described as „clusters" rather than „spots". Inspite of this, 3% of the spot observations we used do not show single spots. For example, when a spot seen as a single component for some time would split into more parts, we generally used one or two further observations of the altered object. In the case of whole spot groups, we used the coordinates at their center to determine the $|L_{CM}| < 60°$ limit; we figured each position individually for their separate components.

To all intents and purposes we have to add another *564* to the number of spot group observations; these represent the number of the $U+P=0$ observations of intermittent spot groups. 250 groups out of the 1919 disappeared temporarily within the 120° observing limit; no measurable spot was noted for one to several days. The distribution of all the *12026* spot group observations by year is shown in Figure 3.

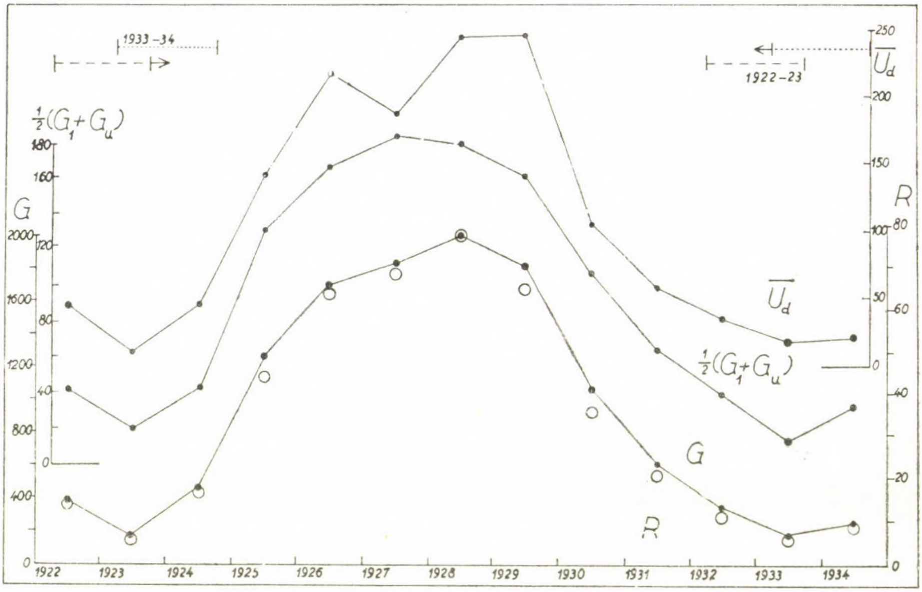

Fig. 3. *Sunspot activity from 1922—1934.*

The number of first (*l*) and last (*u*) observations of groups ($\frac{1}{2}G_l + \frac{1}{2}G_u$) and the number of all spot group observations (*G*). For comparison, the annual averages of the daily umbral area ($\overline{U_d}$) and those of the Zurich daily relative Wolf numbers (*R*) are also given. The length of the ordinates of the maxima are taken as equal for all four spot indices. Only those observations which satisfy the limit $|L_{CM}| < 60°$ are included in the G frequencies.

§ 5. Different categories of classified observations

Using the considerations outlined above and the relevant numerical data in the *Greenwich Photo–Heliographic Results,* we divided the observations into the following categories, which we will give along with our most frequently-used symbols and notations. We shall use these, with such additions as are practical and easy to memorize, not only in this paper but in future articles as well. *The letters S and s refer to single spots, while the G and g always mean spot groups,* although not always those which show more than one spot.

*

G_s and G_p are single spots that form simultaneously a spot group; G_p has a p spot. We found only a very few clear examples of G_f (a group with an f spot), so they are included among the G_s. G_s indicates in general spots which cannot be positively identified as either p or f. An object is called G_s or G_p only if it may be classified not only as a spot group, but unambiguously as a spot also. In the overwhelming majority of cases, result of the classification was identical for both spot group and spot. Evidently, if on three consecutive days (*D–1, D, D+1*), the same spot forms the same group, the *D*-day classification is valid equally for the spot and the group. This is not true when on either $D-1$ or $D+1$, or both, the group is no longer composed of a single spot.

To make this quite clear, we shall give a concrete example. Let us consider a spot group which on the days $D-1$ and D consists of a single spot, *s*, but on $D+1$ shows another spot *s*'. We denote the umbral values of the group on these three days respectively by U_{D-1}, U_D, U_{D+1}. The classification for the umbra on day *D*, if $U_{D-1} < U_D > U_{D+1}$, will apparently be *Max* both for the group and the spot; if $U_{D-1} < U_D < U_{D+1}$, it is *Asc* for the group and may either *Asc* or *Max* for the spot, depending on whether the umbra ($U_{s, D+1}$) of the spot on $D+1$ is larger or smaller than U_D. If none of the data of $U_{s,D+1}$ and $U_{D+1} - U_{s, D+1} = U_{s,D+1}$, are known, we cannot make a clear classification of the spot for the *D*-day observation. For this reason, this case cannot be included in the G_s category, and we introduced a further category:

G_{sc} is a spot group consisting only of a single spot, which is classified (c) simply according to group. We included in this category all cases where the group developed at least one other spot a day earlier or later (or both) and we had incomplete data on the spot classification.

G_{sm} is a small spot group, observed as a single rapidly-moving (m) spot. These were all spots which, in addition to any change in area from day to day, showed a so-called daily proper motion (ΔM) greater than 1° and which had an umbra smaller than *10* on one of the two days. (Proper motion here is motion parallel to the sun's equator and is expressed in heliographic longitude differences.) The classification of the G_{sm} observations may generally be regarded as unquestionably valid only for spot groups. It is possible, and the great change in position increases the probability, that the spot disappeared and a new one formed nearby, rather than that the spot actually moved. (This could happen also among groups which showed no such rapid motion, but, of course, it is much less probable.) The *Greenwich Photo-Heliographic Results* gives M values only for spot groups and fairly important spots (included in the so-called Ledger I and II), and these only from 1924 on. In the other cases, we calculated M ourselves by methods completely in accordance with Greenwich practice. Thus we allowed for differential rotation by the same formula or constant used at Greenwich [3].

G_σ is a spot group consisting of one large and one or more less important smaller spots, where the big spot occupies approximately 90% of the area of the group. This proportion could be determined for certain in over one-third of the observations in this category, where separate measurements of the large spot had been made. In other cases, we estimated the ratio of the area of the large spot to the rest of the group with varying degrees of accuracy. (The average size of the umbrae in groups in this category was *53*.)

G_g is a spot group which has more than one spot. Every group which could not be included in one of the foregoing categories is added to this one.

G_o is a spot group of zero total area — i. e., any $U+P=0$ observation of an intermittent spot group. It is obvious that the only $U+P=0$ cases in this category are those where there were both earlier and later $U+P>0$ observations of the group.

*

G_1, G_u and G_x : stand for the first (*1*), the ultimate (*u*) and any intermediate ($x=2, 3, \ldots, u-1$) observation respectively. These may belong to any of the above categories, with the evident exception that G_1 and G_u cannot be G_o as well. Accordingly, G_1 is the first $U+P\neq 0$ observation following the birth of the group while G_u is the last one before its complete disappearance. $G_2, G_3, \ldots, G_{u-1}$ denote in turn the second, the third, ... and the next to last observations of the spot group; G_x can be any one of them.

*

S_s, S_p, S_{p-}, S_{f-} and S_f are the important single spots of principal groups. These symbols are used only for details of groups. When we could not decide for sure the spot was p or f, we included it in the category S_s. In other cases suitable p or f index was applied. The dashes ($-$) indicate simutaneously observed p and f spots -that is, the individual members of p—f spot pairs- in a group for which we used observations (S_{p-}, S_{f-}).

*

The symbols defined above were used not only to show the characteristics of individual observations, but to give the overall number of observations or their relative frequency as well. In short, we may write: $S_{p-}=S_{f-}$, $G_x= =G_2+G_3+ \ldots +G_{u-1}$, $G_1+G_x+G_u=G_s+G_p+G_{sc}+G_{sm}+G_\sigma+G_o+G_g= =G$. G denotes the spot group observations in general, or more frequently, all the group observations during one or more calendar years.

Along with these categories, and some of the more general ones (like G), *the observations of the different evolutional phases may be given in brackets after the category symbol.* For example, $S_p(U,Asc;P,Des)$ means that in these S_p observations the umbra was increasing and the penumbra shrinking. When we are interested only in the formation of the umbral or penumbral area, *U* or *P* alone is shown. Thus we can designate all the S_p observations of growing umbra by $S_p(U, Asc)$; in this case $S_p(U,Asc)= =S_p(U, Asc; P,Asc)+S_p(U,Asc; P,Des)+S_p(U,Asc; P,Max)+S_p(U,Asc; P, Min)$. Where there is no danger of confusion, the simplest notations are used. It is sufficient in most cases only to write $S_p(Asc)$, and once in a while the mark (*Asc*) is enough too. As another example, we shall generally use $S_p/Asc/$ instead of $S_p(U,Asc; P,Asc)$.

*

We use g in a system roughly analogous to capital G to denote observations covering the whole life-span of the group. Henceforward, $g_1{}^1$ denotes groups which were outside $|L_{CM}|<60°$ during the first and last observation, and g_{1u} refers to groups in which we used the data of both the first and last observational days. $g_{1/u}$ shows recurrent groups of this class. Further, g_1 and g_u denote groups where only the first (1) or the ultimate (u) observation fell within our set limits. To sum up: $G_1 = g_1 + g_{1u}$, $G_u = g_u + g_{1u}$ and $g = g_1 + g_{1u} + g_u + g_{11}$.

TABLE 1

Numerical distribution of the sunspot observations for 1922—1934, classified into evolutional phase by both umbral and penumbral areas. Data on distribution of these spot observations by groups is included. (See Section 5 for definition of the symbols.)

<table>
<tr><td colspan="8">NUMBER OF SPOT GROUP OBSERVATIONS</td></tr>
<tr><td>G_g</td><td>G_o</td><td>G_σ</td><td>G_{sm}</td><td>G_{sc}</td><td>G_s</td><td>G_p</td><td>G</td></tr>
<tr><td>8318</td><td>564</td><td>310</td><td>259</td><td>242</td><td>1931</td><td>172</td><td>11 796</td></tr>
<tr><td colspan="8">NUMBER OF SPOT OBSERVATIONS</td></tr>
<tr><td colspan="5">components of spot groups</td><td colspan="2">isolated spots</td><td>total</td></tr>
<tr><td>S_s</td><td>S_p</td><td>S_{p-}</td><td>S_{f-}</td><td>S_f</td><td>G_s</td><td>G_p</td><td>S''</td></tr>
<tr><td>804</td><td>1011</td><td>762</td><td>762</td><td>162</td><td>1931</td><td>172</td><td>5604</td></tr>
<tr><td colspan="8">NUMBER OF SPOT GROUPS</td></tr>
<tr><td colspan="3">142 | 108
with
only one | two or more
$U+P=0$ observations</td><td colspan="3">156 | 159
with data of
only one | two or more
components of spot group</td><td colspan="2">127
with
S_{p-}—S_{f-}
pairs of spots</td></tr>
<tr><td>$g_{1u}+g_1$</td><td>$g_{1u}+g_u$</td><td>g_1</td><td>$g_{1/u}$</td><td>$g_{1u}-g_{1/u}$</td><td>g_u</td><td>g_{II}</td><td>g</td></tr>
<tr><td>1157</td><td>1183</td><td>443</td><td>42</td><td>672</td><td>469</td><td>293</td><td>1919</td></tr>
</table>

<table>
<tr><td colspan="10">NUMBER OF OBSERVATIONS PER NUMBER OF GROUPS (cca.)</td></tr>
<tr><td>G_o:</td><td>G_σ:</td><td>G_{sm}:</td><td>G_{sc}:</td><td>G_s:</td><td>G_p:</td><td>S_s:</td><td>S_p:</td><td>S_f:</td><td>S'':</td></tr>
<tr><td>2</td><td>3</td><td>1</td><td>1</td><td>3</td><td>2</td><td>12</td><td>5</td><td>2</td><td>6</td></tr>
</table>

*

All the observational material classified by evolutional phase is distributed in different categories as shown in Table 1. Strictly speaking, Table 1 includes only those observations which could be clearly classified by both types of spot area. Those observations which could be classified only by umbra or penumbra amounted only to 1.6% of all the $U+P \neq 0$ observations. The cases represented in this small percentage were those where $U=O$, $P>0$ (or, much more rarely $U>0$, $P=0$) and in addition the zero area was observed to be zero on all earlier or later days. For these $U=0$ (and $P=0$) observations there is no assigned area classification. (That is why there is no classification mark above the dot representing the last observation of Gr.No.9890 in Fig. 1b; this $U=0$ observation is shown only because at this time it was still $P>0$.) More than 96% of the observations classified only according to umbra or penumbra are classified by the latter. It is evident that nearly all these cases are found only where there are numerous smaller areas as well, and they occur roughly in proportion to the number of small areas. Among the spots which form pure group components we only have two such cases, one an S_s, the other an S_f observation.

*

There are certain questions which arise in connection with our system of categorizing observations. Do our procedures create possible misleading circumstances causing systematic error? Do we need to eliminate, or at least consider, the pernicious influences which may result from our purely statistical point of view? For example, is it dangerous to use G_s observations classified by evolutional spot phases, because we have quite necessarily eliminated the G_{sc} cases? We need not worry about this, because the number of G_{sc} observations is less than 13% of the G_s ones and this proportion remains fairly constant, with only minor fluctuations from year to year. Therefore, the G_{sc} cases may be regarded as omitted at random from the G_s ones. As another example, does our selection of the S_{p-}—S_{f-} spot pairs represent a proper statistical sample? Our answer again is positive. In general, all our categories (including the G_0!) are set up so that the average variation of the yearly number of observations shows the characteristics of spot activity fairly well. This is a very satisfactory criterion. Only the G_σ and G_p observations might possibly be exceptions, and even these only in the year 1926 when there are too few of them. (It is possible, however, that this drop has a real -i. e., a solar- cause.) Otherwise we deal only with categories which have a sufficient number of annual observations. In medium size categories we generally use the observational material for two or more subsequent years compiled together. Nevertheless, we must always consider carefully what peculiarities may arise from the definition of the categories themselves, excluding only the G category, which represents spot group observations in general. In this way, our categories will represent useful selections both from the standpoint of statistics and of solar physics.

The basis of most of our categories is, in practice though not in principle, that the publications used and their measurements must be homogeneous. We used the *Greenwich Photo-Heliographic Results* assuming that there were no modifications in the concise descriptions of groups and in the selection of group components over the years. We have no reason to doubt the integrity of the Greenwich observational material in this respect.

§ 6. Distributions of the classified observations

Let us begin with the G_1 and G_u observations. It is evident that their actual numbers must agree when there are enough observations or, rather, when there is a fairly long time interval. The period of an entire solar cycle should be long enough in any case.

Inspite of this, $G_u - G_1 = 30$ over the almost 11-year period limited by the longest intervals, 27 and 74 days, without sunspots during the 1923 and 1933 minimum years respectively. This number is only about 3% of the G_u or G_1 observations, but it cannot be dismissed, because the annual $G_u - G_1$ differences generally increase with the sum $G_u + G_1$. This leads us to conclude that the $G_u - G_1 \neq 0$ originates in systematic error, rather than accidentally.

If we separate the groups into recurrent and non-recurrent ones, we find that *22* out of the *30* cases -a significant majority- relate to recurrent groups, although less than 10% of the total groups studied recur. This immediately shows us the probable source of error. The difference in question probably arises as a result of some physically defective spatial and temporal arrangement of the individual spots into groups, perhaps because of the lack of magnetic observations or failure to take them into consideration.

We might suspect that this difference is due to the method of recording intermittent spot groups. Is it possible that the Greenwich observers are wrong in not giving a new number to a group which appears on or near the site of a group which had disappeared one or a few days earlier? Provisionally if we divide the intermittent groups into further groups by $U+P=0$ observations (G_0) we find that our difference of $G_u - G_1 = 30$ is hardly affected (it will be *27*). The number of (G_u and G_1) observations of the groups would increase only slightly, since relatively many one-day groups come into existence this way.

If we take the opposite case by considering the „Greenwich revival groups" (which show longer periods between appearances) instead of the recurrent ones, our problematic difference is still insignificant. Although in this case the difference between G_u and G_1 is greater, (numerically, by six) it is still in fair proportion to the 17% decrease in the number of observations.

In any event, these numbers do not give any indication that the Greenwich system of classifying disappearing and reappearing groups is generally incorrect.

*

From our definition of evolutional phase, it follows that the G_1 observations are all either ascending or maximal, while the G_u observations are either descending or also maximal.

Table 2a shows the distribution of the observations by evolutional phases classified by both penumbral and umbral areas. The headings along the top refer to the umbrae; the headings, along the side to the penumbrae. The last row and column, of course, refer only to umbrae and penumbrae

TABLE 2a

Relative frequencies of the first and ultimate observations of spot groups (1922—1934).

P \ U	*Asc*	*Max*	
Asc	48	11	59
Max	7	34	41
	55	45	G_1 *1081*

P \ U	*Des*	*Max*	
Des	54	10	64
Max	13	23	36
	67	33	G_u *1043*

TABLE 2b

Deviations ot the relative frequencies of the first and ultimate observations of spot groups shown in Table 2a, from their average of several years (in percentages; the frequencies below, average are indicated by the minus sign).

G_1 P \ U	*Asc*	*Max*	Years	G_u P \ U	*Des*	*Max*
Asc	0	0	1923—25	*Des*	+1	+4
	−1	0	1926		+1	0
	0	−2	1927		+2	−1
	0	+2	1928		−6	−3
	−2	+3	1929		−2	0
	−1	−1	1930−33		+4	+1
Max	−3	+2	1923—25	*Max*	−2	−2
	+1	0	1926		−2	0
	+1	+1	1927		−1	+1
	+2	−3	1928		+4	+5
	−2	+1	1929		+4	−2
	+1	0	1930—33		−2	−2

respectively. The relative frequencies are given to two decimal places in the squares of our double-entry tables. The decimal point and the zero digits next to it are omitted. In other words these tables, as well as our other similar contingency tables, contain the general percentage distributions. Where the number of observations is considerably over a thousand, the frequencies are expressed in thousandths -i. e., they are given to three decimal places. The number of observations used is shown in the lower right—hand square of each table.

First of all, we call attention to the fact that the distributions in Table 2a are almost constant. If we use only half the G_1 and G_u observations for the years 1922—1927 and 1928—1934, the difference in the relative frequencies of the umbral-penumbral phases of evolution is nowhere larger than ± 0.01, using Table 2a which gives the average. Even if we take a still smaller number of observations, we do not find many very large differences. This can be seen from Table 2b, where the number of observations range between *135* and *216*. There is no deviation from average greater than 0.03 for G_1 and only six cases out of 24 exceed this figure for G_u. (The two largest differences, —0.06 and +0.05, are found in cases where the umbra and penumbra are in the same evolutional phase -just when the frequencies have a higher value- and both occur in 1928, at solar maximum. Thus, these two numbers may be real, rather than an accident due to the restricted observational material.)

*

Let us assume again for a minute that each intermittent spot group is divided into more than one group according to the G_0 observations (omitting the so-created one-day groups). In this way we can produce 121 further „new" groups with a lifetime of at least two days, whose „first" observations are not included among the G_1 of Table 2a. If we prepared a table for these new first observations similar to the left half of Table 2a, we find deviations in numbers of the six-place data of umbra classification of 0.10, 0.05, 0.00, 0.06, 0.10 and 0.10 between Table 2a and the new table. These figures are in general substantially higher than those we noted above. Perhaps these sparse data support to a certain degree the idea that spots seen in an area where there was a group not long before cannot be considered newly-formed. We must assume that every intermittent spot group should be regarded as a single unit. It follows from our definition of evolutional phases that *all the G_0 observations in both the umbra and penumbra classifications should be considered as minima.*

*

While *for both the G_1 and G_u observations, there are only two possible evolutional phases, all four phases (considering only either umbra or penumbra) may occur for the G_x observations* (for each $x=2, 3, \ldots, u-1$). It is easy to see that a $G_1(Asc)$ observation is followed by a $G_2(Asc)$ or $G_2(Max)$, and after a $G_1(Max)$ there comes $G_2(Des)$ or $G_2(Min)$. Consequently, a $G_1(Asc)$ or a $G_1(Max)$ may give rise to a G_3 observation which can show any one of the four evolutional phases.

Therefore, if the direction of the changes in area -that is, the increases or decreases irrespective of their rate- are random after the first day of the life of a spot group, we would expect G_3 observations, in sufficient numbers, to be equally distributed among our four main evolutional phases, inspite of that fact that $G_1(Asc) > G_1(Max)$ according to observational evidence (see Table 2a). But the relative frequencies of the G_3 observations (without distinction between umbra and penumbra) do not have a random character. This is true of all of the G_x observations, which are shown in Table 3. Over 79% of the G_x cases are for groups which lasted at least three days.

TABLE 3

Relative frequencies of all spot group observations with the exception of the first and ultimate ones (1922—1934).

P \ U	*Asc*	*Des*	*Max*	*Min*	
Asc	86	11	43	25	165
Des	14	211	81	63	369
Max	28	59	113	42	242
Min	22	47	38	118	224
	150	329	274	247	G_x 9672

If the G_x-distribution of umbra-penumbra evolutional phases is random, the frequencies should be approximately 0.250 everywhere in the last row and the right-hand column of Table 3. There are enough observations in these categories for a statistical distribution. Further, if the penumbra and umbra are entirely independent of each other, our contingency table which shows the 4×4 double-phases of evolution, should have 0.062 throughout. In the opposite case, if the penumbra always changes simultaneously with the umbra, all observations, with some insignificant exceptions, should fall along the main diagonal of the table -that is, in those four squares which show the same evolutional phases for both areas. The realtiy is far from either of these two extreme cases.

*

The distribution of the evolutional phases of a ,,closed and complete'' series of observations -that is, observations for a whole solar cycle which are as free of selective criteria as possible- is obviously the most interesting. The G observations for the years 1923—1933 fulfill these conditions. The relative frequencies of the different evolutional phases for this cycle are given in Table 4a and illustrated in Figure 4 as well for a better survey.

TABLE 4a

Distribution of spot group observations in 1923—1933 by evolutional phase

P \ *U*	*Asc*	*Des*	*Max*	*Min*	
Asc	114	09	46	21	190
Des	11	221	75	52	359
Max	29	59	143	35	267
Min	18	38	31	97	184
	173	327	295	205	G *11 172*

The distribution of spot group observations is rather stable, as is conclusively shown by Table 4b. The deviations of the relative frequencies from average (i. e., the deviations from the corresponding values for the whole spot cycle) never amount to more than 0.015 out of *1650—2153* observations for the cases of double-phase evolution; when there is only one kind of area, the greatest difference is 0.025 in one single case. It is possible, however, that even these small fluctuations are not due completely to simple

TABLE 4b

Deviations of the distribution of spot group observations from their many-year average shown in Table 4a (the numbers are in thousandths; see also Figure 4).

P \ *U*	Years	*Asc*	*Des*	*Max*	*Min*	
Asc	1923—25	+10	+1	+7	+6	+25
	1926	+1	0	+2	0	+3
	1927	+5	−3	+3	−1	+4
	1928	−10	−2	0	−4	−16
	1929	−8	+3	−1	+4	−2
	1930—33	+3	0	−10	−4	−11
Des	1923—25	−4	−10	+6	+5	−5
	1926	+3	−10	+3	+5	−11
	1927	+1	+11	−2	0	−4
	1928	−1	+15	+3	−4	−3
	1929	+2	−13	−2	0	+9
	1930—33	+2	+2	−7	−6	+10
Max	1923—25	−4	−1	+7	−6	−2
	1926	+6	−3	−8	−5	+1
	1927	−3	+1	−2	+1	+9
	1928	−2	0	−6	+6	+13
	1929	+3	+8	−4	+4	−13
	1930—33	+3	−2	+13	−3	−8
Min	1923—25	−6	+1	−4	−9	−18
	1926	+3	−9	+2	+11	+6
	1927	+5	−3	+1	−12	−9
	1928	−3	+1	+3	+5	+6
	1929	+4	+8	−2	−5	+6
	1930—33	0	+1	0	+9	+9
						G
	1923—25	−6	−8	+16	−4	*1826*
	1926	+11	−22	−1	+11	*1650*
	1927	+7	+5	−2	−12	*1788*
	1928	−18	+14	−1	+4	*1969*
	1929	0	+6	−10	+2	*1786*
	1930—33	+7	+1	−5	−4	*2153*

statistical scattering. The deviations from the mean in the years of high spot activity seem to be systematic according to Figure 4. (In the same figure, the two examples taken from a distribution of G_g observations also point to this). We may assume that the frequencies of our evolutional phases vary somewhat with solar activity.

We now turn to Tables 5, 6, 7A, 7B and 8, in which we present distributions of various categories of observations. Many of the tables refer to „large" -that is, to rapid- changes in area. Since on an average the quotient

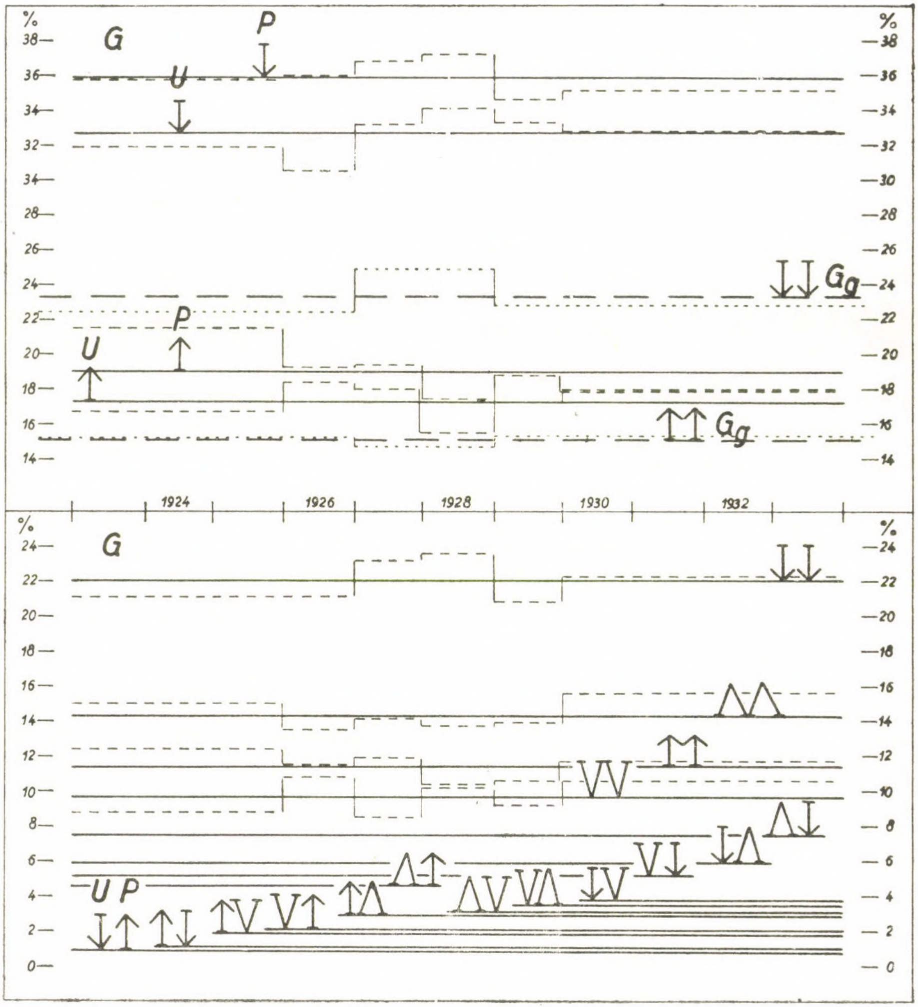

Fig. 4. *Distribution of spot group observations by evolutional phase.* (See Figures 1a and 1b for the meaning of the symbols.)

The left—hand symbol of each pair refers to the umbra, the right—hand one to the penumbra. The heavy straight and dotted lines in the upper half of the figure show average values over 10 years or more (the symbols for evolutional phase are on these lines). The other dotted lines are for a groups of years and show deviations from average. These never exceeded 1 percent in the twelve cases at the bottom of the figure, therefore, they are not plotted of the penumbral to the umbral area is roughly four, *we consider 40 to be a large change for a one-day decrease or increase in penumbra (10 was the value we chose for a large one-day change in umbral area)*. These limits proved to be not only practical, but reasonable as well (see our remarks about foreshortening in Section 8). It should be noted further that the G_p relative frequencies in Table 6 are given for spot groups and not, strictly speaking, for single spots.

