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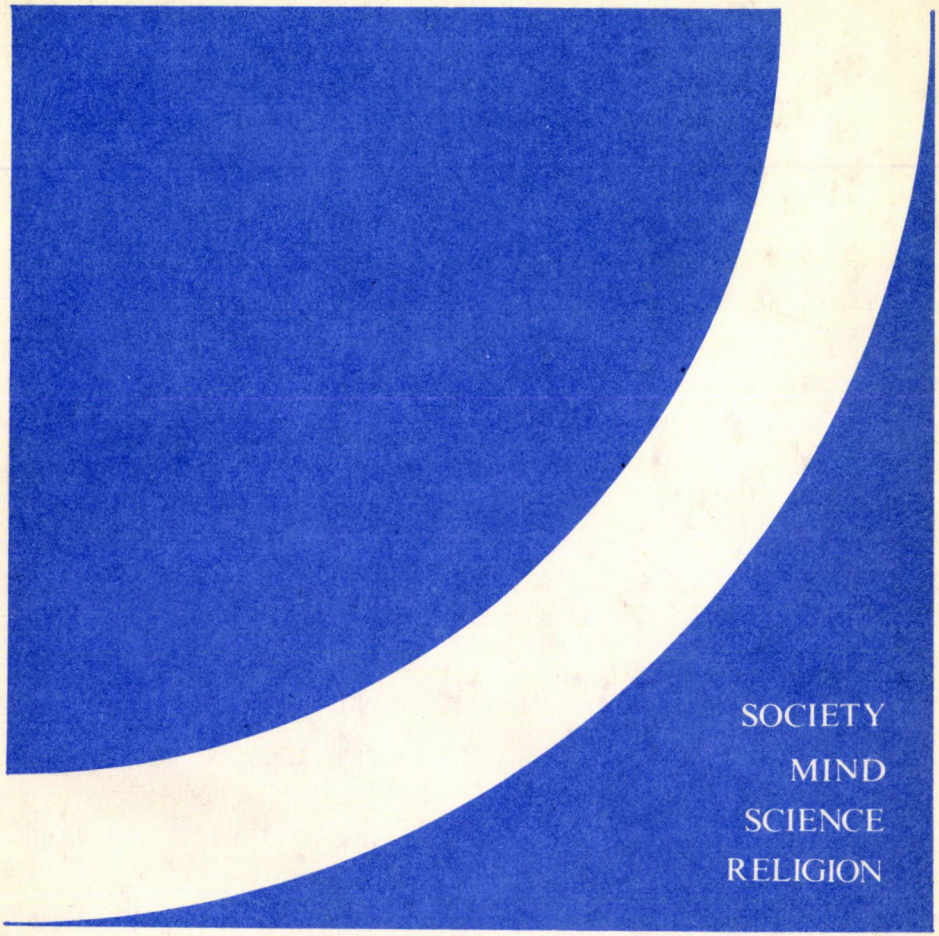

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PREFACE

Hungarian philosophers and historians of science have already published contributions to IUHPS and other conferences in numbers 2, 3 and 6 of DOXA. This volume contains papers presented to the 8th International Congress of Logic, Methodology and the Philosophy of Science (Moscow, 17-22 August, 1987), as well as longer studies written by some participants, which serve as a background to their papers. Editing was done by members of the Department for Epistemology and Methodology of Science at the Budapest Institute of Philosophy, who owe thanks to András Benedek and Balázs Dajka for their assistance.

HEURISTICS OR FOUNDATION OF MATHEMATICS?

Wittgenstein and Lakatos Reconsidered*

by

András Benedek

> "...every rational theory, no matter whether scientific or philosophical, is rational insofar as it tries to *solve certain problems*. A theory is comprehensible and reasonable only in its relation to a given problem situation, and it can be rationally discussed only by discussing this relation."
>
> Karl Popper

1 At first glance Popper's normative criterium sounds persuasive as a scientific observation. But what about as an empirical statement? Couldn't it be false? Couldn't a theory be expressed and discussed independently of the problem situation? Take, for example, mathematical explication. At first, it cannot be described in Popper's words.[1] As Lakatos writes, "...it starts with a painstakingly stated list of *axioms*, *lemmas*, and/or *definitions*... they frequently look artificial and mystifyingly complicated. One is never told how these complications arose. The list of axioms and definitions is followed by carefully worded *theorems*. These are loaded with heavy-going conditions, it seems impossible that anyone should ever have guessed them. The theorem is followed by the proof."[2]

The standard view of mathematics is that it is a deductive system. However, nobody would say that in mathematics we don't solve mathematical problems. In the final analysis *mathemati-*

*Earlier draft of this paper delivered at the Eleventh International Wittgenstein Symposium, August 4 - August 13, 1986 Kirchberg/Wechsel, Austria.

cians go into university lecture halls where, after a couple years, those who file out are mathematicians. They appear to have mastered and comprehended mathematic theories from the presentation. — In today's classroom the discussion, if it can be termed as such - the *style* of presentation - in Lakatos's words, continues to be deductivist. Such a style does not take into account the problem situation, and when it is possible to discuss the problem at all, in terms of the constitution of the subject, at very most it could be spoken about 'heuristically'. *Deductivist* mathematic *heuristic* - if it exists at all - is either rational and deductive at the same time,[3] or "based on the idea that heuristics is not deductive, but is also not rational".[4] It is either impossible or doesn't make sense to discuss.

2 At the same time we have traditional mathematic philosophies-Platonism, Formalism, Empricism, Logicism, Constructivism, etc. which identify and widely discuss - albeit on a meta-level - what the subject of mathematics is and what it concerns, how we are supposed to solve its problems. So now, we are confronted with another problem and this was one of Lakatos's points of contention with Popper - namely, that *the problem situation is usually not given*, it must be constructed or reconstructed.

/ This implies two different assertions on different levels:

(1) That the problem-situation is not given as a *matter-of course* - neither in advance nor afterwards (on the meta-level), nor is it presented analytically or historically for the re/construction.

(2) That problems themselves usually are not given - they are assumed as "eo ipso" given rather than described as "coming to be" and "passing away", their entire history being "ab ovo" omitted.[5]/

We are *forced* to ask where the boundaries of the problem situation ought to be drawn and where they should extend. A strict internalist taking full advantage of the versatility of the term might well argue that the axiomatic foundation of geometry or the set theoretical foundation of arithmetics is external precisely from the point of view of the problem situa-

tion; external postulates, methods, and definitions determine both the explication and mode of presentation. He might then go on the define the 'tradition of foundational studies' as though it were a trend in philosophy of mathematics which attempts to base knowledge on external grounds, whereas - in the usual sense of the word - the foundation of a mathematical theory is an internal part of the subject (if not constituting the subject itself).[6]

2.1 Lakatos, who was a rationalist, worked with the explicit normative criterium that the reconstruction of the problem situation is all the more better when it is able to salvage, to reconstruct a greater part of the history of thought as rational, and "the *rational* aspect of scientific growth is fully accounted for by one's *logic of scientific discovery*".[7] He could not ascribe to the *post festum* view that a theory is given with the very same list of statements, definitions, and postulates which can be identified in books, journals, and documents because *internal* reconstruction *must account* for the *growth* of knowledge and *creation* of the problem situation:

> "A catastrophical consequence of a narrow methodology is that, as well as impoverishing actual problem situations, it invokes external - psychological, sociological - explanations because its internal framework fails too soon."

This is the point of his criticism on Popperian historiography.

> "Heuristic style ... emphasises the problem-situation: it emphasises the 'logic' which gave birth to the new concept." - he writes elsewhere.[8]

Lakatos attempted to extend Polya's heuristic vocabulary so that it could be applied in describing the *process* wherein mathematics and mathematical problems come to exist, *both historically and analytically*.[9]

> "Whatever problem the historian of science wishes to solve, he has first to reconstruct the relevant section of the growth of objective scientific knowledge, that is, the relevant section of 'internal history'" - he continues.[10]

Consequently, Lakatos did not shift the demarcation line horizontally - that is to say, toward sociology, economics, psychology, or even history - but moved it vertically, ranging

history in the *internal* problem situation, that is, including the history of the problem into the problem situation itself.

> "Demarcationist historiography recognizes that all histories of science are inevitably methodology-laden and that one cannot avoid 'rational reconstructions'. Each different type of demarcationism leads to a different 'internal reconstruction', with correspondingly different anomalies and different "external' problems. These 'rational reconstructions', however, can be compared according to well-defined standards and the history of demarcationism...itself constitutes a progressive research programme."[11]

To paraphrase his reply to his critics: 'There is no constructuon without reconstruction'.[12]

It is instructive to compare this with Wittgenstein's words:

> "It is often useful, in order to help clarify a philosophical problem, to imagine the historical development, e.g. in mathematics, as quite different from what it actually was. If it had been different no one would have had the idea of saying what is actually said."
>
> "Our task is, not to discover calculi, but to describe the *present* situation".[13]

2.2 While Lakatos's way of understanding problems was to *reconstruct* their history, Wittgenstein stood in the distance removed from this argument as though to say: 'the problem situation is the *explication* itself'. As early as in the Tractatus he suggested:

> "The existence of an internal relation between possible situations ***expresses itself in language by means of an internal relation*** between the propositions representing them."[14] (Italics mine A.B.)

Nevertheless Wittgenstein and Lakatos raised the same question: How are *problems* expressed *in mathematics* if they are not given?[15] If they are, as the two "preparators" (the 'interpreter' and the 'reconstructor') treated them, "fossilized"? How are they expressed if what we have are only the "impressions"*: *theorems*, *proofs*, and *definitions*?[16]

*Wittgenstein in the *Tractatus* went even further:
"What expresses *itself* in language *we* cannot express by means of language" (*TR* 4.121.)
"A proposition of mathematics doesn't express a thought" (*TR* 6.21)

2.3 There are texts which are good because of how they are written and others which are good because of what is written in them. The former you cannot rewrite, the latter you can - but in either case the text stays the same. In spite of the differences in their styles, Lakatos's dialogues have much in common with Wittgenstein's "monologues". In *these* proofs and refutations, one can never know if they are masking themselves in the how or the what, just as the best theorems in mathematics do - theorems which are not good simply because of what they propose, but because of how they are proved.-Both Wittgenstein and Lakatos became representatives of the view that *the way problems* are expressed is creative in and of itself. Altough Wittgenstein appears to contradict Lakatos when he states, "to say it is the *proof* which gives sense to the question is absurd because it misuses the word 'question'",[17] and they only "give us a sort of hint as to what we are to do, but the proof provides them with content"[18] - the basic thought here is still the same: "The proposition proved by means of a proof ... shows us what is makes SENSE to say".[19] "The proof creates new concept... A proof is a new paradigm."[20] "Could we say: 'mathematics creates new *expressions*, not new propositions'?"[21]

> "*Proof-generated concepts* are neither 'specifications' nor 'generalizations' of naive concepts. The impact of proofs and refutations on naive concepts is much more revolutionary than that: they *erase* the crucial naive concepts completely and *replace* them by proof-generated concepts.*...After Columbus one should not be surprised if one does not solve the problem one has set out to solve."[22]

If and only if mathematics cannot be described in terms used by Popper and Lakatos, Wittgenstein and the so-called inadequacy[23] of his *Remarks* won't escape criticism either, since this criticism starts by repeating the (original) ques-

*This "theory of *concept-formation* (italics mine A.B.) weds concept formation to proofs and refutations" notes Lakatos himself.

tion: 'Well-defined terms express something, so what is expressed then?' The hermeneutic circle of interpretation neglects the *problem* (of "*what* it makes SENSE to say") eventhough its origin is what imbuses the question with meaning[24] when being 'well-defined' is the (internal) problem. "As knowledge grows, languages change. 'Every period of creation is at the same time a period in which the language changes.'* The growth of knowledge cannot be modelled in any given language."

> "That is right. Heuristics is concerned with language-dynamics, while logic is concerned with language-statics,"

remarks one of Lakatos's characters while he notes:

> "Science teaches us not to respect any given conceptual-linguistic framework lest it should turn into a conceptual prison - language analysts have a vested interest in at least slowing down this process, in order to justify their linguistic therapeutics, that is, to show that they have an all important feedback to, and value for, science, that they are not degenerating into 'fairly dried up petty-foggery.'**"[25]

3 Lakatos and Wittgenstein both appeared to be critics rather than creators of an express theory of mathematics, while both focused on the process of mathematic *creation*. Their position can be better described in terms of their opposition to traditional philosophies of mathematics than paraphrased in quotations. They accepted that each of these views contains a certain amount of truth about mathematics,[26] but they generally attacked the *external* grounds of all the traditional philosophies of mathematics and the "what" questions of ontological foundations. This was also the drive of their critique of the language of mathematics and its foundations. For both of them the "problem was to provide a theory of objectivity without a representational theory of truth"[27] and referential concept of

*Félix, L. [1957], *The Modern Aspect of Mathematics*, New York: Basic Books, 1960. p. 10.

**Einstein, A. [1953], Letter to P.A. Schlipp", *Kant Studies*, 51, pp. 430-1, 1959-1960.

meaning. According to I. Hacking, this "is a point that requires elaboration". "Hence in English philosophy knowledge is to be characterized *externally*, in terms of how well it represents reality".[28]

In this sense we may call Lakatos's and Wittgenstein's approaches - which stand in opposition to this tradition - an *internalist philosophical strategy*".[29] Neither ascribed to the view that the "foundation of the solution" and the "methods of investigation" lie outside the problem situation. This places Lakatos's words - ordinarily interpreted as merely a criticism of normative methodologies and, more generally, of pre-critical normative philosophy of science - in new light:

> "There are several methodologies afloat in contemporary philosophy of science; but they are all very different from what used to be understood by 'methodology' in the seventeenth or even eighteenth century. Then it was hoped that methodology would provide scientists with a mechanical book of rules for solving problems. This hope has now been given up: modern methodologies or 'logics of discovery' consist merely of a set of possibly not even tightly knit, let alone mechanical rules for the *appraisal* of ready, articulated theories.* Often these rules, or systems of appraisal, also serve as 'theories of scientific rationality', 'demarcation criteria' or 'definitions of science'. Outside the legislative domain of these normative rules there is, of course, an empirical psychology and sociology of discovery.
>
> *This is an all-important shift in the problem of normative philosophy of science. The term 'normative' no longer means rules for arriving at solutions, but merely directions for the appraisal of solutions already there. Thus methodology is separated from *heuristics*, rather as value judgments from 'ought' statements. (I owe this analogy to John Watkins.)"[30]

Later he even stressed:[31] "I should like to say here that I always had *doubts* about whether this (no doubt progressive) problemshift had not gone a bit too far. This shift had been even more pronounced in the philosophy of mathematics than in the philosophy of science. Following Polya, I have held that there might well be a *limbo* for a 'genuine' heuristic which is rational and non-psychologistic; it was in this vein that I expressed some reservations concerning Tarski's novel use of

the term 'methodology'... But here I cannot pursue this matter further."[31]

4 One of the objectives of this study is to outline some *common* positive prevailing themes concerning this point in their critical philosophy of mathematics and demonstrate how these are interrelated: From this point of view *Wittgenstein's philosophic analysis might be described and serve as meta-mathematical heuristic and Lakatos's heuristic might be termed a kind of meta-historical analysis of producing mathematics.* In other words, Wittgenstein was concerned with *how* mathematical activity actually *operates*, and Lakatos tried to respond to the problem of *what* comprises the *growth* of mathematics. Lakatos frequently took the opportunity to comment on Wittgenstein, but inspite of his sharp criticism, the similarities in their thought are too striking and have too many far-reaching consequences to ignore the impact of Wittgenstein's *Remarks* on Lakatos's *Proofs and Refutations.**

4.1 Each operates from a common point of departure - for both, mathematics is a *human activity*. This simple idea is one of the presuppositions underlying Wittgenstein's *Remarks on the Foundation of Mathematics* and exists in conjunction with his philosophy of action. The same idea crops up in Lakatos's oft-cited words:

> "Mathematical activity is a human activity. Certain aspects of this activity - as of any human activity - can be studied by psychology, others by history. Heuristic is not primarily interested in these aspects. But mathematical activity produces mathematics. Mathematics, this product of human activity, 'alienates itself' from the human activity which has been producing it becomes a living, growing organism, that *acquires a certain autonomy* from the activity which has produced it; it develops its own autonomous laws of growth, its

*Although any Wittgensteinian might certainly ask after having read Lakatos' posthumously published review, *Understanding Toulmin*, why I insist on this 'imbalanced marriage'.[32] (I am fully aware that I neglected their dissimilarities.)

> own dialectic... heuristic is concerned with the autonomous dialectic of mathematics and not with its history, though it can study its subject only through the study of history and through the rational reconstruction of history."[33]

In consequence of this 'simple' idea, both oppose any kind of *statement* or *non-statement* view of mathematics,[34] since

1. informal mathematics is one kind of complex and social activity;[35]
2. actually existing living and growing mathematics is informal and the understanding of a formal system presupposes informal reasoning;[36]
3. the reject the image of mathematics patterned after empirical inductivism consisting of stating (Wittgenstein) and justifying (Wittgenstein-Lakatos) facts or propositions;[37]
4. both reject the correspondence theory of truth[38] and the extensional interpretation.[39]

Naturally, they oppose any kind of normative or epistemological justificationalism irrespective of semantic concepts - truth, inductive evidence, validity, etc. Consequently, they also reject absolutism and mathematical apriorism as the foundation of mathematics since mathematics is a *fallible* human activity and not the ideal but the denial of certain and necessary knowledge.[40] (Note that in the traditional epistemological sense it is impossible to 'justify an activity' just as - for example - Donald Davidson's rationalizations of actions do not signify justifications.[41]) "Justification" even in the case of mathematics is itself a kind of formal description of informal practice which cannot be analytically justified. Descriptions can explain but cannot justify actions, and even the practice of formal description is nothing more than the explication of series of actions in the form of symbolic operations.[42]

4.2 Both oppose mathematic *Platonism* and *Empiricism*. Lakatos opposes it on his quasi-empirical grounds,[43] since "he cannot seriously contemplate objective timeless and spaceless existence in the world of all the entities contained in modern set theory"[44] or ancient geometry, let alone in future theories

yet to be revealed or in a/the world perceivable to none but devoted mathematicians. Wittgenstein writes, "There is no system of irrational numbers - but also no super-system, no 'set of irrational numbers' of higher order infinity." "If you talk about essence, you are merely noting a convention... the depth that we see in the essence... corresponds to the *deep* need for the convention."[45] Although Lakatos in many ways gets involved in Popper's *'third world problem'* and uses the oft-criticized expression "the Platonic world of objective spirit" in connection with it, I go along again with Hacking that this has "little to do with Plato or Platonism".[46] His *Hegelian version of the history of ideas* is what is "independent of the human mind which creates it or understands it".[47] Wittgenstein is considerably more explicit about social interactions in the mathematical community than Lakatos, but less clear about their role and that of history and tradition: "It could be said: a proof helps communication. An experiment presupposes it. Or even 'a mathematical proof moulds our language'."[48] "In a demonstration we *get agreement* with someone. If we do not, then we've parted ways before even starting to communicate in this language. It is not essential that one should talk the other over by means of the demonstration. Both might see it send it, and accept it."[49]

4.3 It will suffice to refer to Lakatos's well-known arguments against *Formalism* that informal mathematics *is* mathematics.[50] Pure formalization is only an abstract possibility "introduced by logicians for their own purposes which no one would be at all able or would want to carry out" and "it is a pity that it is not quite certain - although it is approximately certain - what it is reliable about."[51] Wittgenstein uses similar wording and his arguments here, too, are concerned with the human use of symbols versus meaningless procedures. He writes, "Let's look at what happens... There is a transition from one proposition to another *via* other propositions... There is nothing occult about this process; it is a derivation of one sentence from another according to a rule; it is a comparison of both

with some paradigm or other, which represents the schema of the transition; or something of the kind."[52] This is the idea that he contrasts with "primitive alchemy", "arithmetic as the natural history (minerology) of numbers".[53]

> "The *application* of the calculation must take care of itself. And that is what is correct about 'formalism'.
>
> The reduction of arithmetic to symbolic logic is supposed to shew the point of application of arithmetic, as it were the attachment by means of which it is plugged in to its application. As if someone were shewn, first a trumpet without the mouthpiece - and the mouthpiece, which shows how a trumpet is used, brought into contact with the human body. But the attachement which Russell gives us is on the one hand too narrow, on the other hand too wide; too general and too special. The calculation takes care of its own applications."[54]

4.4 In the area of *Conventionalism*, all this might seem to point to the fact both thinkers emphasize the *creative* character of mathematics which produces new examples and counter-examples, generates or *breaks rules*, and creates meaning. They refute the argument that mathematics is a deductive system based on conventionally accepted definitions and axioms which can be neatly channeled into the conduits of analytic definitions.[55] It is *we* who construct meaning - the extension of mathematical definitions cannot be analytically determined in advance on the basis of convention. *Proof constructs concept - or changes it.*[56] Wittgenstein writes,

> "The agreement of humans that is a presupposition of logic is not an agreement in *opinions*, much less in opinions on questions of logic."[57]

While Lakatos suggests in his dialogue,

> "GAMMA:...There are no *minus* 1-dimensional polytopes!
> EPSILON:...there is one: the empty set.
> GAMMA: You are mad!
> ALPHA: He may not be mad. He is introducing a convention
> ...
> EPSILON: I do not use conventions, and my concepts are not "tools"."[58]

It is precisely this fact which opens the way for counter-examples "potential falsifiers" in mathematics, as Lakatos puts, it, so that we *can* create sentences where we *don't know the*

borders of their extension and we can't supply their analytical meaning in advance.

> "What if someone were to reply to a question: 'So far there is no such thing as an answer to this question'?"[59]

4.4 Opposed to *Instrumentalism* Lakatos and Wittgenstein agree that mathematics is a dialogue, it is interactive, intersubjective, and "*the game has a point*" as well.[60] 'If definitions are abbreviations, then they cannot be false,' - but they cannot be true either.

> "You can scarcely deny that my language, which is the *natural* language reflecting the *essence* (of polyhedra) shows for the first time the deeply rooted essential identity of formerly disconnected, isolated, *ad hoc* criteria!"

- declares EPSILON later in the *Proofs and Refutations* trying to convince GAMMA that his "linguistic instrumentum" is an intuitively "closer translation" of the "true foundation".

> "I like this reformulation which really showed the nature of your simple tools" - comments the TEACHER,[61]

for "how can one refute a language?" - Lakatos queries, if this instrumentalism will mean that we can't criticize, appreciate or change a definition.[62]

Since they gave up absolutism the *ambiguity in articulation* doesn't imply for either of them that absolutely precise and clear-cut inner intuition must exist or if this intuition does not exist, that it is the logician's job to "reincarnate" it. *Instead* of citing the Browerian *introspective constructions* which take shape in the mind of each person who deals with mathematics, it is more plausible (and public) to speak about the *practice* of social agreement.

> "Let us remember that in mathematics we are convinced of *grammatical* propositions; so the expression, the result, of our being convinced is that we *accept a rule*."...
> "The proof places this decision in a system of decisions."[63]

4.6 Wittgenstein's major arguments against *Intuitionism* spring from the conversion of intuitionist arguments[64] and from his *rejection* of the existence of a *private language*. Just as he did in the case of Platonism and Empiricism, he questions any private reality of mathematical objects which can only be penetrated by help of the intuition. Familiarity with *everyday language* which is public, informal, paradigmatic, and adheres to certain rules *is the medium of mathematics*. Linguistic techniques insure a communicative interpretation of mathematical symbols and procedures, techniques, and inferences. They proceed from everyday existence, supplied with the patterns by pre-existing routine of mathematic practice. "Perhaps the fact is that human intellect engages in every game which opens up, but only those games endure which common sense deems practicable"[65], opines one of Lakatos's native colleagues.

> "Do the figures (drawings) in trigonometry belong to pure mathematics, or are they only examples of possible *applications*."[66]

Nevertheless, common sense and practicability could by no means be considered concepts of either linguistics or logic, which doesn't mean we can't use the tools of logic (or linguistics) to examine them. In Lakatos's terms "rational (historical) reconstructions" supply the "paradigms of common sense" (or common non-sense). Whenever Wittgenstein speaks of mental images he usually emphasizes that familiarity with the way a language is used and with its illocutionary actions or speech acts does not lie in comprehension as a consequence of the mental state; nor can it be termed the discovery of properties of an internal entity. Instead, *this familiarity establishes* inferential reception and *the mental state of comprehension.*

Mathematicians create linguistic *expressions and symbols* under the guise of lemmas and definitions, and connect these sentences according to rules in the living language. They create meaning direction and heuristic meaning based on some

sort of procedural technique of expression and explication according to previously accepted rules and, *in part*, to the meaning of previously accepted expressions.[67]

> "When we derive the correctness of one statement from another and put it down in our symbolic way of writing, we have simply shifted from one succession of fixed signs to a different series."

Here, surprisingly enough, it is not Wittgenstein I cite, but the work of Rózsa Péter [1943], the mathematician in her analogy of deductibility and equation-solving:

> "From this, then, we deduced a rule that was purely formal - and henceforward, we applied it without a second thought. We got a mechanical 'rule of the game' which had been deliberately considered, and considered from the point of view of *content* (italics mine): 'Open, fixed signs shifted in a given way from one place to another'. They resemble the rules of chess; the king can be moved one step in whatever direction desired."[68]

That is to say, mathematicians make guesses - they create sentences and/or usages of sentences, and not propositions. The heuristic thinker integrates meaning with accepted sentences and meanings; these create and secure connections with the rest of the meanings. This activity is, in part, constituted by the application of old meanings to new: it might be possible to apply them to the old rules, but a rule may even generate the new meaning of old expressions. "The method of proof is similar to that of the 'contradiction-exemption' [consistency proof] of Bolyai's geometry: Gödel constructed a model in set theory in which the axioms of set theory and the continuum hypothesis exist side-by-side in perfect harmony."[69] That such a construction proves something at all - in the first case - was not yet expressible in the rules of logic. It wasn't a proof. It wasn't even (a) geometry.[70]

Wittgenstein concludes in the *Remarks:*

> "However queer it sounds, my task as far as concerns Gödel's proof seems merely to consist in making clear what such a proposition as: 'Suppose this could be proved' means in mathematics,"[71]

4.7 The fixing and prohibiting of concrete meaning and rules of *exception* is apparently complicated and might even seem like an irrational game to the outside observer. To use a self-contradictory metaphor: It is an interpretational game among "contra-factual mathematical facts" - or in Lakatos' terms - among "quasi-empirical facts".

All this is closely related to what Lakatos variously refers to in the *Proof and Refutations* as the logic or "dialectic" of discovery or heuristics. For him history is present in 'counter-factuals' (sic) - "fossilised examples" or forgotten counterexamples which are not easy to either construct or reconstruct. But by no means are they given *at the outset* of entities in "possible worlds" even if 'post festum' it's easier to hit upon the generative rules than the counterexamples. The "game" goes on in a problem situation where those sentences are selected which are supposed to be accepted as propositions (or 'quasi-propositions') - that is to say, those sentences which are *permissible*: "You can say this sentence: "This is right", "You can't say this", "There is no point to this". Wittgenstein uses these imperatives or commands like a math teacher or Lakatos's imaginary pupils would, but they could have been used by the first devoted interpreters or critics as well. For example - 'the sum of the angles of this triangle is greater than two right angles', or there is no greatest prime number 'the set of all sets', or 'every set can be well-ordered'.

Let me note here that from this point of view mathematicians make their way (the SENSE) through a *jungle of non-referential expressions and/or descriptions*. Wittgenstein says: "...the mathematical general does not stand in the same relation to the mathematical particular as elsewhere the general to the particular".[72] "In one case we make a move in an existent game, in the other we establish a rule of the game."[73] When Alice Ambrose demonstrates that if a statement has a counter-example in mathematics then there is no proposition to which it is a counter-example, and that the acceptance of certain

sentences depends on our decision or, summing up Wittgenstein's words, mathematicians "sometimes... justify acceptance of a new rule and define or *redefine* a term",[74] then she has shown something which is contentually described by Lakatos in various ways under the titles, "piecemeal exclusions", "the method of monster-adjustment", "lemma-incorporations" or "proof-analysis".

4.8 The idea Lakatos and Wittgenstein are talking about here isn't simply the one that knowledge is language and language is knowledge, and mathematics is language created as a usage of language within our human experience. - To paraphrase Imre Tóth[75]: 'Mathematical logos is speech which - *bespeaks* itself'. What Lakatos says in the "dialectical chorus" is that mathematics speaks in the language of *history*, not God, and is articulated by "fallible mathematicians". While Wittgenstein states that : as long as mathematics formulates propositions, in an empirical-referential paradigm he refers to as "extensional interpretation of mathematical statements", this results in the false appearance that something is being claimed about something - as unchangeable Forms or Objects.[76]

Without this interpretation the game looks very sophisticated. But in the majority of cases it goes on in a relatively narrow domain where in 'normal mathematics' as in the case of Kuhn's *normal science* the standard interpretation is given and accepted. To destroy or to change it usually presents greater difficulty than finding or rejecting counter-examples or propositions, but this depends on how big the problem is and how new it is.[77]

The *interpretation paradigm* is revised only when problems of consistency have been pushed to extremes. The debates between mathematicians who otherwise see eye-to-eye have here reached their most heated point. But this is precisely what insures - as Wittgenstein might have put it - that consensus will emerge: "Church's proof is also thought-provoking in that he found it necessary *to express* what we should consider 'today's

mathematical reasoning' when it was this concept he was aiming to apply the mathematical procedures to. No sooner do we express something then we fall victim to circumscribing it. Circumscription is of necessity a narrow enterprise, since undecidable problems always emerge from it."[78] "The motto here is always: Take a *wider* look around" suggests Wittgenstein, and he adds:

> "An undecided proposition of mathematics is something that is accepted neither as a rule nor as the opposite of a rule, and which has the form of a *mathematical* statement. - But is this form a sharply circumscribed concept?"[79]

4.9 If *decision* figures here to such a great extent, then do Lakatos and Wittgenstein believe there are undecidable problems in mathematics? According to the *standard interpretation* of Gödel's theorems, there are - as far as consistency is required. But if problems depend on our decisions, then how can they be undecidable and not - at most - undecided? Lakatos and Wittgenstein reply: If the problem has been stated, then this - this *itself* - is also a matter of decision. "For instance, if we had a formal proof of Fermat's last theorem, then if our formalized number theory is consistent it would be impossible for there to be a counter-example to the theorem *formalizable within the system.*"[80] But "...if we could prove that Fermat's theorem is undecidable, then are we forever helpless to say anything about the truth of it? Not at all.* We may again call informal reasoning to our assistance, and try to operate informally *only* in the intended model. A concrete example of this is Gödel's proof that his undecidable sentences are *true* (i.e. true in the standard model) . But such post-formal proofs are certainly informal and so they are subject to falsification

*Wittgenstein writes: "The problem of finding a mathematical decision of a theorem might with some justice be called the problem of giving mathematical sense to a formula."[81]

by the later discovery of some not-thought-of possibility."[82]

The history of the Axiom of Choice versus Continuum Hypothesis enables us to reinterpret Zenon Paradoxes in Non-standard Analysis, and the Non-Archimedean models of irrational numbers or the history of the Parallel Postulate at very least makes all this plausible. These themselves, however, have become problems of foundation or, if you like, problems that *turn the problem situation into a philosophical problem* as a consequence of the Lakatos-Wittgensteinian internalist strategy. As Wittgenstein puts it:

> Here we stumble on a remarkable and characteristic phenomenon in philosophical investigation: The difficulty - I might say - isn't one of finding the solution; it is one of recognizing something as the solution. 'We have already said everything. - Not anything that follows from this, no *this* itself is the solution!' This is connected, I believe, with our wrongly expecting an explanation, whereas a description is the solution of the difficulty, if we give it the right place in our considerations. If we dwell upon it, and do not try to get beyond it. The difficulty here is to stop."[83]
>
> "The question - I want to say - changes its status when it becomes decidable. For a connexion is made, then, which formerly *was not there*."[84]

"Stopping here" is the crucial factor in determining what the limits of the problem situation shall be. Lakatos might have argued that the reasons are not simply subjective but traditional, and spring from the *historical* problem of their relation to each other. The demarcation here[85] ensures for Lakatos the objectivity of the subjects of mathematics and history. This demarcation also permits Wittgenstein to salvage all those problems from rationalization that we must "pass over in silence". But I am concerned about *how* and *what* it is more difficult to apeak about:

"Etica more geometrico"

or

"Geometria more etico".[86]

One can say that to stop here would make it impossible for Lakatos to write the "Changing Logic of Scientific Discovery" - a book which he always considered writing, but only existed in title form. Perhaps this is why he *maintained a distance* from the problem of *incommensurability* as applied to mathematics and history.[87] *His* "historical paradigm of rationality" was connected with the problem of demarcation between subjectivity and objectivity.[88] For him both kinds of rationality - 'subjective' and 'objectove' - presuppose *some kind of* historically cumulative (critically developing) *continuity* (i.e. growth, and growing) and excludes incommensurable (completely irrational i.e. unanderstable) local changes: 'Rational subjects produce rational history' - 'rational history produces rational subjects'. 'It seems impossible to put forth an explicit definition of his demarcation without an elaborate Philosophy of History (of Thinking) as long as you can ask in the case of ideas and thinkers if they are historical objects or subjects.

This is where contemporary philosophical foundations stop. The separation of the subject from its history is the precondition and presupposition of indirect self-referential paradoxes such as;

> "We can't understand the subject without (understanding) its history."

but

> "We can't understand its history without (understanding) the subject."[89]

5 Thinkers stand at the junction where traditions intersect and ideas at the crossroads where thinkers meet. The point where the roads of Lakatos and Wittgenstein converge is where mathematics, language, and *history* intersect. Wittgenstein's ideas may inspire a meta-linguistic, metamathematical but non-formalist description of mathematical problem-solving praxis. This description characterizes techniques for developing methods of *translation* (transference and conversion) between "what

is formal and informal" - or, if you like, between formal and informal usage of language. For Wittgenstein the process of translation starts with the *interpretation* of signs and symbols and entails that interpretation is itself a particular *human usage of signs and symbols;* but this is exactly what opens the way for description (and for the creation of new signs and symbols as well). *Formalization* is a uniquely mathematical[90] (particular) usage and *way of creation of signs and symbols,* but not (only) in the context of justification. Lakatos, however, disparaged the importance of this *translation,* asserting that in the history of thought (thinking) formalization always assumed a secondary role - as well as in mathematical justification.[91] He undoubtedly adopted a Hegelian schema of "autonomous dialectics" in this conversion of the growth of informal mathematical proofs and theories (pre-formal, formal, post-formal stages),[92] but he conceded that the trial of external (universal and eternal) formalization of the "growth of ideas" or "historical laws of thought" or "methodology of thinking" runs the risk of "devolving" to the "original" seventeenth century program of methodology. Although the problem is not expressed in his works clearly and is far from having been solved, we have to admit that he made it clear: "...the actual historical pattern may deviate slightly from... (the) heuristic pattern".[93] Noteworthy here is the fact that in all the important contexts he uses the word "pattern" and seems to be very careful and conscious about avoiding the use or synonyms of the term "laws of thought" or "necessary development".

