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ON WEAK INTERACTIONS AT LOW ENERGIES

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NEUTRAL WEAK CURRENTS AT LOW ENERGIES SYMPOSIUM IN DEBRECEN

29-30 MARCH 1974

The interest of particle physicists focused on the weak interactions in the last years. There was a certain theoretical indication, that weak reactions (together with the electromagnetic coupling) might become the dominating interaction of matter at superhigh energies, just beyond the reach of our present accelerators, and the "strong but soft" forces would dissolve in that region. The theoretical investigation of these possibilities led to the re-discovery of gauge field theories, which might offer a contradiction-free unified formulation of the fundamental interactions. From experimental point of view the most interesting consequence of some unified gauge models was the prediction of neutral weak currents. The conventional form of weak reactions produces a charge transfer between the conserved consistents of matter (e.g. in the case of K capture one has $e^- + p^+ \rightarrow \nu^0 + n^0$, i.e. a charge transfer from the baryon onto the lepton). In order to arrive at a renormalizable interaction, one needs a weak interaction of similar structure, but without charge transfer, e.g. $\nu^0 + p^+ \rightarrow \nu^0 + p^+$. The experimental discovery of such reaction at CERN in 1973, and its confirmation at NAL and at ANL in 1974 increased the interest in this field. One of the consequences of the neutral current coupling has been a new weak force between different leptons, between different baryons and also between leptons and baryons. This weak force may induce parity impurities not only in nuclei, but also in atomic and molecular systems.

Taking these amazing possibilities into account, the Extended Triangle Collaboration (high energy research institutions in Bratislava, Budapest, Trieste, Vienna, Zagreb) concentrated their March seminar onto the low energy aspects of weak interactions. A special flavour was given to this seminar by the presence of our collegues from the field of nucelar physics, chemistry and biophysics. (They really have collected longer experiences in investigating the reflection asymmetries of composite structures.) The low energy aspect of this meeting explained the circumstance, that the host of this seminar was the Nuclear Research Institute (ATOMKI) of the Hungarian Academy of Sciences. The organizers of

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

the seminar and all the participants of the Extended Triangle Collaboration express the most sincere thanks to Prof. A.Szalay, Director of the ATOMKI for his warm hospitality, to Prof.D.Berény head of the Nuclear Spectroscopy Department of the ATOMKI, for his participation in the organization. We are also indebted to the Hungarian Academy of Sciences, the Roland Eötvös Physical Society, the Central Research Institute for Physics in Budapest, the Biological Research Centre in Szeged and the Eötvös Universit in Budapest for their moral and financial support.

George Marx

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GAUGE THEORIES OF WEAK INTERACTIONS AND ITS LOW ENERGY CONSEQUENCES*

HERBERT PIETSCHMANN

Institut für theoretische Physik, Univ. Wien
and
Institut für Hochenergiephysik, Österr.Ak. d.Wiss.

The topic of this meeting is "Weak Interactions at Low Energies" and the interest is centered around the subject of parity violation. Clearly, the success of Gauge models of weak and electromagnetic interactions (both theoretical and experimental) stimulates all discussions of weak interaction physics. It is therefore quite natural, that also this meeting derives some of its "raison d'être" from these exciting developments.

The task of this talk is to show in a straightforward manner how low energy parity violating potentials are derived from the gauge model of Salam, Ward and Weinberg [1]. Clearly, it is impossible to explain and derive all this model within a talk of 50 minutes. On the other hand, it would be quite unsatisfactory to just scan over most of the derivations. Therefore, as a compromise, I shall give that part of the model in detail, which is necessary for understanding the low energy potential. The rest will be left out and the interested reader is referred to the literature [2].

To be specific, we want to show that the Salam-Ward-Weinberg model predicts a parity violating potential for electrons orbiting a nucleus of Z protons and N neutrons of the following form [3]

$$V_{p.v.} = \frac{G}{4\sqrt{2}m_e}\{\vec{\sigma}\vec{p}\delta^{(3)}(r)+\delta^{(3)}(r)\vec{\sigma}\vec{p}\}Q_W(Z,N) \qquad (1)$$

* Supported in part by "Fonds zur Förderung der wissenschaftlichen Forschung in Österreich", contract Nr. 1905

$$Q_W(Z,N) = C_V Z-N \tag{2}$$

$$C_V = 1-4\sin^2\phi \tag{3}$$

Here, m_e is the electron mass, $\vec{p}$ its momentum and $\vec{\sigma}$ its spin. Terms containing the nuclear spin are left out in eq.(1). The "weak charge" $Q_W(Z,N)$ is, of course, model dependent and we single out the Salam-Ward-Weinberg model throughout this talk.

In order to show how eq. (1) derives from the Salam-Ward-Weinberg model, let us recall the aim of any gauge model of weak and electromagnetic interactions: to formulate an interaction Lagrangian for these interactions in a "gauge invariant" way. What does it mean? We have to construct a space in which the gauge transformations act. It shall be called "weak isospin space" and we begin by spanning this space with the known leptons. Since we are interested in the potential for electrons only, we can also forget about the muon and its neutrino because they just add a trivial generalisation.

It is known since the first days of the celebrated "V-A theory" that the particles enter weak currents only with its lefthanded component. Therefore, we define a doublet in weak isospin space by

$$L(x) = \frac{1+\gamma_5}{2}\binom{\nu_e}{e} \tag{4}$$

It is easy, to form the weak current out of this doublet:

$$\ell_\lambda^{\pm}(x) = 2\bar{L}(x)\gamma_\lambda\tau^{\pm}L(x) \tag{5}$$

In order to be able to construct also the electromagnetic current, we recall that the electron enters the electromagnetic current also with its right-handed component so that we have to define a right-handed singlet in weak isospin space

$$R(x) = \frac{1-\gamma_5}{2}e(x) \tag{6}$$

The electromagnetic current can be now easily formed in the following way:

$$\ell_\lambda(x) = \bar{e}\gamma_\lambda e = \tfrac{1}{2}(\bar{L}\gamma_\lambda L-\bar{L}\gamma_\lambda\tau_3 L)+\bar{R}\gamma_\lambda R \tag{7}$$

We recognize, that it contains a singlet part and the third component of a triplet. The weak currents (5) exactly complete the triplet, and we can therefore write down an invariant interaction Lagrangian

$$L_1(x) = \frac{g}{2}\bar{L}\gamma_\lambda\vec{\tau}L\vec{A}^\lambda + g'[\tfrac{1}{2}\bar{L}\gamma_\lambda L + \bar{R}\gamma_\lambda R]B^\lambda \tag{8}$$

This Lagrangian consits of a singlet interaction and a triplet interaction. Clearly, we are free to attribute two different coupling constants to the two interaction parts. We have done so in eq. (8). As a consequence, the electromagnetic current is, however, not directly contained in eq. (8) and neither the singlet neutral vector boson nor the neutral member of the triplet can be identified with the physical photon.

The difference between singlet and triplet interaction is, however, not a physical one and in reality, we will therefore have a certain mixing between the two mentioned neutral vector bosons. Let us define this mixing in the following way

$$\begin{aligned} A_\mu^3 &= \cos\phi\cdot Z_\mu - \sin\phi\cdot A_\mu \\ B_\mu &= \sin\phi\cdot Z_\mu + \cos\phi\cdot A_\mu \end{aligned} \tag{9}$$

ϕ is called Salam-Ward-Weinberg mixing angle. It is the only hitherto undetermined parameter in the theory. Preliminary determination will be reported in this meeting.

To relate the mixing angle to the two coupling constants in eq. (8) as well as to the electric charge e, let us consider the neutral part of the interaction Lagrangian in closer detail.

$$\begin{aligned} L_1(x) &= \text{charged part} + \frac{g}{2}\bar{L}\gamma_\mu\tau^3L(\cos\phi Z_\mu - \sin\phi A_\mu) + \\ &+ g'(\tfrac{1}{2}\bar{L}\gamma_\mu L + \bar{R}\gamma_\mu R)(\sin\phi Z_\mu + \cos\phi A_\mu) = \\ &= \text{charged part} + \{g'\cos\phi(\tfrac{1}{2}\bar{L}\gamma_\mu L + \bar{R}\gamma_\mu R) - \\ &- \frac{g}{2}\sin\phi\bar{L}\gamma_\mu\tau^3L\}A^\mu + \\ &+ \{g'\sin\phi(\tfrac{1}{2}\bar{L}\gamma_\mu L + \bar{R}\gamma_\mu R) + \frac{g}{2}\cos\phi\bar{L}\gamma_\mu\tau^3L\}Z^\mu \end{aligned} \tag{10}$$

A comparison of eq. (10) and (7) yields the desired result:

$$e = \sin\phi \cdot g = \cos\phi \cdot g' \qquad (11)$$

$$g'/g = \tan\phi \qquad (12)$$

The most interesting question is now, of course, the new interaction of the new neutral weak vector boson Z. It is coupled to the following current:

$$\begin{aligned} & g'\sin\phi(\tfrac{1}{2}\bar{L}\gamma_\mu L+\bar{R}\gamma_\mu R)+\frac{g}{2}\cos\phi\bar{L}\gamma_\mu\tau^3 L = \\ & = \frac{g}{4\cos\phi}\{\bar{\nu}\gamma_\mu(1+\gamma_5)\nu-\bar{e}\gamma_\mu(C_V+\gamma_5)e\} \end{aligned} \qquad (13)$$

where C_V is given in eq.(3).

At this point, it is worth while to digress for a moment and to contemplate, wether the introduction of this extra neutral boson is really necessary. It turns out, that indeed it is not. However, the price to pay is the introduction of new "heavy leptons". The advantage of the Salam-Ward-Weinberg model is exactly that it introduces a new neutral current (which seems to be observed) and no new neutral heavy leptons (which are not observed). Naturally, it is always possible to complicate a model such that it contains both heavy leptons and neutral bosons.

At this point, it is also necessary to investigate the problem of the masses of the vector particles. So far, we have said nothing about this problem. The difficulty is, that the simple adding of mass-terms to the Lagrangian spoils the gauge invariance and therefore the whole game is ruined. Thus one normally invokes the Higgs-Kibble [4] formalism to avoid this difficulty. For our purpose, it is not necessary to go into the details of this formalism. All we have to accept, is the fact that the charged vector bosons and the physical neutral boson Z acquire a mass and that the ratio of these masses is given by

$$\frac{M_W}{M_Z} = \cos\phi \qquad (14)$$

In the language of elementary particle physics, the coupling of two vector currents by means of a massive vector particle is described in momentum space by the propagator

$$\Delta^F_{\mu\nu}(p) = \frac{g_{\mu\nu} - \frac{p_\mu p_\nu}{M^2}}{p^2 - M^2} \qquad (15)$$

If the mass is very large compared to all the momenta, we can write

$$g_{sw}^2 \Delta^F_{\mu\nu}(p) \to -g_{\mu\nu} \frac{g_{sw}^2}{M^2} \tag{16}$$

where g_{sw} is the "semi-weak" coupling constant of the intermediate boson to the weak current. In this limit, we thus have

$$\frac{G}{\sqrt{2}} = \frac{g_{sw}^2}{M^2} \tag{17}$$

It turns out, that the semi-weak coupling constant is given by

$$g_{sw}^2 = \frac{g^2}{8} \tag{18}$$

which can be inferred from eq. (8). Hence the mass of the charged intermediate bosons $W^{\pm}$ is related to the weak four-fermi coupling constant G by

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{e^2}{8M_W^2 \sin^2\phi} \tag{19}$$

Together with eq. (14), this gives a dependence of both masses of charged and neutral bosons as shown in Fig.1. It is clear, that both masses are at least so large, that the approximation of eq. (16) is justified.

In configuration space, the constant of eq. (16) simply leads to the δ-function of eq. (11).

So far, we have only talked about leptons. However, the potential felt by the electron in an atom reguires also consideration of the coupling of the neutral boson Z to the weak current of the nucleons.

The coupling of baryons to the vector bosons is determined by the following requirements: Firstly, the singlet structure has to be such as to yield the electromagnetic current in its usual form, and secondly, the neutral boson Z must not be coupled to a strangeness changing current. This is because neutral lepton currents in strangeness changing transitions have been ruled out experimentally to a very high accuracy.

In fact, the second requirement causes considerable concern. It can be fulfilled only by introducing a new class of particles with a new quantum number, "charm" [5]. However, in our context, even strange particles do not show up, unless we widen our scope to include orbits around hypernuclei. (From the theoretical point of view, this would be extremely interesting, however it seems to be nearly impossible to be done presently).

Following our main line, we shall therefore forget about all these complications. We only note, that the coupling of neutrons and protons is very similar to that of electrons and neutrinos, so that the total electron-nucleon coupling becomes

$$\left(\frac{g}{4\cos\phi}\right)^2 \bar{e}\gamma_\lambda (C_V+\gamma_5) e \,\frac{1}{M_Z^2}\, \delta^{(3)}(\vec{x})\{\bar{n}\gamma^\lambda(1+\gamma_5)n-\bar{p}\gamma^\lambda(C_V+\gamma_5)p\} \tag{20}$$

In order to arrive at the static potential as given in eq.(1), we go back to well-known facts of nuclear beta-decay and remember that in the static approximation

$$\bar{u}_r\gamma_\mu u_{r'} \to \delta_{\mu o}\delta_{rr'} \tag{21a}$$

$$\bar{u}_r\gamma_\mu\gamma_5 u_{r'} \to -\delta_{\mu k}\langle\sigma_k\rangle_{rr'} \tag{21b}$$

Since we neglect terms proportional to nuclear spins, only the vector part of the nucleonic current contributes. This leads directly to the factor Q_W of eq. (1).

From eq. (21a) it is clear, that only the zero-component of the leptonic current is left over. Since we are interested only in the parity violating part of the potential, only the axial vector part has to be taken now. From Dirac algebra we have

$$\bar{u}_2\gamma_o\gamma_5 u_1 = \frac{1}{2m_e}\{(\vec{\sigma}\vec{p})_1+(\vec{\sigma}\vec{p})_2\} \tag{22}$$

Now, the structure of eq.(1) is practically exposed. It remains to compute the numerical factor:

$$\frac{g^2}{16\cos^2\phi M_Z^2} = \frac{g^2}{16M_W^2} = \frac{G}{2\sqrt{2}} \tag{23}$$

Putting everything properly together yields eq. (1) as wanted. The interest which lies in this parity violating potential will be discussed in other talks of this meeting. Our task is fulfilled, but we do not want to close without another remark: There is also a parity conserving part due to the weak interaction Lagrangian, of course. Normally, one would think that this is not interesting because it cannot be disentangled from the ordinary Coulomb potential. However, the Higgs-Kibble mechanism mentioned above requires a coupling of the electrons to the scalar Higgs particle. This coupling is proportional to the mass of the lepton and may therefore be a cause for differences in muonic atoms versus ordinary atoms. May be that this can give a better clue as to the nature of breaking of the symmetry which is possibly the weakest link in the whole chain of arguments leading to the Lagrangian of the Salam-Ward-Weinberg model. Its study is therefore of considerable interest also.

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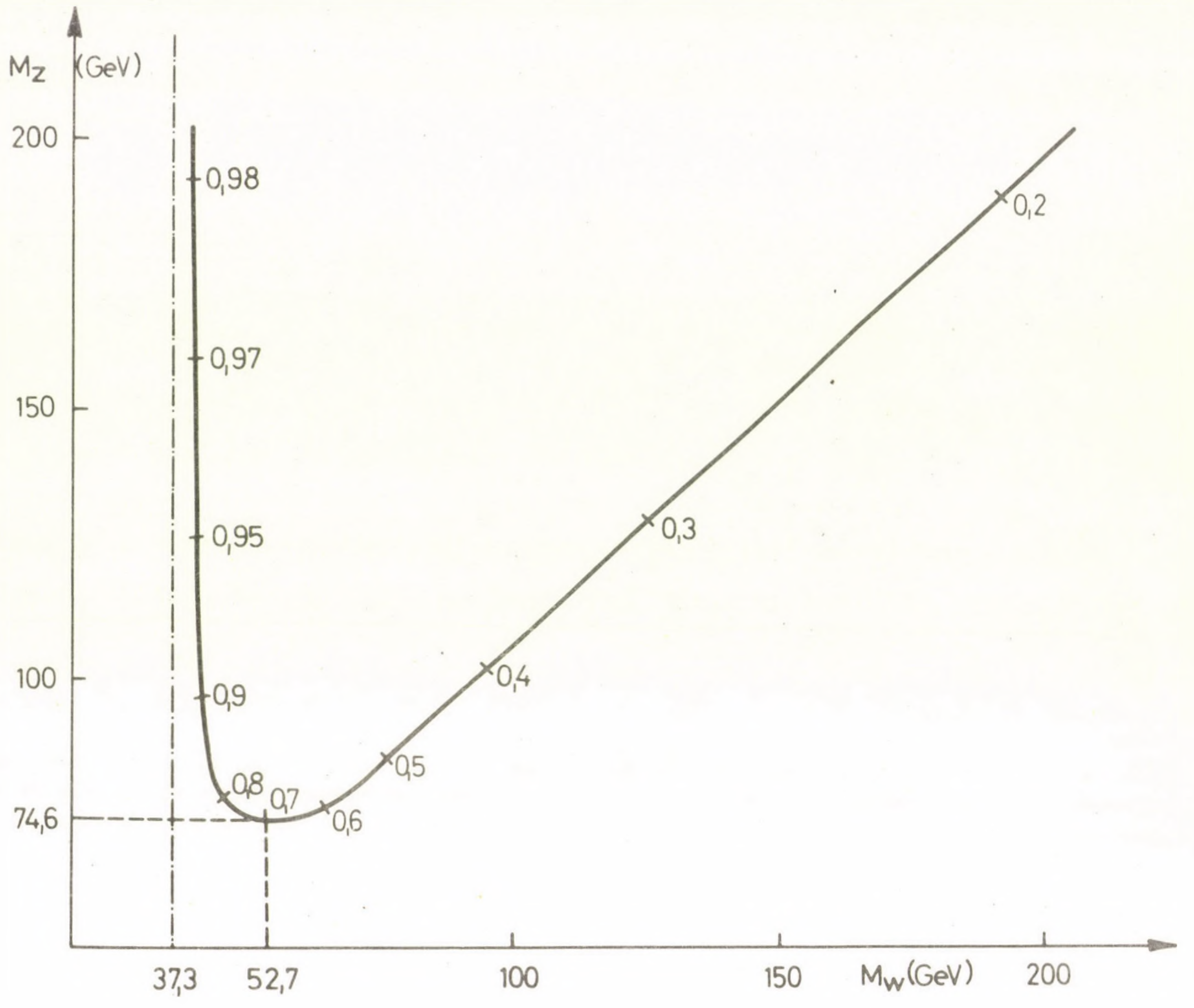

Fig. 1.

Masses of Intermediate Bosons

(Parameter is sinϕ)

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RESULTS OF THE NEUTRAL CURRENT SEARCH IN GARGAMELLE*

JORGE G. MORFIN

III. Physikalisches Institut Technische
Hochschule Aachen - Aachen, W-Germany

Introduction to the Theory.

The weak interaction is phenomenologically described as the coupling of two currents both of which have a vector and an axial vector part. Until the experiment I will describe to you today, only a charge carrying current was necessary to explain all observed experimental data. It has, in fact, been verified that the branching ratio of the neutral current (N.C.) mediated decay process which simultaneously changes strangeness (i.e. $K_L^0 \rightarrow e^+e^-$) is less than $\sim 10^{-9}$, many orders of magnitude below the charged current (C.C.) process.

* The following is an extended review of the analysis performed by the Gargamelle ν-collaboration and published in the two letters:

F.J. Hasert et al., Physics Letters 46B (1973), 121

F.J. Hasert et al., Physics Letters 46B (1973), 138.

Recently there has been a renewal of interest in the question of whether the strangeness non-changing neutral current exists or not. This resurgence is associated with the attempts by the theorists to unify the weak and electromagnetic phenomena in a single renormalizable theory. From an experimental point of view, these models may be categorized as to whether they require new heavy leptons, neutral currents or both. The high statistics $\nu(\bar{\nu})$-experiment, carried out in the Gargamelle heavy liquid bubble chamber, is particularly sensitive to the predictions made by those theories which demand the existence of neutral currents.

Salam and Ward[1] and Weinberg[2] have constructed such a model (SWW model) based on $SU(2) \otimes U(1)$ (i.e., Isospin $\otimes$ hypercharge). In this model[3] there are four, originally massless, vector bosons; an isotriplet (W^1, W^2, W^3) and an isoscalar (B), while the Higgs scalar mesons appear in the from of two isodoublets (ϕ^+, $\overline{\phi^o}$ and ϕ^-, ϕ^o). The scaler field ϕ is assumed to have a self-interaction $V(\phi)$ in such a form that the vacuum expectation value is non-zero. This leads to a shift in the mass spectrum such that three of the four bosons become massive via their interaction with ϕ while one remains massless. The observable fields of direct interest are the charged vector bosons

$$W^{\pm} = \frac{W^1 \pm iW^2}{2} \quad , \qquad (1)$$

the massive neutral vector boson

$$Z^o = W^o \cos\theta_W + B^o \sin\theta_W \quad , \qquad (2)$$

and the massless photon

$$\gamma = B^o \cos\theta_W - W^o \sin\theta_W \quad . \qquad (3)$$

The mixing angle θ_W appearing above is conventionally called the Weinberg angle and is the only free parameter within the theory. There are three coupling constants involved in this unified theory; g--$W^{\pm}$ to the lepton current, g'--B to the lepton current and e--electromagnetic constant, which are related by

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}} \quad . \qquad (4)$$

These constants can be further related to the mixing angle θ_W as

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \quad , \quad \cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \qquad (5)$$

so that

$$\sin^2\Theta_W = \frac{e^2}{g^2} \qquad (6)$$

Due to the equivalence[3] of certain terms in $\mathcal{L}^{int}_{Weinberg}$ and the conventional weak effective Lagrangian $\mathcal{L}^{int}_{weak}$ it is possible to constrain the masses of the intermediate bosons

$$M_W = \left(\frac{\sqrt{2}\, g^2}{8G_F}\right)^{1/2} = \left(\frac{\sqrt{2}\, e^2}{8G_F \sin^2\theta_W}\right)^{1/2} \geqslant 37 \text{ GeV}$$

$M_Z \geqslant 74$ GeV.

Experimental Configuration.

For those of you who have not had the dubious privilege of working with a neutrino beam, I will briefly describe the CERN beam line. A 26 GeV primary proton beam hits a target located within the aperature of a focusing device. This pulsed focuser has the property of creating immense fields for very short periods of time and is thus most efficient in separating positive from negative particles. As table I shows, positive particles are the source for neutrinos while negative particles yield antineutrinos, since incident proton beams produce more positive than negative particles the ν flux is generally higher than the $\bar{\nu}$ flux. The decay kinematics of the $\pi_{\mu 2}$ and $K_{\mu 2}$, which dominate the π and K decays, are such that the π's, which outnumber the K's, contribute the low energy peak while the K's yield the high energy tail. These various characteristics conspire to create the spectra shown in fig. 1.

The heavy liquid chamber "Gargamelle" is schematically shown in fig. 2. It is a cylinder with radius $\sim$ 1 m and length 4.8 m. Out of this total volume, $\sim$ 6 m^3 was defined as the visible volume while 1/2 of this $\sim$ 3 m^3 was designated the fiducial volume. The chamber was filled with heavy freon CF_3Br which has a density of 1.5 g/cm^3, radiation length 11 cm and interaction length 60 cm. This high density gave a fiducial mass of 4.5 tons while the low conversion lengths insured excellent detection efficiency of neutrals (γ's and neutrons). The entire chamber was surrounded by a 20 kg magnetic field which enabled us to measure hadron energy to within 15 %. The sample of analysed film I will report on today varies depending on what aspect of the experiment we are examining, but in the maximal case is 750.000 pictures divided equally between ν and $\bar{\nu}$.

Naturally, such a huge exposure demands a huge collaboration ... and that we have. Fig. 3 shows some of the many physicists associated with the seven universities which make-up the collaboration.

Experimental Results: Purely Leptonic Neutral Currents.

If we limit ourselves to the purely leptonic neutral currents, and avoid the poorly understood hadronic contributions, the cross-sections are exactly calculable from the Weinberg-Salam models.

The neutral current interaction of interest in accelerator experiments is

$$\nu_\mu(\bar{\nu}_\mu) + e^- \rightarrow \nu_\mu(\bar{\nu}_\mu) + e^- \quad . \qquad (7)$$

't Hooft[4] has expressed the amplitude for this interaction as the sum of the Feynman diagrams shown in Fig. 4. The effective Lagrangian has the form

$$\mathcal{L}_{eff} = - \frac{G_W}{\sqrt{2}} (\bar{\nu}_\mu \gamma_\alpha (1+\gamma_5) \nu_\mu)(\bar{e}\, \gamma_\alpha (g_V + g_A \gamma_5) e) \qquad (8)$$

where g_V and g_A in the Weinberg theory are $\begin{pmatrix} \nu \\ - \\ \bar{\nu} \end{pmatrix}$.

$$g_V = 1/2 \mp 2 \sin^2\theta_W$$

$$g_A = \pm 1/2$$

and the differential cross sections can be calculated in terms of the Weinberg angle to be:

$$(\nu) \quad \frac{d\sigma}{d \cos\theta^*_{e^-}} = - \frac{G^2 S}{2\pi} \left[\sin^4\theta_W \sin^4\left(\frac{\theta^*_{e^-}}{2}\right) + \right.$$
$$\left. + \sin^4\theta_W - \sin^2\theta_W + 1/4 \right] \qquad (9)$$

$$(\bar{\nu}) \quad \frac{d\sigma}{d \cos\theta^*_{e^-}} = - \frac{G^2 S}{2\pi} \left[(\sin^4\theta_W + \sin^2\theta_W + 1/4) \times \right.$$
$$\left. \times \sin^4\left(\frac{\theta^*_{e^-}}{2}\right) + \sin^4\theta_W \right] . \qquad (10)$$

For ν-e scattering

$$\frac{G^2 S}{2\pi} = 0.8 \times 10^{-41} \, E_\nu(\text{GeV}) \frac{\text{cm}^2}{\text{electron}} .$$

The observable topology of reaction (7) is a single e^- originating within the liquid, and without any other associated tracks. However, neutrino and neutron interactions in the surrounding magnet provide a great deal of low energy electromagnetic background so that a lower limit on the electron energy was set at 300 MeV. Similarly, from the kinematics of the interaction it is obvious that the e^- will be emitted at a very small angle θ_{e^-} with respect to the incoming ν direction so that an upper limit on this angle was set at 5^o. Using expressions (9) and (10) and folding in the CERN $\nu(\bar{\nu})$-spectrum[5] the predicted laboratory distributions can be determined. Figs. 5 and 6 show the laboratory electron energy and angular distribution for ν-e scattering as a function of $\sin^2\theta_W$ while figs. 7 and 8 show similar distributions for $\bar{\nu}_\mu$-e scattering.

The <u>visible volume</u> of all ν film was scanned twice with a partial third scan while all $\bar{\nu}$ film was fully scanned three times for two categories of events. First was an actual candidate, defined as an isolated e^- without any other possible associated vertex upstream in the same frame, while the second category was any e^+e^- pair again without any visible associated vertex (i.e., source). This second category was used as an aid for background calculations.

In the ν film zero candidates were found, however, two pairs, which satisfied both kinematical cuts, were found. In the $\bar{\nu}$ film one candidate was found satisfying <u>both</u> cuts (fig. 9) while no isolated e^+e^- pairs satisfied both cuts. The candidate itself is located slightly more than five radiation lengths from the upstream end of the chamber and 16 cm from the chamber axis. The energy measured both by curvature and total track length is (385 ± 100) MeV and the angle it makes with the ν direction is $1.4^o\ ^{+1.6^o}_{-1.4^o}$.

There are four possible sources of background that must be considered. The major source comes from the ν_e quasi-elastic interaction

$$\nu_e + n \rightarrow e^- + p \tag{11}$$

where the proton is not observed. It is possible to estimate this contribution by noting that the ν_μ quasi-elastic interaction

$$\nu_\mu + n \rightarrow \mu^- + p \tag{12}$$

is the same as reaction (11) except for the mass of the lepton.

If the shape of the ν_μ and ν_e spectrums are essentially the same, then the final state kinematics of reactions (11) and (12) are the same despite the mass difference of the final lepton. This condition being fulfilled in the experiment, we have examined the ν_μ events found within the fiducial volume. We find 450 events with final state

$$\mu^- + m \text{ protons} \quad (m \geqslant 0) \quad ,$$

while in the same sample we find only three events with a μ^-, no protons, and the angle between μ^- and incoming ν less than 5^o. The scan efficiency for the solitary μ^- topology is 50 %, so that

$$\frac{\mu^-(\theta<5^o) + 0p}{\mu^- + mp} = (1.3 \pm .7)\% = \frac{e^-(\theta<5^o) + 0p}{e^- + mp} \quad . \tag{13}$$

Within the fiducial volume ($\sim$1/2 the visible volume), $15\nu_e$ events with final state

$$e^- + m \text{ protons} \quad (m \geqslant 0)$$

were found so that relation (13) implies a background within the visible volume of the ν exposure of 0.3 ± 0.2 events. Since no e^- + m protons events were found in the $\bar{\nu}$ exposure, we must compensate for the difference in the ν_e flux in the ν_μ beam and the ν_e flux in the $\bar{\nu}_\mu$ beam (see fig. 1). This corresponds to reducing the background, found in the ν film, by an order of magnitude so that the final expected contribution from this source is (0.03 ± 0.02) events in the $\bar{\nu}$ film.

Another possibility is that the candidate is a Compton electron. The cross-section for Compton scattering becomes rapidly suppressed with increasing energy. Using the Klein-Nishina[6] formulae the ratio of Compton and pair production cross-sections, at an energy of 380 MeV, can be calculated to be

$$\left(\frac{\sigma_C}{\sigma_{pp}}\right)_{380\ \mathrm{MeV}} \approx 0.5\ \% \quad .$$

Based on the two e^+e^- pairs found, the background due to Compton scattering is less than 0.01 events in the ν film and negligible in the $\bar{\nu}$ film. An associated possible background contribution is the case of extremely asymmetric pair production. The vertex of the event is obviously clean so as an upper limit on the positron energy we take 2 % of the total energy. With the aid of the Rossi[7] expression and based on the two e^+e^- events found we conclude that the background due to asymmetric pair production is less than .03 event in the ν film and again negligible in the $\bar{\nu}$ film.

The V-A allowed interaction

$$\nu_e(\bar{\nu}_e) + e^- \to \nu_e(\bar{\nu}_e) + e^-$$

hardly contributes since the ratio of ν_e flux to ν_μ flux is less than 1 % as can be seen in fig. 1. The last possible background, the electromagnetic interaction

$$n + e^- \to n + e^- \quad ,$$

is killed by kinematics since to have $E_{e^-} > 0.3$ GeV requires that $E_n > 18$ GeV.

To calculate the cross sections for $\nu_\mu(\bar{\nu}_\mu)$-e scattering, we use the integrated flux of fig. 1 together with the scan efficiency for the single electron topology ($E_{e^-} > 300$ MeV). This has been taken to be the same as the single γ scan efficiency at this energy and has been measured as 86 %. The calculated detection efficiency (fraction that survived the kinematical cuts $E > 0.3$ GeV, $\theta < 5^0$) is model dependent and, in the case of the Weinberg-Salam model, a function of θ_W as in fig. 10. Folding in these factors we find, at the 90 % confidence level, that:

$$\sigma(\nu_\mu + e^- \to \nu_\mu + e^-) < 0.26\ E_\nu \times 10^{-41}$$

$$\sigma(\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-) < 0.88\ E_{\bar{\nu}} \times 10^{-41} \qquad \left(\frac{cm^2}{e}\right)$$

We can interpret these results in terms of the SWW model with the aid of a maximum likelihood technique and find that

$$0.1 < \sin^2\theta_W < 0.6 \quad .$$

What is perhaps more significant than this limit that we place on θ_W is the statement that:

The probability that the single event observed in the $\bar{\nu}$ film is due to non-neutral current background is less than 3 %.

We now turn to the semi-leptonic neutral current channel. The analysis in this case is based on 83,000 ν and 207,000 $\bar{\nu}$ pictures, double scanned in both cases.