We also checked all distributions of over *600* observations in Tables 5 to 8 to find out generally to what degree they represent reliable values. We compared everywhere the tabulated mean and the corresponding relative frequencies of 3 samples for some subsequent years, combining them according to three different phases of solar activity. Thus, for the double-phases of evolution in each contingency table we had 3×16 different possibilities from which we could get information. A brief account of the more important facts we found this way follows:

I) Among the important single-spot components of principal spot groups (S_p, S_{p-}, S_{f-}, and S_S) the relative frequency differences were greater than 0.01 in 20% of the cases. There were 16 of the 4×48 differences which surpassed 0.02.

II) The G_s and G_g observations are roughly twice and ten times respectively more numerous than those which are included in any category under I; on the other hand they also represent incomparably better statistical samples than the others. It is due to this, no doubt, that only two of the differences in question exceed 0.01 in the G_s and G_g categories (we are talking about the G_g observations in the lower right quadrant of Table 7A).

III) In the G_g observations with rapid umbral variations (shown in the left half of Table 7A) these differences exceeded 0.01 in 14×2 cases (14 cases where they occured in each kind). Among these cases, three were over 0.02. Taking a sample of generally slow umbral variations from the G_g observations (the upper right hand quadrant of Table 7A), the differences exceeded 0.02 in ten cases, and were above 0.01 in twelve more. The fact that there are greater differences in the cases of generally slow umbral change than in our other G_g samples of rates of evolution, all of which contain a statistically equal number of observations, is worthy of attention.

There were

a) 16 differences in category I and $3 \times 2 + 10 = 16$ in category III which exceeded 0.02; of these, 8 in each category were over 0.03. The two biggest difference values for group components (I), about 0.04—0.05, were found in p spots. In the G_g cases (III), there were two roughly equivalent differences and, in addition, we found values of 0.06 in two cases and 0.10 in one (i. e., three were still larger). The two greatest differences showed up in $G_g(U, \overline{\mathrm{DES}}; P, \mathrm{D—E—S})$ observations. (In both cases the sign of the difference agreed with Figure 4.) Perhaps it should be noted that 13 of the 16 differences exceeding 0.03 are from periods when the penumbra was decreasing.

b) Up to this point we have been speaking about fluctuations in the double-phases of evolution. The rest are purely an outgrowth of these. There might have been larger differences for either the umbra or penumbra, but generally, this did not happen. This is summed up well in Table 4b, which shows the G observations.

IV) *The distribution in Table* 8 *deserves particular attention.* When we divide its observational material into three year-groups, 1923—1926, 1927—1928 and 1929—1933, having equal numbers of observations, we get exactly the same relative frequencies in all three samples as those in the corresponding squares of Table 8. In the six cases where the area of an integral multiple of the smallest square contains the figure 1, it means that within the boundary lines the relative frequency is 0.01; the position of the number shows its approximate local distribution (or, rather its maximum). Thus, observations might fall with a relative frequency of 0.005 at most to a square left blank.

*

From Tables 4a and 2a and the considerations just discussed above, we can summarize generally the „stability" and, consequently, the reality of the distributions given in our contingency tables as follows:

The fluctuations in distribution tend to decrease as the number of observations increases. This feature, which is an effect of accidental error, shows some systematic character as well. (The observational material is not sufficient to permit us to define this character in more detail, so the dependence on spot activity cannot be regarded as proved.) Frequency distributions which deviate broadly from average vary in number in the different observational categories. (This may arise from the possibility that our categories do not include sufficiently homogenoeus samples statistically.) When we compare relative frequencies for various categories of rapid and slow area changes, or developments and decays, we find that the former data in both cases are more reliable.

Although we must have reservations about distributions based on only a few hundred observations, we can have confidence in those based on at least *500* observations. *The probable error of a relative frequency in our contingency tables which contains more than 700 observations is estimated as* ± 0.01.

It is clear from the above that, since *our evolutional distributions can be assumed to be constant within a small interval, we can take as characteristic data of the sunspot phenomenon.*

TABLE 5

Distribution of the observations of important p and f spots of p-f pairs by evolutional phase (1922—1934).

P \ U	*Asc*	*Des*	*Max*	*Min*	
Asc	13	1	8	4	26
Des	1	13	7	6	27
Max	4	6	10	6	26
Min	4	6	4	7	21
	22	26	29	23	S_p— *762*

P \ U	*Asc*	*Des*	*Max*	*Min*	
Asc	9	1	4	2	16
Des	1	30	11	8	50
Max	4	5	9	4	22
Min	2	3	4	3	12
	16	39	28	17	S_f— *762*

TABLE 5A

Distribution of simultaneous umbral and penumbral observations of important p and f spots of p-f pairs by evolutional phase. (The same observational material as in Table 5; see the text on page 34 for the explanation of the plus and minus signs.)

U; S_p— / S_f—	*Asc*	*Des*	*Max*	*Min*	
Asc	8 **+5**	1 **−3**	4 **−1**	3 **−1**	16
Des	5 **−4**	15 **+5**	10 **−1**	9 **0**	39
Max	6 **0**	6 **−2**	11 **+3**	5 **−2**	28
Min	3 **0**	4 **0**	4 **−1**	6 **+2**	17
	22	26	29	23	*762×2*

P; S_p— / S_f—	*Asc*	*Des*	*Max*	*Min*	
Asc	10 **+6**	2 **−3**	3 **−1**	1 **−1**	16
Des	8 **−5**	17 **+4**	12 **−1**	13 **+2**	50
Max	5 **0**	5 **−1**	7 **+1**	5 **0**	22
Min	3 **0**	3 **0**	4 **−1**	2 **0**	12
	26	27	26	21	*762×2*

TABLE 6

Relative frequencies of different special categories of spot groups and single spots (1922—1934).

P \ *U*	*Asc*	*Des*	*Max*	*Min*	
Asc	10	1	5	0	17
Des	0	33	8	3	44
Max	2	7	16	1	26
Min	2	6	2	4	13
	14	47	31	8	G_{sm} *259*

P \ *U*	*Asc*	*Des*	*Max*	*Min*	
Asc	10	3	2	4	18
Des	2	22	4	10	39
Max	0	3	0	7	11
Min	3	10	6	14	32
	15	38	12	35	G_{sc} *242*

P \ *U*	*Asc*	*Des*	*Max*	*Min*	
Asc	3	1	3	5	11
Des	2	16	9	11	38
Max	3	6	9	8	26
Min	3	7	7	9	25
	10	30	28	32	G_{σ} *310*

P \ *U*	*Asc*	*Des*	*Max*	*Min*	
Asc	2	1	3	1	7
Des	3	22	12	8	45
Max	2	9	14	6	31
Min	2	6	5	4	17
(1923—1933)	9	38	34	19	G_{s} *1789*

P \ *U*	*Asc*	*Des*	*Max*	*Min*	
Asc	4	2	1	2	9
Des	3	28	8	10	49
Max	2	4	6	5	17
Min	2	5	4	13	25
	12	39	19	30	G_{p} *172*

P \ *U*	*Asc*	*Des*	*Max*	*Min*	
Asc	6	1	4	3	14
Des	1	23	10	6	40
Max	4	7	9	6	27
Min	2	4	5	7	19
	14	36	28	22	S_{s} *804*

P \ *U*	*Asc*	*Des*	*Max*	*Min*	
Asc	13	2	8	3	26
Des	2	12	8	8	30
Max	4	6	9	7	26
Min	3	4	4	6	18
	23	24	29	24	S_{p} *1011*

P \ *U*	*Asc*	*Des*	*Max*	*Min*	
Asc	11	2	7	3	22
Des	0	30	8	7	45
Max	2	4	6	8	21
Min	2	2	5	3	12
	15	38	26	21	S_{f} *162*

TABLE 7A

Relative frequencies of spot groups consisting of more than a single spot
(1922—1934)

P \ U	A—S—C	D—E—S	M—A—X	M—I—N	
A-S-C	34	1	7	3	45
D-E-S	0	19	5	3	27
M-A-X	2	3	12	3	20
M-I-N	1	1	2	4	8
	37	23	27	13	G_g *728*

P \ U	$\overline{ASC}$	$\overline{DES}$	$\overline{MAX}$	$\overline{MIN}$	
A-S-C	16	1	7	3	26
D-E-S	1	22	8	6	37
M-A-X	5	6	14	3	28
M-I-N	2	2	2	3	9
	23	31	31	15	G_g *683*

P \ U	A—S—C	D—E—S	M—A—X	M—I—N	
$\overline{ASC}$	17	1	8	4	30
$\overline{DES}$	1	16	9	4	30
$\overline{MAX}$	2	4	18	3	27
$\overline{MIN}$	1	3	4	5	13
	21	24	39	16	G_g *868*

P \ U	*Asc*	*Des*	*Max*	*Min*	
Asc	15	1	5	2	24
Des	1	23	7	5	36
Max	3	6	16	3	28
Min	2	3	3	5	12
	21	33	31	15	G_g *8318*

TABLE 7B

Relative frequencies of important p components of principal spot groups (1922—1934).

P \ U	A—S—C	D—E—S	M—A—X	M—I—N	
A-S-C	48	0	8	1	56
D-E-S	0	9	4	3	15
M-A-X	3	2	7	6	18
M-I-N	3	0	5	3	11
	54	11	23	12	$S_p + S_{p-}$ *104*

P \ U	$\overline{\text{ASC}}$	$\overline{\text{DES}}$	$\overline{\text{MAX}}$	$\overline{\text{MIN}}$	
A-S-C	26	2	12	2	43
D-E-S	3	8	6	5	22
M-AX	4	8	5	8	25
M-I-N	2	4	2	1	10
	36	22	26	16	$S_p + S_{p-}$ *170*

P \ U	A—S—C	D—E—S	M—A—X	M—I—N	
$\overline{\text{ASC}}$	10	1	10	5	26
$\overline{\text{DES}}$	1	6	11	9	27
$\overline{\text{MAX}}$	3	3	14	6	26
$\overline{\text{MIN}}$	3	5	7	6	21
	16	15	43	26	$S_p + S_{p-}$ *221*

P \ U	*Asc*	*Des*	*Max*	*Min*	
Asc	13	2	8	3	26
Des	2	12	8	7	29
Max	4	6	9	6	26
Min	4	5	4	7	19
	23	25	29	23	$S_p + S_{p-}$ *1773*

TABLE 8

Distribution of spot group observations showing generally rapid variations in umbra by evolutional phase (1923—1933).

U / P	$\overline{A—S—C}$			$\overline{D—E—S}$			$\overline{M—A—X}$			$\overline{M—I—N}$			
	A—S—C	A—SC	AS—C	D—E—S	D—ES	DE—S	M—A—X	M—AX	MA—X	M—I—N	M—IN	MI—N	
A-S-C	5	2	1	1			1	1		1			
$\overline{ASC}$	3	3	6				2	2		1		1	30
D-E-S			1	3	1	1	1		1		1		34
$\overline{DES}$				3	7	4	2		4	1	3	1	
M-A-X					1		2	1	1	1			24
$\overline{MAX}$	1	1	1	1	1	1	3	4	2	1	1	1	
M-I-N										1			12
$\overline{MIN}$	1		1	1	1	1	1	1	1	1	1	1	
	8	6	9	8	10	7	12	9	9	6	6	4	
	26			27			30			17			G 4608

§ 7. Some features of the sunspot phenomenon

In this Section, we will give some of the results derived from our classifications of sunspots.

The difference $G_u - G_1$, concerning a period between two longer spotless intervals, may serve to a certain extent as a suitable control for determining the proper arrangement of individual spots into groups. Our experience shows that the group, and not the individual spot, is the primary physical object of the sunspot phenomenon. When there are many spots on the sun in close proximity to each other, it is frequently hard to pick out groups and determine their number (g). Since temporal variation of g is a physical characteristics of spot activity the curve of this variation should agree with that for G_1 and G_u when it is drawn for a long enough interval. In practice, of course (see the first part of Section 6, for instance) it is best to use the mean $^1/_2(G_1 + G_u)$. This is the best form for some indices of spot activity, as can be seen in Figure 3.

The G numbers show us the degree to which the solar surface is spotted and the peak of the G curve coincides with the maximum of solar activity. We can see immediately from Figure 3 that in the 11-year cycle the average number of groups reaches a peak earlier than the spot maximum, while the mean size of groups reaches maximum later. These three maxima seem to have been reached at intervals of about *six* months. (In 1927 there is a high secondary minimum of the $\overline{(U+P)}_d$ curve and the corresponding curve of $\overline{U_d}$ in Figure 3, but the maximum of $\overline{(U+P)}_d$ appears clearly in 1928; the numerical value of the whole spots is already a little lower in 1929 than it was in 1926.)

We found $^1/_2(G_1 + G_u)$ to be *1094* over the entire 1923—1933 sunspotcycle as defined at he beginning of Section 6. Multiplying this figure by three -because we considered spots only within 120° zone of longitude- we at once obtain the total number of spot groups which might have occurred over the entire sun in this 11-year period, about 3300 in all. We have used observational data from those groups which lasted a second day or longer; there were 1807 of them, over half of the 3300 groups. The quotient G/g shows the average duration of each group in days. From this, we get a *mean lifetime* of about *one week for spot groups which last more than one day.*

*

After these introductory remarks, let us see what conclusions we can draw easily from our contingency tables.

First, we will deal with the sums of the values of relative frequencies which lie along the main diagonal of each table (see Tables 2—7).

$G/Asc/ + G/Des/ + G/Max/ + G/Min/ = 0.575$. We get a different numerical value for similar sums of various categories of observations. For whole groups, we have the following data: G_g is 0.59; G_s is 0.42. The former is higher, the latter lower than the general value for G. It is obvious that only the G_s observations really represent the groups consisting of a single spot, because there are not enough G_p observations to be representative. We will return to the important difference of 0.17 between G_g and G_s later. The distributions for G_{sc} and especially for G_σ, which show predominantly the changes in area of a single spot, are low, 0.46 and 0.37 respectively. Our G_p sample shows 0.51 and the G_{sm} distribution is a contradictory 0.63. But this last high figure is reasonable, as we will see below.

We have the following values for the main diagonal of the distributions of single spots forming a portion of a group: S_p is 0.40, S_{p-} is 0.43; we find 0.44 for S_s and the S_{f-} category rises to 0.51. This last figure almost agrees with that for S_f, 0.50. However, the latter covers a very small number of observations and we cannot fully trust it. The other four values and the one for S_{f-} are trustworthy, however.

The sum of the main diagonal frequencies for G_s is almost the same as the average of those for S_p and S_{p-}. This is not surprising, since a single spot comprising a group is most commonly a p spot. It may be that the p spots predominate in the S_s sample or, more likely, that there are relatively few f spots[2]. The fact that our main diagonal value is only slightly higher for the S_s observations than for the S_{p-} and S_p ones, but in comparison, much lover than S_{f-} seems to support this conclusion.

The behaviour of the p and f spots varies to some degree in a way that shows difference in evolution. *The umbra and penumbra of f spots, when they are components of major groups, show pattern of an evolution which is more closely parallel than that of p spots.* By ,,parallel evolution", we mean that the direction of change in area is the same, without considering the rate of change.

We shall now begin to consider rapid area variations. Let $G_g[r]$ be the number of those G_g observations which relate to simultaneous rapid changes in both umbra and penumbra. (These are shown in the upper left quadrant of Table 7A.) According we can write:

$$\frac{G_g/A\text{-}S\text{-}C/}{G_g[r]} > 2\frac{G_g/Asc/}{G_g} \quad \text{and} \quad \frac{G_g/D\text{-}E\text{-}S/}{G_g[r]} < \frac{G_g/Des/}{G_g}$$

Here the G_g may be replaced by G as well. (This can be verified easily by Tables 4a and 8.) The validity of these inequalities seems to be true for p spots also (compare Table 7B). The p spots in general show much less rapid area variations than those required to draw consclusions, but, nevertheless, the first inequality above must apply since the numerical values show a greater difference.

[2] Often a third still fairly important spot appears between the p and f spot in groups spread over a wide area. We may call such a spot the central one or, for the sake of convenience, the ,,c" spot (those designated by the letter c in the Greenwich publications are generally of this type). Presumably many c spots are included among the S_s ones. It is perhaps for this reason that our distribution of S_s observations seems to be somewhat intermediate in comparison with the p and f distributions.
We should note here that, although for the time being our ,,quantitative" data for statistical investigations of sunspot problems come from *Greenwich*, we have used original observational materials ,,qualitatively" as well. Our observatory has the many original photospheric drawings from two now practically defunct Hungarian observatories, *Kalocsa* and *Ó–Gyalla*. This material contains excellent visual observations of spots and faculae over the whole solar disk and cover well over four decades from the middle of 1872. Most of the observations were made by FÉNYI in Kalocsa. We have only 20 years of Ó-Gyalla observations, but they started nearly a decade earlier. Among the Ó-Gyalla observers, we find the names of KONKOLY, KOBOLD and KÖVESLIGETHY. We have our own regular photographic observations beginning with the sunspot minimum of 1954, which will serve us as the main source for studying sunspot group evolution. (We will soon have 10 000 exposed plates, most of them showing the full solar disk.) We try, when possible and reasonable, to distinguish the p, f and c components of each group or, speaking more strictly, we suggest that every group is made up of one or more these ,,basic parts".

These inequalities show that our relative frequency of simultaneous decreases of the umbra and penumbra is lower for rapid area changes, than in general, but the frequency for such incereases is more than twice as high for rapid ones as in the general case. *Umbra and penumbra are much more likely to develop simultaneously when growth in area is swift -that is, when the cause of spot formation has a strong effect. The situation is the opposite in decay, although not to such a strong degree: the umbra and penumbra tend to diminish simultaneously when decline in area is slow.* In short, it seems probable that the development and decay of sunspots are not simply parts of a single phenomenon which rises and then dies out again.

*

We cannot explain the variation in spot area, as we have given it above, purely on the basis of variation in the magnetic fields surrounding the spots. *We might plausibly suppose,* however, that when the magnetic field of a spot is no longer increasing in strength, some other *local conditions, non-magnetic in nature, often serve to play a crucial role in destroing the spot.* Maybe these conditions are more important when the area declines rapidly. This helps to explain why a parallel decrease in umbra and penumbra occurs less frequently in rapid cases than in slow ones, since the two kinds of area might be expected to react simultaneously to the influence of the magnetic field more than to some non-magnetic influence.

We can test our assumption in at least one way. Since the magnetic field is always stronger in the umbra than in the surrounding penumbra, we may consider the umbra to be more resistent to any hydro-mechanical force acting to destroy the spot. Accordingly when the spot or spot group has been reduced to a small area, with simultaneous decrease in the strength of the magnetic field, especially in the penumbra, the penumbra ought to diminish also, particularly by photospheric motions. Consequently, the periods of penumbra decrease ought to be shorter than those of the umbra in these cases compared to the general case of larger areas. In our statistics, these shorter intervals should show up as mean lower relative frequencies. Thus for small spots and spot groups the quotient $(U, Des)/(P, Des)$ should be higher than for the larger spot areas. The contingency tables in this paper are not complete enough to give convincing proof that this is actually so: nevertheless, they do seem to support this assumption.

The G_u and G_{sm} categories should be sonsidered in this regard also. $U>10$ only occasionally in the G_u areas. $U<4$ in more than 2/3 of the $G_u \equiv G_s$ spots and the ratio is about the same for $G_u \equiv G_g$, and again for G_{sm} observations. But the other categories no longer show such a preponderance of small areas. For examples, well over 1/4 of the G_s spots, and 2/3 and 1/2 of the p and f spots respectively are $U>25$. Within the G_u category, the same trend is apparent for both the $G_u \equiv G_s$ and $G_u \equiv G_g$ observations: $G_u(U,Des) > G_u(P,Des)$. The G_{sm} table shows a similar inequality as well. Here the $G_{sm} \not\equiv G_u$ observations give rise to $G_{sm}(U,Des) >$

$>G_{sm}(P, Des)$. If we include the G_{sm} distribution, which is based only on a small number of observations, we have three quite different samples of observations of small spot areas which share this common property. In all the other contingency tables, with a single exception, we find: $(U, Des) < < (P, Des)$. Even in the one exceptional case, the ratio is an equality and, moreover, this distribution (of the $S_p + S_{p-}$ cases) is based only on *170* observations. These ratios seem to more than fulfill our needs, but there are some objections. Certainly these inequalities should be studied further in still more detail based on more observations.

*

Coming back to the sums lying along the main diagonal, we should mention that nearly all of the G observations which show rapid changes both in the umbra and penumbra are in the G_g category. (Tables 4, 7A and 8 also show this.) When we take only simultaneous rapid changes in both areas, the sum of the relative frequencies along the main diagonal becomes higher than any given above. We can see this immediately from our tables for G_g and $S_p + S_{p-}$ (the upper left quadrants of Tables 7A and 7B). We may substitute G for G_g, as we remarked above. We can see immediately that these exceptional sums are from (*U*, A—S—C; *P*, A—S—C) observations and that rapid area changes are found primarily during the development period. Scanning Tables 3—7 we also find that the frequency values along the main diagonal for rapid variations in umbra during the growth period are higher than those during decay only in the case of G_g groups and the significant samples of p spots. This makes it seem as if the p spot generally rule the development of a group. For this reason, we can say that the p spot is not only the leader-spot, the first spot we see as the group comes onto the solar disk, buth the principal spot as well.

Is it possible that the difference between the main diagonal sums of the G_g groups and those of single spots, which we spoke about earlier, is derived simply from the fact that rapid changes take place less frequently in spots? There are, indeed, few cases of rapid change in spots and we have only enough data to deal with the S_p and S_{p-} spots taken together. Nevertheless, we can still say that this assumption is not valid, since 9% of all G_g and 6% of G observations are larger than $G_g[r]$. By the way, the percentage of rapid area variations for both umbrae and penumbrae of p spots is the same as that of G_g.

Although the main diagonals in Table 2a (for G_1 and G_u) are incomplete, we notice that the sums there are the highest. Can our „difference" be explained by this? We must admit that these sums cannot even be taken into account. Almost 20% of the G_1 observations and about 37% of the G_u ones were for single spots. Since less than 1/4 of the groups consist of a single spot, the G_1 and G_u observations tend in general to raise the sums along the main diagonal of spots relative to the G, or rather G_g, categories.

The sum of the main diagonal frequencies seemed as though it might be too high in the case of G_{sm} referred to above. This is due to the G_1 and G_u

observations, since they make up 1/2 of the G_{sm} ones. If we omit the G_1 and G_u observations, our sum drops sharply in the G_{sm} category and decreases in G_p also. On the other hand, in the case of G_x (see Table 3) this diagonal sum is still substantially higher, 0.53, than any of the categories of spots, even if we omit the G_1 and G_u observations.

In summary, we can accept it as an established rule that the sums of the main diagonal frequencies are lower for single spots than for groups containing two or more spots. That is, the development and decay of the total umbra and the total penumbra of a spot group are more closely parallel in course than those of a single spot. From this fact, it follows that *the evolution of spots within a single group is closely connected.* Here, too, we see that the spot group, rather than the individual spot is the basic physical unit of our investigation.

*

Now we shall prove in still another way how strongly the group components are related. Let us take the simultaneously observed p and f spots of p-f spot pairs (shown in Table 5A). Table 5A in general speaks for itself; only the figures marked with a plus or minus sign need an explanation. The observations are divided into two contingency tables by penumbral and umbral classifications and the relative frequencies are given as in Table 5 for exactly the same observational material. Therefore, the last rows and columns of both quadrants of Table 5 and Table 5A are identical in every respect. Supposing that the area variations of f spots are completely independent of those of p spots, we can predict the distributions of Table 5A from Table 5. Thus, as an example, in the upper left hand corner of Table 5A, we should find 22% of the $S_{f-}(U, Asc)=0.16$, or 3, because $S_{p-}(U,Asc)=$ $=0.22$. In actuality we find 8. The numbers calculated in this way (c) and the observational numbers (o) -that is, the real relative frequencies obtained directly from actual observations- are usually not the same. In Table 5A we show the latter figures and the principal differences between o and c. We see that the „main diagonal sum" for both in Table 5A are higher by about 1/3 than they should be if the spots of the pair were independent from each other. The excesses are 0.15 and 0.11 for the umbra and penumbra respectively. Their origin is interesting: at least 2/3 of them come from developments and decays. We note also that the main diagonal sum is lower for the penumbrae (0.36) than for the umbrae (0.40).

To sum up: *the evolution of important p and f spots of a group takes place in parallel over more than 1/3 of their lifetime* (because of the values of 0.40 and 0.36). This high degree of syncronism cannot be merely chance (because of the exesses 0.15 and 0.11). These conclusions hold true both for the umbrae and penumbrae, but *the interconnection of the umbrae is stronger.*

*

We should note some of the properties of our contingency tables outside the main diagonals as well. If we add the relative frequencies of the squares lying to the right of the main diagonal of Tables 3, 4a, 5, 6 and 7, the sum is higher than that of the squares on the other side. Only the G_{sm} distribution is an exception.

We mentoined above that G_{sm} behaves anomalously in many other respects as well and does not share the statistical characteristics of single spots. Unfortunately we have too few observations in this category to draw more definite conclusions. In introducing the G_{sm} category, we assumed that when a spot apparently moves rapidly during a single day, it is the overall area of activity which is in motion. Later in this paper we shall find further support for this explanation of the ,,irregular'' character of the G_{sm}.

To return to our sums of the two ,,triangles'' lying on either side of the contingency tables, the difference results from the fact that the six pairs of squares are placed symmetrically in a perpendicular direction to the main diagonal so that only one pair on the left side and five on the right represent the not smaller frequencies. 19 of our tables follow this rule; among the 19×6 cases there are deviations only in seven and three cases and these amount only to 0.01 and 0.02 respectively. The latter cases occur only in those three contingency tables which show fewer than *172* observations. The other irregular cases may be the result of random error as well. For this reason, the apparent asymmetry in our distributions is most truly manifested in the numerous G observations and probably is present in the other categories. All these rather small systematic differences have been given to show, in advance, that *much of the non-parallel variation in umbral and penumbral areas is not random, but is just as real as the parallel variation* demonstrated above.

*

We now can consider the relative frequency values of the last rows and columns of our contingency tables. We see that $(U, Max) > (P, Max)$ and $(U, Min) > (P, Min)$ for every category of observations except for the minima of G_{sm} and -for reasons discussed above- for G_u. In both kinds of area we used the same unit as a threshold value for determining the evolutional phase (namely, $\Delta U = 1$ and $\Delta P = 1$). Since we can define the degree of fluctuation by the maximum and minimum values, it is certain that *the umbra, rather than the penumbra, fluctuates most broadly both in the group and the spot*. (If we correct for observations whose measurements show no change in area from one day to the next, the inequalities still remain.) The inequalities are even more striking when the variation in area is rapid.

The umbrae show more variations than the penumbrae when we consider the data of the decaying and developing cases as well. We have mentioned already that, with an easily explainable exception, $(U, Des) < (P, Des)$. But $(U, Asc) \leq (P, Asc)$ may also be taken as a general rule. There are deviations only for the G_s and G_p observations in our tables. Taking the increases and decreases together, we observe a two-day continuous monotonic area change in the umbrae less frequently than in the penumbrae, -i. e., here again we have indication of wider fluctuation in the umbra than in the penumbra.

The exceptions $G_s(U, Asc) > G_s(P, Asc)$ are probably real, since we get the same inequalities when we divide the observational material into three equal parts. Only the S_{f-} and S_s categories show an equality. Thus the penumbral cases are the more numerous, particularly for groups of more than one spot. It is important to note that the irregularity in G_s and G_p development -in sharp contrast to what we previously noticed for some declining spots- does not come from small spots, but from the larger ones. This phenomenon, which goes against our rule, originates with $U > 10$ and mainly with $U > 20$ spots and has no relation to the G_1 observations. Moreover, it occurs, with few exceptions, in all of the slow area variations. On this account, we might assume that these groups consisting of a single large spot either cannot develop normally -proof of this would be the fact that they stand alone- or that they are already in the final decaying stages and not subject to causes of spot-formation.

$G(U, Max) + G(U, Min) = 0.50 \pm 0.01$ and $G(P, Max) + G(P, Min) = 0.45 \pm 0.01$ according to Tables 4a—4b. This means that the umbral area of a spot group has a maximum or minimum at least every second day in general -i. e., it increases or decreases on the average for two days in a row; this process lasts about 10% longer in the penumbrae. If we compare these figures with the corresponding data from other observations, we can add the following, always remembering the differing reliabilities of our relative frequencies: The intervals of growth or decrease are definitely higher for the f than for the p spots -at least, among the most important ones. Since there are few observations of the first and last days of p spots, we must also compare the p spot observations with the G_x data. We find the corresponding values are in better agrement, while the f spot data and that for the groups are far from being the same. This difference in the behaviour of p and f spots also serves to indicate that a spot group has more the character of the p than of the f spot.