> The word "pattern" seems to be a pivotal concept in Lakatos's mathematic (as well as historic) heuristics, functioning similarly to Kuhn's "paradigms" in *The Structure of Scientific Revolutions*. Both terms denote the "heuristic" idea of different figures or ways of thinking, i.e. ways of generating problemsolving methods, whereas in the background stands the problem of the 'Wittgensteinian' "gestalt switch". But neither articulates this idea and both fall short of *describing* it. What Kuhn in fact "describes" - perhaps even asserts -

is the existence of different structures (of normal science) without describing the structure and mechanism of change: ways of creating and recognizing patterns and interpretations, and without explaining *how* we develop *new* methods for figuring out problems and solutions and *new* ways of recognizing something as problem or solution. If looking at what Lakatos does "through Kuhnian spectacles" is a "philosophy producing examples"[94] it is by no means the worstof its kind. Lakatos adopted a rationalist and analytic view of this dilemma providing *methodological* descriptions of the "dialectic" change of science, creating not only examples but describing *cognitive patterns* with the assistance of *critical counterexamples*. This locates him at the point of intersection of the rationalist and historicist traditions. He might have replied as a criticism and an excuse to both the endeavors of the *"logical"* and *"historical"* descriptions with Wittgensteinian words:

"both to you and to myself I can only give examples of the application."[95]

"For the point of a new technique of calculation is to supply us with a *new* picture, a *new form of expression*; and there is nothing so absurd as to try and describe this new schema, this new kind of scaffolding, by means of old expressions."[96]

5.1 What Wittgenstein and Lakatos both reject from the *Leibnizian* (and also from the Hegelian) program is *justification*. - We may summarize Leibnizian tradition "unhistorically" and succintly in a Lakatosian vein,[97] as a branch of studies for establishing formal languages to settle disputes and justify truth. This idea was carried over into the post-Kantian folklore of apriorism versus empiricism at the beginning of the century, and it is referred to today as the Frege-Carnap-Quine tradition. It created the formal procedural idea of proof and separated the "empty", "neutral", "meaningless" or "meaning-lost" idea of truth from meaningloaded evidence. Although the constitutive idea of the program was truth, what we got as a result is formal systems with some kind of mechanical checkability. The Leibnizian program exhibited no real intuition for getting at what human language and reasoning is - the modest alternative of the language of God an Certainity would have been enough if it could have been made to be calculable. Here

as well as in the case of formal proofs language is mere metaphor - 'nothing more' than semantic definitions of truth which establish calculations based on formal rules. This image of formal logic as *Science of Thinking* through some "concept-stretching" and "concept adjustment" was declared to be the *Logic of Scientia*. (Forawhile even linguists tried to confine Wittgenstein to their "syntactics versus semantic empire of pragmatics 'cum granulo salis'."[98])

5.2 On the other hand, in the *Hegelian* tradition - which had an obvious impact on Lakatos's and possibly - even if not directly - also on Wittgenstein's *later* works - the content of ideas is historically given but not linguistically produced.[99] This tradition gave rise to the cumulative idea of knowledge and 'laws of *growth*', and the point where it has obviously made an impact has been where twentieth century philosophy of science has reinterpreted the Cartesian idea of methodology and posed the question of incommensurability and the possibility of paradigmatic change in *logic* i.e. in the 'science of thinking' as opposed to the 'laws of thought'. One fundamental problem of this school which has extended to contemporary philosophy of science was that of the rational understanding of *change* in new interpretation *as change of ideas and thinking:* the problem of historical change (and identification) in the case of ideas, concepts and the various ways of presentation (styles, paradigms, standards) employed in thinking.

> Popper - though regarded as the greatest opponent of Hegelian dialectics - saw this clearly: "it is only the *history* and *development of scientific theories* (italics mine - A.B.) which can with some success be described in terms of the dialectic method."[100] But his critique doesn't draw the obvious conclusion that should this be true; then why must the development of scientific theories be commeasured with rules of inference in deductive logic?[101] On the basis of their *own* - critical and comparative - *standards* of rationality, Popper and Lakatos rightly criticize Hegel for his mono-methodological description of change in conceptual framework, omitting from consideration that in the age of post-Newtonianism *thinking* was also a "pre-determined, inevitable process" for the Euclidean and Inductivist programs, where individual creativity or rational criticism "played no essential

role" either. As Popper himself says in the lines preceding the above quote, "...in Hegel's time... logic was the *theory* of thinking and...the laws of logic were usually called the laws of thought;"[102] The work of Whewell - an early representative of the critical and fallibilist tradition - as well as of Popper and Lakatos themselves otherwise reflects the influence of Hegel[103] and his historicism, which put forth an approach for describing the "abstract characteristics of the nature of Spirit," as well as the "means Spirit uses in order to realise its Idea", where "the question of the *means* by which Freedom develops itself to a World conducts us to the phenomenon of History itself."[104]

In other words the problem of interpreting human mental operations rationally may be identified with the Hegelian problem of "logic of philosophy vs. philosophy of logic" and the "philosophy of history vs. history of philosophy" with the rational interpretation of social actions. Both prepared the problem of describing *processes* as developments in human (man-made) systems set up and established by thinkers. Lakatos didn't state this identification as "logic of history versus history of logic" but, similarly to Wittgenstein's *interpretations*, his *methods* of rationally (historically) reconstructing these processes constitute one program of *internal* description as well. *One result of the program is that it proved to be a problem of rationality describing change in systems as change of systems.*

6 Since you've caught me in the act of talking about Lakatos's and Wittgenstein's ideas in terms of quasi-historical and quasi-linguistic "foundation" of mathematics - with a somewhat simplified and metaphorical demarcationist strategy - as *change* in logic and history of thought,... I'll show you there's nothing up may sleeves, just as Lakatos did in his article, *What does a mathematical proof prove?...* Like him, I admit right away, "This, in fact, *is*, just what I should *like* to do."[105] To put it in Wittgenstein's words: "I would like to make a new connection." To outline this formal connection in greater detail, I will now turn to a brief discussion of what (mathematical) heuristics means and what it has to do with foundations and philosophical foundation.

6.1 In condensing what emerges from the thoughts of Wittgenstein and Lakatos into one question - "can heuristics serve as a foundation", the two words - heuristics and foundation - appear contradictory. At first sight nothing is more antinomical to the deductivist style than the process of creation and the process of foundation of ideas.[106]

In Lakatos's view "This controversy between *dogmatists* - who claim that we

> can know - and *sceptics* - who claim that we either cannot know or at least cannot know that we can know and when we can know - is the basic issue in epistemology. In discussing modern efforts to establish foundation for mathematical knowledge one tends to forget that these are *but a chapter* (Italics mine - A.B.) in the great effort to overcome scepticism by establishing foundations for knowledge in general."[107]

In the history and philosophy of mathematics the question is raised whether it is philosophy or mathematics which is related to "logical foundation"? Wittgenstein wrote, "The philosopher must twist and turn about *so as to pass by* the mathematical problems - ...which would have to be solved before he could go further."[108] Lakatos took responsibility for the *normative criteria* stipulating that the deductive style of mathematics had to be exchanged for the language of heuristics. According to Wittgenstein, philosophy should have nothing to say about mathematics - it should leave everything even mathematics as it is. But these words can be interpreted as emphasizing what it "in fact" is. In other words (1) we have to accept mathematics as historical *reality* of human praxis, which might be described as working mathematics; furthermore it does not need to be separated from its history, that is (2) we have to describe it as a *historical* reality of human praxis. I use the term *"descriptive foundation"* in this sense. This "foundation" paves the way for (solving) problems determining whether we accept it as we described it in its historical reality as *our* human praxis. The "empirical basis" of this description is the history of mathematics itself. The contradiction exists only as long as foundation and logical foundation is construed as normative (external) methodological *justification*.

6.2 Heuristics - in Polya's terms - means something different. It may read variously as a "school of thought", a "How to Solve It", or citing Whewell, the "Art of Mathematical Discovery". Polya did not raise foundational questions nor did he propound philosophical problems. Instead, he attempted to describe strategies in everyday language which the mathematician - or rather the math teacher - can use to articulate, define, and solve problems in everyday praxis; he no doubt accepted the fallibility and the differences of human reasoning in mathematics. He shows us *how* to make and improve conjectures and work out methods to find proofs and calculate solutions, providing as well as using *mete* historical *examples* to *illustrate*. To paraphrase his definition of method: "An idea once applied is a trick, twice applied a method". And as Lakatos would have said, applied three times it is "applied" history.[109] Those who are *familiar with* as well as those who *dislike* formalism usually add: 'applied four times it is still an idea, but it's boring - perhaps even tendentious'. - This was a misunderstanding. But the situation changed, it outgrew itself and became a subject for reinterpretation.

7 With the assistance of the Popperian Philosophy of Science and a historicized version of Hegel's dialectics Lakatos attempted to develop the internalist tradition of the school of Hungarian mathematics associated with names like Fejér, Neumann, Rényi, and Polya in describing the rise and appraisal of mathematical theories on the *meta-historical* level as well, but left behind the task of working out a systemized theory of (their) heuristics[110] i.e. *The CHANGING LOGIC of Scientific Discovery*. Pólya had begun working on this *internally* in his methods of teaching mathematics and it remains for us to improve upon it. He didn't like the idea of an express *'theory* of heuristics'. In the preface to his world-famous treatise on *Problems and Theorems in Analysis* before presenting some aphorisms in Wittgenstein's style he expresses his scepticism about its practicibility:

> "General rules which could prescribe in detail the most useful discipline of thought are not known to us. Even if such rules could be formulated, they would not be very useful. Rather then knowing the correct rules of thought *theoretically*, one must have them assimilated into one's flesh and blood ready for instant and instinctive use. Therefore, for the schooling of one's powers of thought only the practice of thinking is really useful. The independent solving and challenging problems will aid the reader far more than the aphorisms which follow, although as a start these can do him no harm."[111]

If he opposed anything at all, he contrasted description to prescription as a 'foundation'.

Some may object to using the word 'foundation' claiming that every foundation takes place on a meta-level and every complete description needs a meta-language and the framework for this in the case of (mathematical) heuristic as well is provided in contemporary logic. Such a view merely reiterates the idea of formal languages. But what the history of mathematics seems to reveal is that if there is a clear description of a problem *defined by the history of ideas*, which is *interpreted* and/or *reconstructed*, then certain external foundations - such as descriptions - may eventually *incorporate from meta-level to the subject itself*.[112] This is a possible outcome of the internalist strategy both in mathematics and philosophy, and might might corroborate Lakatos's as well as Wittgenstein's standpoint. This phenomena is observable in the case of Greek geometry of proportions, seventeenth century algebraic foundation, analytical geometry, set theory, and - it seems to me - it's going on right now with logic in the artificial intelligence framework.

> The question whether we need to have humans as "translators" or "interpreters" in order to convert informal mathematics into formal "languages" in a form to be submitted to a machine has turned into a problem of heuristics: "what is to be converted", that is a problem of the *'how'* or the *'what'*. It provided a theoretical analysis of mathematics to describe that activity which is common to the various theories in *formal representations*.

The job of heuristic isn't one of describing our everyday language, but as Wittgenstein suggested, one of providing a

new informal theory or paradigm of what human languages are - how we work and think with and within them. We can't describe the language of our historical universe because, as Wittgenstein said, "*the limits of my language* mean the limits of *my* world"[113] (italics mine - A.B.) which changes as we do. The implication of Lakatos's ideas is that *'our* rationality is the history of our thinking in this (changing) world'.

Although we may attempt to describe our changing historical logic of this world as the history of *our* rationality, even if we fail we can still have well-expressed problems - that is, we can try to describe and understand them. To draw upon the words of Karl Popper once again: "...every rational theory, no matter whether scientific or philosophical, is rational insofar as it tries to *solve certain problems:* A theory is comprehensible and reasonable only in its relation to a given problem situation, and it can be rationally discussed only by discussing this relation."[114]

8 Heuristics presents the problem situation. *If it is not given* the question is *how* it can be represented - when explication involves the problem of discussion, when explication is itself the problem of the discussion. One way of representation is to describe the language of the *problem;* another is to reconstruct its *history* in an understandable language. When misunderstandings arise they can change the problem situation: they describe a (*new*) *subject* by adding new questions to old ones. This reinterpretation - relating new subjects to old ones - assumes a reflexive usage of language. By means of *formalisation* such language can 'bespeak itself' - can be made self-referential - without problems and without problems having been given.

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Notes

1. In [1972] Popper says, "...the most important mathematical objects we discover - the most fertile citizens of the third world - are *problems*, and new kinds of *critical arguments*." (p. 138.) Popper [1963] doesn't speak *expressis verbis* about *mathematics*, but it's obvious from the context of the motto (pp. 197-200.) that he didn't intend to exclude it from the problems as related to theory and solution. (Cf. p. 230) where he translates "a rhyme for the epistemc-logical nursery" into English by W. Busch the German caricaturist and poet who also wrote such "caprices" as *Hans Huckebein* and *Schein und Sein* (1909, p. 28.),

> "Twice two equals four: 'tis true,
> But too empty and too trite.
> What I look for is a clue
> To some matters not so light."

2. Lakatos, [1976], p. 142.

3. It is *deductive* insomuch as it explains the *formal deduction*, and rational insomuch as it *explains* - makes plain - what is going on in the *deductive* system - and why. In the deductivist style, however, these explanations are alien to the system.

4. *Ibid.*, p. 144.
"What we cannot speak about we must pass over in silence."
The deductivist version of this reads:
"Whereof one cannot speak, thereof one must be silent.":
'The genuine intuition of the good mathematician is inarticulable!
"The logic of discovery is not logic."
For a critique and description of this contrast see Kreisel [1956], [1967a] and also his comments on Mostowski and Bar-Hillel [1967b-c] and cf.
Feferman [1978], *Conclusion*, "...much more from logic will have to be recognised as basic and incorporated into such a theory of mathematics [a more realistic one inspired by Lakatos' failures and successes]. It would be best to reserve the name "the logic of mathematical discovery" for what which is *yet to come*." The deductivist attitude which held heuristics to be rhetoric in opposition to logic is not left by wholly intact these arguments. They say, "Those who have resorted to explanation haven't understood the deduction in the first place."
Cf. Weil [1978] "History of Mathematics: Why and How?" vs. Kreisel [1967d], "Mathematical Logic: What Has it Done for the Philosophy of Mathematics?"

5. Lakatos objected to the latter - that deductivists don't give the problems, moreover they hide them - and both

were the focus of his critique in the case of retrospective historiography and formal foundations. (Cf. Berkson [1976], pp. 47-49., 54.)

6. The "usual sense of the word" *here* is not just the deductivist (formalist) sense according to which, in Lakatos's words, "mathematics is identical with formalised mathematics", but that common (sense) mathematical praxis where the foundation of a theory applies to the subject from Pythagoras' Theorem to Fermat's Last one.

7. Lakatos [1978a], p. 118.
Feyerabend in his commemoration [1975] p. 1. says:
"He was a rationalist, for he thought that man had the duty of using reason in his private affairs as well as in any inquiry concerning the relation between himself, nature, and his fellow man. He was an optimist for he thought that reason was capable of solving most of the problems arising in the course of such inquiry.

8. Lakatos [1978b], p. 202; [1976], p. 144.

9. This "both" is a "clue", if not the "solution" to the problem situation. Ideas can be (presented as being) external from the point of view of different kinds of subjects and for different kinds of subjects - from the point of view of mathematics or history and for our present or past analytical or historical reasoning. Most of the critics of Lakatos ally with one or the other position. Berkson considers Lakatos's critique of Logicism and Formalism external as compared to Gödel's (internal) "critique, for example. (See Berkson [1978], p. 299ff.) Historians - criticizing Lakatos's "historical reconstructions" as caricatures of the 'real history' - haven't seen the point of this "both" in Lakatos's methodology either. In Lakatos's aspiration his "normative-historiographical version of the methodology of scientific research programmes supplies a general theory of how to compare rival *logics of discovery* [italics mine - A.B.] in which (in a sense carefully to be specified) *history may be seen as a "test" of its* rational reconstructions." (Lakatos [1971a], p. 123.) McMullin realized the methodological intention, but the noted historian rejected the "carefully specifiable" ideological implications of the second normative meta-historical comparative function. (Cf. McMullin [1971].)

11. Lakatos [1978b], pp. 110-111.
Lakatos's 'demarcationism', a vestige of the so many "isms" he lived to survive, constitutes - in his own terminology - the protective belt of his "*normative methodology*" as a strategy for defending its "hard core": the rejection of the separation of the problems from their history. (He wasn't explicit, however, as *to what kind of subject* these

demarcations are external to, when he remarked "...the näive school concepts of static rationality like *apriori-aposteriori, analytic-synthetic* will only hinder its emergence." [1965-1978b]) This separation in the case of the subject of mathematics is what he was challenging, in regard to both its history and the "formalist school" of its foundation.

Noteworthy here is that Wittgenstein also remarked, "What I am saying comes to this, that mathematics is *normative*. But "norm" does not mean the same thing as "ideal"." (*RFM*, VII. 63.)

12. See Lakatos [171b] "Replies to Critics", pp. 174-182. Lakatos didn't word this conversion of his dictum as strongly as I have here, paraphrasing his replies and the quotation, where he speaks about a research program which is rather *historiographical* than *methodological*. But he did ask elsewhere, "...what is the 'nature' of mathematics, ...the nature of *informal* theories... of the potential falsifiers...? Are we going to arrive, tracing back problemshifts through informal mathematical theories to empirical theories, so that mathematics will turn out in the end to be *indirectly empirical*,...? Or is *construction* the only source of truth to be injected into a mathematical basic statement? Or *platonistic intuition*? Or *convection*? The answer will scarcely be a monolithic one. Careful historico-critical case-studies will probably lead to a sophisticated and composite solution." (Lakatos [1967b], p. 41.) In paraphrasing Lakatos I have attempted to imply his answer to these questions.

13. *RFM*, III. 80., 81.

14. Wittgenstein [1921], (TR), 4.125 Cf. *RFM*, VII. 8. "Proof must shew an internal relation not an external one.." "The existence of an internal property of a possible situation is not expressed by means of a proposition: rather, it expresses itself in the proposition representing the situation, by means of an internal property of that proposition.

It would be just as nonsensical to assert that a proposition had a formal property as to deny it." (TR, 4.124.)

15. But they were not explicit at all,
"ZETA: Start? Why should I *start*? My mind is not empty when I discover (or invent) a problem.
TEACHER: Do not tease Beta. Here is the problem:..."
"(The dialogue takes place in an imaginary classroom... It is at this point - after the stages *problem* and *conjecture* - that we enter the classroom.)"
"BETA: And where do you get your idea from?
ZETA: It is already there in our minds when we formulate the problem: in fact, it is in the very formulation of the problem." (Lakatos [1976], p. 70. and pp. 6-7.)

Wittgenstein's *Remarks* starts:

1. "We use the expression: 'The steps are determined by the formula...' ... 'Is (this) a formula which determines (the steps)?' One might address this question to a pupil in order to test whether he understands the use of the word 'to determine'; or it might be a mathematical problem to work out...
2. ...What is the criterion for the way the formula is meant" (*RFM*, I.1., 2.)

"BETA: ...I had no problems at the beginning! And now I have nothing but problems!" (*Ibid.*, p. 104).

16. "Even if I think of a proof as something deposited in the archives of language - who says *how* this instrument is to be employed, what it is for?" (*RFM*, III.29.)
"How can one criticize a definition, in particular, if one interprets it nominalistically? A definition is then a mere abbreviation, a tautology. What can one criticize about a tautology? ... But how can a definition have explanatory power or afford us any new insight." (Lakatos, [1974], p. 144.)

17. Wittgenstein [1932], p. 200.

18. *Ibid.*, p. 198.

19. Wittgenstein [1956], III. 28. (*RFM*)

20. *Ibid.*, 41.

21. *Ibid*, 29.

22. Lakatos [1976], pp. 89-90

23. Cf. for example Anderson *Mathematics and the "Language Game*, or Bernays' *Comments on L. Wittgenstein's Remarks on the Foundation of Mathematics* (in Benacerraf [1964], pp. 510-528.): "Wittgenstein argues as though mathematics existed almost solely for the purposes of housekeeping" saying that Wittgenstein didn't know what was going on in the highly mathematized Philosophy of Mathematics."

24. Cf. Stern, L. [1985], "Hermeneutics and Intellectual History", pp. 288-295.
"But are the four terms of the expression 'perfectly well-known terms' really all perfectly well-known terms?" (Lakatos [1962], p. 3.) "interpreted facts have two very different functions in the growth of science. They may serve as tests of already proposed theories ... they may also serve as a stimuli to new theories..." (Lakatos [1978], p. 208)

25. Lakatos [1976], p. 93. and n.1.

26. See Marchi [1976] and Wang [1984] and cf. Hallett [1979] and Machover [1983].

27. Hacking [1979], p. 384.

28. *Ibid.*, p. 385.

29. They are internal from the point of view of philosophy, but argue for internal heuristics from the point of view of mathematics. (Cf. again Berkson [1978] p. 229.) "Lakatos's critique is largely in harmony with Gödel's, but is an external critique..."
Cf. my notes 8. and 10. See also Machover [1983] and Hallett [1979] as to the results of this internal critique and also in connection with its relation to traditional philosophies ot mathematics.

30. Lakatos [1971a], p. 103, n.1.

31. Lakatos [1974], p. 140, n.3.

32. Hacking informs us, "he [Lakatos] read the *Remarks* carefully when writing 'Proofs and Refutations'." [1979],p.391.

33. Lakatos [1976], p. 146.

34. Cf, Niiniluoto [1984] pp. 112ff., 134-137, 201-208; Lakatos [1970], and [1978b] "What does a mathematical proof prove?" pp. 61-69.

35. Cf. Lakatos [1967], pp. 40-42., See Berkson [1978] and Bloor [1984] especially chapters 5 and 6; Cf. M. Leich and S.H. Holtzman [1981]: Communal agreement and objectivity", pp. 20-22.

36. See Dummett [1959], pp. 324-335.; Gordon Baker, "Following Wittgenstein" in Holtzman and Leich [1981] pp. 52-58, 66-69.; (Lakatos *Ibid.*), Berkson [1978], p. 302.

37. Lakatos [1978b], pp. 24-35, A. Ambrose "Is Philosophy of Mathematics 'An Idleness in Mathematics'?" and "Mathematical Generalisations and Counterexamples" (in Ambrose and Lazerowitz [1984], pp. 192-214, 167-175.)

38. Lakatos [1978b], pp. 3-20. "Infinite Regress and Foundation of Mathematics". (See also *Ibid.*, pp. 108-9. n.2., pp. 129-130; and Wright [1980] p. 9.; Klenk [1976] pp. 42-43, 124; Niiniluoto [1984], p. 177.)

39. See Fine [1978], pp. 328., 339.; Lakatos [1976], pp. 103-4 and n.1., 2.; Ambrose [1984] pp. 167-180; and *RFM* V.34, 35.

40. Cf. Lakatos [1978b], p. 43.

41. See Davidson [1968] "Action Reasons and Causes", pp.79-87.

42. For strong arguments see Davis [1972] pp. 258-263.

43. Lakatos [1978b] II. 3. "Mathematic is Quasi-empirical", pp. 30-35.

44. Hersch [1978], p. 149.

45. *RFM* II. 33.; I. 74.

46. Hacking [1979], p. 393.

47. *Ibid.* In the acknowledgements to his PhD thesis Lakatos himself remarked that one of his 'ideological' sources is Hegel's dialectic. (See the editors note in Lakatos [1978b], p. 70.)

48. *RFM* III. 71.

49. *RFM* I. 66.

50. Lakatos [1976], Introduction and pp. 142-143. See also [1978b], p. 40. and G. Radnitzky "Progress and Rationality in Research", [1977], p. 58.

51. Lakatos [1978b], p. 69.; and Hersch [1978], *ibid.*

52. *RFM* I. 6. See also III.10, 28-31.

53. *RFM* V.16., IV. 11., 13.

54. *RFM* III.4.

55. *RFM* III.22., 46., 47., V. 40.; Lakatos [1978b], "The Method of Analysis-Synthesis", pp. 93-94.

56. *RFM* III.25., 28., 31., 41., IV. 20., V. 45.

57. *RFM* VI.49.

58. Lakatos [1976], pp. 114-115.

59. *RFM* V.9. "However queer it sounds the further expansion of an irrational number is a further expansion of mathematics. (*Ibid.*)

60. *RFM* A I. 20. Cf Lakatos [1976] p. 29., n., 38 n.1., and Ambrose [1959], pp. 437 ff.

61. *Ibid.* pp. 114., 116.

62. Lakatos [1968], p. 133., (Cf. note 16. of this paper)

63. *RFM* III.26., 27.

64. According to Heyting, "a linguistic accompaniment is not a representation of mathematics; still less is it mathematics itself." (Heyting, "The Intuitionist Foundation of Mathe-

matics", p. 42. in: Benacerraf [1964]. Wittgenstein says, "To understand a sentence means to understand a language" (*PI* 199.) "*that* is only applying one's understanding." (*PI* 146) "...but then can't an *application come before my mind?*" (*PI* 141) "To understand a language means to be a master of technique." (*PI* 199) "...the process may consist merely in our saying 'Therefore' or 'It follows from this' ..." but "may go on paper, orally, or 'in the head'." (*RFM* I.6, cf. Klenk [1976] pp. 19-25.)

65. Peter [1943], p. 16. (in Hungarian)

66. *RFM* III.50.
"The geometrical illustrations of Analysis is indeed inessential; not, however, the geometrical application. Originally the geometrical illustrations were *applications of Analysis.*" (*RFM* V. 29.)

67. See Lakatos [1976], pp. 127-128. *Editors note:* "In other words this method consists (in part) of producing a series of statements $P_1,\ldots,P_n$ such that P_1 and...and P_n is supposed to be true of some domain of interesting objects and seems to imply the primitive conjecture C. This may turn out not to be the case - in other words we find cases in which C is false ('global counter-examples') but in which P_1 to P_n hold. This leads to the articulation of a new lemma P_{n+1} which is also refuted by the counter-example ('local counter-example'). The original proof is replaced by a new one which can be summed up by the conditional statement
$$P_1 \,\&\ldots\&\, P_n \,\&\, P_{n+1} \rightarrow C.$$
The (logical) truth of this conditional statement is no longer impugned by the counterexample (since the antecedent is now false in this case and hence the conditional statement true)." This "*in part*" just refers to that little "logical" problem that "in this case" why wasn't it the case that we formulated the conditional statement
$$P_1 \,\&\ldots\&\, P_n \,\&\, \neg P_{n+1} \rightarrow C,$$
when the conditional statement is also true (and we would get the 'exception - barring' method). - I.e. what does the WAY, the MODE of "articulation" depend on, and why isn't it articulated?

68. Peter [1943], p. 236.

69. Ibid. p. 242.

70. Cf. Tóth [1977], pp. 396-405.

71. *RFM* VII.22.

72. *RFM* V.25.

73. Zettel § 294.

74. Ambrose [1984], p. 180., (Cf. *RFM* III. 31., I. 166.)

75. Cf. Tóth [1972]

76. "The extensional definitions of functions, of real numbers, etc. pass over - although they presuppose - everything intensional, and refer to the ever-recurring outward form." (RFM V. 36)
"The cut is an extensional *image*." (RFM V. 34)
"Let us not forget that the division of the rational numbers into two classes did not *originally* have any meaning, until we drew attention to a particular thing that could be so described. The concept *is taken over from the everyday use* of language and that is why it immediately looks as if it had to have a meaning for numbers, too." (*Ibid.*)

77. "But that everything can (also) be *interpreted* as following, doesn't mean that everything is following." (*RFM* VII. 47.)
"...it is surely important that I can form the decision with the (general) interpretation so to speak once and for all, and can hold by it, and do not *interpret* afresh at every step." (*Ibid.* 48.)
"It might justly be asked what importance Gödel's proof has for our work... - The answer is that the *situation*, into which such a proof brings us, is of interest to us. 'What are we to say now?' - That is our theme." (*Ibid.* 22.)

78. Peter [1943], pp. 252-253.
"Future development shall certainly broaden the framework, even though we may not be clear as to *how* it do so. (italics mine - A.B.) The moral of the lesson is that mathematics is not static and closed, but alive and constantly in the process of developing: no matter how hard we try to confine it to a closed form, it bursts forth anew, with fresh life and vigor." (*Ibid.*)

79. *RFM* VII. 40.

80. Lakatos [1978b], p. 67. (Italics mine - A.B.)

81. *RFM* V. 42.

82. Lakatos [1978b], p. 69.
Feferman([1978], p. 317.) poses the question: *"Is there no end to guessing?*... The professional mathematician knows rather well what sort of thing will work for certain problems and what won't. ... the guesswork finishes with the mathematician's successful struggle to solve a problem or complete a proof. It is true that *results* are viewed in changing perspective over historical periods. ... But this is quite a different picture from that given by Lakatos

of endless guesswork."
"Interpretation comes to and end." (*RFM* VI. 38.) "Here it is of the greatest importance that all or the enormous majority of us agree in certain things..." (*RFM* VI. 39.)

83. *RFM* A I.2., Cf. *Zettel*, 314.
'Where to stop?' - constitutes a real difficulty in contemporary discussions. (Cf. Feferman's and Lakatos's position and n. 82. of this paper.) Lakatos didn't want to stop - for him the problem of 'where to stop' was a philosophical one. He wanted to stop dogmatism and make it plain: "There is nothing wrong with an infinite regress of guesses". He dwelled upon his philosophical position: *critical fallibilism* was for him one of the leading historical problems of our age. But he attempted to define objective progression. He posed a "new central question" in philosophy and methodology of science: "How do you improve your guesses?", which he viewed as having grave ethical and political implications. (Cf. Lakatos [1962], p. 10.) "In philosophy it is always good to put a question instead of an answer to a question" (*RFM* III. 5.)

84. *RFM* V. 9.
"(I once wrote: 'In mathematics process and result are equivalent'.)" - reiterates Wittgenstein examining his own position. (*RFM* I. 82., Cf. *TR* 6.1261)
"...you are incorporating the result of the transformation into the kind of way the transformation is done." (*RFM* I. 86.)
"When we follow the laws of inference (inference rules) then following always involves interpretation.." (*RFM* I. 114.)

85. Lakatos's demarcation between the Popperian "second world of feelings, beliefs, consciousness" and the "third world of objective knowledge" and the (parallel) one between $history_1$ (the set of historical events) and $history_2$ (a set of historical propositions) plays a crucial role in Lakatos's program of philosophy and methodology of science. (See n. 88. of this paper and cf. Lakatos [1971a], pp. 119-122 and notes.)

86. See Toth [1977] for an internal critique of this distinction.

87. As Feyerabend claimed, Lakatos was a rationalist; sometimes he also had "a queer temptation which expresses itself in my inclination to say: I cannot understand it, because the interpretation of the explanation is still vague" - as Wittgenstein said (*RFM* VI. 40). But whether he would admit - "in a carefully specifiable way" - some *cognitive* or *historical* universals in the explanation or in the understanding of revolutionary changes remains an open question.

88. "Belief, commitment, understanding are states of the human mind. They are inhabitants of the 'second world'. But the objective, scientific value of a theory is a 'third world' matter... Thus demarcationists share *a critical respect for the articulated*. They appraise only what is *articulated* in human knowledge. The demarcationist readily agrees that articulated knowledge is only the tip of an iceberg: but it is exactly this small tip of the human enterprise wherein rationality resides." Lakatos [1978b], pp. 109-110. See also Hacking's account of this point in his [1979], pp. 383-386. and (cf. Lakatos [1970] p. 104., [1971] p.122.)

89. H: The following sentence is meaningless if this sentence is meaningless.
S: The previous sentence is meaningless if this sentence is meaningless.

(There is nothing wrong with the formalization:
$M(x)$ =: 'x is meaningful'
H: $M(H) \longrightarrow M(S)$
S: $M(S) \longrightarrow M(H)$
$\models M(H) \longleftrightarrow M(S)$
'The other sentence is meaningless iff, this sentence is meaningless.'
But: It is not this sentence you don't understand if *this* is the sentence you don't understand!
$(M(H) \longleftrightarrow M(S) \neq H \vee S)$
In a lighter vein, consider the following:
HEGEL: Your theorem is true.
FERMAT: Can you prove it?
HEGEL: No, - but history will...
FERMAT: I'm glad you 're so sure, but can you prove *that*?
HEGEL: That's already been done, Fermat!

Furthermore, if we convert,FERMAT to HEGEL, 'true' to 'false' and 'prove' to 'disprove' an explanation is provided for the discussion above. If HEGEL and FERMAT are two *contemporary mathematicians* they are still looking for the proof. The *Bieberbach Conjecture* and the *Riemann Hypothesis* were proved and the *Mertens Conjecture* was disproved in 1984 - if the proofs proved to be accepted after being examined in detail by the mathematical community at large.

90. "Thus if I say: 'It's as if this proposition expressed the *essence* of form' - I mean: it is as if this proposition expressed a property of the entity *form*! - and one can say: the entity of which it asserts a property, and which I here call the entity 'form', is the picture which I cannot help having when I hear the word 'form'." (*RFM* I. 74.)

91. "I should like to reverse the order: We should speak of formal systems only if they are formalisations of established informal mathematical theories. No further criteria are needed. There is indeed no respectable informal ancestor." (Lakatos [1978], p. 52.) "But all of them are

fallible, not less fallible... than the ordinary classical mathematics which was so much in want of foundations." (Lakatos [1978b], p. 34.) "Triviality and certainty are *Kinderkrankheiten* of knowledge." (Lakatos [1962], p. 3.)

92. See Lakatos [1978b], p. 61.; also [1976], pp. 146-148 and n.1. where - after using the terms "Thesis - Antithesis - Synthesis" - he says: "The Hegelian language which I use here... has, however, its dangers as well as its attractions." Maintaining previously quoted (p. 14-15.) 'Hegelian conception of heuristics' as the "autonomous dialectic of alienated human activity", he notes: "...human activity can always suppress or distort the autonomy of alienated processes and can give rise to new ones" - he can't resist adding: "The neglect of this interaction was a central weakness of Marxist dialectic." (Cf. also Lakatos [1974c-78a], p. 139 and n.1.)