A search has been made for reactions which satisfy the characteristics of the inclusive semi-leptonic neutral current reaction

$$\begin{pmatrix} \nu_\mu \\ \bar{\nu}_\mu \end{pmatrix} + \mathcal{N} \rightarrow \begin{pmatrix} \nu_\mu \\ \bar{\nu}_\mu \end{pmatrix} + \text{hadrons} , \qquad (14)$$

and compared these with the charged current interaction

$$\begin{pmatrix} \nu_\mu \\ \bar{\nu}_\mu \end{pmatrix} + \mathcal{N} \rightarrow \begin{pmatrix} \mu^- \\ \mu^+ \end{pmatrix} + \text{hadrons} . \qquad (15)$$

Reactions (14) and (15) are distinguished by the absence or the presence of a muon. In order to decrease the contamination of selected events of the reaction (15) in the events of the reaction (14) we have accepted as a muon candidate any particle which satisfies one of the following criteria:

- a track which leaves the chamber without interaction or scattering larger than 30^0 ;

- a track which stops and decay into an electron;
- a negative track which stops without interaction and with measured momentum by curvature compatible with the muon mass range-momentum relation

The events themselves were divided into three main categories.

a) NC events

The neutral current (NC) candidates (reaction 14), were defined as events in which no visible muon has been created. The events are characterized by the presence of protons identified by stop or interaction, charged pions or kaons identified by interactions, neutral pions decaying into γ-rays, neutral kaons or hyperons decaying into charged or neutral particles, and neutron induced stars, so that all the secondaries are identified as hadrons.

b) CC events

The NC events were compared to the events where there was one and only one muon candidate (reaction 15). In order to minimize any bias, all secondary products apart from the muon candidate must have the same properties as the NC events. These events are called the charged current (CC) candidates.

c) AS events

As we will see, a possible background may come from neutrons or K_L^0 generated in neutrino interactions in the material surrounding the chamber. In order to estimate this background, we have to evaluate the rate of production of such events by searching for events with two visible vertices.

The first interaction must have at least one charged lepton candidate whereas the second interaction must have the same properties as the NC events. These events are called associated (AS) events.

Among the AS events four identified K_S^0 decays (2 in the ν film and 2 in the $\overline{\nu}$ film) have been removed from the sample.

There exists a low energy neutron flux into the chamber. In fact, in 25% of the pictures containing an event, a short proton track was observed upstream. This fraction is the same for both the NC and CC events. Therefore, any star in which the total visible energy is smaller than 150 MeV was never taken as a primary interaction vertex.

To improve the discrimination of events due to reaction (14) from the background, it was required that the total visible energy in the NC and the accompanying neutron star of the AS events as well as in the hadronic part of the CC events should be greater than 1 GeV. This cut enables also a rather accurate determination of the line of flight of the neutral particle inducing the observed event. This line of flight is obtained from the measurement of the total visible momentum taken off by the secondaries. For the AS events, a mean deviation of 17^0 was found between the line joining both vertices, i.e. the true direction, and the total visible momentum.

The results of the classification of the events are displayed in table II for both neutrino and antineutrino films.

Results

To get the true numbers of NC events a background subtraction must be applied using the best estimate of B/AS ratio rather than the upper limit as in the preceding discussion. The angular and energy distributions of the protons emitted in $\nu(\bar{\nu})$ interactions are taken as neutron distributions. The two relevant parameters n and θ_o are respectively equal to 2.7 and 350 m.r. and the resulting ratio B/AS is 0.8 ± 0.4.

After subtraction:

$$NC\ (\nu) = 90 \text{ events} \ ; \quad NC\ (\bar{\nu}) = 53.4 \text{ events.}$$

Further subtraction must be made from the events to remove the contamination of $\bar{\nu}$ in the ν beam and of ν in the $\bar{\nu}$ beam. This subtraction can be estimated from the numbers of wrong sign muon signatures in the CC control samples, and gives no contamination in the ν and 7.8 events in the $\bar{\nu}$. Thus the real numbers of NC events are:

$$NC_R\ (\nu) = 90 \text{ events} \ ; \quad NC_R\ (\bar{\nu}) = 45.6 \text{ events.}$$

A correction must also be applied to the CC events since a pion from an NC event leaving the chamber before interacting or decaying can be taken as a muon thereby transferring the event to the CC category if the remaining hadron energy is > 1 GeV. This transfer of category can be estimated using the known hadron detection efficiency and the number of charged current events having two lepton candidates where the energy of the extra hadrons is > 1 GeV. This gives the real numbers of CC events as:

$$CC_R\ (\nu) = 398 \text{ events} \ ; \quad CC_R\ (\bar{\nu}) = 98 \text{ events.}$$

Finally we obtain the ratios:

$$\frac{NC_R}{CC_R}(\nu) = 0.23 \pm 0.03; \quad \frac{NC_R}{CC_R}(\bar{\nu}) = 0.46 \pm 0.09.$$

Interpretation

We must emphasize that the neutral current hypothesis is not the only interpretation of the observed events. They could also be attributed to penetrating particles other than ν_μ and ν_e, heavy leptons immediately decaying into hadrons, or to penetrating particles produced by neutrinos and in equilibrium with the ν beam.

Heavy leptons of the Glashow type ($E^{\pm}_{0}$, $M^{\pm}_{0}$), as required in some gauge theories have both hadronic and leptonic decay modes[8]. From the lack of e^+ candidates in the neutrino exposure[9], we conclude that leptons of that specific type are not a possible interpretation of our data.

If we interpret these events as induced by neutral currents, we can, in particular, compare the data with the evaluation by Pais, Treiman, Paschos and Wolfenstein[10] in the frame of the Salam-Weinberg theory. Using the notation of Paschos and Wolfenstein, the cross-sections for CC and NC reactions are given by:

$$CC_{\nu} : \sigma_- = A + I + V$$

$$NC_{\nu} : \sigma_o = 1/2\ (A + x\ I + x^2\ V + y^2\ S)$$

$$CC_{\bar{\nu}} : \sigma_+ = A - I + V$$

$$NC_{\bar{\nu}} : \bar{\sigma}_o = 1/2\ (A - x\ I + x^2\ V + y^2\ S).$$

where A, I, V, S are respectively the contributions from the axial, axial-vector interference, vector and isoscalar terms; and $y = x - 1 = - 2 \sin^2\theta_W$ where θ_W is the Weinberg mixing angle. Upon neglect of the S term, these expressions give lower limits for the ratios $\frac{\sigma_o}{\sigma_-}$ and $\frac{\sigma_o}{\sigma_+}$. However, because the hadronic energy in this experiment is cut at 1 GeV, and also the topology of the hadronic part is chosen to be non-ambiguous, corrections are required before a comparison of the data with these predictions can be made. These corrections are expected not to be very large but cannot be rigorously evaluated without further assumptions or models.

We may circumvent these restrictions by noting that

$$\frac{\sigma_o - \bar{\sigma}_o}{\sigma_- - \sigma_+} = \frac{x}{2}$$

this in turn implies

$$\sin^2\theta_W = \frac{1}{2} - \frac{\frac{\sigma_o}{\sigma_-} - \frac{\bar{\sigma}_o}{\sigma_+}\ \frac{\sigma_-}{\sigma_+}}{1 - \frac{\sigma_-}{\sigma_+}}$$

We have evaluated all of these ratios assuming the same E_H cut and find

$$\sin^2\theta_W = 0.35 \pm 0.07$$

Another means of interpreting the data is with the aid of the Sehgal model[11]. This quark model assumes the existence of four fundamental quarks, the familiar p, n and λ quarks and a fourth quark p' which is similar to the p but has the additional property of "charm". This p' quark is necessary to eliminate the $\Delta S = 1$ neutral current interaction which, as mentioned earlier, is strongly suppressed with respect to the charged current interaction in all experimental results.

With such a model, Sehgal is able to fix the isoscalar term and thus predict the NC to CC ratio itself (as opposed to a lower limit) as a function of the Weinberg parameter θ_W.

Furthermore with the expressions for the cross sections given, we may calculate explicitly the effects of the cut in hadron energy E_H. With the Sehgal model we again find that $\sin^2\theta_W \approx 0.3 - 0.4$.

ADDENDUM

In the subsequent discussion, Prof. M. Melvin (Temple Univ. Philadelphia) mentioned that as early as 1963 he and R. Ingrahm (Il Nuovo Cimento, 29 (1963), 1034) had constructed a model that predicted leptonic neutral currents. In particular, they stressed the necessity of a search for the interaction

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

which I have discussed today. They calculated that the cross section for this reaction should be of the same order as

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

which is also predicted by the SWW theory and which is consistent with our experimental results.

Table I

	ν	$\bar{\nu}$
low E ν_μ	$\pi^+ \rightarrow \mu^+ \nu_\mu$	$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$
hi E ν_μ	$K^+ \rightarrow \mu^+ \nu_\mu$	$K^- \rightarrow \mu^- \bar{\nu}_\mu$
ν_e contribution	$K^+ \rightarrow \pi^o e^+ \nu_e$	$K^- \rightarrow \pi^o e^- \bar{\nu}_e$
"	$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$	$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$
"	$K_L^o \rightarrow \pi e \nu_e$	

Table II

	ν	$\bar{\nu}$
No. NC candidate	102	63
" CC "	428	148 ($35\mu^-$)
" AS "	15	12
" NC/pix	1.2×10^{-3}	0.3×10^{-3}

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(9) H. Deden et al., High energy electronic neutrino (ν_e) and antineutrino ($\bar{\nu}_e$) interactions. Submitted to Phys. Lett. B, (21.8.73).

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E.A. Paschos and L. Wolfenstein, Phys. Rev. D7 (1973) 91.

(11) L.M. Sehgal, Predictions of the Weinberg Model for Neutral Currents in Inclusive Neutrino Reactions. Submitted to Nuclear Physics.

Figure Captions

1) Spectra of CERN ν beam.

2) Gargamelle: chamber and delineated volume

3) Feynman Diagrams for $\nu + \ell \to \nu + \ell$ scattering

4) Monte Carlo Results: Electron energy distribution for $\nu_\mu + e^- \to \nu_\mu + e^-$ (Normalized to 2500 events)

5) Monte Carlo Results: Angular distribution of e^- with respect to incoming ν direction for $\nu_\mu + e^- \to \nu_\mu + e^-$

6) Monte Carlo Results: Electron energy distribution for $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$

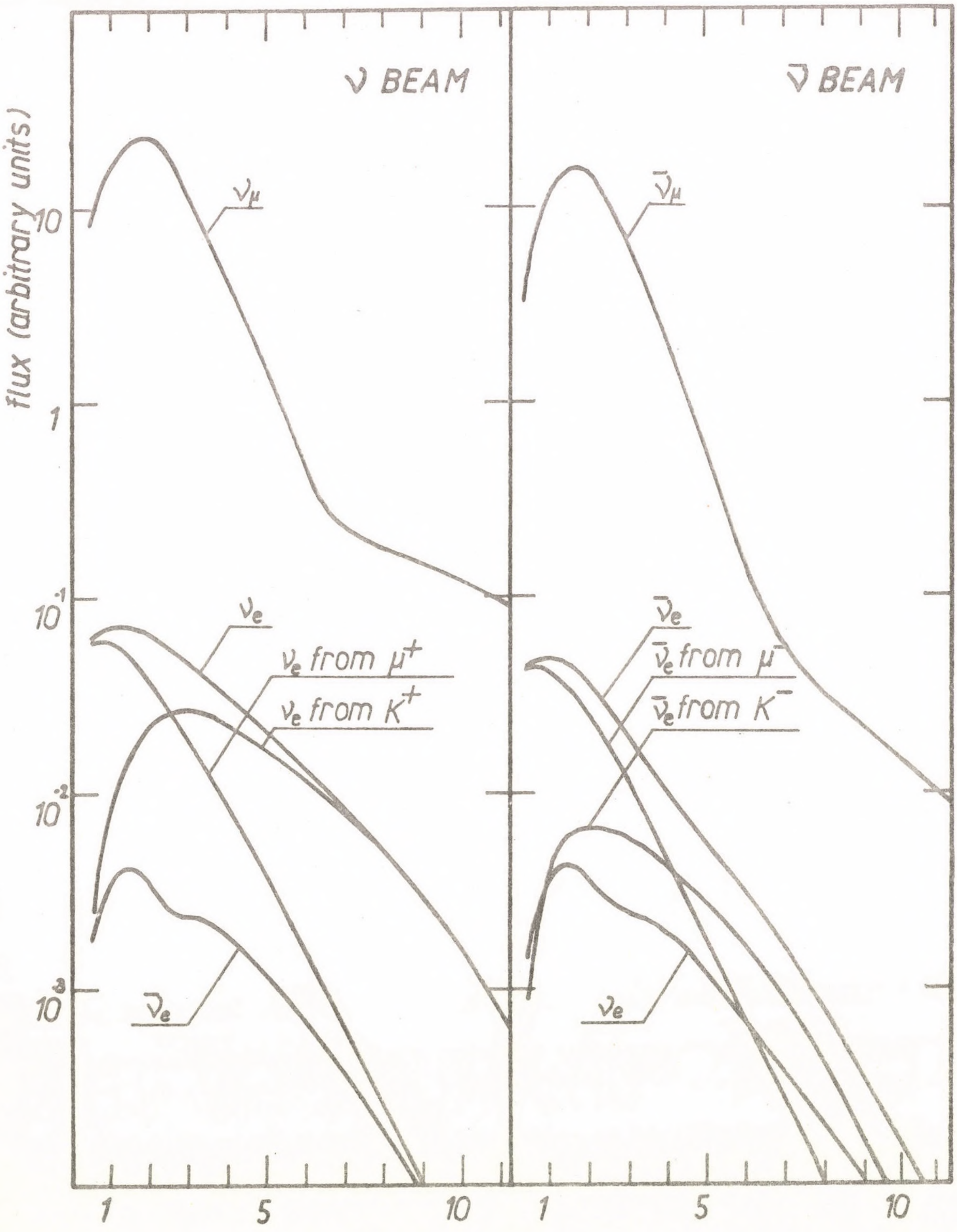
7) Monte Carlo Results: Angular distribution of e^- with respect to incoming ν direction for $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$

8) The candidate found in the $\bar{\nu}$ scan: primary electron plus bremsstrahlung

9) Detection efficiency ($e_D \equiv$ % events retained after kinematical cuts) as a function of $\sin^2\theta_W$.

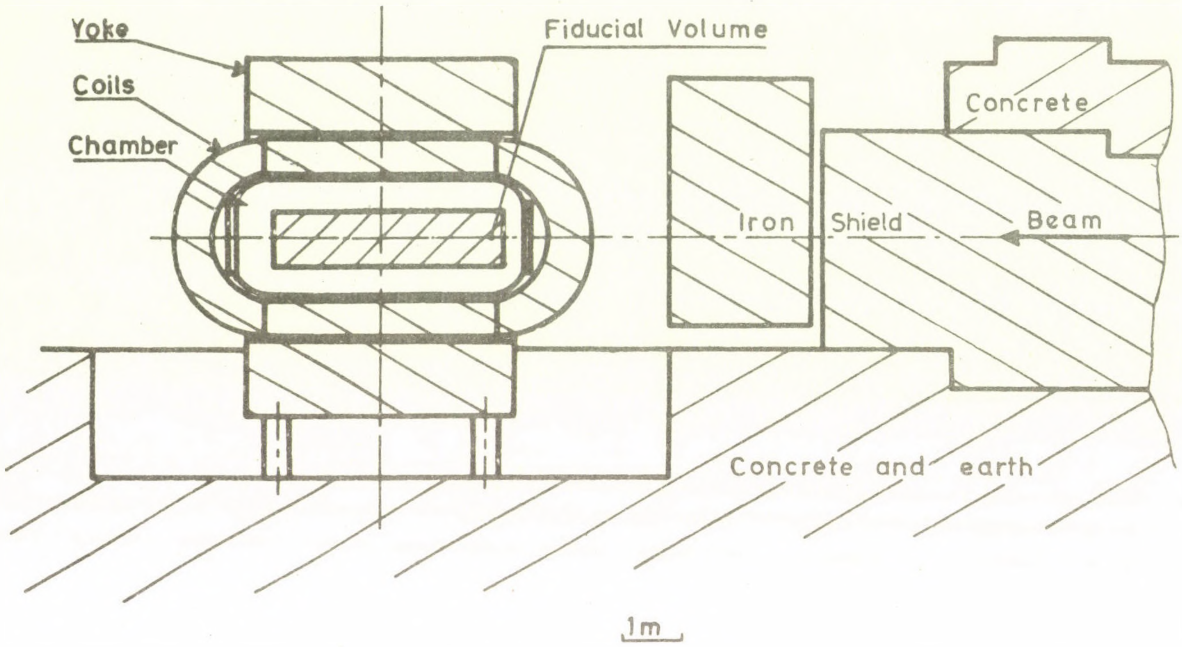
10) Variation of log likelihood function with $1/\lambda_a$ for CC, NC and AS event samples.

Figure 1



ν BEAM
ν̄ BEAM
flux (arbitrary units)
10
1
10⁻¹
10⁻²
10⁻³
ν_μ
ν_e
ν_e from μ⁺
ν_e from K⁺
ν̄_e
ν̄_μ
ν̄_e
ν̄_e from μ⁻
ν̄_e from K⁻
ν_e
1
5
10
1
5
10

Side view



Top view

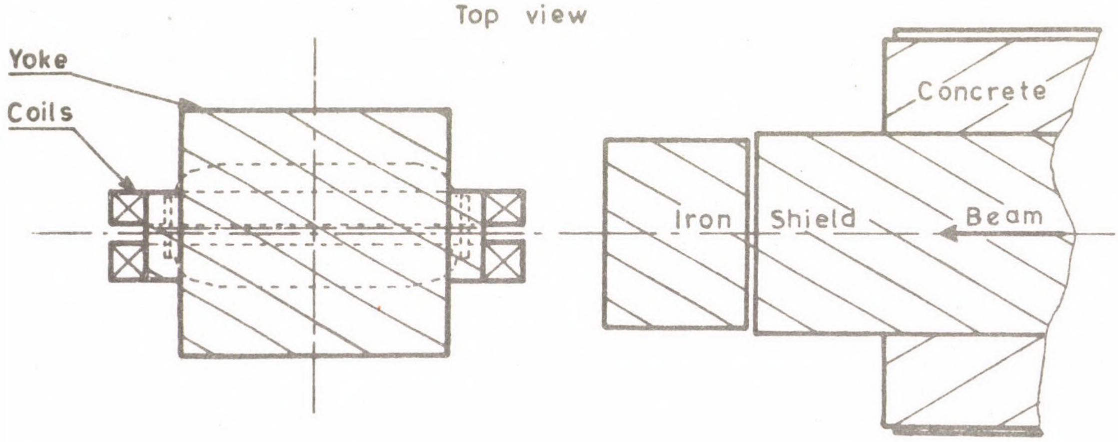
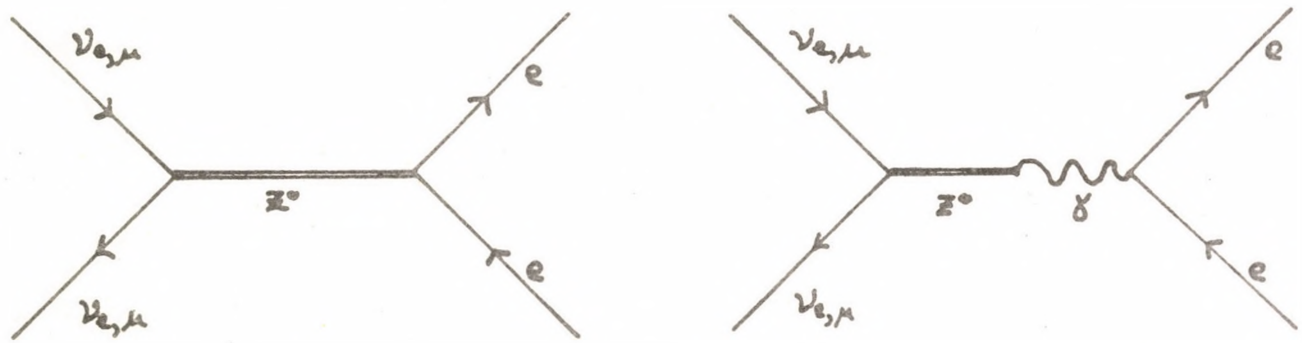


Figure 3



$g_V = g_A = 1/2$ $\qquad$ $g_V = \frac{2e^2}{g^2}, g_A = 0$

Diagrams for ν+ℓ → ν+ℓ scattering

Figure 4

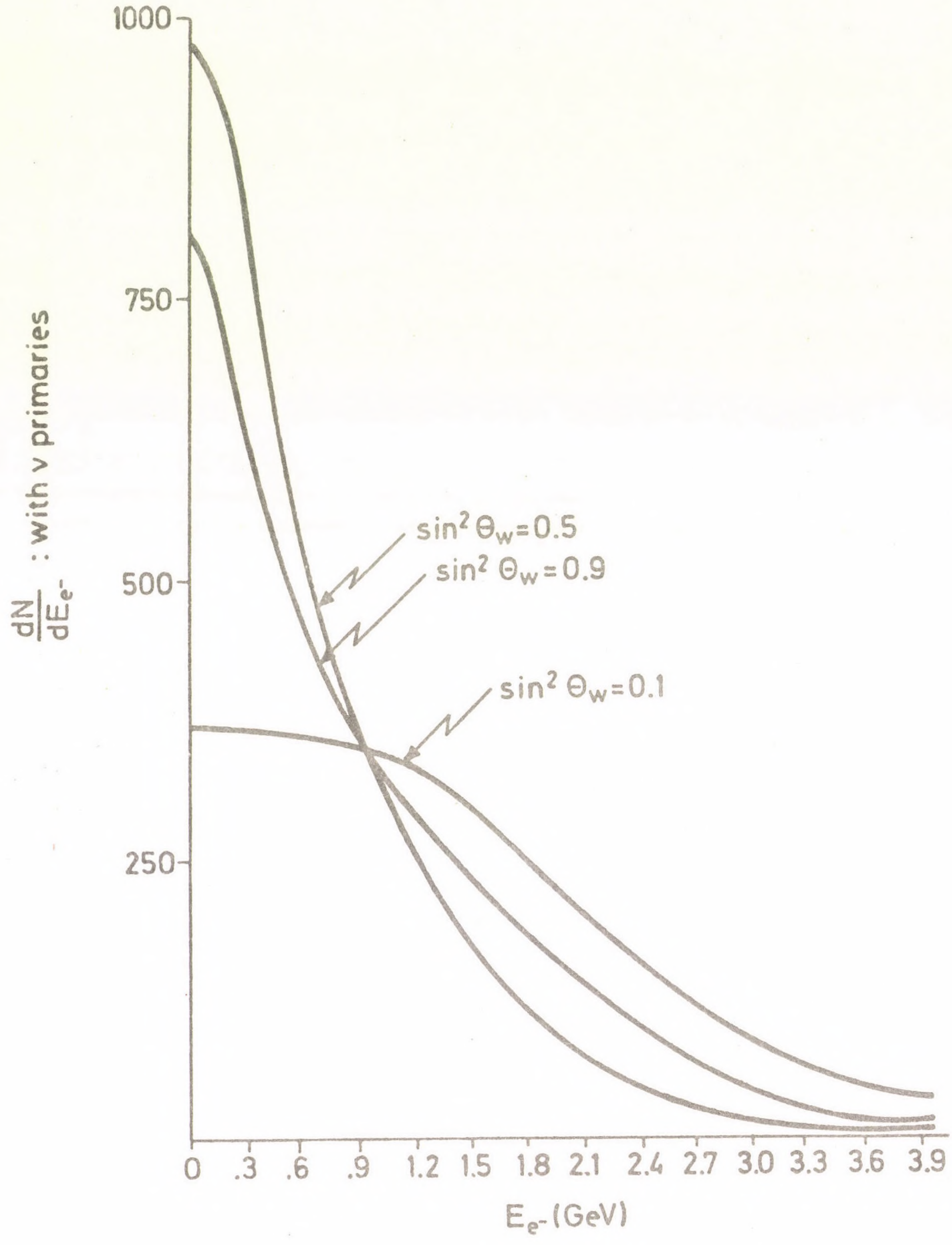

1000
750
500
250
$\frac{dN}{dE_{e^-}}$: with ν primaries
$\sin^2\Theta_W=0.5$
$\sin^2\Theta_W=0.9$
$\sin^2\Theta_W=0.1$
0 .3 .6 .9 1.2 1.5 1.8 2.1 2.4 2.7 3.0 3.3 3.6 3.9
E_{e^-} (GeV)

Figure 5

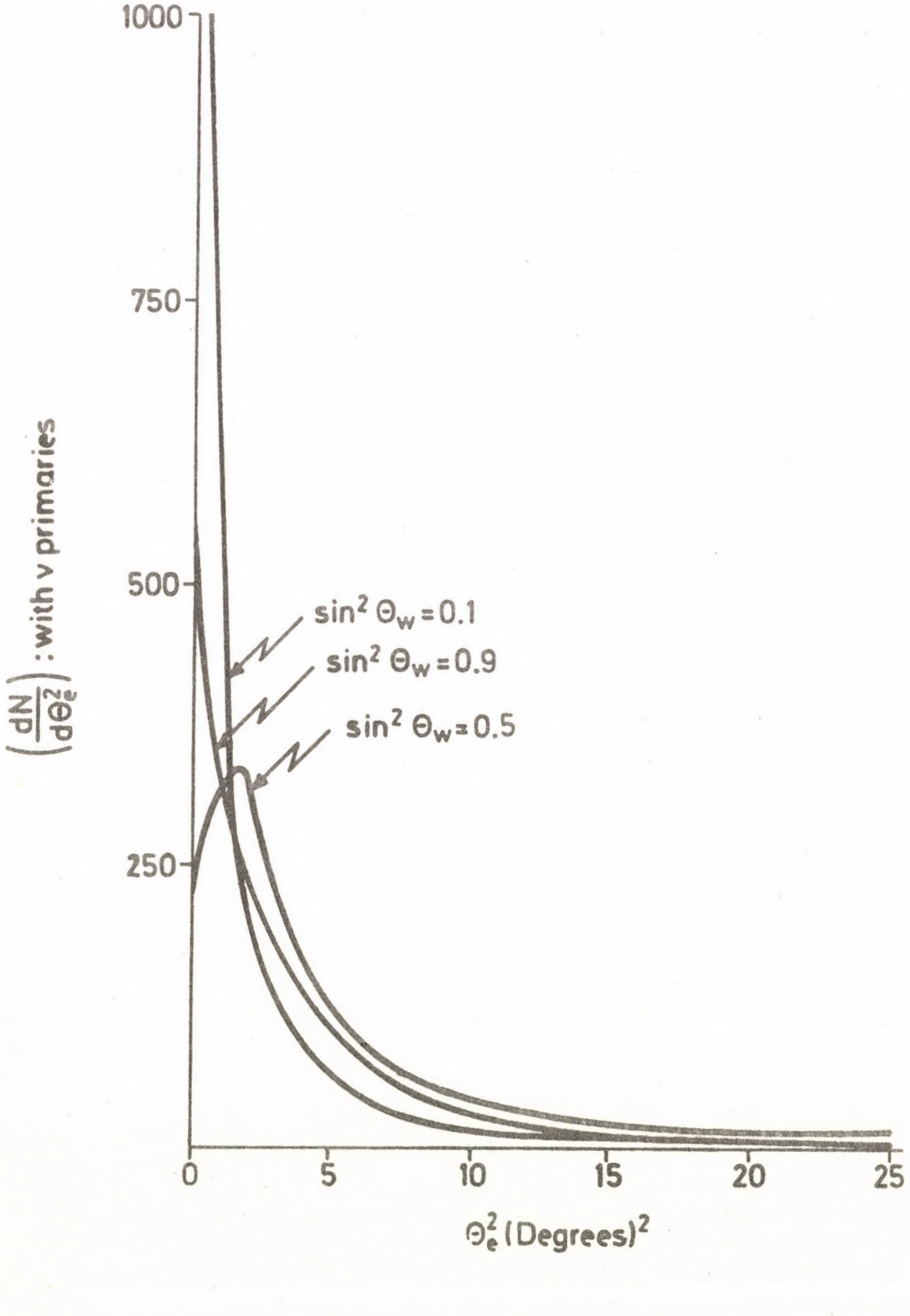
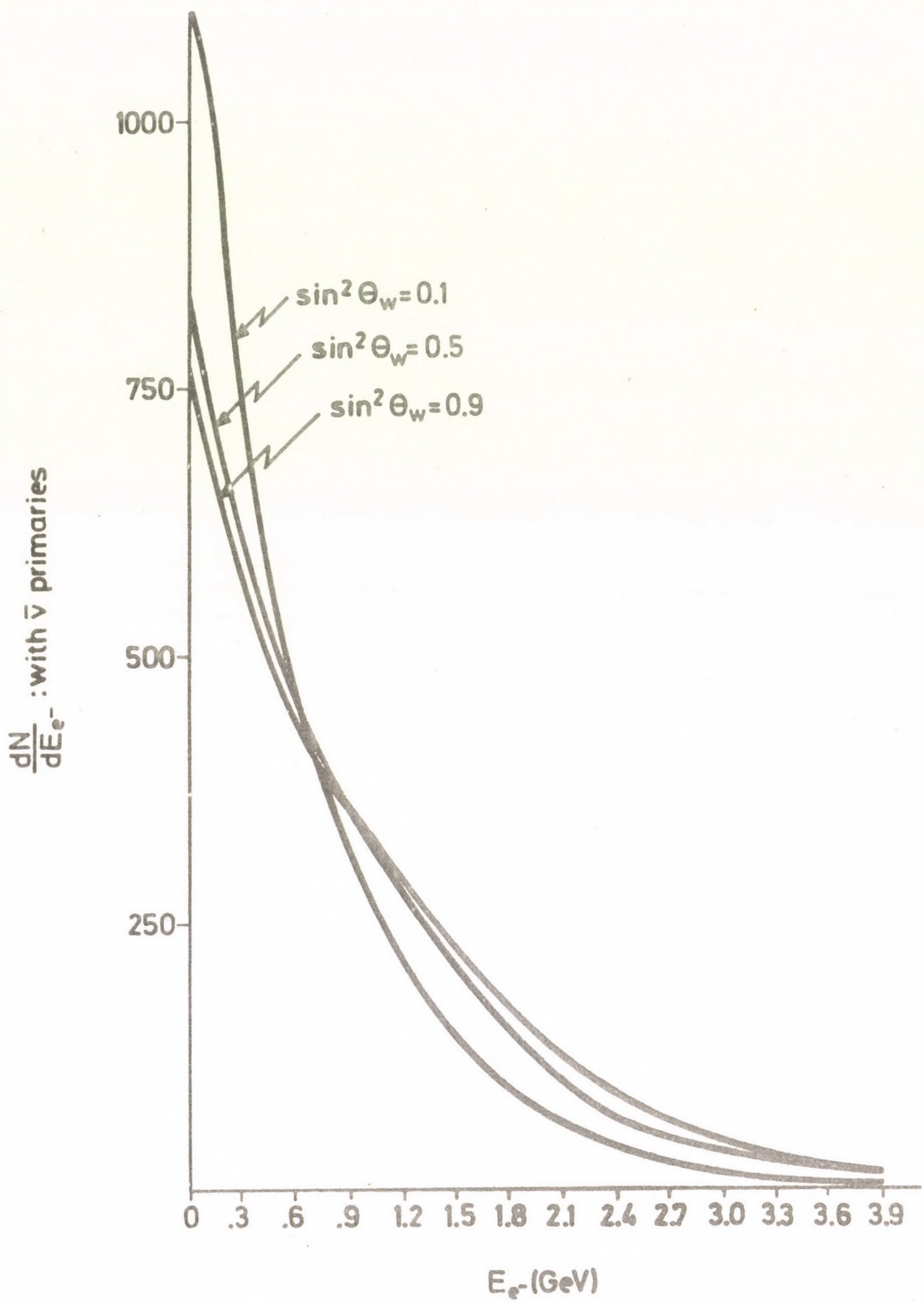