$G_g(U, Max) + G_g(U, Min) = 0.46$ and $G_g(P, Max) + G_g(P, Min) = 0.40$ according to Table 7A. Both figures are lower than in the case of G. That is, when a group consists of more than one spot, the total umbral and the total penumbral areas of the group increase or decrease without interruption, on the average, for a longer time than those of a group consisting of a single-spot. Here we conclude for the third time that the spots of a group are closely inter-related.

*

Some observational evidence show directly, even without measurement that increases in the entire area of a spot group take place generally in a shorter period of time (t) than decreases. This fact, which has been well-known for years, may also be expressed by the formula:

$$\left|\frac{d(U+P)}{dt}\right|_{Asc} > \left|\frac{d(U+P)}{dt}\right|_{Des}$$

Introducing the notation ΔU_i for a two-day (Δt) change of the umbra, we may write:

$$\left|\frac{dU}{dt}\right|_{Asc} = \frac{1}{G(U, Asc)} \frac{\Sigma \Delta U_a}{\Delta t} \text{ and } \left|\frac{dU}{dt}\right|_{Des} = \frac{-1}{G(U, Des)} \frac{\Sigma \Delta U_d}{\Delta t}$$

where the possibility is from *1* to G(*U, Asc*) and G(*U, Des*) for *a* and *d* respectively. Here, of course, we are using the number of observations, not the relative frequencies.

If the observational material is large enough and covers all observations: $\Sigma \Delta U_a + \Sigma \Delta U_d = 0$. Thus the ratio of the average numerical values of the umbral *"Asc* and *Des"* velocities will be G(*U,Des*) per G(*U, Asc*). The same can be sadi for the penumbra as well and for all the

TABLE 9

The ratio of the number of observations of area decrease to those of area increase. (Descending per ascending frequency values for umbrae and penumbrae.)

Years	G		S_p		S_{p-}		S_{f-}		Years
	U	*P*	*U*	*P*	*U*	*P*	*U*	*P*	
1923—25	*1.9*	*1.7*	*1.0*		*1.6*		*2.1*		1922—26
1926	*1.7*	*1.9*		*0.9*		*1.3*		*2.9*	
1927	*1.9*	*1.9*	*1.1*		*0.9*		*2.1*		1927—28
1928	*2.2*	*2.1*		*1.4*		*0.9*		*2.9*	
1929	*1.9*	*1.8*	*1.1*		*1.1*		*3.2*		1929—34
1930—33	*1.8*	*2.0*		*1.3*		*0.9*		*3.9*	
1923—33	*1.9*	*1.9*	*1.1*	*1.2*	*1.2*	*1.0*	*2.5*	*3.2*	1922—34

categories of spot observations where the latter equation is fulfilled. It should be considered true in cases where many observations relate to relatively few individual objects. That condition seems to be satisfied in the case of G, S_{p-}, S_{f-} and S_p and at first sight also for the S_s observations, as we read from Table 1. In Table 9 we compiled the quotients $(U,Des)/(U,Asc)$ and $(P,Des)/(P,Asc)$, used in determining the ratios of the area velocities. Notwithstanding the small number of p and f spot observations compared to G, we give here for the spots both the best means and some further values of these quotients which may help us to determine the reliability of the numerical data. We find that the figures for f spots are much higher than those for p; for this reason the S_S observations were excluded from Table 9, ince in that category we cannot fix the proportion of the three different pot types.

From the table we can sum up the following results:

$$\overline{\left|\frac{dU}{dt}\right|}_{Asc} = K_U \overline{\left|\frac{dU}{dt}\right|}_{Des} \quad \text{and} \quad \overline{\left|\frac{dP}{dt}\right|}_{Asc} = K_P \overline{\left|\frac{dP}{dt}\right|}_{Des}$$

where generally $K_U \approx K_P \approx 2$ *for spot groups, while for important spots of group components* $K_U \approx K_P \approx 1$ *for the p spots and* $K_U \approx 2.5 < K_P \approx 3$ *for the f spots.* That is, there is a significant difference between the ratio of the average area velocities of the development and decay of the umbra compared to the penumbra only in the case of f spots. It should be noted that great care must be taken in comparing the K_U and K_P values of G groups to those of f and p spots, because we do not have enough first and last spot observations, especially for the p spots.

*

It is not surprising that *the statistics concerning the birth and death of spots and the main periods of spot-life in between are somewhat different in several respects.* Some rough, but enlightening, data shows some of these differences.

At least 70% of the G_0 observations were immediately preceded by G_1 or followed by G_u or another G_0 observation. Over 10% of the G_2, and still more of the G_{u-1} observations, are of the G_0 type. There are about 50% more $G_{u-1} \equiv G_0$ observations than $G_2 \equiv G_0$. During G_0 — that is, when $U+P=0$ — the spot group, or rather the center of activity, was shifted by more than one heliocentric degree in 2/3 of the cases.[3] This holds both for one-day and for longer spot group disappearances (more than half of the disappearances lasted one day, and there are more than twice as many two-day disappearances as longer ones). But the proportion of

[3] These data are based on a rough guess with the aid of our graphs showing all measured areas and positions of the Greenwich groups from 1922—1934.

the number of large position changes to small ones, using 1° as a limiting value, is considerably different when we take the beginning and last observations into account. Comparing every earliest ($U+P\neq 0$) position after a $G_2\equiv G_o$ observation to the position of the G_1 group and every last position before a $G_{u-1}\equiv G_o$ observation to the position of the G_u, we find that large changes in position are only twice as frequent as the small ones in the early period, but in the decaying phase the number of these large shifts is over four times the small ones.

Turning back to the G_{sm} observations, we note first that their number is about 30% higher than that of the one-day disappearance of spot groups. But, as we mentioned earlier, half of the G_{sm} category comes from G_1 and G_u observations. It is interesting that the ratio of the number of these observations –i. e., $G_1\equiv G_{sm}$ to $G_u\equiv G_{sm}$ – resembles the ratio of $G_2\equiv G_o$ to $G_{u-1}\equiv G_o$ which we gave above. If we consider that *both of these observational samples relate to large scale motions (those special* G_{sm} *and* G_o *ones)*, it is quite plausible for us to regard G_{sm} observations in general as the result of a „short period" of intermittence -that is, of a disappearance shorter than a day. Since there are more short disappearances than long ones, there is good reason to suppose that in reality a disappearance may occur every time a spot is formed or dies out, not just in the 10% of the cases where this phenomenon is actually observed and that we do not see them because the observations are far from being continuous. From our statistical data there is no doubt that every Greenwich *intermittent spot group should indeed be considered as a single physical unit.*

There is another question to be considered in connection with the intermittent groups, however. When there are G_0 observations on two days together, which happens rather frequently, we count them as two minima for the sake of homogeneity. There is some danger that this may introduce a source of error into our calculations. When we correct for it, however, we find that only the G(*U, Min; P, Min*) frequency and the similar one for G_x are altered noticeably, by about 0.02. There is no change in any of our qualitative statements. Even if we omit all G_0 observations (which would certainly be wrong), there are no important modifications.

There are other examples which show a conspicuous difference between the beginning and end of spot groups. More than 10% of the G_1 observations show rapid variations in the umbra; only 1% of the G_u cases are this type. Among the groups with a lifetime of more than three days, these rapid evolutional phases are six times more frequent among the G_2 observations than among the G_{u-1} ones. There are twice as many $G_u\equiv G_s$ as $G_1\equiv G_s$ observations. In the latter sample about 90% of the spots are $U\leq 4$ and $5G_s(U, Asc; P, Asc) < G_s(U, Max; P, Max)$. This inequality exhibits a great deviation from the general distributional rule (shown in Table 2a). On the other hand, nearly every $G_{sc}(U, Asc; P, Asc)$ observation is a G_1 one. We see from all this that *the initial development of a spot group is generally weak if the group consists only of a single spot on its first day.*

*

Now we shall give some proofs of a characteristic of the spot phenomenon which may be quite important. It is easy to show with our various observational samples that any *change in the evolutional phase of the penumbra mostly lags in time behind the similar change in the umbra.*

First we must define our main relative frequencies -or rather the individual observation classified by different evolutional phases- a little more extcaly. When we classify an object as „ascending" or „descending", it might, indeed, be developing or decaying at the moment of observation. Certainly this is true of rapid area variations almost without exception and there is a great probability that it is true in the alternative variations. A significant number of exceptions is possible only in the case slow area changes. On the other hand, we can only define a maximum or minimum observation by saying that the area had an extreme value, relative or absolute. We do not even know how near the real time of maximum or minimum was to the moment of observation; certainly it must have taken place within a two-day interval. Within this period, there might occasonally have been more than one maximum or minimum in area. An „observed" extreme is evidence, at any rate, for at least one maximum or minimum, even though at the moment of obseervation the spot or spot group might still be growing or diminishing. It is clear from the above that *our classifications of development-decay and maximum-minimum have to some degree quite different meanings,* but the relative frequencies of our maxima and minima still furnish useful and fairly reliable data.

In certain cases, from the observed shape of the area curve near maximum or minimum, we can estimate the state of evolution with a high degree of probability. Let us look at the G(*U*, MA–X) and G(*U*, M–IN) observations for example. Considering that the periods of development are shorter as a rule than those of decay, we can conclude that the umbrae were probably diminishing at the moments of both types of observations (see Figure 2). Some of the figures in Table 8 support this idea, also, since the umbra and penumbra show the tendency to change in a parallel way. The frequency values alone G(*U*, MA—X; *P*, *Asc*) = G(*U*, M—IN; *P*, *Asc*) = 0 prove that actually we have rather sharp decrease in the umbrae rather than the apparent extremes in umbra area.

From Table 8 we see that G(*U*,M–IN; *P*,*Des*) = 0.04 > $^1/_2$ G(*U*,M—IN) = = 0.03 -i. e., the penumbra was still in decreasing over half of these special cases when the umbra was at minimum. In such instances we have very few cases where the minimum in the penumbra was observed. This may be attributed to the lag in evolution of the penumbra.

We can take another case from Table 8. G(*U*, A—SC; *P*, A—S—C) = = 2 G(*U*, AS—C; *P*, A—S—C), inspite of G(*U*, *A*–SC) = 2/3 G(*U*, AS–C). That is, early rapid developments in the umbra are followed by a large increase in the penumbra on the next day with a much higher frequency than late rapid developments. This also shows the time-lag.

We can take further examples from Table 8 or other statistical samples to support the idea that penumbral evolution is delayed in relation to that of the umbra. It is not difficult to see this in the following frequencies for example: $G_1(U,$ A–S–C; $P,$ A–SC)$=4G_1(U,$ A—S—C; $P,$ AS—C) while $G_1(U,$ A—S—C; $P,$ A—S—C) $\approx G_1(U,$A—S—C; $P,$ A—SC)$+$ $+G_1(U,$ A–S–C; $P,$ AS—C). It is only to be regretted that we have too few data for the years 1922—1934 to prove it (there are only *84* observations showing this distribution).

Table 4a gives: 0.090=G(U, *Max*)–G(U, *Min*)≈G(P, *Max*)– –G(P, *Min*)=0.083. These figures, or rather differences, tell us how many absolute maxima could have occurred among our spot observations. Many (0.06) of the numerical values given here originate from those non-recurrent groups which had both G_1 and G_u observations. We have 672 groups (see Table 1) where we can be sure that the time of an observed absolute maximum coincides with a real extreme in area with error of less than a day. In 2/3 of these 672 groups the absolute maxima for both umbra and penumbra fell on the same day, and in 2/3 of these cases where the maxima did not coincide, the penumbral maximum lagged. In more than 60% of the delayed cases, the penumbra was still increasing on the day that the umbra reached absolute maximum and reached its maximum one day later. Consequently a real retardation seems to predominate. In about 10% of our 672 groups the umbra showed its absolute maximum for more than one day, sometimes on successive days. We included data only for the earliest of these maximum days.

All these data present convincing evidence of a time-lag between the evolution of the umbra and the penumbra. The penumbra seems to follow changes in the umbra; it shows a sort of „inertia".

*

We should note the inequality G(U, A—SC)<G(U, AS—C), which contrasts sharply to G(U, D—ES)>G(U, DE—S) (Table 8). This means that the rate of development of spot groups usually accelerates with time, while the rate of decay decelerates.

*

Up to this point we have dealt exclusively with observations which could be classified by both umbral and penumbral phases. Now we shall talk about $[U=0, P\neq 0]$ observations too. There are *221* of them which all except *14* are also G_1 and G_u observations. The other *14* adjoin G_u or G_1 or G_o; nine of them are the type G_{u-1}. The main distribution is $G_1=70$ and $G_u=137$. These *70* first and *137* last observations were followed and preceded by G_o observations in *19* and *22* cases respectively. If we eliminate these cases, the ratio $G_u/G_1\approx 2$ for these $[U=0, P\neq 0]$ observations increases still further.

$G_u[U=0]=137$ means that almost 12% of the spot groups were last observed without umbra. If we consider that this figure comes from daily single observations, it seems quite probable that spot groups in their last hours as a rule show only (one, or more than one,) penumbrae.

But having only penumbrae is not a characteristic quality only of the ultimate period of the spot group. The $G_1[U=0]$ and $G_u\,[U=0]$ observations on the whole may be regarded as developing and decaying cases respectively. The ratio of the area velocities of these two opposite evolutional phases are approximately the same as the G_u/G_1 quotient in the penumbral case; for this reason, we can assume that the penumbral appearance came first at the beginning of the life of the spot group, also.

We should add that there are another *58* $[U=0, P\neq 0]$ cases among our G observations which are neither G_1 or G_u and can be classified by both umbra and penumbra. They are evenly divided between the types G_2 and G_{u-1}. This fact also suggests that a *non-umbral phase occurs at both the beginning and end of the spot lifetime.* If we compared frequency data for the penumbral spot groups with that for „intermittent" spots, we migth find further support for this idea. We would expect the distribution of penumbral spot groups by phase by the group lifetime to somewhat resemble the corresponding distribution of the G_o and G_{sm} observations.

*

Earlier we gave evidence for definite and close connections between umbra and penumbra. Although we did not say so explicitly, it seems from this evidence quite probable that *the umbra alone should be considered the „real spot"*, with the penumbra as a much less important and essentially different photospheric change. If this is true, what about spot observations which show no umbra? Do penumbral spots really exist?

§ 8. Penumbral spots

In order to study the penumbral spots statistically, we had to choose observational material from an earlier period than 1922—1934. Up to 1916 in Greenwich most spots in a group were measured separately, only a few small close spots grouped together were measured together and the relative data published; after that time only important spots were handled individually. For this reason we used observations for the years 1901—1913 from the *Greenwich Photo-Heliographic Results* as the basis for a thorough investigation of penumbral spots. This period gives us observational material for an entire solar cycle (*no. 14*). We included every penumbra recorded as having no accompanying umbra, or over *14 000* in all. We must emphasize that the conclusions reached of course refer to individual spots.

We studied distributions of the areas of penumbral spots (P), and their heliocentric angular distances (Φ) reckoned from the center of the solar disk. Here again we used only areas corrected for geometrical foreshortening -that is, $P=P_{pr} \cdot \sec \Phi$, where P_{pr} denotes the areas observed directly on the disk.

Figure 5 gives us some direct first hand information. Every dot represents one separate penumbra recorded in 1904; together they cover all such cases. (The six spots in the upper right-hand corner ought to be far above the margin, as indicated by the arrows.) At first glance we can tell that very large penumbral spots appear only toward the solar limb, indicating that the main problem that faces us must be purely a visibility effect. What does the well-marked minimum frequency of penumbral spots at the center of the solar disk mean? It is obvious that this may come simply from the relation of the solar equator to the position of the spots as we see them from earth.

The center of the sun's disk comes into a spot zone only from time to time for a short period. We may take $6.0^\circ \leq B_o < 7.3^\circ$ as the necessary condition if B_O is the heliographic latitude of the earth. In the upper left corner of Figure 5 we plotted the penumbral spots observed in the years 1900—1916 which fulfilled both this condition and the following ones at the same time: $6.0^\circ \leq B < 7.3^\circ$, sg B = sg B_o and $|L_{CM}| < 20^\circ$, where B is the heliographic latitude of the spot. The distribution obtained this way proves that if penumbral spots occur just as often near the center of the sun's disk, where their maximal frequency appears in Figure 5, we should not observe a diminished number of spots even at the center of the disk. It was necessary to explore this point as there is a possibility that we observe umbrae in some apparently „penumbral" spots when they approach the center of the disk. We should mention also that twice as many of the spots shown in the upper left hand corner of Figure 5 were observed in the period from August-September as from February-March. We conclude from this that observations of such small spots depends to a large degree on meteorological factors. (By the way, in the former period we have all $B > 0^\circ$ spots; in the latter, all $B < 0^\circ$.)

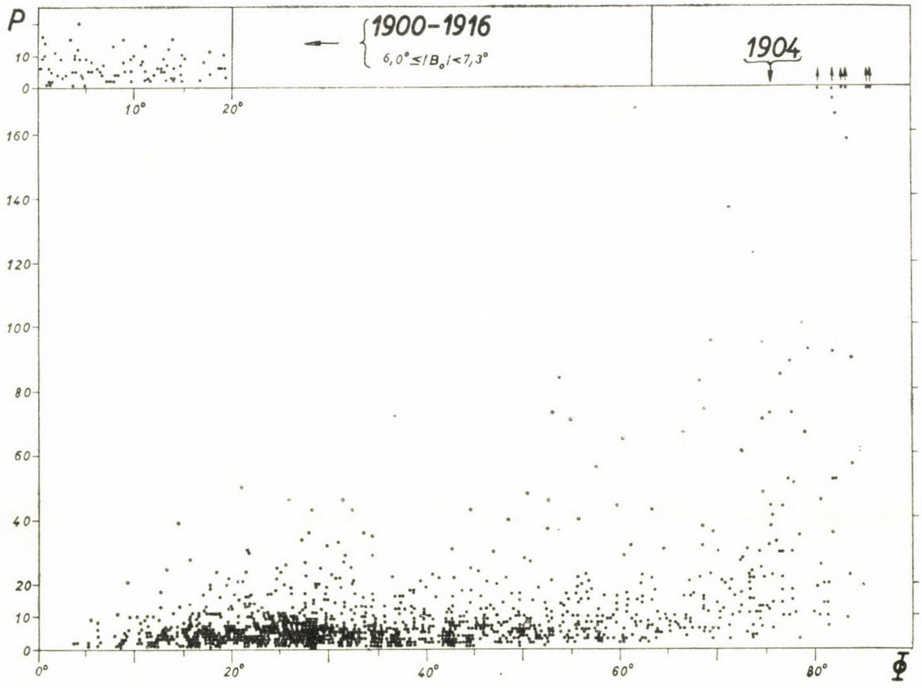

Fig. 5. *Penumbral spots observed in 1904.* (See text above for an explanation of the dots in the upper left hand corner.)

The histograms of Figures 6 and 7 show the frequency distributions of penumbral spots of different sizes and apparent positions. In Figure 6 the relative frequencies in $\Delta\Phi = 5°$ intervals are given separately for each of three size categories. In Figure 7 the relative frequencies are counted separately for each of six position categories, but not all of them are plotted. On the right hand side outside the margin there are a considerable number of spots of $P > 74$ near the solar limb ($\Phi > 81°$). (Compare also Figure 5.) In Figure 7 the frequencies of the three $\Phi > 60°$ samples are given in different ΔP intervals, while at $\Phi \leq 60°$ the frequencies are counted for every P value under $P = 51$. (Unfortunately, it turned out at the end that it would have been more advantageous to choose shorter Φ-distances, which would

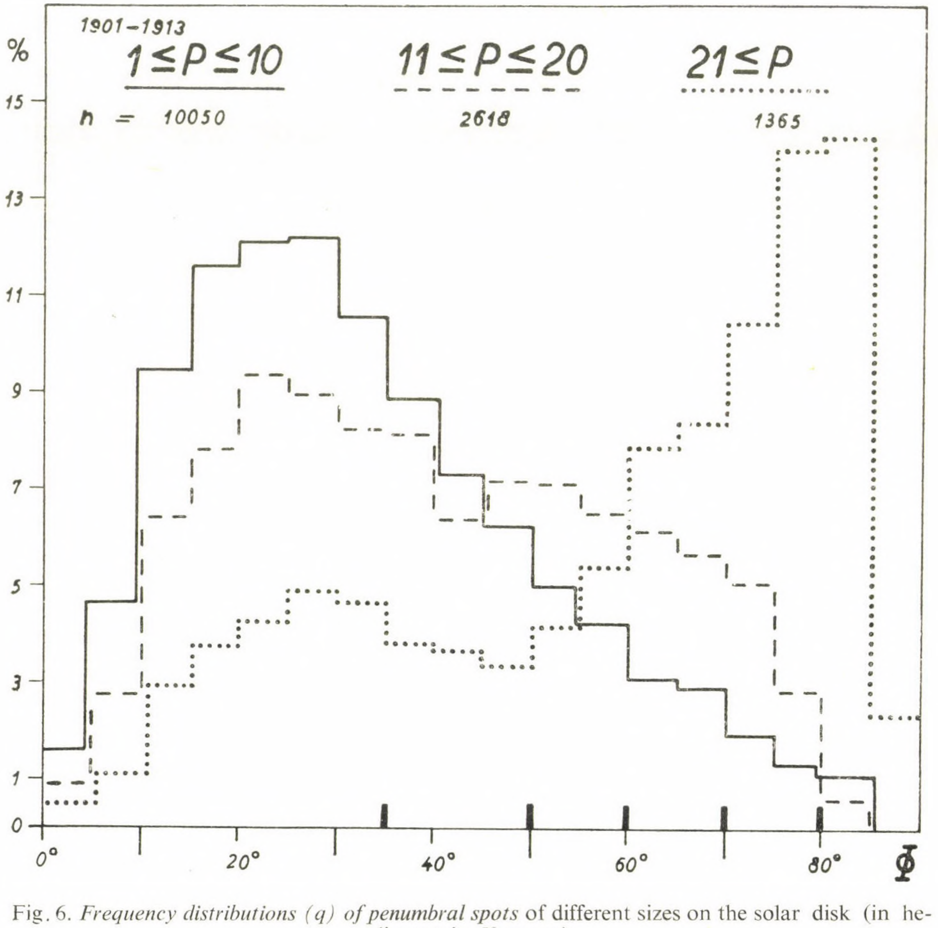

Fig. 6. *Frequency distributions (q) of penumbral spots* of different sizes on the solar disk (in heliocentric 5° zones).

have given better distributions in the central part of the solar disk.) The three frequency distributions of Figure 6 together contain every individual penumbral spot recorded in Greenwich from 1901 through 1913. The relative frequencies of the six curves in Figure 7 include the same observational material. In both figures we show the number of observations (n) used for the frequency determinations of each curve. The short heavy lines perpendicular to the axes of the abscissae facilitate comparison of the data in the two figures.

It is clear from what we said earlier about Figure 5 that the slopes on the left side of the histograms in Figure 6 have no real „solar'' meaning, if $0° < \Phi < 20°$. But we can draw important conclusions from the other parts ($\Phi \geqq 20°$) of these mean frequency curves.

A sharp break near $\Phi = 50°$ in the step curve of $P > 20$ spots is readily noticeable. There is a similar tendency in the $P \leq 10$ curve and this feature can be traced in spots of intermediate size also. The slopes of the histograms ($\Delta q/\Delta\Phi$) in the $25° < \Phi < 50°$ and the $50° < \Phi < 75°$ intervals may be approximated by straight lines. In Table 10 we show the results of linear approximations by the method of least squares. As a consequence, $(\Delta q/\Delta\Phi)_{I} < (\Delta q/\Delta\Phi)_{II}$

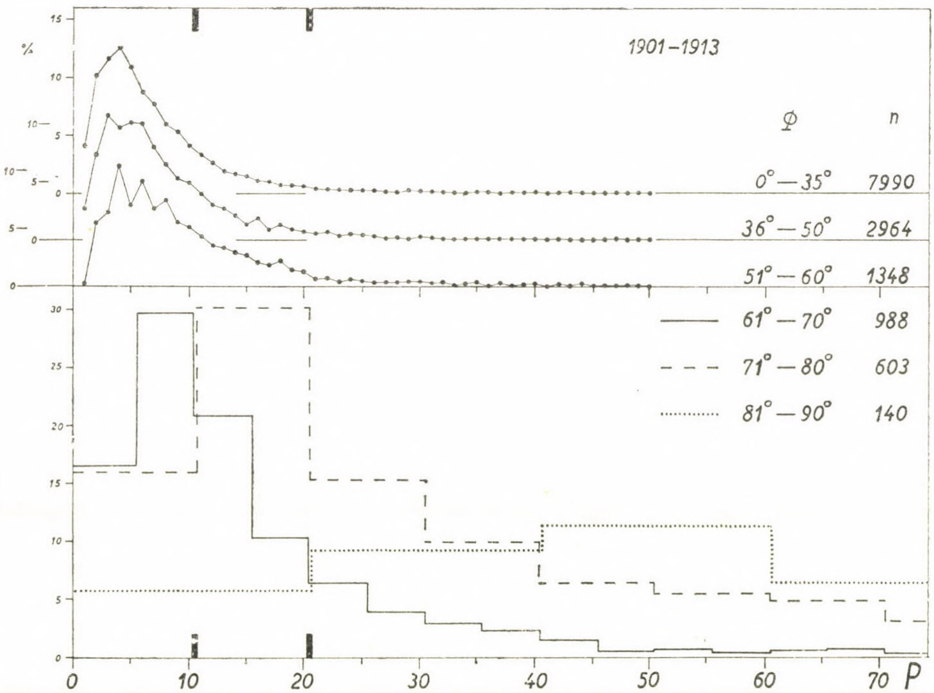

Fig. 7. *Frequency distributions (Q) of penumbral spots* by size in six zones of the solar disk.

TABLE 10.

Ratio ($\Delta q/\Delta \Phi$) of the relative frequencies of penumbral spots (q) with heliocentric angular distances (Φ) measured from the center of the solar disk and the mean error (1901—1913).

Φ \ P	$1 \leq P \leq 10$	$11 \leq P \leq 20$	$21 \leq P$
I. $25° < \Phi < 50°$	-1.51 ± 0.07	-0.53 ± 0.19	-0.41 ± 0.06
II. $50° < \Phi < 75°$	-0.74 ± 0.08	-0.49 ± 0.01	$+1.55 \pm 0.15$

for all size-categories of penumbral spots, where I and II are the intervals (given above) near the center and the limb of the solar disk respectively and the data relating to them.

It is quite evident that we see the effect of physical foreshortening in the $\Phi > 30°$ area of Figure 6. The quantitative differences between cases I and II are an essential manifestation of this effect (from a qualitative point of view). Even if the lines of case I which fit best represent truly penumbral spots, the frequency data above the $\Phi > 50°$ extrapolated trend lines cannot originate from such spots. There is no doubt that in reality these spots, with some unimportant exceptions, are all larger ones which have umbrae und appear as penumbral only due to observational conditions. Accordingly, speaking broadly, interval I shows penumbral foreshortening, while interval II mainly shows foreshortening of the umbrae. In the latter case the increase of penumbral frequencies leads to a decrease in the number of observable umbrae.

From Table 10 we can also see that $(\Delta q/\Delta \Phi)_k < (\Delta q/\Delta \Phi)_l$, where the subscripts k and l distinguish data relating to smaller and larger spots respectively ($P_k < P_l$). This inequality is apparently not valid when the penumbrae are too small; they must have a certain minimum size.

From the fact that the histograms of Figure 6 show a definite diminuation of frequency in interval I even for large spots ($P \geq 21$), it is apparent that when Φ increases we do not observe fewer penumbrae but smaller areas. A great many of the areas observed as $P < 21$ and $\Phi > 30°$ are really $P > 20$, if $\Phi < 30°$. Consequently, true $P = 1$ spots (those near the limits of measurement) are no longer observed at about $\Phi = 33°$ and greater distances from the center of the solar disk, even though they must be very numerous.

Figure 6 indicates that the sunspot area measurements are rather disappointing from at least one view point of physical reality, since it proves that *physical foreshortening makes about 2/3 of the area values somewhat ambiguous. The area is probably incorrect from $\Phi \sim 33°$ for penumbrae and $\Phi \sim 53°$ for umbrae*, and the uncertainty increases rapidly with Φ. Perhaps the value for the umbrae should have been 3° less and it is possible that more accurate data would result in a lower Φ value for the penumbrae also.[4]

It is necessary to consider the following decisive factors in forming an opinion of the true nature of the physical foreshortening effect: *1*) The foreshortening of the umbrae begins to be conspicuous at a considerably greater distance from the center of the solar disk than that of the penumbrae. There is at least $\Delta\Phi \sim 20°$ between the two. *2*) When a penumbra lying outside the interval $0° \leq \Phi \leq 30°$ rotates away from the earth by about $\Delta\Phi \sim 20°$ (in only 36 hours!), its area appears to decrease strikingly. A single example will convince us of this: the number of $P \leq 10$ penumbrae is halved between $\Phi \sim 28°$ to $\Phi \sim 48°$ (see Figure 6). *3*) Last but not least, it is well known from observation that the common boundary lines between the umbra and penumbra of spots and between the penumbra and the photosphere are generally clearly defined. They show a high contrast which is visible at $\Phi < 60°$ and is lost only near the limb of the solar disk.