93. Lakatos [1976], p. 127 n.

94. Cf. Kuhn [1970b].

95. *RFM* VI. 40.

96. *RFM* II. 46.

97. ...i.e. with "tons of salt", as Kuhn parodizes Lakatos. (See Kuhn [1970a] in: Lakatos [1970], p. 256.)- "I look at continuity in science through 'Popperian spectacles'. Where Kuhn sees 'paradigms', I *also* see rational 'research programmes'. (*Ibid* p. 177.) One aspect of the Kuhn-Lakatos debate is the *problem of* (historicizing!) *rationality*, as Kuhn himself realizes (see Kuhn [1970b], pp. 143-144.); but there is yet another feature of historicizing rationality: the rational - internal or external - *description* of change (in scientific thought-processing) (Cf. *Ibid.*, p. 93.)

98. Cf. Whorf [1956] and Smith [1972], pp. 48-51. vs. Naess [1968], pp. 155-157.

99. ...at least not in the sense of formal languages or pre-Chomskian grammatics... This doesn't imply that this tradition has no general theory of language of its own with its specific problem of representing interpreted systems of (highly organized) knowledge, *and thinking as a "process"*. (Cf. Gauthier [1984], pp. 308-309, and Niiniluoto [1984], "The Evolution of Knowledge", pp. 61-71.).

100. Popper [1963], p. 328.

101. Cf. Popper [1963], pp. 317-323. and p. 327.

102. *Ibid.*, Cf. Lakatos [1976], p. 104.

103. See Whewell [1860], pp. 307ff.

104. Hegel [1956], pp. 17., 20.

105. Lakatos [1978b], p. 61. added: "I am quite convinced that even *the poverty of historicism is better then* the complete absence of it - always providing of course that it is handled with the care necessary in dealing with *any explosives*." (Italics mine - A.B.) It can help to realise that we can try to describe the role that heuristics plays both on "metaphysical" (i.e. methodological) and "meta-mathematical" level (i.e. in the process of formal interpretation).

106. "Justificationist epistemology has... two main problems: how to discover (ultimate) Truth and how to prove that it *is* the Truth... (1) the problem of foundations of knowledge (the logic of justification) and (2) the problem of the growth of knowledge (the problem of method, logic of discovery, heuristics)." (Lakatos [1978a], p. 196.)

107. Lakatos [1978b], pp. 3-4.

108. *RFM*

109. Cf. Polya-Szegő [1972], iii.

110. The problem (of *heuristics*) is presented in our contemporary history and philosophy of science, as well as in model theory, in non-Cantorian set theoretical methods of *forcing*, and in terms of the problem of structural representation of knowledge in the artificial intelligence framework.

111. Polya-Szegő [1972], *Preface* to the First German Edition. [1924] "One should try to understand everything: isolated facts by collating them with related facts, the newly discovered through its connection with already assimilated...

    There is a similarity between knowing one's way about a town and mastering a field of knowledge, from any given point one should be able to reach any other point... If one is very well informed indeed, one can even execute special feats, for example, to carry out a journey by systematically avoiding certain forbidden paths which are customary - such things happen in certain axiomatic investigations." (*Ibid.*)

112. This was always the case - although to varying degrees - in new informal branches of mathematics: the "theoretical" and the "empirical bases" were the same: namely the history of mathematical reasoning itself. This history was "alienated" in the form of operations and *"heuristical"* methodological reflections in formalized or symbolic languages and structures, even though mathematics was involved in rational and not historical re-construction. To

provide a mathematical interpretation of historical representation of knowledge what we require is a good *heuristical concept* of non-linear, pattern-generated thought-processing as a *counterexample* (to deductive logic).

113. Wittgenstein, *Tractatus*. 5.62.

114. Popper [1963], p. 199.

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THE ROLE OF A METAPHOR IN THE BIRTH OF GENERATIVE GRAMMAR

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Noam Chomsky is not only the most influential linguist of the second half of the century, but also a leading political writer. So the question may be put if there is a connection between his political and linguistic ideas. At first sight, the answer seems to be no. He never appeals to political philosophy in order to justify his linguistic theories. Nor does he claim that his linguistic findings prove that American foreign policy should be changed. However, he himself believes that there is a connection, even if not a direct one. He says both his linguistic and political ideas derive from the same general view of human nature. This paper is an attempt to scrutinize the heuristic role Chomsky's view of man played in the birth of generative grammar. First, I will describe Chomsky's view of man. Then I will sketch how these ideas came to be connected to linguistics; and finally, the particular points of connection will be considered.

Chomsky's rather implicit philosophy of man is built on three notions: creativity, freedom and constraints. It may be best summarized perhaps by saying that man is a being that creates freely within constraints. This may sound somewhat paradoxical. First freedom is most commonly regarded as the absence of constraints. Second, constraints seem to restrict creativity. Nevertheless, the air of paradox disappears as soon as we begin to understand these notions in the same way as Chomsky does. The constraints in question are our common biological characteristics, which are determined genetically. They do not prevent us from being free. The fact that we may not have wings does not make us unfree. Hence constraints like this do not contradict freedom. On the contrary, Chomsky says they are constitutive of it. Freedom involves capability to resist external pressure. A free man cannot be controlled and coerced

at will. He is always capable of saying no. A particularly important source of this resistance, Chomsky says, is our biological nature. No external pressure can make us do certain things, just as no propaganda can make us grow wings. So our freedom stems partly from our biological characteristics. Biological constraints are essential to creativity as well. An activity can only be called creative, if it takes place within a framework of rules. Activity without regularity is just acting at random. Throwing patches of paint at a canvas is not artistic creativity unless it shows some pattern. Chomsky is convinced that some of the rules that govern our activities, and make them, in this way, creative are supplied by our innate biological characteristics. However, these biological constraints are not to be mistaken for the external ones, which are imposed on us by certain social institutions. The latter are always harmful. They restrict our freedom, and thereby stand in the way of our spontaneous creativity. They are only legitimate in so far as they facilitate the survival of the human species.

How does this view of human nature connect to the study of language? The connection might have been set up by Chomsky's definition of the subject of linguistics. Linguistics is asserted to be the science of the idealized native speaker's linguistic competence. This definition may give rise to a very special kind of analogical reasoning, which can be schematically described as follows: Premise 1: *p* is a part of *w*. Premise 2: *w* has property *A*. Conclusion: *p* has property A_1, which is similar to property *A*. It is a non-demonstrative inference from a property of the whole to a property of the part. Chomsky's definition of the subject of linguistics establishes whole-part relationship between man and his linguistic competence. As a result, it facilitates inferences from the properties of human nature to the properties of linguistic competence. Consequently, ideas about human nature may infiltrate into linguistic theory. What I will try to show is that the birth of some of Chomsky's linguistic ideas might be explained in this way.

Five particular points will be examined. The first one is what might be labelled as the "principle of regularity". It involves a theoretical assumption and its methodological consequence. The theoretical assumption is that in language regularity prevails. Systematicity belongs to the very nature of language. It is this assumption which is at work when Chomsky accuses traditional grammar and structuralist linguistics of not discovering enough regularities. The assumption implies the methodological rule that irregularities are not worth investigating. They are peripheral phenomena, which cannot help us to a deeper understanding of language. They should be relegated to the lexicon or to the theory of markedness, and should not be dealt with any more. Since the validity of this rule depends on the soundness of the assumption we have to return now to the assumption. It may strike us at once that it is not directly testable. Imagine a debate between radical regularists, like Chomsky, and moderate ones who are convinced that there is no regularity in language over and above the obvious cases of agreement, comparison of adjectives, etc. The opposing parties would soon be involved in vicious circles. The radicals would argue from the underlying regularities they unearthed. The moderates would dismiss these examples as artefacts. In fact, the very acceptability of the data relevant to the question seems to depend on a prior decision concerning the degree of systematicity of language. Hence the assumption of regularity cannot be regarded as a generalization from hard facts. I would like to suggest that it may be understood, partly, in terms of Chomsky's views on human nature. He thinks that creativity presupposes rules. So if language use is creative, it has to take place within a system of rules. Thus it might be the rules governing our creative activities where the principle of regularity comes from.

With this, I have arrived at the second point of possible connection, namely, the idea of the creative use of language and its explanation. Chomsky was the first to attribute great significance to the widely known fact, that we are able to produce and understand sentences we have never heard before,

i.e. our language use is in a sense creative. This fact could be most easily explained perhaps by saying that language is not regulated strictly. Grammatical rules are too few and too loose to prevent innovations. Chomsky's explanation is markedly different. Innovations do not occur inspite of the rules. On the contrary, those are exactly the rules that make them possible. In his theory, the rules of grammar specify an infinite number of sentences. So all the sentences that are regarded as innovations are given in advance by a system of rules. This conception of linguistic creativity may be readily accounted for in terms of Chomsky's view of man. Linguistic creativity is important, since creativity is a fundamental characteristics of the human species. It is explained by the rules of grammar, since human creativity in general is governed by rules. This explanation seems to be supported by the fact that Chomsky is fully aware of the analogy between the creative use of language and human creativity in general. He considers the former a particularly crucial realization of the latter.

The third point is the notion of universal grammar. This notion appears already in Chomsky's doctoral dissertation under the name "general theory of linguistic form". Universal grammar consists of constraints imposed on the form of grammars. The set of all these constraints defines the class of possible human grammars, which, in turn, defines the class of possible human languages. The qualification "human" is important. There could be languages violating these constraints which could be used for the very same purposes as languages that do not violate them. These languages, however, would not qualify as human languages, since people could not possess them. The reason is that universal grammar reflects the innate structural characteristics of human mind. And if human mind has got a definite structure, this structure restricts the kinds of information it can accomodate. We cannot put a pair of skis into a bookcase. In the same manner, languages incompatible with the structure of human mind cannot be accomodated in it. So the languages violating universal grammar cannot be used by human beings. This should suffice to see that universal grammar

is a good exemplification of Chomsky's idea of biological constraints characteristic of the human species.

The fourth point, which heavily relies on the previous one, is Chomsky's theory of primary language acquisition. He has vehemently criticized Skinner's behavioristic approach, in which language is impressed on the child's brain by means of selective reinforcement. Even though his criticism is strictly factual and methodological, his motivations, he admits, were partly political. What worried him in Skinner's account was that it implies that human beings are infinitely malleable. That is to say, their knowledge and personality is determined fully by the external influence they have been subjected to. This idea is said to have dangerous political implications, namely, that manipulation and coercion are always successful. As a result, this view is suitable for laying the foundations of doctrines of manipulation and oppression. From Chomsky's own theory an entirely different view of man can be gathered. In his opinion, the process of language learning is by and large determined by universal grammar, which he calls language acquisition device in this context. The role of external factors is reduced to supplying linguistic data. So the child is not the passive subject of external influence, whose knowledge can be arbitrarily shaped by careful doses of reinforcement. He is an active agent, who looks for the best way to adapt the data to his innate mental structures. He is not infinitely malleable either. If a language does not fit these structures, i.e. it runs counter to universal grammar, he simply will not learn it. This picture of the child constructing the grammar of his mother tongue in accordance with the principles of universal grammar resembles strongly Chomsky's vision of man creating freely within the constraints of his biological nature. Thus it seems reasonable to suppose that his theory of language acquisition might have been suggested by his views on human nature.

The last point is the idea of evaluation measure, which figured in Chomsky's earlier accounts of language learning. It was introduced so as to solve the following problem. There may

be several grammars which are compatible both with the constraints of universal grammar and the linguistic data. The child has to choose somehow from among them. The problem is how he makes this choice. Chomsky's solution was that universal grammar contains an evaluation measure which selects the best grammar. He said the selection was based on a criterion of simplicity. He emphasized, however, that this kind of simplicity. He emphasized, however, that this kind of simplicity is not an a priori aesthetic category which is understood in advance outside of linguistic theory. It has to be discovered empirically just as the value of a physical constant. It may even run counter to our intuitive judgements of simplicity. So why did Chomsky call it a criterion of simplicity at all if it has nothing to do with our intuitive understanding of simplicity? The answer may lie in the idea he took over from his teacher, Nelson Goodman, that simplicity and systematicity are more or less synonymous. So what Chomsky's claim that the evaluation measure utilizes a criterion of simplicity really amounts to is that the child chooses the most systematic grammar. Thus we once again ran into the idea of systematicity, which we have already tried to derive from his view of man.

Finally, I would like to forestall a possible objection. My reconstructions presupposed that Chomsky believed in the psychological reality of grammar at the time of the conception of these ideas. There is, nevertheless, a widespread conviction that it happened only in 1959 or 60 that Chomsky began to interpret generative grammar mentalistically. It was even argued that his mentalistic turn was brought about by his entering into controversy with Skinner. If this conviction is correct all my reconstructions fall, except the one which was concerned with Chomsky's theory of primary language acquisition. However, this conviction captures only the half of the truth. It is right in so far as there was really a turn in Chomsky's thought at that time, and it was probably brought about by the controversy with Skinner. Nevertheless, what happened was not that Chomsky gave up his antimentalistic position. What he did was rather to make his mentalism explicit. To be more exact, his

mentalism became a programmatic claim from an insignificant assumption. This interpretation may be supported by two arguments. First, Chomsky himself says that the mentalistic position was taken for granted as early as his 1955 doctoral dissertation. Second, his doctoral dissertation, in fact, contains numerous mentalistic formulations which are clearly not used for the sake of convenience.

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LAWS OF LOGIC FROM THE POINT OF VIEW OF PHILOSOPHY OF SCIENCE

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> "...the most certain principle of all is that about which one cannot be mistaken; for such a principle must be both the most familiar (for it is about the unfamiliar that errors are always made), and not based on hypothesis. For the principle which the student of any form of Being must grasp is no hypothesis; and that which a man must know if he knows anything he must bring with him to his task."*

In present day philosophy of science the question arises whether it is possible that a principle has those properties which Aristotle attributed to the principle of contradiction. Is there any principle which is the most certain, about which one cannot be mistaken, which is, at the same time, the most familiar and not based on hypothesis and a man must know it if he knows anything? Or, is it the case that every principle, and therefore Aristotle's principle of contradiction, is a standard way of looking at the world, a paradigm shared by scientists for a long time without questioning its validity?

Nowadays we recognize that there is no *one* Aristotelian logic in general but, rather, several different ones, and that each of them has a special formulation of the principle of contradiction. But, in this case, there arises another question, namely *which* Aristotelian principle of contradiction is the most certain, the most familiar, etc.?

The generations following Aristotle read his work from the point of view of their own paradigms and regarded their own interpretation as "the authentic reading of the principle of

*Aristotle, *Metaphysics*, IV. III. 7-8.

contradiction", as "the really Aristotelian principle of contradiction". Some of them thought that the principle of contradiction was an ontological principle, a most general principle of being. Others held that it was about language use, while the adherents of a third approach thought that it was a general methodological rule for discussion, etc... Which, if any, of these different interpretations is the fundamental one?

I think, that, although Aristotle gives several and, to a certain extent, different formulations of the principle of contradiction, these formulations are not independent of each other. More over they are closely connected. Aristotle believed that Being as Being had certain peculiar modifications and from them there followed the principles of sound reasoning. That is why, if we want to answer the above stated questions, then at first we must study the connection between ontologically interpreted logics and reality.

Logic and reality

In order to understand the relation of an ontologically interpreted logic to reality we have to take into account that the recreation of reality in the mind is accomplished by the activity of consciousness. One of the manifestations of the activity of mind is the process of homogenization. The term "homogenization" was first introduced in a philosophical sense by G. Lukács. In explaining the meaning of this term I will rely on G. Lukács's suggestions.*

Homogenization is a necessary concomitant of every cognition, it is a form of abstraction. In the process of cognizing heterogenous reality man views various things, by emphasizing some of their characteristics, and relations, as bearers of

*See, Lukács, G. *Hegel's False and His Genuine Ontology*. Transl. by D. Fernbach, London, Merlin Press, 1978.

these characteristics and relations. He identifies them on the basis of some common characteristics while disregarding others in which these things differ from one another. The identification is always effectuated on the basis of characteristics which are essential in the given process of cognition, while the characteristics which are left out of consideration are such that, from the point of view of the process of cognition, theyare inessential, they are negligible. It is possible that something on the basis of which this identification is effectuated is not essential as far as the thing or its objective existence is concerned, and it is possible that something, from the difference of which we make an abstraction in certain concrete cognitive situations, is necessarily an inherent property of the objective existence of the thing.(Primitive man, in order to select from among the stones that happened to be lying around those suitable for his cutting requirements and to leave the unsuitable behind, had to separate the contingent, natural form of stones - that is, a contingent, not essential, form of existence of a stone "as stone" - from their other properties, and had to view them as bearers of this property. This homogenization is the precondition of establishing the suitability or unsuitability for cutting. Stones, by virtue of their natural form, are suitable or unsuitable for cutting, but this property can only be actualized in and by human labour. In the simple existence of the stone it would remain a never realized possibility.)

If we refer Lukács's statement concerning homogenization to logical laws interpreted as ontological laws we have to admit that each of them is only an abstract, idealized apprehension of certain relations between things. Thus, for example in the laws of two-valued first-order predicate logic (PL) heterogenous reality is homogenized first of all by virtue of the condition - condition exists in every classical, two-valued logical system - that we presuppose the existence of individuals having sharply different properties and relations. This

presupposition is the generalization of man's practical experience of cognition. In order to judge things, man has to distinguish, has to separate them from one another. The objective basis of this distinction, or separation, lies in their inherent relative otherness. (This otherness is relative because every individual as far as its properties and relations are concerned, is such that it differs from other individuals and that it also has identical traits.)

Heterogeneous reality becomes homogenized in PL because PL disregards the time factor. This presupposition is also the generalization of man's practical experiences. In order to be able to think man has to disregard, to a certain degree, the eternal change of things. The objective basis of this lies in the relative constancy of things. (Things preserve some of their characteristics for a relatively long time.)

PL homogenizes again because in investigating individuals and their properties (relations) it disregards the properties of properties (e.g. relations of relations), it disregards the modality of properties (e.g. necessary, possible properties) and so on.

As a result of what we have said above, logical laws expressed in the language of PL reflect reality in the condition of abstraction, and on the level of homogenization which is a peculiar quality of PL. In a similar way the laws of Aristotle's ontology are about peculiar modifications of Being as Being, i.e. Being as conceived of by consciousness in specific concepts and propositions in the language of Aristotle's ontology.

Unlike Aristotle, PL presupposes that for every x either F or its privation (not-F) is true. In PL, but not with Aristotle, asserting that x has not the property F is equivalent to asserting that x has the property not-F. This means that in PL, $\forall x(Fx \vee \bar{F}x)$ is a law and the formula $\overline{\exists x}(Fx \& \bar{F}x)$, which is called "the law of contradiction in PL" (CPL) is also valid. But CPL is valid only in the framework of logics where it is presuppoed that for each object it is significant to predicate ei-

ther F or its privation (not-F). CPL is not valid in logics which presuppose the possibility of situations with objects having neither a given property F nor its privation. However, in these logics, the principle of contradiction in Aristotelian ontology can be valid. Namely: "It is impossible for the same attribute at once to belong and not to belong to the same thing and in the same relation."*

In the time of classical logic, man - relying on objective bases - created a world concept in which individuals possess sharply defined characteristics, relations, etc., as I have mentioned before. With the advancement of the science of logic - led by needs of certain cognitive objectives as, for example, the problems of microphysics and microbiology - there evolved investigations which ventured to establish a world concept resting on different principles than those of the world of classical logic. In various systems of logic and in virtue of various cognitive aims the 'world of beings' becomes a differently homogenized world which is selected by the conscious activity of the cognitive man. The 'different worlds' which appear in logical systems exist together, constitute the 'World' together, not separate from one another in objective reality and independent of human cognition. We have to allow for this in order to be able to formulate more clearly the problem at issue. The problem of the general validity of logical laws is a question which refers to individuals already homogenized in thinking, to individuals conceptualized in some way for a goal of cognition.

The laws of some logical systems - in case of ontological interpretation - reflect reality on a level of homogenization which is defined by the abstractional conditions of the system. Various systems of logic differ from one another not in the fact that their scope of validity belongs to different sub-classes of the class of beings but, rather, in the fact that

their scope of validity is the reflection of the *totality of beings* on various levels of homogenization. (Of course, we can say that some homogenization is suitable for certain investigations, in other cases, however, it is not relevant.)

Therefore, when examining reality on a given h_1 level of homogenization, the laws of a given L_1 ontologically interpreted logical system are generally valid, universal laws in every field of reality. If, however, certain objectives of cognition require us to change over to another examination on h_2 level of homogenization and we employ the corresponding L_2 system, it is possible that we get out of the scope of the laws of the L_1 system. The laws of both L_1 and L_2 systems are equally valid in every field of reality if the conditions of employing them on the corresponding h_1 and/or h_2 level of homogenization are fulfilled. Otherwise, however, I think there is no, and that it is impossible to create, such an ontologically interpreted logical system, such a universal logic, the laws of which, examining reality on any level of homogenization, would be valid. The existence of a logical system like this would mean that there is cognition the results of which convey closed, final, non-improvable knowledge of reality in every aspect.

If we wish to introduce logic in some field of inquiry (e.g. quantum-mechanics) as a tool of cognition by which we could express the specificity of objects in that field, it would be more expedient to employ more refined means than the abstractional conditions of two-valued logic, means which do not presuppose such an extent of homogenization as does two-valued logic.

The name 'logic of micro-world' is related to what we have said above. The term 'micro-world' refers to the fact that, in the process of cognizing the structure of the material world, we penetrate ever deeper into the structure of things and we separate the image which we have obtained as an other 'world'. Among these 'worlds' there are some (e.g. the 'micro-world') in the examination of which it is useful to employ other logical means than those of two-valued logic. Of course, the de-

termining factor of what logic can be suitably employed in examining micro-world is the nature of the micro-world itself, namely, what specificities of the World make the objects of our consciousness. In my opinion, however, this does not mean that we could speak of the 'logic of the micro-world' as different from the logic of other fields which would mean the *refutation* of the general validity of logical laws in the sense of what we have said above. It is not at all proved that we shall never need the 'logic of micro-world' in some other fields, namely, in fields where the means and abstractions of two-valued logic seem to be appropriate. Taking mathematical logic in general as an analogy we can mention that it is not exclusively the logic of the "world" of mathematics. To date, the results of investigations in exploring the logic of mathematics are applied in several fields of research and if they are not applied in some field it does not mean that in this special field we have reached at opposite results, it means at best, that in some cases it is not relevant to apply them or the conditions of their adoption do not exist.

The principle of contradiction as a general metalogical principle

We must distinguish the Aristotelian law of contradiction as a part of his ontology from another Aristotelian principle of contradiction, or, it may be better to say: another possible interpretation of the Aristotelian principle of contradiction, which is a metalogical principle. It is a formulation of a general precondition of understanding and communication.

Aristotle says: "Now those who intend to join in discussion must understand one another to some extent; for without this how can there be any common discussion between them?" ... "Now he who says that A is and is not denies that the term signifies what it does signify. But this is impossible. Therefore if 'to

be so and so' has a definite meaning, the opposite statement about the same subject cannot be true."*

Here I would like to point out that Aristotle formulizes this principle in the framework of terms which signify something and have a definite meaning. I think that in this framework the Aristotelian principle of contradiction interpreted as the principle of understanding (GMPC) is a general principle of sound reasoning. In this framework it really is a principle which is the most certain, about which one cannot be mistaken, etc... It will have these characteristics as long as the nature of human beings is such that man necessarily builds his knowledge on the pillars of terms used with definite meanings in a universe of discourse.

GMPC is not an axiom of one or another logical theory which may be questioned or omitted by a rival theory. It is a final precondition to any theory, at least up to our time and in the framework of our cultural community.

J. von Neuman called the logic of central nervous system, viewed as language, a primary language. He wrote that this primary language must structurally be essentially different from the one to which common experience refers. When we talk about logic we discuss a secondary language which is, although built on the primary lanquaga, not absolutely relevant from the point of view of evaluating the primary language that is truly used by the central nervous system.**

I think the line started by von Neumann should be continued. What is at stake in the science of logic is indeed the logic of thought expressing itself in natural languages, but we speak about these second-order languages in a third-order language. Third-order languages may have essential variants. One of them is the language of the Aristotelian syllogism, another one is,

*See, von Neumann, J. *The Computer and the Brain*. Yale Univ. Press, 1959. p. 82.

**Aristotle, *Metaphysics*, XI. V. 4-5.

for example, classical predicate logic, etc. These various third-order languages are suitable means of revealing certain characteristics of logic expressed in secondary language, i.e. some characteristics of thought expressed in natural language. But a particular third-order logical language constitute only one approach to concepts and propositions having concrete contents in second-order language. On the other hand, these third-order logical languages extrapolate. They deal with the forms and laws of possible operations which can be used to attain knowledge independently of whether anyone uses them or not in the practice of thinking.

On the page preceeding the one quoted above from von Neumann's work, we can read the following:

> "Just as languages like Greek or Sanskrit are historical facts and not absolute logical necessities, it is only reasonable to assume that logics and mathematics are similarly historical, accidental forms of expression. They may have essential variants, i.e. they may exist in other forms than the ones to which we are accustomed."*

My question is: which logics does von Neumann speak about here? First let us see if he means what we have called third-order languages, when logics are viewed as languages. In this case we can bring up as an argument in support of his hypothesis any many-valued logic or paraconsistent logic etc., which is really other than that to which we have been accustomed for centuries and about which we have learned that it originated from Aristotle and developed on a higher level through Russell and Whitehead's *Principia Mathematica* to gain its full-blown form as classical mathematical logic.

On the other hand, if he thinks here of the logic of thought as it is expressed in natural languages (in secondary languages) then could it, or could it not, have essential vari-

*von Neumann, J. *Op. cit.* p. 81.

ants. This is a question to which I cannot give a general answer, but I think that at least the natural languages of our cultural community are subsumed under some common laws one of which was that formulated by Aristotle as GMPC. These laws are the objects of the third-order languages. The third-order languages refer to these laws through different homogenizations and presuppositions following from the nature of the given third-order language.**

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**On the validity of APC in paraconsistent logics see Havas K.G., "Differences in the Unity". In: *Logique et analyse*, 1986. N° 114. (June), pp. 149-160.

CHANGING EPISTEMOLOGICAL PERSPECTIVES ON SCIENTIFIC COGNITION - WHITHER NOW?

Imre Hronszky

It was commonly held for a long time that scientific research is an exceptional, self-sufficient and closed intellectual enterprise in which mind works in itself for itself by using proper rational argumentation (based on experimental as well as theoretical considerations). This assumed self-sufficient character of scientific argumentation was taken to be the basis of the necessary autonomy within society of the institution called science. Nowadays the situation has slightly changed. A quite strong stream of epistemologically relevant sociologies has been attacking the old positions during the last 15-20 years.

One of the aims of this paper is to summarize the historical process of changing the perspectives in the theory of explanatory historigraphy of science as a *gradual deconstruction of* that basic *mystifying attitude* in which scientific cognition has been seen as a self-sufficient intellectual enterprise (and consequently, society as its bare vehi-

*There had been, it is true, some sociology of science even earlier which in its "externalistic" approach had attacked this or that element of the prevailing rationalistic position of its time in exploring history of science. There had even been some epistemologically relevant sociology of science already in the first half of the century. Nevertheless the debate has reached a new level by now. To name just one difference in advance let me mention that any normative theory of science does not seem to be able to abandon the sociological challenge without answering it if it does not want to loose some credit.

icle.* Having acknowledged then the development of the rationalist perspective on science (from the possible narrowest conceptualization realized by positivist thinking to the much more flexible ideas of Larry Laudan or Ernan McMullin, just to name some of the modern leading rationalists) the question arises how the idea of *a full-blooded analysis* (which is not willing to exclude any part of scientific reasoning ab ovo a priori-istically from such an analysis) *should be realized.*

Based on the validity of some conception of a sociology *of* the rational (that means: denying the validity of the idea of social *or* rational) the paper tries newertheless to be critical of some assumed weaknesses and overtones in modern epistemologically relevant sociology of science.**

*Sociology *of* scientific knowledge does not mean validly, at least to my mind, a sociological *reductionism*. That accepts either pure social causation without the mediating effect of rational argumentation as a general model or reduces the social information of scientific knowledge to the determination of the cognitive agent as if he were doing nothing but using his action for fitting into the social interactional network and forgets about the other side that the activity of this agent is aimed at *cognition* of the world.

**What is given here is a very elementary outline of the process, a phenomenal description. On a deeper level a sociological analysis and evaluation of the story of science research should be given. In that even the internalist analyses of science could be demonstrated as not pure cognitive activities but also as justificatory efforts for ideological goals set within the scientific "society" or sometimes explicitely in the broader society. An element of this here neglected level would analyse the effect on Sarton's historiography of his commitment to his idea that science is to be seen as the most humanistic effort. The trial of healing social life and the way he explicated history of science were consciously connected by him. Another element could be an analysis of how B. Hessen was influenced by the slogan of Bucharin that "great practice requires great theory" and by the task made explicit with Bucharin that scientists in a new socialist society should be aware of the practical functioning of scientific knowledge when he formulated his Newton-understanding. The bewildering tone of the debate about the "sociological turn" gives another immediate example that not only pure cognition is at stake.

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The epistemological basis for modern theory of science laid down by the neopositivists of the Vienna Circle aimed at the fullest *insulationist, demarcationist* approach to science. It even claimed that science was the only true form of knowledge, secured by its unquestionable fundament and strict methodology. Individual mindcontents were assumed to be the valid object of research in philosophy of science. In a long process of a step by step giving up of this original position only minor modifications were introduced initially. Popper's reorienting thinking in methodology from inductivism to hypothetico-deductivism was based on his antifundamentalism. He reformulated a moderate task when he raised the problem and asked the question of qhy science should be seen as an exceptionally adequate mode of cognition, besides others. Science became seen no more as the only real form of knowledge. Research on scientific cognition was later reoriented to a new item, the assumed "objective knowledge", a special world, of its own, as it was claimed, with the idea in mind of defending "objectivity". From the early aprioristic preconceptions aprioristic normativism as basic attitude for the theory of science was preserved. This attitude kept philosophy of science as normative issue separated from the historigraphy of science as a descriptive one.

Coming back for a moment to the beginnings I have to draw attention on the case that positivist intellectual historians of science became more immediately challenged by historians who tried to describe and explain history of scientific cognition as determined by social factors. Just take the strange example of B. Hessen, who preserving the, then prevailing, positivist idea of scientific cognition tried to emphasize the decisive role of factors from "outside" (technological needs, mediated by technological cognitive needs, or metaphysics) as "driving forces" (for example of the motivation of scientists), on the choice of research topics within science,

or on the formulation of scientific ideas effected by "good" or "bad" metaphysics.

Following the advices of vulgarmarxism Hessen tried to exemplify a onesided determinism concerning the "interaction of science and material production" presuming, of course, that science has its own laws of development, laid down in a pure cognitive situation. Denying the strong reductionistic tendencies and the one-sided conceptualization what the effect of society is on science-development, R. Merton conceptualized science-development as also influenced by social values, securing the acknowledgement of scientific activity as long as it is not able to rely on its own values and merits. He also assumed that some special groups carried with them some certain general ontological ideas about the world which were favourable for scientific reasoning.

The "externalism" of Hessen or Merton challenged the prevailing "internalist" attitude to history of science at some important points (whether acceptably or not for us is not the problem we are dealing with here) but *left intact the positivist understanding of scientific reasoning process.* This was successfully challenged and broadened later by the new historiography of Koyré type that made intellect the basic cognitive item for historical research, and the advance of objective spirit became the leading idea in research front for a long time, criticising successfully the positivistic-empiricist conceptualizations of scientific reasoning and the sociological approaches based on this sort of epistemology.

One more remark on this situation. Already G.N. Clarck was ready to accept the idea sketched by earlier German economy historians, that basic attitudes from outside, such as the calculative one of the merchants, may have informed scientific cognitive attitude. Nevertheless he did not think that scientists accepted this point of view *for some social reasons* having function on the socialinteractional layer of social life and he also held that the scientific cognitive attitude,

once informed, followed its own closed dynamics.*

Let's return from descriptive historiography of science to the crisis caused by the *historical turn* made by Kuhn. It threw out a serious challenge with its questioning the meaningfulness of supposed existence is *history-independent* norms, an idea that continuously served for justification of the task of philosophers of science so far. Based on insights reached in philosophy of science he recalled into question many parts of the earlier images of scientific activity.

A lot of old presuppositions were challenged by the rich content of the Kuhnian model (for example: the cumulativity assumption). The problem also emerged: *how to learn from history* in developing philosophy of science, what is the validity of factual history for normative issues? (It was not true that Kuhn simply changed normativism to descriptionism of factual dynamics, evaluating it in a simple Darwinistic way, as some German constructivists believed.)

Some sort of apriorism had to be given up after the Kuhnian model. History had to be included in some way into rationality of science considerations when scholars wanted to get credit for their rationality models. Apriorism had to be weakened but the efforts at overall models of rationality equally valid for all periods of history of science were not given up. To overcome the "irrationality" to be found in the paradigm model, because of the necessarily further not rationalizable step made as decision at paradigm change M. Hesse developed a very abstract model of a "learning machine" based on an "inductive logic" in order to save the rationality of decisions in the development of scientific cognition.

*I think one of the equivocal presumptions in science research has until now been the idea of growing insulation and autonomy of science, presuming that science became increasingly self-developing. The differentiation of the meanings of this concept seems to be one of the basic conditions for meaningful discussions nowadays.

Lakatos acknowledged that historiography and philosophy of science should be seen as interrelated ones and tried to adjust on Popperian methodology so that it could be believed appropriate to reconstruct history of scientific cognition. It seems that he took his quasi-Kantian phrase seriously enough when he divided the tasks of both parts of the connection driven in on each other. Normative rationality approach was preserved and historiography was kept in the role of *ancilla* by him. I think the weaknesses of his approach were prettily discussed by Larry Laudan and Ernan McMullin, respectively. Let me only mention McMullin's criticism to which he introduced the term "*imputed rationality*" criticising Lakatos saying that latter tried to "impute" rationality into science and believed that, according to his model, history really behaved rationally on a deeper level. McMullin reformulated the division of labour laid down earlier for philosophy and history of science and advised intellectual historians to explore what he called the *implicit rationality* of historical events. The task of intellectual historians was formulated by him as *historicized* one, following the tradition of intellectual historiography of the type begun with Koyré.