1000
750
500
250
0
5
10
15
20
25
$\left(\frac{dN}{d\Theta_e^2}\right)$: with ν primaries
$\sin^2\Theta_W = 0.1$
$\sin^2\Theta_W = 0.9$
$\sin^2\Theta_W = 0.5$
Θ_e^2 (Degrees)^2

Figure 6



1000
750
500
250
$\frac{dN}{dE_{e^-}}$: with $\bar{\nu}$ primaries
$\sin^2\Theta_w = 0.1$
$\sin^2\Theta_w = 0.5$
$\sin^2\Theta_w = 0.9$
0 .3 .6 .9 1.2 1.5 1.8 2.1 2.4 2.7 3.0 3.3 3.6 3.9
E_{e^-}-(GeV)

Figure 7

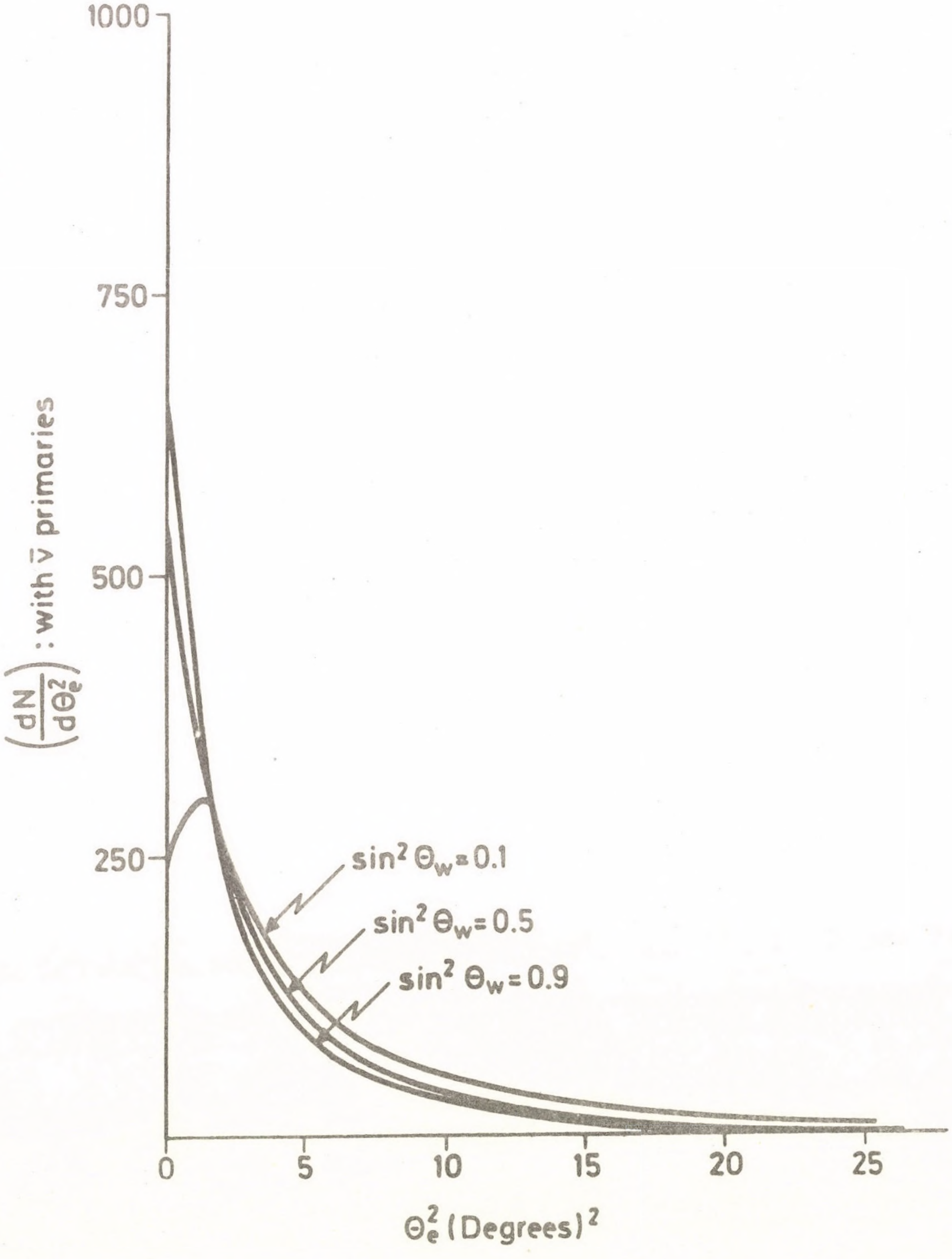

1000
750
500
250
$\left(\frac{dN}{d\Theta_e^2}\right)$: with $\bar{\nu}$ primaries
$\sin^2\Theta_w = 0.1$
$\sin^2\Theta_w = 0.5$
$\sin^2\Theta_w = 0.9$
0
5
10
15
20
25
Θ_e^2 (Degrees)^2

Figure 8

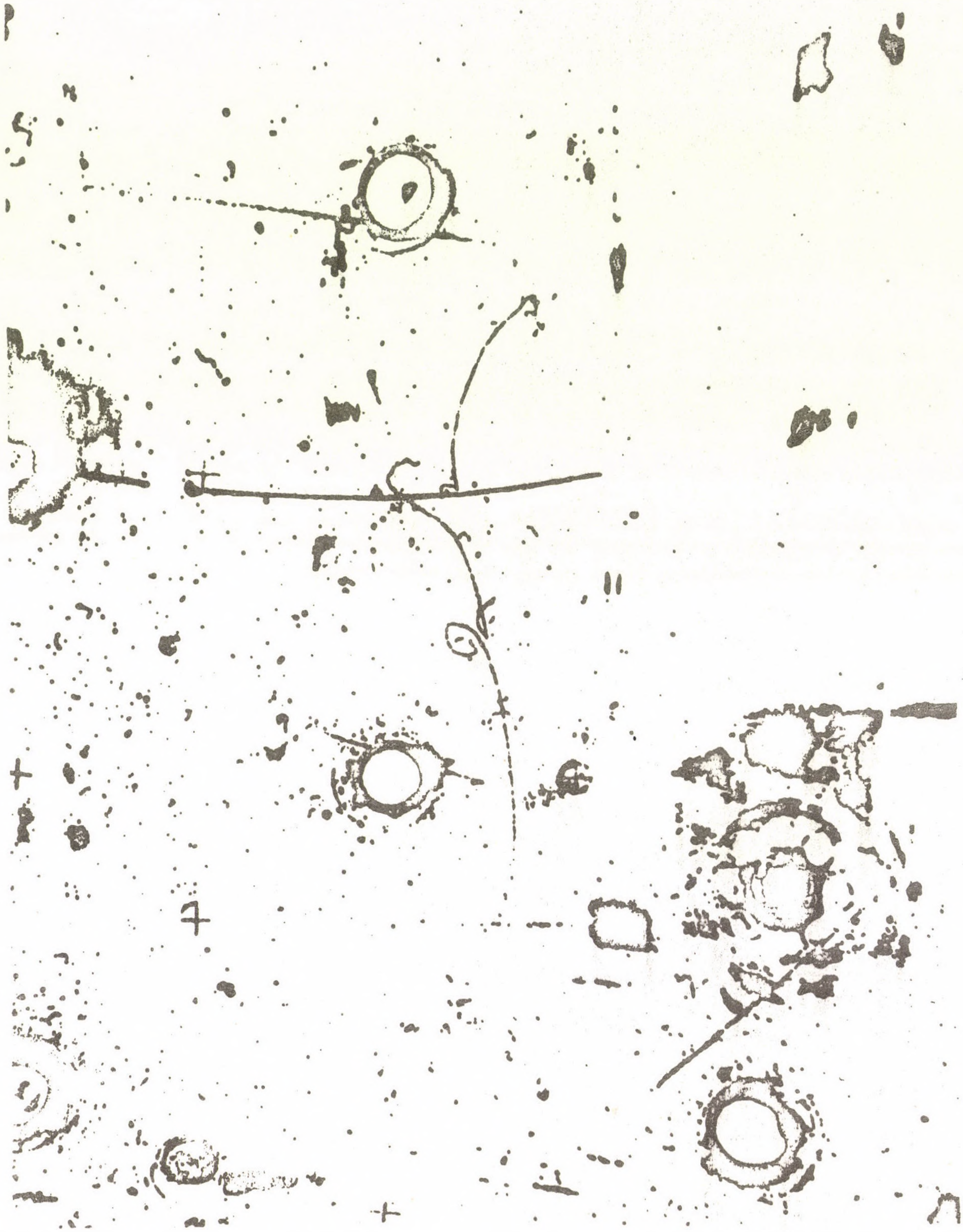
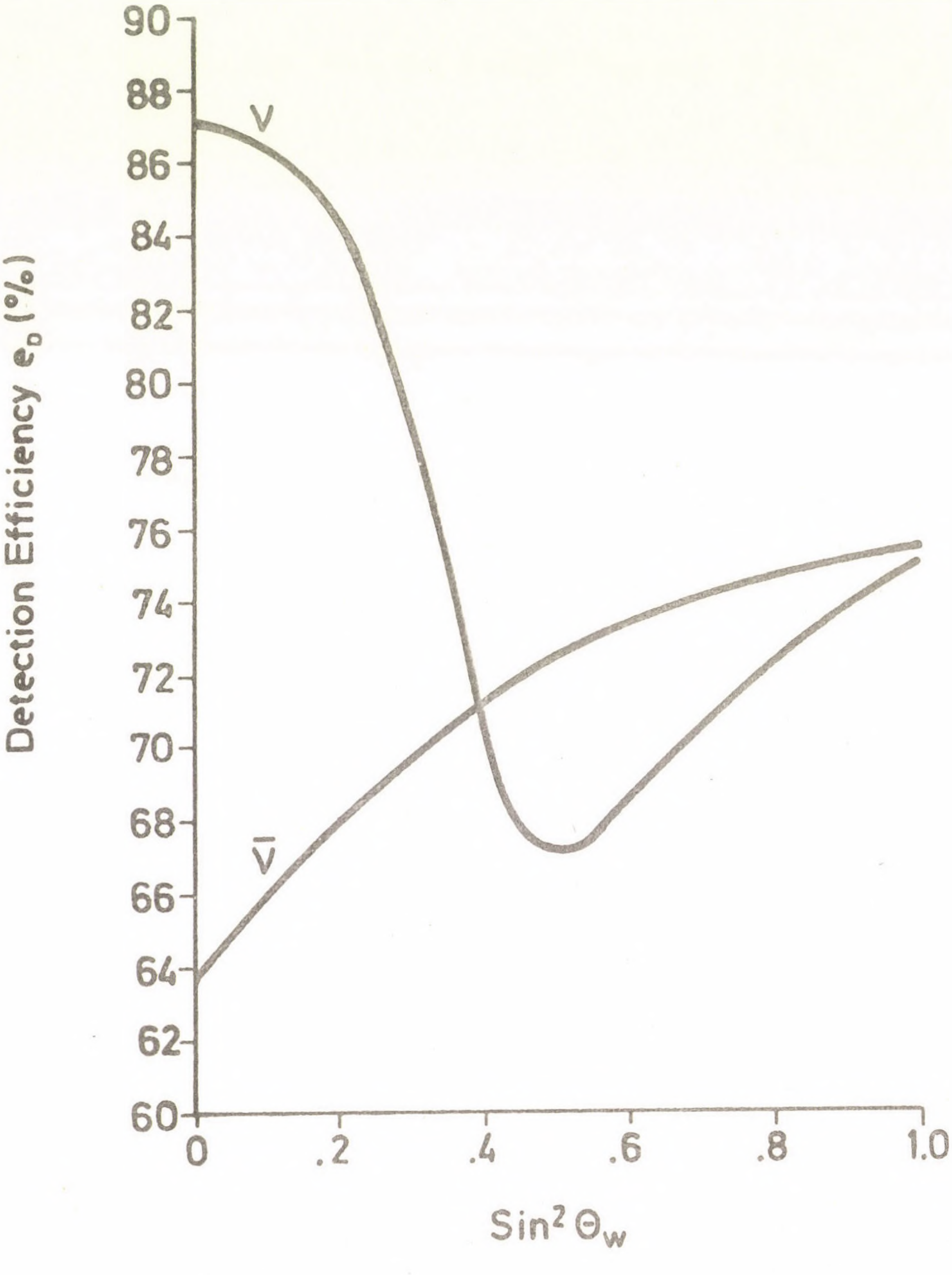


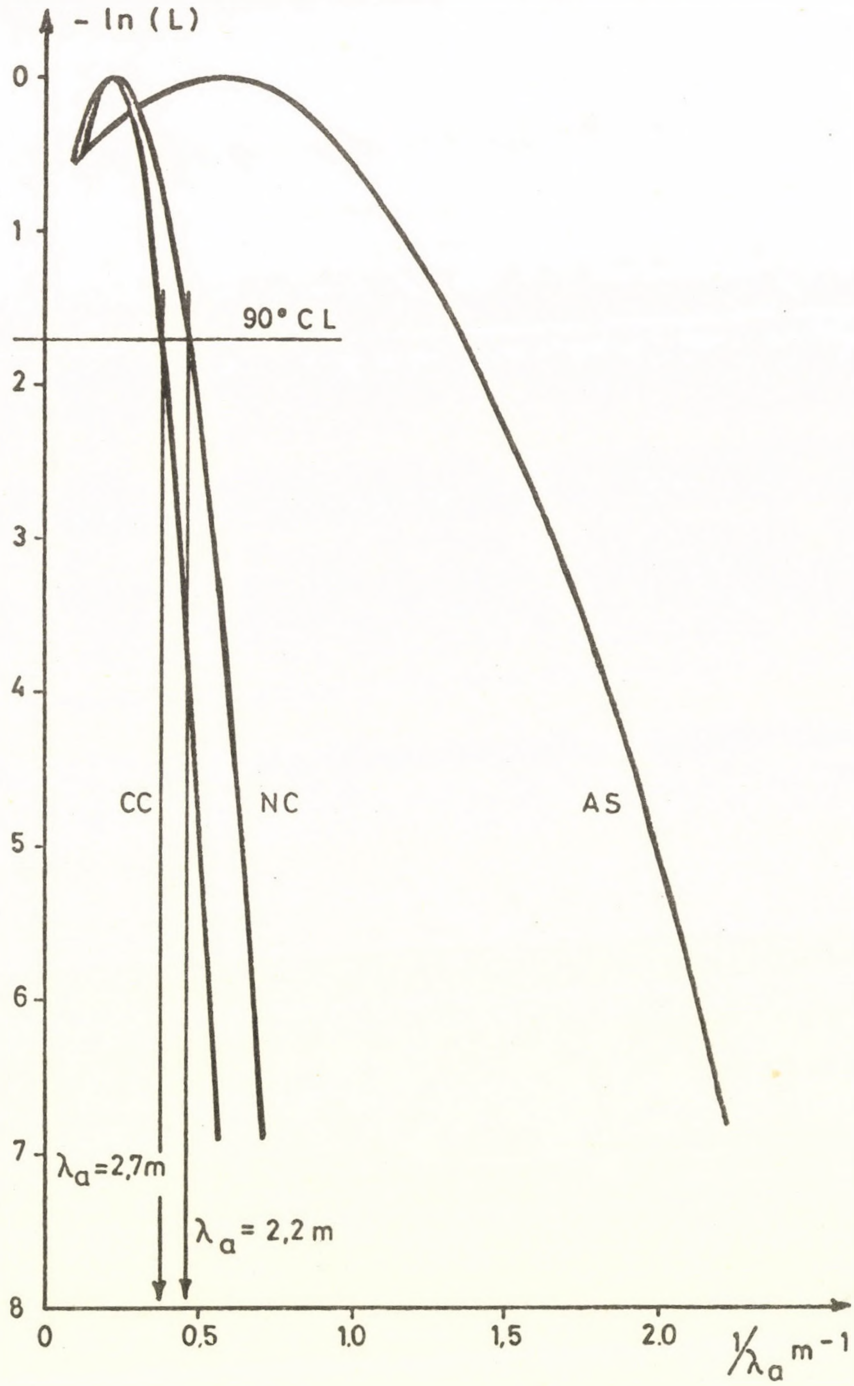
Figure 8a

Figure 9



90
88
86
84
82
80
78
76
74
72
70
68
66
64
62
60
Detection Efficiency e_D (%)
ν
$\bar{\nu}$
0
.2
.4
.6
.8
1.0
$\sin^2\Theta_W$

Figure 10



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LEPTONS BOUND STATES AND GAUGE MODELS

P. FURLAN

Instituto di Fisica Teorica, Universitá degli Studi, Trieste

Introduction

This brief talk is a summary of the work done during these two last years by P. Budini and myself about the following subject: is it possible that the gauge fields are not elementary but composite ones? and is it possible that their constituents are the same fermions to which they couple?

Since the complexity of the physical world decreases in the same way as the strength of the interactions involved in it, we began our analysis starting from the leptonic world. The justification for this choice comes not only from a simplicity reason, but essentially from our conviction that the fundamental fields of the physical world are simply two-component spinor fields.

The particles that most resemble the quanta of these fields are leptons and, between them, massless neutrinos. These are certainly abundant in the universe, but they appear to have a fundamental role only in particular phenomenon like radioactive decays and perhaps some peculiar astrophysical transitions.

The question then arises: could it be that, despite their apparent weak interaction at low energy, they have nevertheless a fundamental role in determining the forces and quanta which generate the variety of material phenomena we have learned to know?

To answer this question, we started a phenomenological study about the possible consequences one could draw from the experimentally well-estabilished Fermi lepton weak-interaction Lagrangian, which is the only Lagrangian containing exclusively two-component lefthanded lepton fields.

$$L_I(x) = - 2\sqrt{2}\, G_F j_\lambda(x) j^{\lambda +}(x) \qquad (1.)$$

where

$$j_\lambda(x) = \sum_{i,j} :\bar{L}^{(i)}(x)\gamma_\lambda L^{(j)}(x): \qquad (2.)$$

and

$$L^{(i)}(x) = \tfrac{1}{2}(1+\gamma_5)\psi^{(i)}(x) \qquad (3.)$$

$\psi^{(j)}(x)$ represents the Dirac spinor fields of lepton j.

Starting from this Lagrangian, the first question we asked was: which are the forces that this Lagrangian implies between leptons? Obviously they can only arise because of lepton pair exchanges; and of the leptons the massless neutrinos give the forces of the longest range, such that they will be the most effective in giving consequence, if any. These forces have been studied. The most interesting feature is that while being repulsive between two leptons they are attractive between lepton-antilepton.

Their long-range behaviour is well known ($\sim x^{-6}$) and computable from the local weak Lagrangian and is not dependent on the possible refinement introduced in this Lagrangian to make the the theory finite or renormalizable. This will instead determine the behaviour of the Green function at short distances. With the faith that the weak Lagrangian can be in fact made renormalizable, we will suppose that the short-distance behaviour of the Green function is of the Coulomb type ($\sim x^{-2}$)(if the theory is super-renormalizable it will behave as x^{-n} with $n<2$ or as a power of $\log x^2$).

The next question has been: are these forces sufficient to give composite systems built up of leptons-anti-leptons, and if so, could these composite systems have something to do with what we call the elementary particles?

To examine if (1.) may give rise to lepton composite states we have used the Bethe-Salpeter equation:

$$\begin{aligned}\Gamma(X,x) &\equiv (\overrightarrow{\hat{p}}+\tfrac{1}{2}\overrightarrow{\hat{P}}-m)\chi_P(x)(\tfrac{1}{2}\overleftarrow{\hat{P}}-\overleftarrow{\hat{p}}+m) = \\ &= -\frac{Gp^2}{2}\gamma^\rho(1+\gamma_5)\langle 0|T[j_\rho(x_1)\times \\ &\quad\times \psi(x_1)\bar{\psi}(x_2)j_\sigma(x_2)]|P\rangle\gamma^\sigma(1+\gamma^5)\end{aligned} \qquad (4.)$$

with

$$X = \frac{x_1+x_2}{2}, \quad x = x_1-x_2, \quad P = p_1+p_2, \quad p = \frac{p_1-p_2}{2}$$

and

$$\hat{p} = i\gamma^\mu\frac{\partial}{\partial x^\mu}, \quad \hat{P} = i\gamma^\mu\frac{\partial}{\partial X^\mu}$$

and we defined:

$$\chi_P(x_1,x_2) = <0|T[\psi(x_1)\overline{\psi}(x_2)]|P> = e^{-iP\cdot X}\chi_P(x) \qquad (5.)$$

as the bound state wave function.

It can be easily shown from eq.(4.) that in the Dirac space $\Gamma(X,x)$ has the form

$$\Gamma(X,x) = \gamma^\mu(1+\gamma_5)\; B_\mu(X,x) \qquad (6.)$$

where $B_\mu(X,x)$ multiplies the unit matrix. To attempt an actual solution of eq. (4.) we have to go in the ladder approximation:

$$<0|T[j_\rho(x_1)\psi(x_1)\overline{\psi}(x_2)j_\sigma(x_2)]|P> \simeq \Pi_{\rho\sigma}(x)\chi_P(x_1,x_2) \qquad (7.)$$

where

$$\Pi_{\rho\sigma}(x) = <0|T[j_\rho(x)j_\sigma(0)]|0> \qquad (8.)$$

represents the Green function of the force due to the exchange of lepton loops between the lepton L and $\overline{L}$. The force will be long range for massless leptons, then in this first phenomenological approach we will take for $j_\rho(x)$ the neutrino current.

In space time $\Pi_{\rho\sigma}(x)$ has the form, valid for $x^2\neq 0$:

$$\Pi_{\rho\sigma}(x) = -\frac{2}{\pi^4}\,\frac{2x_\rho x_\sigma - g_{\rho\sigma}x^2}{(x^2-i\varepsilon)^4} \qquad (9.)$$

and gives to the lepton anti-lepton neutrino forces a behaviour of the type $\sim x^{-6}$ for $x^2\to\infty$. This behaviour of the Green function will not depend on the possible change brought to the Lagrangian (1.) in order to make it renormalizable or simply to regularize it, provided (1.) represents correctly the low energy limit of the lepton weak interaction. This change of the Lagrangian (1.) will instead influence the behaviour of $\Pi_{\rho\sigma}(x)$ at $x^2\sim 0$ where $\Pi_{\rho\sigma}(x)$ is simply not defined. One could certainly compute $\Pi_{\rho\sigma}(x)$ for $x^2\sim 0$ using for example an intermediate vector meson model of weak interactions and possibly a renormalizable form of it. For our present tentative approach we will not undergo such a burdensome and somehow not very significant work and will suppose instead that, for $x^2\sim 0$

$$\Pi_{\rho\sigma}(x) \underset{x\simeq 0}{\simeq} g_{\rho\sigma}\frac{M^4}{x^2} \qquad (10.)$$

where M is a parameter having the dimension of l^{-1} for a renormalizable model and

$$\Pi_{\rho\sigma}(x) \underset{x\simeq 0}{\simeq} g_{\rho\sigma}\frac{M^{6-n}}{x^n} \quad \text{with } n<2 \tag{11.}$$

or

$$\Pi_{\rho\sigma}(x) \underset{x\simeq 0}{\simeq} g_{\rho\sigma}M^6 \ln(Mx) \tag{11.'}$$

for a super-renormalizable theory.

For a renormalizable model we have taken the spectral representation

$$\Pi(x^2;M) = i\int_0^\infty d\mu g(\mu,M)\Delta_f(x^2,\mu^2) \tag{12.}$$

where

$$\Delta_f(x^2,\mu^2) = \frac{i\mu}{4\pi^2}\frac{K_1(\mu\sqrt{-x^2})}{\sqrt{-x^2}} \tag{13.}$$

which guarantees the behaviour (10.) at $x^2<M^{-2}$, and have chosen the spectral function $g(\mu,M)$ such that it represents correctly the behaviour $\sim x^{-6}$ at $x^2\to\infty$. In order to obtain super-renormalizable behaviour (11.) one can simply introduce derivatives of Δ_f with respect to μ^2 in (5.).

In the computations in which the results will be summarized below we have chosen:

$$g(\mu,M) \equiv \frac{1}{(2\pi)^2}\mu^5 e^{-\mu/M}\left(-\frac{\partial}{\partial\mu^2}\right) \tag{14.}$$

It can be seen that the parameter M^{-1} represents the length at which the Green function deviates from the long distance behavoiur $\sim x^{-6}$ to go to the less singular behaviour. In an intermediate boson theory M would represent the mass of the intermediate boson.

Solutions of the B.S. equation

Since we want to study the possible existence of massless bound states, we start from the B.S. equation (4.) in ladder approximation (9.), with $P_\mu \equiv 0$ i.e., after Wick rotation has been performed

$$(\gamma\cdot p+im)\chi^{(r)}(p)(\gamma\cdot p+im) = i\frac{G_F{}^2}{2}\int d^4q\tilde{\Pi}[(p-q)^2;M]\times \\ \times\ \gamma_\rho(1+\gamma_5)\chi^{(r)}(q)\gamma_\rho(1+\gamma_5) \tag{15.}$$

or for the vertex (4.)

$$\Gamma^{(r)}(p) = i\frac{G_F{}^2}{2}\int d^4q\ \frac{\tilde{\Pi}[(p-q)^2;M]}{(q^2+m^2)^2}\ \gamma_\rho(1+\gamma_5)(\gamma\cdot q-im)\times \\ \times\ \Gamma^{(r)}(q)(\gamma\cdot q-im)\gamma_\rho(1+\gamma_5) \tag{16.}$$

where $\tilde{\Pi}(k^2;M)$ is the Fourier transform of (12.) with the choice (14.), and r denotes a collection of labels (spin, helicity, parity, ...) apt to distinguish the various degenerate bound-states having $P_\mu=0$.

The set $\{\chi^{(r)}(p)\}$ must form the basis for a reducible representation of $O(4)^{(\Pi C)}$, since O(4) is the little group belonging to $P_\mu=0$, and Eq.(15.) is invariant also under (ΠC) (Π=three-space reflection, C=charge conjugation); therefore we want to expand the B.S. amplitudes $\chi^{(r)}(p)$ in a base apt to split their various irreducible components. But we know from eq. (6.) that the vertex has a simpler structure in Dirac space; therefore, we prefer to analyse it instead of the amplitude.

To this aim we introduce to following four-vector hyper-spherical harmonics

$$Y^{2j^+2j^-}_{j\ j_3;n}(\Omega_p) \equiv \sum_{\substack{s,s_3\\ \ell,m}} \begin{bmatrix} \frac{1}{2}\frac{1}{2} & \frac{n}{2}\ \frac{n}{2} & j^+\ j^- \\ s s_3 ; & \ell\ \ m & j\ \ j_3 \end{bmatrix} Y^n_{\ell m}(\Omega_p)\mathbf{e}_{ss_3} \tag{17.}$$

where

$\{\mathbf{e}_{ss_3}\}$, $s=0,1$ $-s\le s_3\le s$ (the fat letter denotes a fourvector) are four orthonormal fourvectors belonging to the representation $(\frac{1}{2},\frac{1}{2})$ of O(4). $Y^{n}_{\ell m}(\Omega_p)$ are the scalar hyperspherical harmonics and the square bracket is an O(4) Clebsh-Gordon coefficient explicitly given by

$$\begin{bmatrix} j_1^+ j_1^-, & j_2^+ j_2^- & j^+ j^- \\ \ell_1 m_1 & \ell_2 m_2 & j \ j_3 \end{bmatrix} = (\ell_1 m_1 ; \ell_2 m_2 | j j_3) \times \times \sqrt{(2\ell_1+1)(2\ell_2+1)(2j^+ +1)(2j^- +1)} \begin{Bmatrix} j_1^+ j_1^- \ell_1 \\ j_2^+ j_2^- \ell_2 \\ j^+ j^- j \end{Bmatrix} \qquad (18.)$$

In relation (17.) (j^+,j^-) define the particular O(4) representation to which $Y^{2j^+2j^-}_{j\ \ j_3;n}$ belongs, n is a degeneracy quantum number, apt to distinguish the various equivalent representations having the same (j^+,j^-), j gives the total angular momentum content of the particular harmonic and j_3 is its projection.

It is easy to see that the harmonics (17.) form the basis for three irreducible inequivalent representations of $O(4)^{(\Pi C)}$ (for fixed n).

Once the base vectors are fixed, it is possible to expand the vertex (4.)(written in momentum space for $P_\mu=0$), in such a way that eq. (16.) decouples into <u>three</u> systems of coupled integral equations, each system corresponding to one irreducible representation of $O(4)^{(\Pi C)}$ (for every fixed n). These equations have been solved using a method based on the spectral representation and solving the integral equations in the first Fredholm approximation of the vertex. The condition of existence of massless bound states establishes for every n a connection (quantum condition) between the Fermi coupling constant G_F and the free parameter M which, in the kernel $\Pi(x^2;M)$establishes the distance at which the Green function (8.) deviates from its long range behaviour.

Therefore the kernel $\tilde{\Pi}[(p-q)^2;M]$ turns out to be n-dependent through M, since for every irreducible representation of $O(4)^{(\Pi C)}$it is possible to establish the relations

$$M^{(v)}(n) = 2\pi\left(\frac{n(n+1)}{6G_F^2}\right)^{\frac{1}{4}}$$
$$M^{(e)(b)}(n) = 2\pi\left(\frac{n+1}{6G_F^2}\right)^{\frac{1}{4}} \qquad (19.)$$

Every solution of eq.(16.) will then belong to a single quantum number n, characterizing, together with j and j_3, the particular bound state. This means that it is possible to have the following three sets of vertices:

$$\Gamma(p)^{(v)}_{njj_3} = \gamma_\mu(1+\gamma_5)[G^{(-)}_n(|p|)Y^{nn}_{jj_3;n-1;\mu}(\Omega_p)+G^{(+)}_n(|p|)Y^{nn}_{jj_3;n+1;\mu}(\Omega_p)]$$

$$\Gamma(p)^{(e)}_{njj_3} = \gamma_\mu(1+\gamma_5)G^{(e)}_n(|p|)\frac{1}{\sqrt{2}}[Y^{n+1\,n-1}_{j\;j_3;n;\mu}(\Omega_p)-Y^{n-1\,n+1}_{j\;j_3;n;\mu}(\Omega_p)]$$

$$\Gamma(p)^{(b)}_{njj_3} = \gamma_\mu(1+\gamma_5)G^{(b)}_n(|p|)\frac{1}{\sqrt{2}}[Y^{n+1\,n-1}_{j\;j_3;n;\mu}(\Omega_p)+Y^{n-1\,n+1}_{j\;j_3;n;\mu}(\Omega_p)] \quad (20.)$$

Since the B.S. equation (15.) (or (16.)) is a homogeneous integral equation, it gives the hyperradial functions $G^{(b)}_n(|p|)$ except for a normalization factor. To determine it, the normalization condition must be imposed. Because of this condition it turns out that the only acceptable solution for <u>sector (v)</u> is <u>n=1</u>, <u>j=1</u>.