With these three considerations, it is easy to understand that *the apparent foreshortening effect cannot be interpreted merely as an extinction process, in which Φ increases the decline in area. But we can explain it as an ,,occultation'' by a ring of non-transparent faculae surrounding the spot.*

In Figure 8 we indicate the most plausible model of a penumbral spot. (The arc depicted with a heavy line indicates the average level of the undisturbed photosphere, while the two hatched columns perpendicular to it represent a cross-section of the ring of faculae. In this example just half the penumbra can be observed. Three different possibilities for the penumbra are drafted with dotted lines; we can obviously omit the possibility that it is a conspicuous bulge.) It is clear that if the spot and facula have a common boundary, then whatever the depth and shape of the penumbra, its directly observable size (P_{pr}) depends only on Φ, on the height (F_H) of the facula and the angle of inclination between its inner wall and the photosphere. Supposing this angle to be 90°, we can at once estimate the average minimal F_H value of the $P=1$ spots. In this way we obtained $F_H \geq 1500\ km$ for the true $P=1$ spots, since an area observed as $P=1$ at $\Phi = 0°$ diminishes to $P \leq 0.5$ when $\Phi = 33°$. Accordingly *the height of the facula will be about 3/4 the diameter of the spot.*

[4] We need to make the following remarks about the ,,umbral'' spots -i. e., spots recorded as having no penumbra: in the observational material used in Section 7 there are only *17* such cases. $G_1 = 6$, $G_x = 8$, $G_u = 3$ is their distribution. Not a single umbral spot was recorded at Greenwich until 1913; from the beginning of 1916 to the end of 1943, there were nine years with no recorded cases (from scanning the 1874—1943 volumes of the *Greenwich Photo –Heliographic Results*). All other years showed at least one case and the most, eight, occurred in 1929 (only two of them fell into our used G cases). On the other hand, from July 1914 to October 1915 there are *81* umbral spots recorded! For this reason, we cannot draw conclusions about the umbral spots from the *Greenwich Photo – Heliographic Results*. Incidentally, although most of the *81* spots were observed at $15° < \Phi < 60°$, with a fairly smooth distribution in this interval, some have been recorded at $\Phi > 60°$ as well. We consider it indisputable that most of these *81* spots were really only darker penumbral spots; we are inclined to assume that this is true generally in all the cases of other umbral spots as well.

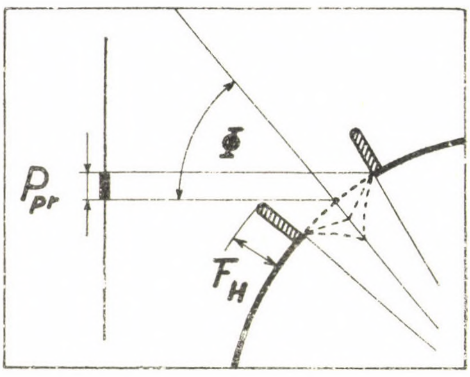

Fig. 8. *An explanation of the physical foreshortening effect.*

Since umbrae disappear at a greater Φ, our observations of them may be influenced by the non-transparent faculae as well. This can be explained qualitatively when both the umbra and the penumbra of a spot are at a lower level than the photosphere, assuming that the depressions are unbelievably deep (like those obtained by measurements of the Wilson effect). It is most likely that both reasons play a role for the umbrae.

The physical foreshortening effect becomes a geometrical one, if we assume that the ef-ective height of the non-transparent faculae varies somewhat with spot size. Perhaps eventually this effect may be taken into account in reduction of area measurements.[5]

From these consideration, it is obvious that only the upper curve of Figure 7 shows the frequency distribution of the area of penumbral spots with sufficient accuracy. The most important data of this distribution may be read from the $0° \leq \Phi \leq 35°$ interval of Figure 9. The other sections of the figure ($\Phi > 35°$) show all other objects which were observed as penumbral spots. (Figure 9 can be regarded partially as a combination of Figs. 6 and 7.) The thickness of the lines with two breaks in Figure 9 shows the error of the calculated mean and the dotted lines show the standard deviation. Disregarding the observations near the limb of the sun ($\Phi > 80°$), which are untrustworthy, the maxima, medians and means seem to follow a regular course. From each of these three distributional characteristics, we obtain almost the same rate of change with Φ, in the $55° < \Phi < 75°$ range, for the physical foreshortening. From this, it can be proved that *the choice of* $|\Delta U| = 10$ *as the lowest limit for daily change in area introduces no error due to physical foreshortening into our classifications of rapid variations,* even if $|L_{CM}| \sim 60°$.

Let us return to Figure 7. The decrease in relative frequencies (Q), as we go from the maxima along the curves toward the right in the direction of increase in *P*, obviously comes from the fact that there are more small spots than large ones. The slope to the left, if $\Phi > 35°$, results from the difficulty in observing spots due to physical foreshortening. But why does the curve of $0° \leq \Phi < 35°$ have a maximum where there is no physical fore-shortening effect? Often small spots, particularly $P < 4$, often pass undetected due to meteorological factors. If we add in addition that on the average

[5] Since we would like to study further both the foreshortening and this „height-depth" question we do not mention here some additional numerical data which still seems rather obscure.

the lifetimes of smaller spots are believed to be shorter than those of larger ones and that we have only a single solar observation for each day, it becomes obvious that many small spots are not included in the observational material. As a result the relative frequencies, at least of the $P<4$ spots, must be erroneous. We have no way of correcting these wrong values. We can only assume that the maximum of the upper curve of Figure 7 does not tell the full truth. It seems more probable that in reality the Q-curve has no maximum at all. Its course, in rough approximation, could possibly be shown by a curve such as that we would get from the observed curve by extrapolating the $P>4$ branch through the maximum at $P=4$ to the $P<4$ spots.

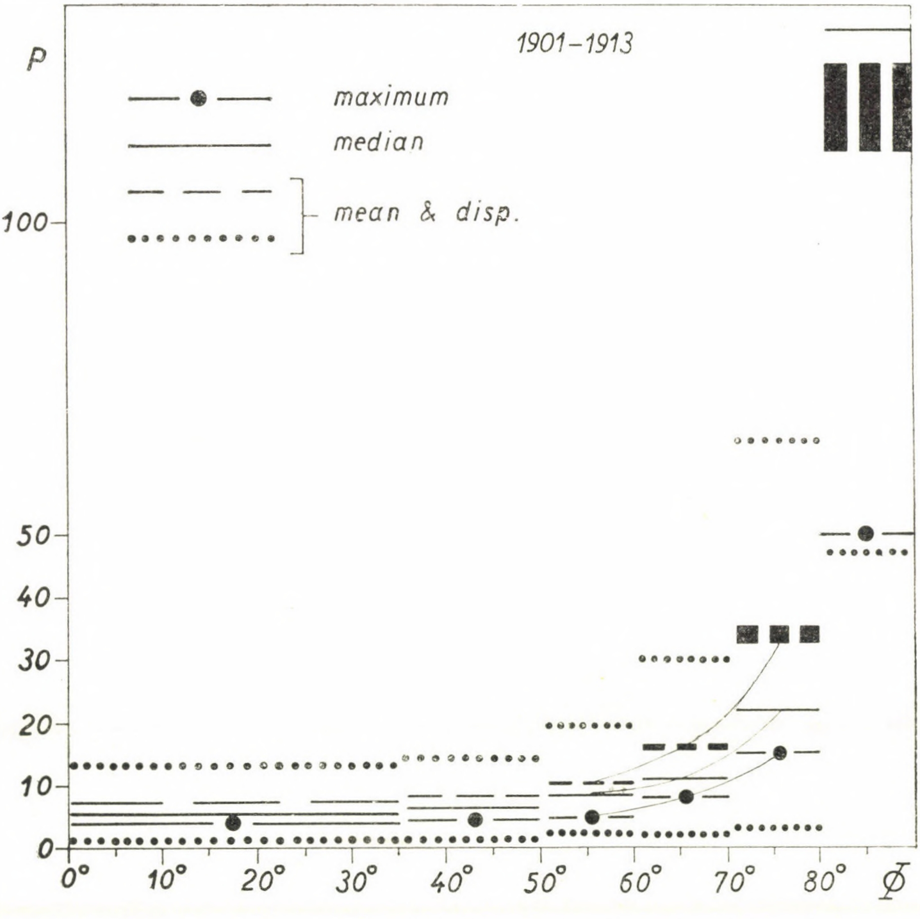

Fig. 9. *Distribution of penumbral spots over the solar disk* ($0° < \Phi < 90°$).

If we consider that this extrapolation is at least partially true, we come to the conclusion that, contrary to direct observation, small penumbral spots are more numerous than large ones -that is, that $n_k > n_l$ is true for all P values. The relative frequencies of Q decrease very rapidly with increase in area until about $P=10$. Even in Figure 7 the $P=16$ spots represent only 1% of all the observed ones. Further, we must keep in mind the following: In spite of the fact that no umbra is seen in some large spots, they must certainly have umbrae. Cases where the umbra is not observed are not due solely to larger Φ. From spots observed at the center of the sun's disk, it is well known that the umbra is sometimes quite asymmetric in relation to the penumbra. Last but not least we are referring to the range of the ratio of the penumbral-umbral areas in general.[6] Taking all things into consideration it stands to reason that, strictly speaking, *there is no such thing as a true penumbral spot. At best, there may be a short beginning and ultimate period in the life of a spot which may be called a true penumbral phase.* It is likely that in penumbral appearances there is a facula above the umbra which is just splitting or strongly shrunken blocking observation.[7] It is probable that even in medium size spots we do not see the umbra because of occulting faculae in the so-called „non-umbral" cases. At $U<1$, we can no longer measure the umbra.

Even if we assume the maximum of the Q-curve at $0° \leq \Phi \leq 35°$ to be real, the conclusions given above are still substantially valid; this maximum is at the $P \sim 4$ point, which makes it just compatible with P/U data.

§ 9. Is there a closer connection between spot group?

We tried to answer this question in the following way: Is it mere chance or not that the total $(U+P)$ area of two or more groups often reaches absolute maximum on the same day?

Again we used Greenwich observations of spot groups which were seen at least on two days in the years 1922—1934. For each Carrington rotation, we counted the occurrences of $U+P$ absolute maxima, in which the maximum was observed within our limit of $|L_{CM}| < 60°$. Each recurrent group was represented by a single maximum and if it was observed at $|L_{CM}| \geq 60°$ we eliminated the group altogether.

According to Section 7, it would make more sense to use the U and not the $U+P$ maxima. But, as we have seen, 2/3 of the U and $U+P$ absolute

[6] *These Publications*, No. 2 (in the press).

[7] Is the simple view presented here acceptable or not? This question, as well as that concerning connections between a spot and nearby faculae, may be cleared up only by an extensive series of continuous observations made probably in violet light of objects not far from the center of the solar disk (at $\Phi < 35°$). We hope to enlarge our observational program at *Debrecen* for this purpose.

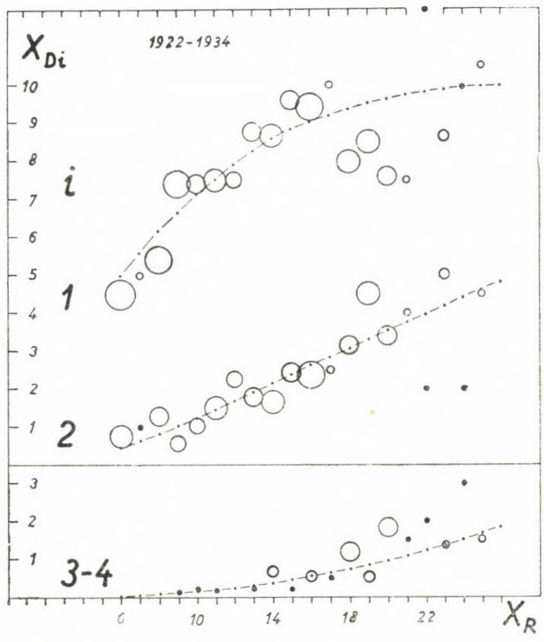

Fig. 10. *Frequency of the maxima of spot groups occuring daily (X_{Di}) in different number (i) versus the number of maxima (X_R)*

maxima[8] fall on the same day and the time-lag between umbra and penumbra presumably ran only to a few hours in general.

Let X_R be the number of maxima observed during a rotation as defined above; X_{Di} is the number of days on which such i maxima occurred during the rotation. That is, $X_R = \Sigma\, i \,.\, X_{Di}$. Our period of 13 years contains 174 complete rotations (the beginning of the earliest and the end of the last fall just within 1922 and 1934 respectively). Omitting 84 rotations where $X_R < 6$, we had 90 rotations containing altogether *1241* maxima. The related X_R and X_{Di} values are represented by circles of different size in Figure 10. Their diameter is in direct proportion to the number of used X_{Di} values. The smallest black circles indicate a single case, while the largest circle includes eight rotations. Since $i = 4$ in only seven rotations, with a single occurrence in each rotation, we put them together with the $i = 3$ cases. $i > 4$ was not found at all.

Assuming that there is no connection between spot groups, we would expect to find the X_R maxima distributed at random over the 27 days of a solar rotation. Accordingly we can say that the probability is

$$\left(\frac{1}{27}\right)^{i}\left(1-\frac{1}{27}\right)^{X_R-i}$$

that i-maxima fall to one of the 27 days, but there is no $X_R - i$ maximum on the same day. Since we would like to know how often we may expect i-maxima

[8] We confess that dealing with the $U+P$ maxima was very convenient and that is why we made use of them here. The epochs of these values were already plotted on the special synoptic solar maps which we had drawn originally for a work in progress on statistical investigations of solar promicences.

to occur, we must multiply this expression by 27 and the number of sets of X_R maxima which contain different i-maxima. We obtain for this value:

$$X_{Di} = 27 \cdot \frac{X_R!}{i!\,(X_R - i)!} \left(\frac{1}{27}\right)^i \cdot \left(1 - \frac{1}{27}\right)^{X_R - i}$$

The three dotted curves of Figure 10 were plotted by this formula. The bottom part of the figure shows the $X_{D3} + X_{D4}$ sum. In 22 of our 90 rotations there were 28 days of observation, not 27, but this is not important (making separate calculations, we found that the ordinates of the abscissae $\sim$15—25 of the $i = 1$ curve are somewhat altered in these 22 rotations; on the scale of the figure, they rise by about 1—2 mm).

The observed and calculated data of Figure 10 agree fairly well if $X_R < 18$. For rotations which had more numerous maxima, it seems as if the number of days with two-four maxima is a little higher than it ought to be. But generally a larger X_R means that there are not only a larger number of spot groups, but that there are more lying near each other, especially since we considered only a third of the solar surface. Under these circumstances, it is possible that a group was sometimes accidentally divided into more than one. And since the different parts of a group tend in general to be in the same evolutional phase, as we pointed out in Section 7, these deviations from random distribution could be the result of inaccuracy in grouping the spots. The strength of the connecting between two groups depends on their distance, but this does not show up in the above.

It appears even from the brief discussion above that *the highest stage of spot development of groups does not show any strong time-correlation.* As a result, it is our guess that local factors play the main role in the evolution of a spot group. *The spot phenomenon and its development should be considered in first approximation as a spatial rather then a temporal phenomenon.*

§ 10. Postscript and acknowledgements

There are obviously more conclusions to be drawn from such studies of sunspot observations; there is still more to be said about the observations included in this paper. But at present we are trying only to point out and to insist upon the most obvious properties of sunspots (this is true of our next articles, which essentially are a continuation of this one). We will temporarily postpone a critical comparison of these works with other results concerning the same subject or closely related to it.

For the time being, we certainly do not regard these sunspot investigations as final. We are continuing to study the spot phenomenon by evolutional phase wherever possible both on the basis of our own observations and of published foreign material (at the moment, mainly still that from Greenwich).

Our statistical investigations of sunspots involved a great deal of work, especially in arranging the raw material into categories, etc. Since all of this was done without any special instrumental facilities except ordinary calculating machines, particular acknowledgement should be given to individuals who participated in the program. We will mention those making significant contributions to this point, although the bulk of the material prepared is far from being fully utilized.

All the members of our observatory staff shared in the work of preparing sunspot statistics. Mr. O. GERLEI participated with great efficiency in every phase of the investigations. In the beginning, calculator J. MERSITS and (technical) observer L. NAGY and, later on, Miss E. HORVÁTH compiled most of the necessary special catalogs and made the reductions and the first calculations from them. Dr. I. GUMAN is making preparations to extend these investigations to include several solar cycles. Mr. V. SIPOS and Mr. I. DUCHNOVSZKY also contributed to the results.

Most of the graphs used were plotted by volunteer collaborators. First Miss BERNADETTE LOVAS and Mr. J. MAGYAR, and later on GYÖRGYI BADI, ILONA HUNYADI, B. KÁLMÁN, I. KÁNYA, ÁGNES KOVÁCS and I. TÖRÖK (all of the latter were students from the KOSSUTH University of Debrecen) assisted us in obtaining important data.

Finally we should like to take the opportunity of expressing our gratitude to Prof. S. SZALAY, Director of the Institute of Nuclear Research of the Hungarian Academy of Sciences in Debrecen for offering us the use of the facilities of his institute to support our observing program.

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A Connection Between the Motion and Evolution of Bipolar Pairs of Sunspots

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(Publ. Debrecen Obs., Volume 1, PAGES 55—108)

PUBLICATIONS OF DEBRECEN HELIOPHYSICAL OBSERVATORY
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RELATIVE SIZE OF THE PENUMBRAL AND UMBRAL AREAS OF SUNSPOTS

by

L. DEZSŐ and O. GERLEI

Summary: Using Greenwich observational material covering an entire solar cycle and our own classification of evolutional phases in sunspot development, we have investigated the ratio of penumbral-umbral areas and its variations. Most attention was given to spot groups. From our results, it seems that the average penumbral-umbral quotients most of the time decreases continually and steadily during the solar cycle no. 16. The penumbral/umbral values of a spot or group depend primarily on the evolutional phase and, consequently, may change considerably even in a short period of time. The quotients are usually somewhat higher for the larger areas. Most of the time the daily change in umbral area goes in the opposite direction than the penumbra-umbra ratio. In other words, changes in size in the penumbra are usually "backward" (lag) in these periods compared to the umbra. In some well-defined cases, it is possible to use the one-day change in the penumbral-umbral area quotient to predict the immediate future (or evaluate the past) development of a spot group. The penumbral-umbral value proved to be a significant parameter of sunspot evolution and may eventually aid in predicting spot and group development.

Л. ДЕЖЁ и О. ГЕРЛЕИ:

ОБ ОТНОСИТЕЛЬНЫХ РАЗМЕРАХ ПЛОЩАДЕЙ ПОЛУТЕНИ И ТЕНИ В СОЛНЕЧНЫХ ПЯТНАХ

Резюме: На основании Гринвичских данных, охватывающих полный солнечный цикл, мы исследовали отношение площади полутени к площади тени в солнечных пятнах и изменения этого отношения, главным образом, с помощью введенных нами ранее фаз эволюции. Прежде всего были исследованы группы пятен. Получилось, что по видимому отношение полутени к тени непрерывно уменьшается в течение цикла № 16. Величина отношения полутени к тени сильно зависит от фазы развития и, следовательно, может заметно изменяться даже за короткое время. Это отношение обычно больше для больших пятен. Часто знак суточного изменения отношения площадей полутени к тени противоположен знаку соответствующего изменения площади тени. Это означает, что в такие моменты развитие полутени опаздывает по сравнению с развитием тени. В некоторых четко выраженных случаях суточные изменения этого отношения позволяют определить вероятную тенденцию развития группы на короткий срок вперед или эпигнозировать предшествующие значения. Во всяком случае, отношение площадей полутени к тени является важным параметром солнечных пятен, и, возможно, он будет даже полезным для прогностических целей.

DEZSŐ LORÁNT és GERLEI OTTÓ:

A PENUMBRA ÉS UMBRA NAPFOLT TERÜLETEK VISZONYLAGOS NAGYSÁGÁRÓL

Összefoglalás: Egy teljes napciklust felölelő greenwichi megfigyelési anyagon nagy részletességgel tanulmányoztuk a penumbra és umbra napfolt területek arányát és ennek változásait az általunk bevezetett umbra fejlődési fázisok segítségével. Legfőbbképpen foltcsoportokat vizsgáltunk. Eredményeink szerint nagyban és egészben véve úgy látszott, hogy a penumbra per umbra hányadosok átlagértéke az 1923-as foltminimumtól a következőig általában csökkent. Ezen sajátosság egyébként a fejlődési megkülönböztetések nélkül is megmutatkozott. A penumbra per umbra értékek legerősebben a foltok fejlődési állapotától függenek és ezért már „rövid" idő alatt nagymértékben megváltozhatnak. Bizonyos határokon belül ezek a hányadosok általában nagyobbak a nagyobb területek esetében. Leggyakrabban a kétféle foltterület arányának egy napi megváltozása az umbra terület változásával ellentétes értelmű. Ez arra vall, hogy a penumbra fejlődése ilyenkor „elmarad" az umbráéhoz képest. A penumbra és umbra területek hányadosainak egy napi változásai meghatározott esetekben felismerhetővé teszik a foltcsoport „közeli" jövőjének (és múltjának) valószínű fejlődési irányát. A penumbra per umbra érték mindenesetre a napfolt jelenség igen fontos paraméterének bizonyult, amely esetleg még prognosztikai célokra is felhasználható lesz.

§ 1. Preliminary numerical data

Let U_{jk} be the value of the entire umbral area of a spot group (k) observed among the n_j groups on the solar disk on day j. $\bar{U}_j$ and U_{dj} will be respectively the mean and the sum of these n_j total areas; that is, $n_j\bar{U}_j = \sum_k U_{jk} = U_{dj}$. This means that U_{dj} is the so-called "daily umbral area" (U_d) of the day j. Considering the observational data of d_n successive days, for which there was only a single daily observation of the solar disk, $n = \sum_j n_j$, $d_n\bar{n}_d = n$ and $d_n\bar{U}_d = \sum_j U_{dj}$, which means that, during the period defined by d_n, n is the total number of observations of sunspot groups, $\bar{n}_d$ is the daily average number of groups and $\bar{U}_d$ is the mean of the daily umbral areas.

The single suffix i may also be used to denote the n "individual" observations. For penumbral data, we use the letter P, and all averages of P_i, U_i and P_i/U_i for d_n days will be called $\bar{P}$, $\bar{U}$ and $\overline{(P/U)}$. For example: $n\bar{U} = \sum_i U_i$. The summations in this paper are always from 1 to n_j, d_n and n, for k, j and i respectively.

By replacing $\overline{(P/U)}$ with the roughly approximated quotients $\bar{P}_d/\bar{U}_d$, which, for convenience, will be written as $[P/U]_d$, we obtain a weighted mean in place of the averages of P_{jk}/U_{jk} and P_{dj}/U_{dj} for one-day and d_n-days. This mean is attained by using

$$\frac{\bar{n}_d U_{jk}}{\sum_k U_{jk}} \quad \text{and} \quad \frac{d_n U_{dj}}{\sum_j U_{dj}}$$

respectively as weights. In this way:

$$\overline{\left(\frac{P}{U}\right)} = \frac{1}{n}\sum_i \frac{P_i}{U_i} = \frac{1}{n}\sum_j \sum_k \frac{P_{jk}}{U_{jk}} \approx \frac{\bar{n}_d}{n}\sum_j \frac{P_{dj}}{U_{dj}} \approx \frac{\bar{P}_d}{\bar{U}_d} \equiv \overline{\left[\frac{P}{U}\right]}_d$$

Such $\overline{[P/U]}_d$ values obviously will be closer to the $\overline{(P/U)}$ means, as the weights approach unity. At any event, we can replace $\overline{(P/U)}$ with $\overline{[P/U]}_d$ whenever the dispersion of the distribution of weights around unity becomes low enough. A sufficient condition for this could be when, within a d_n period, the differences are only slight in each of the three sets of values: the size of groups on a single day, the daily umbral area and the daily number of groups. In other words, if the variations of U_{jk} with k and of U_{dj} and n_j with j are not too large we may use $\overline{[P/U]}_d$ instead of $\overline{(P/U)}$. It is evident that, if we want to use such "incorrect" means, the above-stated conditions must be regarded as not only sufficient, but necessary.

The $\overline{[P/U]}_d$ approximate averages are never applicable — strictly speaking — to our problem, because the U_{jk} areas differ widely from one another on any single day. Even if the spot activity is high and, therefore, both the U_{dj} and the n_j values remain nearly constant for weeks, the disadvantageous effect of the U_{jk} numbers is still felt. Actually, however, in observational material for times when there are numerous groups on the disk, the influence of U_{jk} probably does not affect our rough approximation. Presumably *we can use the* $\overline{[P/U]}_d$ *values* in cases of high spot activity, at least *for general information.*

It is easy to see that our second weight helps to smooth our data, while the first may at times have an impact on the results which is hard to estimate, especially since n_j appears in the denominator as the range of the index k. When spot activity is low or, in particular, if only n_j varies rapidly with time (as an illustration, we refer to the steep slopes during two intervals of about two years each of the $\frac{1}{2}(G_1+G_u)$ curves in Fig.3 of the first paper of this series [1]), $\overline{[P/U]}_d$ does not furnish us with useful information either about some of the years near the end of the 11-year sunspot cycle or, mainly, about its early years. We can improve the reliability of these incorrect means somewhat by choosing a sufficiently large d_n, however.

*

In order to obtain some preliminary numerical data, we shall now consider Figures 1 and 2 and Table 1, always keeping in mind the considerations given above. In this Section we have used Greenwich daily areas (U_d and U_d+P_d) exclusively. The observations are for a 56-year interval through 1949.

The $\overline{[P/U]}_d$ values are shown in the upper half of Figure 1, while in the lower, the $\bar{U}_d$ means for 1902—1944 are given for comparison. Each dot represents three consecutive Carrington rotations, so that d_n in general $=3\times 27$. Nevertheless, U_d and, in most cases, $\overline{[P/U]}_d$ are given for each rotation; in this way moving averages are presented. Points of $\overline{[P/U]}_d$ are omitted only when there was a value of $\bar{U}_d<10$ on the x-axis for observations of an entire rotation.

From a brief glance at Figure 1, without noticing the last spot cycle presented there, one might think that the $\overline{(P/U)}$ values vary in a direction parallel to the spot activity. The solar cycle before the earliest one shown in Figure 1 — i.e., before 1900 — shows the same characteristic. However, after 1938 this parallel appearance disappears completely. Comparing the upper and lower parts of Figure 1 in more details we see that, as the $\bar{U}_d$ curve becomes greater, the $\overline{[P/U]}_d$ curve becomes correspondingly smaller in amplitude. This makes it appear that the amplitude of the P/U mean becomes less as the spot maximum becomes higher. We should note, however, this is not true, as data of the spot cycles (from 1944) show.

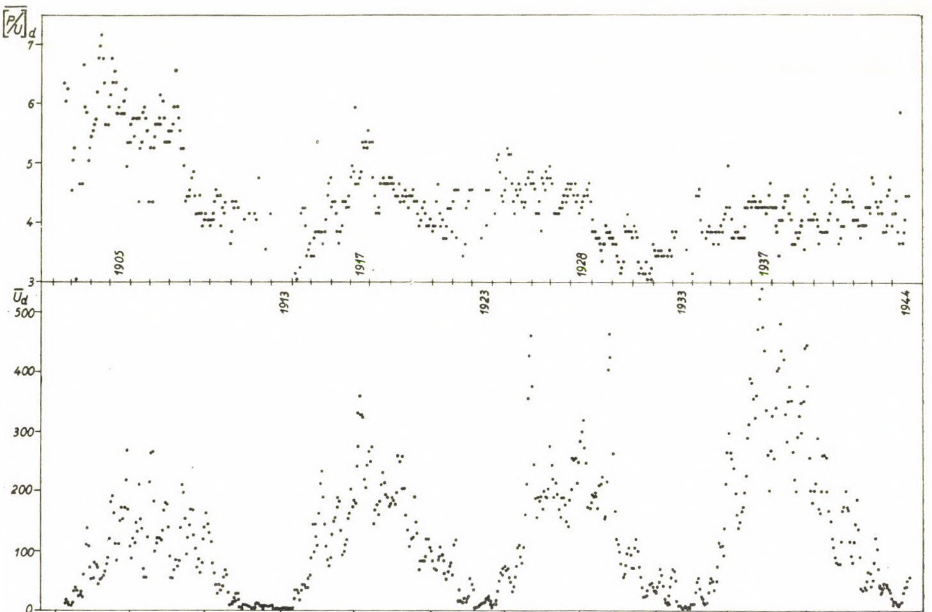

Fig. 1. *Approximate average values of the penumbral-umbral area ratio of sunspot groups,* $\overline{[P/U]}_d$, and the means of the daily umbral areas, $\bar{U}_d$

It is quite clear from our introductory remarks explaining the derivation and meaning of the $\overline{[P/U]}_d$ points of Figure 1 that their distribution represents to some extent the true variation of the averages of P/U only in those intervals of the abscissae where the $\overline{U}_d$ points are more or less evenly scattered about a trend line parallel to the x-axis. This condition is roughly fulfilled around the years of spot maxima. In such areas we can recognize as an unmistakable characteristic that the $\overline{[P/U]}_d$ values, and probably the true averages of P/U as well, show a general decreasing tendency with time at least for some years. The fact that this tendency appears more conspicuously in Figure 1 at lower maxima of spot activity is due to the simple reason that the useful segment of the trend line is shorter when the maximum is more intense.