It is now a platitude to say that Lakatos's efforts belonged to a period of a philosophy of science that was still led by a sort of *methodologism*, an idea that the rationality of scientific enterprise relies on the (assumed) existence of a *generally valid methodology*, an ahistorical ideal of scientificity ("logic" of science).

Giving a *pragmatic* characteristic to what makes scientific reasoning rational Laudan also *historicized* it. The overall problemsolving activity, the general feature of scientific activity was assumed being realized in historically various ways and advancement of scientific reasoning in a historical learning process. Laudan's purpose was to unify pragmatistic understanding of rationality (of science) with historicity and to save in this way the idea of rational advancement of scienti-

fic thinking, given that there is a valid overall definition of rationality throughout history: problem solving activity.

Rationality came to be seen as a self-developing process the framework of which became the historically varying broader cognitive culture. The attack on self-sufficient rationality of science (made decidedly but somehow restrictedly by Kuhn and in a much broader way by the new generation of knowledge-sociologists to be discussed later) was intended to avert by him broadening the concept of rationality. But the concept of rationality could be saved only in its more moderate form bringing in foreground the problem of a sociology of cognition looking for *preconditions* of the historical forms of rationality in form of cognitive attitudes characteristic for special layers within society or for special periods of history and so on. In that meaning some effortst of *earlier* sociologists became implicitly acknowledged by the problem-solving model.

A long period of a theory of science seems to have reached its endpoint, for abstract, autonomous mind has became, within the barrier of being autonomous, *practical and historicized*. The self-development of mind through continuous feedback loops to the factual world through solving "empirical problems" and to itself through "theoretical problems" has become the last word for those who insisted on the presumption of the autonomous development of cognition. Not science but broadly: mind became the active subject. It is not a special methodology that secures the rationality of knowledge anymore, but the continuous double feedback. But instead of an idea of "knowledge without knower" the idea of a *purely cognitive practice* got the task of preserving the validity of the idea of the *autonomous* development of cognition.

This point has not been attached. Society could not change its moderate position of being a bare vehicle, and realists and pragmatists could jointly command sociology to reduce its activity to dealing with the "deviants". But anyway, they might have forgotten that a metaphysical reason, that they allow,

can have the form of reason, nevertheless as metaphysics it is *ideology*, too, in some way. Through this characteristic a reasoning process using metaphysical starting points has also a necessary ideological side role. *The picture of rational reasoning process as a closed process of reasoning could have been preserved but its functioning, has to be seen as including an ideological role, too.* Their working included what I would call, an *implicit social reasoning.*

With the renewal of the knowledge-sociological approach to scientific cognition a new perspective has challenged the philosophy of science. To sum up briefly, what became at stake was the cognitive sociology *of* the reasoning process when sociology has been defined as a research dealing with *social causation (included social reasons).* To speak of sociology *of* the reasoning process means that it is assumed a valid task of science research to look for social causation of choosing objects of knowledge in science. And it is also valid to look for the social causation of the reasoning process itself, including the acceptance of the results. But to put it this way, that means: emphasising the reasoning process, could be misunderstandable. Let me summarize the main point of recent knowledge-sociological approach a bit broader. It allows for the validity of knowledge sociological exploration of *all possible knowledge items;*

1. to see the possible variety of cognitive goals as being caused or at least effected by genuine social effects, from social cognitive needs to ideological ones, by social effects coming from "outside" or inside within the "scientific community", 2. to explore the necessarily open structure of scientific reasoning (being open because of the indeterminacy of advancing and applying rules) as permeated by sociological effects. That idea commits knowledge sociologists to see the product of cognition, the knowledge as having internalized a series of decisions. Those decisions may seem irrational or impossible to explore further for rationalists or even denied

as fact by them. And further, 3. that the reasons that may have been developed to validate the process of cognition by the actors themselves (or later on) should be seen as *genuine social action*, open to sociological analysis.

An enormous number of case studies argue for the validity of the basic knowledge sociological tenets. They force, to say the least, to rethink rationalistic reconstructions of the same events. And some of them seem to be really persuasive and bringing important differences in the description of the events in comparison to the rationalistic reconstructions not only a sociological explanation.*

Let me draw the attention on some justifications of knowledge sociology, that means on the philosophy of science of knowledge sociology. Mary Hesse lately drew the conclusion of the *necessary* sociologization of epistemology from an analysis of the failure of validating efforts based narrowly on the bare historicizing or naturalizing of epistemology. (The same conclusions were reached by e.g. M. Fehér.) As Hesse argued scientific cognition should be opened for all those perspectives of investigation that have been developed for the analysis of the other forms of cognition. For, scientific cognition can get all the functions that knowledge can get at all. If I understand her correctly, she states that the difference of the presence of these factors is not of yes or no but of intensity or mode. This perspective justifies the opening the field in its whole range for the various social disciplines.

No part of the field could be saved by the rationalists as untouchbar. Science, just as any other forms of cognition, is made by concrete social agents and its peculiarity does not make it an exception of being socially permeated and formed. Of course, Larry Laudan (and other rationalists) are right in

*The purpose of the paper allows not to deal with the problem that knowledge sociology has developed into some different mainstreams by now and their tenets are different enough. I think there is a broadening of the perspective; we return to this problem later.

a minor point when they argue for *the fullest possible (rational) reconstruction of the reasoning processes*. The reconstruction can be fatal to those knowledge sociological perspectives which would make a short-circuit and would give social causal explanations *instead* of reconstructing a framework of rational argumentation *itself fully open to sociological analysis*. And taking as a starting point a vision of scientific cognition that looks at scientists as real social subjects there is all reason to draw attention on those social effects, too, that are caused by the simple fact that a scientist has got to preserve its position (at least within the "scientific community"). This "overdetermining" social causation of scientific cognition can be in a very intermingled, interprenetrated relation to other types of social causation of ideas, e.g. to the socially fixed goals of cognitive activity.

The very task of a knowledge sociologist is to point out why and how the cognitive action is governed and permeated by its concrete social milieu. The commitment to *microsociological* perspective seems vitally important here. It is a point of second rang I think what is sometimes emphasized that minor causes can effect major results and microsociologically oriented sociologists should be cautious with macrosociological conclusions because the latter are all to overgeneralizing or at least too fast drawn. (I think knowledge sociology makes itself a lot of difficulties when trying to omit more general social determinations onesidedly tries to explain by contingencies, exclusively by factors of short range. Put this way of doing microsociologically relevant knowledge sociology together with a one sided effort to take into account microsociological only social interactional efforts as social effects - say, the scientists trial to fit into the acknowledged core of experts in the field - may even help to develop its self-caricature.) But the more important point with microsociology is that it makes the working of rationalists reconstruction *really concrete*. (And, as Bloor made clear sometimes, the knowledge sociologists reconstruction of the rationality of the event, the rational

frame of the reasoning process may show the incorrectness of the rationalist's reconstruction having stopped too early in searching for further important differences in the reconstructed argumentation.)

Science and the cognitive effort should just be seen as one peculiar sort of socially formed activity. And social activity is *social practice by nature.* Perhaps those perspectives within knowledge sociology can be seen the most progressive ones, the most promiseful ones that try to take into account this social practice nature of scientific cognition. As far as I see two different approaches try to utilize this practice perspective. The first identifies practice as something not fully *rationalizable.* If so then scientific reasoning should be seen as a genuine psychological item and persuasion as only partly based on rational argumentation. For it is not fully propositional. But the idea that the nature of scientific cognition is social practice can also be utilized in a different way, too, when pointing out that cognitive activity always has its even in words inarticulable knowledge ingredients. This perspective leads to the rethinking of the "tacit knowledge" dimension of scientific knowledge from a sociological point of view.

But there is another possibility of utilizing the mentioned perspective and that is I intend to follow here a bit more detailed. We have dealt so far with conceptualisations of *descriptive* art.* They aimed at the reconstruction of the assumed rationality of scientific cognition or the reconstruction of how scientific cognition was constructed by its social milieu. The latter had critical potential without aiming at

*Knowledge sociology has so far mostly aimed at an explanative descriptive reconstruction. It concentrated on the refusal of the claim of the rationalist that the story of scientific cognition can be meaningfully reconstructed evaluating and comparing it to some aprioristic rationality models. But it is not necessary to reduce the set of possible tasks to the descriptive one.

being of *social practical-critical* nature. But science itself and meta-scientific analysis, too are (conscious or uncon-cious) practice in them values, with social effect are set, realized, validated, reset, reevaluated an so on. This characteristic may sometimes be strongly mediated to be seen evidently, nevertheless the metaanalyst may have the task to reflect on, make conscious and be critical of this social value content of scientific cognition. Science and metascience are both types of ideologies, too and some sort of cognitive sociology should *evaluatedly* analyse this social value commitments of cognitive actions. That is the point where a new program of the theory of science should have its starting point. A normative theory of science (and hence a normative methodology) can only be based on the sociological analysis of modern science and can be hoped to be valid for modern science (as far as it is not changed) trying to make *evaluations of alternatives of investigation*. I think this type of analysis can not be imagined as just an application of some general theory of scientific growth, say e.g. that one that prefixes the essence of scientific cognition in its problemsolving rationality.

Sal Restivo and Daryl Chubin seem to have given a promising characterisation of the task when they ask a general question as follows. "In what kind of world would what kind of epistemic activity lead an epistemic community to conclude that it was (according) to its own definition (improving its model of the world and what are the implications for those inside and outside of the epistemic community?") This *practice* perspective seems to be able *to unify the normative and descriptive task*.

The lastly mentioned perspective seems to relativize the perspective radically. It really does in some respect. And, according to its proponents it does in that direction, too, that it includes the rather ad hoc characterisation "insight" instead of the idea of "truth". Knowledge systems are nothing but more or less successful "insights". This is the point where I would like to differentiate my point of view from theirs. Since

I think that even when we acknowledge the validity of the knowledge sociological turn (in the meaning I have been trying to sketch throughout the present paper) we seem not allowed simply to neglect the acknowledgement of some sort of truth conception. The last part of the presentation tries to sketch this opinion.

It seems that knowledge sociologists pointed out successfully that there is nothing in scientific cognition that makes it apart from a full-blooded sociological analysis, an analysis of full scale. As a *method* of analysis sociology seems to have proved its right basically. But having developed this side of analysis, another side of the *uniqueness* of scientific cognition seems to have lost. To make the problem a bit clearer it is now necessary to bring into mind that all the perspectives developed so far have forgotten that modern science (since the mid-17th century) has became such a sort of practice, in relation to Nature, in which, if it was possible the external world has been simultaneously reproduced not only in mental but in mental and *material models* by which the "appropriateness" of the mental has been measured.* I suggest looking at science as something the fullest expressed, realized in the branches which can connect theoretical thinking with experimentation (even: with experimental production of their object).

Then science is to be seen as a joint effort in understanding and transforming Nature both in mind an material reality. Theorists of science may have "forgotten" about this characteristic in some way. The old prejudice has been prevailing until

*It is perhaps possible to argue (in the case of special types of knowledge) for the falsity of the commonly accepted preconception that the practical success of knowledge utilization brings with itself nothing more argumentative power than instrumental validity. This demarcationism seems to be vased, at least partly, on a deeper one that keeps doing and understanding apart. Assuming that the natural world is *immediate* object of knowledge deprives philosophy of science of arguin for the thesis that practical success in transforming materially the natural processess (into "material models" of these processes) can have argumentative power for more than the idea of a bare instrumental success.

now that cognition is something to be characterised in separation from the material transformation of Nature.*

Put otherwise, cognition is made within the *subject*, in mind which is connected to the external world only outwardly. The conceptualisation of this sort of subject could be changed from the individual to the collective one, yet all this has not changed this sort of *demarcationistic* tendency in understanding cognition. And this failure seems to have the necessary consequence that the cognitive practice of the collective subject had to be understood as nothing but one type of shared convention. It seems that the other way of conceptualizing our image of cognition has not been utilized until now, at least not in connection to the achievements of modern theory and cognitive sociology of science. I mean especially that mental models seem to reflect not Nature as such, but immediately our practically prepared material models. Pairs of the mental and material models have realized jointly the advancement of man -- Nature relation, the "success" of transformation of Nature by collective human practice.

It seems true, scientific cognition is tied to, informed by a complexity of social activities. But within them it is a special type of social activity, *a material transformatory effort with aiming at knowledge*, unique in this meaning. All this, I think, should force us to rethink the sociological achievements. Nature, through the peculiar material practice of science, *does* influence cognition in a very influential way, in comparison to other types of cognition. If this is true, we have to rethink the truthproblem. This rethinking should be oriented on the problem: How some sort of thinking about the characteristics of truth relation, realized in scientific cog-

*Other cases seem to be deducible from this main core. To make another remark there has not been place to analyse whether an argumentation based on the idea sketched here can give reasons for the idea that because material models "mediate" spontaneous natural processes hence something can be stated validly about how Nature per se is.

tion, can be unified with the recently reached knowledge-sociological achievements.

It seems we are allowed to speak of the progress of the "appropriation" of Nature, even in a mental form, having been developing adequate mental models for the parts of the world which are successfully reproduced in our material practice. So the truth problem should be set within the joint problem of "appropriation" of Nature and objectification of our socially-historically acquired mental and material reproductive capacity. Some sort of "moderate" realism should then be brought back into the considerations without committing ourselves to being "arrogant" or "ignorant", as Newton-Smith suggested. Mentioning this possible perspective I do not want to forget about the logical problems raised by this assumption especially and by all sorts of "realism" in general. (To put just one: the relative indeterminatedness of the reference relation.) I only insist on the idea that in the recent crisis we clearly need *some further "new" visions* of scientific cognition.

To sum up: The progress in the "sociological turn" seems partly not to have been running far enough in giving up old prejudices and so it has been necessarily overhasty in relativizing scientific cognition. The social nature of scientific cognition, that informs it in historically varying ways, does not make disappear the determinations, rooted in Nature, that are effected on it, but it transforms them into socially digested form.

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EXISTENCE, QUANTIFICATION AND THE MEDIAEVAL THEORY OF AMPLIATION

Gyula Klima

1. Introduction

(1) Bucephalus is dead
(2) What is dead does not exist
Therefore,
(3) Bucephalus does not exist
Therefore,
(4) something does not exist

In my opinion this is a conclusive argument for the thesis that something does not exist. As is well-known, however, many philosophers regard this thesis as paradoxical in a way, and, consequently, they would make several objections to the simple reasoning that led to it above.

In what follows I firstly deploy some typical objections to this reasoning. After this I give replies to these objections. The informal discussion will be followed by the description of the syntax and semantics of a formal language, AMPL, which, in my opinion, can serve as a suitable frame of reference for handling the problems that occur in the course of the informal discussion. As the language AMPL is a certain extension of the language MPL I used in an earlier paper for reconstructing mediaeval logical theories concerning extensional contexts,* I shall relate here the language AMPL to a mediaeval theory concerning intensional contexts, namely to the theory of ampliation. This will be followed by a formal reconsideration of the introductory objections and replies. The paper will end with some historical and philosophical concluding remarks.

*Klima [10].

2. Objections

1. In the argument above, if conclusion((3) is true, then it is false, therefore it is false. (By the "consequentia mirabilis": $(p \supset \sim p) \supset \sim p$.) For if a statement is true, then that which it is about must exist. (About something non-existent one cannot make a true statement.) So, if (3) is true, then Bucephalus exists. But so (3) is false. Therefore (3) is false, of necessity.*

2. In the whole argument existence is treated as a predicate. But existence is not a predicate. For if it were, then, since a statement implies the existence of what it is about, all positive existential statements would be necessary, and all negative ones would be contradictory. But this is surely not the case: "There are extra-terrestrial intelligent beings" is contingent.**

3. "Exists" in the argument is used as if it were a first-level predicate. But it is a second-level predicate. So the argument is senseless. That existence is a second-level predicate may be seen from the following: "'pink iguanas exist' means nothing more nor less than 'Something is a pink iguana'; since the latter is formed by wrapping the second-level predicate 'something' around the first-level predicable 'is a pink iguana' it seems all but certain that the 'exist' in the former sentence must also be deemed a second-level predicate."***

4. The inference from (3) is invalid. For "to be is to be the value of a bound variable", therefore, if (3) is true, then Bucephalus cannot be a value of a bound variable. So the bound variable in (4), if (4) is to be thought of as "For some

*Cf. Plato [17] 236E-239B, Quine [19] pp. 1-2., Russell [25] Linsky [11] pp. 1-2.

**Cf. Ayer [4] pp. 57-58., Kiteley [9] pp. 365-366.

***See Flint [7] pp. 131-132.

x, x does not exist", cannot take up the value Bucephalus, and so, even if (3) is true, (4) may be false.*

5. The claim: "Something does not exist" is simply inconsistent. For "Somethint does not exist" is equivalent to "There is something that does not exist" and this further to "There exists something that does not exist" and this is clearly inconsistent.**

3. Replies

1. To the first objection we may give a twofold answer. First, we can point out that the rule "if a statement is true, then that which it is about must exist" holds only for positive statements. For a negative statement is true, iff the corresponding positive statement is not true, i.e., if *tertium non datur*, is false. But the non-existence of its subject is just one of the possible reasons on account of which a positive statement may be false. (It is not true that the present King of France is bald, just because there is no such a person.) So, if its subject does not exist, i.e. if its subject term fails to refer to anything, then a negative statement is true. As the schoolmen put it: Negativa cuius subjectum pro nullo supponit est vera.*** Secondly, we may point out that one must distinguish between referring to what does not exist (non-existence of the reference) and failing to refer altogether (failure of reference).**** As Wittgenstein says: "That is to confound the reference (Bedeutung) of the name with the bearer of the name. When Mr. N.N. dies, we say that the bearer of the

*Cf. Quine [20] p. 145., Routley [24] p. 133., Linsky [11] pp. 110-111.

**See Parsons [14] p. 365.

***See e.g. Ockham [13] P.II. c.12. pp. 284-285., Buridan [6] c.2., Albert of Saxony [1] Tr.4. c.24., Ashworth [3] p. 145.

****See Linsky [11] p. 18., Parsons [14] p. 366.

name dies, not that the reference dies. And it would be nonsensical to say that, for if the name ceased to have reference, it would make nonsense to say 'Mr. N.N. is dead'."*

But we can quite sensibly say that Mr. N.N., or Bucephalus is dead. So in (1) the name "Bucephalus" refers to Bucephalus, even if Bucephalus, since perished, i.e. ceased to exist, does not exist. So here "Bucephalus" refers to something that does not exist. Therefore it is true that the reference of "Bucephalus", i.e. Bucephalus, does not exist.

2. "Bucephalus existed, then died, i.e. ceased to exist, and now does not exist." This is a perfect example of the use of existence as a contingent predicate of things. So any argument trying to demonstrate the slogan "Existence is not a predicate" is simply futile. The objection, as it proceeds from the false assumption that "a statement implies the existence of what it is about", proves nothing.

3. Again, since the example above beyond being an example of contingent existence-predication makes also perfect sense, therefore, any argument trying to prove *in genere* the senselessness of existence-predications is doomed to futility. The *causa apparentiae* in the objection lies in our liability to take "Something is a pink iguana" to *mean* the same as "Pink iguanas exist". Now, it is true that in *most* cases a statement of the form "Something is an F" is ***equivalent to*** "F's exist", or "an F exists". But this by no means implies that these two forms of statement are synonymous. What is more, in some cases "Something is an F" and "an F exists" are not even equivalent to each other.
For example, "Something (or, rather, somebody) is dead" is true, but "A dead (man) exists" is false.**

*Wittgenstein [30] I. §40. as quoted by Geach [8] p. 58.
**See again Ashworth [3] p. 143. and p. 145.

4. Since to be a value of a bound variable is to be an element of the universe of discourse, and what can be referred to in a language is an element of the universe of discourse, therefore, the sloqan "to be (or to exist) is to be the value of a bound variable" reduces to the claim that everything that can be referred to in a language exists. But this cannot be true already for an artificial modal language (as Nicholas Rescher pointed out, (x)(Ex) (Everything exists) leads necessarily to (x)(◊Ex ⊃ Ex) (Everything that can exist exists), i.e. to the unpalatable: *a posse ad esse valet consequentia,** and *a fortiori* it cannot be true for a natural language, as it was shown in the preceding replies. But so, since Bucephalus, even though does not exist, can be the value of a bound variable, therefore, if (3) is true, i.e. "x does not exist" is true for Bucephalus, then "For some x, x does not exist", i.e. "Something does not exist" must also be true.

5. In reply to the fifth objection first we should ask: on what grounds does one say that "There exists something that does not exist" is inconsistent? For if you analyse this sentence as "For some x, x does not exist" (i.e. "Something does not exist"), then, provided that "exist" is not a necessarily universal predicate, as it was argued for above, it is surely not inconsistent.

It *would* however, be inconsistent if you analysed it on the analogy of, say, "There runs something that does not run", i.e., if you treated "exist" as a predicate in *both* of its occurrences in "There exists something that does not exist". For Then this sentence would be equivalent to "Something exists and does not exist". But in this case it cannot be thought to be equivalent to "Something does not exist". So, if "There exists something that does not exist" is analysed as equivalent to a contradiction, then it is not equivalent to "Something does not exist", and, on the other hand, if it is analysed as

*Rescher [23] p. 161.

equivalent to "Something does not exist", then "There exists something that does not exist" is not equivalent to a contradiction.

Now, whether or not these objections or replies are convincing, in my view, depends on certain rather vague and conflicting intuitions. In the following two sections, therefore, I shall describe the syntax and semantics of a formal language, AMPL, which, I hope, will be of some help in clarifying these intuitions.

4. AMPL Syntax

The language AMPL is defined as follows. AMPL:=<C,P,V,T,F>. Here C:={$\sim$,&,=,$\exists$,$\neg$,.,α,(,)}, P is a denumerably infinite set of parameters, V is a denumerably infinite set of variables (*proper* variables, as I shall call them), T is the set of terms and F is the set of formulae of AMPL. P detailed: $P:=P^p \cup P^i$, where P^p is the set of predicate parameters and P^i is the set of individual parameters of AMPL. ***Note:*** $P^p \cap C := \{\exists\}$.
T is defined as follows: $T := P^i \cup V^{ar}$ is the set of ***variables*** of AMPL. V^{ar} is defined in the following manner. $V^{ar} := V \cup V^r$, where V^r is the set of ***restricted variables*** of AMPL. V^r is defined as follows.

If $v \in V$ and $Av \in F$ in which some occurrences of v are free, then $\ulcorner v.Av \urcorner \in V^r$, in which all occurrences of v are bound, and any occurrence of any other variable is free, iff it is free in Av.

No other strings of signs are elements of V^r.

The set of formulae is defined by the following clauses.

(F1) If $t_1,\ldots,t_n \in T$ and $P^n \in P^p$, then $\ulcorner P^n(t_1)\ldots(t_n)\urcorner \in F$ (in case $P^n = \exists$, n=1), in which, for any i,j ($1 \leq i \leq n$; $1 \leq j \leq n$), if $t_i \in V^{ar}$ and $t_j \in V^r$, then any occurrence of t_i which is not an occurrence of t_i in t_j is free, and any other occurrence of a variable is free, iff it is free in t_j.

(F2) If $t_1, t_2 \in T$, then $\ulcorner t_1 = t_2 \urcorner \in F$, in which, for any i and j ($1 \leq i \leq n$; $1 \leq j \leq n$), if $t_i \in V^{ar}$ and $t_j \in V^{r}$, then any occurrence of t_i which is not an occurrence of t_i in t_j is free, and any other occurrence of a variable is free, iff it is is free in t_j.

(F3) If $t_1, \ldots, t_n \in T$ and $P^n \in P^p$, then $\ulcorner (\sim P^n)(t_1)\ldots(t_n) \urcorner \in F$, in which the free occurrences of a variable are the same as those in $\ulcorner P^n(t_1)\ldots(t_n) \urcorner$.

(F4) If $A, B \in F$, then $\ulcorner \sim(A) \urcorner$, $\ulcorner \alpha(A) \urcorner$, and $\ulcorner (A \& B) \urcorner$ are elements of F, in which the free occurrences of a variable are the same as those in A or B.

(F5) If $v \in V^{ar}$ and $Av \in F$ in which some occurrences of v are free, then $\ulcorner (\exists v)(Av) \urcorner \in F$ and $\ulcorner (\forall v)(Av) \urcorner \in F$, in which all occurrences of v are bound, and an occurrence of any other variable is free, iff it is free in Av.

No other strings of signs are elements of F.

Further connectives and the universal quantifier are regarded as shorthands for the usual definitions.

5. AMPL Semantics

The definition of a model for AMPL runs as follows. $M_{AMPL} := \langle U, S, a_s, D, R \rangle$, where U and S are nonempty sets, a_s is a distinguished element of S, D is a function from S to the set of all subsets of U, i.e., if $s \in S$, then $D(s) \subseteq U$, where D(s) may also be designated as D_s and R is a function assigning semantic values to the parameters of AMPL.

Intuitively, U is the *universe of discourse* of M, S is a set of *situations* or states of affairs, a_s is the *actual situation* and D is the *domain assignment of situations* so that D(s), i.e. D_s is the *domain of the situation s*.
R is defined by the following clauses.

(R1) If $a \in P^i$, then $R(a) \in U$.

(R2) If $P^n \in P^p$, then $R(P^n)_s \subseteq U^n$.

(R3) $R(\exists)_s = U$.

Let us define further *the extension of P^n in the situation s*, $E_s(P^n)$ in the following manner. $E_s(P^n) := R(P^n)_s \cap D_s^n$. Given a model M, *an assignment* f is defined for proper variables as follows.

(f) If $v \in V$, then $f(v) \in U$.

Given an assignment f, for every situation s and for every subset B of U we define *a restricted assignment associated to f in the situation s*, f_{sB}, for proper variables as follows,

(f_{sB}) If $B \subseteq U$ and $v \in V$, then $F_{sB}(v) \in B$, if $B \neq \emptyset$, otherwise $f_{sB}(v) = U$.

An *assignment in the situation s associated to the assignment f* is a function $f_s \in (U \cup \{U\})^T$, i.e. a function from the set of terms to $U \cup \{U\}$ satisfying the following conditions.

($f_s 1$) If $v \in V$, then $f_s(v) = f(v)$.

($f_s 2$) If $a \in P^i$, then $f_s(a) = R(a)$.

($f_s 3$) If $v \in V$ and $Av \in F$ in which some occurrences of v are free, then $f_s(\ulcorner v.Av \urcorner) = f_{sE^v_{M,f_s}(Av)}(v)$,

where $E^v_{M,f_s}(Av)$ a subset of U, *the extension of Av with respect to v in M according to f_s* is defined in the following manner: $E^v_{M,f_s}(Av) := \{u \in U : |Av|_{f_s[v:u]} = t\}$, where $f_s[v:u]$ is the same as f_s except that to v it assigns u, i.e.

$$f_s[v:u](w) = \begin{cases} f_s(w), & \text{if } w \neq v \\ u & \text{otherwise,} \end{cases}$$

and $|Av|_{f_s} = t$, which reads "Av is true according to f_s" is defined by the following clauses.

(T1) If $t_1, \ldots, t_n \in T$ and $P^n \in P^p$, then

$$|\ulcorner P^n(t_1)\ldots(t_n)\urcorner|_{f_s} = t, \text{ iff } \langle f_s(t_1), \ldots, f_s(t_n)\rangle \in E_s(P^n) .$$

(T2) If $t_1, \ldots, t_n \in T$ and $P^n \in P^p$, then

$$|\ulcorner(\sim P^n)(t_1)\ldots(t_n)\urcorner|_{f_s} = t, \text{ iff } \langle f_s(t_1), \ldots, f_s(t_n)\rangle \in D_s^n - E_s(P^n) .$$

(T3) If $t \in T$, then

$|\ulcorner \exists(t) \urcorner|_{f_s} = t$, iff $f_s(t) \in D_s / {=} E_s(\exists)$

(T4) If $t_1, t_2 \in T$, then

$|\ulcorner t_1 = t_2 \urcorner|_{f_s} = t$, iff $f_s(t_1) = f_s(t_2) \neq U$

(T5) If $A \in F$, then

$|\ulcorner \sim(A) \urcorner|_{f_s} = t$, iff $|A|_{f_s} \neq t$.

(T6) If $A, B \in F$, then

$|\ulcorner (A \& B) \urcorner|_{f_s} = t$, iff $|A|_{f_s} = t$ and $|B|_{f_s} = t$

(T7) If $A \in F$, then

$|\ulcorner \alpha(A) \urcorner|_{f_s} = t$, iff there is an s' such that $|A|_{f_s'} = t$.

(T8) If $v \in V^{ar}$ and $Av \in F$ in which some occurrences of v are free, then

$|\ulcorner (\exists v)(Av) \urcorner|_{f_s} = t$, iff there is an assignment in s g_s such that g_s differs from f_s at most in the value of v and $|Av|_{f_s[v:g_s](v)]} = t$.

Note: If $v \in V^{ar}$, f and g are assignments, s and s′ are situations, f_s, is an assignment in s′ associated to f and g_s is an assignment in s associated to g, then

$$f_{s,[v:g_s(v)]}(w) = \begin{cases} f_{s'}(w), & \text{if } v \neq w \\ g_s(v) & \text{otherwise} \end{cases}$$

that is, $f_{s,[v:g_s(v)]}$ is the same as $f_{s'}$ except that to v it assigns the same value as g_s.

(T9) If $v \in V^{ar}$ and $Av \in F$ in which some occurrences of v are free, then

$|\ulcorner (\imath v)(Av) \urcorner|_{f_s} = t$, iff there is exactly one g_s such that g_s differs from f_s at most in the value assigned to v and $g_s(v) \neq U$, and $|Av|_{f_s[v:g_s(v)]} = t$.

Truth in a model M is defined as follows.

(T) If $A \in F$, then $|A|^M = t$, iff there is an f_{a_s} such that $|A|_{f_{a_s}} = t$.

If $A \in F$, then A is *satisfiable*, iff there is a model M such that $|A|^M = t$. A formula is *valid*, iff its negation is not satisfiable. If $A,B \in F$, then $A \Rightarrow B$, iff $\ulcorner A\&\sim B \urcorner$ is not satisfiable.

6. *AMPL and the Theory of Ampliation*

α, the intensional operator of AMPL, is called - barbarously enough - the *ampliator* of AMPL. As to its intuitive meaning, it has no single natural language equivalent, instead, in several applications it may serve for several intensional operators.

For example, in modal contexts it may serve for the familiar possibility operator, ('◇' or 'M'), while in tensed contexts it may serve for the past or the future tense operator ('P' or 'F'). Of course, accordingly, the set of situations S is to be interpreted as the set of possible, past or future situations.

As an indication of the actual intended interpretation I shall use subscripts to α. (Even if these *stricto sensu* do not belong to the vocabulary of AMPL.) For example, for the sentence "A white thing can be not white" I shall write $'(\exists x.Wx)\alpha_M\sim(W(x.Wx))'$, while for the sentence "A white thing will be not white" I shall write $'(\exists x.Wx)\alpha_F\sim(W(x.Wx))'$. Of course, in "mixed contexts", where e.g. both tense and modal operators are involved (as in an argument concerning the future contingents) the single α would not suffice. In such a case further operators should be introduced, and, accordingly, the set of situations should be further specified. (E.g. in the case of tensed modal contexts the elements of S should be ordered pairs of time points and possible worlds *à la* Montague.) But for our present purposes the single α with its multifarious

intuitive interpretation will suffice. For with the aid of α in AMPL we can represent that common feature of intensional sentences that an adequate statement of their truth conditions involves reference to *situations beyond the actual one*, and that, accordingly, it contains (at least implicit) reference to *individuals beyond the actual ones*.

Now, according to the mediaeval analysis of sentences of this kind, the common feature of these sentences is that the range of reference of their subject terms is extended, *ampliated* beyond the sphere of actual entities to several past, future or possible non-actual ones. For example, while in the sentence "A man runs" (Homo currit) the subject term "man" refers to what *is* a man, in the sentence "A man ran" (Homo cucurrit) it refers to what *is* or *was* a man, so that according to this analysis the sentence "A man ran" could be true even if actually there were no men.*

Now this analysis can be represented in AMPL as follows. $(\exists x.Mx \vee \alpha_p(Mx))\alpha_p(R(x.Mx \vee \alpha_p(Mx)))$, i.e. a thing that *is* a man or *was* a man ran (quod est homo vel fuit homo cucurrit). This might be called the *common* analysis of a sentence containing an *ampliative* verb (verbum habens vim ampliandi)** But, as it is well-known concerning modal sentences, the mediaevals also made a distinction between *de dicto* and *de re* sentences, or *de dicto* and *de re* readings of sentences containing such ampliative verbs.

For example, let us take the following sentences.

(1) A white thing can be balck (*Album potest esse nigrum*)

(2) A thing that is white can be black (Quod est album potest esse nigrum)

(3) It is possible that a white thing is black (Possibile est album esse nigrum)***

*See e.g. Buridan [5] c.6. p. 349.

**For a "catalogue" of these see Buridan ibid.

***See Buridan [6] pp. 68-69., Albert of Saxony [1] Tr.2. c.10. ff. 15-16. and Tr.5. c.4. f. 40.

(2) is a *de re* modal sentence, and (3) is a *de dicto* modal sentence. (2) can also be regarded as the *de re*, while (3) as the *de dicto* reading of (1); or, according to another terminology, (2) can be regarded as expressing the content of (1) *in sensu diviso*, while (3) can be regarded as doing the same *in sensu composito*. Of course, if we take (2) and (3) as expressing different readings of (1), then we must hold that (1) is ambiguous, and, consequently, we cannot hold that there is a single *common* analysis of (1). If, on the other hand, we accept that (1) has a single *common* analysis, then we cannot take (2) and (3) as expressing different senses or readings of (1), instead, we must regard them as being different sentences in their own right.*
Which way we choose, I think, is a matter of convention.

What concerns us more in the present context, however, is that the differences of (1), (2) and (3) can be brought out very clearly im AMPL. For the sake of simplicity interpreting "black" as "not-white", we can write:

(1') $(\exists x.Wx \vee \alpha_M(Wx))(\alpha_M((\sim W)(x.Wx) \vee \alpha_M(Wx)))$

(2') $(\exists x.Wx)(\alpha_M((\sim W)(x.Wx)))$

(3') $\alpha_M((\exists x.Wx)((\sim W)(x.Wx)))$.