Concerning the other two sectors, the only positive norm, normalizable solutions correspond to the choices

(a) n = 2, j = 1

(b) $\begin{cases} n = 2 \;;\; j = 2 \\ n = 1 \;;\; j = 1 \end{cases}$

To decide if these solutions have a physical meaning, it remains to evaluate the renormalized effective coupling constans corresponding to the vertices (20.). We will define as renormalized effective coupling constant $g^{(k)}(n;j)$ the value of the vertex (20.) on the mass shell of the external leptons, that is for $p^2=-m^2$. It turns out that (k=v,e,b)

$$g^{(k)}(n;j) = k(n)\frac{(mG_F^{\frac{1}{2}})^{n-1}}{I^{(k)}_n(mG_F^{\frac{1}{2}})} \quad (21.)$$

where $k(n)$ is some numerical constant depending only on n and $I_n^{(k)}(mG_F^{\frac{1}{2}})$ takes on the form

$$I_n^{(v)}(mG_F^{\frac{1}{2}}) = \begin{cases} A & n = 1 \\ \dfrac{1}{mG_F^{\frac{1}{2}}}\sqrt{B-\ln(mG_F^{\frac{1}{2}})} & n = 2 \\ \dfrac{C}{mG_F^{\frac{1}{2}}} & n \geq 3 \end{cases}$$

$$I_n^{(e)(b)}(mG_F^{\frac{1}{2}}) = \begin{cases} \sqrt{D-\ln(mG_F^{\frac{1}{2}})} & n = 1 \\ E & n \geq 2 \end{cases} \qquad (22.)$$

where A, B, C, D, D, E are finite or infinite (in the non-normalizable cases) constants independent of m. From relations (21.) and (22.) it appears that, due to the smallness of the quantity $mG_F^{\frac{1}{2}}(\sim 10^{-6})$, the only physically acceptable solutions correspond to the cases

$$(v),(b): \; n=1, \; j=1; \; \Pi C=+1$$

From relations (21.)and (22.) it turns out also that $g^{(v)}(1;1)$ is <u>simply a number</u>, while $g^{(b)}(1;1)$ is logarithmically dependent on the mass of the constituent leptons. Explicity ($g^{(b)}(1;1)$ evaluated for the electron)

$$|g^{(v)}(1;1)| = 2\pi\sqrt{3}[1+\frac{m^2}{M^2}\ln(\frac{m}{M})+0(\frac{m^2}{M^2})] \approx 10.9[1+0\left(\frac{m^2}{M^2}\ln(\frac{m}{M})\right)]$$

$$|g^{(b)}(1;1)| = \frac{\pi}{\sqrt{C-\ln(\frac{m}{M})}}[1+0\left(\frac{m^2}{M^2}\ln(\frac{m}{M})\right)] \approx 0.70[1+0\left(\frac{m^2}{M^2}\ln(\frac{m}{M})\right)] \qquad (23.)$$

And taking the limit of $p^2=-m^2$ for the physically meaningful vertices of (20.) (i.e. cases (v) and (b) with $j=n=1$, $\Pi C+1$), one obtains

$$\Gamma^{(v)}_{11j_3}(p) \underset{p^2 \simeq -m^2}{\simeq} \frac{g^{(v)}_{(1;1)}}{(2\pi)^4}\left\{[1+O(\frac{p^2+m^2}{m^2})]\delta_{\mu\nu}-[O(\frac{p^2+m^2}{m^2})](\delta_{\mu\nu}-\frac{4p_\mu p_\nu}{p^2})\right\} \times$$

$$\times \quad \varepsilon^{(j_3)}_\nu \gamma_\mu (1+\gamma_5) \qquad (24.)$$

$$\Gamma^{(b)}_{11j_3}(p) \underset{p^2 \simeq -m^2}{\simeq} \frac{g^{(b)}_{(1;1)}}{(2\pi)^4}[1+O\Big(G_F(p^2+m^2)\Big)]\frac{1}{2\pi}\vec{\varepsilon}^{(j_3)}\frac{\vec{p}}{|p|}\wedge\vec{\gamma}(1+\gamma_5)$$

As we will see, in order to get a local vertex connecting the free states with the composite vector-axial state (what we will call the local limit) one has to integrate in the relative coordinates of composite state. Then in the local limit of the two, only $\Gamma^{(v)}_{(p)}$ will survive.

We see that the coupling constant is nearer to the strong coupling than the electromagnetic one which, apart from the limits of validity of our approximations, means that if this is the origin of the electromagnetic vertex one cannot neglect the internal symmetry of the leptons, in analogy to what happens in the gauge models of weak and electromagnetic interactions where the coupling with the non-Abelian gauge fields is stronger than the electromagnetic coupling.

Before closing this part we wish to point out that for $\Gamma^{(v)}(p)$ the limit $m \to 0$ exists and is finite. This is somewhat surprising since one would rather expect an infrared divergence.

One could think that this result depends on the approximation adopted. Otherwise, if we perform the same calculation with the same approximation for a vertex when the chiral project $\frac{1}{2}(1+\gamma_5)$ is eliminated, we obtain in fact the expected infrared logarithmic divergence.

This could mean that the elimination of the infrared divergence results from the coherent use of two-component spinor fields.

Gauge Invariance

We have imposed zero mass for the bound state and the only reason was that of the simplicity of the integral equations. But if we could insert the field B_μ in a gauge invariance field theory then masslessness would be dictated by gauge invariance. Being now the field B_μ composite we should have that its gauge transformation should follow from that of the field $L(X)$.

That is from:

$$L(X) \to L'(X) = e^{i\varepsilon(X)} L(X) \qquad (25.)$$

$$B_\mu(X) \to B'_\mu(X) = B_\mu(X) - \frac{1}{g}\frac{\partial\varepsilon(X)}{\partial X^\mu} \qquad (26.)$$

should follow.

In order to show the way this could happen we have first to try to connect the phenomenological calculation of the previous paragrapha with a field theoretical model; i.e. to give a Lagrangian formalism from which equations of motions for the fields $L(X)$ and $B_\mu(X)$ in interaction could be deduced.

A possible way with simple physical meaning is to start from the Lagrangian.

$$L = L_o + L_I = \frac{i}{2}\bar{L}(X\overleftrightarrow{\partial}_X L(X) - G_F \int \bar{L}(X)\gamma^\mu L(X_1)\bar{L}(X_2)\gamma^\rho L(X_3) \times$$
$$\times F_{\mu\rho}(X-X_1, X-X_2, X-X_3)\, d^4X_1 d^4X_2 d^4X_3 \qquad (27.)$$

with the condition

$$\int F_{\mu\rho}(X-X_1, X-X_2, X-X_3)\, d^4X_1 d^4X_2 d^4X_3 = 2\sqrt{2}g_{\mu\rho} \qquad (28.)$$

which guarantees with low energy local behaviour. The Lagrangian has not to be interprented as a non-local Lagrangian but rather as an effective one. The function $F_{\mu\rho}$ represents the summation over all possible internal virtual lines connecting the free fields $L(X)$ in interaction either through the weak Lagrangian (1.) or through its renormalizable intermediate vector boson version.

From (27.) one gets the equation of motion:

$$i\hat{\partial}L(X) = G_F\int\gamma^\rho F_{\rho\sigma}(\xi,\eta,\zeta)L(X+\xi)\bar{L}(X+\eta)\gamma^\sigma L(X+\zeta)d^4\xi d^4\eta d^4\zeta \qquad (29.)$$

To connect the present approach with the previous calculation one has simply to set:

$$F_{\rho\sigma} = \delta^{(4)}(\xi)\delta^{(4)}(\xi-\eta)\Pi_{\rho\sigma}(\eta) \qquad (30.)$$

and

$$\Pi_{\rho\sigma}(\eta) = G_F g_{\rho\sigma}\,\Pi(\eta^2) \qquad (31.)$$

Should we go in second quantisation, i.e. consider the fields in (29.) as operators, then $\Pi_{\rho\sigma}$ would represent the casual Green function of neutrino pair exchange and we could further have a chronological operator T in front of the integrand. Equation (29.) then becomes:

$$i\hat{\partial}L(X) = G_F{}^2\int\Pi(\eta^2)T\gamma^\rho L(X)\bar{L}(X+\eta)\gamma_\rho L(X+\eta)d^4\eta \qquad (32.)$$

(here X represents the coordinate x_1 of paragraph 2; and $T\equiv 1$ in first quantization).

For the sake of simplicity, we will discuss gauge invariance on equation (32.) instead of eq. (29.).

Let us suppose that, near $\eta_\mu = 0$, $\Pi(\eta^2)$ behaves as supposed in paragraph 2 formula (10.)

$$\Pi(\eta^2) \underset{\eta\simeq 0}{\sim} \frac{M^4}{\eta^4} \qquad (33.)$$

Substituting this expression in (32.) we see that as far as the contribution of the singularity at $\eta_\mu \sim 0$ is concerned the equation (32.) becomes scale invariant (this can also be verified a posteriori in $M^4G_F{}^2\equiv$adimensional) and we can apply the Wilson expansion to the product of the three L fields near the point $\eta_\mu = 0$:

$$\lim_{\eta_\mu \to 0} [\psi^\alpha(X)\bar{\psi}^\beta(X+\eta)\psi^\gamma(X+\eta)] = \tag{34.}$$

$$= \lim_{\eta_\mu \to 0} [P_f : \psi^\alpha(X)\bar{\psi}^\beta(X+\eta)\psi^\gamma(X+\eta) : + iE(\eta)\frac{\gamma_\mu^{\alpha\beta}\eta^\mu}{\eta^4}\psi^\gamma(X+\eta)]$$

where $E(\eta)$ is either an arbitrary renormalization constant or an arbitrary at the most logarithmically divergent function of η for $\eta_\mu \to 0$.

Let us now perform the gauge transformation (25.) on eq. (32.).

The left hand side will become as usual:

$$i\hat{\partial}L = ie^{-i\varepsilon}\hat{\partial}L' + (\hat{\partial}\varepsilon)e^{-i\varepsilon}L' \tag{35.}$$

From

$$\psi(X) = e^{-i\varepsilon(X)}\psi'(X)$$
$$\bar{\psi}(X) = e^{i\varepsilon(X)}\bar{\psi}'(X) \tag{25'}$$

and from (34.) we will have:

$$\lim_{\eta_\mu \to 0}[\psi^\alpha(X)\bar{\psi}^\beta(X+\eta)\psi^\gamma(X+\eta)] =$$

$$= e^{-i\varepsilon(X)}\lim_{\eta_\mu \to 0}[P_f : \psi'^\alpha(X)\bar{\psi}'^\beta(X+\eta)\psi'^\gamma(X+\eta) : + iE(\eta)\frac{\gamma_\mu^{\alpha\beta}\eta^\mu}{\eta^4}\psi'^\gamma(X+\eta)] =$$

$$= \lim_{\eta_\mu \to 0}[e^{-i\varepsilon(X)}P_f : \psi'^\alpha(X)\bar{\psi}'^\beta(X+\eta)\psi'^\gamma(X+\eta) : +$$

$$+ (1 + i\frac{\partial\varepsilon}{\partial X_\rho}\eta_\rho)iE(\eta)\frac{\gamma_\mu^{\alpha\beta}\eta^\mu}{\eta^4}e^{-i\varepsilon(X+\eta)}\psi'(X+\eta)] \tag{36.}$$

In order to test if the singular term proportional to the arbitrary function $E(\eta)$ may give rise to gauge invariance we have to substitute (36.) in the right hand side of (32.). Near $\eta_\mu = 0$

we have supposed that $\Pi(\eta)$ has the form (33.) and then it is convenient to divide the domain of integration (whole space-time in the right hand side of (32.) in two parts. An infinitesimal domain σ around the origin where (33.) and (36.) are both valid and the rest of space-time V-σ and we have:

$$e^{-i\varepsilon(X)}[i\hat{\partial}L'(X)+\hat{\partial}\varepsilon(X)L'(X)] = G_F{}^2 e^{-i\varepsilon(X)}\int\Pi(\eta^2)\gamma^\rho L'(X)\bar{L}'(X+\eta)\times$$

$$\times\ \gamma_\rho L'(X+\eta)d^4\eta+\gamma_\mu\frac{\partial\varepsilon}{\partial X_\rho}\lim_{\sigma\to 0}\times \qquad (37.)$$

$$\times\int_\sigma 2G_F{}^2M^4E(\eta)e^{-i\varepsilon(X+\eta)}L'(X+\eta)\frac{\eta^\mu\eta_\rho}{\eta^6}d^4\eta$$

It is clear then that gauge invariance is satisfied if

$$\lim_{\sigma\to 0}\int_\sigma 2G_F{}^2M^4E(\eta)\frac{\eta_\mu\eta_\rho}{\eta^6}d^4\eta = g_{\mu\rho} \qquad (38.)$$

which is well possible considering that the integral is logarithmically divergent at $\eta_\mu=0$ and the domain of integration is infinitesimal. The result of the integration is $g_{\mu\rho}$ times an arbitrary constant. An appropriate choice of the arbitrary constant E may then satisfy (38.) and the gauge invariance of eq. (32.)

It has to be stressed that in order to obtain gauge invariance the constituent fields must be two components fields. Further, it is also necessary to have the form (10.) for $\Pi(\eta^2)$, i.e. to have a Green function of the renormalizable type (otherwise one could not satisfy (38.) at least if the limit $\sigma\to 0$ is used).

Now the problem is: <u>can we ascribe the compensation of the second term on the left hand side of (37.) to a gauge transformation</u> of a vector (pseudovector) boson field?

Let us suppose that because of a force (neutrino pair exchange) represented by the Green function $\Pi(\eta)$ a massless lepton anti-lepton composite state exists, as discussed in the previous paragraphs. In this case

$$T[\psi(X)\bar{\psi}(X+\eta)] = B(X,\eta) \qquad (39.)$$

represents the composite state operator and $\langle 0|B(X,\eta)|P\rangle$ its wave function, and both will go rapidly to 0 for $\eta\to\infty$. Then for these states one could write:

$$
\begin{aligned}
& G_F{}^2 \int \Pi(\eta^2) T\gamma^\rho L(X)\bar{L}(X+\eta)\gamma_\rho L(X+\eta) d^4\eta = \\
& = G_F{}^2 \int \Pi(\eta^2) T\gamma^\rho L(X)\bar{L}(X+\eta) d^4\eta \gamma_\rho L(X) + \\
& + G_F{}^2 \int \Pi(\eta^2) T\gamma^\rho L(X)\bar{L}(X+\eta)\eta_\sigma d^4\eta \gamma_\rho \frac{\partial L}{\partial X_\sigma} + \ldots \simeq \\
& \simeq G_F{}^2 \int \Pi(\eta^2) T\gamma^\rho L(X)\bar{L}(X+\eta) d^4\eta \gamma_\rho L(X).
\end{aligned} \qquad (40.)
$$

We will call the equality (40.) the local limit, and putting

$$G_F{}^2 \int \Pi(\eta^2)\gamma_\rho L(X)\bar{L}(X+\eta) d^4\eta = gB_\rho(X) \qquad (41.)$$

we shall find that in the local limit eq. (32.) becomes

$$i\hat{\partial}L(X) = g\gamma^\rho B_\rho(X)L(X) \qquad (42.)$$

and the field $B_\rho(X)$ will be proportional to the unit matrix in the Dirac space and because of (37.) and (38.), it will transform due to the gauge transformation (25.) as:

$$B_\rho(X) \rightarrow B'_\rho(X) = B_\rho(X) - \frac{1}{g}\frac{\partial\varepsilon}{\partial x^\rho} \qquad (43.)$$

That is $B_\rho(X)$ is, in the local limit, a gauge field and the equation (32.) becomes eq. (42.) and is rigorously gauge invariant.

From (40.) one can easily see that the local limit is a good limit so far the wave lengths in $L(X)$ are large with respect to the dimension of the bound state wave function which in our model is of the order of $\sqrt{G_F}$. So that for all known experimental situations it is a good limit (far lower than the present lowest limits of validity of quantum electrodynamics).

Non-Abelian gauge

Leptons can be grouped into multiplets and these multiplets will have, in our scheme, a fundamental role in determining both the parity conservation in leptons quantum electrodynamics and the actual value of the electric charge. We shall suppose that they build up the basis for a representation of an intrnal non-Abelian symmetry group for the weak Lagrangian.

Let $L(X)$ represent a lepton multiplet and let the weak Lagrangian be

$$L_I = G_F^2\int\Pi(\eta^2)\bar{L}(X)\gamma_\mu\vec{\tau}L(X)\bar{L}(X+\eta)\vec{\tau}\gamma^\mu L(X+\eta)d^4\eta \quad (44.)$$

where the components of $\vec{\tau}$, operators in the multiplet space, are the elements of the algebra of the symmetry group. This Lagrangian is invariant with respect to the non-Abelian gauge transformation:

$$L(X) = S(X)L'(X) = e^{-i\vec{\tau}\cdot\vec{\varepsilon}(x)}L'(X) \quad (45.)$$

As in the Abelian case the wave equation

$$i\hat{\partial}L(X) = G_F^2\int\vec{\tau}T\Pi(\eta^2)\gamma_\mu L(X)\bar{L}(X+\eta)\gamma^\mu\vec{\tau}L(X+\eta)d^4\eta \quad (46.)$$

is gauge invariant provided

$$\lim_{\sigma\to o}2\int_\sigma G_F^2M^4\frac{\eta_\mu\eta_\rho}{\eta^6}E\vec{\tau}\cdot\vec{\tau}S(X+\eta)L(X+\eta)d^4\eta = g_{\mu\rho}S(X)L(X) \quad (47.)$$

and the gauge invariance can be here also ascribed to a composite gauge field

$$gB_\mu(X) = G_F^2\int\Pi_{\mu\nu}(\eta)T\gamma^\nu\vec{\tau}\ L(X)\bar{L}(X+\eta)\vec{\tau}d^4\eta \quad (48.)$$

which in the local limit will interact with the lepton multiplet L according to the equation

$$i\hat{\partial}L(X) = gB_\mu(X)\gamma^\mu L(X) \quad (49.)$$

In reality the interaction is non-local, as described by (46.).

As seen above, because of the transformation (45.), the field $B_\mu(X)$ will transform according to:

$$B_\mu(X) \to B'_\mu(X) = S^{-1}(X)B_\mu(X)S(X) + i\vec{\tau}\frac{\partial\vec{\varepsilon}(x)}{\partial x^\mu} \qquad (50.)$$

and ensures gauge invariance to (49.). $B_\mu(X)$ is then an ordinary gauge field of Yang and Mills. It can be decomposed:

$$gB_\mu(X) = g_1\vec{\tau}\cdot\vec{V}_\mu(X) + g_2S_\mu(X) \qquad (51.)$$

where

$$g_1\vec{V}_\mu(X) = \int\Pi_{\mu\nu}(\eta)T\gamma^\nu Tr[L(X)\bar{L}(X+\eta)\vec{\tau}]d^4\eta \qquad (52.)$$

and

$$g_2S_\mu(X) = \int\Pi_{\mu\nu}(\eta)T\gamma^\nu Tr[\vec{\tau}L(X)\bar{L}(X+\eta)\vec{\tau}]d^4\eta \qquad (53.)$$

If we consider the Yang-Mills gauge fields as composite fields, we can construct the gauge models of weak and electromagnetic interaction, starting from a fundamental weak Lagrangian between Weyl massless spinors.

Gauge models

We shall now give an example of derivation of a model of the Salam-Weinberg type. The model fixes the massless multiplet in the lepton space: they are a left-hand massless doublet and a right-hand singlet:

$$L(X) = \frac{1+\gamma_5}{2}\begin{pmatrix}\nu(X)\\ e(X)\end{pmatrix}, \quad R(X) = \frac{1-\gamma_5}{2}e(X)$$

where $\nu(X)$ and $e(X)$ represent the neutrino and electron fields, respectively.

We postulate that the Lagrangian is given by the sum of the free Lagrangian

$$L_o = \frac{i}{2}\bar{L}\overleftrightarrow{\hat{\partial}}L + \frac{i}{2}\bar{R}\overleftrightarrow{\hat{\partial}}R \qquad (54.)$$

plus the weak effective interaction Lagrangian

$$L_I = L^{(1)} + c_2 L^{(2)} + c_3 L^{(3)} \qquad (55.)$$

where

$$L^{(1)}(X) = -2\sqrt{2}G_F\bar{L}(X)\gamma^{\mu}\vec{\tau}L(X)\bar{L}(X)\gamma_{\mu}\vec{\tau}L(X) \qquad (56.)$$

is responsible for the known standard weak interaction between electron and neutrino, while

$$L^{(2)}(X) = -G_F\bar{L}(X)\gamma^{\mu}L(X)\bar{L}(X)\gamma_{\mu}L(X) \qquad (57.)$$

and

$$L^{(3)}(X) = -G_F\bar{R}(X)\gamma^{\mu}R(X)\bar{R}(X)\gamma_{\mu}R(X) \qquad (58.)$$

contribute only to the unknown diagonal weak Lagrangian. c_1 and c_2 are arbitrary parameters. The Lagrangian L_I is invariant with respect to an SU(2)non-Abelian group times two U(1) Abelian groups: one for the left-handed $L^{(2)}$ and one for the right-handed $L^{(3)}$. In this way, the invariance group of $L_{(I)}$ is SU(2)×U(1)×U(1).

Starting from these Lagrangian and iterating them, one arrives at the following effective Lagrangians:

$$L_I^{(1)} = -G_F{}^2\int \bar{L}(X)\gamma^\mu \vec{\tau} L(X)\bar{L}(X+\eta)\gamma_\mu \vec{\tau} L(X+\eta)\Pi(\eta^2)d^4\eta$$

$$L_I^{(2)} = -\beta G_F{}^2\int \bar{L}(X)\gamma^\mu L(X)\bar{L}(X+\eta)\gamma_\mu L(X+\eta)\Pi(\eta^2)d^4\eta \qquad (59.)$$

$$L_I^{(3)} = -\gamma G_F{}^2\int \bar{R}(X)\gamma^\mu R(X)\bar{R}(X+\eta)\gamma_\mu R(X+\eta)\Pi(\eta^2)d^4\eta$$

The vertex obtained from these Lagrangians will be a sum of three vertices: one right-handed singlet and two left-handed, of which one triplet and one singlet. In the local limit we shall then have:

$$L_{eff} = g(\beta)\bar{L}\frac{\vec{\tau}}{2}\gamma^\mu L\vec{A}_\mu + g_2(\beta)\frac{1}{2}\bar{L}\gamma^\mu B_\mu^{(2)}L + g_3(\gamma)\bar{R}\gamma^\mu B_\mu^{(3)}R, \qquad (60.)$$

where

$$g(\beta)\vec{A}_\mu = (\beta-1)2G_F{}^2\int \Pi(\eta^2)\gamma_\mu \mathrm{Tr}[\vec{\tau}L(X)\bar{L}(X+\eta)]d^4\eta$$

$$g_2(\beta)B_\mu^{(2)} = (\beta+3)2G_F{}^2\int \Pi(\eta^2)\gamma_\mu \mathrm{Tr}[L(X)\bar{L}(X+\eta)]d^4\eta \qquad (61.)$$

$$g_3(\gamma)B_\mu^{(3)} = 2\gamma G_F{}^2\int \Pi(\eta^2)\gamma_\mu \mathrm{Tr}[R(X)\bar{R}(X+\eta)]d^4\eta$$

are Yang-Mills gauge fields which are subjected to gauge transformation making

$$L = L_o + L_I^{(1)} + L_I^{(2)} + L_I^{(3)} \qquad (62.)$$

$SU(2)\times U(1)\times U(1)$ gauge invariant. The parameters β, γ will depend on c_2 and c_3 and are themselves arbitrary. We can use this arbitrariness to obtain.

$$g_1(\beta) = g_2(\gamma) = g' \qquad (63.)$$

In this way we obtain the Salam-Weinberg effective Lagrangian:

$$L = L_o + L_{eff} = \frac{i}{2}\bar{L}\overleftrightarrow{\hat{\partial}}L + \frac{i}{2}R\overleftrightarrow{\hat{\partial}}R + g\bar{L}\frac{\vec{\tau}}{2}\gamma^{\mu}L\vec{A}_{\mu} + g'(\frac{1}{2}\bar{L}\gamma^{\mu}B_{\mu}L + \bar{R}\gamma^{\mu}B_{\mu}R) \qquad (64.)$$

where

$$B_{\mu}(X) = B_{\mu}^{(1)}(X) + B_{\mu}^{(2)}(X) \qquad (65.)$$

From this point one can proceed as customary in the gauge models; i.e. break the symmetry with the Higgs mechanism and give appropriate masses both to the bosons and to the leptons. The electromagnetic potential will then be given by the linear combination

$$A_{\mu}(X) = \cos\Theta A_{\mu}^{3}(X) - \sin\Theta B_{\mu}(X) \qquad (66.)$$

and provided

$$g\cos\Theta = g'\sin\Theta = e = \frac{gg'}{\sqrt{g^2+g'^2}} \qquad (67.)$$

will interact in the standard way (in the local limit) with the field $e(X)$ only

$$L_{eff\ e.m.} = e\ \bar{e}(X)\gamma^{\mu}e(X)A_{\mu}(X) \qquad (68.)$$

The limit of validity of the local quantum electrodinamics expressed by (68.) is of the order of $\sqrt{G_F}$, i.e., far beyond the present experimental limit. Also, the gauge model thus derived presents the advantage of finiteness. In fact, the known triangular diagrams are finite in the present approach (they are actually hexagonal diagrams and finite in perturbation expansion).

In principle, one could now compute the values of g following the procedure used in paragraphs 2 and 3 and then take the free parameter γ in order to make g' satisfy (67.). In such a way, one could compute the value of

$$\sin\Theta = \frac{g'}{\sqrt{g^2+g'^2}}$$

which gives measure of the neutral currens, and of the meson masses. We find

$$\sin\Theta \simeq 0.14$$

$$M_W \simeq 100 \text{ GeV}$$

$$M_Z \simeq 200 \text{ GeV}$$

One could now use this procedure to start a bootstrap mechanism and to use the gauge models to make the Lagrangian(1.) renormalizable, then fix the masses of the intermediate bosons to give massless composite gauge fields, and from these reconstruct the gauge model one has started from.

This could be applied to different models and try to get self-consistency conditions. One unpleasant feature of this procedure is the introducion of new boson fields to break the symmetry, as requested by the Higgs mechanism. In the present frame it would be much more coherent to use the original weak Lagrangian, not only to generate composite states, but also to break the symmetry in the spirit of the idea of Nambu-Jona Lasinio recently reconsidered by S. Coleman and E. Weinberg and R. Jackiw and K. Johnson.

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THE PARITY-VIOLATING ELECTROMAGNETIC FORM-FACTOR OF THE ELECTRON IN THE WEINBERG MODEL

L. SCHÜLKE, T.H. REID

Institut für Theoretische Physik II, Ruhr-Universität Bochum, 463 Bochum-Querenburg, GERMANY

This talk is just a statement of some preliminary results obtained in a calculation of the parity-violating part of electron-electron scattering calculated in fourth order in the Weinberg model. In this talk we are just interested in those contributions to electron-electron scattering in which a photon is exchanged. In other words we are calculating contributions to the parity-violating electromagnetic form-factor of the electron.

The most general form for the parity-violating electron-photon coupling is

$$T = G(q^2)\bar{u}(p_2)(2mq_\mu - q^2\gamma_\mu)\gamma_5 u(p_1) \qquad (1)$$

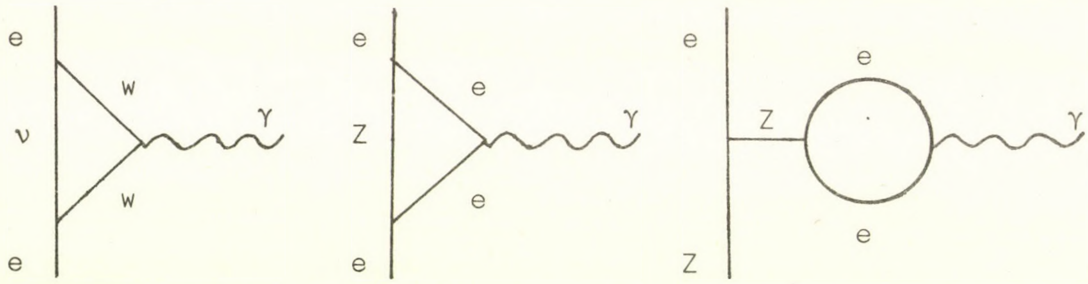
with $q=p_1-p_2$, m the electron mass.

The photon exchange terms in electron-electron scattering have the general form

$$T = G(q^2)(\bar{u}\gamma_\mu\gamma_5 u)(\bar{u}\gamma_\mu u) \qquad (2)$$

From this it is clear that it is the q^2 dependence of $G(q^2)$ that determines the range of the parity-violating electron-electron coupling.

In this talk we shall simply summarize our results for the contribution to $G(q^2)$ from the following diagrams;



(a) (b) (c)

We have calculated these diagrams using the generalized renormalizable gauge formulation of Fujikawa, Lee and Sanda [1]. Here one must calculate with propagators for the vector bosons which depend upon a parameter that determines the non-abelian gauge. As a consequence only certain groups of diagrams, and not individual diagrams, are gauge independent and thus of physical significance. However, in seeking the long-ranged contributions to $G(q^2)$, we have found a gauge-independent result.

Let us now give the results. From diagram (a) we obtain a contribution i.e. (i) gauge dependent for all q^2, (ii) short ranged, on account of the two charged bosons in the intermediate state. So this contribution is of no further interest here. It is clear that it must be taken with some other scattering diagrams (perhaps box diagrams) in electron-electron scattering in order to yield a gauge independent contribution. Moreover, the actual result is so complicated that we shall not reproduce it here.

Next, we have the diagram (b). We find that (i) $G_b(q^2)$ is gauge independent for $q^2=0$, (ii) in the approximation (a very good one) that $M_Z^2 >> q^2$, m^2; $G_b(q^2)$ is gauge independent for all q^2 (iii) $G_b(q^2)$ has a rapid variation for q^2 of the order of m^2 and therefore will lead to a long-range contribution. We have in the approximation $M_Z^2 >> q^2$, m^2

$$G_b(q^2) = \frac{eG_F}{8\sqrt{2}\pi^2}\left(\cos^2\theta-\frac{3}{4}\right)\left[\frac{2}{q}-\frac{4}{3}\ln\left(\frac{m^2}{M_Z^2}\right)+8\int_0^1 dx(x-1)x\ln\left(1-\frac{q^2}{m^2}x(1-x)\right)\right] \quad (3)$$

Expanding eq. (3) in $\frac{q^2}{4m^2}$ we obtain

$$G_b(q^2)=\frac{eG_F}{8\sqrt{2}\pi^2}\left(\cos^2\theta-\tfrac{3}{4}\right)\left[\frac{2}{q}-\frac{4}{3}\ln\left(\frac{m^2}{M_Z^2}\right)+\frac{16}{15}\left(\frac{q^2}{4m^2}\right)+\frac{16}{35}\left(\frac{q^2}{4m^2}\right)^2+\ldots\right] \quad (4)$$

Here θ is the Weinberg angle, m is the electron mass, M_Z is the Z mass and G_F the fermi constant.

This contribution is gauge-independent and long-ranged.

Finally, we find for the contribution from (c) that (i) $G_c(q^2)$ is gauge-independent for all q^2 (ii) $G_c(q^2)$ has a rapid variation for $q^2 \approx m^2$.

In fact one has the following simple relation between the contributions from diagrams (b) and (c), namely,

$$G_c(q^2) = G_b(q^2) - \frac{eG_F}{8\sqrt{2}\pi^2}(\cos^2\theta - \tfrac{3}{4})\left[\frac{2}{q} - \frac{4}{3}\ln\left(\frac{m^2}{M_Z^2}\right)\right] \tag{5}$$

Thus when we add G_b to G_c we still obtain a long-range contribution.

It is interesting to compare the contribution to parity-violating electron-electron scattering from the simple exchange of Z with that from $G_b + G_c$. One obtains

$$T_Z = iG_F\sqrt{2}(\cos^2\theta - \tfrac{3}{4})(\bar{u}\gamma_\mu u)(\bar{u}\gamma_5\gamma_\mu u)\frac{1}{1 - \frac{q^2}{M_Z^2}} \tag{6a}$$

$$T = \frac{iG_F e^2}{8\sqrt{2}\pi^2}(\cos^2\theta - \tfrac{3}{4})(\bar{u}\gamma_\mu u)(\bar{u}\gamma_5\gamma_\mu u)\frac{31}{1 - \frac{q^2}{(5\,\mathrm{MeV})^2}} \tag{6b}$$

at $q^2 = 0$ $\quad \frac{T_Z}{T} \approx 30$ but T has a range 10,000 times longer than T_Z.

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COMMENTS ON PARTIAL TRANSITIONS IN THE NORMAL NUCLEAR MUON CAPTURE

ZBIGNIEW OZIEVICZ

Institute for Theoretical Physics Wrocław University*

Contents

1. Multipole amplitudes for normal muon capture

We call the muon capture process, with no break-up of the daughter nuclear state, normal process.
This is the simplest channel, and I would like to restrict myself to this particular but important case. The normal partial lepton

* Postal adress: ul. Cybulskiego 36, 50205-Wrocław, Poland

(electron or muon) capture process could be described kinematically as the two body decay of the corresponding (mu-mesic) atom. The phenomenological description of the normal lepton capture should have an analogy with the multipole analysis of the nuclear photo-excitation and deexcitation processes. Therefore I should like to introduce the multipole amplitudes for the electron and muon captures as a generalization of the well known procedure for nuclear electromagnetic radiation.
As we see later such a parametrization is a very convenient one.