The data presented in the histograms of Figure 2 are calculated in the same way as those of the upper half of Figure 1, but d_n here represents a calendar year. Each step curve coveres a three-year interval; the first step on the left is the year of maximum sunspot activity, which was determined as being the year when the annual means of both $\overline{U}_d$ and $\overline{(U_d+P_d)}$ daily areas reached their maximum in the same year.

In Table 1 we show data calculated in a way similar to those of Figure 2 separately for the northern and southern hemispheres of the sun from 1922—1934. The decreasing tendency of $\overline{[P/U]}_d$ show no difference between the two hemispheres within this interval, which covered one complete spot cycle and a short part of two others. In Table 1, moving from left to right, we find for both hemispheres two monotonically decreasing sets of numbers

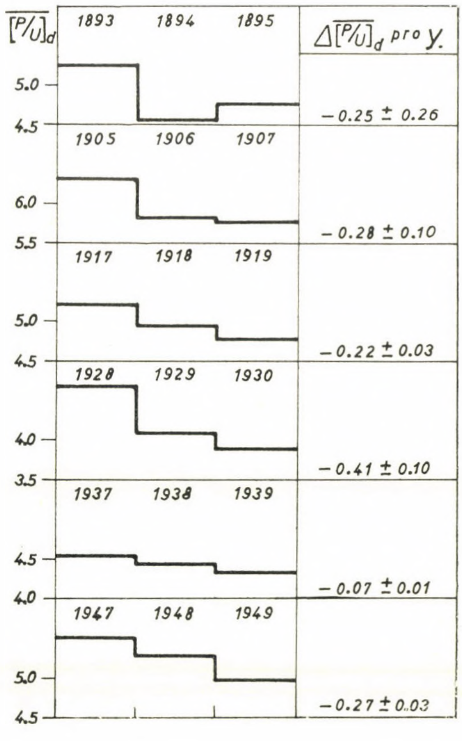

Fig. 2. *Approximate average values of the penumbral-umbral area ratio of sunspot groups* for years of maximum spot activity and further two-two years

covering three and six years respectively. In the intervals defined by these sets, we used linear approximation by the method of least squares to derive some data for the annual variation of $\overline{[P/U]}_d$; these are also given in the table, while similar values, obtained in the same way, have been incorporated into Figure 2. The average rate of change is —0.25±0.04 per year from data of Figure 2; it is valid for the year or so following maximum.

TABLE 1

Approximate average values of the penumbral-umbral area ratio of *sunspot* groups of the northern (N) and southern (S) hemispheres, $\overline{[P/U]}_d$, and their average yearly variation.

Years	1922	1924	1925	1926	1927	1928	1929	1930	1931	1932	1934
N	4.37	5.11	4.74	4.62	4.92	4.91	4.13	4.02	4.00	3.73	4.50
			−0.24 ±0.08				−0.25 ±0.05				
S	5.07	5.43	5.02	4.40	5.12	4.67	4.01	3.89	3.69	3.44	4.29
			−0.52 ±0.05				−0.28 ±0.06				

It is striking that in Figures 1 and 2 the approximate average values of P/U are not the same for the maxima of different solar cycles. For the time being it is questionable whether this is real characteristic.

In summary: *In spite of the fact that the statistical data used above is somewhat untrustworthy*, it is clear that *there is a good deal of evidence for a variation of the penumbral-umbral ratio of spot groups*. The means of the quotients of the areal ratio decrease over a great part of a solar cycle.

§ 2. The three main variation of the penumbral-umbral ratio

It was already obvious from the considerations of Section 1 that *the penumbral/umbral quotients may somehow be regarded as a characteristic parameter of sunspot activity;* even our first paper on evolutional distinctions of sunspots (§ 7 of [1]) suggested that the ratios of the two kinds of sunspot area should reveal interesting properties. It seemed plausible that studying this quotient by the same methods might be worthwhile. Since the umbra, rather than the penumbra, should be considered the basic area of a sopt, *we used the evolutional phases of the umbra to classify these areal ratios.*

In this Section (and in § 4), *we* will *deal with exactly the same Greenwich observational material* for spot groups and single spots, covering the years 1922—1934, *that we used in Section 7 of our first paper* [1] *and we will use the same notations and symbols in general.* We first of all determined the daily P/U value to two decimal places and then studied the means and distributions of the obtained data. We used only simple arithmetical averages; for the calculations [2] the class interval used was *0.5* and *1.0*, respectively for P/U (in this Section) and for the daily changes of P/U (in § 4). We omitted the $U=0$ (and also the few $P=0$) observations, because of some considerations (given in [1]), and the $P/U>12$ values, because we thought that they are no longer real figures. The total number of cases of the latter is less than two dozen out of the more than 11 000 observations of spot groups and spots which we studied.

The histograms in Figure 3 show P/U averages for spot groups (G). Throughout this article the different evolutional phases — development (*Asc*), decay (*Des*), absolute or relative maximum (*Max*) and minimum (*Min*) —, which are shown as different kinds of lines in Figure 3, are for the umbrae. The averages of P/U values for two of four successive years are given for the eight principal years of solar activity, while the three years around minimum are omitted due to insufficient data. The first four years of the eight precede sunspot maximum. The following characteristics of Figure 3 should be mentioned before we go any further: we have chosen six size categories of U for which we can obtain reliable averages. Thus, in the cases of $U\leq 60$, omitting the G(*Min*) observations for the other three evolutional phases there are $4\times4\times3=48$ size-year-evolutional categories, each for a mean value of two years. Of these 48 categories for $\overline{(P/U)}$, only seven were calculated from less than fifty individual P/U figures. Every four-year average was calculated from much more extensive observational materials. More than 90 daily P/U were used to determine the mean value in over half of the 72 steps in Fig.3. The $U>210$ observations for the eight years in question are not included, because there were only 124 of them and any data based on so few observations would be unreliable.

Figure 3 shows us very strikingly the three main variations of the penumbral-umbral area ratio. In the *first* place, the most obvious characteristic is *a strong dependence on the evolutional phase.* Regarding the means of P/U of the four main evolutional phases, one can see that generally for nearly all size categories of spot areas and at different phases of spot activity:

$$\overline{(P/U)}_{Max}<\overline{(P/U)}_{Asc}<\overline{(P/U)}_{Des}<\overline{(P/U)}_{Min}.$$

The only exception to this rule is probably in the case of small ($U\leq 10$) groups and only to the extent that the middle inequality is not valid. The *second* kind of variation is the one *outlined in Section 1.* From Figure 3, it seems that at least comparable averages of P/U are larger for some years before spot maximum than in the period immediately following it. The *third* feature which Figure 3 reveals is that *the average P/U value depends on the size of the spots* also.

We should add that single spots, broadly speaking, seem to show about the same properties as spot groups. This is true for the $U\leq 10$ observations also. Looking closely at Figure 3, we see that there is an apparent exception to the rule in the $U\leq 10$ case, e.i. $\overline{(P/U)}$ $>\overline{(P/U)}_{Des}$; however, this is probably because the G(*Des*) figures are too low. If this is true, we can explain the possible exception as being due to local hydromechanical factors, which frequently play a role in destroying a spot. One of us (L.D.) has already discussed these factors (in § 7 of [1]). In the case of small spot areas the immediate surroundings probably contribute more to decrease the penumbral areas than in general, because of the very weak magnetic fields in the penumbra. Consequently in the $U<10$ groups the penumbral areas should, on an average, be smaller relative to the umbrae during decrease than during their developmental period, exactly as is shown in Figure 3.

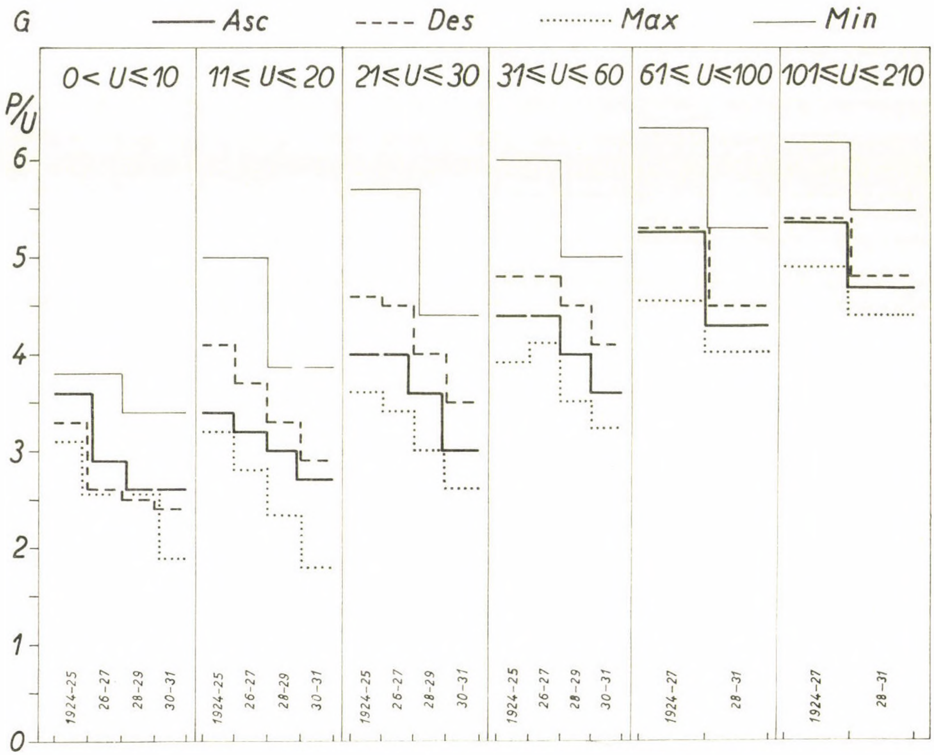

Fig. 3. *The averages of the daily penumbral-umbral area quotients of sunspot groups* for different sizes of umbra and evolutional phases, according to phase of the solar cycle

We should emphasize that the ensemble of the histograms in Figure 3 shows a regular and well-defined course of the step curves, inspite of the fact that a single plotted value may be based on a number of observations which is statistically very limited. Among the 72 levels shown, there are only eight based on more than 200 P/U values. Nevertheless, the mean error of each step is quite small; nearly 2/3 of them have an error of less than ± 0.1. But the standard deviations are quite large. It is striking that they decrease roughly from ± 2 to less than ± 1 if we proceed from the $U < 10$ areas towards the $U \sim 100$. For this reason, it seems evident that the penumbral-umbral ratio becomes more stables as the areas increase in size.

We would like to stress the following features of the obtained numerical values of P/U and their concrete means and mean variations. The value of $\overline{(P/U)}$ between the small and very large areas as a rule is at least doubled. The rate of change of $\overline{(P/U)}$ with U is fastest within the $10 < U < 30$ interval where the value for $\overline{(P/U)}$ ranges mainly from three to four. It seems in general as if the area quotient is tending toward a level of about five, while U goes on increasing over 100. But even from one day to the next an individual P/U can change considerably, even doubling, since the phase of spot evolution can alter considerably even during such a short interval. An example of alternation of a group between a state of maximum and minimum development for days was shown in Figure la of [1], and these cases for both groups and single spots are not at all rare. Consequently, *high dispersions of the P/U distributions are entirely natural.*

The results of some least square computations of the annual variations of $\overline{(P/U)}$ are given in Table 2. Each value represents a linear approximation from four data of developing or decaying spot groups. But regarding the observations of groups as a whole, (G), we should consider a somewhat higher figure for the general rate of change than the one (*0.16*) shown by the two equal mean values of the table. It can be seen from Figure 3 that, at least in the cases of $U \leq 60$, the curves of the G(*Max*) and G(*Min*) steps are slightly steeper than the G(*Asc*) and the G(*Des*) ones. (The numerical data from Section 1 also give evidence of a faster change.)

TABLE 2

Average yearly variation of means of the penumbral - umbral area quotient of developing (*Asc*) and decaying (*Des*) sunspot groups ($U \leq 100$) observed in the years 1924—1931.

U	*Asc*	*Des*
1— 10	−0.17 ±0.06	−0.14 ±0.05
11— 20	−0.12 ±0.02	−0.20 ±0.01
21— 30	−0.17 ±0.05	−0.18 ±0.04
31— 60	−0.14 ±0.03	−0.12 ±0.03
61—100	−0.20 ±0.07	−0.17 ±0.06
1—100	−0.16 ±0.01	−0.16 ±0.01

In Figure 4, some P/U distributions are given as typical examples. Each one of these six frequency polygons is based on 100—300 individual P/U figures from the year 1928. All the white and black circles represent moving averages of a smaller ($U \leq 20$) and larger ($21 \leq U \leq 60$) size category of spots respectively and every circle covers a unit segment of the x-axis. Two or three points were omitted at $P/U \geq 9$, but their values were used to determine the relative frequency. In Figure 4 the following results can be noted: the frequency maxima for both developing (*Asc*) and declining (*Des*) groups consisting of two or more spots (G_g) fall to the same abscissae

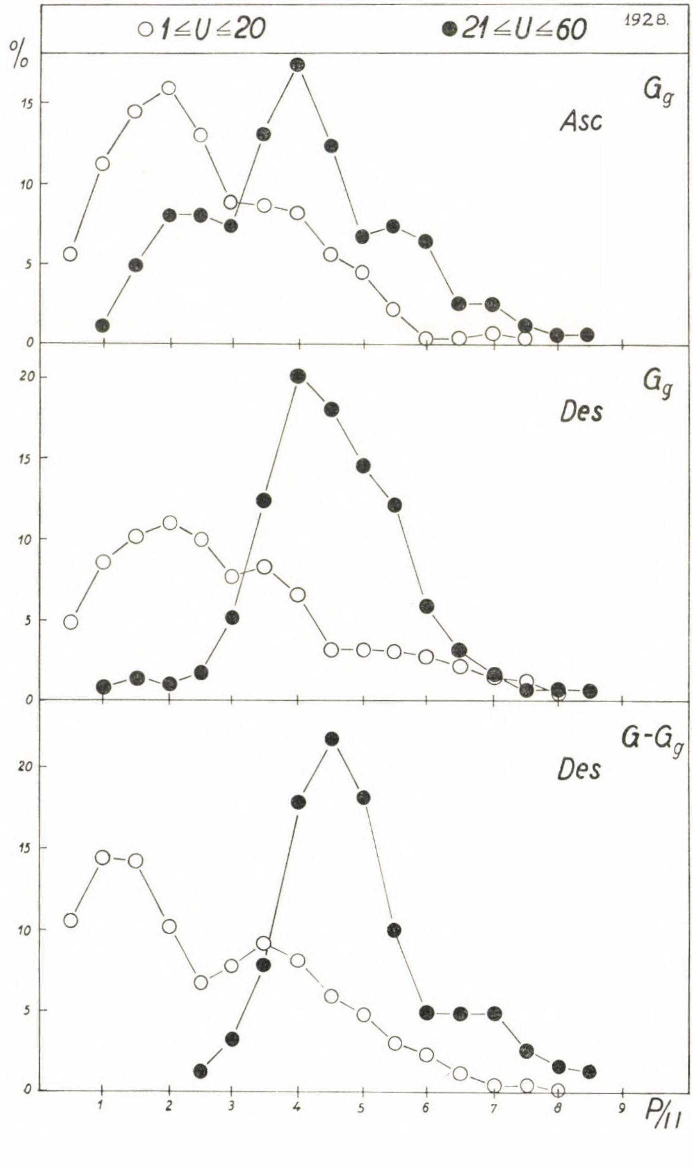

Fig. 4.
Examples of frequency distributions of daily penumbral/umbral values of different umbra sizes for groups consisting of a single spot (G—G_g) and more than one spot (G_g), observed in phases of development (*Asc*) or decay (*Des*)

(*2* and *4*), but the corresponding means for P/U do not. In comparing the groups of a single spot ($G—G_g$) with the others (G_g) in a state of decay, we observe that the maxima are slightly shifted. A similar comparison for groups in the developing state would also show some displacements of the maxima. On the whole, distributions of such a kind show somewhat smaller dispersions for single spots than for "true" groups of spots and, with the exception of the smallest spots, the maximal frequencies and means of these distributions, including those for the important p and f spots taken separately, have as arule at least slightly higher P/U value than that of the comparable groups. (These slight difference between groups and single spots of the same size category can be more easily understood if we consider that the groups consist of numerous spots that belong to a size category of small umbrae.) For small spots, we found a tendency toward lower numbers only in cases of decay; Figure 4 shows this clearly. But it is obvious that this exception is the same as that for $\overline{(P/U)}_{Asc} > \overline{(P/U)}_{Des}$. Since the lowest of the three curves of $1 \leq U \leq 20$ in Figure 4 has its maximum shifted to the left relative to the other two, which peak at the same P/U, we may evidently assume that the conspicuous exception $\overline{(P/U)}_{Asc} > \overline{(P/U)}_{Des}$ in the case of $U \leq 10$ is due to isolated spots after all. This supports further the explanation for this exception given above.

In Table 3, some particularly reliable P/U averages of developing groups and their mean error are given; they show convincingly that *the P/U values depend greatly on the rapidity of umbral i n c r e a s e*. We did not notice such marked deviations (δ) between the similar G(*Des*) and G(D—E—S) figures, which might be due to the secondary, but still important, diversity in nature between spot development and decay. (Compare § 4 and § 7 of [1]).

TABLE 3

The deviation (δ) between the means of the penumbral-umbral area quotients of rapidly developing (A—S—C) sunspot groups and all developing groups (*Asc*).

$\overline{(P/U)}$	$11 \leq U \leq 30$		$31 \leq U \lesssim 210$	
Years	1924—27	1928—31	1924—27	1928—31
G(*Asc*)	3.56 ± 0.11	3.14 ± 0.09	4.92 ± 0.08	4.17 ± 0.07
G(A—S—C)	2.72 ± 0.17	2.27 ± 0.17	4.54 ± 0.11	3.76 ± 0.11
δ	−0.84	−0.87	−0.38	−0.41

A proper statistical investigation of the true characteristics of the penumbral-umbral area quotient is very difficult because of the three main variations, especially since the range of all three is of the same order of magnitude. We have in general too few observations for each sample to calculate reliable means and other data; it is not possible to sort out all the necessary parameters for each different kind of change. The greatest difficulty in ascertaining the most typical and significant values for the penumbral umbral area ratio arises from the fact that this ratio for both groups and single spots depends on the instantaneous changes in evolutional phase. We know (from [1]) that these evolutional phases may vary rapidly, in consequence *an individual P/U value can change considerably even within a short time* — as much as 100% in a few hours. On the other hand, for exactly these reasons, it is probable that the quotients of the two kinds of sunspot area will, at least in practice, have an important role during future observations and investigations. *This variable quotient*, which has been almost entirely ignored to the present time, *should be recognized as an important parameter of the spot phenomenon.*

§ 3. Earlier investigations

The relative sizes of the penumbral and umbral areas of sunspots has been investigated very little up to the present time. Only the Institute of Theoretical Astrophysics of Oslo has published detailed investigations dealing expressly with this problem in papers by E. JENSEN, J. NORDÖ, T. S. RINGNES and E. TANDBERG-HANSSEN, [3]—[7]. They used data taken from Greenwich observational material from 1878—1954.

The fact that an average P/U varies during a solar cycle was noticed right at the beginning of our statistical investigations of sunspots [8]. But at first we saw only, what the Norwegian authors pointed out at the same time, that these values are larger at sunspot maximum than during the years around minimum. This formulation is in accordance with an investigation of A. W. F. EDWARDS [9]; Greenwich large spots ($U+P \geq 500$) have been used, observed between 1878 and 1954.

JENSEN, NORDÖ and RINGNES investigated the variations of some means of P/U, using nearly 4600 ($U+P > 50$) individual spot observations connected with 653 [3] and 845 [4], in Greenwich terminology, "regular" spots. In addition, they used smaller spots and U_d+P_d and U_d data. TANDBERG-HANSSEN [5] studied about 160 p—f spot pairs and dealt only causally with small spots. In some figures in these papers, there is some indication of P/U dependence on the size of the spot, as well as on spot activity in general. These authors more than once express the opinion that the averages of P/U on the whole are smaller for regular and "individual" spots than for the "composite" spots of Greenwich and groups of spots, respectively. We believe, as we mentioned in Section 2, that exactly the opposite is true. TANDBERG-HANSSEN, in this connection, found a slight difference between the p and f spots, which he attributed to the regular (p) and composite (f) character of these objects. Nevertheless, we cannot consider even this difference as firmly established. TANDBERG-HANSSEN gave some examples of larger daily changes in the P/U of a sunspot also. He regarded these variations as being related to a more or less periodic phenomenon, with periods of 1—5 days. Actually, they are only the concomitants of changes in the evolutional phase (see Section 2).

The P/U values of regular spots were also investigated by JENSEN and RINGNES in two further respects. They concluded [7] that the P/U depends on heliographic latitude much less than on the phase of spot activity; they also determined [6], in another connection, how the P/U quotient varies with spot size in zones concentric to the center of the solar disk. In the latter case, the foreshortening effect of the spots was actually studied by means of the P/U figures. These conclusions fit in with ours concerning the penumbral spots at least qualitatively (see § 8 of [1]) and may be explained in the same simple way.

Papers [3]—[7] dealt neither with the individual P/U values of the observations nor their means, but with a genarel type of "average", that we have used only in Section 1. Otherwise, in all the works cited except [9], [12] and ours, the penumbra-per-umbra value always refers to $(1+P/U)^{\frac{1}{2}}$.

Similar types of linear spot sizes had already been given earlier by M. WALDMEIER [10]. He also published a formula by which P/U decreased with increase in $U+P$. This formula is based only on 82 photographs of large $(50<U+P<620)$ round spots chosen from Zürich observations during four sunspot maxima. Among these 82 observations, probably the majority of the maximum evolutional phases occurred when the area was largest, namely the 82 data do not actually relate to 82 different spots, since some spots are represented by observations taken over several days. All these factors considered, our Figure 3 even at a glance shows why WALDMEIER's selected spots showed this peculiarity. WALDMEIER also presents a figure with A. WOLFER's micrometer measurements of spot diameters during 1882—1883 (see Abb.5, loc.cit., p.446). This observational material, which is much more extensive than the other Zürich material, as well as EDWARDS' investigation [9], gives no indication that the P/U of larger spots decreases with increasing spot size. The general rule is exactly the opposite as one of us has pointed out earlier [11]. The maxima of the frequency distributions of P/U values for different umbral sizes may be stated: maximum of $(P/U_k)<$maximum of (P/U_l) if $U_1 \leq U_k \leq U_2 \leq U_3 \leq \leq U_l \leq U_4$. In the same article [11] another tantative P/U characteristic, to be discussed in Section 4, was given.

As far as we know, except for the papers discussed above and one by S. B. NICHOLSON [12], there have been no other original data published on the ratio of the two kinds of sunspot area. NICHOLSON was the first to publish such data. He found the average proportion of umbral area to that of the entire spot (that is, $U+P$) to be 0.175 on the basis of Greenwich measurements of the important p spots of the years from 1917 to 1920. But he found no noticeable variation in the P/U values with the size of the spot. NICHOLSON's 0.175 corresponds to an average P/U value of *4.71*.

From our Figure 1 (or, rather, from the large-scale original drawing from which this figure was taken) we obtain a value of *4.85* for this quantity for spot groups occurring in the same four years. These two figures are in good agreement. We cannot draw any conclusions about the *0.14* difference, since NICHOLSON does not say how his mean was obtained.

§ 4. Daily variations of the penumbra-umbra ratio

We distinguished between the observations of one day, j, and the next, $j+1$, for each spot group or spot according to the notations and suffixes used previously: $\Delta U = U_{j+1} - U_j$, $\Delta P = P_{j+1} - P_j$ and $\Delta(P/U) = P_{j+1}/U_{j+1} - P_j/U_j$; that is, let ΔU, ΔP and $\Delta(P/U)$ be the daily variations. Further we can put $U_{j+1} = \nu U_j$ and $P_{j+1} = \mu P_j$. Therefore: $\Delta U = (\nu - 1)U_j$ and $\Delta(P/U) = (\mu/\nu - 1)P_j/U_j$. Then $\nu = \mu$ means that the umbral area changes in equal proportion to the penumbra. In these, and only in these cases, is $\Delta(P/U) = 0$. It may also be said that the two kinds of spot areas have an "equally strong" parallel evolution if $\nu = \mu$. We can say that the evolution of the penumbra is "stronger" than that of the umbra in the following cases: during development of the spot ($1 \leq \nu$) if $\nu < \mu$; during decay ($1 \geq \nu$) if $\nu > \mu$. (This will be called stronger penumbral evolution for short when there is no danger of confusion.) It is clear that, regarding the "spot area velocities" (which are defined by the temporal rate of change of the areas) and regarding the cases of $sg\Delta U = sg\Delta P$, the absolute value of the penumbral area velocity is higher than the umbral for a stronger penumbral evolution.

From these considerations, the following should be said: if $\Delta U > 0$, then we have a stronger penumbral development in the case of $\Delta(P/U) > 0$, while for $\Delta(P/U) < 0$ the penumbra is not developing more strongly than the umbra or is in a state of decay. If $\Delta U < 0$, then we have a stronger penumbral decay in the case of $\Delta(P/U) < 0$, while for $\Delta(P/U) > 0$ the penumbra does not decline more strongly than the umbra or is in a state of development. To sum up: *the penumbra is in a stronger evolution when and only when* $sg\Delta U = sg\Delta(P/U)$.

Let us denote by *As* and *De* a *one-day ascending and descending branch of the curve of areal variations* in the umbra. Each *As* or *De* is determined from two successive observations. We shall also use these symbols in a system analogous to that for the three-letter symbols introduced in Section 3 of [1] to characterize daily spot evolution in general. Accordingly, *As* means a $\Delta U > 0$ case, while we write *De* for $\Delta U < 0$. We may also speak of an *As* or *De* "transition" between two observations, in order to distinguish among the pairs of observations. When we write, for example, $G(As) = G_j(Asc) \rightarrow G_{j+1}(Max)$, or $(As) = (Asc) \rightarrow (Max)$, this only means that the *As* transition was such that an *Asc* observation was followed by *Max*. Obviously, in the case of G(*As*) the G_j can only be an *Asc* or *Min* observation; while for G_{j+1} there are also only two possibilities: *Max* or another *Asc*. Within both the *As* and *De* cases, only 4—4 different types of transitions may occur. There are *no Des* observations in the *As* transitions and the *De* cases have *no Asc* ones. Finally, it is useful to consider how many "extremes" are related to a transition. As an example of the transition of zero, one and two extremes, we can mention the following *De* cases respectively: $(Des) \to (Des)$, $(Max) \to (Des)$ and $(Max) \to (Min)$.

*

After these preliminary remarks, we can now consider the histograms of Figs. 5—7. All of these show frequency distributions of daily changes in the individual P/U values of the years 1922—1934 (as described in Section 2).

In Figure 5 all the very large $20 < |\Delta U| < 150$ changes are included without regard to the different transition possibilities observed in these 13 years. The lower part of the figure includes groups of more than one spot (G_g) and the upper part contains single spot (S″) observations. Most of these single spots are components of groups. (Compare Table 1 of [1].) Each G_g histogram was made from approximately 500 pairs of observations; while each histogram of S″ is based on almost 180 data.