Now as it is easily seen, while (2') is satisfiable, (3') is not. For (2') is true in a model M, iff there is an f_{a_s} in M such that (2') is true in f_{a_s} in M. But (2') is true in

*As from the texts referred to above is quite clear, this was the attitude taken by Buridan and Albert of Saxony. In his *Tractatus de Fallaciis*, Buridan explicitly states: "Ista enim propositio 'laborans sanabatur' non est distinguenda: habet enim unicum sensum qui explanandus est per propositionem unam de disiuncto subiecto sc. istam 'qui est vel fuit laborans sanabatur'." See Pinborg [16] p. 155.
On the other hand, Ockham writes thus: "Et ideo quaelibet talis propositio est distinguenda, eo quod talis terminus potest supponere pro his quae sunt vel pro his quae fuerunt." See Ockham [13] P.I. c.72. p. 216.
Again, for a "catalogue" of the technical terms for such and closely related distinctions, see St. Thomas Aquinas [27] I. q.14. a.13. ad 3-um and [28] lb.1. c.67. n.10.

an f_{a_s} in M, iff there is a g_{a_s} in M such that it differs from f_{a_s} at most in the value assigned to 'x.Wx' and $|\alpha_M((\sim W)(x.Wx))|_{f_{a_s}[x.Wx:g_{a_s}(x.Wx)]} = t$. Let such a g_{a_s} be such that $g_{a_s}(x.Wx) = d$. (Of course, by $(f_s 3)$, $d \in E_{a_s}(W)$.) Now $|\alpha_M((\sim W)(x.Wx))|_{f_{a_s}[x.Wx:g_{a_s}(x.Wx)]} = t$, iff there is an s' such that $|(\sim W)(x.Wx)|_{f_s,[x.Wx:g_{a_s}(x.Wx)]} = t$, i.e. iff there is an s' such that $f_{s,}[x.Wx:g_{a_s}(x.Wx)](x.Wx) = g_{a_s}(x.Wx) =$ $= d \in D_s, - E_s,(W)$. So, in a model M in which there is an s' such that there is a $d \in E_{a_s}(W)$ which is also an element of $D_s, -E_s,(W)$ (2') is true. But since there *is* such a model, (2') is satisfiable. On the other hand, (3') is true in a model M, iff there is an f_{a_s} such that (3') is true in f_{a_s} in M. Now, (3') is true in an f_{a_s} iff there is an s' such that $|(\exists x.Wx)((\sim W)(x.Wx))|_{f_s,} = t$. But this condition is satisfied in an s' iff there is a g_s, such that it differs from f_s, at most in the value assigned to 'x.Wx' and $|(\sim W)(x.Wx)|_{f_s,[x.Wx:g_s,(x.Wx)]} = t$.

But in any M, there cannot be such a g_s, , so (3') can be true in no M. For R(W) in a model M is either empty or not. If it is, then - since in this case for any s' $E_s,(W) = \emptyset$, and, therefore, for any g, $g_s,(x.Wx) = U$, for any g_s, $|(\sim W)(x.Wx)|_{f_s[x.Wx:g_s,(x.Wx)]} \neq t$. (Since for any g, $g_s,(x.Wx) = U \notin D_s, - E_s,(W)$.)

If, however, R(W) in not empty, then for any s' in which $E_s,(W) \neq \emptyset$, for any g_s, $g_s(x.Wx) \in E_s,(W)$. So, for any g_s, $g_s,(x.Wx) \notin D_s, - E_s,(W)$. So, again, for any g_s: $|(\sim W)(x.Wx)|_{f_s,[x.Wx:g_s,(x.Wx)]} \neq t$. Therefore, for any model M, there is no such a g_s, that would make $(\sim W)(x.Wx)$ true in an f_s, in M. So there is no model M in which (3') is true. Therefore it is not satisfiable.

Now, since (2') is satisfiable, and (3') is not, therefore $\ulcorner(2')\&\sim(3')\urcorner$ is satisfiable, i.e. an argument from (2') to

(3′) is fallacious. In particular, such an argument from a proposition *in sensu diviso* to a proposition *in sensu composito* would be a case of a fallacy termed by the mediaevals as *fallacia compositionis et divisionis*.

A notable example of this fallacy would be an argument from

(4) It is possible that the number of the planets is less than seven

to

(5) That number which *is* the number of the planets is possibly less than seven

That is, from

(4′) $\alpha_M((\iota x.Nx)(L(x.Nx)(s)))$

to

(5′) $(\iota x.Nx)\alpha_M((L(x.Nx)(s))$

(Vocabulary: 'N' = "is the number of the planets"; 'L' = = 'less than"; 's' = "seven")

Clearly, (4′) does not imply (5′), and so they cannot be regarded to be equivalent. So even if the argument from (5) with

(6) The number of the planets is nine

to

(7) Nine is possibly less than seven

is valid, still as (5) is false, (7) may also be false. On the other hand, the argument from the true (4) with (6) to (7) is not valid. So the falsity of (7) is in no conflict with the truth of (4). Therefore, if we carefully distinguish between the true *de dicto* [(or opaque, (4)] and the false *de re* [or transparent, (5)] readings of "The number of the planets is possibly less than seven", then, contrary to Quine's claim,* no inconvencience follows from the "essentialism" involved in (5), or in its existential generalization, the false

(8) A number which *is* the number of the planets is possibly less than seven

(8′) $(\exists x.Nx)\alpha_M((L(x.Nx)(s)))$

* Quine [20] p. 148.

(Of course, (8′) follows from (5′), but does not follow from (4′), neither is it implied by (9′) $\alpha_M((\exists x.Nx)(L(x.Nx)(s)))$ - - (9) It is possible that a number which is the number of the planets is less than seven.)

According to Quine, another inconvencience which follows from "Aristotelian essentialism" is that, at least on Quine's interpretation of this view, the same attribute of an object, depending on which description of the object is considered, must be regarded both as essential and not essential to the object.* For example, Socrates' wife is necessarily Socrates' wife. So being Socrates' wife is an essential attribute of Socrates' wife. But Xanthippe is not necessarily Socrates' wife (she might have married another man), so, being Socrates' wife is not an essential attribute of Xanthippe. But Xanthippe is Socrates' wife. So, being Socrates' wife is both essential and not essential to the same person, namely, Xanthippe.**

Now this line of reasoning involves two fallacies at once. One is the already familiar *fallacia compositionis et divisionis*, and the other is an *ignoratio elenchi*. The *ignoratio* consists in ignoring the proper definition of what it is to be an essential attribute of something. The *fallacia compositionis et divisionis* (hand in hand with the above-mentioned *ignoratio*) is involved in the transition from "Socrates' wife is necessarily Socrates' wife" to "Being Socrates' wife is essential to Socrates' wife".

Now, let us introduce a necessity operator in AMPL in the following way:

(DN) $|^{\ulcorner}N(A)^{\urcorner}|_{f_s} = |^{\ulcorner}\sim(\alpha_M(\sim(A)))^{\urcorner}|_{f_s}$

As one can see, the following formula is not valid.

(10′) $N((\imath x.Wx)(W(x.Wx)))$

(10) It is necessary that Socrates' wife is Socrates' wife
(Vocabulary: 'W' = "is Socrates' wife")

*Quine [21] p. 199. For an informal criticism of Quine's original argument, see Rasmussen [22].

**For the sake of formal simplicity I have changed Quine's original example, the cycling mathematician. See note above.

For in a model M in which there is a situation s in which $E_s(W) = \emptyset$ (i.e. in which Socrates has no wife) (10′) is false. The following (*de dicto*) formula, however, is valid.

(11′) $N((\eta x.Wx)(\exists(x.Wx)) \supset (\eta x.Wx)(W(x.Wx)))$

(11) It is necessary that if Socrates' wife exists, then she is Socrates' wife

(Or, more smoothly: It is necessary that if Socrates has a wife, then she is Socrates' wife - whoever Socrates' wife may be.)

But the corresponding *de re* formula is not valid.

(12′) $(\eta x.Wx)(N(\exists(x.Wx) \supset W(x.Wx)))$

(12) Of that person, who actually *is* Socrates' wife, it is necessarily true that if she exists, then she is Socrates' wife

(i.e., this is true of Xanthippe)

For in a model M in which there is exactly one d, such that $d \in E_{a_s}(W)$, and there is an s such that $d \in D_s$ but $d \notin E_s(W)$ (i.e., in which Xanthippe exists, but is not Socrates' wife) (12′) is false.

Now a predicate of an object is essential to this object, iff it is necessary that if the object exists, then the predicate is true of the object.*

This definition can be introduced in AMPL in the following way

(DNe) $|^{\ulcorner}Ne(P(t))^{\urcorner}|_{f_s} = |^{\ulcorner}N(\exists(t) \supset P(t))^{\urcorner}|_{f_s}$

As it is easily seen, the argument from (12′) to

*It is interesting to see that Rasmussen, while criticizing Porphyry for being unfaithful to Aristotle's original doctrine, embraces the same definition (see Rasmussen [22] p. 323.), which, however, is a quite straightforward consequence of Porphyry's definition of an accident. As it was cited by the mediaevals (in Boethius' translation): "quod adest et abest praeter subiecti corruptionem". Cf. Ockham [13] P.I. c.25. pp. 81-84.

(13′) $(\iota x.Wx)(Ne(W(x.Wx)))$

(13) Of the person, who *is* Socrates' wife it is essentially true that she is Socrates' wife

is valid, still, as (12) is false, (13) may be, and actually *is*, false too.

But the argument from (11′) to (13′) is not valid.

So the transition from the true (11) to the false (13) is clearly fallacious. Therefore the argument proves nothing against Aristotelian essentialism.

Now in the foregoing formulations we find a strange use of the symbol $\exists$. What justifies this queer use is, in my view, the following metatheorem.*

(EQ) If $v \in V$ and $Av \in F$ in which some occurrences of v are free, then for every model M

$|^{\ulcorner}(\exists v)(Av)^{\urcorner}|^{M} = |^{\ulcorner}\exists(v.Av)^{\urcorner}|^{M}$, iff $E^{V}_{M,a_s}(Av) = \emptyset$ or $E^{V}_{M,a_s}(Av) \cap D_{a_s} \neq O$,

where

$E^{V}_{M,a_s}(Av) := \bigcup_{f_{a_s} \in \varphi_{a_s}} E^{V}_{M,f_{a_s}}(Av)$, in which φ_{a_s} is the set of all assignments in a_s in M.

Proof

To simplify the proof first I prove two lemmas.

Lemma 1. $|^{\ulcorner}(\exists v)(Av)^{\urcorner}|^{M} = t$, iff $E^{V}_{M,a_s}(Av) \neq \emptyset$.

Proof

a. Suppose $E^{V}_{M,a_s}(Av) \neq \emptyset$. Then there is an f_{a_s} such that $E^{V}_{M,f_{a_s}}(Av) = \{u \in U : |Av|_{f_{a_s}[v:u]} = t\} \neq \emptyset$. So there is a $u \in E^{V}_{M,f_{a_s}}(Av)$ such that $|Av|_{f_{a_s}[v:u]} = t$. Therefore, there is a g_{a_s} such that $|Av|_{f_{a_s}[v:g_{a_s}(v)]} = t$ and g_{a_s} is the same as f_{a_s} except that to v it assigns u. But so $|^{\ulcorner}(\exists v)(Av)^{\urcorner}|^{M} = t$.

b. Suppose $E^{V}_{M,a_s}(Av) = \emptyset$. Then there is no f_{a_s} such that $E^{V}_{M,f_{a_s}}(Av) \neq \emptyset$. Therefore, for any g_{a_s} $|Av|_{f_{a_s}[v:g_{a_s}(v)]} \neq t$. So $|^{\ulcorner}(\exists v)(Av)^{\urcorner}|^{M} \neq t$.

Q.e.d.

*Cf. the last two sentences of Section 7.

Lemma 2. $|\ulcorner\exists(v.Av)\urcorner|^M = t$, iff $E^v_{M,a_s}(Av)\cap D_{a_s} \neq 0$.

Proof

a. Suppose $E^v_{M,a_s}(Av)\cap D_{a_s} \neq \emptyset$. Then there is an f_{a_s} such that $E^v_{M,f_{a_s}}(Av) = \{u\in U: |Av|_{f_{a_s}[v:u]} = t\}\cap D_{a_s} \neq \emptyset$. So there is an f_{a_s} such that $f_{a_s}(\ulcorner v.Av\urcorner) = f_{a_s}E^v_{M,f_{a_s}}(Av)(v)\in D_{a_s}$. So $|\ulcorner\exists(v.Av)\urcorner|^M = t$.

b. Suppose $E^v_{M,a_s}(Av)\cap D_{a_s} = \emptyset$. Then there is no f_{a_s} such that $E^v_{M,f_{a_s}}(Av) = \{u\in U: |Av|_{f_{a_s}[v:u]} = t\}\cap D_{a_s} = \emptyset$. So there is no f_{a_s} such that $f_{a_s}(\ulcorner v.Av\urcorner)\in D_{a_s}$. So $|\ulcorner\exists(v.Av)\urcorner|^M \neq t$.

Q.e.d.

Now, suppose that 1. $E^v_{M,a_s}(Av) \neq \emptyset$ and 2. $E^v_{M,a_s}(Av)\cap D_{a_s} \neq 0$. Then, by 1. and lemma 1. $|\ulcorner(\exists v)(Av)\urcorner|^M = t$ and, by 2. and lemma 2. $|\ulcorner\exists(v.Av)\urcorner|^M \neq t$.

Suppose further that 1. $E^v_{M,a_s}(Av) = \emptyset$ or 2. $E^v_{M,a_s}(Av)\cap D_{a_s} \neq 0$.

From 1. by lemma 1. it follows that $|\ulcorner(\exists v)(Av)\urcorner|^M \neq t$. From 1. it also follows that $E^v_{M,a_s}(Av)\cap D_{a_s} = \emptyset$. Therefore, by lemma 2. also $|\ulcorner\exists(v.Av)\urcorner|^M \neq t$.

From 2. by lemma 2. it follows that $|\ulcorner\exists(v.Av)\urcorner|^M = t$. From 2. it also follows that $E^v_{M,a_s}(Av) \neq \emptyset$. Therefore, by lemma 1. also $|\ulcorner(\exists v)(Av)\urcorner|^M = t$.

And this completes the proof.

The significance of this metatheorem is that it shows the close connection, but without blurring the distinction, between an existential statement and an existential, or perhaps it is better to say, a particular quantification.

What metatheorem (EQ) actually says is that if the actual extension of the open sentence ($E^{v}_{M,a_s}(Av)$) involved in the quantification and in the restricted variable of the existential statement is not *ampliated* to non-actual individuals, then these two forms of statement are equivalent, i.e. interchangeable *salva veritate*, but if it *is* ampliated to non-actual individuals and does not contain actual ones, then they are not equivalent.

For example, in virtue of metatheorem (EQ) the following formulae are equivalent.

(14') $(\exists x)(Cx)$

(15') $\exists(x.Cx)$

And this is how it should be. Clearly, the sentence

(14) Something is a centaur

is equivalent to

(15) A centaur exists*

This is why we are entitled to use, even in our logical practice, the types of statements represented by (14') and (15') interchangeably.

But let us take the following two sentences:

(16) Something is destroyed

(17) A thing that is destroyed exists**

These are clearly not equivalent.

*Or "There is a centaur" or "There exists a centaur". I do not believe that these sentences are totally synonymous. Nor do I believe, however, that "There is ..." or "There exists ..." can never carry exactly the same idea as "Something is...". But for the present I am only concerned with the *conceptual* difference between existence and quantification, with no special regard to linguistic form.

**In this sentence "thing" is not to be thought of as a predicate, but as a natural language equivalent of a proper variable. See Appendix.

Let us introduce a distinguished predicate 'D' ("is destroyed"] in AMPL in the following manner.

(D) $|\ulcorner D(t)\urcorner|_{f_s} = |\ulcorner \alpha_p(\exists(t) \;\&\; \sim(\exists(t))\urcorner|_{f_s}$

(i.e., a thing is destroyed, iff it existed and does not exist.)*

Now with this interpretation of 'D' the following formulae are not equivalent:

(16') $(\exists x)(Dx)$

(17') $\exists(x.Dx)$

For

(18') $(\exists x)(\alpha_p(\exists(x)) \;\&\; \sim(\exists(x)))$ (Something existed and does not exist) is evidently not equivalent to

(19') $\exists(x.\alpha_p(\exists(x)) \;\&\; \sim(\exists(x)))$

(A thing that existed and does not exist exists]

Actually (19') is inconsistent, i.e. not satisfiable, while (18') is satisfiable.

And this is why we cannot use (16) and (17) interchangeably.

7. *Objections and replies reconsidered*

After this rather prolonged overview of how AMPL is supposed to work, I think it is time to see how it resolves the informal objections listed at the beginning of this paper. The main argument can be formalized as follows.

(1') Db

(2') $(\forall x.Dx)(\sim(\exists(x.Dx)))$

(3') $\sim(\exists b)$

(4') $(\exists x)(\sim(\exists x))$

*Of course, accordingly, the definition of $E_s(P^n)$ is to be understood with the exception of 'D'.

(Vocabulary: 'b' = "Bucephalus"; 'D' = "dead".)
The argument is valid.

The first objection, as the reply rightly states, is a blatant *non sequitur*. From ~(∃b) it does not follow that ∃b. In general, from ~(Pa) it does not follow that ∃a.
The second objection, again, assuming the false principle: '~(Pa)' => '∃a', proves nothing.

An analogous true principle would be the following: '(~P)(a)' => '∃a' . (Or better: 'Pa v (~P)(a)' => '∃a'; cf. Strawson's existence-presupposition.*) The interesting case of which is '(~∃)(a)' => '∃a', which shows the inconsistency of '(~∃)(a)' ("a is nonexistent"). But this does not show the inconsistency of '~(∃a)' ("a does not exist").

To the third objection the reply is evident from the results of the preceding section. The equivalence of "Something is a pink iguana" with "Pink iguanas exist" is not sufficient to show that existence is a quantifier, a second-level predicate.

In reply to the fourth objection I can simply point to the fact that the inference from (3') to (4') is valid in AMPL. Of course, the construction of AMPL in itself is such that it involves the denial of Quine's slogan. For this denial, however, I can find no better justification than the simple fact that we *do* quantify over individuals that do not exist. Just consider "Some of the things which existed before the Second World War do still exist, but some do not".

Lastly, the fifth objection, as the reply states, rests on a confusion between '(∃x)(~(∃x))' and '∃(x.~(∃x))' (or the equivalent '(∃x.~(∃x))(∃(x.~(∃x)))', the former of which is satisfiable, while the latter is not, and so the latter is equivalent to '(∃x)(∃x & ~(∃x))'. I think it is also worth noticing that while in '(∃x)(~(∃x))' the first occurrence of '∃' is a

*Cf. Linsky [11] p. 90.

quantifier and the second one is a predicate, in '∃(x.~(∃x))' the same symbol functions as a predicate in *both* of its occurrences.

8. Concluding remarks

In the preceding sections I investigated the interconnections between the notions of existence and quantification with the help of the interpreted language AMPL. The basic result of these investigations, precisely formulated in metatheorem (EQ), is that while in purely extensional contexts, where no reference is made to non-actual individuals, an existential statement and an existential (particular) quantification (existentially quantified statement) are interchangeable *salva veritate*, the same does not necessarily hold for non-extensional contexts.

Now, I think, it is for this reason that in *mathematical* logic, where there is no need of, indeed, no place for, a distinction between actual and non-actual elements of the universe of discourse, the notion of existence could be analysed in terms of existential quantification. (Such and such a number exists, iff some number is such and such.) But this analysis, backed up by the Kantian tradition on the one hand, and by the amazing successes of mathematical logic on the other, led to an overall identification of the two notions. This situation, however, unavoidably led to certain "anomalies" in the logical analysis of non-extensional contexts.*

But as soon as we realize the proper connection between the two *distinct* notions, most of our most recalcitrant puzzles get solved. Not all, however.

*For an imposing list of these "anomalies" and an abundance of "epicycles", so to speak, "to save the phenomena" see Williams [29].

On the preceding pages I have taken my examples from modal and tensed contexts. I have also indicated how AMPL could be extended to cover also mixed contexts.

There are, however, several further intensional contexts, e.g. those created by intentional verbs.* One of the main characteristic features of these verbs is that they may be true of objects that do not exist.

Now, let us introduce into the language AMPL an intentional predicate 'P^{α}' in the following manner.

$$'P^{\alpha}' \in P^{P}.$$

In any model **M** $E_s(P^{\alpha}) = R(P^{\alpha})_s$, for any s.

An *ampliated assignment in the situation s associated to the assignment f*, f^{α}_s is defined in the following way.

$(f^{\alpha}_s 1)$ $\quad f^{\alpha}_s(a) = f_s(a)$

$(f_s 2)$ $\quad f^{\alpha}_s(v) = f_s(v)$

$(f_s 3)$ $\quad f^{\alpha}_s(\ulcorner v.Av \urcorner) = f_s(\ulcorner v.\alpha(Av) \urcorner)$**.

Let (T1) be modified so that it concern only elements of $P^{P}-\{'P^{\alpha}'\}$. And for P^{α} let us formulate the following clause.

*There are even further (and broader) intensional contexts, like the context of a myth, legend or any fiction. (Or, for that matter, the context "in the legend".) These all may be represented, through some proper modification of AMPL, by a situation s and a corresponding special α which "takes over" the discourse from a_s into that s. In general, here I take sides with Leonard Linsky, who writes:
"I would not, in my own ontology, divide objects into ideal objects which subsist and real ones which exist, but into objects which are, e.g., characters in fiction, legendary figures, mythological figures, comic strip characters, make-believe figures, as well as abstractions, mathematical objects, concepts, etc." See Linsky [11] p. 20.

**Note:

$$f_s[v:g_s,(v)](w) = \begin{cases} g_s,(v), & \text{if } v = w \\ f^{\alpha}_s(w) & \text{otherwise} \end{cases}$$

$(T1^{x})$ $\quad |^{\ulcorner}P^{\alpha}(t_i)^{\urcorner}|_{f_s} = t, \quad \text{iff} \quad f_s^{\alpha}(t_i) \in E_s(P^{\alpha})$

Now, with these formulations at hand we can see that from

(1) I seek a unicorn

(1′) $P^{\alpha}(x.Ux)$

it does not follow that

(2) Something that is a unicorn is such that I seek it

(2′) $(\exists x.Ux)(P^{\alpha}(x.Ux))$

Neither does it follow that

(3) Something is a unicorn

(3′) $(\exists x)(Ux)$

nor that

(4) A unicorn exists

(4′) $\exists(x.Ux)$

These do not follow, because (1′) may be true in a model M in which $E_{a_s}(U) = \emptyset$, i.e. in which unicorns do not exist, but in which there is an s, such that $E_s(U) \neq \emptyset$ and there is a $d \in E_s(U)$, such that $d \in E_{a_s}(P^{\alpha}) = R(P^{\alpha})$. And this is possible because $E_{a_s}(P^{\alpha})$ *may extend beyond* D_{a_s}, i.e. because such a predicate can be true of anything that can be referred to, with no regard to whether it exists. And this is so, because such predicates signify certain acts of the mind, which, by means of its abstract concepts, can think of things indifferently whether they exist or not.

As Buridan's pupil, Albert of Saxony puts it:

> "All verbs, even in the present tense, which of their very nature can concern future, past and possible things as well as present ones (habent naturam transeundi super rem ita futuram vel praeteritam vel possibilem sicut super praesentem) ampliate their terms to all times, future, past and present, like these: think, know, mean and the like. And what accounts for this is that a thing can be thought of without any difference of time

(sine differentia aliqua temporis), sc. abstracted from any place or time. And so, when a thing is thought of in this way, then a thing which was, or will be, or can be may be thought of as well as a thing which is. Therefore, if I have the common concept from which we take (a quo sumitur) this name 'man', then I can think indifferently of all men, past, present and future. And this is why these verbs can concern past or future things as well as present ones."*

There remain, however, problems in connection with intentional verbs which can not be handled even with this extended apparatus of AMPL. Such is e.g. the problem of non-interchangeability of identicals in the context of such verbs.

For example, the following argument is not valid:

(i) You do not know the man with his face covered

(ii) The man with his face covered is your father

Therefore

(iii) you do not know your father**

Now, for a solution of this sophism we need some more sophisticated devices, namely the devices of the mediaeval theory of *appellation*.***But for a reconstruction of *this* theory first we need a reconstruction of a more basic mediaeval theory, *the inherence theory of predication*.****

*Albert of Saxony [1] Tr.2. c.10. 8^{a} regula.

**Cf. Aristotle [2] ch.24. 179a27-179b33. For this version of the sophism see Lucian's *Philosophies for Sale*, ch.22-3.

***See Buridan [6] pp. 59-83. esp. pp. 72-74.

****The job will be done in my paper currently being prepared - "The Inherence Theory of Predication."

APPENDIX

In note 22 above I asked the reader to understand the word "thing" in "A thing that is destroyed exists" not as a predicate, but as a natural language equivalent of a proper variable. Let me explain why.

If we understood "thing" as a predicate in this sentence, then the AMPL equivalent of the sentence in question, one might think, should be something like this.

(i) $\exists(x.Tx \ \& \ Dx]$

But, in my view, this formulation is mistaken. For just like a fictitious centaur is not something that *is* a centaur *and* is fictitious, so a thing that is destroyed is not something that *is* a thing (a being) *and* is destroyed. (Ens et res convertuntur.) Formally, the problem is this.

If $'T' \in P^p$, then $E_s(T) \subseteq D_s$, for any s. So for any f_s, $E^x_{M,f_s}(Tx) \subseteq D_s$. But given the definition of 'Dx', for any f_s $E^x_{M,f_s}(Dx) \cap E^x_{M,f_s}(Tx) = E^x_{M,f_s}(Tx \ \& \ Dx) = \emptyset$. But so, for any f_s, $f_s(x.Tx \ \& \ Dx) = f_{sE^x_{M,f_s}(Tx \ \& \ Dx)}(x) = U$. And so, even

(ii) $(\exists x.Tx \ \& \ Dx)\alpha_p(\exists(x.Tx \ \& \ Dx))$

i.e. on the intended reading,

(iii) A thing that is destroyed existed

could not possibly be true.

So we cannot take (ii) as being a correct formalization of (iii), or, conversely, we cannot take (iii) as being a correct reading of (ii). Indeed, a correct reading of (ii) would be rather this.

(iv) Something that *is* a thing (a being) *and* is destroyed existed

And *this* surely cannot be true. But, then, what would be the correct formalization of (iii)?

For an answer to this question we need a slight modification of AMPL. First, in its syntax, we should permit restricted variables to be the operator variables of other variables. Secondly, we should work out the semantics for the newly got variables that makes possible the distinction between, say, a fictitious centaur and something (an x) that *is* a centaur *and* is fictitious.

The required modification is served by the following clauses.

Syntax

If $v \in V^{ar}$ and $Av \in F$ in which some occurrences of v are free, then $\ulcorner v.Av \urcorner \in V^{r}$, in which all occurrences of v are bound and an occurrence of any other variable is free, iff it is free in Av.

Semantics

$(f_s B)^*$ If $B \subseteq U$ and $v \in V^{ar}$, then $f_{sB}(v) \in B$, if $B \neq \emptyset$, otherwise $f_{sB}(v) = U$.

$(f_s 3)^*$ If $v \in V^{ar}$ and $Av \in F$ in which some occurrences of v are free, then

$$f_s(\ulcorner v.Av \urcorner) = f_{sE^{v}_{M,f_s}(Av)}(v) ,$$

where

$E^{v}_{M,f_s}(Av) := \{g_{s'}(v) \in U : |Av|_{f_s[v:g_{s'}(v)]} = t\}$. (Here $g_{s'} \in \varphi$, the set of all assignments in all situations.)

$(f^{\alpha}_s 3)^*$ If $v \in V^{ar}$ and $Av \in P$ in which some occurrences of v are free, then

$$f^{\alpha}_s(\ulcorner v.Av \urcorner) = f_s(\ulcorner v.\alpha(Av) \urcorner) .$$

Now with these definitions at hand we can give as an answer to the above question the following formula.

(vi) $(\exists x.Tx.D(x.Tx))\alpha_p(\exists(x.Tx.D(x.Tx)))$

Substituting the *definiens* of 'Dx' we get

(vii) $(\exists x.Tx.\alpha_p(\exists(x.Tx))\&\sim(\exists(x.Tx)))\alpha_p(\exists(x.Tx.\alpha_p(\exists(x.Tx))\&$
$\&\sim(\exists(x.Tx))))$

that is

(viii) A thing that existed and does not exist existed

And this is surely true.

Pari ratione the following sentence

(ix) A fictitious centaur is fictitious

that is,

(x) $P^{\alpha}(x.Cx.P^{\alpha}(x.Cx))$

(Vocabulary: 'P^{α}' := "fictitious", 'C' := "centaur") is true. But the following sentence,

(xi) A fictitious centaur is a centaur

that is

(xii) $C(x.Cx.P^{\alpha}(x.Cx))$

there being no centaurs, not even fictitious ones, is false, So even if a fictitious centaur is not a centaur, a destroyed thing is not a thing, a dead man is not a man, a potential being is not a being, still, these *are* fictitious, destroyed, dead and potential, respectively. See Peter of Spain [15] pp. 158-9. "Et gaudeo haec intellexisse" (Buridan [6] p.31.)

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ON THE HIDDEN ARISTOTELIAN THERMODYNAMICS

Katalin Martinás and László Ropolyi

Aristotle's ideas appear in modern physics, too. Nevertheless we think that the similarity between Aristotelian and modern physics is more important and more fundamental than it is usually supposed. The reason why this deep similarity is never recognized is what might be called the "mechanicist interpretation" of Aristotelian physics. According to this interpretation, which is prevalent in the standard textbooks on the history of physics, Aristotle's physics is essentially a kind of mechanics. As a result, Aristotelian physics is reduced to an immature or even primitive form of Newtonian mechanics. We think, however, that the mechanicist interpretation does not do justice to Aristotle's complex and highly organized system of thought.

We shall break with this tradition in pursuit of an interpretation which leans more toward thermodynamics. In doing so it is not our wish to gloss over the distinctions between the Aristotelian and thermodynamic concepts. We are well aware that there is an incommensurable gap between Aristotelian physics and macroscopic phenomenological thermodynamics and the fact that while centuries of reflection on Aristotelian physics figured prominently in the development of today's mechanics, no such activity can be observed in thermodynamics. That is why any thermodynamic interpretation would seem startling beside the mechanistic one which wins universal acceptance. Although thermodynamic interpretations may at first glance appear unhistorical, they put the history of thermodynamic concepts in new perspective.

1. On the dynamical aspects of Aristotle's natural philosophy

It is an important feature of Aristotle's natural philosophy that nature exhibits an all-embracing universal order. This

is not simply given. It is continuously created and recreated. So this order is always in the state of becoming. This might be expressed by saying that Aristotle's world is stationary. It is in state of an equilibrium which exists and goes on existing as the result of the constant movement of its parts. One manifestation of this idea of the world is the principle that nature and movement are indivisible.

Aristotle classified movement and change along several lines: He distinguished between *first*, the movement of living creatures and inanimate bodies; *second*, the movement of celestial and earthly bodies; *third*, natural and constrained movement. The circular movement of celestial bodies and the movement of earthly things striving upwards and down is natural, while constrained movement is produced when one body is directly influenced by another one. A *fourth* kind of classification is based on his theory of categories. According to this, movement (or change) can be divided into 4 types: a) *substantial* change (coming to be and passing away); b) *quantitative* change (increase and decrease); c) *qualitative* change (such as the change of color); and d) locomotion (movement where *place* is changed). The most general one of these last four kinds of changes is locomotion.

In all movements we can identify the following factors: mover or movent (*action*), moved (*passivity*), moving in something (*date*), and moving to somewhere (*relation*) [1].

In what follows we will concentrate on earthly locomotion of unanimate bodies. The reason is that Aristotle investigated the quantitative aspects only for locomotion.

1.1 On natural locomotion

Problems in Aristotelian *dynamics* are usually divided into two parts: problems of natural and problems of constrained movement. It is this division that sometimes seems to legitimate the mechanicist interpretation. E.g. Árpád Szabó [2] writes: "Aristotle differentiated between 'natural' and 'con-

strained' movement (movement 'caused by force'). Natural movement more properly corresponds to our concept of motion occuring under the impact of gravity." We find here two essential components of the mechanicist interpretations. First, force is regarded as if it were the cause only of constrained movement. Second, natural movement is to some extent equated with movement due to gravitation.

Nevertheless, Aristotle's concept of force is somewhat more complicated. As Jammer [3] points out there are *two kinds of force* in Aristotle's work: "...the Platonic conception of force inherent in matter, which he (i.e. Aristotle) calls 'nature' (physis), and force as an emanation from substance, the force of push and pull, causing the motion in a second object, and not in itself." When we speak about natural locomotion, we must pay special attention to this first kind of force, i.e. force inherent in matter.

However, it is not quite clear what Aristotle thought about this force. Mary Hesse [4] asks the following question: What does this force essentially belong to? To the body or to its natural place? Is it the "wish" of the body to come to rest in its natural position? Or is it a kind of attraction exercised by the natural position on the moving body? One way to solve this dilemma is to consider the body and the environment which includes its natural position *a single system*. Now we may regard natural motion as the common-existence of elements in this system. In other words we may make an attempt at a thermodynamicist interpretation instead of the mechanicist one.

When bodies have come to rest in their natural position the system can be said to be in a state of equilibrium. A moving thing embodies the passive principle of motion in so far as it undergoes movement only as a result of its lightness or heaviness. That is to say, it is "spurred" to action by its relation to its environment, namely that it is not in its natural position. In this way the force or "nature" inherent in the matter can be said to have a twofold character. On the one hand, it is an internal characteristic of the body, i.e. its lightness or heaviness. On the other hand, it belongs to the natural po-

sition. This double faced force strives to return the body to its natural position, and thereby to restore order or equilibrium.

Equilibrium is not the only thermodynamical concept that can be discovered in Aristotle's physics. The distinction between natural and unnatural positions amounts to the introduction of the principle of *inhomogeneity*. These questions are discussed at length in Book IV. of his Physics, where he argues against the possibility of void. Aristotle writes,

> "But how can there be any natural movement in the undifferentiated limitless void? For *qua* limitless it can have no top or bottom or middle, and *qua* vacancy it can have no differentiated directions of up and down..."

In other words space is isotropic. Furthermore,

> "in vacancy neither of these agencies would be in operation, so that nothing could go on moving unless it were carried. Nor (if it did move) could a reason be assigned why the projectile should ever stop - for why here more than there. It must therefore either not move at all, or continue its movement without limit, unless some stronger force impedes it." [5]

Aristotle explains the various kinds of resistance that can be encountered during motion as resulting from the differing densities of various media. Thus we can posit the existence of varying densities from the existence of motion itself and apply this in a more general sense to include the inhomogeneous distribution of matter in the world.