The partial wave expansion of the matrix element for the nuclear gamma emission $|I_i\mu_i> \overset{\gamma}{\to} |I_f\mu_f>$ in terms of the multipole amplitudes (i.e reduced matrix elements for the Poincaré group) $A_{L\eta}$ looks like (Frauenfelder and Steffen [1]).

$$<\vec{k}[\eta];-\vec{k}[I_f]\mu_f|S-1|[I_i]\mu_i> = \sum_L (-)^{L+M} C^{I_f M_f}_{L-M I_i \mu_i} A_{L\eta} D^{*L}_{M\eta}(\vec{k}) \qquad (1)$$

Where L is the total angular momentum (multipolarity of the emitted gamma radiation, and $\eta=\pm 1$ for right and left-polarized radiation, respectively.
The state of the lepton in the atom could be described by the total angular momentum I' and component μ' (e.g. in the 1s state $I'=\frac{1}{2}$) or alternatively by component ν of the momentum I' in the direction of the neutrino linear momentum $\vec{\nu}$. Such states[1] are connected as follows

$$|\vec{\nu},[I'],\nu> := \sum_{\mu'} D^{I'}_{\mu'\nu}(\vec{\nu})\,|[I']\mu'> \qquad (2)$$

1. All the states are supposed to be normalized in noninvariant manner "per one particle in the interaction volume". Corresponding to this normalization, the nonrelativistic phase space factor for normal partial muon capture is

$$S = \frac{d\hat{\vec{\nu}}}{4\pi}\,\frac{\nu^2}{\pi}\left(1+\frac{\nu}{E}\right)^{-1}$$

where ν and E are the total energies of neutrino and recoil nuclear state in the rest system of the mu-mesic atom.

The matrix elements for the nuclear lepton capture process $I_i \xrightarrow{e^- \text{ or } \mu^-} I_f$ from atomic shell I' with emission of a neutrino in the sharp helicity state $|\vec{\nu}[h]\rangle$ could be written by standard but tedious calculations analogouslv to (1)in the form [2]

$$\langle \vec{\nu}[h]; \quad -\vec{\nu}[I_f]\mu_f \quad |S-1|[I_i]\mu_i; \quad \vec{\nu}[I']\nu\rangle$$

neutrino	final nuclear state	initial nuclear state	lepton on shell

$$= \sum_L C^{I_f \mu_f}_{I_i \mu_i LM} \sqrt{2L+1}\, T^{\eta}_{L} D^{L}_{M\eta}(\vec{\nu}) \tag{3}$$

This partial wave expansion for electron as well as for the muon capture processes could be treated as a definition of the weak multipoles T^{η}_{L} (which are just complex numbers), where $\eta := \nu - h$.

For fixed L and the neutrino helicity h the capture process is fully described by the n independent multipole amplitudes, where

$$\begin{aligned} n &= 2I' + 1 && \text{if } L \geqslant I' + |h| \\ &= 2L + 1 && \text{if } L \leq I' - |h| \\ &= I' - |h| + L + 1 && \text{if } I' + |h| \geq L \geq I' - |h| \end{aligned}$$

and $|h| \geq \frac{3}{2}$

(neutretto)

Taking $|h| = +\frac{1}{2}$ we have e.g. for K-capture

$n = 1 \quad$ if $L = 0$

$n = 2 \quad$ if $L = 1$

<u>for L-capture:</u> $\quad n = 1 \quad$ if $L = 0$

$n = 3 \quad$ if $L = 1$

$n = 4 \quad$ if $L = 2$, etc.

It is convenient to speak about the "transverse" lepton fields in which case the interaction is described in terms of the amplitudes with $\eta \neq 0$ and the "longitudinal" lepton field when $\eta = 0$.

For K capture we define

Table 1.

	$\Delta\pi = (-)^L$	$\Delta\pi = (-)^{L+1}$	Remarks
T_L^{trans}	$\sqrt{\frac{L+1}{L}}\, M_L$	$i(-)^{\frac{1}{2}+h} \sqrt{\frac{L+1}{L}}\, E_L$	$\lvert\eta\rvert = 1$; $L \neq 1$ Magnetic and electric weak multipoles
T_L^{long}	V_L	$i(-)^{\frac{1}{2}+h}\, A_L$	$\eta = 0$ The statellite multipoles

It is easy to show that the weak E_L and M_L multipoles, when restricted to contributions coming only from polar vector nuclear current are built up from exactly the same nuclear operators as the usual electric EL and magnetic ML multipoles. Obviously T_L^{long} has no analogy in the electromagnetic radiation processes It is an easy task to calculate these amplitudes explicitly, e.g. in the standard current x current theory. Morita and Fujii's classification [3] of allowed and forbidden transitions in the muon capture follows from Table 1. and looks like those for nuclear radiation. As an illustration I give the contributions from the interested terms calculated in the impulse approximation in Table 2.. The statements in Table 2. are strictly correct up to αZ correction (the relativistic corrections to muon wave function). It should be noted that the exact values of the contributions depend on the nuclear structure assumed.

Table 2.

Multipole amplitudes	Pseudoscalar	Weak Magnetism	$\partial_\mu V^\mu$	Remarks
E_L	FORBIDDEN	LARGE	FORBIDDEN	$E_o \equiv 0$
A_L	LARGE	FORBIDDEN	FORBIDDEN	Vector Forbidden
M_L	FORBIDDEN	LARGE	SMALL	$M_o \equiv 0$
V_L	FORBIDDEN	FORBIDDEN	LARGE	Axial current Forbidden

Clearly, the multipole amplitudes (Table 1.) can and should be deduced from the experimental data as kinematically independent quantities together with the relative phase between them. In this sense these amplitudes are the only measurable quantities (by means of complete set of experimental data). Keeping this in mind it is convenient to take the full theoretical analysis on each of these amplitudes separately, just as on the measurable (not directly) quantities. Obviously, much more dynamical information is contained in the set of these amplitudes than e.g. in the capture rate alone (=definite sum).

2. What do we lose considering only the partial capture rate?

It is instructive to see explicitely why the angular distribution or polarization measurements are so important for the future developments of the theory. My aim is to show this in the simplest manner. Let us consider the muon capture by spinless nuclei $0 \xrightarrow{\mu} I \neq 0$.

Denoting

$$xe^{i\phi} := \begin{cases} \dfrac{A_I}{E_I} & \text{for} \quad I = 1^+, 2^-, 3^+ \ldots \text{(electric = unique transitions)} \\ \dfrac{V_I}{M_I} & \text{for} \quad = 1^-, 2^+, 3^- \ldots \text{(magnetic = non-unique transitions)} \end{cases}$$

and, moreover, $N := S(E_I^2 + M_I^2)$, where S is a statistical factor given earlier[1], the normal capture rate could be written as follows

$$W(0 \xrightarrow{\mu} I) = N\left(\frac{I+1}{I} + x^2\right) \qquad (4)$$

Such factorization of the capture rate is strictly correct and allowed due to fact that the second factor could be independently determined from angular distribution measurements. It is well-known that the first factor depends very strongly on the structure of the nuclear levels involved, so we will call this factor conveniently as "nuclear physics". At the same time, x^2 is only an independently measurable quantity in muon capture by spinless nuclear which (besides the phase ϕ) can give us come information on the weak form factors, etc. So we can call this second factor as "particle physics" in the muon capture. It is obvious and very important that such factorization is not an approximation and does not depend on theoretical models at all.

For the data of a given capture rate the allowed region on the "nuclear physics" and "particle physics" plane is bounded by two hyperbolas. Therefore the capture rate data alone are not so much restrictive to nuclear models. Let us look for examples. In Fig. 1. the partial muon capture data for $^{16}O(g.s) \xrightarrow{\mu} {}^{16}N(1^-)$ transition are presented. The points (a), (b) and (c) refer to M. Rho's (1967) calculations [4] in the framework of the Migdal theory: (a) means independent quasiparticle approximation and (b) and (c) correspond to different sets of the nuclear parameters (see Table III and IV in [4] for details). Obviously x^2 contains (for this particular transition) the important information on the polar vector current contribution (by the way $x^2>0$ directly proves the existence of the vector current in the nuclear muon capture). Moreover x^2 is very sensitive to CVC predictions because the divergence of the weak vector current contributes essentially to the V_3 multipoles. The need for the independent experimental determination of x^2 by means of the e.g. gamma-neutrino angular distribution [9] is evident.

3. Exotic currents (Impulse approximation)

This comment is based on calculations given by G. Hock and myself [10].

I will speak on the weak operators with

$$\Delta Q = \pm 1$$

$$|\Delta T| = 1, \underbrace{2, 3, \ldots}_{\text{exotic}}$$

In the strict impulse approximation theory such exotic currents are absent.

The experimental evidence of such exotic currents will serve equally well to demonstrate the existence of the meson and nucleon isobar terms in the <u>nuclear</u> weak-current operator (exchange currents).

So we have "nuclear physics motivation" in systematic search of the effects by exotic hadron weak currents in e.g. <u>nuclear</u> muon capture. It was Primakoff the first who pointed out the importance of these currents.

Example: Muon capture in ^{19}F [10].

$$(\tfrac{1}{2}^+,\tfrac{1}{2}) \xrightarrow{\mu} (\tfrac{1}{2}^+,\tfrac{3}{2}),\ E^* = 1.47\ \text{MeV or } 3.24\ \text{MeV} \quad \gamma \quad ;$$

$$W(\vec{k}\cdot\vec{\nu}) = 1 + B_1 a_1 (\vec{k}\cdot\vec{\nu})$$

B_1 depends on the mixing ratio of the gamma transition and could b measured independently from γ-γ transition.

Circular Polarization:

$$P = B_1 \cdot a_1(\vec{k}\cdot\vec{v})$$

(The sign of P may also give information on the exotic currents.)

Muon capture depends on three multipole amplitudes:

$$V_o,\ E_1,\ A_1\ .$$

For $\Delta T=1$ we have $V_o=0$, so all the informations on the $\Delta T=2$ are involved in the amplitude V_o. For example, for statically populated hf levels [2]

$$a_1(\text{stat}) = \frac{2E_1{}^2 - 2\sqrt{3}\ A_1 V_o}{2E_1{}^2 + A_1{}^2 + 3V_o{}^2}$$

From this formula it follows that if $a_1(\text{stat})<0$ then we have exotic currents to appear.

4. Induced pseudoscalar

In order to be more explicit I am going to use the customary impulse approximation (Primakoff [4]). Obviously a more accurate treatment would include terms arising from presence of mesons and nucleon isobars inside the nucleus. However , even in the framework of the impulse approximation theory it is convenient to introduce the sequence of simplifications. Morita and Fujii's [3] approximation (MFA) neglects the small component of the muon wave function.

If we treat muon consequently to be nonrelativistic (muon wave function can be considered as a constant) then we arrive at the e.g. Luyten, Rood and Tohoek [12] formulas (LRTA).

The last step in this sequence of the approximations is the Fujii and Primakoff approximation (FPA) when we neglect all nucleon recoil terms [2/].

Pictorially:

$$\text{Impulse Approximation} \rightarrow \text{MFA} \rightarrow \text{LRTA} \rightarrow \underset{\text{(only for unique transitions)}}{\text{FPA}} \qquad (5)$$

After the theory of partial transitions was done by Morita and Fujii (1960) the FPA was extensively discussed in the literature. It was expected that this approximation should be very good for any unique (both allowed and forbidden) transitions. Consequently for unique transitions, any angular and polarization distributions should be (if the approximation is good) completely independent of nuclear structure contrary to the capture rate. This is the main idea of our previous papers initiated by Popov [13]. We wonder to know to what extent it is possible to separate problems related to muon-proton primary interaction and to nuclear structure. Obviously the capture rates generally give information only on nuclear structure (with the exception of the time dependence) but it is extremly interesting to know what information on the weak vertex can be inferred from angular and polarization distributions in unique transitions (which is the question of the validity of FPA).

I should like to discuss in detail the old problem of the induced pseudoscalar interaction as an example. In what accuracy can we in principle determine phenomenologically this form factor from nuclear capture?

Even in the MFA our multipole amplitudes (Table 1.) have the remarkable property (i.e. up to the very small corrections due to the small component of the muon wave function):

induced pseudoscalar term contributes only
to the satellite axial A_L multipole.

2/ The FPA we call sometimes as "Nuclear Model Independent Approximation". However, strictly speaking, this approximation gives predictions which are independent of the nuclear structure only as far as the angular distributions are considered and only in the unique transitions. A Russian group calls the FPA as [101] - approximation.

I would like to stress once more that all informations on the C_P are contained only in the numbers A_L.

Let us look at the angular distributions or polarizations, which depend on the ratios of our multipoles. So we are interested in the ratio $A/E=x\cdot\exp i\phi$ which can be written in the LRTA for $L=1^+, 2^-, 3^+, \ldots$ as:

$$x^2 = (1-\varepsilon)\left\{\frac{1+(1-\frac{C_P}{C_A}-y)\frac{\nu}{2M}+\ldots}{1-(\mu+\delta)\frac{C_V}{C_A}\frac{\nu}{2M}+\ldots}\right\}^2 \qquad (6)$$

Here for simplicity we disregard the phase being very important for other reasons (see e.g. Ref. [10]). The dots ... in (6) are for terms of order $(\frac{\nu}{2M})^2$ and higher, and $\mu=1+\mu_p-\mu_n=4.706$ as the consequence of the isotriplet structure of the weak vector current[3/].

The nuclear physics enters x in LRTA by means of three ratios of the nuclear matrix elements ε, y and δ. Here ε represents the "higher order forbidden contribution"

$$r_L \equiv \sqrt{\frac{L}{L+1}} \cdot \frac{[1L+1L]}{[1L-1L]} \sim \frac{\int j_{L+1}}{\int j_{L-1}} \qquad (7)$$

and y and δ are the recoil terms.

In the Morita and Fujii notations [3] we have explicitely:

$$\varepsilon = -2\left(\frac{2L+1}{L}\right) r_L \cdot \frac{(1+\frac{1}{2L}r_L)}{(1-r_L)^2} \qquad (8)$$

3/ Evidently, $\mu=1+\mu_p-\mu_n$ is not a consequence of the conservation of the weak vector currents $\partial_\mu V^\mu=0$. The weak magnetism term μ and the conservation of the weak vector current $\partial_\mu V^\mu=0$ should be tested experimentally separately.

$$y = -\frac{2}{\nu}\left(3\frac{2L+1}{L}\right)^{\frac{1}{2}} \frac{[0LLi\vec{\sigma}\cdot\vec{p}]}{[1L-1L]}\left(1+\frac{L+1}{L}r_L\right)^{-1} \qquad (9)$$

$$\delta = +\frac{2}{\nu}\left(\frac{2L+1}{L+1}\right)^{\frac{1}{2}}(1-r_L)^{-1}\cdot\frac{[1LLi\vec{p}]+\mu O\{(\frac{1}{2}M)^2\}}{[1L-1L]} \qquad (10)$$

In formula (9) we recognize the trivial generalization of the parameter y (of) Foldy and Walecka's [19] analysis of the muon capture in carbon ^{12}C.

For exapmle, in the notations of Foldy and Walecka the formulas (8-10) for $L=1^+$ looks like

$$\varepsilon = \frac{|\int\vec{\sigma}|^2-3|\vec{\nu}\cdot\int\vec{\sigma}|^2}{|\int\vec{\sigma}|^2-|\vec{\nu}\cdot\int\vec{\sigma}|^2} \qquad (11)$$

$$\delta = -\frac{2}{\nu}\mathrm{Re}\frac{\vec{\nu}\wedge\int\vec{\sigma}\cdot(\int i\vec{p})^*}{|\int\vec{\sigma}|^2-|\vec{\nu}\cdot\int\vec{\sigma}|^2} \qquad (12)$$

$$y = \frac{2}{\nu}\mathrm{Re}\frac{(\vec{\nu}\cdot\int\vec{\sigma})^*(\int\vec{p}\cdot\vec{\sigma})}{|\vec{\nu}\cdot\int\vec{\sigma}|^2} \qquad (13)$$

The important observation of Foldy and Walecka is that ε and δ could be determined[4/] experimentally separately from the pure Gamow-Teller β-decay $^{12}B \to {}^{12}C + e^- + \vec{\nu}$.

From muon capture experiment alone (actually from angular distributions only) we can deduce x values for $L=1^+,2^-,3^+,\ldots$ but not ε, y, δ and C_{P}/C_{A} separately.

The trivial consequence of the formula (6) is then, even if we know accurately the term ε from the reversed β-decay, the muon capture could give us no more than

4/ In elementary particle approach the same idea was exploited by Kim and Primakoff [15] and Galindo and Pascual [16]. However, the experimental data allow to determine x^2 with very big errors. For example from [16] follows that $x^2=0.58\pm75\%$.

$(\frac{C_P}{C_A}+y)$ only, i.e. the sum of the pseudoscalar plus Foldy Walecka's parameter y and the sum $(\mu+\delta)$. So the accuracy of the C_P value depends in a most essential way on the knowledge of parameter y and the weak magnetism term is always jointed to δ.

Unfortunately the Foldy Walecka parameter y can not be deduced from other independent experimental data. Theoretical estimations of y depend strongly on the assumed nuclear structure.

As an example in the Table 3., the theoretical estimations of the nuclear parameters ε, y and δ are presented, using the intermediate coupling wave functions of Cohen and Kurath [17]. Table 3. is essentially recalculated from Table 1. of the Mukhopadhyay's paper [18].

Theoretical calculations [18].

Table 3.

Partial Muon Capture	ε	Recoil terms	
		y	δ
$^6Li(1^+)\rightarrow {}^6He\ (0^+1{:}g.s.)$	0.34	- 0.4	- 0.005
$^{10}B\ (3^+)\rightarrow {}^{10}Be^*(2^+1{:}3.37)$	0.52	+ 0.4	- 2.28
$^{10}B\ (3^+)\rightarrow {}^{10}Be^*(2^+1{:}5.96)$	0.33	+ 0.5	- 0.26
$^{11}B\ (\frac{3}{2}^-)\rightarrow {}^{11}Be^*(\frac{1}{2}^-\frac{3}{2}{:}0.32)$	0.44	+ 1.4	+ 0.21
$^{12}C\ (0^+)\rightarrow {}^{12}B\ (1^+1{:}g.s.)$	0.40	+ 3.6	+ 0.04
$^{13}C\ (\frac{1}{2}^-)\rightarrow {}^{13}B\ (\frac{3}{2}^-\frac{3}{2}{:}g.s.)$	0.37	+ 3.3	- 0.555
$^{14}N\ (1^+)\rightarrow {}^{14}C^*\ (2^+1{:}7.01)$	0.26	- 1.1	- 0.26

From these theoretical calculations the inadequacy of FPA follows at once. Obviously, FPA means (from formula (6)).

$$|\varepsilon| \ll 1$$
$$|y| \ll \left|\frac{C_P}{C_A}-1\right| \qquad (14)$$
$$|\delta| \ll \mu=4.7$$

Let us recall the Foldy and Walecka conclusions for the transition $^{12}C \to {}^{12}B/1^+, g.s/$:

$$+0.03 \lesssim \varepsilon \lesssim +0.35$$
$$+0.1 \lesssim \delta \lesssim +0.6$$
from EXP.(errors are purely experimental in origin).

$y = 5.4 \pm 25\%$ - the nuclear model calculations

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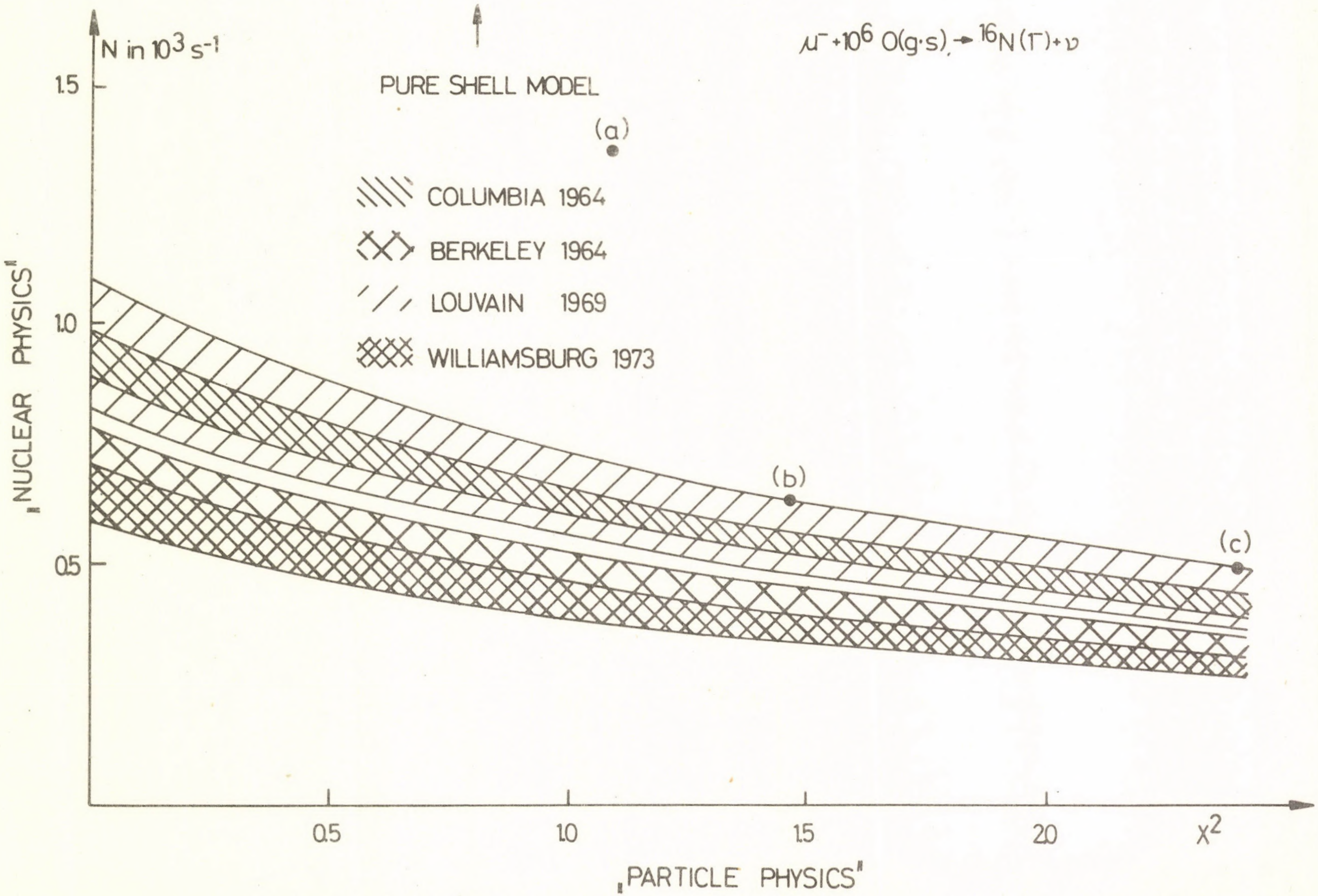
Figure caption

Fig. 1. Partial muon capture data for the transition $^{16}O(gs) \rightarrow {}^{16}N(1^-)$ are plotted on the model independent "nuclear-particle" plane. The experimental data are taken correspondingly:

Columbia from [5]
Berkeley from [6]
Louwain from [7]
Williamsburg from [8]

Fig 1.



N in 10^3 s^{-1}
$\mu^- + {}^{16}O(g.s.) \rightarrow {}^{16}N(1^-) + \nu$
PURE SHELL MODEL
(a)
(b)
(c)
COLUMBIA 1964
BERKELEY 1964
LOUVAIN 1969
WILLIAMSBURG 1973
1.5
1.0
0.5
0.5
1.0
1.5
2.0
X^2
„NUCLEAR PHYSICS"
„PARTICLE PHYSICS"

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AN ESTIMATION FOR THE POSSIBILITY TO USE SEMICONDUCTOR X-RAY DETECTORS TO HAVE AN EVIDENCE FOR DOUBLE BETA PROCESSES

D. BERÉNYI and G. HOCK

Institute of Nuclear Research of the Hungarian Academy of Sciences Debrecen (Hungary)

The possibility of using the X-rays from a certain type of double beta processes and the recent techniques of X-ray semiconductor detectors to obtain an evidence for the existence of double beta processes has been examined.

1. Introduction

Since the first experimental search of double beta decay in 1948 [1], numerous attempts have been made with a variety of techniques. In one of the most impressive experiments, recently Fiorini proved the non-existence of neutrinoless double beta decay with a high reliability in the case of ^{76}Ge using an ingenious arrangement with a special low background Ge(Li) detector situated in the tunnel below the Mont Blanc [2] [3]. Another recent experiment is that of Wu and her collaborators [4] in which they could give an order of magnitude greater limit on the lifetime for the neutrinoless double beta decay of ^{48}Ca (and similarly for the two-neutrino process) than previous experiments using a rather sophisticated instrumentation in a deep salt mine.

Contrary to the vast effort, however, there are only two indirect reliable positive experiments on the exsistence of double beta decay of ^{82}Se and ^{130}Te, i.e. the change in the isotope composition of krypton and xenon has been shown in a geological sample by means of mass spectrometric method [5] [6]. It means that the direct experimental evidence for the double beta decay is missing even now.

Having new semiconductor X-ray detectors with very high resolution recently, it has appeared to be reasonable to examine the possibility to use these techniques in search of double beta processes. It is all the more justified to examine this possibility because detecting the X-rays in a double beta process followed by X-ray emission (double electron capture or capture of an electron and emission of a positron in one process), one can get monoenergetic peak in the X-ray spectrum both in the case of no-neutrino and two-neutrino processes.

2. Theoretical half-lives for double beta processes with X-ray emission

X-rays are emitted after the following double beta processes

$$2e^- + (A,Z) \rightarrow (A,Z-2)\ (+2\nu_e)$$

$$e^- + (A,Z) \rightarrow (A,Z-2) + e^+\ (+2\nu_e)$$

The limits for the half-life obtained in the experiments of Fiorini [2], Wu et al. [4] and others [7] appear to show that the existence of no-neutrino double beta process has a very small probability. At the same time, the method under consideration is equally suitable to obtain an evidence for double beta process with and without neutrino emission, as it has been mentioned before. So, it seems to be reasonable to take into consideration in the calculations only the two-neutrino processes all the more because the half-life values for these processes are the higher in every case.

In table 1 some candidates for double beta process with X-ray emission are given, having different atomic numbers and transition energies [8] [9]. Using the formulas of Rosen and Primakoff [10] and further necessary data from ref. [8] [9] and [11] [12], the results for the half-life of processes with X-ray emission given in table 2 and fig.1 were obtained.

We can see that the most important precesses for X-ray emission is that with the capture of one electron and the emission of one positron in the same transition ($K\beta^+$). Furthermore, the probability of the process increases (i.e. the half-life decreases) very rapidly as a function of the energy. The table and the figure prove very convincingly that the situation is more favourable here than it was believed before [3] [8].

It can be also seen that the only experimental study carried out by searching for X-rays after double beta process is not a favourable case at all (^{64}Zn). The ^{106}Cd is a much more favourable candidate for such an experiment, especially because strongly enriched material of it (for nearly 100 per cent) is available [13].

3. Suggested experimental arrangement and effect to be expected

A rather simple arrangement seems to be the most suitable. A certain number of Si(Li) X-ray detectors of good resolution (line-width cca. 200 eV) have to be used with separate preamplifiers, amplifiers and with a proper computer as an analyser to record the spectrum region concerned separately from the individual detectors and after recording to sum them up. In this way the good resolution of the individual detectors can be preserved. The material to be studied would be placed as a thin sheet between the detectors arranged in two groups face-to-face to give practically 4π geometry.

The quantity of the material, i.e. the total number of the candidate nuclei is of crucial importance. The thickness of the sheet is determined by the self-absorption of that for the K-X-rays from the process (in the case of Cd: Pd-K-X-rays of about 20keV). It is not reasonable to use a thicker than 0.05 mm sheet which has an about 50 per cent minimum transmission in the perpendicular direction. It means 430 g of the material (to 100 per cent enriched ^{106}Cd) if we suppose that the size of the sheet is $1 m^2$.

To a sheet of $1 m^2$ surface, an unreasonable number of detectors would be necessary to view, because the area of the detectors with good resolution is about $1 cm^2$. Even at these conditions, presumably not attainable really, the maximum number of K-X-rays from the double beta process of ^{106}Cd is less than

2 counts/year

according to our estimation (supposing 10^{24} half-life, see table 2).

A further difficulty is the question of the background. Even in optimal case (as in the Fiorini experiment), the background might be orders of magnitudes higher than the above figure. It is obvious not only on the basis of different conside rations but also from data of preliminary measurement for the background spectrum of a Si(Li) detector in the low backgr chamber of the ATOMKI [14].

4. Conclusion

The use of X-rays followed a certain type of double beta processes seemed to be very remarkable because in this case monoenergetic peaks could have been produced in the spectrum both in no-neutrino and two-neutrino cases, as the Fiorini method showed in no-neutrino double beta decay.

Contrary to the fact that the situation in connection with the probability of the double beta processes with X-ray emission is better than it was believed earlier, and contrary to the possibility of using the most contemporary techniques of semiconductor X-ray detectors, to obtain a direct experimental evidence for double beta processes seems in this way to be out of reach according to our present estimation.

The autors are obliged to Dr.J.Bacsó for carrying out the preliminary experiment on the low background behaviour of the Si(Li) X-ray detectors.

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Table 1

Some possible double electron captures and double positive beta decay nuclei

Transition	Atomic number	Energy difference (keV)
^{54}Fe - ^{54}Cr	26	681
^{64}Zn - ^{64}Ni	30	1103
^{112}Sn - ^{112}Cd	50	1929
^{136}Ce - ^{136}Ba	58	2440
^{130}Ba - ^{130}Xe	56	2582
^{96}Ru - ^{96}Mo	44	2730
^{106}Cd - ^{106}Pd	48	2772
^{78}Kr - ^{78}Se	36	2880
^{124}Xe - ^{124}Te	54	3070

Table 2

Half-lives for the different double beta processes with minus two changes of the atomic number with neutrino emission and the partial half-live for processes with K-X-ray emission at the nuclei concerned

Nucleus	τ /years/				
	$\beta^+\beta^+$	β^+K	KK	for K-X em.	total
^{54}Fe *	-	-	9.10^{30}	9.10^{30}	9.10^{30}
^{64}Zn	-	2.10^{33}	2.10^{28}	2.10^{28}	2.10^{28}
^{112}Sn	-	2.10^{26}	2.10^{26}	1.10^{26}	1.10^{26}
^{136}Ce	3.10^{31}	3.10^{24}	2.10^{25}	3.10^{24}	3.10^{24}
^{130}Ba	1.10^{30}	2.10^{24}	2.10^{25}	2.10^{24}	2.10^{24}
^{96}Ru **	1.10^{29}	2.10^{24}	2.10^{26}	2.10^{24}	2.10^{24}
^{106}Cd	3.10^{28}	7.10^{23}	4.10^{25}	7.10^{23}	7.10^{23}
^{78}Kr	8.10^{27}	2.10^{24}	4.10^{26}	2.10^{24}	2.10^{24}
^{124}Xe **	3.10^{27}	4.10^{23}	2.10^{25}	4.10^{23}	4.10^{23}

For each value of the half-lives τ a factor of $10^{\pm 2}$ is to be added due to the uncertainties in estimating the nuclear matrix elements [10].

* An average excitation energy of the 0^+, 1^+ levels in ^{54}Mn as much as 2MeV was taken [12].

** The half-lives refer to the case when the lowest 0^+, 1^+ levels of the intermediate nucleus lie below 2.5MeV while for spacing these levels in the region 2.5 ÷ 10MeV they may be raised by one order of magnitude (c.f. Fig.).