The frequency distributions of Figures 6 and 7 are shown only for the $-4 \leq \Delta(P/U) \leq +4$ abscissa intervals, since all the maxima fall within $|\Delta(P/U)| < 2$; even the maxima of the curves of p and f spots determined the same way (which are not shown here) fall between the $\Delta(P/U) = \pm 2$ points. The few values at $|\Delta(P/U)| > 4$ are obviously included in the y-axes of the step curves of these figures. The calculated average of $\Delta(P/U)$ for every observational sample is indicated by the 5%-line perpendicular to the x-axis in each histogram. Rough data on the mean errors of these $\overline{\Delta(P/U)}$ averages follow. If the number of $\Delta(P/U)$ data was between 100 and 300, the numerical error was at most *0.15*. The error was less than *0.10* when the individual data exceeded 300 and less than *0.20* even there were fewer than 100 data. The Roman numerals in the figures, multiplied by 100, show the approximate number of $\Delta(P/U)$ values which were used for the calculations of means and frequencies. If there were less than 100 data available, there is no Roman numeral. The least number of $\Delta(P/U)$ which we used was 57 in a single case.

The $\Delta U > 0$ changes are shown in the four quadrants of Figure 6, while Figure 7 gives the $\Delta U < 0$ cases, separated by the 4—4 transitions for all available spot group observations (G) of the years 1922—1934. The data for each transition are separated into three size categories of ΔU. For example, the histogram on the left at the bottom of Figure 7 shows variations of $G_j(Max) \to G_{j+1}(Des)$ and $\Delta U \leq -9$.

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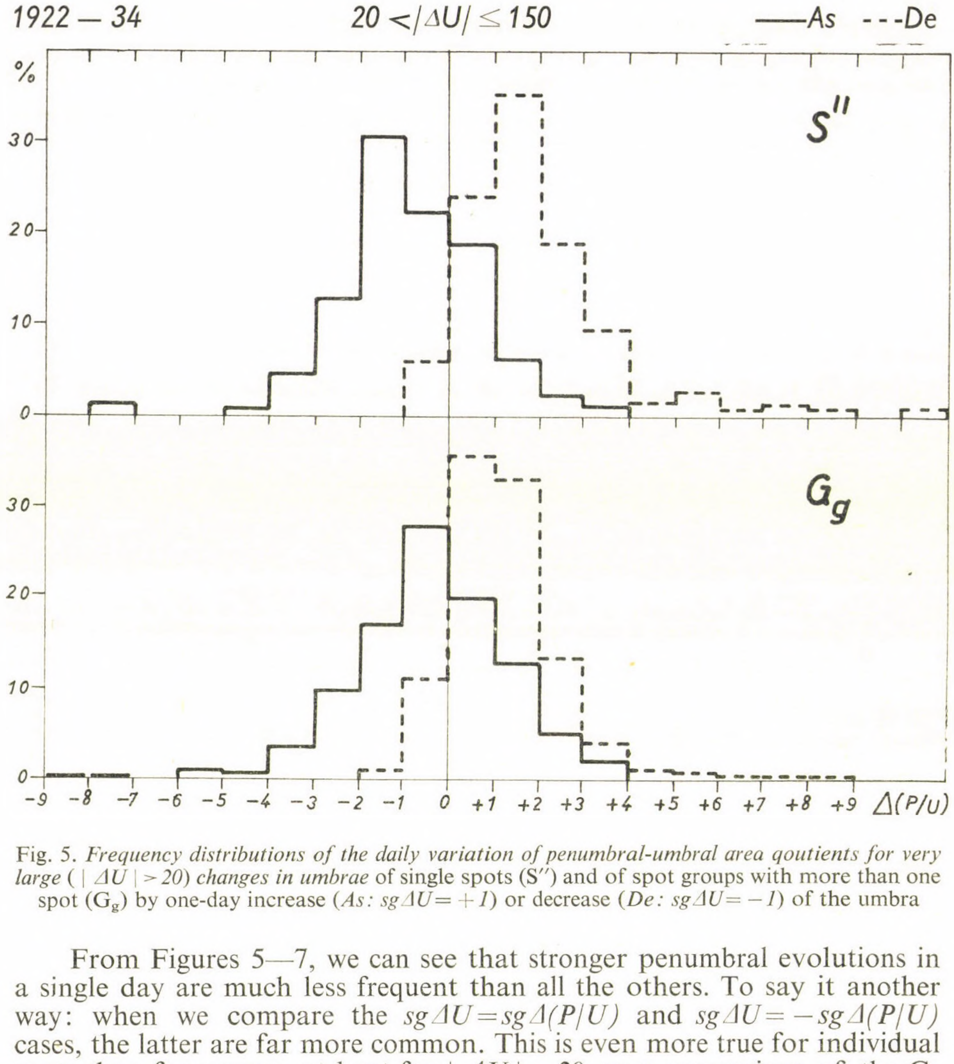

Fig. 5. *Frequency distributions of the daily variation of penumbral-umbral area qoutients for very large* ($|\Delta U| > 20$) *changes in umbrae* of single spots (S'') and of spot groups with more than one spot (G_g) by one-day increase (*As*: $sg\Delta U = +1$) or decrease (*De*: $sg\Delta U = -1$) of the umbra

From Figures 5—7, we can see that stronger penumbral evolutions in a single day are much less frequent than all the others. To say it another way: when we compare the $sg\Delta U = sg\Delta(P/U)$ and $sg\Delta U = -sg\Delta(P/U)$ cases, the latter are far more common. This is even more true for individual spots than for groups, at least for $|\Delta U| > 20$, as a comparison of the G_g curves of Figure 5 with the S'' curves shows. The $G_j(Des) \rightarrow G_{j+1}(Des)$ transitions for $|\Delta U| \leq 9$ and the $G_j\,(Asc) \rightarrow G_{j+1}\,(Asc)$ transitions seem to be exceptions to this rule, and in the latter case we detect a clear exception only if $\Delta U > 9$.

Accordingly, stronger penumbral evolutions are in the majority only in the case of transitions of zero extreme — that is, where the umbral area decreases or increases on at least three succeeding days. This rule does not work for decreasing groups of $\Delta U < -10$ (e.i. $|\Delta U| \geq 10$), these are obviously not the small areas, since the smallest G_j in the class of $10 \leq |\Delta U| < 20$ has an area of at least $U=11$, while among the $G_j(Des) \rightarrow G_{j+1}(Des)$ transitions of $\Delta U \leq -20$, each umbra of a G_j is over 20. The histogram of the latter case shows strikingly that there are only a relatively low number (about 20%) of stronger penumbral evolutions. In another formulation: the numerical value of the penumbral area velocity in those cases where the umbral development ($\Delta U > 0$) continues three days is greater than the umbral area velocity for more than half of the G cases and this holds for all the three classes of ΔU (e.i. even for $U > 20$ areas). This is not true of the analogous decays ($\Delta U < 0$), in general, since the larger ($U > 10$) groups of rapid decline ($\Delta U \leq -10$) make an exception. This is another attribute of the fact previously outlined (in § 7 of [1]) that development and decay of the sunspots are not simply and solely similar events going in the opposite direction. It can be recalled that earlier, both in Section 2 and in paper [1] we tried to explain some special characteristics of spot decay by assuming that significant hydromechanical processes are active in the surrounding area. Using the same assumption, we can more easily understand the difference between the $|\Delta U| \leq 9$ and the $|\Delta U| \geq 20$ cases of the $G_j(Des) \rightarrow$ $\rightarrow G_{j+1}(Des)$ transitions. In the latter case, we have larger penumbrae surrounded by strong magnetic fields, which effectively prevent mechanical conditions from influencing their decay to any significant degree.

From Figs. 6 and 7 it is apparent that most spot groups tend to show a stronger penumbral evolution, e.i., $sg\Delta U = sg\Delta(P/U)$, when the daily change in the umbra is smaller. Only those cases which have an umbral development lasting more than one day, i.e., the $G_j(Asc) \rightarrow G_{j+1}(Asc)$, and very possibly the $G_j(Asc) \rightarrow G_{j+1}(Max)$ and perhaps the $G_j(Min) \rightarrow$ $\rightarrow G_{j+1}(Asc)$ cases, are exceptions to this rule. This contrast also suggest that spot increase and decrease are not merely opposite processes, and Figure 5 further supports this idea. There is a conspicuous difference between the $sg\Delta U = sg\Delta(P/U)$ occurrences of *As* and *De* transitions; the proportion is approximately 4 to 1 for the S″ and 3 to 1 for the G_g observations. It should be noted that observational errors or other misleading factors play no significant role in the quantitative characteristics of Figure 5, since the areas and their changes are sufficiently large. The difference between the two proportions is not important; it may be a simple arithmetical consequence of the fact that each G_g transition originates from at least two S″ spots. The daily changes of P/U showed no substantial difference between the so-called important p and f spots as far as we can tell.

It should be emphasized once more that, apart from the few exceptions given above, *the general rule is that the cases of $sg\Delta U = -sg\Delta(P/U)$ have higher frequencies than the rest*, or, in other words, *most of the penumbral areas are n o t in a state of stronger evolution than the umbrae and may even be changing in the opposite direction.* But we also know that a state of rapid umbral development, A—S—C, or decay, D—E—S, tends to involve increase or decrease in the penumbra as well, this is true in nearly 100% of

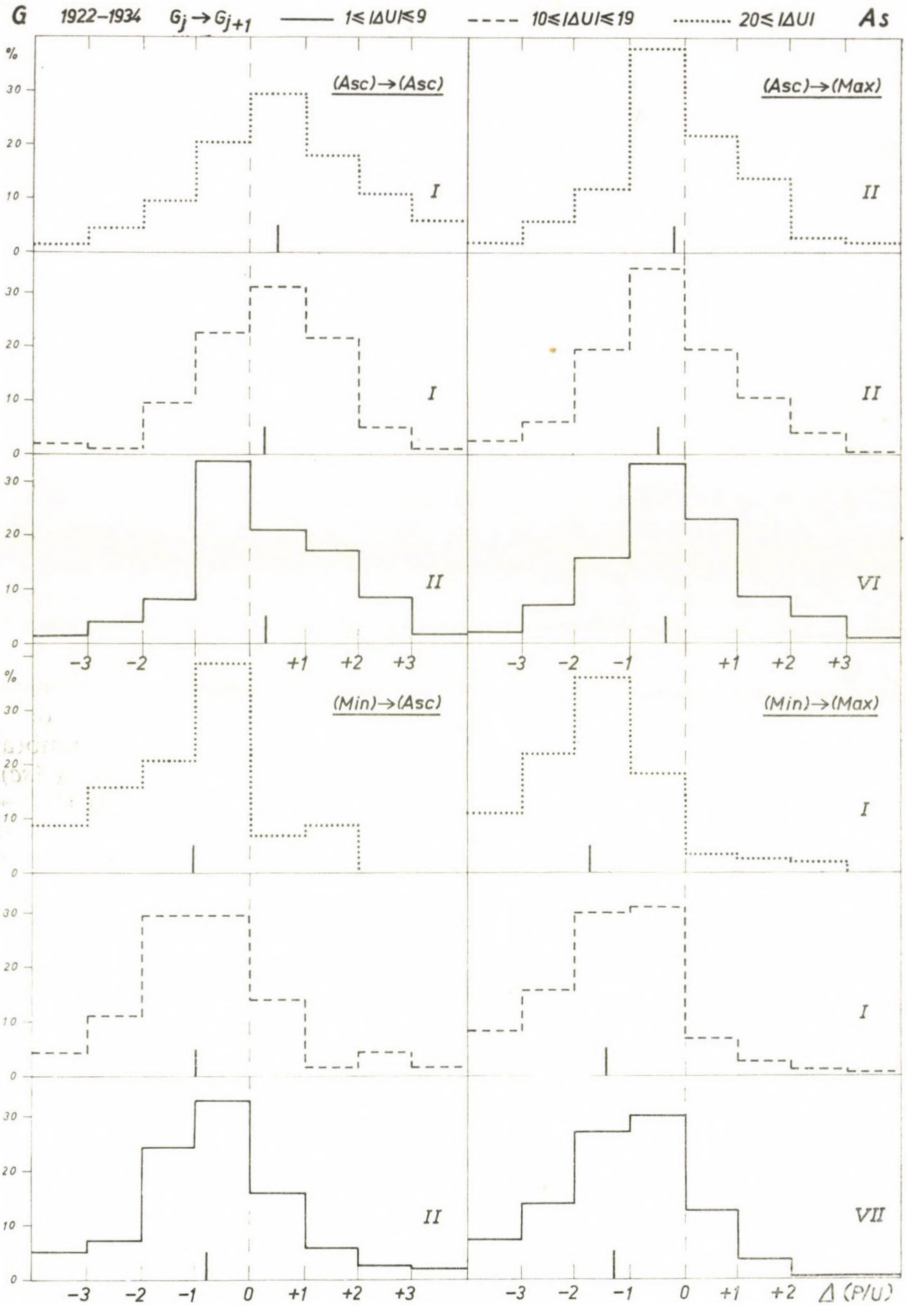

Fig. 6. *Frequency distributions for daily variation of the penumbral-umbral area quotients of spot groups* (G) *in which the umbra* *i n c r e a s e d* *from one day to the next*

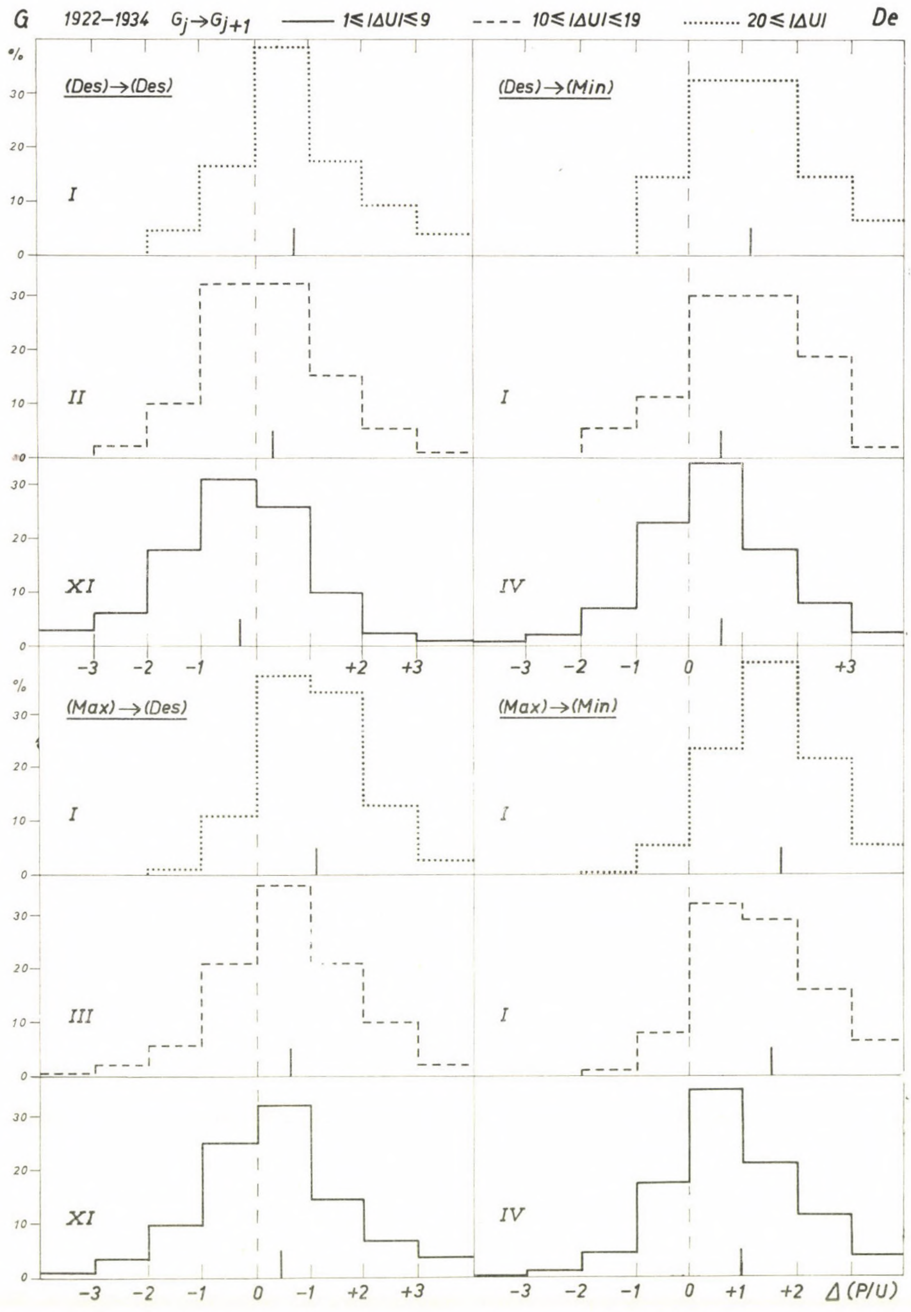

Fig. 7. *Frequency distributions for daily variation of the penumbral-umbral area quotients of spot groups* (G) *in which the umbra d e c r e a s e d from one day to the next*

developing areas and in 75% of declining ones (as may be seen from the first and fourth columns of numbers in Table 8 of [1]). Thus the $|\Delta U| > 20$ occurrences shown in Figure 5 must in practice all be cases where the umbra and penumbra changed in a parallel direction. So must the overwhelming majority of the $|\Delta U| \geq 10$ cases of Figures 6 and 7. Considering all these factors, we can finally formulate a general rule for the average area velocity of sunspots as follows:

For the daily changes of $|\Delta U| \geq 10$: $$\overline{\left|\frac{dU}{dt}\right|} > \overline{\left|\frac{dP}{dt}\right|}$$

and more generally for $\Delta U \gtrless 0$: $$\overline{\frac{dU}{dt}} \gtrless \overline{\frac{dP}{dt}}\,.$$

These three inequalities are valid both when they are averaged over all spot group observations (G) and also, with some exceptions which we have discussed, for averages of individual transitions.

*

We can obtain further important information by a more detailed examination of the histograms in Figures 6 and 7. Let us first notice the arrangement of the step curves in each figure. In every horizontal pair of quadrants, the G_j observations are for the same evolutional phase, while the phase of the G_{j+1} on the left is always different from that on the right. On the other hand, when we compare a vertical pair of quadrants, we see that the evolutional phases of the G_j observations are different and the G_{j+1} are the same. In other words, arranging the quadrants into sets by rows and columns, we can say that the immediate past of the G_{j+1} groups in a row was qualitatively the same, while the G_j groups in a column had the same future, in both cases, over a two-day interval. Studying the pairs of histograms of the same ΔU class, among the 12 histograms, in each figure, where either the G_j or the G_{j+1} observations are for identical phases of spot evolution, we detect generally a slight but systematic difference in all the two-two frequency distributions. Among these pairs of histograms of the same transitions somewhat more definite difference are shown for a class of greater $|\Delta U|$.

Since the relative frequencies represent empirical probabilities, in principle we should be able to predict spot evolution for some brief period of time. If, for example, on a j and $j+1$ day we have successive observations of sunspot group area and, in addition, we know whether on the $j-1$ day the umbra was larger or smaller than on the j day, then in many cases we have a possibility of predicting increase or decrease of the umbra for the $j+2$ day.

For even a rough prognosis of this type, the frequencies shown in histograms like those of Figures 6 and 7 are far from sufficient, we need to consider the distributions of data for histograms. In special cases, however, a single histogram pair from one of our figures may already furnish enough information for a forecast. For example, let us suppose that for a spot group we have from observations: $U_{j-1} < U_j$, $U_{j+1} > U_j + 20$ and $\Delta(P/U) \equiv$ $\equiv P_{j+1}/U_{j+1} - P_j/P_j \approx -0.5$. This is a $G_j(Asc)$, $\Delta U > 20$ case, which relates to the two uppermost curves of Figure 6. But an $(Asc) \rightarrow (Max)$ transition occurs about twice as often as an $(Asc) \rightarrow (Asc)$ one, which can be easily seen from the Roman numerals in these curves. Further, the relative frequency for a $\Delta(P/U) = -0.5$ value is again approximately twice as high in the first case as in the second. Our probability for a $G_{j+1}(Max)$ occurrence is thus four times (or, by more accurate calculation, three times) that for a $G_{j+1}(Asc)$. Under these conditions, we can expect a decay of this group — that is, $U_{j+1} > U_{j+2}$ will probably be observed the next day.

So *we do have some possibilities for forecasting the evolution of sunspots* with the aid of rough empirical probabilities similar to those in Figures 6 and 7. At the very least, it cannot be denied that *three successive spot area observations show the impact of both the near future and past.* It is to be hoped that eventually we will be able to estimate both in advance and retrospectively the evolution over short periods of sunspots and spot groups. (Retrospective analysis would help us in filling in gaps in available observational data). Since the P/U quotients may change considerably even during a single day, it would be very useful for forecasting to have several different observations for each day to work from.

For practical purposes, the P/U parameter alone will probably not suffice and other solar data will be needed for forecasting. First of all, magnetic measurements would help. The umbral area, as well as the penumbral, is undoubtedly a function of the field strength of sunspot magnetic fields (H). But umbra and penumbra are different formations and have different magnetic fields, so our P/U parameter is also a function of H. (Possibly the variation of P/U averages during a solar cycle bears a relation to variation in the overall solar magnetic field which is superimposed on the field of the spots.) Consequently, we should presumably be able to extrapolate the future of a sunspot, at least in principle, not only from $\Delta(P/U)$ but also from adequate ΔH data. Practically, the use of both indices together will perhaps yield results.

*

The possibility of forecasting sunspot evolution showed itself in Figures 6 and 7 clearly in the fact that the $sg\Delta U = sg\Delta(P/U)$ cases are different for every pair of histograms, vertical and horizontal, of the same ΔU class. The sums of the relative frequencies of these pairs are always lower for the twelve histograms on the right and the twelve at the bottom, that is, where transitions have more extremes. Accordingly, as the *As* and *De* cases of the same ΔU class go to more extreme transitions, we find that the relative frequencies of stronger penumbral evolution get lower. In other words, we can state it still more precisely by saying that, not counting the fact that the numbers of data for the different transitions vary, stronger penumbral development or decay has less chance of occurring as the umbra changes more strongly in the opposite sense. The probability of such an occurrence is considerably less for greater $|\Delta U|$, so much so that, in the case of $|\Delta U| \geq 10$, and transitions of two extremes, there are only an insignificant number of stronger penumbral evolutions.

It was found earlier [1] that the variations in penumbral evolutional phases mainly show a time-lag relative to the similar moments of umbral development. The evidence just mentioned above also supports this finding. Namely, when there is no stronger penumbral evolution, the penumbra "lags" in comparison with the umbra. As the evolutional direction alters and the magnitude of the umbral change increases, the penumbra has an increasing tendency to evolve in the opposite direction to the umbra. Indeed, some inequalities already lead us to suspect that the penumbra usually evolves this way [1]. The fact that the P/U values on an average are much lower for *Max* cases than for *Min*, which is clearly demostrated by Figure 3, proves also this. Certainly these values show that *the penumbra in general does not follow changes in the umbra in the same proportion and/or simultaneously*. It may be recalled that we did find that *the penumbra can have a stronger evolution frequently only when there is a monotonic umbral evolution over a long enough period.*

All things considered, we believe that the penumbra shows a kind of inertia relative to the umbra. It is as if the penumbra is influenced by causes altering the direction of sunspot evolution only with some retardation compared to the umbra. But we can also suppose that the umbra also is somewhat, although to a lesser degree, retarded. This assumption makes the possibility of forecasting sunspot development more easily understandable; the lag between umbra and penumbra may be a predictable phenomenon. At any rate, both kinds of sunspot areas should be considered as secondary manifestations of a more essential primary magnetic and hydrodynamic effect.

*

In conclusion, from Table 4 and Figure 6a we would like to call attention to a suspected further variation which has yet to be confirmed. $\overline{\Delta(P/U)}$ averages are given in Table 4 for the transitions in Figures 6 and 7 for two significant intervals of the sunspot period and two classes of $|\Delta U|$. In Figure 6a we show the same distributions and observational material for the $\Delta(P/U)$ of $G_i(Min) \rightarrow G_{i+1}(Max)$ transitions only. (The Roman numerals in Table 4 are defined like those in Figs. 6 and 7.)

According to Table 4 and Figure 6a, there is some evidence that even the daily changes of the penumbral per umbral area quotient depend on the phase of sunspot activity, although the effect is so small that we cannot decide for sure. In the table, the numerical values of $\overline{\Delta(P/U)}$ are smaller in 14 of the 16 pairs of data for the four-year interval before sunspot maximum.

While the figure shows a fairly uniform shift between frequency distribution to the two-year groups in both classes of ΔU. (The two upper histograms of Fig. 6a are based on 89 and 92 $\Delta(P/U)$ data respectively.) We could not pick out such systematic differences in similar pairs of histograms

TABLE 4

The means of daily variation of the penumbral-umbral area quotients of sunspot groups before the year of maximum sunspot activity (1924—1927) and in the next years (1928—1932)

$\overline{\Delta(P/U)}$	$G_j \rightarrow G_{j+1}$	$1 \leq \lvert \Delta U \rvert \leq 9$		$10 \leq \lvert \Delta U \rvert \leq 19$	
		1924—27	1928—32	1924—27	1928—32
As	$(Asc) \rightarrow (Asc)$	+0.28	+0.22	+0.19	+0.17
	$(Asc) \rightarrow (Max)$	−0.52 III	−0.33 III	−0.64	−0.38 I
$\Delta U > 0$	$(Min) \rightarrow (Asc)$	−0.78 I	−0.74 I	−0.79	−1.07
	$(Min) \rightarrow (Max)$	−1.43 III	−1.25 III	−1.51	−1.34
De	$(Des) \rightarrow (Des)$	−0.36 IV	−0.30 V	+0.38 I	+0.20 I
	$(Des) \rightarrow (Min)$	+0.55 I	+0.65 II	+1.46	+1.02
$\Delta U < 0$	$(Max) \rightarrow (Des)$	+0.36 IV	+0.29 V	+0.81 I	+0.59 II
	$(Max) \rightarrow (Min)$	+1.15 II	+0.79 II	+1.82	+1.08

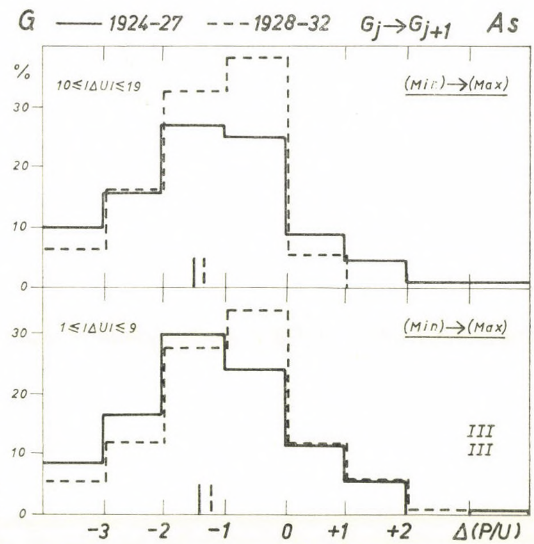

Fig. 6a. *Different distributions in two samples of observations* of Fig. 6, one for years before sunspot maximum (1924—1927) and one for the next years (1928—1932)

for other transitions. Of course, the small differences in the $\Delta(P/U)$ data of Table 4 are still less than the mean error, but this is true even for the two cases which do not follow the pattern. (The one showing greater deviation relates only to 31 and 37 cases respectively.) At any rate, observational material for a single sunspot cycle, including only a single observation per day, does not furnish a solid enough basis to permit further resolution.

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* We should take the opportunity to mention why we have not published anything in detail on this question until the present time. First of all, we have had some doubts about the validity of this assumption. Second, in recent years our solar team has left the Observatory of Budapest to set up a new solar observatory in Debrecen, so that our main concern has been setting up our instruments, etc., in order not to lose too many observations of the last extraordinarily high solar cycle.

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A CONNECTION BETWEEN THE MOTION AND EVOLUTION OF BIPOLAR PAIRS OF SUNSPOTS

by

L. DEZSŐ, O. GERLEI and V. SÍPOS

Summary: By determinig the relative velocities between the two important components of principal sunspot groups, based on Greenwich observations, extended over an entire solar cycle, and using our evolutional distinctions introduced previously, we found that these velocities depend on the state of spot evolution. The concerning average velocity of developing spots is definitely higher, in general, than that of the declinings.

Л. ДЕЖЁ, О. ГЕРЛЕИ и В. ШИПОШ:

О СВЯЗИ МЕЖДУ ДВИЖЕНИЕМ И РАЗВИТИЕМ БИПОЛЯРНЫХ ПАР СОЛНЕЧНЫХ ПЯТЕН

Резюме: На основании Гринвичских данных, охватывающих полный солнечный цикл, исследованы нами относительные скорости главных пятен важнейших групп солнечных пятен; при этом использованы введенные нами ранее фазы эволюции. Нами найдено, что эти скорости зависят от фазы развития пятен. Вышеуказанные скорости обычно являются безусловно большими у развивающихся пятен, чем у распадающихся.