Actually, he proves there the more general thesis that movement is impossible in homogeneous world. If natural locomotion is to be subject to mechanistic interpretation as motion under the law of gravity, we must bear in mind that only the "inhomogenitive" character of the impact of gravity is of primary importance in Aristotle's system. Aristotle was interested in primarily the particularity of inhomogeniety, its disappearing, its counterbalancing phenomena. Aristotle put down a classical version of non-equilibrium thermodynamics in several key ideas: the concept of natural position or *equilibrium* of the body undergoing natural locomotion; the proposition that natural and unnatural must be differentiated (*inho-*

mogeneity); and the law that bodies strive toward natural position - toward counterbalancing the inhomogeneity (*equilibrium*).

Book IV. is the place that Jammer uses, when he shows that Aristotle investigated the *quantitative* aspect of the dynamics of natural movement as well [6]. If this quantitative aspect is also taken into consideration, we can develop the following thermodynamicist interpretation of natural movements. (See Table 1)

Table 1

The Thermodynamic Interpretation of Natural Movement

Aristotle's Concepts	Their Thermodynamic Interpretation
Moving body and its environment	Thermodynamical System
Natural locomotion	Transport process
Velocity of moving body (v)	Flow of extensive quantity (j)
Natural position	State of Equilibrium
Moving toward natural position	Process aimed at counterbalancing inhomogenious distribution
Moving force (A) (heaviness proportionate to density, or lightness proportionate to low density)	Driving force (gradient of intensive parameters of state) (X)
Resistance yielded by the "environment" (B)	Specific resistance (R)
Dynamic relation	Dynamic formula
$v \quad \frac{A}{B}$	$j = \frac{X}{R}$
Unnatural forces can assist or hinder natural movement	The processes of thermodynamic system can be influenced by external constraints, forces

1.2 On constrained locomotion

In the standard mechanistic interpretation of constrained locomotion, the force exercised by other objects on the moving body is termed the cause of movement. (This is the second kind of Aristotelian forces, i.e. force of push and pull.)

Constrained locomotion, too, can be understand in terms of thermodynamics. (See Table 2 and Table 3.)

Table 2

Interpretations of the Basic Elements of Constrained Locomotion

The Four Factors of movement	Aristotle	Standard Mechanical Interpretation	Aidun's Mechanical Interpretation [7]	Thermodynamic Interpretation
A	mover	force	power	driving force
B	moved thing	weight or mass	frictional resistance	specific resistance
C	displacement	displacement	displacement	change of the extensive quantity
D	time interval	time interval	time interval	time interval

In favour of the thermodynamical interpretation the following argument can be adduced. In the mechanicist interpretation of constrained movement Aristotle's dynamical law is the following:

$$v = \frac{A}{B} \qquad \left(\text{velocity} \sim \frac{\text{force}}{\text{mass}}\right).$$

This law is false. However, if we take the thermodynamicist interpretation the same law can be formulated as follows:

$$j = \frac{X}{R} \qquad \left(\text{flow} \sim \frac{\text{driving froce}}{\text{resistance}}\right).$$

Table 3

Similarities in the Dynamics of Constrained Movements

Aristotelian (Peripatetic) Dynamics [8]	Newtonian (Mechanical) Dynamics [8]	Thermodynamics
to sustain movement an acting force is needed	acting force is needed to change state of movement	to sustain movement driving force is needed
$v \sim F$	$\frac{d}{dt} v \sim F$	$j \sim X$
if $F = 0$ then $v = 0$	if $F = 0$ then $v =$ constant	if $F = 0$ then $j = 0$
the movement: process	the movement: state	the movement: process

This equation is not false any more. So while the mechanicist interpretation has to attribute a mistake to Aristotle, our interpretation shows that Aristotle was right. So the thermodynamicist interpretation leads to a unified view of the dynamics of natural and constrained movements. The content of the dynamical principles is an essence identical with the Second Law of thermodynamics. We can therefore regard the entire Aristotelian world system as an antique thermodynamics system.

2. *On the Organizing Principles of Aristotle's Natural Philosophy*

In addition to the principles and statements we have already dealt with, Aristotle's work is replete with references to the organizing role of thermal effects. The heat of the sun is preminent in the development and sustenance of universal order. Thus, the Physics tells us, "In Nature man generates man; but the process presupposes and takes place in natural material already organized by the solar heat..." [9].

Furthermore, according to Aristotle, things with various qualities can be constructed of elements characterized by primary qualities. These primary qualities must be tangible, since touch is the primary sence. Primary qualities can be classified as pairs of opposites: hot and cold, wet and dry. Hot and cold are described as active, wet and fry as passive primary qualities [10]. So thermal qualities are both primary and active. Mechanical qualities, such as lightness and heaviness, are, in contrast, only derivative ones.

The notion of temperature, too, is reduced to the primary thermal qualities. The temperature of a body expresses the proportion in which they are combined [11]. When Aristotle calls these two primary qualities hot and cold, he speaks literally and not metaphorically. Solmsen argues that "hot" as one basic quality generally "does not really act as hot; it does not burn, scorch, consume, harden, or exsiccate. These are the functions which had been of interest to earlier physicists and physicians; "active power" as such would have meant nothing to them. Where Aristotle introduces hot and cold into the present discussion, he defines their specific, "active" powers as capacities of bringing together (i.e. "associating")... [12].

In other cases - for instance, in biological applications - heat fulfills these two roles; it generates warmth and imposes order on the development of living creatures [13].

Aristotle uses thermal qualities to explain principles in a dual sense. As we saw in Solmsen's explanation earlier, hot and cold are possessed of an active force which draws things

together; as such they are the medium for function. At the same time they retain much of their concrete nature and exert this inherent force. Most examples drawn from Aristotle exhibit this dual character. Heat restores the part of the body afflicted by disease by causing it to grow warm. [14]

The pre-eminence of hot/cold also ties in with ideas on material structure. The fall of Democritus' atoms through space can be described as having rectilinear, vertical motion. At the same time Aristotle found many arguments in support of the impossibility of void. As a result, in Aristotle's way of thinking matter is constantly filling the world. Because of the nature of this matter all different sorts of objects existing in the world simply cannot be generated according to Democritus' method. Consequently, Aristotle must turn to new areas of experience where hot and cold become the main organizing and moving principles. And these principles are not only preeminent in material structural processes, but play a prime role in organizing the world order. Thus the sun's heat is essential to the great natural cycles governing the circulation of water [15].

Aristotle's ideas on the active character of thermal qualities sometimes sound startlingly modern. They appear, e.g. in Prigogine's and Stenger's recent book on thermodynamics [16]: "We know now that non-equilibrium, the flow of matter and energy, may be a source of order...this... leads to a new concept of matter, matter that is 'active', as matter leads to irreversible processes and as irreversible processes organize matter."

3. Describing a Particular Change

Aristotle considers movement as a process in which potentialities become actual. He describes each particular process of this kind in three concepts: dynamis, energeia, and entelekheia. Dynamis means roughly ability to change [17]. Energeia stands for the working of this ability [18]. Finally, entelek-

heia is best construed as realized ability [19]. (Table 4.)

Table 4

Interpretations of the Concepts Concerning a Particular Change

Aristotle's Concepts	Their Meaning	Mechanicist Interpretation	Thermodynamical Interpretation
dynamis	*ability* the potentiality of change	*force* (associated to constrained movement)	*driving force* caused by inhomogeneity
↓	↓	X	↓
energeia	*working of the ability* the actualization of change	*energy* < potential / kinetic	*the process of homogenization*
↓		X	↓
entelekheia	*realized ability* change which has been actualized	-	*equilibrium* as the actualized homogenization

Another shortcoming of the mechanicist interpretation is that it devotes too much attention to dynamis and neglects or misinterprets the other two concepts. In the thermodynamicist interpretation, nevertheless, these concepts regain their full importance. The fact that in each of Aristotle's analysis these concepts appear in dynamis - energeia - entelekheia order can also be readily explained. It is a central idea in thermodynamics that natural processes have directions. It is exactly this idea of direction which is signified by this order. We would like to add here, that the direction of natural processes

has been already touched upon. We said that natural locomotion is a movement towards the state of equilibrium. What we can see now is that *all* Aristotelian processes have definite directions.

A brief comment on Aristotle's theory of causality is in order to illustrate the efficiency of thermodynamic interpretation. We shall only deal with one specific problem in relation to the four causes (material, formal, effective, and final). [20] The presence of the effective and final causes merits special consideration. Applying a purely mechanistic interpretation to the principle of final cause would lead us nowhere. However, this is not the case with a thermodynamic interpretation, since thermodynamics takes a basically teleological view of the problem. This teleological approach stems from the uniquely global character of the thermodynamic thought-process.

4. Aristotelian Thermodynamics

We have strived so far to present arguments in favour of a possible thermodynamicist interpretation of Aristotelian physics. If this interpretation is accepted, it will turn out that nearly all the *experiences* that are essential to thermodynamics were described by Aristotle. His descriptions include biological, medical, meteorological and technical observations, as well as knowledge necessary for cooking.

Most of the basic thermodynamic *concepts* such as equilibrium inhomogeneity, irreversibility, driving force, thermal interaction [21], self-organization can be found in his dynamical views. We have hinted at his recognition of the content of the Second Law of thermodynamics, and it is clear that he was familiar with the essence of the First one, too. Evidence for this is the knowledge of the fact: heat could be produced by friction [22].

It does not require much effort to identify in his work important features of modern thermodynamical *theories*.

Some very important *problems* are also present in his work. For instance the very fascinating question why there is no heatdeath. His solution runs as follows. If in the course of natural movement bodies moving into natural position have come to a state of equilibrium, one wonders why they do not stay there. In his explanation the heat of the sun makes the water evaporate. This water then falls down to the earth when it rains, and it does so again and again. Thereby the equilibrium is broken again and again. However the sun acting as a foreign external force transforms water into vapour. Furthermore the sun moves itself, and its movement was started by and maintained by the motionless mover. Thus the motionless mover governs the world in such a way that he is constantly jarring it from its state of equilibrium! Aristotle's God is the great jumbler of the Universe - an entropic God. He is not at all like the clockmaker of the seventeenth century - the magnificent constructor, the mechanistic God. So the Aristotelian world was simultaneously closed and open; the motionless mover closed the world logically and made it open physically.

Aristotle arrived at great many thoughts on thermodynamics by subjecting all earlier theories to critical summary. He summed them up in a single theory and added a lot of ideas that were completely new. These new concepts and ideas proved to be of such enormous social and cultural value that they completely overshadowed older theories, or at least threw them into new perspective.

Aristotelian physics contained the possibility of both thermodynamics and mechanics. Nevertheless, it was only the latter possibility that was eventually realized. While centuries of reflections on Aristotelian physics figured prominently in the development of today's mechanics, thermodynamics was born independently from it. It emerged only in the nineteenth century. But why did not the nineteenth century thermodynamicists consider Aristotle their predesessor? As the mechanical world view gained wide spread acceptance, the evolution, interpretation and explanation of Aristotelian physics also become subject to mechanistic thought. As a result, by the time

scientific thermodynamics had begun to develope the chance for using Aristotelian physics to interpret thermodynamics was lost.

So we can say that there is a close analogy between Aristotelian physics and modern thermodynamics, but we cannot say that it was Aristotelian physics that developed into modern thermodynamics.

Nevertheless, we may put the following question. What impeded the continuous development of thermodynamics and supported the development of mechanics in particular? There were a lot of causes, but we have only a highly tentative suggestion:

The relation between man and his natural and social environment has radically changed: "the active man in an active environment" view was superseded by "active man in a passive environment" view. A parallel process was the transition from physics of the world of *observers* to the physics of the world of *handicraftsmen* [23]. This new physics facilitated the development of mechanics, since its content harmonized with the handicraftsmen's world view; and hindered the development of thermodynamics because its content would not harmonize with this new world view. The transition from the physics of the world of observer to that of physics of the world of handicraftsman was so alluring and successful, that the thermodynamic elements of Aristotelian physics were relegated to the background until they completely disappeared or were submerged in mechanistic propositions.

5. Summary

We have shown that if we break with mechanicist interpretation, lots of new features of Aristotle's physics will be revealed. We have found fairly strong analogies between Aristotelian physics and modern non-equilibrium thermodynamics. These analogies are so strong, that we may try to reconstruct *the Aristotelian thermodynamics*. Constituents of Aristotelian

thermodynamics can be identified in different levels of abstractions. First of all there is an adequate thermodynamic interpretation of *Aristotelian dynamics* leading to a unified view of natural and constrained movement, and the content of his dynamic principles is in essence identical with the Second Law of thermodynamics. Furthermore in Aristotle's natural philosophy the *hot* and *cold* are the organizing principles. So the name "Aristotelian thermodynamics" was coined so as to refer to two things. First, to an ancient period of thermodynamics which remained largely unnoticed for a long time. Second, to a relatively neglected part or aspect of Aristotle's physics.

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BEMERKUNGEN ÜBER DIE ÖKONOMISCHE ANSICHTEN VON LEONARDUS LESSIUS

Károly Redl

Der langwierige und in Weltmaßstäben bis heute auch noch nicht abgeschlossene Prozeß bürgerlicher Umwälzung setzte irgendwann im Mittelalter ein. Diejenigen großen Kämpfe und Revolutionen, die die Entwicklung vorwärtstrieben, brachen nicht auf einen Schlag durch, sondern arbeiteten unter sich ändernden historischen Bedingungen und mit mannigfachen Methoden an der Entfaltung dieses welthistorischen Prozesses. Die ersten Zusammenstöße zwischen der alten Ordnung und den neuen Kräften gingen noch inmitten eines erdrückenden Übergewichts der feudalen Verhältnisse vor sich, als die ideologische Form des Kampfes noch eine religiöse war; in unserem Zeitalter jedoch, in welchem der kapitalistischen Entwicklung gegenüber auch schon die Möglichkeit anderer Alternativen sich zeigt, sind verspätete bürgerliche Revolutionen auch gezwungen, unter der Hülle sozialistischer Ideen zu erscheinen. Mochte es aber die Mönchskutte oder die Toga eines römischen Republikaners gewesen sein, was der Bürger in seinem Kampf für die Macht anzog, er wählte diese Rollen nicht willkürlich, er war im Gegensatz dazu gezwungen, diese Rollen zu spielen, weil er anderswie die Unterstützung der Massen nicht hätte erwerben können. Die Bedeutung der Massen wuchs jedoch stets, um so mehr, weil um ihrer Interessen willen auch sie den Kampf immer besser erlernten.

Die niederländische Revolution und der Freiheitskrieg, die Spinoza juristisch und moralisch legitimiert, galten jedoch nur als Ouvertüre von dem gewaltigen historischen Prozeß der bürgerlichen Unwälzung, sie galten als die erste groß Explosion, die die Feudalordnung erschütterte. Die Kraft des Feudalismus war in dieser Zeit noch bei weitem nicht gebrochen und die Herrschaft von mittelalterlichen Formen durchdrang noch je-

de Lebensspäre. Währendder "goldenen" Periode der Niederlande ging jedoch der erste große Versuch vor sich, diese Formen zu verändern, sie umzuwälzen und neue Formen herauszuarbeiten. Dies ist der Grund dessen, warum die Ergebnisse und Erzeugnisse dieser Periode in Hinsicht auf die bürgerliche Entwicklung für lange Zeit eine paradigmatische Bedeutung erlangten.

In seinen Vorlesungen über die Philosophie der Weltgeschichte würdigte Hegel diese große Umwälzung und setzte ihr mit folgenden Worten Denkmal: "Die Niederlande aber brachen gegen Spanien in förmliche Empörung aus; ihr Aufstand war Lossagung vom Glaubensjoche, aber auch politische Befreiung vom Drucke der Fremdherrschaft. Belgien war der katholischen Religion noch zugetan und blieb unter spanischer Herrschaft; der nördliche Teil dagegen, Holland, hat sich heldenmütig gegen seine Unterdrücker behauptet. Die gewerbetreibende Klasse, die Gilden und Schützengesellschaften haben die Miliz gebildet und die damals berühmte spanische Infanterie durch Heldenmut überwunden. Wie die schweizerischen Bauern der Ritterschaft standgehaltan haben, so hier die gewerbetreibenden Städte den disziplinierten Truppen. Währenddessen haben die holländischen Seestädte Flotten ausgerüstet und den Spaniern ihre Kolonien, woher ihnen aller Reichtum floß, zum Teil genommen Ewig denkwürdig ist dieser Kampf betriebsamer Bürger gegen die Herren der Reichtümer Mexikos."

Holland erkämpfte sich jedoch seine Selbständigkeit nicht allein durch das protestantische Prinzip, auch wenn dieses die "Rechtschaffenheit" mit enthielt, wie Hegel an demselben Orte sagt. Die Erkämpfung der Selbständigkeit und die in ihren Spuren einsetzende stürmische Entwicklung hatten auch andere, zum Teil internationale, zum Teil aber aus den Eigenschaften der lokalen Wirtschaft stammende Vorbedingungen.

Zeitgenossen über den hollandischen "Wunder"

Zeitgenössische Beobachter, die sich gewöhnlich bewunderungsvoll über die Entwicklung der Niederlande äußerten, waren auf diese Zusammenhänge aufmerksam geworden. Der englische Ökonom, Thomas Mun, sagt beispielsweise folgendes: "Von der Zeit an, seit diese (die Holländer) das Joch spanischer Knechtschaft abschüttelten, kann man nur bewundern, wie sie jede ihrer Fähigkeiten entfalteten. Was für gewaltige Mittel sie sich erwarben, um ihre Freiheit vor der Kraft eines so großen Feindes zu schützen! Und ist all dies nicht Frucht ihres im Handel entfalteten unermüdlichen Fleißes? Sind ihre Länder vielleicht nicht zu Warenlagern für die Mehrheit von den Ländern der christlichen Welt geworden, demzufolge ihr Reichtum, ihre Schiffahrt, ihr Seehandel, ihre Industrie, ihr Volk und folglich ihre öffentlichen Einkünfte, sowie Steuereinnahmen eine be wunderungswürdige Größe erstiegen? Vergleicht man die Zeit ihrer Knechtschaft mit ihren gegenwärtigen Zuständen, so bieten sie das Bild eines anderen Volkes."

William Petty, durch den, wie Marx festsellt, die politische Ökonomie in England ihren Anfang nahm, befaßt sich in seiner zwischen 1671 und 1676 verfaßten "Politischen Arithmetik" (Political Arithmetic) eingehend mit den wirtschaftlichen und politischen Ursachen des holländischen Aufschwunges. Petty, der die Ansicht vertritt, daß "ein kleines Land von niedriger Einwohnerzahl durch seine geographische Lage, durch seinen Handel und seine Politik an Reichtum und Kraft gleichwertig mit einem viel größeren Volk, bzw. Gebiet werden kann", bemerkt über die Holländer folgendes: "Viele Autoren, die über dieses Thema schreiben, loben die Holländer so sehr, wie wenn sie sich in übermenschlichen Höhen befänden und die übrigen Nationen gleichzeit (in Handel und Politik) unter dem menschlichen Niveau lägen; sie lassen die Holländer als Engel auftreten, während die anderen gleichzeitig als Dummköpfe, Tiere und Trunkenbolde beschrieben sind; ich bin aber der Ansicht, daß der

Grund ihrer Ergebnisse ursprünglich die geographische Lage des Landes war; erst dadurch wurden sie in die Lage gesetzt, unnachahmliche Dinge vollbracht und beispiellose Vorteile erworben zu haben."

Diese Vorteile erblickt Petty in den folgenden Komponenten: weil das Land "flach, reich und fruchtbar ist, kann es deshalb viele Menschen ernähren", "die Menschen können in der Nähe voneinander wohnen, weil sie mit Hilfe des Handels einander gegenseitig helfen können", überall können Windmühlen aufgestellt werden, demzufolge "viele tausend Arbeiterhände frei werden", ferner ist es so, weil man "mit der Manufaktur viel mehr gewinnen kann als mit der Landwirtschaft und mit dem Handel viel mehr als mit der Manufaktur" und "Holland und Zeeland befindet sich jedoch bei der Mündung dreier großer Flüsse, die reiche Länder überqueren und dies behält die gesamte Einwohnerschaft von den Ufern dieser Flüsse im Zustand der Landwirtschaft: während die Holländer selber all diese Produkte in Manufakturen aufarbeiten und in die ganze Welt in allen Richtungen auseinandertragen und die Preise von den Waren, die man ihnen für sie als Rückvergütung gibt, im ganzen nach ihrem Belieben festsetzen können." Petty erwähnt darüber hinaus noch eine ganze Reihe von ökonomischen Komponenten (die Nähe der schiffbaren Handelswege, die geringen Kosten der Landesverteidigung, die vor allem der günstigen Verteidigungslage des Landes entspringen, die niedrigen Landungskosten in der Schiffahrt, das Einkommen aus der Fischerei, die Erkenntnis von den Schätzen und Bedürfnissen ferner Länder durch Schiffahrt und eine aus fremden Rohstoffen genährten und die Bedürfnisse der verschidensten Länder befriedigende Produktion) und dann geht er auf die Erforschung der politischen Gründe ein und hebt die große Bedeutung von der Religions- und Gewissensfreiheit hervor.

Auch den Grund von der Loslösung von Spanien erblickt Petty darin, daß die Holländer, die "zumeist wirtschaftende, nüchterne und geduldige Menschen und als solche der Ansicht sind, daß die Arbeit und der Eifer ihre Pflicht dem Gott gegen-

über ist" ("so irrig auch solche Ansichten sind" - fügt er hinzu), bestrebt waren, "der Steuerveranlagung des Klerus" aus dem Wege zu gehen und die Pfarrer für "überflüssige Last" hielten.

Religiöse Toleranz gründet bei ihnen auf folgende Erwägungen: "Ihre Überzeugung ist, daß vergeblich, absurd und der Achtung Gottes fremd ist, wenn niemand seinen Neigungen nachgehen und glauben darf, was er will, ferner, wenn die Menschen gezwungen seien zu glauben, was sie nicht glauben." Die Holländer, die sich für keine "unfehlbare Kirche" halten, "beobachteten" auch, "daß dort, wo die Aufrechterhaltung der Gleichförmigkeit in religiösen Dingen" am meisten forciert wird, die Heterodoxie am schönsten blüht". In diesem Zusammenhang macht Petty, dessen Humor schon Marx pries, die folgende Bemerkung: "Wenn ein Viertel der Bevölkerung heterodox wäre und diese Gruppe vermittels irgendwelchen Wunders verschwände, so oder so würde ein Viertel der gebliebenen Bevölkerung wieder heterodox werden, da es ein natürlicher Zug des Menschen ist, daß sich die Auffassungen über die übersinnlichen und übervernünftigen Dinge voneinander unterscheiden und insbesondere die, die über geringeres Vermögen verfügen, diejenige Auffassung für selbstverständlich halten, daß in ihnen mehr Geist und Vernunft hauptsächlich in Fragen über den Gott existiert, weil sie diese Fragen als solche ansehen, die vor allem eine Angelegenheit der Armen ist."

Die scharfblickenden Zeitgenossen wiesen in der Tat auf richtige Momente hin, indem sie das Geheimnis des "niederländischen Phänomens" unter die Lupe nahmen. Die von Petty in die Rede gebrachte geographische Lage spielte tatsächlich eine große Rolle in dieser Entwicklung. Die Entfaltung der internationalen wirtschaftlichen Relationen, in der ersten Linie die Verschiebung der großen Handelswege verlieh diesem Gebiet eine eigentümliche Bedeutung. Die Bedeutung des Mittelmeers als jenes Wasserweges, welcher Europa mit dem Osten verbindet, verblaßte, infolge der großen geographischen Entdeckungen gegen-

über dem Atlantischen Ozean, welcher nun zur Landstraße des Verkehrs mit Amerika geworden ist. Dadurch ist die Bedeutung des atlantischen Ufers, sowie die der Niederlande natürlich größer geworden. Die Ausnützung von dieser günstigen welthistorischen Position wurde aber erst durch weitere besondere Komponenten ermöglicht. Unter diesen sind die mittelalterliche Entwicklung des Landes, sowie der Erfolg des Freiheitskriegs gegen die spanische Herrschaft, sowie der Sieg der Revolution die wichtigsten. Das Flandern des Mittelalters gehörte zu entwickeltesten Gebieten im feudalen Europa, und zwar vor allem wegen seiner Tuchindustrie. Kein Zufall, daß dieses Gebiet, wo die städtische Leben blühte und der Demokratismus der Stadtrepubliken lebendig war, gleichzeitig als Brutstätte von Ketzerbewegungen galt. Der Freiheitskrieg gegen Spanien war nicht allein politisch und wirtschaftlich, sondern damit eng verbunden auch religiös von Bedeutung: der Gegensatz von Katholizismus und Reformation war ideologische Form der Kontroverse zwischen feudalen Kräften und bürgerlicher Ordnung. Durchaus wichtig war auch der historische Umstand, daß England, die klassische Heimat der ursprünglichen Kapitalakkumulation und der frühen bürgerlichen Entwicklung sich in dieser Epoche noch auf seine innere Entfaltung konzentriert und sich noch in der Phase der Kräftesammlung befindet, in der - wie Marx darauf hinweist - auch die Wirkung eben von Holland eine überaus wichtige Rolle spielte, England trat auf dem Schauplatz der internationalen Wirtschaft also noch nicht mit seinem vollen Gewicht auf. Das geschieht erst seit der zweiten Hälfte des 17. Jahrhunderts, es ist die Zeit, nach der Holland auch in Weltmaßstäben in den Hintergrund gerückt und aus seiner führenden Position verdrängt wird.

Die Scheidung von Süd und Nord

Die Entfaltung der niederländischen Revolution und des Freiheitskrieges war nicht geradlinig, sondern ließ eine innere Polarisation zwischen Süd und Nord entstehen. Die Wurzeln derselben reichen in die Vergangenheit zurück und entstammen u.a. den Unterschieden von der früheren Entwicklung der beiden Teritorrien. Die militärische Niederlage des Südens vollendete nur die in der Union von Arras (1579) deklarierten Loslösung von den von Vallonen bewohnten Gebieten. Durch den Fall von Gent, Bruxelles und Antwerpen (1584-1585) endete sich die Eroberung der Süd-Niederlande durch die Spanier und die im Mittelalter einheitliche "natio belgica" entzweite sich endgültig: der Süden, d.h. Belgien, setzte seine Existenz unter spanischer Vorherrschaft fort und unter Albert von Habsburg (1596-1621), sowie seiner Frau, der Tochter von Phillipp II, Isabella (1595-1633) ging die Restauration von den feudalen Verhältnissen, sowie die von der katholischen Kirche vor sich. Im Norden jedoch setzten die Vereinigten Länder, die nach ihrem führenden Gebiet Holland genannt wurden, den Kampf fort und so entstand die erste unabhängige Bourgeois-Republik Europas. Der Prozeß von der Entstehung der bürgerlichen Nation sowie der belgischen und holländischen Nation, der am Ende des 16. Jahrhunderts seinen Anfang nahm, erlebte jedoch mehrere Stadien und ließ sich erst im 19. Jahrhundert vollenden.

Die divergierende Entwicklung von Belgien und Holland offenbarte sich nicht nur auf dem Gebiet der Wirtschaft und der Politik, sondern auch auf dem der Religion und Kultur. Aus möglichen Gegensätzen und Paralleler es wäre sehr interessant z.B. diejenige zu untersuchen, die zwischen den beiden Denkern Lessius und Spinoza, die in gewissen Sinne die belgische bzw. holländische Entwicklung representieren, zu ziehen wäre. Dies-

bezüglich erwähnen wir hier nur einen, aber wichtigen Vergleichspunkt: wie die Relation der beiden Denker zu der kapitalistischen Entwicklung gestaltet ist.

Kapitalistische Entwicklung im neuscholastischen Spiegel: Lessius und Antwerpen

Albert von Habsburg übernahm die Regierung des südlichen Landteils, Belgiums, als Protektorat und stellte sie in den Dienst von den Interessen des spanischen Absolutismus. Seine Verwaltung unterdrückte die Opposition mit Waffengrwalt, unterordnete die ständischen Organe der Zentralgewalt, hob die politischen Rechte von Zünften und die Tätigkeit von den demokratischen Institutionen der Städte auf, rief die Ständerversammlung der Länder durch zwei Jahrzehnte hindurch nicht zusammen, setzte Jahressteuern von riesiger Größenordnung systematisch aus und trieb außerdem auch noch außerordentliche militärische "Hilfen" ein. Grundlegende Maxime seine Politik war es, wie er das Land in einen geeigneten Aufmarschraum gegen die "aufrührerischen" Kräfte des Nordens verwandeln kann.

Die politische Restauration begünstigte die Erstärkung der feudalen Verhältnisse, sowie die der katholischen Kirche. Ihre Folgen stellten in wirtschaftlicher Hinsicht auch einen Rückfall dar, unter anderen auch aus dem Grunde, weil viele protestantische Kapitalisten, Handwerker und Bürger vor religiösen Verfolgungen nach Norden flüchteten.

Die katolische Restauration, auf deren Spitze die Jesuiten standen war mit dem materiellen und gleichzeitig mit dem geistigen Machtzuwachs der Kirche gleichbedeutend. In jesuitischer Hand waren die Universitäten von Douai und Leuven, die als geistige Zentren der Restauration galten. So wurde Belgien zu einem Hauptbollwerk der Restauration.

Eine der hervorragenden Persönlichkeiten katolischer Restauration war Lessius, der zu der Spinoza vorangehenden Generation gehörte und sich ernsthaft mit den neuen Erscheinungen des zeitgenössichen Wirtschaftslebens auseinandersetzte. Leonardus Lessius (de Leys) (1554-1623), der Meister scholastischer Analyse der Wirtschaft, ist in Brecht geboren, in der Nähe Antwerpens und wurde von seinen Eltern zum Beruf eines Kaufmanns ausersehen. Nach dem Abschluß der Universität Leuven trat er in seinem 17. Lebensjahr in den jesuitischen Orden ein und lehrte zwischen 1575 und 1581 Philosophie im englischen Kolleg in Douai. Nachher studierte er zwei Jahre lang in Rom unter der Führung des berühmten Roberto Bellarmini (1542-1621) und Francisco Suárez (1548-1617), die damals als die bedeutendsten Denker des jesuitischen Ordens galten. Aus Rom zurückgekehrt, erhielt er 1584 einen Lehrstuhl an der Universität Leuven, wo er sich bald eine große Berühmtheit verschaffte. Wegen seines schwachen gesundheitlichen Zustandes jedoch, der die Folge seiner Flucht vor den "geux" im Jahre 1578 war, zog er sich in seinem 46. Lebensjahr von dem Unterricht zurück und widmete den weiteren Teil seines Lebens der literatischen Tätigkeit. Außer seinen theologischen Arbeiten, die sich mit den Fragen den Gnade, des freien Willens, sowie der Prädestination befassen, ließ er seine Stimme auch in aktuellen Problemen der politischen Theorie hören und griff - wie auch Suárez - die absolutische Theorie von dem göttlichen Recht der königlichen Macht und der aus Gottes Gnade stammenden unbegrenzten Macht des Königs an, wie diese Theorie von Jakob dem Ersten, dem englischen König vertreten wurde. Das wichtigste Werk Lessius ist *De Justitia et Jure* (1605), welches die Gerechtigkeit und das Recht behandelte und in ganz Europa in zahlreichen Ausgaben erschien. Der dritte Abschnitt von dem zweiten Buch diese Werkes (17-28. Kap.), welcher sich mit den Fragen der Verträge beschäftigt (*De contractibus*), enhält die Erörterungen von Lessius über

wirtschaftliche Thematik. Albert der König von Belgien, hielt der Überlieferung nach dieses Werk von Lessius stets bei der Hand und sah in ihm einen soliden Ratgeber.

Lessius, der einer reichen Kaufmannsfamilie entstammte, hatte Gelegenheit im Laufe des 16. Jahrhunderts das wirtschaftliche Leben des sich zu einem internationalen kommerziellen und geldwirtschaftlichen Mittelpunkt entwickelnden Antwerpen aus unmittelbarer Nähe zu beobachten und er beruft sich tatsächlich auf seine diesbezüglichen Kenntnisse. Unter den im 16. Jahrhundert emporgekommenen internationalen kommerziellen Zentren wie Kastilien, Lyon, Frankfurt, Besançon - errang Antwerpen eine führende Rolle für Jahrzehnte, seine Bedeutung wurde später durch die holländische wirtschaftliche Blockade in den Hintergrund gedrängt.

Unter der Hülle von den überlieferten Kategorien der scholastischen Analyse der Wirtschaft zeichnet sich bei Lessius die Bedeutung von der Geldwirtschaft, sowie den Marktverhältnissen bereits klar aus. Lessius' Protagonist ist nicht mehr der idealisierte Polis-Bürger antiker Philosophen, der als wohlhabender Herr seiner Wirtschaft, *oikos*, sich von den niedrigen Handwerkerberufen vornehm fernhalten konnte, sowie über die genügende Freizeit verfügte, um mit der Pflege des eines freien Menschen würdigen Künste und durch das Studium der Philosophie sich solide Seelenruhe und erhabene rationale Anschauung der kosmischen Zusammenhänge und Gesetzmäßigkeiten zu erwerben. Er ist aber auch nich mehr eine Verkörperung von dem christlichen Lebensideal mittelalterlicher Doktoren, nich mehr der Gläubige, der aus den Früchten seiner Arbeit bescheiden lebt, anderen gern schenkt, sich an weltlichen Gütern nich festhält und insbesondere den Geiz und die Geld- und Vermögensakkumulation ablehnt, das diesseitige Jammertal zugunsten des zukünftigen himmlichsen Glücks vernachlässigt und der Pflicht der persönlichen Dienstübung und dem Befehl der Liebe lebt. Der Protagonist von der wirtschaftlichen Welt ist bei Lessius bereits der *negotiator diligens*, der fleißige Geschäftsmann, der

flinke Kaufmann, der der Abstraktion des "homo oeconomicus" schon n:her als jenem adeligen Ideal von den herrschenden Klassen der Naturalwirtschaft betreibenden Epochen steht, welches die vollständige und vielseitige menschliche Persönlichkeit in den Mittelpunkt stellt und ist bestrebt, in seinen Berechnungen den erreichbar größten Gewinn zu erzielen.

Symptomatisch ist es schon auch, daß in seinem erwähnten Hauptwerk Lessius das Geld nicht allein als Wertmesser und Tauschmittel betrachtet, wie es die mittelalterlichen Doktoren gewöhnlich taten, sondern auch als Ware, was im Gegensatz zum römischen Recht und zur feudalen Auffassung stand.