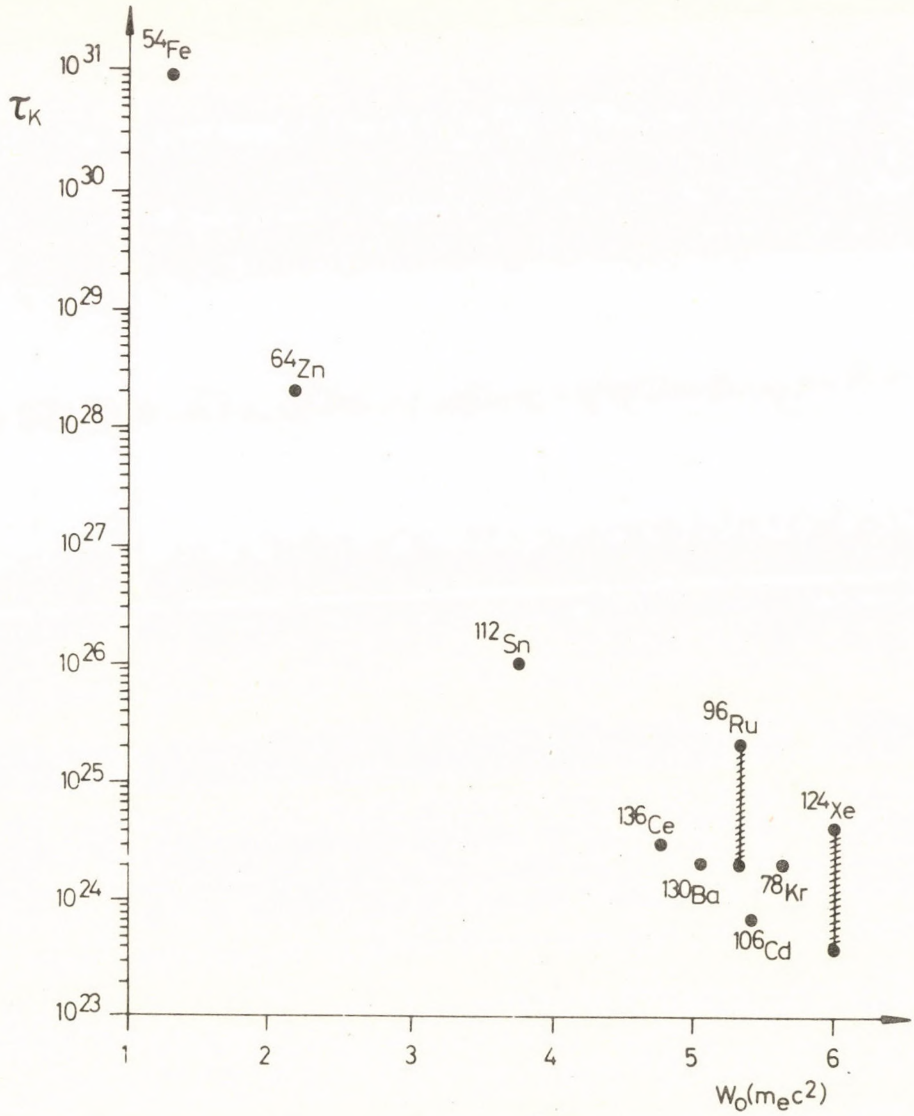

Fig. 1.

Partial half-lives for two-neutrino double beta processes with K-X-ray emission as a function of the energy release in the transition. The possible error at every point is ±2 orders of magnitude as usual because of the uncertainty in the transition matrix elements. In the case of ^{96}Ru and ^{124}Xe the uncertainty is even higher because of the lack in reliable data on intermediate states.

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FURTHER INVESTIGATIONS ON NON-UNIQUE FORBIDDEN ELECTRON CAPTURE RATIOS

E. VATAI

Institute of Nuclear Research of the Hungarian Academy of Sciences Debrecen (Hungary)

Electron capture (EC) occurs when one of the atomic electrons is captured by the radioactive nucleus and a neutrino is emitted; i.e. in the capture process no detectable particle is emitted. We get information about EC only through the study of subsequent processes. X-ray and Auger electron emission. Nevertheless, the EC enables us to solve a variety of problems concerning the atom, nucleus and weak interactions. These are the following:

It has been proved that atomic electrons take part in EC not separately, but the total atomic cloud and nucleus as an interacting system [1-3]. It means that the approximation in which the EC probability proportional to the density of a given electron inside the nucleus, is only the first approximation. It has been also proved that the effect of the vacancy in the final state is not negligible [4]. An important consequence of this is that in atomic processes the initial and final states do not overlap perfectly therefore the antisymmetric wave function of the initial and final atoms should be used in the calculation. The discrepancy between the experimental and theoretical K_α/K_β ratios has been explained this way by Scofield just recently. [5].

The K/β^+ ratios are sensitive to the presence of second class currents in the weak interaction. Comparisons of theoretical and experimental K/β^+ ratios were used to disprove [6] or prove [7] the existence of these currents.

We showed last year that the EC ratios can be used to investigate the nuclear matrix elements (NME-s) in nonunique EC [8]. It turned out that the $(L_1+L_2)/K$ ratio does not depend on NME-s, and the L_3/K ratio which depends on them can be determined by subtracting the theoretical $(L_1+L_2)/K$ ratio from the experimental L_{total}/K ratio. The L_3/K ratio is large enough to be determined in this way for certain isotopes. We have for second forbidden nonunique EC: In ξ approximation where we neglect terms with W_eR, qR and m_eR as compared with αZ

$$\frac{L_3}{K} = \frac{(p_{L_3} q_{L_3} \beta_{L_3})^2}{(q_K^2 \beta_k)^2} \left(\frac{\sqrt{\frac{5}{2}} - \frac{\alpha Z}{5} R_2}{\sqrt{\frac{5}{2}} - \frac{\alpha Z}{3} R_2} \right)^2$$

where $R = ({}^V F^o_{220} - \sqrt{\frac{3}{2}}\, {}^A F^o_{221}) / {}^V F^o_{211}$ is the ratio of nonrelativistic and relativistic FFC-s or NME-s.

p - momentum of electron

q - neutrino momentum

β - Coulomb amplitude

We note, that the accuracy of the expression above is much better than that estimated earlier by us: 10% for ^{36}Cl.

The second forbidden nonunique EC ground state-ground state transition is given in table 1. Measurements of L/K ratios for ^{36}Cl [9] and ^{59}Ni [10] were made by the so-called multiwire proportional counter technique. The excess of the experimental ratios over the theoretical $(L_1+L_2)/K$ ratio can be explained by the existence of L_3 capture, from which the ratio of nuclear matrix elements can be calculated. The results are given in the last column. The case of the ^{36}Cl is especially important, because certain theoretical estimations of NME-s exist in this case. The independent particle model is valid for ^{36}Cl with good accuracy, therefore ${}^A F^o_{22} = 0$ i.e. the axial vector matrix element is absent in this case. The CVC theory gives the following relation between the vector ME-s [11]

$$\int \frac{S\, i\, A_{ij}}{S\, R_{ij}} = \frac{\sqrt{10}}{R} \frac{{}^V F^o_{211}}{{}^V F^o_{220}} = W_o + 2.5 - \xi \Lambda_1$$

where $\xi = \alpha Z/2R$, R - the nuclear radius and W_o the transition energy. According to different theoretical calculations Λ_1 has the value $1.9 \leq \Lambda_1 \leq 2.4$ (for references see [12]). Using the ratio R_2 given in table 1 we can calculate the following value for Λ_1: $0.61 \leq \Lambda_1 \leq 0.78$ or $0.78 \leq \Lambda_1 \leq 1.15$, which is smaller than the theoretically expected value. This discrepancy might indicate either some systematic error of the experimental L/K ratio or the fact that the assumption about the applicability of independent particle model is not valid. In any case, further experimental and theoretical investigation of the L/K ratio pf ^{36}Cl is desirable.

Comparison of experimental R_2 with the theory for ^{59}Ni is difficult because no calculation exist for these nuclei.

Table 1.

Second forbidden non - unique ground state EC transitions

Transition	Q_{EC} lg ft	L/K exp.	$(L_1+L_2)/K$ theor.	R_2
$^{36}Cl \rightarrow {}^{36}S$ $2^+ \rightarrow 0^+$	1144.1±1.7 3.5	.112 ±.008	.0966	$30 < R_2 < 38$ or $38 < R_2 < 43$
$^{53}Mn \rightarrow {}^{53}Cr$ $7/2^- \rightarrow 3/2^-$	597.3±1.2 12.6	-	.1105	-
$^{59}Ni \rightarrow {}^{59}Co$ $3/2^- \rightarrow 7/2^-$	1073.1±1.1 11.6	.121 ±.002	.113	21±1 or 25±1
$^{97}Tc \rightarrow {}^{97}Mo$ $9/2^+ \rightarrow 5/2^+$	346±9 3.1	.83* ± .08	.83*	≦ 11 or ≦ 18

* K - capture probability P_K

Finaly we note that positron emission is possible energetically for both ^{36}Cl and ^{59}Ni, and in these cases we can use the K/β^+ ratio as well for the determination of NME. Preliminary calculations indicate that the K/β^+ ratio gives an independent equation for the determination of NME such that using the L_3/K and K/β^+ ratios and lg ft values together we can determine all the three dominant NME-s.

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THE MASS OF THE RELICT NEUTRINOS AND THE MISSING MASS PHENOMENA IN THE UNIVERSE

A.S. SZALAY and G. MARX

Department of Atomic Physics Eötvös University, Budapest

The rest masses of the neutrino (ν_e) and neutretto (ν_μ) are rather poorly known from laboratory experiments [1]: $m_{\nu e}<60eV$ $m_{\nu\mu}<0.8MeV$. The aim of this note is to discuss the astronomical information about these mass values, offered by the normal gravitational interaction of these particles as suggested by Zel'dovich and others [2].

When in the seconds following the Big Bang, the temperature of the Universe was above $10^{11}K$, all the leptons were in thermal equilibrium with the photons. The annihilation of muons and positrons, the decoupling of neutrettos, neutrinos and photons occured rather soon after this equilibrium period. Before decoupling the behaviour of matter had been described by the adiabatic equation. After having decoupled, the particle number has been frozen and the momenta of the individual particles started to decrease according to the Hubble law

$$p = h/\lambda \sim R^{-1} \tag{1}$$

(Here R means the curvature radius of space). For massive particles the mass density ρ_a obeys the equation $\rho_a R^3$=const. For massless particles, e.g. for photons the Hubble shift is equivalent to an adiabatic drop of temperature:

$$R.T_\gamma = const \tag{2}$$

consequently their mass density decreases as $\rho_\gamma R^4$= const. In the case of neutrinos, possessing a tiny, but possibly non-vanishing rest mass m_ν the occupation number is given by the formula

$$n = [\exp\frac{\sqrt{m_\nu^2+p^2}}{kT} + 1]^{-1} \quad \text{with } pR = \text{const} \qquad (3)$$

By knowing the dependence of the total mass density on R one is able to integrate the Einstein equation of gravity for a homogenous-isotropic model. The starting point of integration is the decoupling time t_d of neutrettos. The computed time evolution R(t) will depend on the parameters $m_{\nu e}$, $m_{\nu\mu}$ and on the initial value $R(t_d)$. The integration stops at t_o, when the photon temperature, given by eq. (2) drops to its present empirical value $T(t_o)=2.7K$. The computer can be programmed to print the values t_o, $R(t_o)$, $H_o=\dot{R}(t_o)/R(t_o)$ and $q_o=-\ddot{R}(t_o)R(t_o)/\dot{R}^2(t_o)$. If one knows the present empirical values of the age of universe t_o, Hubble parameter H_o and deceleration q_o one can find out the correct input parameters $m_{\nu e}$, $m_{\nu\mu}$, $R(t_d)$. Our astronomical knowledge about t_o and q_o is rather limited, so we have plotted the computed relation between these data on Fig.I, by assuming for simplicity $m_{\nu e}=m_{\nu\mu}$. From the lower limit of $t_o>10^{10}$ years and from the upper limit of $q_o<1$ one may conclude [3], that the neutrino and neutretto rest masses cannot exceed 16eV. This is a much lower limit than those obtained by laboratory experiments.

There is indication, that q_o is about 0,5 (flat universe). The universe opens at $\rho_{dyn}=5.10^{-30}g/cm^3$. This dynamical mass density is definitely larger, than the optically observed mass $\rho_{opt}=3\cdot 10^{-31}gcm^{-3}$. The "missing mass" $\rho_{dyn}-\rho_{opt}$ might be explained by the relict neutrino gas possessing a rest mass of the order of 1eV.

As neutrinos loose their kinetic energy, they become unstable for large scale fluctuations. As a consequence, the density excess will pull atomic matters to the denser places. The formation of some large visible objects can be explained on this way. From a given neutrino mass the separation of these objects, their mass, radius and density distribution can be calculated (using simple approximations), and as we shall see, they agree surprisingly well with those of clusters of galaxies (or at least they do not contradict).

The critical point for the occurence of fluctuations is the temperature $kT_c = m_\nu c^2$, where the neutrinos suddenly become non-relativistic particles. The critical length for fluctuations at that temperature is approximately $\lambda_J = \left(\frac{\pi c^2}{G\rho}\right)^{\frac{1}{2}}$ *. Projected to its present size today we obtain the average separation of the above objects. The result is $D = 3{,}6 \cdot 10^{26}$ cm for $m_\nu = 1$ eV which is the right separation for clusters of galaxies. [Peebles: Physical Cosmology, p. 67].

We can expect, that today the density and the mass is dominated by the neutrinos within the cluster, so the dynamical mass will be much larger, than the visible mass. This effect, the "missing mass phenomenon" is well-known among astronomers, Encouraged by this indication we shall discuss the calculation of the equilibrium cluster shape in details.

The local gravitational potential $\phi(r)$ is given by

$$\nabla^2 \phi(r) = 4\pi G(\rho_G + \rho_\nu) \qquad (4)$$

Here $\phi \to 0$ if $r \to \infty$. The mass density of galaxies may be approximated by the isothermal distribution

* From the theory of fluctuations the definition of the Hans-length is

$$\lambda_J = \left(\frac{\pi k T_c}{\rho G m_\nu}\right)^{\frac{1}{2}} .$$

Since the fluctuations become important only when the temperature is about $kT_\nu = m_\nu c^2$

$$\lambda_J = \left(\frac{\pi c^2}{G\rho_\nu}\right)^{\frac{1}{2}} .$$

We obtain the same result, taking a sphere of radius λ_J, and putting the kinetic energy of the neutrino equal to its potential energy (neglecting factors of order 1)

$$\frac{4\pi}{3}\lambda_J^3 \frac{G\rho m_\nu}{\lambda_J} \approx \varepsilon_{kin} \approx \frac{3}{2} kT .$$

$$\rho_g(r) = \rho_g(\infty)\exp\left(-\frac{M\phi(r)}{kT_c}\right) \quad \text{with } kT_c = \tfrac{1}{2}M\langle v^2\rangle \tag{5}$$

where $\langle v^2 \rangle$ is the mean square of the observed random radial velocities of the galaxies within the cluster. The neutrino mass density will take a very simple form, if the cool neutrino gas is approximated by a degenerate Fermi gas:

$$\rho_\nu(r) = \rho_\nu(\infty)\left[1-\frac{2^{5/3}m_\nu^{8/3}}{3^{2/3}\pi^{4/3}\hbar^2}\,\frac{\phi(r)}{\rho_\nu(\infty)^{2/3}}\right]^{3/2} \tag{6}$$

Putting the formulas (5) and (6) into (4) one arrives at an equation for $\phi(r)$. For simplicity one can write $\rho_\nu(\infty) \ll \rho_\nu(r)$. Introducing convenient units, eq. (4) will take the form

$$\nabla^2\psi(x)+\psi(x)^{3/2} = \frac{3\pi^2}{32}\left[m_\nu\left(\frac{\hbar}{m_\nu v}\right)^{-3}\right]^{-1}\rho_g(\infty)\exp\psi(x) \tag{7}$$

It is shown by this form, that the relative importance of ρ_ν and ρ_g in shaping the local field of gravity depends on the relative magnitude of $\rho_g(r)$ to $m_\nu\left(\frac{\hbar}{m_\nu v}\right)^{-3}$. For values characteristic for the Coma cluster one expects a dominance of neutrino halo upon the galaxies, if $m_\nu > 1$ eV.

The shape of the cluster may be used to test this hypothesis. By taking the central galaxy density from observations, with the help of the formula (6) one can get the initial value $\phi(0)$. The solution of eq. (4) or (7) gives the self-consistent potential and finally the formula (5) gives the galaxy distribution which can be directly compared with the astronomical evidence. The size of the cluster is very sensitive for the neutrino rest mass. For a given $\rho_g(0):\rho_g(\infty)$ ratio the cluster radius is proportional to $m_\nu^{-4/3}$. In the case of the Coma cluster, where the "missing mass" is estimated to be about 85-90%, a good fit is obtained with $m_\nu \approx 0.5$eV. [On Figure II $\rho_g(0)=5\cdot 10^{-26}$ g/cm^3. $\rho_g(\infty)=3\cdot 10^{-31}$ g/cm^3. $v=1000$ km/s for the Coma cluster. The points are taken from Peeble: Physical Cosmology, p.69.]

The system is unstable for $m_\nu=0$, it is stable for $m_\nu\sim 1$ eV and nearly singular for $m_\nu\sim 2$eV. Since other observed forms of matter and evolutionary effects cannot be excluded, the neutrino and neutretto rest mass value $m_\nu=2$ eV may be considered only as an upper limit. Assuming a non-vanishing neutrino rest mass we have been considering astrophysical phenomena on different scales, looking for contradictions with astronomical observations in order to find an upper limit on the ν mass. However, all the phenomena, discussed above allow a small, but nonzero neutrino mass, <u>of the same order of magnitude,</u> $m_\nu \sim 1$ eV.

The authors are sincerely indebted to S.A. Bludman for valuable discussions and criticism.

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Figure Captions

Fig.I The dependence of the deceleration parameter q_o and the age of the Universe t_o on the Hubble constant H_o and the neutrino rest mass m.

An upper limit on m can be obtained using some restrictions on H_o, q_o, t_o.

Fig.II The density profiles for galaxies (solid lines) and neutrinos (dashed lines) within the Coma cluster for the neutrino rest mass values m=0.5 and 1eV.

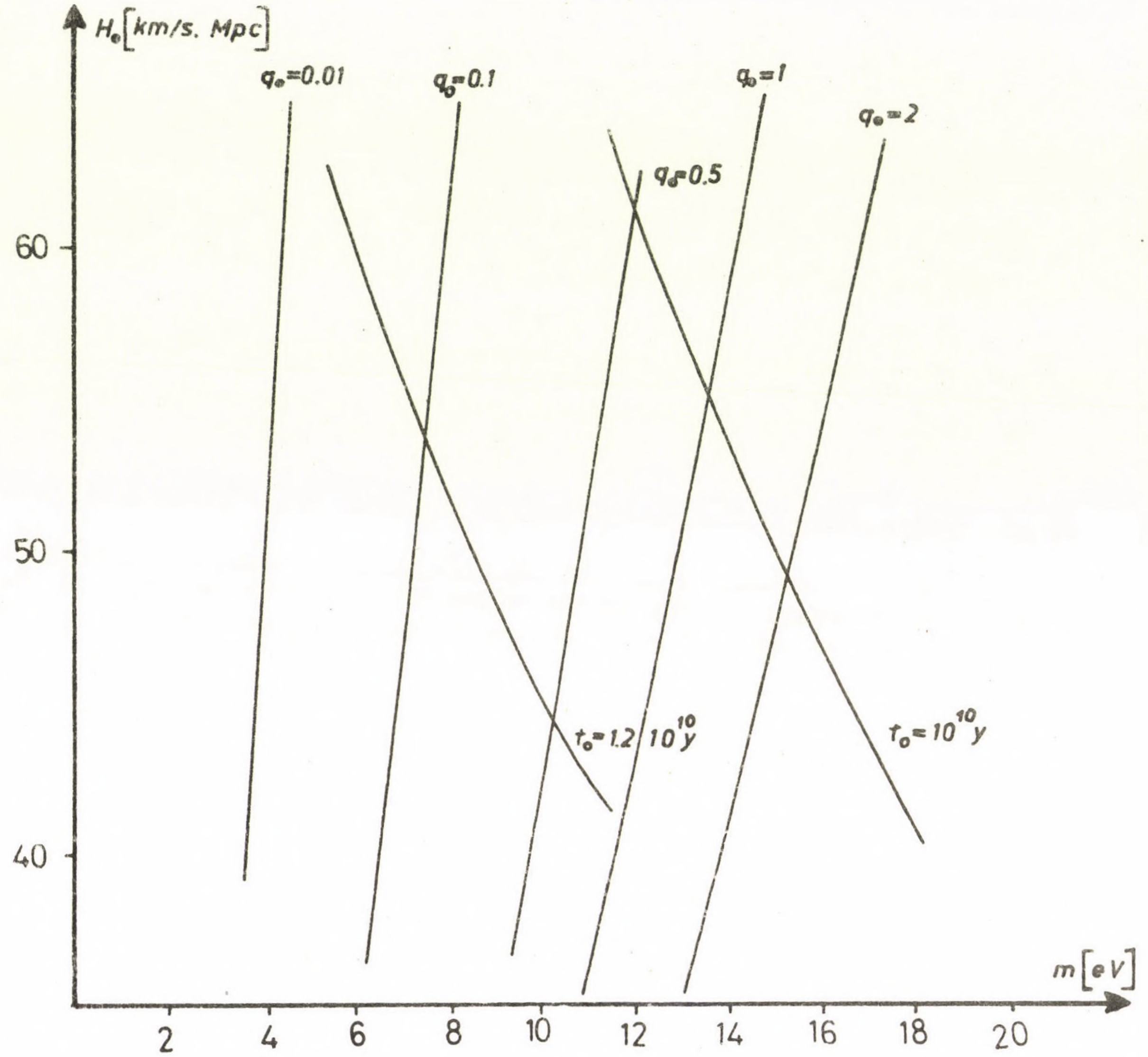

Figure I

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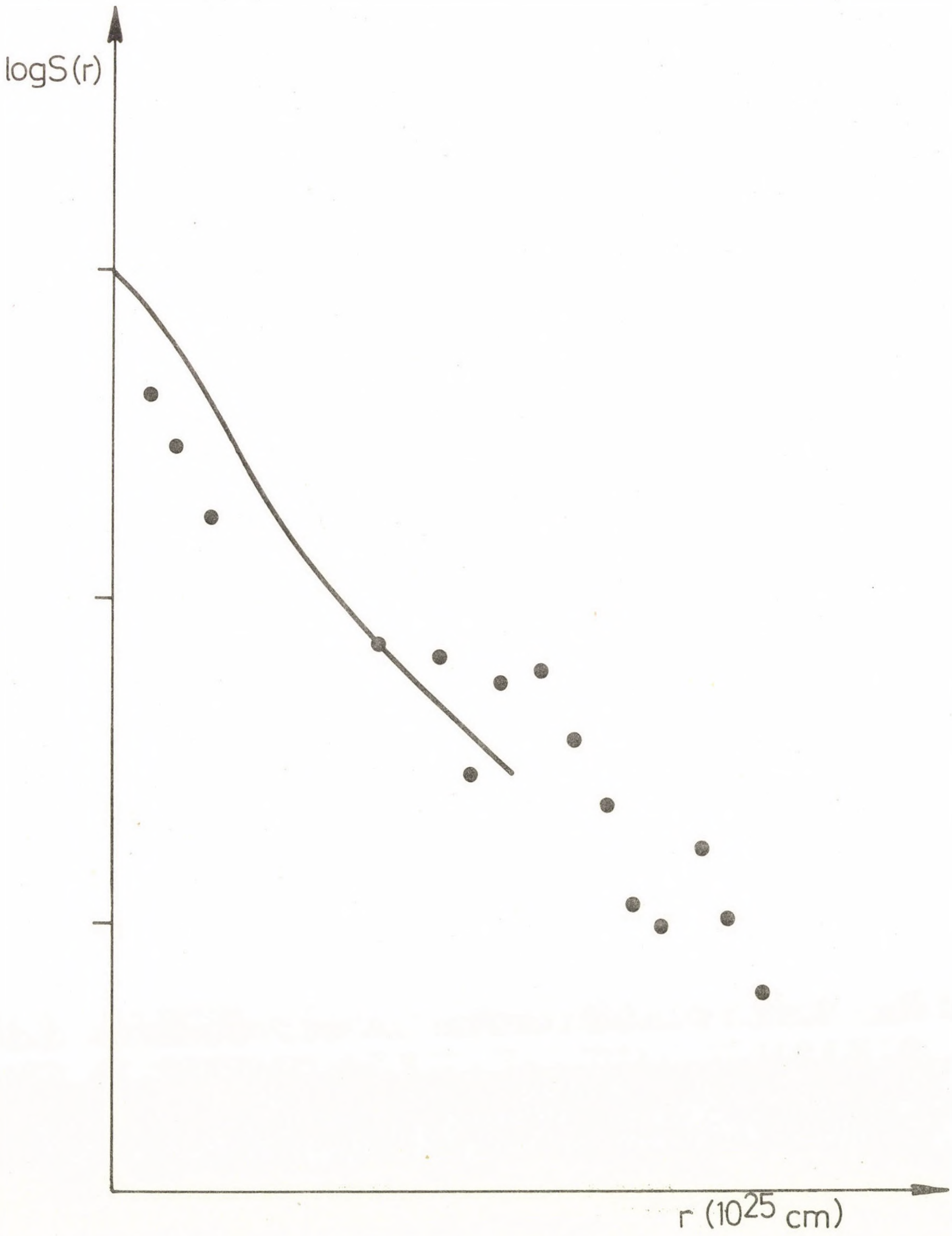

Figure II

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THEORETICAL PREDICTION OF PARITY VIOLATING NUCLEAR POTENTIAL

D. TADIĆ*

Institute "Ruder Boskovic" and
University of Zagreb - Zagreb, Yugoslavia

Introduction

The main intention of this introduction is to highlight some interesting and relevant facts from the vast subject of the parity-violating nuclear interactions. Anybody interested in the systematic and comprehensive presentation of the subject should consult some of the existing review papers [1-8].

The one meson exchange contributions to the parity violating (P.V.) nucleon-nucleon potential can be divided in two groups according to the isospin (I) selection rules. The pion exchange potential, V_π, allows for $|\Delta\vec{I}|=1$ when nucleons are on the mass shell, while with one nucleon or both off the mass shell, admixtures of $|\Delta\vec{I}|=0,2$ potentials are possible. The vector meson exchange potential, V_ρ for short, allows for $|\Delta\vec{I}|=0,2$.

Existing experiments on parity violation can be, roughly speaking, divided into two groups: Those involving mainly complex nuclei (characteristic representative is γ-decay of Ta^{181}) measure both $|\Delta\vec{I}|=0,2$ and $|\Delta\vec{I}|=1$ parts of the P.V. nuclear potential.

There are two experiments which measure only parts of the P.V. potential analogous to V_ρ:

(i) $O^{16} \to C^{12}+\alpha$

(ii) $n+p \to d+\gamma$

Although of fundamental importance, each of these two very difficult and very beautiful experiments has been performed only once. Obviously, further experimental work is needed, especially on processes in which only $|\Delta\vec{I}|=1$ part of the potential contributes, as in the experiment proposed in Professor Fiorini's talk.

* On leave at the Physics Department, University of Cincinnati Cincinnati, Ohio

While for the process (i) the standard one vector boson exchange potential V_ρ leads to the theoretical prediction which is in very good agreement with the experiment, the theoretical value of the circular γ-ray polarization process (ii) has the wrong sign and is off by an order of magnitude. The same situation is found for the Ta^{181} decay, where both V_π and V_ρ contribute.

Several possible ways out have been under discussion:
- As (ii) and Ta^{181} decay involve γ emission, parity is possibly violated by the electromagnetic interaction itself [9,10]. A lucid exposition of this question is to be found in the lecture by L. Wolfenstein. From the theoretical nuclear physics point of view, one should re-examine existing experiments on the circular polarization of γ-rays, especially the ones in which result was negative.

- The vector meson exchange potential V_ρ is not correctly given by the factorization approximation. A short summary of the question, that is discussed at more length in [4], [7] and [11-13], will be presented in the next section.

As the determination of the V_π depends on the hyperon non-leptonic decays, one has to take into account recent fits of the experimental decay amplitudes. New, unified field theory based models of the H_W can also lead to new predictions. Finally, the off mass shell effects (OMS) can be of importance, leading to the one pion exchange contribution even in processes (i) and (ii). The discussion of the one pion exchange P.V. potential is going to be the main topic of the present article.

Two pion exchange contribution (or generally multiboson exchange) and the boson exchange contributions to the P.V. γ-decay have also been under intensive examination. They are to some extent reviewed in [7]. Recently, a systematic work on the 2π exchange [14] has appeared. In the present exposition we do not intend any further discussion of these effects, what should not be taken as judgment about their importance.

By the methods described elsewhere [4], one pion exchange potential is easily calculated once the P.V. NNπ amplitude is known. The result is:

$$V_\pi = \alpha\Gamma(\vec{\sigma}_1+\vec{\sigma}_2)[\vec{p}_{12}, \exp(-m_\pi r_{12})/r_{12}]T^{(-)}$$

$$T^{(-)} = \tau_1^{(+)}\tau_2^{(-)} - \tau_1^{(-)}\tau_2^{(+)} \qquad (1)$$

The constant Γ measures the strength of the potential, σ_i and τ_i are the usual Pauli matrices associated with nucleon spin isospin, respectively, and α depends on the H_W. (For the Cabibbo model $\alpha=(tg\theta_c)/\sqrt{2}$). The vector meson exchange potential has a similar form.

Vector meson exchange potential

Straightforward application of the current algebra to the local current-current interaction weak Hamiltonian

$$H_W \sim J^+_\mu(x) J_\mu(x) \qquad (1)$$

leads to the so-called factorization-approximation. The existence of such term can be established using field theory arguments [4]. It leads to the already mentioned agreement and/or disagreement with experiments (i), (ii) and Ta^{181}. However, McKellar and Pick [13] were able to show, by assuming $SU(6)_W$ symmetry for nonleptonic decay amplitudes, that in the Cabibbo model parity-violating $NN\rho^0$ and $NN\omega$ amplitudes cannot both vanish, as follows from the factorization approximation. Using in addition various dynamic assumptions, like meson pole dominance, etc., they deduced coupling constans which were of opposite sign and larger up to a factor 8 than the factorization results. Danilov [11] proposed a large increase of the $NN\rho$ amplitude coming from the divergencies inherent in weak interactions. However, in his approach, which involves dubious shift of coordinates (similar to the one encountered by the Adler anomaly [15]), both $NN\rho^0$ and $NN\omega$ amplitudes vanish. An alternative analysis starting from

$$H_W \sim S_{ab} \int d^4y \Delta_{\mu\nu}(y-x) T\{V^a_\mu(y) A^b_\nu(x) + A^a_\mu(y) V^b_\nu(x)\}$$

where $\Delta_{\mu\nu}$ can be envisaged ad the intermediate vector boson propagator recovered the factorization result and led to the ratio [12]:

$$R = \frac{\text{divergent contribution}}{\text{factorization approximation}} = -\frac{1}{(4\pi)^2} f^2_\nu \frac{\Lambda^2}{M^2}$$

Here f_ν is the strong vector meson coupling constant, M is the nucleon mass and Λ is the weak interaction cut-off. Depending on the cut-off, the ratio is

$$R \sim -100(\Lambda = 15\,\text{GeV});\ -7(\Lambda = 4\,\text{GeV}).$$

As both $NN\rho^0$ and $NN\omega$ amplitudes exist, this analysis appears in surprising agreement with the $SU(6)_W$ based one. However, it is dangerous to put ones trust entirely into inherently divergent and nonrenormalizable theory. A new approach, based on the unified field theory of weak and electromagnetic interactions, is needed.

While improving the situation in the case of experiments (i) and Ta^{181} considerably, these new estimates[12], [13] completely destroy agreement in the case of the α-decay (ii). Should this be taken as a definitive indication that speculation [12] and [13] are wrong depends, among other things, also on one's trust in the existing α-decay calculations. New light on this involved question of nuclear theory is shed in the contribution by G. Fai, B. Apagyi and J. Nemeth.