DEZSŐ LORÁNT, GERLEI OTTÓ és SÍPOS VIKTOR:

ÖSSZEFÜGGÉS BIPOLÁRIS NAPFOLT-PÁROK MOZGÁSA ÉS FEJLŐDÉSE KÖZÖTT

Összefoglalás: Felhasználva egy teljes napciklust felölelő greenwichi mérési adatokat azt találtuk, hogy a legjelentősebb napfoltcsoportok két főfoltjának relatív sebessége a foltok fejlődési állapotától függ. Ha összehasonlítjuk a kifejlődési és visszafejlődési időközökre vonatkozó ezen sebesség adatokat azt láthatjuk, hogy ezek az első esetben határozottan nagyobbak, mint a másikban.

§ 1. Data used

In our search for possible links between motion and evolution of sunspots, we first studied the important p—f spot pairs of principal spot groups, using *Greenwich observations for the years 1922—1934*. The changes in the relative distance (r) of simultaneously observed p and f spots of a single group were determined by diagrams like those shown in Figure 1. We will use *the symbols and definitions* for various evolutional phases which *we introduced in the first two papers of this series*, [1]—[2].

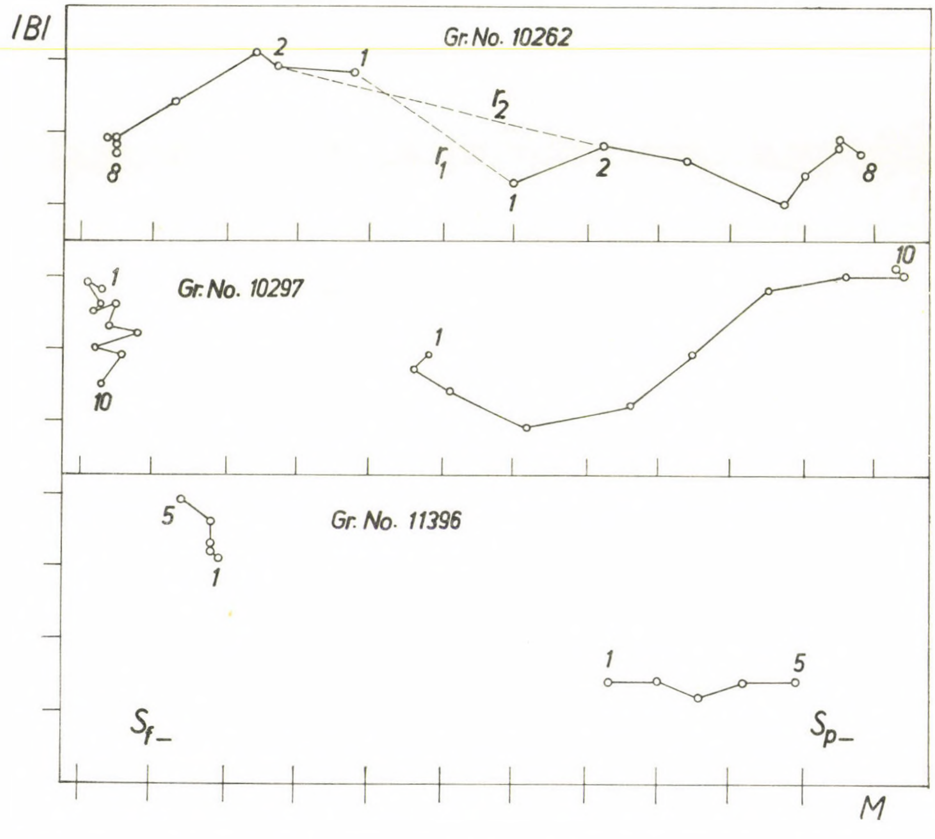

Fig. 1. *Three typical examples of diagrams which were used for graphic determinations of distances* (r_i) between simultaneously observed p and f spots of a sunspot group. Proper motions, ΔM, are plotted on the x-axis and changes in heliographic latitude, $|B|$, on the y-axis. The divisions correspond to 1° and the spot pair is plotted according to the measured distance. The Greenwich serial numbers (*Gr.No.*) are given for each group.

TABLE 1

The number of data included in the used observational material (1922—1934).

groups (g)	observations (S_{p-} and S_{f-})	distance difference of		
		$r_{j+1}-r_{j-1}$	$r_{j-1}-r_j$	
70	*521*×2	379	406	N
64	*477*×2	349	386	S

We calculated the approximate one- and two-day mean relative velocities of the p and f spots, using the formulae:

$$\frac{\Delta r}{\Delta t}=\frac{r_{j+1}-r_j}{\Delta t} \quad \text{and} \quad \frac{\Delta r}{\Delta t}=\frac{r_{j+1}-r_{j-1}}{\Delta t}$$

in which Δt was always taken for *24* and *48* hours respectively.

We dealt separately with sunspots of the northern (N) and southern (S) solar hemispheres and used only the evolutional phases of the umbrae. Table 1 gives information on the observational material utilized. To gain additional data, we used observations for the entire solar disk, rather than restricting ourselves to those within 60° heliographic longitude from the central meridian (as we did in [1] and [2]).

§ 2. Velocities of the two-day means

The averaged velocities are arranged in Table 2 by evolutional phases of simultaneously observed p and f spots of spot pairs. We calculated these velocities by the formula:

$$\overline{\left(\frac{dr}{dt}\right)}=\frac{1}{n}\sum\frac{\Delta r}{\Delta t}\,.$$

One can notice immediately from Table 2 that there must be a connection between the motion and evolution of the pjincipal members of spot groups.

Let us regard, for example, only the different evolutional phases of p or f spots — that is, the velocity values in the lower part or on the right side of our double-entry table. The speed of general divergence is higher when one area of a spot pair is increasing than when this area of a pair is decreasing. This is true for maxima and minima alike, but the differences in these cases are much smaller.

TABLE 2

Average velocities in km/sec of the divergent motions of p and f spots of bipolar sunspot pairs, based on two-day means. The figures in parantheses (here and in Tables 3 and 3a) show the number of velocity measurements (n) included in the average (1922—1934).

f \ p	*Asc*	*Des*	*Max*	*Min*		
Asc	0.16 (18) 0.10 (22)	0.01 (9) 0.06 (5)	0.06 (21) 0.08 (12)	0.10 (15) 0.06 (9)	0.09 (63) 0.09 (48)	N S
Des	0.05 (25) 0.04 (21)	0.03 (53) 0.05 (54)	0.03 (35) 0.06 (40)	0.03 (31) 0.03 (27)	0.03 (144) 0.04 (142)	N S
Max	0.08 (17) 0.10 (22)	0.04 (27) 0.04 (23)	0.07 (37) 0.07 (32)	0.05 (16) 0.07 (16)	0.06 (97) 0.07 (93)	N S
Min	0.05 (16) 0.10 (12)	0.03 (19) 0.06 (17)	0.06 (14) 0.06 (14)	0.03 (26) 0.05 (23)	0.04 (75) 0.06 (66)	N S
	0.08 (76) 0.08 (77)	0.03 (108) 0.05 (99)	0.06 (107) 0.07 (98)	0.05 (88) 0.05 (75)	0.05 (379) 0.06 (349)	N S

Taking the average velocities of the main diagonal, which are obviously he most significant, we find:

$$\overline{\left(\frac{dt}{dt}\right)}_{|Asc|} \gtrsim 2\overline{\left(\frac{dr}{dt}\right)}_{|Des|} \quad \text{and} \quad \overline{\left(\frac{dt}{dt}\right)}_{|Max|} > \overline{\left(\frac{dr}{dt}\right)}_{|Min|}$$

The mean errors of the velocity values of the left and right side of the first inequality are ± 0.03 and ± 0.01 respectively. The differences seem to be greater for the northern, than for the southern hemisphere.

§ 3. Velocities of the one-day means

In order to prove in a more satisfactory way that there is a real relationship between the direktion of area changes and the relative motion of p and f spots of a spot group, we have studied the *As* and *De* transitions. (See the definitions in § 4 of [2].)

We find from Table 3, in a manner similar as above:

$$\overline{\left(\frac{dr}{dt}\right)}_{|As|} \gtrsim 2\overline{\left(\frac{dr}{dt}\right)}_{|De|} .$$

Accordingly, there is good agreement with the conclusions drawn above.

It is not likely that these differences in average velocities is due to some coincidental circumstances. The indicated connection between the evolution of sunspots and their motions is supported by the fact that the difference in question becomes twice as great when we consider only those cases where the area variations are larger. This is shown in Table 3a, which was obtained by using all the daily changes in umbra from Table 3 which exceeded an area difference of 9. It is to be seen from Table 3a that:

$$\overline{\left(\frac{dr}{dt}\right)}_{/\mathrm{A-S}/} > 4\overline{\left(\frac{dr}{dt}\right)}_{/\mathrm{D-E}/} .$$

We distinguished here the large one-day increases and decreases, from the general *As* and *De* cases by A—S and D—E respectively. (This notation is analogous to that introduced in § 3 of [1].)

TABLE 3

Average velocities in km/sec of the divergent motions of p and f sunspots of bipolar pairs, based on one-day means (1922—1934).

f \ p	*As*	*De*	
As	+0.10 (99) +0.10 (92)	+0.06 (69) +0.05 (56)	N S
De	+0.05 (98) +0.07 (94)	+0.03 (140) +0.05 (144)	N S

TABLE 3A

Average velocities similar to those in Table 3 for the whole sun (N+S) using only the cases of large daily area changes ($|\Delta U| > 9$).

f \ p	A—S	D—E
A—S	+0.13 (59)	−0.03 (16)
D—E	+0.05 (26)	+0.03 (47)

There are other examples which show just as clearly that the relative motions of the p and f spots of a group depend on the direction of spot evolution. In Figure 2 the maxima of relative frequencies for the *As* transitions lie at higher velocities than those for the *De* ones. It is even more important to note that some data fall outside the figure on both the right and left sides, but only the extreme right side of the frequency polygon of each *As*, which is not shown here, contains more than 12% of the total occurrences, while in the other six cases it was no more than 1 or 2%. Further, considering the data for the upper left corner of Table 3a, approximately 1/3 of the 59 cases show a greater velocity than 0.16 km/sec. This is true separately for both hemispheres. Only four individual velocities in the lower right corner of Table 3a, all relating to the southern hemisphere, have higher values.

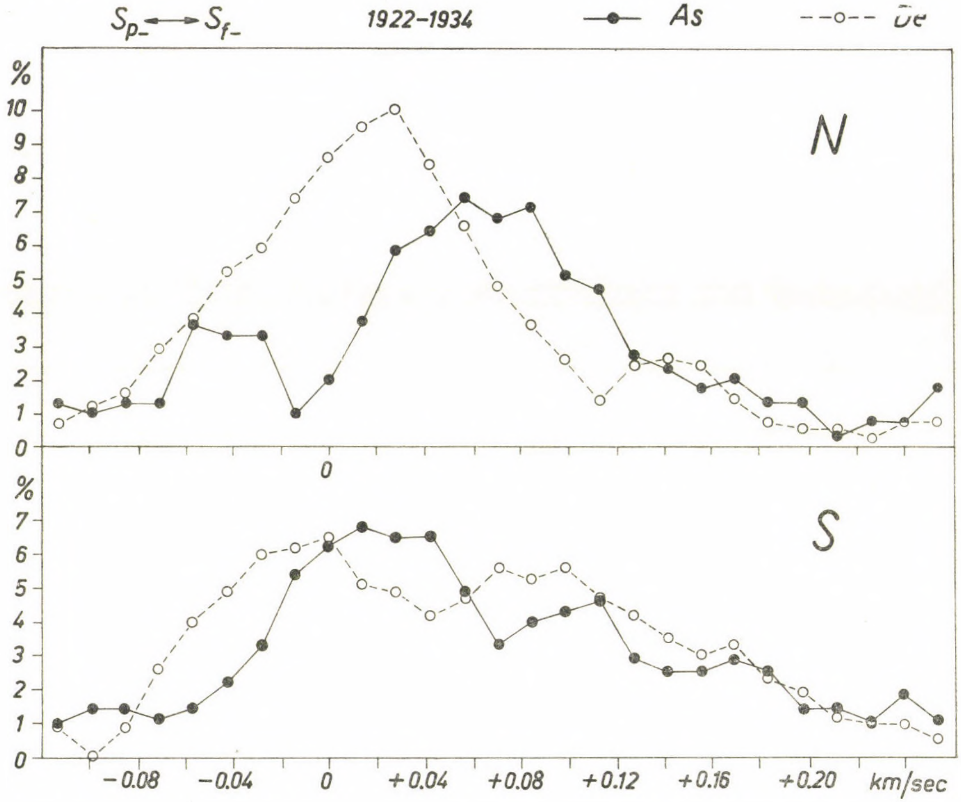

Fig. 2. *Frequency distribution of velocities of the relative motion of* p *and* f *spots of bipolar sunspot pairs, when the* umbrae of both the p and f spot are simultaneously increasing (*As*) or decreasing (*De*). The plus figures represent divergent motions. Each dot represents the average value over a velocity interval of approximately 0.04 km/sec.

The data of Figure 3 furnish an additional statistical consideration also: it is quite noticeable that all levels of the *As* cases for both hemispheres individually lie higher than the *De* ones.

Figure 3 shows another property as well. It indicates that average divergent velocities of the p and f spots of bipolar spot pairs are lower for larger sunspots than for small ones. We should remark, however, that four

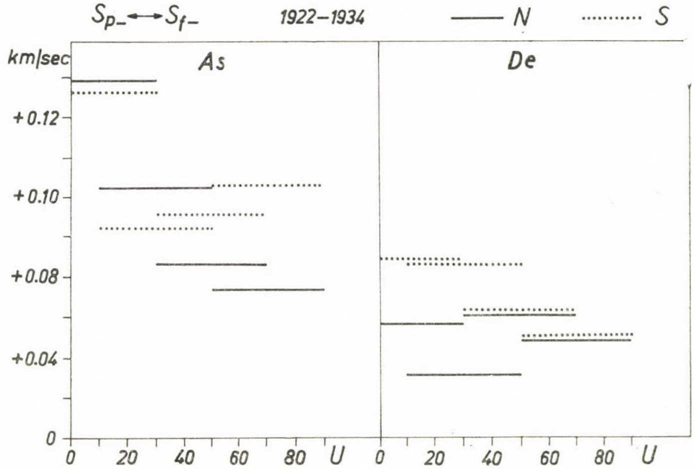

Fig. 3. *Average velocity levels of relative motions of* p *and* f *sunspots* of approximately equal size, plotted for different intervals of umbral (U) sizes, when the umbrae of both spots are simultaneously increasing (*As*) or decraesing (*De*).

of the 16 velocity values of Figure 3 are based only on 7—15 single data and the greatest number used anywhere in calculating the means was 54.

§ 4. Some conclusions

Since SCHEINER's time, it has been fairly well known from observations that (in today's terminology) a bipolar pair of spots forming a group shows a definite and large divergent motion, at least for some period after its birth, when the spot sizes are still small. Our conclusions show that the tendency to diverge continues during the later life of important p—f pairs also and the average velocity is about 0.06 km/sec. Table 2 shows 0.05 and 0.06 km/sec for the northern and southern hemispheres and a simple calculation based on Table 3 gives 0.06 and 0.07 respectively; these figures serve as evidence that the noticed motion is real.

From Section 2 and 3, there is a good indication that *the velocity of relative motion of a bipolar spot pair,*

$$\frac{dr}{dt}, \textit{ must somehow be a function of both } \frac{dU}{dt} \textit{ and } U,$$

and an important characteristic of this function seems to be that,

$$\text{if } \frac{dU}{dt} \gtrless 0 \text{ then the value of } \frac{dr}{dt} \text{ is } \begin{cases} \text{larger} \\ \text{smaller} \end{cases}$$

than when the umbra is changing in the reverse direction. These facts agree with the long-known high divergent velocities of p and f spots during their initial periods. It is quite plausible that spot areas increase more rapidly and for longer duration in the beginning than later in their lifetimes.

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NEW FACTS ON THE EAST-WEST ASYMMETRIES OF SUNSPOTS

by

L. DEZSŐ and O. GERLEI

Summary: Investigating distributional problems of sunspot groups and single sunspots over the solar disk, and using our previously introduced distinctions of evolutions and Greenwich observational material over an entire spot cycle, it was found that the asymmetric characteristics of distributions depend strongly on the phase of evolution. The developments reveal a definite preponderance on the eastern half of the sun's disk over that of the west. Declining spots show on the whole the opposite features. A great deal of evidence is given which altogether indicate fairly well that probably there exists only a single kind of east-west distributional asymmetry relating to sunspot phenomenon. At present we do not know any interpretation that could account even for an approximate explanation of the various facts observed.

Л. ДЕЖЁ и О. ГЕРЛЕИ:

НОВЫЕ ФАКТЫ О ВОСТОЧНО-ЗАПАДНОЙ АСИММЕТРИИ СОЛНЕЧНЫХ ПЯТЕН

Резюме: Нами исследовано распределение групп и одиночных солнечных пятен на солнечном диске и использованы при этом Гринвичские данные, охватывающие полный солнечный цикл и введенные нами ранее фазы эволюции. Нами найдено, что асимметрия распределения сильно зависит от фаз эволюции. Развитие преобладает на восточной половине солнечного диска, а распадающиеся пятна преобладают на западной половине. Даются нами многочисленные доказательства того, что вероятно, существует только единственная восточно-западная асимметрия относительно явлений солнечных пятем. В настоящее время авторы не могли даже приблизительно указать причины, вызывающие указанные факты наблюдения.

DEZSŐ LORÁNT és GERLEI OTTÓ:

ÚJ TÉNYEK A NAPFOLTOK KELET-NYUGAT ASZIMMETRIÁJÁRA VONATKOZÓLAG

Összefoglalás: Az általunk bevezetett fejlődési klasszifikációk segítségével tanulmányozva a napfoltok és napfoltcsoportok napkorongon való eloszlását — felhasználva egy teljes napciklus greenwichi észlelési adatait — azt találtuk, hogy az eloszlásokban mutatkozó aszimmetriák legfőbbképpen a foltfejlődési fázisoktól függenek. A fejlődések határozottan túlsúlyban vannak a napkorong keleti felén, míg a visszafejlődések nagy általánosságban éppen ellenkezőleg látszanak viselkedni. Sikerült sok észlelési tényt felsorakoztatnunk amelyek együttvéve nagy valószínűséggel arra utalnak, hogy (nem három fajta, hanem) csupán egyféle napfolt kelet-nyugat aszimmetria létezik. Jelen pillanatban azonban az észlelési tények elfogadhatóan még megközelítőleg sem értelmezhetők.

§ 1. Introduction

In 1907 A. S. D. MAUNDER [1] published some surprising results of her careful and detailed investigation of sunspot statistics. She pointed out that *the sunspot phenomenon shows an east-west asymmetry in respect to the solar central meridian.* Since then this fact has been confirmed by other authors and through sunspot observations from different sources and periods [2].

It should be emphasized that *other kinds of solar phenomena also show a similar type of asymmetry.* I. SYKORA [3], a decade before Mrs. MAUNDER, already observed such a feature in prominences. As far as we know, E. W. MAUNDER [4], A. ROMAÑA and J. N. TORROJA MENÉNDEZ [5], F. LINK and J. KLECZEK [6], J. S. HEY, S. J. PARSONS and J. W. PHILLIPS [7] and J. BOUSKA [8] were the first to discover east-west asymmetry in faculae, in calcium flocculi, in flares, in some emissions on radio wavelengths and in the optical region of the coronal spectrum respectively.

All of these solar phenomena tend to appear more on the east than on the west half of the solar disk. But since this excess on the east which has also been studied by many other authors, has generally been found to be quite small, its existence has often been disputed, expecially since some samples of observations do not show such asymmetric distributions at all. Nevertheless, the problem of such asymmetries should be considered seriously, as at least much of the data relating to them is undoubtedly real.

Up to the present time, *even the facts about these solar asymmetries are not well enough understood, much less their underlying causes.* Each manifestation of asymmetry was treated separately and interpreted in different ways. There are two main trends in the explanations: according to one, the east-west effect is purely apparent, while the other considers them the result of some external influences upon the sun. The latter idea has been considered by such authorities as Kr. BIRKELAND [9a], E. W. BROWN [9b] and A. SCHUSTER [9c], and many others, who based their studies chiefly on Mrs. MAUNDER's paper and the other numerous papers written since about these asymmetries. A critical study of all this extensive literature shows no definite results, either positive or negative; that is to say, we believe that, on the basis of observational evidence of these solar asymmetries, it is not yet possible to decide whether solar activity is ruled from outside the sun or not.

If we presume that the cause of asymmetries favoring the east side of the solar disk has its origin outside the sun and, consequently, is due at least partly to the same source which causes solar activity in general to vary in intensity, then some manifestations of these variations should somehow show up in the degree of east-west asymmetry also. At present, sunspot observations form the richest available material for investigation of the problems of solar asymmetry; our working method of evolutional classifications has proved to be a useful supplement in the study of sunspots, as our earlier papers in this series, [10a], [10b], [10c], show. We used our different categories and evolutional phases of spots and spot groups as defined in § 3 and § 5 of [10a] to test modifications of east-west asymmetries with the life-span of sunspot groups. Some of the results obtained so far are given in the following Sections; we have already published a brief summary of our result that there is, indeed, a significant difference in the asymmetry of developing and declining spots and spot groups, [10d]. Of course, the fact that developing spots and groups do show higher numbers on the east side of the disk does not verify the existence of an external influence on the sun.

On the whole, the Zürich classification of sunspot groups represents the average types of evolution. Considering the development and decline of sunspot groups by Zürich types, L. PAJDUSÁKOVÁ-MRKOSOVÁ [11] pointed out that the east-west asymetry reverses itself in these two periods of spot life. This result agrees with ours. Thus, since we know that the excess of spot groups on the east side of the disk is due principally to developing groups, it is easy to understand why flare observations, as well as solar radio wavelengths, show a similar characteristic. We think that probably all east-west asymmetries of distributions of different solar phenomena on the sun's disk have the same final cause, which is closely connected with sunspot group evolution, although only a thorough investigation will finally confirm this supposition.

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We can summarize briefly *the basic facts of the asymmetric east-west distribution of sunspots* as they have long been known. This distribution generally shows up in any sufficiently large sample of observational material and is usually classified into three different kinds (See [2]).

First, *observed sunspot activity is slightly higher on the eastern half of the solar disk;* this is true both for the numbers and the sum of the areas of sunspot groups.

The second kind of asymmetry, which is much more conspicuous than the first, relates to spot groups in their initial and final stages. *The positions of the groups at their first and last observations show different distributions; in the latter case, there is a larger number of groups on the western half of the disk*, as M. MINNAERT [12] and M. WALDMEIER and A. LIEPERT [13] have pointed out.

A. S. D. MAUNDER originally spoke of a third type of solar asymmetry on the disk, namely, that *more spot groups rotate onto the east limb than disappear on the west limb.* According to Mrs. MAUNDER, early observational material from Greenwich, covering an entire sunspot cycle, showed this property, which is past all understanding. In any event, W. GLEISSBERG [14] studied Mt. Wilson observations for the years 1934—1945 and could not find the slightest trace of such a distribution. We think that Mrs. MAUNDER's remark is *easy to interpret;* it is exactly these two contradictory findings which helped us to understand Mrs. *MAUNDER's third type of asymmetry.*

Let us imagine a bipolar spot group which originates somewhere on the western side of the sun's disk and has a lifetime of between 3/4 and one solar rotation. It develops to a larger group after its disappearance over the west limb and is already in the decaying stage when it reappears on the disk. Further, let us consider the tendency of p and f spots to diverge, which often leaves a group looking like two widely separated groups in its later lifetime. In this case, which evidently occurs rather frequently, a single group observed disappearing on the west limb of the sun may be counted as two groups when it reappears on the east two weeks later. Mrs. MAUNDER's third kind of asymmetry is apparently at least partly a result of such circumstances. GLEISSBERG's negative results, using more recent observations made at Mt. Wilson, in which there are certainly fewer errors in picking out true bipolar groups, due to classification by magnetic polarities, also suggest that this was the case. Consequently, we will ignore this 'observational' asymmetry in our study of the other types, which reveal some important and really puzzling phenomena. (Nevertheless, the considerations given above can serve as a control for the proper grouping of single spots into groups in some cases.)

§ 2. Some characteristics of the east-west distribution of sunspots

W*e studied the east-west asymmetry of sunspot distribution from Greenwich observational material for 1922—1934* which is described in detail in § 4 of [10a]. All observations utilized are restricted to within 60° heliographic longitude of the solar central meridian and the *symbols and notations* used here are those *defined in the first paper of this series* [10a]. Since in general our studies in the present paper are based only on umbral evolution, all classifications relate to the umbra, except where a P is used to specify the penumbra insted. Different statistical samples of the number (n) of daily observations of spot groups and single spots were determined for equal longitudinal zones lying symmetrical on either side of the central meridian. We calculated the *frequencies to the east,* $n_E/(n_E+n_W)$, in which n_E is the number of spot groups or single spots observed east of the central meridian, and n_W is the same number west of the meridian. Many of the frequencies for the eastern part of the solar disk are contained in Tables 1—8. (They are given to two decimals, omitting the decimal point and the zero digit, i.e., they are expressed in precentages.)

The reliability of the data of Tables 1—9, of course, depends on the number of observations on which they are based; the tables of [10a] show these numbers. On the whole, there were a sufficient observations to serve as a good basis for calculating frequencies and some data on areas. However, we could not find enough observational material for all cases in our present paper. For example, there was not enough data to establish the majority of the east frequencies of Table 6 reliably; hence, we included the n_E+n_W values in parantheses also.

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In order to characterize the east-west distributions quantitatively, we will in addition define the *excess on the east* as $(n_E-n_W)/(n_E+n_W)$. *Its numerical value*, which we will call the *asymmetry factor*, serves as a plausible measure of the degree of asymmetry.

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The primary data on the east-west distribution of sunspots are shown in Tables 1 and 2. They show unmistakably that the excess on the east *depends on the phase of sunspot evolution. It is greatest for well-defined, growing* (*Asc*) *umbrae; its value apparently refuses to be positive for similar cases of decrease* (*Des*). The figures in the last column of these two tables are those for asymmetry of all observed sunspots, the first kind of asymmetry spoken of above, which has been studied earlier by other investigators. Not every sample of this type of observation shows asymmetry at all — for example, the last column of Table 1 for 1923—1926 —, but the developing groups always show a significant excess on the east, even for these exceptions. Comparing the spot group observations classified by evolutional phase of Table 1 with those for single spots shown in Table 2, we find that the frequencies to the east are on the whole about equal for single spots and for groups; we should note that G. H. ARCHENHOLD [15] obtained a contradictory result* (quoted in [2c]).

An example may prove useful in explaining the significance of these frequencies arranged by evolutional phase. Thus, the number 60 in the upper left corner of Table 1 means the following: 60% of all spot groups observed during 1923—1925 on at least two days to be increasing in umbral area had a mean position in this two days on the eastern half of the solar disk within 60° of the central meridian. This makes the frequency for the eastern part of the disk 0.60 and the excess on the east, +0.20, or 60% and 20% respectively.

The distribution on the solar disk of the positions of first observations (G_1) of spot groups and the positions of last observations (G_u) both have a greater asymmetry than all other spot observations combined (G_x). Compare the data of Tables 1 and 3. It is easy to see that the G_x observations should have similar frequencies on the east to those in Table 1; the G_1 and G_u observations together (G_1+G_u) result only in a small negative frequency according to Table 3 and the number of G_1+G_u is considerably lower than G_x. Since we have seen that the developmental phase of sunspots shows a

* Note added in proof: ARCHENHOLD' s result,that the asymmetry is somewhat less for single spots than for spot groups, is in accordance with some of our recent conclusions.

significant positive eastern excess, which is not present during decrease, it is understandable that there is no such excess when the first and last observations are taken together, since these two periods of the life-history of groups always show development or decay, even when the observation actually shows a local maximum in umbral area.

TABLE 1

Relative frequencies of the numbers of sunspot groups observed east of the solar central meridian by years and groups of years, using our evolutional phases and, in the last column, when evolutional distinctions are omitted. Each frequency includes only spot group observations within 60° of the central meridian. (The frequencies are given to two decimal places, the decimal point and the zero digit next to it are omitted.)

Years \ $G-G_0$	*Asc*	*Des*	*Max*	*Min*	
1923—25	60	45	49	49	50
1926	55	43	53	50	50
1927	59	45	52	52	51
1928	60	49	51	53	52
1929	57	40	55	57	51
1930—33	63	46	51	52	52
1922—34	59	45	52	52	51

TABLE 2

East frequencies — that is, percentages — similar to those of Table 1 for single sunspots. (The data of Table 2a—8a also show similar east frequencies for different samples of spot observations.)