Lessius erkennt aber auch die Kapitalfunktion des Geldes an. Während seiner Beschäftigung mit dem alten Problem des *lucrum cessans*, des "ausgebliebenen Nutzens", die ursprünglich die Berechtigung der Zurückgabe der in Leihgeschäften gelegentlich erlittenen Schäden ausdrückte und sich immer mehr zu einem Prinzip entfaltete, welches die Rechtsmäßigkeit der nach jeder Geldanleihe erforderlichen Zinsen aussagte, hebt Lessius diesen "fruchtbaren" Charakter des Geldes hervor und hält ihn für vollkommen akzeptabel. Dies tut er im Gegensatz zur Auffassung des Mittelalters, welche das Geld im Grunde als "unfruchtbares", "unproduktives" Ding ansah und dementsprechend den Geldwucher, in dem Geld Geld zeugt, als "unnatürlich" und als Sünde abqualifizierte, Lessius das Geld als den "fruchtbaren Kern des durch Eifer erzielten Gewinns" (*semen foecundum lueri per industriam*) definiert.

Von dem Entwicklungsstand von den geldwirtschaftlichen und kommerziellen Operationen, sowie von dem institutionalisierten und organisierten Charakter der Marktverhältnisse zeugt auch diejenige Überlegung Lessius', daß - wenn auch das Darlehen nicht identisch mit der Aufopferung grwissen potentiellen Profits ist, jeder Kredit für den Darleiher mit einer Aufopferung, bzw. Entbehrung des zur Verfügung stehenden und liquiden Geldsumme gleichbedeutend ist, was unter den Verhältnischen eines gut ausgebauten Geldmarktes den Darleiher in ungünstige Situation zu bringen imstande ist und gerade deshalb gewährt der

Verlust des liquiden, mobilisierbaren Kapitals im Laufe des Darlehens einen neuen Rechtstitel auf die Zinsen, die nach der verliehenen Summe verlangt werden dürfen. Diesen Fall nennt Lessius das Fehlen oder die Entbehrung des Geldes (*carentia pecuniae*).

Unter so entwickelten wirtschaftlichen und kommerziellen Verhältnissen, wie diejenigen, die im Antwerpen dieser Zeit vorherrschend waren, bedeutete die Verfügung über Bargeld, über liquides Geldkapital offensichtlich einen besonderen Vorteil, für welchen die Geschäftsleute bereit waren, Zinsen zu zahlen. Infolge dieser Nachfrage nach Bargeld wurde der Anspruch des Darleihers auf Kompensation, auf Zinsen als berechtigt anerkannt, worin Lessius mit den Geschäftsleuten vollkommen übereinstimmt. Er beschwört in diesem Zusammenhang das Beispiel der Antwerpener Bröse herauf: "Und es scheint, dies Praxis der Antwerpener Börse oder Peristylium, wo die Kaufleute täglich zusammenkommen und nachdem sie mit der Fülle oder dem Mangel des Geldes, mit der Anzahl der valutarischen Angelegenheiten (*cambia*), mit der Masse der Waren und mit jedweder Art der Profitmachung (in denen Bargeld - *pecunia praesens* - erfordert wird), gerechnet hatten, setzten sie entweder allein oder durch ihre Makler den Preis von der Entbehrung des Geldes fest, den zu fordern die Kaufleute berechtigt sind, weil sie ihr Geld so lange entbehren und es vor dem festgesetzten Termin nicht zurückverlangen können. Dieser Preis war manchmal 6% für ein Jahr, in anderen Fällen 7%, 8%, 9%, 10%, 11% oder 12%. Nach Anordnungen von Karl V., die er am 4. Oktober 1540 Brüssel traf, war es verboten, mehr als 12% zu verlangen und wenn irgendwo mehr verlangt wurde, faßte man es als Wucher auf. Ferner kann diesen Preis auf demselben Ort jeder Kaufmann verlangen, auch wenn ihm kein Nutzen ausbleibt (*nullum lucrum cesset*) ... Den es ist dies der gerechte Preis für die Entbehrung des Geldens unter den Kaufleuten; denn der gerechte Preis von irgendwelcher Ware (*res*) oder Verbindlichkeit (*onus*) ist in einer Gemeinschaft, was dieselbe im Interesse des Gemeinwohls bei Er-

wägung aller Umstände festsetzt... Wenn auch also mein Nutzen infolge der Entbehrung des Geldes durch ein Jahr hindurch nicht ausbleibt und mein Kapital nich gefährdet wird, kann ich sie (die Zinsen) ebenso verlangen, wie die anderen, weil für diese Entbehrung aus rechten Gründen so ein Preis ausgesetzt ist."

Hier können wir selbstverständlich auf eine Darlegung von den wirtschaftlichen Ansichten des Lessius nicht eingehen. So eine Untersuchung würde jedoch ergeben, wie dies vielleicht schon auf Grund des Bisherigen zu vermuten ist, daß die Anerkennung, sowie die theoretische Bestätigung von der Bedeutung der Marktverhältnisse eine beträchtliche Leistung des belgischen Theologen war. Nich nur Fragen des Warenmarktes, des Marktes, für Gelddarleihen, sowie des Devisenmarktes erscheinen in seinen Untersuchungen (was den Devisenmarkt anbelangt, ist es merkwürdig, daß er die Wechselkurse von Antwerpen (*cursus cambiorum*) nach der Situation vom 7. Dezember 1600 mitteilt), es tauchen aber auch einzelne Fragen des Arbeitskraftsmarktes auf. Sehr wichtig ist, daß das Wechselverhältnis, die Interdependenz verschiedener Märkte auch schon in seinen Sichtkreis gerückt wird und dadurch scheinen die inneren Zusammenhänge der wirtschaftlichen Sphäre als solcher, sowie die von den ersten Gesetzmäßigkeiten der wirtschaftlichen Organisation auf. Die Untersuchung von dem Problem des Preises, insbesondere die Unterscheidung des "legitimen" Preises (*pretium legitimum*) von dem in markwirtschaftlichem Konsensus entstehenden "natürlichen" Preis (*pretium vulgare seu naturale*) weist darauf hin, daß die autonome Logik der Marktverhältnisse gelegentlich sich den Entscheidungsmechanismen der politischen Sphäre gegenüberstellen kann und Lessius erwähnt auch Fälle, in denen im Gegensatz zum offiziellen Preis gerade der natürliche Preis bevorzugt werden muß. Lessius billigt die Rolle der unternehmerischen Tätigkeit (*industria*), sowie deren Anspruch auf das Surpluseinkommen und erörtert merkwürdige Ansichten über die Monopole, die er zwar wie auch die Mehrheit der Scholastiker im allgemeinen verurteilt, aber auch auf Fälle hinweist, in denen der Monopolreis begründet ist, wie

es beispielsweise bei den von dem Fürsten im Interesse des Gemeinwohls gegebenen Konzessionen oder wegen der Größe der Transportkosten bei Importmonopolien geschah. Auch seine Behandlung des Getreidemonopols beweist klar, daß Lessius diejenige, für die mittelalterlichen Doktoren charakteristischen Anschauung überwand, welche die zwischen partikularen Einzelnen zustandekommenden wirtschaftlichen Verbindungen vor Augen hielt. Er wird schon auf den unpersönlicheren, objektiveren Charakter der marktwirtschaftlichen Zusammenhänge, bzw. Verhaltensweisen bereits theoretisch aufmerksam.

Die Anerkennung von der Objektivität des Marktes enthält in der Tat einen "subjektiveren" Gesichtspunkt. Es ist so einerseits dem "Objektivismus" der mittelalterlichen Doktoren gegenüber, die den Gesichtspunkt der Nutzens dem nicht in bloß juristischem, sondern auch in religiösmoralischem Sinne aufgefaßten Kriterium der Gerechtigkeit streng unterordneten und andererseits ist sie "subjektiver" auch im Vergleich zu den später ausgearbeiteten Ergebnissen der politischen Ökonomie. In den Arbeiten von den Klassikern dieser Wissenschaft werden die Gesetzmäßigkeiten der ökonomischen Prozesse tiefer beleuchtet und es wird eine objektivere analytische Ebene ausgearbeitet, die jedoch in der Zeit von der Herrschaft des Handelskapitals noch unerreichbar war und erst mit der Entfaltung von der führenden Rolle des Industriekapitals möglich wurde. Es kann nicht als bloßer Zufall betrachtet werden, wenn Barry Gordon oder Raymund de Roover bei ihrer Untersuchung der Ansichten von Lessius auf Parallelen zu Standpunkten von solchen Ökonomen des 19. und 20. Jahrhunderts hinweisen wie die Vertreter der subjektiven ökonomischen Richtung Jevons, Walras, Marshall, sowie Keynes.

Nachdem es am Beispiel Lessius' kurz demonstriert ist, wie sich Phänomene des frühen kapitalistischen Wirtschaftslebens in der Neuscholastik widerspiegeln und bis zu welchem Punkte die Denker dieser Richtung in Erkenntnis, Analyse und Verteidigung der kapitalistischen Entwicklung gelangten, können

wir nun auch ihre Mängel nicht unerwähnt lassen. Raymond de Roover, der sich eigehend mit dieser Richtung des ökonomischen Denkens befaßte und ihre Bedeutung in der Geschichte der Nationalökonomie in mehreren Studien würdigte, weist darauf hin, daß so große aufsehenerregende Ergebnisse diese Literatur in der Feststellung und Erkenntnis von den Tatsachen der ökonomischen Realität auch erreicht haben mag, erschöpfte sich diese Richtung im 16. und 17. Jahrhundert bald und verfiel in Kasuistik. In den späteren gewaltigen Aufschwung von der Wissenschaft der Nationalökonomie konnte sie sich nicht mehr aktiv einschalten, obwohl seine Wirkung ganz bis Adam Smith reicht. Der Grund dessen wäre vor allem daring zu suchen, daß die Neuscholastik, obwohl ihr Aufschwung während der Gegenreformation eine gewisse Rezeption von den Errungenschaften des Nominalismus und der *via moderna* voraussetzte, eigentlich nicht fähig war, die überlieferten konzeptuellen Rahmen wirklich erneuert zu haben. Was die Nationalökonomie anbelangt, führten die neuen Einsichten nicht - und wegen ihres auf Grund des mit den feudalen Kräften geschlossenen politischen Bündisses konservativen und defensiven Programms konnten es auch nicht tun - zu einer grundsätzlichen Umformung des theoretischen Instrumentariums. Auf diese Weise war diese Richtung nicht imstande, die faktischen Beobachtungen in eine neue Theorie zu generalisieren. Das neuscholastische ökonomische Denken verblieb so auf der Bahn von der legalistischen und moralisierenden Logik, die von dem Mittelalter an charakteristisch für dasselbe war.

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LANGUAGE, PICTURES, AND TRUTH

Kornél Solt

Pictura est
laicorum literatura

1. Two ways of representing the world

1. Of the various modes of thinking and communicating, two stand out. These are: (i) *description* and (ii) *depiction*. We are able to think and to express our thoughts either by means of *language*, or by means of *pictures*. It is, of course, also quite natural to combine these two ways, the discoursive and the non-discoursive modes.

> One is able, for example, to call one's friend John to mind either by saying John's name to oneself, (without imagining his face), or by imagining John's face (without saying internally John's name). These are two different ways of thinking about an *individual object*. (See FODOR [5], p. 180.)
>
> Similarly, we can also represent in our mind a *fact* either by means of propositions (statements) which denote a given fact or by imagining it pictorially. For instance, if it is raining we can represent this fact internally either by the proposition "It is raining", or by imagining an inner picture of a falling rain.

Likewise, we can consider two kinds of *communication*, (expression of our thought). It is possible to communicate some information to addressees either by means of *words* or by means of *pictures*. These two types of communication have several common features, as well as several points of difference.

2. Descriptions and depictions are among other kinds of world-representation, two types of "messages" in HOFSTADTER's sense. ([8], pp. 158-175, 369-390.) A message has form and content. Its content is its information. All messages have a code. Decoding a message is equivalent to understanding its content, the item of information expressed in it.

3. What is a picture? I do not try to define it. Let me only mention two types of pictures.

(i) There are pictures which represent something from *the real world*, for instance, internal images of objects visually perceived, memory-pictures, mirror-images, photographs, graphics, paintings, statues, films etc.

(ii) Other pictures represent objects, or courses of events which, in this way, *do not exist* in the real world. These include, for instance, phantasy images, some images in our dreams, films which show us fictive stories, illustrations for science-fiction books etc.

4. It is sometimes a characteristic feature of certain good pictures that there is a strong *likeness* between the picture itself and its depictum.* It is trivial, on the other hand, to hold that there is no "likeness" between a name and its denotatum, or between a statement and the fact represented by it.

2. *Static and dynamic messages*

5. Let us distinguish between the *issuing* of a message by its author (sender), and the *decoding* of its content by its addressees.

From the point of view of their authors, messages are either *static* or *dynamic*. Paintings and statues are static messages because they are, as information bearers nearly independent of the passage of time. On the other hand, e.g. radio reports or moving pictures are dynamic messages because the sending of them is itself a process "moving" in time.

However, the *decoding* of a message is always a dynamic process because the interpretation of a message necessarily needs time, independently of the static/dynamic character of the message bearer.

*There may be a nearly "perfect" likeness between a photograph and e.g. the person represented by it. A photograph, says WALTION in [12], may be "transparent" in the sense that one can see the real object itself "behind it". - It is, however, clear that "likeness" alone is generally not a sufficient condition for a picture's goodness.

6. An addressee of a static message has a great liberty concerning the *order of succession* in which he decodes the message. However, the order of succession of the decoding of a dynamic message is, generally, determined by the sequence of the information given in the message. We are free to chose where to begin and where to finish our observation of a picture by TITIAN, but if we read a novel by Thomas MANN, then the rational sequence of its decoding is determined by the work, by the author, and not by us.*

3. Abstractness and concreteness in representation

7. Generally speaking, by means of depiction we can come nearer to the *concreteness* of a real object than by linguistic means. Thus, a good painting of the Kirov Bridge in Leningrad is able to show many more features of it than any verbal description is able to do.

On the other hand, by means of language, we can, generally, reach a level of *abstractness* higher than by means of depiction. However, there are a lot of exceptions to this rule. For instance, the statue *Le Penseur* (RODIN) expresses abstractness in an impressive way and to a high degree.

8. An object in the real world either has a given property or does not have it. However, if we try to represent an object, then we are, of course, always unable to show it with *all* its properties. The real world is infinitely *richer* than any message concerning it. Consequently, no information is "complete". Some features of the real object are always missing from its representation. In such cases we can speak of "lack of some information".

*A painter, however, knows many of ways in which he can draw one's attention to the most important features of his picture. A good example is *The Breakfast* by VELAZQUEZ (1617, Hermitage). If we follow the painter's "instructions" then our eyes will move round the table in the middle from figure to figure.

4. The lack of information

9. This phenomenon differs profoundly from the *truth value gap*.* A proposition falls into the truth-value gap if it is neither true nor false. A well-known example of it is the sentence "The present King of France is bald".

If something is missing from a piece of information then it is the *real world* which is richer than the information about it. If, however, a proposition falls into the truth-value gap, it is the *information* which is *"richer"* than the real world because, in this case, something is missing from the actual world that is "given" in the sentence (e.g. the present King of France). Where is a lack of information, there is no truth-value gap.

10. Theoretical discussion about the "hattedness of a man" is already familiar. (See SHORTER [11], DENNETT [2].) We can be in no doubt that a man either has a hat or does nct have a hat on his head. However, in the sentence "Yesterday I met John" there is no information about John's "hattedness". This information is missing from John's portrait, too, if his forehead is obscured by a branch of a tree also in the picture. A lack of visual information is also possible in a picture without obscuring the facts in question. (See BLOCK [1].)

> The pictograph of bison in the Lascaux caves contains much precious information concerning bison, however, much more information on bison is missing from it.**

It may be, of course, that a mirror-image of an object or a photo of it (without re-touching) is nearly "complete" as regards visual information. Even so, a lot of other information is lacking from it (e.g. all tactile information).

*On the truth-value gap see, e.g., von WRIGHT [13].

**A remark: if one has a black and white photo of the pictograph in the Lascaux caves, this does not *lie* concerning the colours of the original. It simply gives no information concerning the colours of the pictograph. This is a lack of information.

11. The sender of a linguistic or of a pictorial message has plenty of liberty in *selecting* the features and details of the object concerning which he intends to inform his addressees.

Photos and films, however, have a peculiar status in this regard. The sender of a linguistic message (e.g. a writer) has more liberty in selecting than the maker of a photo or film (thus e.g. a director of motion pictures). (See DENNETT [2], p. 136.) Here is an example:

> Let us call to mind of the scene on TOLSTOY's *War and Peace*, when Prince Andrei Bolkonsky is lying wounded on the battlefield at Austerlitz with the bare flagstaff (the flag having already been taken away by French soldiers). Napoleon just glances at him and says: "Voilà une belle mort". TOLSTOY describes this scene in a wonderful way. He tells only what is important for him and for us. Now let us consider the same scene in BONDARCHUK's film. BONDARCHUK deals with it in a very impressive way, but he is forced to present to us many unimportant details also. This is, of course, not his fault, but rather the "fault" of every kind of photograph and film. BONDARCHUK was obliged to show us nearly "all", TOLSTOY had more freedom to select.* (Abstractness in the novel, singularity in the film.) For instance, the *flagstaff* is "only"a *type* of entity in TOLSTOY's work, but it is a *token* of the same type in BONDARCHUK's film.**

5. Truth and falsehood in depiction

12. The most difficult and the most interesting problem concerning pictures (from the point of view of logic) is, undoubtedly, whether they may have certain truth-value or not. Many authors categorically deny this possibility. They argue that only propositions (accordingly descriptions) may be either true or false. (E.g. FODOR [5], GOODMAN [6].) Others

* Where some information is lacking from a text or from a picture, we are in the area of "open interpretation". (Cf. Umberto ECO's idea of the "Open Work" in [3].) - I do not deal here with the "vagueness" of some information. I contend that this offer a possibility for the "open interpretation" of a message.

**The flagstaff in BONDARCHUK's film is a strictly *singular* (unique) object, whereas in TOLSTOY's work it is "only" *non-particular* flagstaff.

(these are fewer) accept that, in some sense also pictures may be true or false. (E.g. ROSKILL - CARRIERE [10].)

I agree with this last opinion. If descriptions and depictions are merely two ways of representing the world, then we have no reason to accept the legitimate applicability of truth-values in one of these areas and to refute it in the other.

There exists a great variety of truth-theories and, therefore, also of definitions of truth. Whether the informative content of a picture may be labelled with the words "It is true" depends mainly on the kind of definition of truth from which we start out.

13. Let us now adopt TARSKI's partial definition of truth, according to which the proposition "Snow is white" is true in English if and only if snow is white. If we use the word "true" in this sense (in accordance with the correspondence theories of truth) then, for example, a narrative description of an historical course of events is true if and only if what is described by it is really the case and, analogously, a picture (e.g. a photograph or a portrait) is true if and only if what is represented by it is really the case.

> Using the word "true" in this very narrow sense we can state that, for example, the well-known portrait of Modest MUSSORGSKY painted by REPIN (Tretyakov Gallery) is a true representation of the famous composer. It is true on a concrete level. It shows him shortly before his death. But it is also true on a higher level of abstraction, as it represents generally his whole tragedy.
>
> It is quite clear that the *w h o l e t r u t h* of this portrait does not consist merely in the simple addition of the truth of its parts. Thus, the *Principle of Compositionality* does not hold for depictions.

14. Now let me quote as examples of unquestionably *false* depictions pictures painted by sailors hundreds of years ago representing *sea-serpents*. These are false because what they actually depict are not sea-serpents but only dolphins playing. In these cases, the painters' intentions were to represent something from the real world. Therefore, we see that the truth or falsehood of a picture depends on the authors' pragmatic attitude also.

15. We have, however, no doubt that the above notion of truth is too narrow for artistic (narrative) descriptions and for many kinds of depiction. If we use the word "true" only in the Tarskian sense then the information content of nearly all literature and the overwhelming majority of paintings would be neither true nor false. Accordingly, we need a broder notion of truth concerning works of art. This is the "artistic truth". But what is meant by this?

Before touching briefly upon this problem, let us dwell for a while on the theme of the truth-value gap in connection with descriptions and depictions.

6. The truth-value gap in descriptions and in depictions

16. Let us now define the "truth-value gap" in a piece of information as the lack of the logical values "true" or "false", in the Tarskian sense. In this case, the greatest part of the information contained in the works of literature falls into the truth-value gap. If a narrative description (e.g. a novel) informs us about fictive persons, objects or stories, then this information is neither true nor false. For instance, all the information we receive from A. CONAN DOYLE about Sherlock Holmes or from Thomas MANN about Aschenbach (*Death in Venice*) or from SHAKESPEARE in *A Midsummer Night's Dream* fall into the truth-value gap.

The case seems to be similar concerning the great majority of paintings too. If a picture shows us "only" a *fiction* (*i.e.* a segment of an imagined world), then its information content may be labelled only by the truth-value gap, for example, the information contained in *The Last Judgement* by Hieronymus BOSCH (Alte Pinakothek, Munich), or in modern illustrations for science fiction books.*

*There are many paintings representing the *unicorn*. If their artists' intention was to show us something from an imagined world then the information content falls into the truth-value gap. If, however, the artists' intention was to depict something from the real world, then the picture may be beautiful but its information content will be false.

17. There are several different levels at which fiction can be said to exist. For example, the whole of *Macbeth* is a fiction compared to the actual world, but Macbeth's dagger is a fiction within a fiction. The dagger is a fictitious object compared to the non-actual world represented in SHAKESPEARE's tragedy.*

Although a fiction is neither true nor false, its interpretation may be either true or false. It is possible to misinterpret a fiction also.

7. What is "artistic truth"?

18. It is clear that Tarskian truth is too narrow for the characterization of works of art. However, an acceptable conceptual framework in which the "artistic truth" or the "artistic falsehood" of a novel or of a picture might be placed is unknown. Here I only venture a few remarks concerning this theme.

It is most likely that there are several different kinds of artistic truth. A novel or a picture has many ways of expressing thoughts, sentiments, criticisms and so on. Thus, we can speak of the symbolic, allegorical, metaphorical (etc.) truth of a novel or picture. (An artist is able to show us some characteristic features of *our actual world* in the "mirror" of a non-actual world.)

I suppose that there is no clear-cut demarcation line between an *objective* report and a *subjective* fiction. It is probable that every kind of description and of depiction is a *mèlange* of non-fictive and of fictive elements in a large variety of different proportions. If so, it is reasonable to claim that the non-fictive elements of a work of art must be true in the Tarskian sense. But what can we say concerning a

*David LEWIS writes concerning Macbeth's dagger:

> "It is a perfectly ordinary dagger. But it is part of another world, floating before the eyes of an otherworldly alternative Macbeth whom the real Macbeth takes himself to be."([9], p. 4.)

"real" fiction? Perhaps this problem can be solved by means of the semantics of possible model worlds.

19. If a novel or a picture does not represent the real world directly, but creates a fantasy-world, can we reasonable assert *that it is false*? In such a case is a discrepancy possible between the representing and the represented? There is nothing compared with which a picture can be said to be false if its painter intended it to be a fiction.

> M. ROSKILL's examples of false depictions are not convincing. He mentions, for instance, that a picture is false if lines are made to appear to diverge, when they are in fact parallel to one another. ([10], p. 83.)

20. Let us now turn our attention to AIVAZOVSKI's famous picture *The Ninth Wave* (Russian Museum, Leningrad). This does not represent something which has actually happened but something what was non-actual and, at the same time, logically and also physically possible.*

> In this picture we see a turbulent sea and some shipwrecked people struggling for life. A very big wave is rolling on. We come to know only from the picture's title** that this is the critical *ninth wave*. If they survive *this* wave, they will be saved later. What will be the "actual" future of these shipwrecked people?
>
> We can interpret the picture in two ways, in an optimistic way and in a pessimistic way. Which interpretation is the true one? There is a well-known tradition in European painting that warm yellow-red colours always suggest some optimism. And it is just these colours which prevail in the background of the picture. The sun is breaking through the darkness. Therefore, the optimistic interpretation seems to be true, and the pessimistic one false.

21. It is, of course, far from my intention to state that the *entire content* of a picture consists merely of true or

*The model world of some science fiction books is only logically but not physically possible.

**Also the *title* (inscription) of a portrait (statue) belongs to the informative content of the portrait (statue).

false information. There are a lot of other very important messages in a picture. In first place, a picture may be *beautiful*. Or it may be ugly. It may be interesting or annoying. We can enjoy it or hate it.

In pictures there are *artistic-aesthetic-moral* values and these are not measurable in terms of truth-values.

A good picture has always not only a rational content but also is capable of moving us emotionally. It is well-known that sometimes a picture is also emotionally laden. Observing it, we often feel its emotive force. A picture can be strongly provocative. It can influence us to change our outlook on the world or even our way of living.

22. I must stress here that an interpretation of a picture (like a novel) is always *context-dependent* and *background knowledge-dependent*. If we are to interpret a picture correctly, we need always to have a deep background knowledge. The richer this knowledge is, the richer the interpretation we are able to arrive at.

8. Paradoxes in descriptions and in depictions

23. By means of words we are able to create real paradoxes. The content of a paradox is true and false synchronically. For example, the following statement

(1) *This sentence is false*

is paradoxical. (This is an instance of a *Liar-type* paradox.)

I believe paradoxical information (in the sense of *Liar-type*) is expressible only by means of descriptive messages. Pictorial messages never express real synchronical paradoxes in the above sense. It is known that many excellent paintings by Victor VASARELY allow us two different, alternative interpretations.* They are, separately, both consistent. We are able, however, to see at first only *one* visual variant in the

*E.g. V. VASARELY: *Figure III.* (*1969*).

picture and, later, *another*, but never synchronically *both*. Our *visual cortex* is unable to decode alternative visual messages in a really paradoxical way.

Another example for alternative visual messages is the curious picture of a "duck-rabbit" which offers us two alternative schemes for perceptual organization, each leading to a meaningful figure. (See E.H. GOMBRICH [7], p. 165.)

This is, I contend, also a basic difference between discoursive and non-discoursive thinking. Only *rational* thinking is able to create *Liar-type* paradoxes, the earlier and much more conservative *visual* thinking is unable to do it.

24. Some very impressive paintings by VASARELY which offer us more alternative ways to decode them are not analogous to the sentence (1) mentioned above. They are paradoxical only in a weakened sense and are similar, I consider, to the following sentence:

(2) *Reginam occidere nolite timere bonum est si omnes consenserint ego non contradico*

This message is ambiguous. It has two contradictory meanings, rather like the Dodonian oracles. There is no punctuation in it.

Some believe that (2) was the message given by Archbishop JÁNOS to the so-called "malcontents" who later killed Queen GERTRUDE.*

The use of differing punctuation allows two opposite interpretations.

(i) The *prohibitive* version is: *Reginam occidere nolite.* (Do not kill the Queen) *Timere bonum est.* (It is good to be afraid) *Si omnes consenserint, ego non.* (If everybody consents, I do not.) *Contradico.* (I oppose it)

(ii) The *permissive* version is: *Reginam occidere nolite timere,* (Do not fear to kill the Queen) *Bonum est,* (It is

*GERTRUDE was the wife of ANDREW II, King of Hungary (1205-1235).

good.) *Si omnes consenserint, ego non contradico.* (If everybody is consenting, I do not oppose it.)

I think VASARELY's paintings are only in a weakened sense paradoxical like (2), and not like (1). (1) contains a *single* self-contradictory message but (2) contains *two* alternative and separately non-self-contradictory messages - like VASARELY's alternative paintings.

9. Impossible worlds represented by pictures

25. Earlier I mentioned that by visual means we are unable to express and to decode *Liar-type* paradoxes. This does not mean, of course, that there are not some very strange pictures expressing enigmatic, astonishing contents. For istance, a lot of excellent paintings depict some *impossible worlds* (impossible not only physically but also from the point of view of classical logic).

For example, numerous graphics by M.C. ESCHER are of this type, e.g. the lithographs *Relativity* (1955), (See in [4], p. 47.), *Concave and Convex* (1955), (p. 83.), *Cube with Magic Ribbons* (1957), (p. 85.), *Belvedere* (1958), (p. 87.) etc.

Their characteristic feature does not consist merely in the possibility of interpreting them in several different ways but in the logical impossibility of giving them at least *one consistent interpretation.**

I have tried here to analyse some aspects of description and of depiction. My paper is, however, only exploratory. This stimulating topic needs further examination.

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*For example, the PENROSE impossible triangle (see [7], p. 87) transgresses the boundaries of rationality.

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PHYSICAL THEORY AND PHILOSOPHICAL VALUES

Philosophical Debates on the Theory of Relativity and L. Jánossy's Interpretation

László Székely

> I do not know what I may appear to the World; but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.
>
> [Newton]

The physics of the present century revolutionized our world picture to such an extent that perhaps even the Copernican turn of thought cannot be resembled to it. At any rate, there is a great qualitative difference between these two shifts of outlook: while Copernicus substituted another picturesque world picture for a given picturesque world picture - the world picture of the 20th century means a radical break from picturesquesness. Galileo still illustrated the relativity of motion with a moving ship drifting along a ship at rest. But Einstein's formula concerning the relativistic addition of velocities cannot be made picturesque anymore (not even by two spacecrafts that pass along one another).* Similarly, in the present-day conceptual framework of modern physics one cannot resolve by means of explanation in terms of picturesquesness e.g. that contradictory nature of the experimental results in quantum mechanics which follows from the wave-particle dualism,** and

*Albert EINSTEIN: *Über die spezielle und allgemeine Relativitätstheorie.* 12. Auflage. Braunschweig: Vieweg, 1921.p.24-28

**For instance: Lajos JÁNOSSY: *A filozófia jelentősége a természettudományos kutatásban.* In: JÁNOSSY L.- ELEK T.: A relativitáselmélet filozófiai problémái. Budapest, Akadémiai Kiadó, 1963. p. 19-34.

the finite but boundless Universe of modern cosmology cannot be made pisturesque either.*

The revolution in physics taking place at the turn of the 20th century was started by Planck's quantum hypothesis and Einstein's theory of relativity. From a physical point of view, the two theories can be regarded as of equal importance, but as regards the sensation and popularity they aroused in a wider intellectual and cultural context, the former theory is eclipsed by the latter: both meant a revolution in the development of physical thought, though what threw out a challenge for the world picture (and, in general, for the thought) of the natural sciences was not Planck's but Einstein's theory. It was Einstein who *turned against the physical world picture* which had earlier been believed to be natural and *the only possible one*, and by this turn realized a new Copernican revolution of the world picture of the natural sciences, - however the fate of Einstein's thoughts has been just the opposite than the afterlife of Copernicus' theory. The theory of relativity became fashionable, which was perhaps the manifestation of the claim to novelty of the thought of the turn of the century, that of the longing to get off the limits of reality and rationality on the part of the intelligentsia of the age, that of the claim to some myth - and all this was then offered by science. The slowing down and acceleration of time, the declaring the lack of any absolute reference point as a thesis of natural science and the possibility of curved space recalled the vein of myths, and that on the part of science and *rationality*, which so far had been characterized just by expelling myths from the world and by restricting the free soaring of fancy.

Einstein's theory was a breakthrough, in that he surpassed the outlook of the 19th-century natural sciences and of the common sense that had been shaped during centuries, and he

*Albert EINSTEIN: *Uber die...* p. 105-110.

laid down several theses as science and rationality such that would have been classified as myth and irrationality by 19th-century thought. This aspect of Einstein's theory soon turned out not to be an extraordinary or isolated episode in the history of physics but such a new factor in the development of physics which appears in other spheres of the general physical theories - in the first place in quantum mechanics - too. By the 1930s it had become evident and natural that the outlook and statements of fundamental physical theories run contrary to everyday thinking and that the statements that can be made in their frameworks are absurd for everyday experience. Progress in this respect 'surpassed' even Einstein himself, who - as it is known - could not accept the outlook of quantum mechanics in all his life.

In the name of dialectical materialism the first definite and sharp criticism against Einstein's theory was written by A.K. Timiriazev, in his book entitled *Natural Science and Dialectical Materialism*, published in 1925.* The title is highly expressive and foreshadows the main line of the debate on the theory of relativity: Timiriazev criticizes and rejects Einstein's theory using the categories and ideas of dialectical materialism, while the authors arguing with Timiriazev and the representatives of standpoints similar to his one reason the other way round, trying to defened the theory of relativity against attacks of ideological motivations by emphasizing the theory's natural scientific success and at the same time trying to prove its dialectical and materialistic character. The concernstone of Timiriazev's criticism is that the theory's system of ideas and categories is Machian and as such it is the manifestation of Einstein's idealistic views. His book tries to follow the tradition started by Lenin and criticizes Einstein's theory as the newest version of physical idealism,

*A.K. TIMIRIAZEV: *Estestvoznanie i dialekticheskii materializm.* Moskow, 1925.

but in spite of this with him we cannot find that sharp distinction which Lenin made between the physical and philosophical viewes of his opponents. Timiriazev's viewes were criticized by S.I. Semkovskii, stressing that Einstein's philosophical conception must not be mixed up with the world of thought of the theory of relativity, which, breaking loose from the earlier metaphysical physics, gives a dialectical materialistic description of physical reality.* A decade later V.F. Mitkevitch took a similar view to that of Timiriazev's; Mitkevitch was criticized by S.I. Vavilov and A.F. Ioffe in the review *Pod znamenem marksizma.***

The second phase of the debate concerning the theory of relativity in the Soviet Union took place between 1947 and 1955. In terms of content the turning-point in the debate was the article of academician V.A. Fock, published in the review *Voprosy filosofii* (1943 No. 1) under the title 'The Incompetent Criticism of Modern Physical Knowledge'.*** Here Fock re-

*S. Iu. SEMKOVSKII: *Dialekticheskii materializm i printsip otnositel'nosti.* Moskow and Leningrad, 1926.

**V.F. MITKEVICH: *Osnovnie fizicheskie vozhreniia.* Moskow and Leningrad, 1926; S.I. VAVILOV: *Po povodu knigi V.F. Mitkevicha.* in: Pod Znamenem Marksizma, 1937/7.; A.F. IOFFE: *O polozhenii na filosofskom fronte sovietskoi fiziki.* in: Pod Znamenem Marksizma, 1937/10-12.