One pion exchange and the hyperon nonleptonic decays *

In order to find P.V. $NN\pi$ vertex, one has to connect strangeness conserving amplitude ($\Delta S=0$) to the strangeness changing ($\Delta S\neq 0$) decay amplitudes. Initial attempts were done using SU(3) transformation properties obeyed by H_W and by the state vectors [16, 17, 4 Appendix A] or using standard soft-pion method and SU(3) based current algebra [18, 19]. When the latter method is combined with the octet dominance hypothesis, it gives exactly the same sum-rule as the former one

$$A(\Xi^-_-)-2A(\Lambda^o_-)-\frac{\sqrt{3}}{2\alpha}A(n^o_-) = 0 \qquad (1)$$

The constant α appearing here is identical with the constant in formula (1.1). The s-wave amplitudes correspond to the following processe

$$\Xi^-_- \sim \Xi^- \to \Lambda^o + \pi^- \quad ; \qquad \Lambda^o_- \sim \Lambda^o \to p + \pi^-$$

$$n^o_- \sim n \to p + \pi^-$$

Both method implicitly assume:

i) The amplitude is a slowly varying function of the pion momenta q such that

$$A(q^2=M_B^2) = A(q^2=0)$$

In the derivation based on SU(3) the masses associated with the initial and final state vectors respectively are also equal, as they have to belong to the same multiplet.

ii) They imply that the effective masses of the initial and the final baryon respectively are the same. If neutron-proton mass difference is neglected that might mean (but need not) that the nucleons are on the mass shell.

In order to fit simultaneously both s and p wave decay amplitudes, it appeared necessary to relax assumptions (i) and (ii) [20, 21]. Current algebra contributions (in future A_{cc}) was used in combination with the one particle pole to fit experimental results (in future A_{pp}). The s-wave amplitude was given by

$$A = A_{cc} + A_{pp}$$
$$A_{pp} \underset{q\to 0}{\to} 0 \tag{2}$$

In the Gronau's fit [20] pole term came from the vector meson exchange. The baryon-vector meson vertex is strong while vector meson interacts weakly with the pion.

For the Cabibbo model, one has

$$A(p^{+}_{+}) = A_{cc}(p^{+}_{+}) + A_{pp}(p^{+}_{+}) =$$
$$= -\frac{\sqrt{2}}{f_\pi}\, tg\Theta_c (F+D) - \frac{c}{6\sqrt{2}}\, \frac{2\cos^2\Theta_c - \sin^2\Theta_c}{\sin\Theta_c \cos\Theta_c} \times \tag{3}$$
$$\times \frac{M^2_{K^*}}{M^2_\rho}(1+\frac{\delta}{\phi})(N_f - P_i)$$

and

$$A_S(n^{0}_{-}) = -A_{cc}(p^{+}_{+}) - \frac{c}{6\sqrt{2}}\, \frac{2\cos^2\Theta_c - \sin^2\Theta_c}{\sin\Theta_c \cos\Theta_c}\, \frac{M^2_{K^*}}{M^2_\rho}(1+\frac{\delta}{\phi})(P_f - N_i) \tag{4}$$

In the expression (3) and (4), parameters F and D were determined to be consistent with both s and p wave nonleptonic hyperon decays [20]. The parameters c, δ and ϕ were also determined from experimental numbers. The vector meson mass ratio appears because in $\Delta S=0$ transitions ρ meson is exchanged instead of K^*. N,P denote masses of the initial and final nucleons. The Cabibbo angle factors are self explaining. (See, for example, [17].)

If nucleons are on the mass shell

$$N_f - P_i = -(P_f - N_i) = \Delta M(p,n) \tag{5}$$

the A_{pp} contribution which is proportional to the proton neutron mass difference ought to be small. As in the $\Delta S \neq 0$ transitions, mass differences are larger; it is obvious that the newly determined $A(n^0_-)$ shall be smaller than the one found by using formula (1). It is also worth mentioning that the pole term in (3) and (4) gets contribution only from the isoscalar part (I=0) of the weak Hamiltonian. Nevertheless, as the mass breaking term, responsible for (5), is an isovector (I=1), one finally obtains

$$A(n^0_-) = -A(p^+_+) = -1.4\cdot 10^{-8} \tag{6}$$

where

$$\begin{aligned} A_{cc} &= -1.61\cdot 10^{-8} \\ A_{pp} &= 0.23\cdot 10^{-8} \end{aligned} \tag{7}$$

This turns out to be almost three times smaller than the old result which would follow from the sum-rule (1) alone

$$A(n^0_-) = -4.1\cdot 10^{-8} \tag{8}$$

This new value for Γ (1.1) can only worsen the experiment theory disagreement.

However, one can assume that nucleons are off-mass shell (OMS) in such a way that the effective mass of one nucleon, say the final one, is larger. According to the way this game is usually played [22, 23], we have to take

$$N_f - P_i = P_f - N_i = \Delta M(OMS) \tag{9}$$

We do not attempt here to discuss the meaning or the credibility of this game, but mention that estimates of ΔM vary roughly, in the range [4, 23]

$$8 MeV \le \Delta M(OMS) \le 30 MeV \qquad (10)$$

As the A_{pp} contribution is now isoscalar, this leads to the potential

$$V_\pi(OMS) = \gamma(\vec{\sigma}_1-\vec{\sigma}_2)[\vec{p}_{12}, \exp(-m_\pi r_{12})/r_{12}]T^{(+)}$$
$$T^{(+)} = \tau_1^{(+)}\tau_2^{(-)} + \tau_1^{(-)}\tau_2^{(+)} \qquad (11)$$

The new values for A_{pp} are

$$1.8 \cdot 10^{-8} < A_{pp}(OMS) < 6.6 \cdot 10^{-8} \qquad (12)$$

Thus the OMS contribution can be quite large in the case of the Cabibbo model. The values quoted (12) are of about the same order of magnitude as the ones found in [23] on the basis of a field theory model. (See also [7]). Moreover, vector meson pole allows the emission of the neutral pion if and when nucleons are OMS. The amplitude for π^0 emission is comparable with (12).

Relative importance of the vector meson pole is very much reduced for the models of the weak Hamiltonian having isovector part multiplied by $\cos^2\theta_C$. In the case of, for example, d'Espagnat model, one obtains

$$A_{cc}(n^{\underline{o}}) = 3.5 \cdot 10^{-7}$$
$$A_{pp}(n^{\underline{o}}) = 1.3 \cdot 10^{-9} \qquad (13)$$
$$A_{pp}(OMS) \sim 0.1 \cdot 10^{-7} \sim 0.4 \cdot 10^{-7}$$

The old result was, again, about three times larger.

$$A(n^{\underline{o}}) = 9 \cdot 10^{-7} \qquad (14)$$

There is another simultaneous fit of the s and p wave decay amplitudes [21] in which decuplet resonance pole is used. Although it is possible to argue that the vector meson pole fit and the decuplet resonance pole fit are equivalent in the duality sense [21, 24] we will show shortly that the consequences for the

$\Delta S=0$ amplitudes are very much different. Scadron and Thebaud [21] fit the s-wave decay amplitude by

$$A = A_{cc} + A_{10} \qquad (15)$$

where

$$A_{10} = \frac{1}{3}(M_i - M_f)[g_{B_f D\pi} \frac{M_D + M_i}{M_D^2} H^{PV}_{DB_i} + + H^{PV}_{B_f D'} \frac{M_D' + M_f}{M_D^2} g_{D' B_i \pi}] \qquad (16)$$

Here M_i, M_f, M_D are masses, g are strong baryon, pion, resonance coupling constants, and H are matrix elements of the weak Hamiltonian. This contribution is again propotional to mass difference and vanishes for $q \to 0$.

As the nucleon has isospin ½ and for the decuplet resonance I=3/2, only isovector part of the H_W can contribute. Thus the situation is very much different from the vector meson pole fit, as the enhancement factor $ctg^2\theta_C$ is now missing. The result, for the Cabibbo model, is

$$\begin{aligned} A_{cc}(n^0_-) &= -1.1 \cdot 10^{-8} \\ A_{10}(n^0_-) &= -3.7 \cdot 10^{-11} \\ A_{10}(n^0_0) &= -7.5 \cdot 10^{-11} \Delta M(OMS) \end{aligned} \qquad (17)$$

The last row corresponds to the π^0 vertex. The contribution from the pole term is obviously negligible, and this situation won't change for the d'Espagnat model, where both A_{cc} and A_{10} are enhanced by the same factor. The new value for A is even smaller than the one given by (6).

In principle, the study of the OMS effects [25] could distinguish among the two models for the hyperon nonleptonic decay amplitudes. The practicability of that endeavour is however questionable.

In the end, let us mention the work by Reid [26] where, by taking into account the final state interactions, a result in close agreement with (6) was obtained:

$$A(n^0_-) = -1.66 \cdot 10^{-8} \qquad (18)$$

P.V. NNπ Amplitudes and the unified field theory*

In the previous section we saw how NNπ P.V. amplitude can be connected with the experimentally accessible amplitudes, using SU (3) transformation properties of the weak Hamiltonian. The unified field theory models are based on the other symmetry than SU (3), what leads to a complicated situation in which H_W behaves as a sum of the nonequivalent SU (3) tensors.

Generally, the baryon pion P.V. amplitude in the unified theory has the form

$$A \sim i\int d^4k \frac{\delta_{\mu\nu} + \frac{k_\mu k_\nu}{m_V^2}}{k^2+m_V^2} \int d^4x e^{-ikx} <f|T\{J_\mu^V(x), J_\nu^{-V}(0)\}|i> + \quad + i\int d^4k \frac{1}{k^2+m_\phi^2} \int d^4x e^{-ikx} <f|T\{J_\phi(x), J_\phi(0)\}|i> \tag{1}$$

The first term comes from the vector meson exchange with V=W and/or Z, while the second term is due to the exchange of the Higgs scalar ϕ.

The Salam-Ward-Weinberg model [27] has been already discussed in [7]. It allows for the W and Z exchange. As J_ϕ is scalar [29] the last term does not contribute to P.V. amplitudes. In the four quark version [30, 31] of the model, the bilinear product of the quark fields

$$M_i^j = \psi_i\bar{\psi}^j - \frac{1}{4}\delta_i^j\psi_K\bar{\psi}^K \tag{2}$$

transforms as a four by four meson matrix appearing instead of the three by three SU(3) one [24]. New "mesons" are

isosinglet $S_p^+ = p'\bar{\lambda}$

isodoublet $D_p^+ = p'\bar{n}$; $D_p^o = p'\bar{p}$

and the SU(3) singlet χ. New quark p' carries charm and charge and is an SU(3) scalar. It is convenient to denote the transformation properties of the weak current by "mesons", like, for example,

$$J_\mu^W = \cos\Theta_c(\bar{p}n)+\sin\Theta_c(\bar{p}\lambda-\bar{p}'\lambda)+\cos\Theta_c(\bar{p}'n)$$

$$\Rightarrow \cos\Theta_c\ \pi^- + \sin\Theta_c(K^- - S_p^-)+\cos\Theta_c D_p^- \qquad (3)$$

The γ-matrices determining spatial transformation properties of the current were suppressed.

The products of currents can be further decomposed in SU(3) tensors. Altough, for example, both K^-K^+ and $D_p^- D_p^+$ products lead to the SU(3) tensors transforming as isovectors, their reduced matrix elements are obviously different. K^- and D_p^- can, in principle, experience very much different strong interactions. We list tensors, and the corresponding products of currents as follows:

$$T_{\Delta S} \sim \pi^- K^+ \qquad T''_{\Delta S} \sim S_p^- D_p^+$$

$$T_1 \sim K^-K^+,\ \pi^o\eta \qquad T_1'' \sim D_p^- D_p^+ \qquad \hat{T}_1 \sim \pi^o\chi \qquad \hat{\hat{T}}_1 \sim \pi^o U$$

U is SU(4) scalar

Tensors in the same row have the same SU(3) transformation properties, but only the tensors in the same column have, via Wigner-Eckart theorem, the same reduced matrix elements. The effective interaction for $\Delta S \neq 0$ and for $\Delta S = 0$ transitions, respectively, transform as

$$\Delta S \neq 0 \quad H_W \sim \sin\Theta_c\cos\Theta_c(T_{\Delta S} - T_{\Delta S}'') \qquad (4)$$

$$\Delta S = 0 \quad H_W \sim (\sin^2\Theta_c + a)T_1 + \cos^2\Theta_c T_1'' + b\hat{T}_1 + c\hat{\hat{T}}_1 \qquad (5)$$

The constants a, b and c do <u>not</u> have any Cabibbo angle dependence and are of the order one. It is immediately obvious that the effective interactions (4) and (5) respectively lead to matrix elements that cannot be connected by SU(3) transformation properties alone. As (5) contains terms propotional to $\cos^2\Theta_c$ or 1, the NNπ amplitude might be (<u>but need not be</u>) enhanced in comparison with charged currents only model. Similar conclusions have been reached in ref. [32] for the three triplet quark model and seem to hold for the Georgi-Glasgow [31] model, too.

<u>Georgi-Glashow model</u> [31] as investigated in ref's [29] and [33] necessitates inclusion of even larger number of quarks with the following charges

$$Q = \begin{matrix} 1 & pp' \\ 0 & n\lambda qq' \\ -1 & rr' \end{matrix}$$

While p,n and λ are the usual SU(3) triplet quarks, all other quarks are SU(3) singlets. Quarks p,r,r' are charmed while q has zero hypercharge.

In ref. [33] was assumed that nonleptonic hyperon decays were dominated by the Higgs boson exchange contribution. The part of the scalar density which contributes to P.V. processes is of the form

$$J_\phi = \sin\beta\cos\beta[m\bar{N}_c\frac{1+\gamma_5}{2}q - m_q\bar{N}_c\frac{1-\gamma_5}{2}q + m\bar{\lambda}_c\frac{1+\gamma_5}{2}q' - m_{q'}\bar{\lambda}_c\frac{1-\gamma_5}{2}q' + h.c.] \qquad (6)$$

Here the SU(3) symmetry breaking is neglected, i.e., $m=m_\lambda=m_p=m_n$, β is a model parameter and

$$\begin{aligned} N_c &= n\cos\Theta_c + \lambda\sin\Theta_c \\ \lambda_c &= -n\sin\Theta_c + \lambda\cos\Theta_c \end{aligned} \qquad (7)$$

Assuming m_q, $m_{q'} >> m$ and keeping only dominant terms, Lee and Treiman [33] could reproduce $|\Delta\vec{I}| = \frac{1}{2}$ rule for the nonleptonic hyperon decays. Using Fierz reordering theorem, one can write

$$J_\phi J_\phi = T_{\Delta S \neq 0} + T_{\Delta S = 0} + \ldots \qquad (8)$$

$$\begin{aligned} T_{\Delta S \neq 0} \sim \sin\Theta_c\cos\Theta_c\bar{n}\gamma_\mu(1+\gamma_5)\lambda[m_q^2\bar{q}\gamma_\mu(1-\gamma_5)q - \\ - m_{q'}^2\bar{q}'\gamma_\mu(1-\gamma_5)q' + h.c.] \end{aligned} \qquad (9)$$

$$\begin{aligned} T_{\Delta S = 0} \sim \bar{n}\gamma_\mu(1+\gamma_5)n[\cos^2\Theta_c m_q^2\bar{q}\gamma_\mu(1-\gamma_5)q + \\ + \sin^2\Theta_c m_{q'}^2\bar{q}'\gamma_\mu(1-\gamma_5)q' + h.c.] \end{aligned} \qquad (10)$$

In (8) we have left out charmed parts which do not contribute. Actually we obtain two different tensors, depending on whether we combine normal quarks with q or q' quarks

$$T_{\Delta S \neq 0} = \sin\Theta_c \cos\Theta_c (m_q^2 T_{\Delta S \neq 0} - m_{q'}^2 T^{q'}_{\Delta S \neq 0}) \tag{11}$$

$$T_{\Delta S = 0} = \cos^2\Theta_c m_q^2 T^q_{\Delta S = 0} + \sin^2\Theta_c m_{q'}^2 T^{q'}_{\Delta S = 0} \tag{12}$$

Thus the knowledge of the $T_{\Delta S \neq 0}$ matrix element does not allow us any prediction for $T_{\Delta S = 0}$ without some additional dynamical assumptions. However, again the enhancement for the $NN\pi$ amplitude is possible. It would be (for example) clearly the case if $m_q >> m_{q'}$. Inclusion of the other terms from (6) does not, obviously, improve predictability. The same goes when one considers vector meson exchange contribution. As in the model under consideration, there are no neutral weak currents we have to consider only decomposition of the charged one

$$J^w_\mu = J^{(8)}_\mu + J^{(3)}_\mu + J^{(3*)}_\mu + J^{(singlet)}_\mu \tag{13}$$

Here, the upper indices denote SU(3) transformation properties. The parts are as follows

$$\begin{aligned} J^{(8)} &= \cos\Theta_c \bar{p}n + \sin\Theta_c \bar{p}\lambda \\ J^{(3)} &= -\bar{N}_c r - \bar{\lambda}_c r' + \mathrm{ctg}\beta(\bar{p}_q) + \frac{1}{\sin\beta}(\bar{p}_q)_- \\ J^{(3*)} &= \bar{p}'\lambda_c \end{aligned} \tag{14}$$

The minus sign in the second row indicates that the particular bilinear combination comes with $1-\gamma_5$ instead of $1+\gamma_5$ projection operator. The products of currents have the structure

$$\bar{J}^W J^W = \bar{J}^{(8)} J^{(8)} + \bar{J}^{(3)} J^{(3)} + \bar{J}^{(3^*)} J^{(3^*)} + \bar{J}^{(singlet)} J^{(singlet)} + \ldots \tag{15}$$

The left out terms do not contribute. The first term is the one appearing in the standard Cabibbo-model. The second term already leads to the combinations of tensors which do not allow for any predictions. We have

$$\begin{aligned} \bar{J}^{(3)} J^{(3)} &= \sin\Theta_C \cos\Theta_C [T^r_{\Delta S \neq 0} - T^{r'}_{\Delta S \neq 0}] + \\ &+ \cos^2\Theta_C T^r_{\Delta S=0} + \sin^2\Theta_C T^{r'}_{\Delta S=0} + T^q_{\Delta S=0} \end{aligned} \tag{16}$$

Superscript indicates quarks involved in a particular term. Even with $T^q \equiv 0$ we are still unable to predict $\Delta S=0$ amplitude.

Outlook

It should be obvious from the last section that P.V. nuclear effects are even more important for studying unified field theory models of weak interactions than they were for the conventional models. In the latter case, model could be used, in principle, to predict experimental results. In the former case, nuclear P.V. experiments are our only means, so far, to learn something about particular aspect of the model. It is, therefore, very urgent to have accurate experiments in which only $|\vec{\Delta I}|=0$ or only $|\Delta\vec{I}|=1$ isospin changes are possible [8] . The theoretical nuclear physics part of the calculation has to be perfected to allow unambiguous extraction of the weak model parameters from the experimental data. Better understanding of the strong interaction dynamics, especially in connection with the unified field theory, is needed for a more dependable derivation of the nuclear potentials. Talks presented in this seminar indicate efforts and advances made or planned in the research of parity violating nuclear forces so we might hope to learn some very interesting facts in the near future.

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DIRECT PARITY VIOLATING TRANSITIONS IN NUCLEI

ETTORE FIORINI

Istituto di Fisica dell'Universitá di Milano
Istituto Nazionale di Fisica Nucleare- Sezione di Milano

Various and often beautiful experiments on parity non conservation in nuclei have been carried out in recent years as the only way to investigate weak interactions involving only non-strange hadrons [1], [2]. Various experiments have been carried out on the existence of pseudoscalar terms like circular polarization of γ-rays, angular asymmetries in γ-emission from polarized nuclei or in scattering of protons. As an example I would like to recall the pioneer experiment by V.M. Lobashov [3] on circular polarization of γ-rays from ^{181}Tl (Fig.1.). The polarizations of photons from the source was measured by Compton scattering in iron placed in a magnetic field whose direction was reversed periodically (τ=2seconds). This reversal induced an alternating component on the current from the photomultiplier viewing the CsI(Tl) crystal. This current is amplified and transformed into a mechanical force that drives an astronomical pendulum tuned on a period of 2 seconds. Note that the experiment allows also to determine when the pendulum started its oscillations and therefore to determine the sense of the polarization and therefore to determine the sign of the parity violating parameter F.

Of the various experiments on angular correlations I would like to mention those that found a very relevant effect in ^{180m}Hf (1.6%!). A high polarization of the nucleus has been obtained by refrigerating [4] a Hf, Zn, Fe alloy which is ferromagnetic at low temperatures. The source was polarized with an external field, which was periodically rotated among angles of 0°, 90° and 180° and the γ-rays were counted by two Ge(Li) detectors placed on either side of the source.

An elegant experiment of a different type is presently carried on at Seattle [5] on the angular correlation of the 120keV $(\frac{1}{2})^-$ state of ^{19}F. This nucleus is polarized by producing it in the ^{22}Ne(p,α)^{19}F reaction using a transversely polarized beam of 4.96 MeV protons. The parity violating anisotropy is measured by placing identical thin window Ge(Li) detectors on either side of the gas target and by flipping the beam ten times per second.

Experiments of these types have shown the presence of pseudoscalar terms in ^{2}H, ^{41}K, ^{75}Ar, ^{114}Cd, ^{175}Lu, ^{180m}Hf, ^{181}Tl and ^{203}Tl. These effects seem, however, one or two order of magnitude higher than those predicted theoretically. This could be attributed if not due to "trivial" nuclear effects, to parity violation in electromagnetic interactions [6], [7] or to the presence of weak neutral currents [1].

It is therefore important to investigate direct parity violating alpha decays, or alpha interactions, whose rate is however very low, of the order of the square of the parity violating constant. Most of the experiments have been carried out on ^{16}O, whose exited levels are shown in Fig.2., where one can see that three levels look interesting at 8.872, 10.952 and 11.080 MeV, with $J^{\pi}=2^{-}$, 0^{-} and 3^{+}, respectively.

The 8.872 MeV level is the easiest to obtain, since it can be produced by beta decay of ^{16}N. The branching ratio for β decay to this level is 1.1%, while that to the "parity allowed" level at 9.597 MeV is about three orders of magnitude lower (1.2×10^{-3}).

Two experiments have yielded positive results on alpha decay from the 8.872 MeV level. In the experiment by E.L. Sprenkel <u>et al</u> [8] the radioactive ^{16}N gas produced by the $^{19}F(n,\alpha)^{16}N$ reaction is frozen at 15 m from the fluorine target on a surface in proximity of a Si(Li) detector. The α particle spectrum is dominated by the allowed decay from the nearby 9.597 MeV level. A peak of 2.5 standard deviations appears however in the region corresponding to the decay from the 8.872 MeV level ($E_{\alpha}=1.280 \pm 0.002$ MeV).

In the experiment by H.Hatting <u>et al</u> [9], [10] the parent ^{16}N nucleus was obtained by the reaction $^{15}N(d,p)^{16}N$ by bombarding nitrogen enriched in ^{15}N. The radioactive gas was allowed to flow (Fig.3.) into two identical detection chambers, located 1 metre apart, each containing 4 thin windows viewed by surface barrier detectors. After 2000 hours of measurements and 1000 hours of testing a spectrum corresponding to 2.5×10^{8} alpha particles was obtained, which is dominated by the broad distribution from the 9.597 MeV level. The deviations from the fit to this distribution in the region of the parity forbidden transition are given in units of mean standard deviations in Fig.4. From a χ^2 test one obtains a value of 76 on 34 free parameters when one tries to fit the experimental data in the region of interest with a smooth background curve. The contribution of the channels 30-34 to this value is 34. As a consequence, the authors take the bump in the region of interest as evidence for the parity violating transition, corresponding to $9538 \pm 1150\alpha$ particle decays. The corresponding value for the parity violating width Γ^{PV} is $(1.03 \pm 0.25) \times 10^{-10}$eV.

Detailed theoretical calculations on this width will be reported by G. Fai [11], let me therefore attempt here only a very rough calculation. Since the 2^{+} level at 9.847 MeV is only 1.1 keV wide, the main contribution to the parity admixture with the 2^- level at 8.872 MeV should occur with the 2^+ level at 11.52 MeV. The two widths could therefore be related by the expression:

$$\Gamma_{\alpha}^{PV}(8.872) = F^2\Gamma_{\alpha}(11.52)\ \frac{P(8.872)}{P(11.52)}$$

where P are the penetrabilities.

By using the most recent result on the width of the 11.52 MeV state (110±10 keV) [12] one has $F \simeq 2\times10^{-7}$.

The other parity forbidden levels in Fig.2. look very promising:

a/ the (0^-) level at (10.952±0.003) MeV should be favoured by parity admixture with 0^+ states at 11.26 MeV (Γ_α=2.5 MeV) and at 14.0 MeV (Γ_α=4.8 MeV) [13]. The existence of these levels however has not been confirmed in the recent experiments by T. Marvin and P.P. Sigh [12], who have, on the other hand, proved the existence of a 0^+ state at 14.07 MeV with Γ_α=260±25 keV.

b/ the 3^+ level at (11.080±0.003) MeV, whose parity violating width should be rather large due to parity mixing with the near state at 11.44 MeV [14], [15].

Since these levels cannot be produced by α decay of ^{16}N, the Milano group [16] is studying them in the reaction [16]

$$\alpha + {}^{12}C \rightarrow {}^{16}O^* \rightarrow {}^{16}O + \gamma\text{-rays}$$

by exposing thin self-sustaining carbon targets to single charged ^4He beams from the CN Van de Graaff of the Laboratori Nazionali di Legnaro. Currents from 6 to 8μA were used during the experiment. γ-rays were measured by a 75 cm^3 coaxial Ge(Li) detector, whose pulses were sent, after amplification, to a 4096 channels Laben analyser.

In order to calibrate precisely the energy of the machine which is essential in this experiment, we have used a 10μgr/cm^3 target enriched to 90% ^{13}C and we have searched for the threshold of the reaction

$$\alpha + {}^{13}C \rightarrow {}^{17}O^* \rightarrow {}^{16}O(6.131\ \text{MeV}) + n$$

followed by emission of 6.131 MeV γ-rays. The spectrum for this reaction is shown in Fig.5.

The parity violating processes have been studied using $20_{\mu}gr/cm^3$ targets enriched to 99.9% ^{12}C by repeating continuously the following series of measurements: a run, totalling 20mC of alpha particles, at an energy immediately below the resonant energy, 40 mC at the right energy and 20mC immediately above. Data were reported for runs totalling 1.6 C and 0.8 C for the 10.952 MeV and 11.080 MeV resonances, respectively. The spectra at the energies below and above were subtracted from the spectrum at the "right" energy. No evidence for parity non conserving effects appears in these "difference spectra" for both levels. The following limits on parity non conserving widths were obtained with 80% confidence level:

$$\Gamma_{\alpha}(10.952) < 6\times10^{-4}eV \text{ and } \Gamma_{\alpha}(11.080) < 5\times10^{-4}eV$$

The corresponding limits on parity non conserving parameters have been evaluated as for the 8.872 MeV level. Let us note that here the penetration factors for the parity forbidden states are practically equal to those of the parity allowed ones. In the case of the 0^- level at 10.952 MeV the difficulty lies in the lack of knowledge of the 0^+ states in the nearby energy region. If only the narrow level at 14.07 MeV would exist, the limit on F^2 would be 2.8×10^{-9}; if, however, levels like those at 11.26 of 14.00 MeV would be present, this limit would be lower by 10-20 times.

The parity violating process for the 11.080 MeV level should be dominated by the well-established level at 11.44 MeV and should be 6×10^{-10}. It could be lower by a factor of two if also the rather ambigous 3^- level at 11.63 MeV would exist. Measurement on these levels are in progress.

The Milano group is also beginning an experiment on the reaction

$$\alpha + {}^2H \rightarrow {}^6Li(3.562\ MeV) \rightarrow {}^6Li + \gamma$$

where parity is violated and isospin is changed by one, since ^4He, ^2H and $^6Li(3.562$ MeV) are $(0^+, I=0)$, $(1^+,0)$ and $(0^+,1)$, respectively. The only result obtained so far is an upper limit by D.H. Wilkinson [17] in 1957 ($F^2<10^{-7}$), and is too high for a meaningful comparison with theory.