Years \ S''	*Asc*	*Des*	*Max*	*Min*	
1923—25	57	50	55	51	52
1926	59	45	57	54	53
1927	64	49	57	54	55
1928	60	50	54	51	53
1929	64	44	55	56	53
1930—33	57	47	54	48	51
1922—34	60	48	55	51	53

The heading $G-G_o$ in Table 1 shows that it does not contain the observations of zero areas (G_o), e.i., when both the umbra and penumbra of a group was not visible. The omitted zero observations also show a definite excess on the east. The frequency, which we obtained by counting all group disappearances lasting more than one day — that is, two or more consecutive G_o observations — as a single occurrence, turned out to be 0.53. This frequency agrees rather well with that for minimum phase as shown in Table 1; this is logical, since, according to our classifications [10a] every G_o observation represents a minimum in group area. This alone confirms the fact that the G_o observations do in general represent minima. The two figures of Table 3 support this view even better. They show for all $G_I + G_u$ observations on the eastern half of the disk a slight negative eastern excess of -0.03. It seems that if we consider each G_o-period of a group as a sign of the real end of the group and, at the same time, another new group being formed, we should expect no positive excess on the east. The positive excess of G_o observations on the east is inconsistent with this supposition. In short, this data furnish two additional examples for considering every intermittent spot group to be a single unit, as we maintained in [10a].

TABLE 2a

Two main samples of east frequencies of single sunspots for two size categories of umbrae, using the same observational material as that in the last row of Table 2. The figures at the bottom of the table show the approximate proportions of observations of small umbrae to larger ones.

S″ 1922—34	*U*	*Asc*	*Des*	*Max*	*Min*
$\frac{n_E}{n_E + n_W}$	*1— 50*	63	48	57	51
	51—100	60	44	52	55
n_{1-50}/n_{51-100}		*3*	*8*	*4*	*5*

TABLE 3

East frequencies for the first (*I*) and ultimate (*u*) observations of sunspot groups.

G_1			Years	G_u		
Asc	*Max*			*Des*	*Max*	
65	63	64	1922—27	36	29	34
60	63	61	1928—34	34	34	34
62	63	63	1922—34	35	32	34

The high figures of the absolute values for the eastern excess of G_l and G_u observations might make one think that this excess may come alone from the fact that the areas at the beginning and end of the lifetime of groups are generally quite small in comparison with the intermediate observations. Such a cause may be not only secondary but can have significance for the asymmetry factor. This factor, according to Table 2a, is higher for smaller spot areas than for larger ones only in the case of *Asc* and *Max* observations, while observations of the other evolutional phases (*Des* and *Min*) show exactly opposite behaviour. The main characteristic of east-west distribution — its strong dependence on evolution — holds for different size categories of group areas; Table 2a shows this property, too.

That the conditions of evolution are the dominant elements in the frequencies of groups may already be seen convincingly from Table 4. Eastern frequencies are given in Table 4 for cases in which there was a large daily change of umbral areas (ΔU) lasting over two days on the average and for all other observations. (A large daily change is defined by $|\Delta U| \geq 10$.) It is striking that both in developing and declining spot groups, the factor of asymmetry becomes greater with larger variations in area. This rule is valid for single spots as well as for groups, as we may guess from the example of the p spots. (The single exception to this rule is shown by numbers in parantheses, but there were only *48* observations in this category, as compared to well over *100* in the others, so it cannot be considered reliable.)

TABLE 4

East frequencies for sunspot groups and p spots divided according to rapid and generally slow evolutional patterns. All observations for the first and last days of group-life are omitted.

	A—S—C	$\overline{ASC}$	$\overline{DES}$	D—E—S	Years
G_x	57	54	47	35	1922—27
	62	60	50	33	1928—34
	60	57	48	34	1922—34
$S_p + S_{p-}$	59	53	38	(44)	

It is obvious that asymmetry should increase with distance from the central meridian, $|L_{CM}|$. Therefore we divided the solar disk into zones of heliographic longitude (L) lying symmetrically on either side of the central meridian, forming areas $\Delta L = 13.2°$ in width (the average daily synodic rotation of the sun), and counted the number of observations within each of the first four zones to the east and to the west of the central meridian and calculated the relative frequencies (q) for some statistical samples in all eight zones and the values of $\Delta q/\Delta L$. The results of liner approximations by the method of least squares for six samples of observations are given in the last column of Table 5.

First of all, we note that the average rate of change of q frequencies, $\Delta q/\Delta L$, not considering direction, is the same for both the $G_1(Asc)$ and $G_u(Des)$, within the limits of mean error. Second, one might suppose that the $\Delta q/\Delta L$ values should increase successively from top to bottom of Table 5, but it is just as possible that the value for G_x(D—E—S), which is slightly higher than that for $G_u(Des)$, is real. The $\Delta q/\Delta L$ data for the G_x observations show not only the difference between the developments and declines, but also, taking rapid and generally slow area changes, show a great deviation in value within the cases of decay (0.09), which is not seen in developments. This also indicates, as we have remarked earlier (§ 7 of [10a]), that direction of evolution is not the single difference between developing and declining sunspots.

TABLE 5

East frequencies of sunspot groups for two zones of heliographic longitude, lying symmetrically on either side of the solar central meridian. In the last column averages of the rate of change of relative frequencies with 1° of longitude are given. The observations are divided according to the first (*l*), last (*u*) and intermediate (*x*) days of group-life and different samples of evolutional phase.

	$n_E/(n_E+n_W)$				$\Delta q/\Delta L$
$\lvert L_{CM} \rvert$	0°—26.4°		26.5°—52.8°		0°—52.8°
Years	1922—27	1928—34	1922—27	1928—34	1922—34
$G_1(Asc)$	54	56	70	65	−0.12 ±0.01
G_x(A—S—C)	57		63		−0.08 ±0.02
$G_x(\overline{ASC})$	50	53	59	63	−0.07 ±0.01
$G_x(\overline{DES})$	50	53	44	48	+0.02 ±0.01
G_x(D—E—S)	40		30		+0.11 ±0.02
$G_u(Des)$	54	39	30	35	+0.10 ±0.02

There is another manifestation of different behaviour in increasing and decreasing spot areas to be seen in the asymmetry factor. It becomes successively greater in $G_x(\overline{DES})$, $G_x(\overline{ASC})$, G_x(A—S—C) and G_x(D—E—S) samples — that is, there is a considerably larger difference between the two declining cases than the two developing ones. This is shown both in Table 4 and, more reliably, in the columns of Table 5 covering longitudes from 26.5—52.8°, reckoned from solar central meridian.

Table 5 includes observations of $|\Delta L_{CM}| \leq 52.8°$, so that the data are more reliable than that containing observations of umbrae from $52.9° \leq |L_{CM}| \leq 60°$, which may have a bad influence on the results (as we saw in § 8 of [10a]). We suspect that there is at most a slight foreshortening effect acting on the data of Table 5, not enough to influence the results. The east frequencies of this table are therefore quite characteristic. It is striking that east-west asymmetry of sunspots appears on the solar disk even within the central $|L_{CM}| \leq 26.4°$ zone. (The observational material is almost evenly divided between two year groups, 1922—1927 and 1928—1934, and, where 2 figures are given separately for these periods in Table 5, their simple mean represents for the entire 1922—1934 period.)

The dependence of east-west asymmetry on evolution is most convincingly proved by such eastern frequencies as those given in Table 6. In this double-entry table, all figures are for important spots in principal spot groups. The cases when both the spot and the group were in the same evolutional phase are given in the upper squares, while the lower squares contain the cases where the phase of spot and group were different for an average of two days. The heading "*no Asc*", for instance, includes the G(*Des*), G(*Max*) and G(*Min*) cases combined.

TABLE 6

East frequencies of p and f sunspots, divided by cases in which the spot evolutional phase coincided with that of the group and those in which it differed. (Each figure in parantheses shows the number of observations used in calculating that frequency.)

1922—34	S_p		S_{p-}		S_{f-}	
G	*Asc*	*Des*	*Asc*	*Des*	*Asc*	*Des*
Asc	63 (*135*)		61 (*107*)		72 (*88*)	
Des		32 (*165*)		32 (*141*)		41 (*180*)
no Asc	46 (*85*)		37 (*51*)		46 (*35*)	
no Des		51 (*65*)		61 (*59*)		53 (*119*)

The factor of asymmetry is never larger in any of the cases in Tables 1—4 than it is in the upper squares of Table 6, where an important spot is in the same evolutional phase as the group to which it belongs. Certainly the data in the upper half of Table 6 generally represent groups in intensive development or decay, but the G_I and G_u observations show high asymmetry also. It is not surprising that both samples have eastern frequencies of approximately the same value. Consequently, we recognize here again an indication (and quite a strong one) that *the east-west asymmetry of the first* (G_1) *and final* (G_u) *stages of sunspot groups is not extraneous, but part of the general phenomenon.*

Comparing the frequency values of Table 6 in vertical pairs, we see very considerable differences. Considering the east frequencies for single spots, we see that figures given in the lower squares do not follow the patterns we have found to be characteristic up to this point at all. In other words, if we take only those observations in which the spot is in the same phase as the group and divide them into two parts, one for developing areas and one for declining, we find that the eastern excess values are, respectively, positive and negative, and quite strongly so. The numerical values of the east frequencies in the bottom squares of Table 6 are reasonable, if they are considered to be data of groups. Among the "*no Asc*" cases, the majority are G(*Des*) occurrences with east frequencies below 0.50, while the "*no Des*" cases (that is, the *no* G(*Des*) occurrences) belong to the other three main evolutional phases, which generally have east frequencies higher than 0.5. Consequently, the evolution of important single spot of principal groups is far from being the decisive factor in the east frequencies of the spots, since they seem to depend on the evolutional phase of the group as a whole.

TABLE 7

East frequencies for simultaneously observed p and f spots of p—f spot pairs of sunspot groups.

1922—34	*Asc*	*Des*	*Max*	*Min*	
S_p—	52	41	47	52	48
S_f—	65	46	60	60	55
G (containing the S_p— and S_f— spots)					51

TABLE 8a

Further examples of east frequencies of G_x observations of sunspot groups (for the years 1922—1934).

P \ *U*	*Max*	*Min*
Asc	57	59
Des	50	49

Two of the three samples of single spots in Table 6 (S_p— and S_f—) relate to simultaneously observed p and f spots of p − f pairs. Due to this, the selection of both samples already involves an internal east-west asymmetry, but we can easily see from Table 7 that this selection does not influence the basic results given above. The east frequencies, regardless of evelutional phase, for all the p and f spot observations in question and for their groups are given in the last column of Table 7. The average longitude interval between the p and f spots was 9° and the difference in east frequencies of the spots is an evident consequence of our selection ($L_{CM} = \pm 60°$). The 0.51 value for groups shows that all data of S_p— and S_f— in both Tables 6 and 7 should be adjusted by 0.03 or 0.04. If we introduced this numerical correction, the difference between east frequencies of the p and f spots should be smaller for all evolutional phases. The residual difference in frequency is probably a consequence of spot size (see Table 2a), since the p spots are on an average larger than the f ones. Thus, we finally conclude that it is not possible to establish any essential difference between the east excess of the p and f spots. The difference which GLEISBERG [16] found in this regard was probably due to selection.

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Let us now consider some penumbral evolution also. In Table 8 we give two values for the east frequencies of the $G_x(U, Asc; P, Asc)$ observations. Comparing the value of 0.53 which we obtained for the east frequencies of all $G_x(U, Asc)$ observations at $\lvert L_{CM} \rvert \leq 26.4°$ with the corresponding figure in Table 8, we find a definite difference, which, however, is easily explainable. An event in which the umbra and penumbra are both developing is evidently more significant than an event in which the penumbra is in another evolutional phase. There was no such difference in the two observational samples in the zone $26.4° \leq \lvert L_{CM} \rvert \leq 59.9°$. Accordingly, we see that the majority of the east excess in the central zone is due to clearcut developments, when both umbra and penumbra are growing. At a greater distance from the central meridian, the asymmetry is already quite large and does not appear to be affected by ambiguous cases where only the umbra is developing.

TABLE 8

East frequencies and means of the umbral areas of sunspot groups for different zones of heliographic longitude lying symmetrically to the solar central meridian for three different G_x observational samples

1922—34	$n_E/(n_E+n_W)$		$\bar{U}$			
$\lvert L_{CM} \rvert$	0°—26.4°	26.5°—59.9°	0°—26.4°		26.5°—59.9°	
			E	W	E	W
$G_x(U, Max)$	51	53	*49*	*48*	*49*	*54*
$G_x(U, Min)$	52	52	*39*	*37*	*40*	*39*
$G_x(U, Asc; P, Asc)$	55	61	*48*	*39*	*42*	*40*

Further, it is interresting to notice that we have the same value when we compare the east frequencies of $G_x(U, Asc; P, Asc)$ with $G_1(Asc)$ for the $\lvert L_{CM} \rvert \leq 26.4°$ in Tables 5 and 8 respectively. Since nearly 7/8 of the $G_1(Asc)$

observations are $G_I(U, Asc; P, Asc)$ ones, this gives additional support to the idea that the east-west asymmetry of both G_I and G_x observations is a single phenomenon.

§ 3. Some discarded interpretations

Summing up the results of the foregoing Section, we believe that there is a good deal of evidence that east-west asymmetry of sunspots is seemingly connected with the general evolution of spot groups and that all east-west asymmetry is essentially a single phenomenon. One might object that the large asymmetries obtained in our data are due to our working method; we think a careful study in detail of the three preceding papers in this series, [10a], [10b], [10c], will be enough to refute these objections. Nevertheless, we have looked for additional proofs that these asymmetries are both real and significant.

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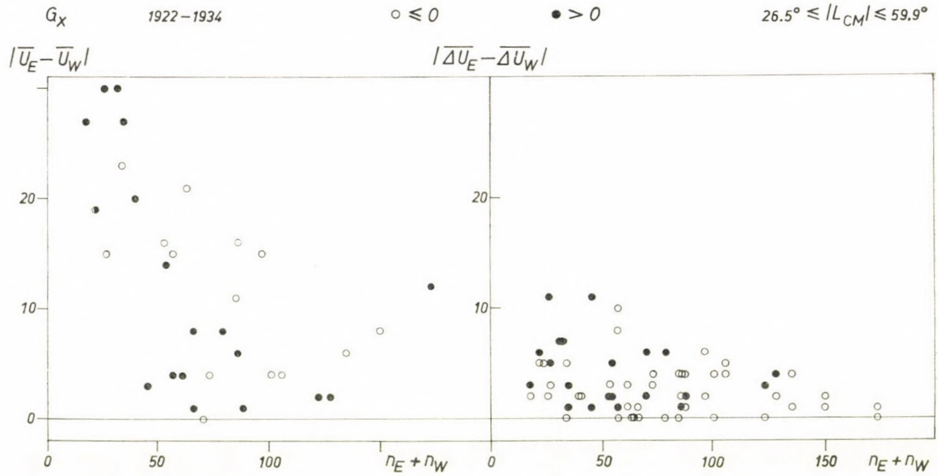
One might suppose that the observational material we used has considerable latent error introduced by the "physical" foreshortening effect (spoken of in [10a] and regarding it symmetric to solar central meridian). Superficially this error, together with our evolutional selections, would appear to lead qualitatively to such asymmetries as we found. But if it really played a role, we should observe for example a general systematically larger increase or smaller decrease in daily area east of the central meridian than west of it. However, we found no trace of any such systematic tendency, as we show in Table 9 and in Figure 1b.

It is easy to see how reliable the data in Table 9 really are. The $\overline{\Delta U_y}$ and $\overline{\Delta U_z}$ average values of daily area change show some conspicuous differences according to the phase of penumbral evolution, but no significant difference according to east-west distribution. ($\Delta U_y \equiv U_j - U_{j-1}$ and $\Delta U_z \equiv U_{j+1} - U_j$; the $U_2 = 0$ and the $U_{u-1} = 0$ observations have been omitted from the G_I and G_u observations in Table 9.) The fact that both scatter of data (which is generally small) and the numerical values of differences decrease as the number of observations used increases in Figure 1b proves that the observed average daily area changes for the east and west hemispheres of the solar disk are really equal; $\overline{\Delta U_E} = \overline{\Delta U_W}$. Consequently, the *foreshortening effect*, mentioned above, *in itself hardly account for the east-west asymmetry of sunspots.*

Still referring to Table 9, we see that the average daily change in umbral area seems to be around *10*. This agrees with the figure we used as a limit between large and small variations in umbrae, which shows that our choice was not only practical, but reasonable as well. We also notice the east-west differences in Table 9 and Figure 1b are quite small, compared to this limiting value *10*, even where the number of observations is small.



Figs. 1a and 1b. *Numerical differences between east and west average areas* (1a) *and averages of daily change in area* (1b) *plotted againts the numbers of G_x observations of spot groups* near maximum or minimum phases of umbral evolution. Both Figs. 1a and 1b are based on the same observational material as that used in the G_x parts of Tables 8 and 9. Each circle shows the difference relating to a certain phase of umbral-penumbral evolution and some year-groups. The observational data of 1922—1934 were divided into four different year-groups).

Even if some of the differences were real, they would not affect the asymmetry factor, since it is the large daily changes in umbral area (over nine) which generally show the properties of asymmetry most clearly.

Some other examples may also be given to show that the dependence of asymmetry on our evolutional phases of sunspot areas is not caused by a simple area foreshortening effect. First, we again call attention to Table 6. Each vertical pair of east frequencies obviously shows the same effect of the foreshortening, but there is still an eastern excess which varies in each pair with the evolutional phase of the group. A second example is presented in Table 5. The absolute values of $\Delta q/\Delta L$ of the $\overline{G_x\ (\mathrm{ASC})}$ and $\overline{G_x\ (\mathrm{DES})}$ observations differ, although they should agree if foreshortening were the only effect involved. Further, in the central zone of $|L_{CM}| \leq 26.4°$, where foreshortening obviously plays no role at all, we have comparatively a high factor of asymmetry in a majority of cases.

*

TABLE 9

Average values of the daily rate of change of sunspot group umbral areas at extremes of area change for east and west solar hemisphere. They are given both for the main phases of penumbral evolution and omitting these distinctions. The observations of the 26.5°≤| L_{CM} |≤59.9° zones were used. (See text for fuller particulars.)

$\overline{\Delta U}$		G_I		G_{II}		G_x			
U / *P*		*Max*		*Max*		*Max*		*Min*	
		ΔU_y	ΔU_z	ΔU_y	ΔU_z	ΔU_y	ΔU_z	ΔU_y	ΔU_z
Asc	E	+7.5	−2.0			+21.0	−11.5	−10.0	+18.0
	W	+10.5	−3.0			+20.5	−10.0	− 9.0	+19.0
Des	E			+1.0	−4.5	+ 7.5	−11.5	−11.0	+ 6.0
	W			+1.5	−4.6	+ 5.5	−14.0	−13.0	+ 5
Max	E	+7.0	−3.5	+2.0	−4.0	+11.0	−10.5	−10.0	+ 8.5
	W	+8.0	−3.7	+2.5	−4.9	+12.5	−11.0	−10.5	+10.0
Min	E					+ 9.0	− 9.0	− 9.5	+ 9.0
	W					+ 8.0	−11.5	−10.0	+ 9.0
1922—1934	E	+7.1	−3.1	+1.8	−4.3	+11.4	−11.0	−10.1	+ 9.3
	W	+8.8	−3.7	+1.9	−4.7	+10.7	−11.9	−11.2	+ 9.4

SCHUSTER [9c] has proposed an explanation of the fact that more sunspot groups are seen to form on the east side of the solar disk than on the west. He assumes this to be a rotational effect, caused by variations in visibility at different distances from the central meridian. GLEISSBERG [17] and MINNAERT [12] have gone into more detail of this possibility and MINNAERT decided that, as a result, the real distribution of the last observations of spot groups should be asymmetric also, but in the opposite direction.

It is indisputable that *SCHUSTER's reasoning may account qualitatively for the observed G_I and G_u distributions, but*, notwithstanding their large excess on the east, *it hardly explains them quantitatively*. In the central zone of the solar disk, $|L_{CM}| \leq 26.4°$, the visibility of sunspots is practically constant. On the other hand, both the G_I and G_u east-west distributions resemble our other samples so much, as we saw in the foregoing Section, that there is no reason to regard them as a special case. Even in principle, SCHUSTER's explanation is valid only for the G_I and G_u observations, but it should be carefully considered for all observations of $|L_{CM}| \gtrsim 50°$ (but probably only for these observations). A small part of the eastern excess for some G_I and G_u observational samples may originate in such a visibility circumstances.

The distribution of penumbral spots on the solar disk, which is shown in Figure 9 of [10a], serves as an indication that there may be a difference in visibility of less than two units at different points within the interval $26.5° \leq |L_{CM}| \leq 52.8°$. The umbral values of the G_I and G_u observations, given in the columns of Table 9, are respectively $\overline{\Delta U_y} = \overline{U}_1 \approx 8$, $\overline{\Delta U_z} = \overline{U}_u \gtrsim 4$. Both of these mean values in area are higher than our visibility limit of two, which was probably an overestimate anyway, and so once again we see that differences in visibility do not greatly influence east frequencies such as those in Table 5.

The $\Delta q/\Delta L$ values for average rate of change and the east frequencies in Table 5 are conclusive evidence that SCHUSTER's interpretation is far from adequate to explain the G_I and G_u east-west distributions. The speed of area variation is on the average at least twice as fast at the beginning as at the end of the life of a spot group; this may be rapidly estimated by comparing the $\overline{\Delta U_y} = \overline{U}_1$ figure with the $\overline{\Delta U_z} = \overline{U}_u$ mentioned above. A direct consequence should be that both the absolute value of $\Delta q/\Delta L$ and the asymmetry factors of the G_u observations should be higher than those of the G_I observations, if we use the SCHUSTER—MINNAERT reasoning [12]. There is no evidence that this really happens. The east frequency pairs of both the G_I and G_u observations for $26.5° \leq |L_{CM}| \leq 52.8°$ in Table 5 differ exactly with the same value from 0.50.

To sum up: the main cause of the asymmetry factor for both G_I and G_u observations must be the same as that of all the similar asymmetries which we find in other observational samples of sunspots.

*

Mrs. MAUNDER mentioned [1] that the observed asymmetric east-west distribution of sunspots might be explained by an inclination of the axis of the spot with respect to the normal line to an average surface of the photosphere — that is, the spot appearance we observe is not even in the first approximation to be regarded as surface markings.

If this hypothesis that there is a systematic westward tilt of sunspot axes were true, there would be two obvious consequences: (I) the number of spot groups, and especially of single spots, should be higher on the east half of the solar disk than on the west, since the very small areas would be more visible on the east and less on the west; (II) the observed average areas of both spots and spot groups should be somewhat larger to the east of the central meridian than to the west. There is no observational evidence to support this second conclusion.

We could not find any systematic differences between corresponding east and west average areas. Inspite of the fact that the eastern umbrae were slightly larger in five of the six samples given in Table 8, this cannot be due to a westward tilt of the sunspot axes, since the area values in the region $26.5° \leq | L_{CM} | \leq 59.9°$ should all have a larger east-west difference than those in the middle zone. But this is not true. We should like to call attention to the $G_x\,(U, Asc; P, Asc)$ statistical samples, which should show the property in question best of all. The fluctuation of average areas due to chance, like those in Table 8, may be seen quite well in Figure la. The differences in area between east and west decrease as the number of observations in the sample increases.

By the way, we should mention that even the observational material used by Mrs. MAUNDER [1] (see, e.g., p. 163 of [2b]) which covered thirteen years does not show on the average larger spot group areas on the east half of the solar disk. Mrs. MAUNDER's material shows the opposite to be true. When we divide the sum of the total areas by the number of spot groups for all 14 ΔL zones symmetric to the central meridian, we get a somewhat higher value for the average area of spot groups in all the seven pairs west of the central meridian without exception. Using the figure 4 for the P/U ratio (see [10b]) we have on the whole umbral areas for the west which are about 2 units higher than for the east, while Mrs. MAUNDER's data gives east frequencies of 0.52 and 0.51 for the $| L_{CM} | \leq 66.0°$ and $| L_{CM} | \leq 52.8°$ zones respectively.

Let us assume the existence of a spot umbral area U which lies on the solar equator at some $| L_{CM} |$ distance with its axis tilted at angle I toward the west. In this case, the area which we observe, the so-called projected area (U_{pr}), should be reduceable to east ($L_{CM} < 0$) and west ($L_{CM} > 0$) when it is multiplied by sec ($| L_{CM} | - I$) and sec ($| L_{CM} | + I$) respectively. If this constant I really exists, by designating the area quantities of $L_{CM} < 0$ and $L_{CM} > 0$ respectively by E and W, we could write

$$U = U_E \cos L_{CM} / \cos(| L_{CM} | - I) = U_W \cos L_{CM} / \cos(| L_{CM} | + I)$$

where $U_E = U_{prE} \sec L_{CM}$ and $U_W = U_{prW} \sec L_{CM}$. Consequently, taking observations covering two equally wide zones at an average distance $|\overline{L_{CM}}|$ on either side of the central meridian, the ratio of the two average areas will be:

$$\overline{U}_E/\overline{U}_W = \cos(|\overline{L_{CM}}| - I)/\cos(|\overline{L_{CM}}| + I).$$

Numerically accoding to this formula, as an example, we obtain a difference between the average size of sunspots on the east and on the west at $|L_{CM}| = 60°$ of over *10*, if $I \geq 5°$ and $U \approx 35$ (the approximate average size of spot groups in Mrs. MAUNDER's material). It may be seen that *small inclinations of the axes of sunspots does not contradict the general east-west asymmetry, but they hardly serve to explain it.*

In addition, if Mrs. MAUNDER's hypothetical tilt were real, the directions of the inclination of sunspot axes would vary in a complicated manner through the different *U* and *P* phases of evolution (see, for example Table 8a). These conditions again make any connection between sunspot axes and asymmetry unbelievable.

*

A general westward inclination of spot axes could evidently influence our evolutional classifications. In principle, systematic inclinations would make the one-day increases in areas appear larger to the east of the central meridian and smaller to the west, and, therefore, might cause the asymmetric east-west distribution of developing spot areas. In cases where the sunspot is declining, the tilted axes should obviously cause apparently larger daily area decreases to the west and smaller ones to the east. Quantitatively, it is easy to see that any reasonable inclination could not affect our obtained east-west distributions.

In order to estimate the approximate magnitude of the effect of inclination upon the various classifications of sunspot evolutional phases, we shall calculate the one-day change in umbral area as a function of I. Using the same notations and formulae as above, we have for an apparent one-day change in umbral area (ΔU_I) on account of $I > 0$, taking the $\Delta U_E = 0$ and $\Delta U_W = 0$ cases:

$$\frac{\Delta U_I}{U_E} = \frac{\cos(|L_{CM}| - 13.2°)}{\cos(|L_{CM}| - 13.2° - I)} - \frac{\cos L_{CM}}{\cos(|L_{CM}| - I)}, \text{ if } L_{CM} < 0 \text{ and}$$

$$\frac{-\Delta U_I}{U_W} = \frac{\cos L_{CM}}{\cos(L_{CM} + I)} - \frac{\cos(L_{CM} - 13.2°)}{\cos(L_{CM} - 13.2° + I)}, \text{ if } L_{CM} > 0.$$

The ΔU_I data calculated by these formulae yield only quite insignifican values for $I < 5°$ and thus proves that *sunspot inclinations gen erally do no influence the evolutional classifications.* Even at $I = 5°$ in the distant one-day zone of $|L_{CM}| \approx 66.6°$, the tilt effect cannot alter a $U = 10$ spot more than $\Delta U_I \sim 2$. Or, to take a more general case of $|L_{CM}| \approx 36°$, $I = 5°$ would cause $\Delta U_I \sim 1$ only at $U \approx 40$ or larger areas. In considering the $I = 5°$ cases, we have already overestimated any possible inclination. Any angle this high should already reveal an observable difference between the east and west area averages which could also be detected in other ways.

Our final conclusion is that there is no possibility that any feasible axis inclinations can influence our results of the east-west asymmetry of sunspots. All distributions show a higher degree of asymmetry when the daily area changes are large ($\Delta U \geq 10$). Even the largest possible, rarely occurring ΔU_I values are too small to affect the main asymmetry characteristics we have pointed out in this paper. Finally, we should mention that A. BRUZEK [18], using entirely different consideration, found no trace of any asymmetric tilt of the axes of sunspots.

*

We think that it is quite clear, on the basis of the data and discussions in this paper, that *the east-west asymmetric distribution of the sums of sunspot areas is a mere consequence of the gerenal asymmetry of the number of sunspot groups* as a whole and that *there is a single essential cause for at least the major part of these asymmetries.* The most reasonable possible explanations of this cause given to data, which we have discussed at length in this Section, cannot account for the great east — west asymmetries which we observe.

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