***A.A. MAKSIMOV: *Marksistkii filosofskii materializm i sovremennaia fizika.* in: Voprosy Filosofii, 1948/3. --: *Borba Leninas fizicheskim idealizmom.* in: Velikiia Sila idea leninizma. Moskow, 1950.; G.A. KURSANOV: *Dialekticheskii materializm o prostranstve i vremini.* in: Voprosy Fil. 1950/3; L.I. STORCHAK: *Znachenie idei Lobachevskogo v razvitii predstavlenii o prostranstve i vremeni.* in: Voprosy Fil., 1951/1.; M.M. KARPOV: *O filosofskikh vzgliadakh A. Einsteina.* in: Voprosy Fil., 1951/1.; G.I. NAAN: *K voprosu o printsipe otnositel'nosti v fizike.* in: Voprosy Fil. 1951/2. D.I. BLOKHINTSEV: *Za leninskoe uchenie o dvizhenii.* in: Voprosy Fil., 1952/1.; G.A. KURSANOV: *K kritisheskoi otsenke teorii otnositel'nosti.* in: Voprosy Fil., 1952/1. V. SHTE V. SHTERN: *K voprosu o filosofkoi storone teorii otnositel'nosti.* in: Voprosy Fil., 1952/1.; M.B. VIL'NINSKII: *Za posledovatel'no-materialisticheskuiu traktovku printsipa otnositel'nosti.* in: Voprosy Fil., 1952/1. M.F. SHIROKOV:

futes the ideological charges raised in connection with quantum mechanics and the theory of relativity. Fock's article was followed by that of A.A. Maksimov, who was the most extreme representative of the counter-camp and used the ideological 'clichés' in the most prominently* - but an editorial note was attached to it to the effect that his article was published only for the sake of the correctness of the debate. In a later number of the same review, A.D. Aleksandrov criticized Maksimov, and the editorial that formally closed the debate demarcated itself from the latter's standpoint too.** (From the aforegoing it may be clear that the discussion of the theory of relativity was free from any such tragical turn as that in the debate on genetics. How much role did Stalin's death play in this? And how much can this be attributed to the special strategic function of modern physical research, as regards e.g. atomic energy or atomic bomb? These are interesting questions

O preimuchestvennikh sistemakh ottschota v nutonskoi mekhanike i teorii otnositel'nosti. in: Voprosy Fil., 1952/3.; Ia.P. **TERLETSKII:** *O soderzhanii sovremennoi teorii prostranstva i vremeni.* in: Voprosy Fil., 1952/3.; P.G. **KARD:** *O teorii otnositel'nosti.* in: Voprosy Fil., 1952/5.; I.P. **BAZAROV:** *Za dialekticheskо-materialisticheskoe ponimanije i razvitie teorii otnositel'nosti.* in: Voprosy Fil., 1952/6.; A.A. **MAKSIMOV:** *Provitiv reaktsionnogo einshteiniantsva v fizike.* in: Krasnyi Flot, June 23, 1952.; I.V. **KUZNETSOV:** *Sovetskaia fizika i dialekticheskii materializm.* in: Filosofskie voprosy sovremennoi fiziki, (ed.: by A.A. MAKSIMOV) Moskow, 1952.; N.V. **MARKOV:** *Znachenie geometrii Lobacheskogo dlia razvitii fiziki.* in: Filosofskie voprosy ...; M.M. **KARPOV:** *Kritika filosofskikh vzgliadov Einshteina.* in: Filosofskie voprosy...; R.Ia. **SHTEINMAN:** *Za materialisticheskuiu teoriiu bystrykh dvizhenii.* in: Filosofskie voprosy...; A.I. **UIEMOV:** *Geliotsentricheskaia sistema Kopernika i teoriia otnositel'nosti.* in: Filosofskie voprosy...; V.A. **FOCK:** *Protiv nevezhestvennoi kritiki sovremennoi fiziki.*-in: Voprosy Fil., 1953/1.

*A.A. **MAKSIMOV:** *Borba za materializm v sovremennoi fiziki.* in: Voprosy Filosofii, 1953/1.

A.D. **ALEKSANDROV: *Po povodu nekotorykh vzgliadov na teorii otnositel'nosti.* in: Voprosy Fil., 1953/5.; --: *K itogam diskussii po teorii otnositel'nosti.* in: Voprosy Fil., 1955/1.

of the history of science but they cannot be answered from here, Hungary.)

The articles that defended the theory of relativity, such as G.I. Naan's 1951 one or Fock's 1953 one, emphasized the importance of differentiating between the natural scientific theory and its ideological-philosophical interpretation. Maksimov also admitted the difference between the philosophical and the natural scientific level, but according to him Fock's chief mistake was just that Fock admitted the opposition between the two fundamental trends or attitudes of philosophy, namely materialism and idealism, only with regard to philosophical interpretations, denying that there was a permanent struggle between idealism and materialism within physics itself and within the theory of relativity.* This is a statement which today sounds rather astonishing and runs contrary even to Lenin's standpoint concerning physics. It appears as an unambiguous evidence of Maksimov's philosophical and physical incompetence and we can regard it as a matter of fact as a cliché aiming at the fulfilment of the ideological claim of (political) power. But if we go beyond this point the following question might be raised at once: how does Maksimov try to prove this statement which at first sight seems to be, anyhow, surprising and bold? Answering this question will at the same time offer a key to a deeper understanding of the debate on the theory of relativity.

First I am going to quote Maksimov:

> "The Lorentz transformation, which was discovered even before the creation of the theory of relativity, is a rational thought. The phenomenon was given a materialistic explanation by Lorentz and an idealistic one by Einstein."** "Lebegyev examined the relation between mass and energy in a materialistic way. Einstein explained this relationship idealistically."***

*Voprosy Filosofii, 1953/1. p. 179.
**Voprosy Filosofii, 1953/1. p. 194.
***ibid.

On the base of these quotations it will be obvious that when Maksimov (as well as other critics of the theory of relativity) speaks about the struggle between two trends within physics, he differentiates between them not on the base of some artificial construction taken from without, but he has in mind two actually existing physical standpoints that run contrary to one another. That is all right. But one might ask: is not it just the fact that he attaches a positive sign to that very interpretation of Lorentz's which was 'refuted' by Einstein's theory that proves Maksimov's incompetence?

If we approach the problem on the base of the cumulative conception of the development of science, this question will be warranted, since looking at the situation from this aspect we can see that the representatives of Lorentz's theory criticize such a theory, which, to use V.A. Fock's words, has been confirmed so strongly as the roundness of the earth.* But on the base of the modern theory of science it will be clear that the critics of the theory of relativity try to defend a defeated paradigm - or, in Lakatos' terminology, a defeated 'research programme' - and revive it. And in this respect the debate in the Soviet Union is similar to the western ones, where attempts have also been made at working out some alternative to (or against) Einstein' theory.** What was specific in the debate at issue was that, due to the particular internal conditions of power and ideology in the Soviet Union, the debate concerning the theory of relativity gained considerable ideological charge: the representative of the defeated para-

*Voprosy Filosofii, 1953/1. p. 172.

for example: A. **EHRENFEST: *Zur Kriese der Lichtäther-Hypothese*. Berlin, Springer, 1913.; A.A. **ROBB**: *The Absolute Relation of Time and Space*. Cambridge, 1921.; E.A. **MILNE**: *Kinematic Relativity*. Oxford, 1948.; H. **LAAGE**: *Der innere Mechanismus der Gravitationskraft*. Hamburg, Laages Verlag, 1985.

digm turned for help to the politically victorious ideology in order to strengthen their own position and to call into question the scientific character of the victorious paradigm. In this sense, the charge of 'scientific incompetence' will also gain a new meaning: this is a typical charge, on the part of the representatives of the victorious paradigm, against those committed to standpoints that oppose this paradigm.

Naturally, beyond a certain point it is indeed unreasonable and unscientific to commit oneself to a refuted and rejected paradigm. It applies particularly to those cases when, facing facts raised a novel social practice more expanded than the earlier one, the old paradigm proves to be absolutely inefficient. But, as for the theory of relativity, in this respect the situation in again specific. For, the relation between Lorentz's and Einstein's views is characterized by such a structural factor which offers the possibility of making Lorentz's theory just as efficient as Einstein's theory. This factor became quite clear and explicit only by virtue of Lajos Jánossy's research to this effect, though latently and vaguely it was lurking in the studies of the Soviet critics of the theory of relativity. G.I. Naan and V.I. Fock were stressing the importance of the distinction between the physical theory and its interpretation in terms of world view, while A.A. Maksimov denied the possibility of making this distinction. But neither of them recognized that what is at stake here is a triple structure, whose members are as follows: (1) the mathematical apparatus of the theory; (2) the *physical* interpretation of the mathematical apparatus; (3) the interpretation in terms of *ideology and philosophy* of the theory that consists of factors (1) and (2). The attack of those criticizing Einstein was challenged by factor (2). They demanded the substitution of this factor and they were speaking of the struggle between the two trends at this level. But since they did not make this triple distinction, their attack was directed to the physical theory that consisted of factors (1) and (2).

On the other hand those defending the theory stressed its fruitfulness and were bound first of all to the first factor, but, in the lack of making these distinctions, they took factor (2) as well for irrefutably proved and hence undebatable. So, as the triple structure at issue remained unrecognized, the questions bearing real points of confrontation became blurred. Contrary to the analyses of Naan and Fock, the problem of interpretation has two layers and hence it is deeper than they supposed to be: it appears not only in the relation between the physical theory and the philosophical standpoints of ideological nature, but also in the relation between the mathematical apparatus and the physical interpretation. Therefore the demand for a new physical interpretation of the mathematical apparatus does not necessarily mean the calling into question of the achievements of the theory or their rejection. Though Maksimov in this respect goes one step further than Naan or Fock, since he admits that the mathematical formulae describing the Lorentz transformation are right and regards as mistaken only their Einsteinian interpretation - yet he does not recognize the central importance of making this distinction. Thus the possibility and necessity of this distinction does not become the focal point of his criticism. But it follows also from the lack of making this distinction that e.g. G.I. Naan, who was an advocate of Einstein's theory and thus rejected Lorentz's standpoint, at the end of his article passes to the direction of the Lorentzian interpretation (though he himself does not recognize this).*

The clearcut distinction among the above-mentioned three factors was made by Lajos Jánossy. It was he who emphasized, from a theoretical point of view, that the mathematical apparatus of Einstein's theory could be preserved even on the base of Lorentz's view, and at the same time he proved it 'in practice' by working out, in the form of a correct physical theory,

*G.I. NAAN: Voprosy Filosofii, 1951/2, p. 76-77.

the Lorentz-Jánossy alternative of Einstein's theory.* The essential core of his standpoint is that according to him factor (2), namely the physical interpretation of the theory, was considerably influenced by Einstein's world view, and that the theory can be transformed into a materialistic one only if the physical interpretation of the mathematical apparatus is worked out in a Lorentzian vein. This standpoint of Jánossy's is significant even from the point of view of György Lukács' *Ontology:* the reflections on the theory of relativity in his *Ontology* cannot be understood without Jánossy's views and the distinctions Jánossy makes between the mathematical apparatus and its physical interpretations.**

Naturally, these trials at the reinterpretation may be rejected in the narrower frameworks of natural scientific thought, or on the base of the Popperian criterion of falsification. (Hungarian physicists reject Jánossy's theory in fact on the base of reasons of these kinds.***) But if we expect more from natural science than the fruitful and useful description of physical reality from a pragmatic point of view, if we do not regard it as a mere instrument but we have ontological expectations from it too - as Lukács did in his *Ontology* - then Jánossy's interpretation becomes relevant at once. In connection with this we have to mention that after the victory over the extremist critics of the theory of relativity, a certain differentiation followed also among those Soviet scholars and scientists who defended the theory. For example, A.D. Aleksandrov emphasized that Einstein's conception should be surpassed in that the theory of relativity should be reinter-

*Lajos JÁNOSSY: *Theory of Relativity Based on Physical Reality*. Budapest, Akadémiai Kiadó, 1971.

**Georg LUKÁCS: *Zur Ontologie des gesellschaftlichen Seins*. I. and II. Darmstadt, Luchterhand, 1984. (I), 1986. (II.) I.: 345., 353., 359-363., II.: 388., 391.

***for example: NOVOBÁTZKY Károly: *A filozófia jelentősége a fizikai kutatásban*. in: Magyar Tudomány, 1956/7-12, --: *A Lorentz-elv a kritika mikroszkópja alatt*. ibid. 1966/6.

preted as an absolute space-time theory determined by matter.* And in V.A. Fock's later view of the theory of relativity, the principle of the equivalence of inertial and gravitational mass is valid only locally and even that just approximately.**

Analyzing the debate concerning the theory of relativity, a new and important recognition was made by István Farkas. He pointed out that even from the point of view of physical interpretation (factor (2)) neither Einstein's nor Jánossy's theory can unambiguously be considered idealistic or materialistic. According to him, both Einstein's and Jánossy's interpretation can be given a materialistic interpretation, and thus the correspondences Einstein → idealist, Lorentz → materialist are not warranted.*** At the end of his analysis he states that choosing between the Lorentz-Jánossy theory and the Einsteinian theory is a task of physics and not of philosophy, which has to be performed using the usual means of natural scientific cognition.****

As it may be clear from the aforegoing, the philosophical debate on the theory of relativity was centred around the question whether the theory interprets physical reality in a positivistic-instrumentalistic way, or the other way round, according to the dialectical materialistic outlook upon nature. The critical remarks concerning the theory's non-picturesque nature (i.e. that it seems to be absurd in more than one respect for everyday consciousness) were embedded in and subordinated to this sphere of thought. But Timiriazev, who was the first

*see for instance: A.D. ALEKSANDROV: *Teoriia otnositel'nosti kak teoriia absoliutnoge prostranstva-vremeni*. in: Filosofskie voprosy sovremennoi fiziki. Moskow, 1959.

**for instance: V.A. FOCK: *The Theory of Space, Time and Gravitation*. New York, 1959.

***István FARKAS: *Marxizmus és relativitáselmélet*. in: Magyar Tudomány, 1979/6.

****Magyar Tudomány, 1976/6. p. 457.

prominent Soviet critic of the theory of relativity, had already raised the problem of breaking loose from the old, picturesque concepts: to understand the physical phenomena Einstein explained "we do not need to accept the lot of paradoxical hypotheses in which Einstein's theory abounds".*

In what follows, I am going to examine that shift on turn which I think 20th-century physics, and particularly the theory of relativity and quantum mechanics, meant in the history of natural scientific thought, or, in general, rational thought aiming at a scientific physical world picture. Then I shall point out that Jánossy was prompted to work out his alternative theory of relativity by the recognition - and rejection - of precisely this turn. This will be followed by analyzing the relation between Einstein's and Jánossy's theory of relativity; this section of the study will be closed by stating that, unlike the conclusion of the debate on Jánossy's theory, we cannot choose conclusively between these two theories on the base of physical or natural scientific considerations.

*

The 19th century was the period of the ripening of the natural science of Modern Times, which was developing parallel to bourgeois civilization. Though the beginnings of the rise of the natural scientific thought of Modern Times are usually marked by the Copernican revolution which threw out a challenge to that picturesque world picture which was based on the daily observable motion of celestial bodies, yet in the course of further development materialism and natural science opposed just the criterion of rationality which was entwines with picturesquesness and understandability to the religious and mystic thought of the Middle Ages (constituting the ideological basis of feudalism), which briefly and symbolically can be

*A.K. TIMIRJAZEV: *Estestvoznanie i dialekticheskii materializm.* Moskow, 1925. p. 259.

characterized by Tertullianus' famous thesis: 'credo quia absurdum est'. (The developing natural scientific thought was arguing directly not with the Tertullianus-like mysticism but with Aristotelian Scholasticism, and that is the reason why we can use here Tertullianus' words only symbolically.) Bruno rejects Aristotle's cosmological model on the grounds of common and natural sense; in this model, space is taken to be finite and bounded, and hence it is incompatible with our spatial notion. The same 'common sense' is present somewhere in Galileo's words when he is mocking the Aristotle of the Scholastics:

> "For instance, if he moon is invariant, how would you have the sun or any other star act upon it? The action would doubtless have no more effect than an attempt to melt a large mass of gold by looking at it or by thinking about it. Besides, it seems to me that at such times as the celestial bodies are contributing to the generations and alterations on the earth, they too must be alterable. Otherwise I do not see how the influence of the moon or sun in causing generations on the earth would differ from placing a marble statue beside a woman and expecting children from such a union."*

We can find two important theoretical factors in Galileo's ironical simile, due to which we can find his reasoning rational and natural. One of them is the principle of interaction which is confirmed again and again by everyday as well as by scientific practice; the other is his conjecture according to which between things of in principle different species there cannot be any connection, which latently involves the principle of the world's material unity. It is quite different why Bruno's reasoning seems to be 'common-sensical': he rejects what appears absurd for our notions and mind just on the grounds of this absurdity.

*Galileo GALILEI: *Dialogue Concerning the Two Chief World Systems*. (translated by S. Drake) Berkeley, Los Angeles, London: University of California Press, 1967. p. 60.

"If we conceive of the Universe as boundless in the sense we propose, then it will comfort our mind, while from the opposite view there always follow innumerable difficulties and absurdities",*

as he sums up the essence of his reasoning. The 'common sense' of Modern Times and the rationality connected with it eventually can be defined by the two factors at issue: first, by the claim that the statements concerning empirical nature, empirical-sensual reality should be compatible with empirical reality itself, i.e. that they should be based on actually observed phenomena and processes and not on a prioristic speculation; secondly, by the rejection of everything that is in a certain intuitive yet definite sense illogical, absurd and inconceivable. Here we have to recognize that the latter is a principle which is independent of empirical reality and it can essentially be deduced from the rejection of the Scholastic-Aristotelian thought which may be characterized, as mentioned above, by Tertullianus' thesis. (The rather interesting paradox that it was just Bruno, an advocate of hermeticism, and inclined to mysticism, whose reasoning rested on the base of 'common sense', might deserve some investigation into the history of science.) Yet the connection between this principle and empirical reality is interesting, since the latter proved to be processable by means of the conformity to this principle, it 'fitted in' the demand involved in the principle. It would be a mistake to identify the rationality or common sense at issue with everyday consciousness or everyday thought, which may involve even elements of superstition. In the age of Copernicus his theory, among others, throw out a challenge just to everyday thought, to the 'ordinary mind' of the age. This kind of rationality and common sense rather meant a claim to some conscious, des-

*Da quel, che, dicendo il mondo interminato, nel modo nostro séguito quiete nell' intelletto, e dal contrario sempre innumerabilmente difficultadi ed inconvenienti."
G. BRUNO: *De l'infinito, universo e mondi.* in: Bruno: Dialoghi Italiani, 3. ed. Firenze, 1958. p. 350-351.

anthropomorphic, universal and coherent world picture that goes beyond the phenomenal aspect of reality and which aims at a full accordance with empirical-sensual world. On the other hand, it is also important that the criteria of this rationality are in rather close connection with those elements of everyday thought which have been shaped and have become natural evidences during hundreds of years in the course of socio-historical practice.

The physics of the 20th century, and especially Einstein's theory of relativity, came into antagonism with the intuitive rationality-criterion analyzed above that had been one of the most important ordering principles of the natural science of Modern Times so far. What is paradoxical in this antagonism is that at first approximation it seems to be the case that even if in other respects there was a break with the outlook of 19th-century natural science, in the case of this criterion there was a full continuity: it is the inadmissible condition of the scientific character of theories that they be built up rationally as well as that they should make statements that are in every respect rational and 'this-worldly'. For instance, in the case of Einstein's theory what is usually stressed is just its simplicity and 'beauty'. So the criteria of the creation of scientific theories did not change, and the way how some theory should relate to empirical reality is the same as it has been shaped during centuries in natural scientific thought. Yet a fundamental turn followed just in the question of how the rationality-criterion should relate to empirical reality. For, the possibility of curved spaces, of the finite but boundless universe, the consequences of the uncertainty relations of quantum mechanics, or the possible explanations of their respective experimental results (traced back to the wave-particle dualism) are absurd for our thinking. And not in the sense as Copernicus' theory seemed to be absurd for the everyday thought of the Middle Ages, but in a much deeper and much more fundamental sense, though this can be grasped only intuitively. On

the one hand, in Copernicus'case the matter in question was only that a world conceived of having a given structure had to be conceived of, on the base of novel thoughts, having another (different) structure. The problem did not lie in the fact that this other structure cannot be pictured or it is unconceivable but rather in that, because of the connection to the old structure, it seemed to be incredible that this other structure can describe the world. On the other hand, the problematic statements of 20th-century physics are simply unconceivable for us and their absurdity follows from this fact. If we accept these statements we also have to accept that the structure of the world is contradictory. That is, modern physics found itself face to face with that element of the rationality-criterion, which in the present paper is mentioned in connection with Bruno (cf. his reasoning against Aristotelian cosmology).

It is highly essential that, as we have already mentioned, the criteria of the creation of theories, of their empirical testing and of their selection are the same even in 20th-century physics as those having been shaped during a number of centuries. It is precisely this fact that endorses, renders scientific and inappealable these scientific results which are otherwise absurd for our thinking. Yet behind this continuity there lurks the change of the relation between physical theory and physical reality. In the great theories of 20th-century physics the mathematical apparatus is of chief importance, and the relation of this apparatus to the physical reality under examination has just the opposite sign as in the case of Newton's mechanics. The Newtonian and Leibnizian differential calculus came into being for the mathematical treating of the mechanical phenomena under physical examination. In this mathematical apparatus the given sphere of reality under examination is mapped in an abstract and mathematical form. Hence, the relation between the apparatus and the physical reality under examination did not (and does not) raise any specific

epistemological problems. But the mathematical apparatus used by the theory of relativity or quantum mechanics came into being not by mapping the physical reality examined therein, but those creating the theories depicted them from among the mathematical apparatuses that came into being in the course of mathematics' internal self-development, in order to solve their problems. What happened in the course of the theory-creation was in fact that a mathematical apparatus was attached to certain empirical facts and theoretical presuppositions as input, and after 'operating' the mathematical apparatus there came into being an output which, on the one hand, contained empirically measurable data which therefore were easy to test; on the other hand, such mathematical relationships which concerned physical reality and had to be provided with physical interpretation. Thus here the applied mathematical apparatus had already existed before its physical application was raised; its selection and 'survival' was based on its efficiency, i.e., on its heuristic power. Hence, the mathematical relationships appearing at the output had to be related to physical reality, they had to be given physical content, or had to be provided with physical interpretation. Slightly exaggerating we might say that while in the case of Newtonian mechanics we get the mathematical apparatus by means of the mapping of the physical reality - in the case of modern physics we get physical reality by means of the mapping of the mathematical apparatus. As Heisenberg wrote about the already complete mathematical apparatus: "Schrödinger's procedure considerably simplified the process of a number of computations which were extremely complicated in quantum mechanics. But the *physical interpretation of the mathematical system* met with serious difficulties."* (Italics mine

*"...man konnte nach dem Schrödingersehen Verfahren viele Rechnungen durchführen, die in der Quantenmechanik ausserordentlich kompliziert gewesen waren. Die Schwierigkeiten begannen aber bei der physikalischen Interpretation des mathematischen Schemas." (W. HEISENBERG: *Der Teil und das Ganze*. München, Piper Verlag, 1971. pp. 102-103.)

- L.Sz.) Heisenberg recalled the period following the working out of the mathematical apparatus of quantum mechanics as follows: "During the following few months Bohr and I were speaking in fact of nothing but the *possible physical* interpretation of quantum mechanics."* (Italics mine - L.Sz.) And the situation is complicated further by the fact that the consequences concerning physical reality, which are gained by the interpretation of the mathematical relationships we have got, are related to such spheres about which we cannot have any direct empirical experience; we can gain information about their behaviour and the processes that take place in them only with the help of complicated physical measuring apparatuses and empirical data, gained by the mediation of different theoretical presuppositions. The statements of 20th-century physics that are problematic for our outlook and 'natural' thinking appear when we interpret the relationships gained by applying the mathematical apparatuses, or at the physical interpretation of the latter ones themselves. Hence, with regard to modern theoretical physics, applying the thesis of 'credo quia absurdum est' is demanded by the mathematical apparatus applied therein. The conflict we are faced lies in the question whether we should believe our common sense or mathematics? Common sense would imply the continuation of the materialistic tradition of the natural sciences, but the high efficiency of the mathematical apparatus is for it, as well as, first of all, the accordance of the information at the output with the results of the empirical measurements.

*

* "In den folgenden Monaten bildete die physikalische Deutung der Quantenmechanik das zentrale Thema der Gespräche zwischen Bohr und mir."
(W. HEISENBERG: *Der Teil und das Ganze.* München, Piper Verlag, 1971. p. 109.)

*

When Lajos Jánossy called into question that the physical interpretation Einstein gave the theory of relativity was right, he recognized the dilemma mentioned above and tried to resolve it. As he summarizes the essential core of his standpoint: *"The scientific way of thinking cannot be but the dialectical refinement, deepening and further development of everyday thinking."** The second thesis or element of his theoretical starting point is the conjecture that the mathematical apparatus of the theory of relativity does not necessarily generate that Einsteinian interpretation which contains the statements absurd for everyday consciousness. He claims that it is possible to give such a physical interpretation to the Einsteinian mathematical apparatus of the theory of relativity which corresponds to his conception of the relation between scientific and everyday thinking. As he writes,

> "The best method to prove the thesis that scientific thinking is the dialectical improvement of everyday thinking is to point out that the whole complex of the theory of relativity can be built up by means of natural methods in conformity with everyday thinking. At the beginning of the [present] century a great sensation was created by the statement that the scientific analysis of the experiments necessitates that we should break loose from the usual thinking and should introduce new, 'revolutionary' concepts concerning space and time. - We are going to point out, by an objective and impartial analysis of the facts, that these sensation-creating statements were unfounded."**

Hence, Lajos Jánossy's approach to the Einsteinian theory of relativity is far from some damnation-like rejection of it, but rather outlining a positive programme: he aspires to work out such a new interpretation which corresponds to his concep-

*JÁNOSSY-ELEK: *A relativitáselmélet filozófiai problémái*. Budapest, Akadémiai Kiadó, 1963, p. 9.

**ibid.

tion of the relation between everyday and scientific thinking. Note that this decision is eventually a value-choice, which in the given situation goes beyond the scope of natural scientific and, more concretely, physical judgement. What is at stake in this value-choice is the tackling of the traditional rationality-criterion of materialism and natural scientific thought, or its rejection, according to the widely spread attitude of modern physics - and on the part of natural science the first alternative could be excluded only if it might be proved conclusively and in an exact manner that there are no other possible interpretations than those prevailing today. But such a proof might be worded only within a given conceptual or theoretical framework, therefore its conclusive nature might always be called into question. Jánossy realized the programme put forward in the context of the theory of relativity in a concrete and full-blown theory, and thus his value-choice was supported from the side of natural science. Jánossy's interpretation is not free from problems, but the same holds for the Einsteinian interpretation either. At those points which are connected with empirical reality and at which the two interpretations generate different consequences, we have not yet succeeded in gaining empirical data such that would enable us to choose between the two interpretations on the base of natural scientific criteria. The traditions of natural scientific thought and materialism speak in favour of Jánossy's interpretation, while what speaks in favour of Einstein's is the simplicity and 'beauty' of his theory, as well as its high heuristic power which manifested itself in the history of physics. As for its (theoretical) 'beauty', obviously it cannot be the criterion of theory choice in a theory of science that rests on the materialistic theory of reflection. But no doubt it remains a fact that this feature of scientific theories, as regards the history of natural scientific thought, is one of the traditional criteria of choice - and all this has been somewhat neglected so far by the Marxist theory of reflection. Es-

sentially the same might be said about the heuristic power, too.

Hence, Jánossy's attempt rests on the tackling of certain traditions as values. His choice is in accordance with that attitude which Lenin represents in his *Materialism and Empirio-criticism* when analyzing the problem of the crisis of physics and the 'disappearance of matter'. But in the conceptual frame-work of materialistic dialectics there is a possibility to warrant a decision that runs contrary to his choice, i.e. to warrant the acceptance of the statements of modern physics that are absurd for our outlook. According to this philosophical conception, our cognitive ability and frame of mind have de-velopment in the course of biological evolution and then socio-historical development, during millions of years. In this de-velopmental process a decisive role was played by the relation between the subject and his biological and socio-historical surroundings, which, however, have been changing, yet the re-lation itself has remained continuous. Since this relation was connected to the world of metres - mankind got in touch with the realm of microphysical and cosmological scales only in the natural scientific cognition of the present century - on the base of the epistemological conception of materialism it seems to be necessary that our outlook has been adapted to the con-ditions of the world of metres. As on argument might go, in accordance with the principles of dialectics, in the world of the microphysical and cosmological scales the relations are highly different from those of the 'metric' sphere, hence it seems natural that our outlook - precisely because it has been shaped by the world of metres - is unable to picture or con-ceive of those relations. Thus, to conclude this line of thought, it cannot be regarded as mere chance that in our know-ledge of the world of microphysical and cosmological measure-ments picturesquesness has been lost, what is more, this is to be considered a necessary process. Hence, the idea or convic-tion that at some higher level the picturesquesness of the

physical world picture may be revived again - by the further progress of cognition, or as a result of some new interpretations worked out for the mathematical formulae of theoretical physics - seems to be not more than a desire after a simple world which is easy to survey, after a harmonic and a picturesque natural world picture which characterized the natural science of the 19th century.

At first approach, this line of thought is convincing and conclusive, but one might raise the objection whether the existence of different conditions in different spheres may concern such fundamental categories as space and time, determinism and unambiguity, and whether e.g. the existence of milliards of light years can warrant such conjectures as that of finite but boundless three-dimensional space, which is rather absurd at least for everyday thinking? Jánossy refutes the physical reality of concepts of just this kind - on the grounds that here we are dealing with mathematical formulae and concepts having only smybolic significance. Hence, even if we accept, on the base of the aforementioned line of thought, the disappearance of the picturesquesness of the physical world picture as a necessity, there will hang over our decision Jánossy's conception as Damocles' sword, only to disturb us: does dialectics not play the same role in our argumentation as, according to Marx, it played with Hegel when, as regards the Prussian conditions, he tried to grasp as reasonable the unreasonable Prussian reality? Did we not degrade dialectics in this line of thought to become such a tool with the help of which we try to conceal, even from ourselves, that we have become the Tertullianuses of the present?

We cannot choose between Jánossy's conception and the standpoint put forward in the aforegoing (which tries to accept and explain the disappearance of the picturesquesness of the physical world picture in the framework of materialistic dialectics) categorically and conclusively neither on the base of natural scientific nor on that of philosophical criteria.

Even if we managed to gain such empirical data which would speak in favour of Einstein with regard to the theory of relativity, this would not call into question Jánossy's conception. This would only mean that that very concrete physical interpretation which Jánossy worked out has not proved fit for the realization of their programme. But this would not still mean the in principle impossibility of a picturesque interpretation of modern physics which is acceptable for everyday thinking as well, and so Jánossy's programme would remain valid. Therefore, our choice may be but a value-choice. What, from the point of view of physics, yet speaks against Jánossy and in favour of the other alternative is the extraordinary efficiency of the theoretical apparatus of modern physics - i.e., of the Einsteinian theory of relativity and quantum mechanics. But it is only an instrumental factor. Giving an answer to the question whether on the base of the instrumental efficiency to what extent the physical world picture may be regarded as correct and adequate knowledge in an ontological - and not only in an instrumental - sense has not a foundation without choosing between the two standpoints. It has not because here the theories under examination are eventually mathematical apparatuses, whose interpretation and possible physical contents are influenced to a considerable extent just by the choice between these two standpoints. And at this point there appears a new, serious and profound problem that has not been answered so far: how much can the theories of modern physics be regarded as the ontologically correct grasp or 'mapping' of the spheres of nature under examination - and how much are they mere instruments, which serve for the theoretical handling of physical reality and for the manipulation with physical reality?

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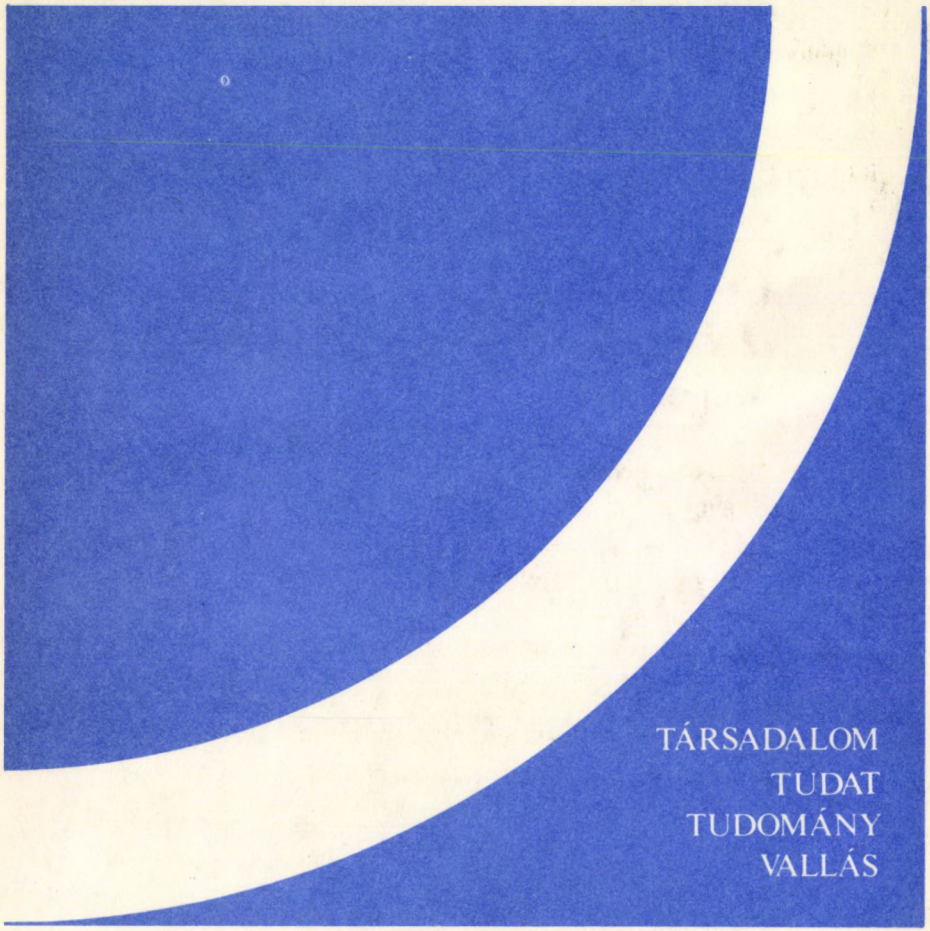

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