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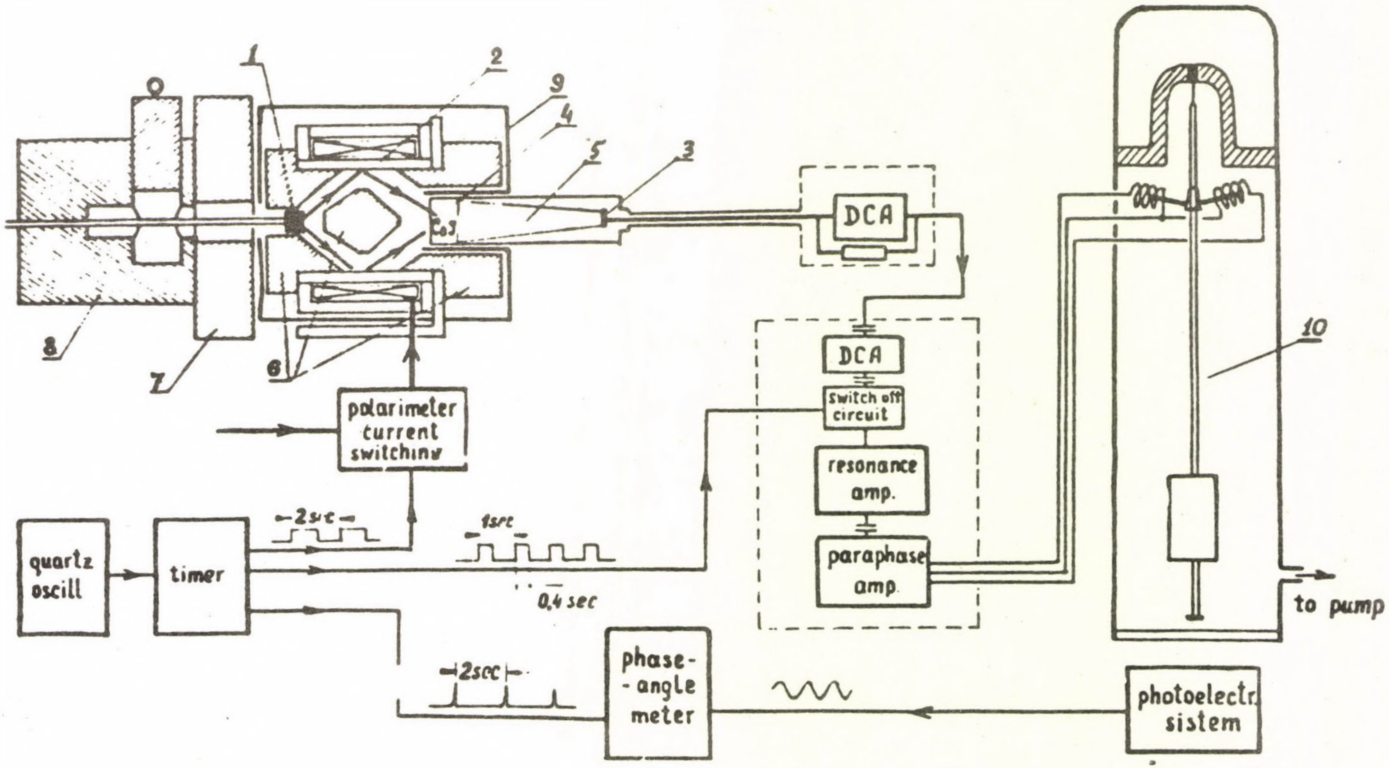
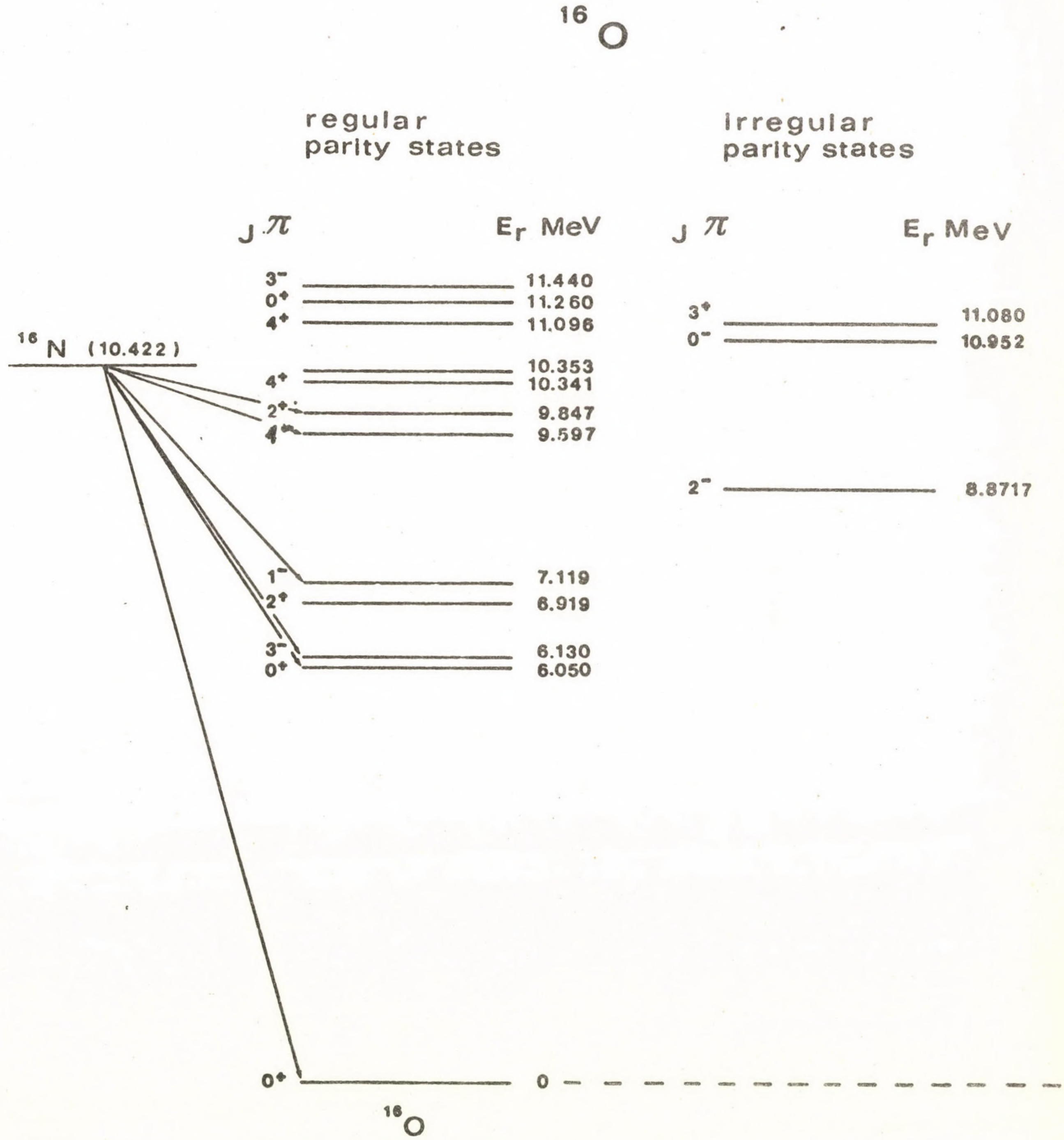
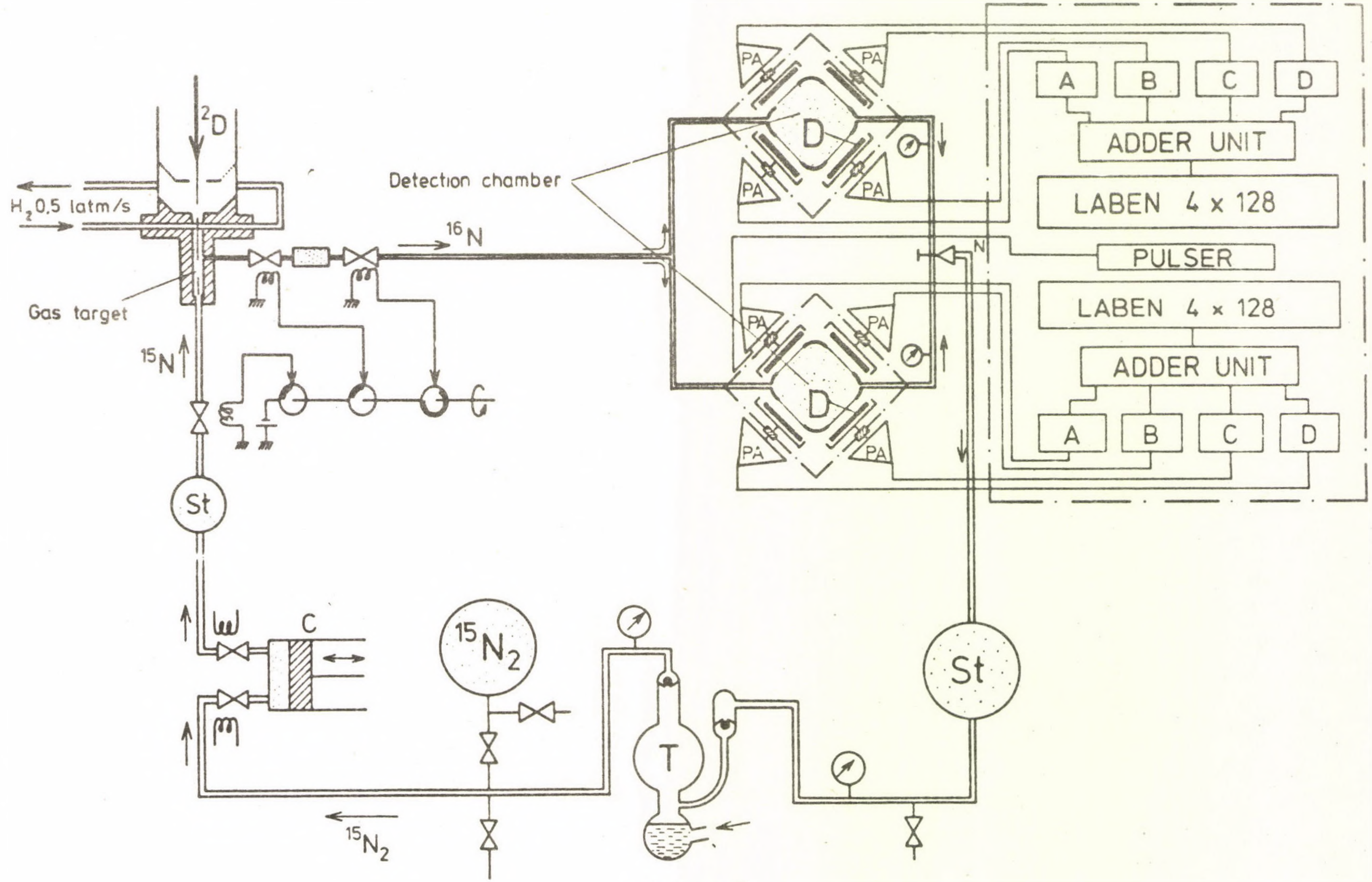

1
2
9
4
5
3
DCA
DCA
switch off circuit
resonance amp.
paraphase amp.
10
to pump
8
7
6
polarimeter current switching
quartz oscill
timer
2sec
1sec
0,4 sec
2sec
phase-angle meter
photoelectr. sistem

Fig.1

Fig. 2.



16 O
regular parity states
irregular parity states
J π
E_r MeV
J π
E_r MeV
3⁻ 11.440
0⁺ 11.260
4⁺ 11.096
10.353
4⁺ 10.341
2⁺ 9.847
9.597
1⁻ 7.119
2⁺ 6.919
3⁻ 6.130
0⁺ 6.050
0⁺ 0
3⁺ 11.080
0⁻ 10.952
2⁻ 8.8717
16 N (10.422)
16 O



²D
H₂ 0,5 latm/s
Gas target
¹⁶N
¹⁵N
Detection chamber
PA
D
A
B
C
D
ADDER UNIT
LABEN 4 x 128
PULSER
LABEN 4 x 128
ADDER UNIT
N
St
C
¹⁵N₂
T
St
¹⁵N₂

Fig. 3

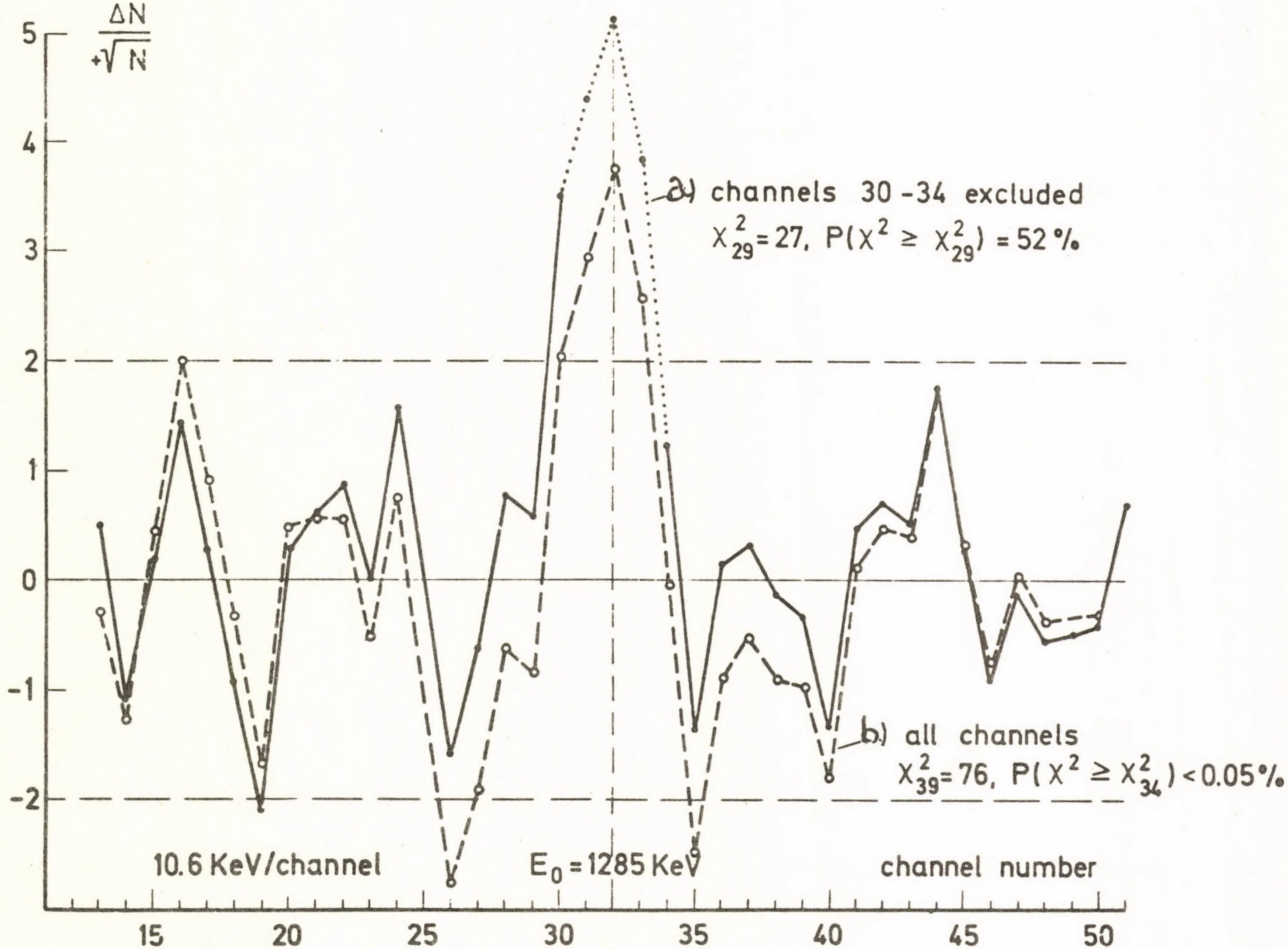

$\frac{\Delta N}{+\sqrt{N}}$
5
4
3
2
1
0
-1
-2
a) channels 30 - 34 excluded
$\chi^2_{29} = 27$, $P(\chi^2 \geq \chi^2_{29}) = 52\,\%$
b) all channels
$\chi^2_{39} = 76$, $P(\chi^2 \geq \chi^2_{34}) < 0.05\,\%$
10.6 KeV/channel
$E_0 = 1285$ KeV
channel number
15
20
25
30
35
40
45
50

Fig. 4

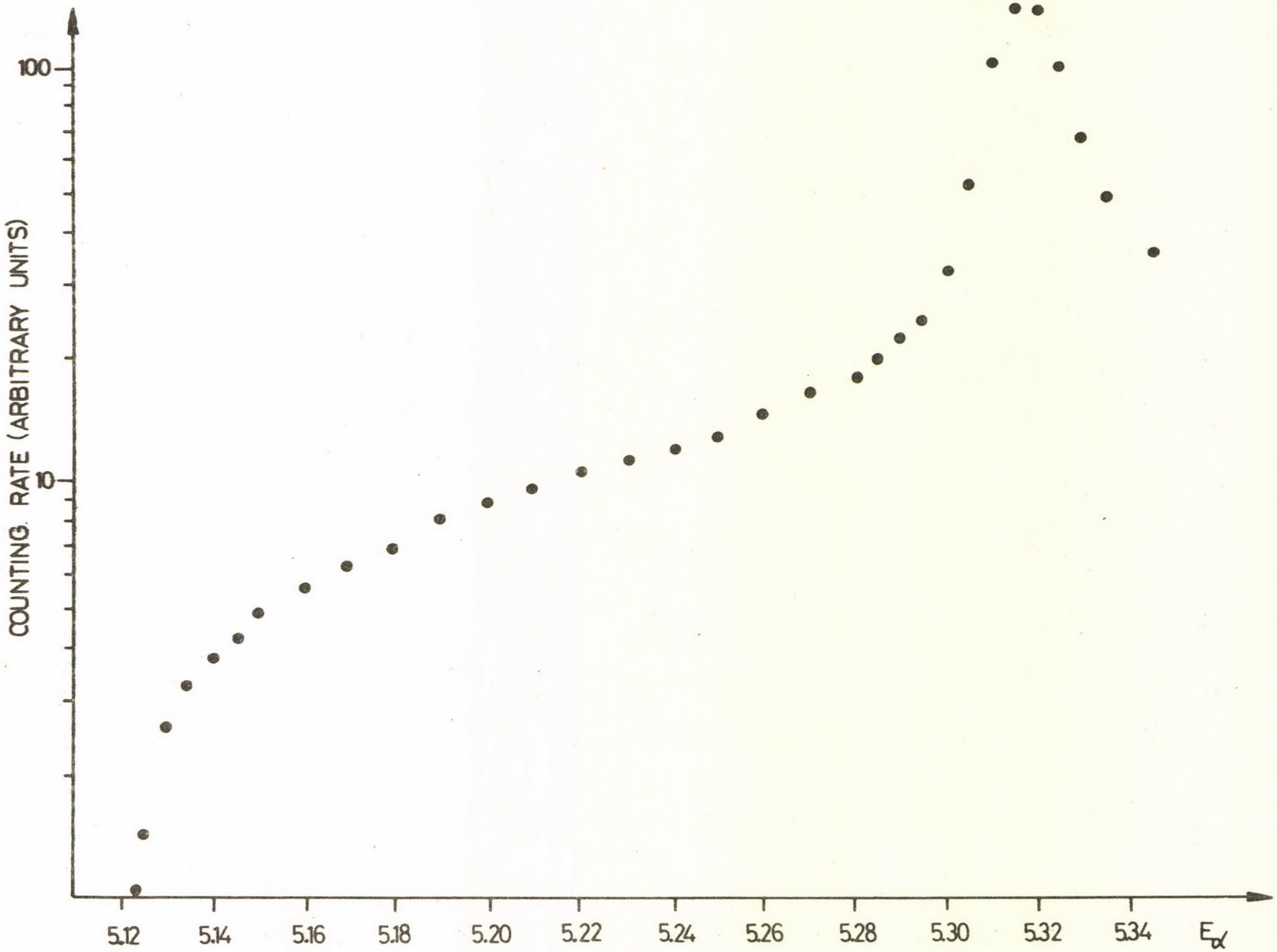

COUNTING RATE (ARBITRARY UNITS)
100
10
5.12
5.14
5.16
5.18
5.20
5.22
5.24
5.26
5.28
5.30
5.32
5.34
Eα

Fig. 5.

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PARITY-FORBIDDEN ALPHA DECAY OF ^{16}O

G. FÁI and J. NÉMETH and B. APAGYI
Institute for Theoretical Phisics, L. Eötvös University, Budapest
Physical Institute, Technical University, Budapest

Introduction

The nuclear forces have a small parity-violating part due to the parity-violation of the weak interaction. There are different methods to derive the parity-nonconserving (PNC) two-nucleon potential, some of which was outlined in a former talk [1]. The effects of the PNC forces can be examined in different phenomena, first of all in the alpha-decay and gamma-transition, as described before [2].

We want to concentrate on the parity-forbidden alpha-decay of the E=8.87 MeV 2^- level of ^{16}O nucleus. This excited state can decay into an alpha-particle and the ground state of the ^{12}C nucleus with L=2 relative angular momentum. The irregular alpha-decay width of this level has been found experimentally [3] to be:

$$\Gamma_{\alpha}^{PNC}(2^-, E=8.87\,MeV) = (0.98\pm0.30)10^{-10}eV$$

The outcome of our theoretical calculation will depend on the PNC potential used, and it will be strongly influenced by the nuclear physical approximations. In order to be able to say anything abouth the weak interaction models we decided to investigate the nuclear physical calculations and, if possible, to improve them. In the following we shall consider the PNC potential as an input.

Perturbation theory

Since the effect of weak interaction is small compared to that of strong interaction, i.e.

$$\frac{G}{g} \approx 10^{-6}$$

$$Gm_N^2 = 1.02 \cdot 10^{-5} \qquad \frac{g^2}{4\pi} \approx 14.4$$

(with G weak coupling constant, g strong coupling constant, m_N nucleon mass) we expect first order perturbation theory to apply in our case. Therefore we write the total Hamiltonian of the ^{16}O nucleus as

$$H(^{16}O) = H_o + V^{PNC} \tag{1}$$

where the PNC potential V^{PNC} is the sum of the weak PNC nucleon-nucleon potentials v_{ij}^{PNC}, i.e.

$$V^{PNC} = \sum_{i<j} v_{ij}^{PNC}$$

acting as a small perturbation. The usual parity-conserving (PC) nuclear Hamiltonian H_o is given as

$$H_o = \sum h_i + V_{res} \tag{2}$$

a sum of single particle Hamiltonians

$$h_i = t_i + U_i$$

(with t_i and U_i being the single particle kinetic and potential energy operator, respectively), plus some residual interaction V_{res} , which takes into account the effects, that cannot be described by the single particle potentials U_i, i.e.

$$V_{res} = \sum_{i<j} v_{ij}^{PC} - \sum U_i$$

where v_{ij}^{PC} is the strong PC nucleon-nucleon potential.

If we suppose that the spectrum of H_o is known

$$H_o |J(\Pi), i\rangle = E_{oi} |J(\Pi), i\rangle \tag{3}$$

(meaning that nuclear physics in a sense has been solved, what happens not to be the case), the eigenfunctions of the total Hamiltonian (1) can be written as

$$|J, i\rangle = |J(\Pi), i\rangle + \sum_n \frac{\langle J(-\Pi), n | V^{PNC} | J(\Pi), i\rangle}{E_{oi} - E_{on}} |J(-\Pi), n\rangle \tag{4}$$

Here $|J(\Pi), i\rangle$ means an eigenfunction of H_o with a given parity Π, E_{oi} and E_{on} are the energies of the states i and n, respectively.

In the wave function (4) parity is no longer a good quantum number, to the i-th eigenstate of H_o given by (3), all the opposite parity states of the same angular momentum are admixed with the admixing coefficients containing all information about parity-violation. It should be noted that (4) is an eigenfunction of the total Hamiltonian $H(^{16}O)$ up to first order in G if the summation appearing in the expression is carried out completly. However, this is clearly not possible, therefore, following Gari [4] we limit ourselves to the terms in which the absolute value of the denominator $|E_{oi} - E_{on}|$ is not too big.

Microscopic theory of the alpha-decay

To determine the parity-forbidden alpha-decay width one has to consider first the general theory of the allowed alpha-decay. There are several theories of the alpha-decay [5], [6]. The theories agree upon that they devide the space into an inside domain and an outside one. Roughly speaking the inside domain corresponts to the parent nucleus of radius R_o, and to get into the outside domain the alpha-particle has to penetrate the Coulomb barrier. To derive an executable expression for the alpha-decay width, one has to make approximations. The final result of the theories can be given as follows: the decay width Γ_n of the parent nucleus being in the n state is equal to the product of two factors, the penetrability P and the reduced width $|\gamma_n|^2$, i.e.

$$\Gamma_n^{PC} = P(E,R_o)\cdot|\gamma_n(R_o)|^2 \quad (5a)$$

where E is the kinetic energy of the emerging alpha-particle and R_o is the channel radius, where the inner and outer domains join. The penetrability is the probability that the alpha-particle penetrates the barrier (see fig. 2b). The reduced width represents the probability of an alpha-particle to be formed at the nuclear surface (see fig. 2a). This factor contains all informations about the nuclear structure and can be expressed as the overlap integral on the surface R_o:

$$\gamma_n = \langle \left(A^{daughter}+\alpha\right)_L \, | A_n^{parent}[J(\Pi),E_n] \rangle_{R_o} \quad (6a)$$

where $J(\Pi)$ is the angular momentum and parity, E_n is the energy of the n state of the parent nucleus A_n^{parent}, L is the relative angular momentum of the daughter nucleus $A^{daughter}$ and the alpha-particle α. In the following we assume that $A^{daughter}$ is in its ground state. In the calculation of γ_n one generally assumes that the center of mass of A^{parent} and $A^{daughter}$ are the same, while the center of mass of the alpha-particle is at R distance apart, and one has to calculate the overlap integral at $R=R_o$. This assumption is reasonably good for heavy nuclei but one cannot apply it for ^{16}O, where the recoil effect must be taken into account.

Knowing the alpha-decay theory for allowed transitions, one can calculate the PNC alpha-decay width of the 2^-, $E=8.87$ MeV level of the ^{16}O nucleus by substituting Eq. (4) into Eq. (6a). One gets as a result [4]

$$\Gamma_{8.87}^{PNC} = P(E,R_o) \; |\sum_n F_n \gamma_n(R_o)|^2 \quad (5b)$$

where the overlap integrals

$$\gamma_n = \langle \left(^{12}C+\alpha\right)_{L=2} | \, ^{16}O(2^+,E_{on}) \rangle_{R_o} \quad (6b)$$

and the admixing coefficients

$$F_n = \frac{\langle 2^+,E_{on} | V^{PNC} | 2^-,8.87 \rangle}{8.87-E_{on}} \quad (7)$$

are to be calculated by considering the nuclear many-body problem.

It is clear from Eq. (5b) what we are facing two difficulties: the calculation of the quantities P and γ_n using some kind of alpha-decay theory, and the determination of the numbers F_n.

One should think at first sight that the best thing to do is to extract the reduced widths $|\gamma_n|^2$ from experiments. Looking at the level scheme of ^{16}O (fig.1), one realizes that this does not work since the reduced width of the E=6.90MeV level below treshold cannot be measured. On the other hand there is no reason why it should contribute less than the levels above threshold being at about the same energy distance apart from the 2^- level than the one at E=6.90 MeV. Moreover, since γ_n enters quadratically the measured regular widths (Eq.(5a)), its sign cannot be determined from experiments. This sign is nevertheless important, because interference terms show up in Eq. (5b) as soon as one takes not only one 2^+ level into account.

Unfortunately, we do not have a fairly reliable alpha-decay theory for light nuclei. If we calculate the decay width of light nuclei using prescriptions of the existing alpha-decay theories, we find that even for the relative decay widths they give results which do not agree with the experimental ones. Recently, Arima and Yoshida have shown [7] that these difficulties arise from the approximations made by the alpha-decay theories going from the exact starting point to the final result (Eq.(5a)). Although this final result is very clear, the theory is good only if the width Γ_n^{PC} does not depend too sensitively on the radius R_0. Making Eq. (5a) less dependent on R_0 we hope that we get a more reliable value for the regular reduced width of the E=6.90MeV 2^+ level below threshold.

The improvements of the alpha-decay theory are in progress.

Nuclear many-body problem

We want to calculate the admixing coefficients F_n (Eq.(7)). In order to do this we need "good" wavefunctions for the 2^+ and the 2^- states involved. For this purpose as a first approximation we use Zuker's wavefunctions as given in [8]. The main components of these wavefunctions are 2 particle-2 hole and 4 particle-4 hole excitations of the ^{16}O core in the case of the 2^+ states, while 1 particle-1 hole and 3 particle-3 hole excitations in the case of the 2^- state. More precisely speaking four nucleons are distributed in all possible ways among the 1p1/2, 1d5/2, 2s1/2 subshells leaving a ^{12}C core inert. The total wavefunction of the state is constructed as a linear combination of these several excitations:

$$\bar{\Psi} = \sum c_i \psi_i \qquad (8)$$

The coefficients c_i are determined by diagonalization. The predictions of the Zuker wavefunctions seem to agree with experimental data in a number of cases fairly well (e.g.[9]).

Starting from these wavefunctions one is able to reduce the matrix element in eq. (7) to a complicated sum where PNC effects enter the form of PNC two-body matrix elements only. The value of these two-body matrix elements depends sensitively on the wavefunctions used for the description of the two-particle states. Taking harmonic oscillator single-particle wavefunctions, the two particle state can be easily transformed into relative and center of mass coordinates [10] . In this case, however, also the relative wavefunction will be a harmonic oscillator one, sketched in fig. 3 for the simplest case. Figure 3 shows that this relative wavefunction is considerable within the repulsive core of the strong nucleon-nucleon interaction, in other words, it does not account for the short range correlation between the nucleons. On the other hand, short range correlations should be included because of the extreme short range behaviour of v^{PNC}. In principle one should carry out a complete Bruckner-Hartree-Fock calculation to get the correlated wavefunctions.

Another problem with the harmonic oscillator wavefunctions is that they do not describe well the nucleus on and outside the surface. This is because the harmonic oscillator potential has a large positive value in this domain instead of almost zero.

Therefore we use as a first approximation the single-particle wavefunctions of a density dependent Hartree-Fock calculation. These wavefunction give good total and single-particle energies, radii and densities of the ground state spherical nuclei (including ^{16}O) [11].

To get these wavefunctions one made the following approximation. In the Brueckner-Hartree-Fock theory the relative two-body wavefunction is determined from the integral equation

$$\psi = \phi + \frac{Q}{e} v\psi = \Omega\phi \tag{9}$$

where ψ and ϕ are the correlated and uncorrelated wavefunctions, respectively, v is the strong two-body interaction, e means the energy denominator and Q is a projection operator due to the Pauli principle: it projects into states which are above the Fermi surface. Ω is an operator which changes ϕ into ψ.

Using (9) an effective two-body potential can be defined by the requirement

$$\left.\begin{aligned} \langle\psi|v|\psi\rangle &= \langle\phi|v_{eff}|\phi\rangle \\ v_{eff} &= \Omega^{-1} v \Omega \end{aligned}\right\} \tag{10}$$

After deducing v_{eff} from nuclear matter many-body calculations, the ϕ wavefunctions were determined. To take into account the short range correlations one has to use either ψ instead of ϕ, or v_{eff}^{PNC} (calculated the same way as v_{eff}) instead of v^{PNC}. Doing so one expects to improve on the shortcomings mentioned above.

After having completed the present computation we want to calculate the predictions of different models of v^{PNC}. We also plan to give a theoretical pediction for the irregular decay of the 3^+ and 0^- levels investigated experimentally by Fiorini et al. [2].

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Figure captions

Fig. 1. Partial level scheme of ^{16}O. Levels not relevant to our considerations are omitted.

Fig. 2. Sketch of the separation of the alpha-decay problem into two parts:

a/ the probability of the formation of the alpha-particle on the surface of radius R_o (inner problem characterized by the reduced width $|\gamma(R_o)|^2$),

b/ the probability of the penetration of the alpha-particle with energy E on the potential barrier plotted (outer problem described by the penetration factor $P(E,R_o)$).

Fig. 3. Short range behaviour of the uncorrelated relative wavefunction of two nucleons. It does not vanish within the repulsive core of the strong nucleon-nucleon interaction.

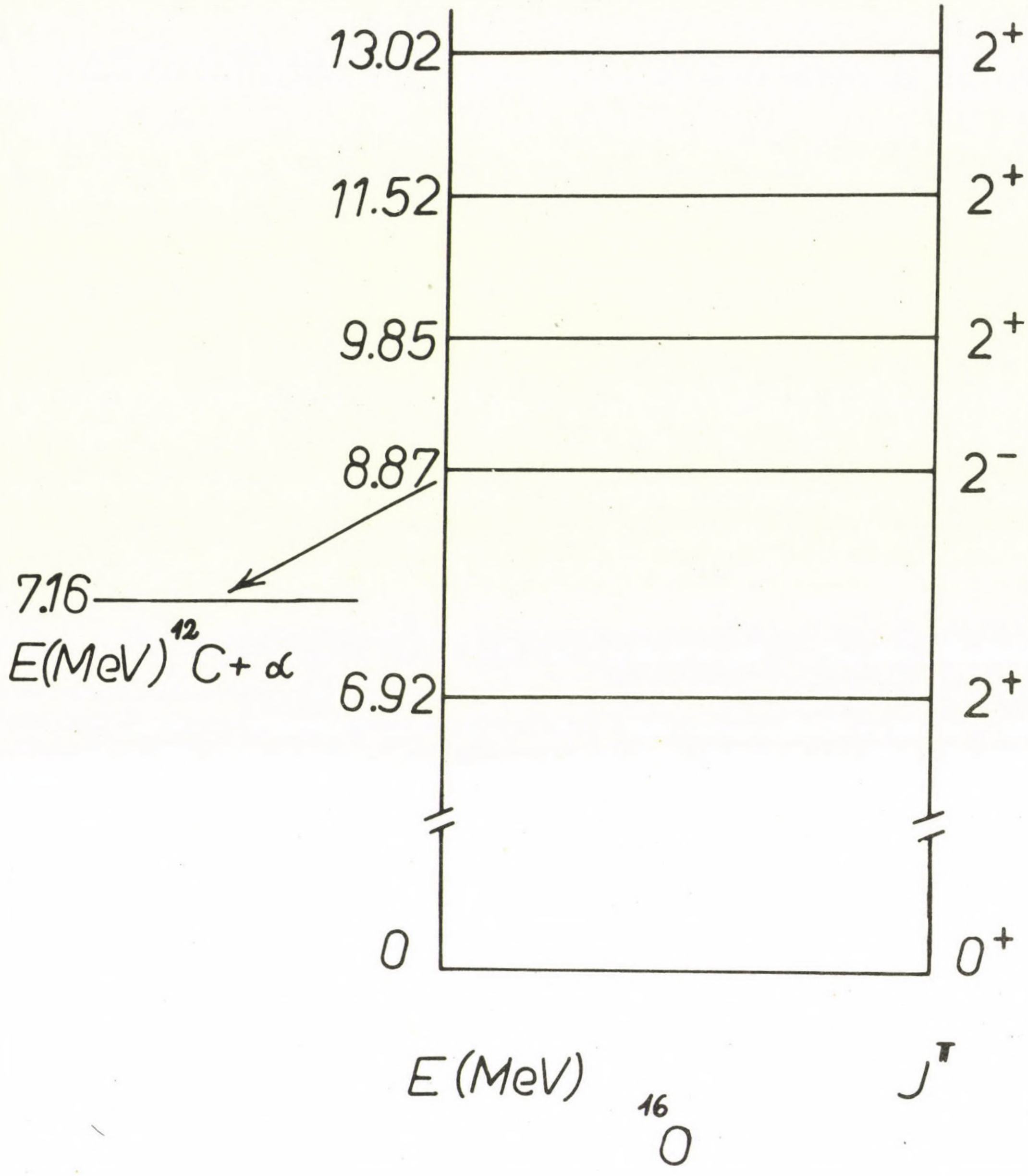

FIG. 1.

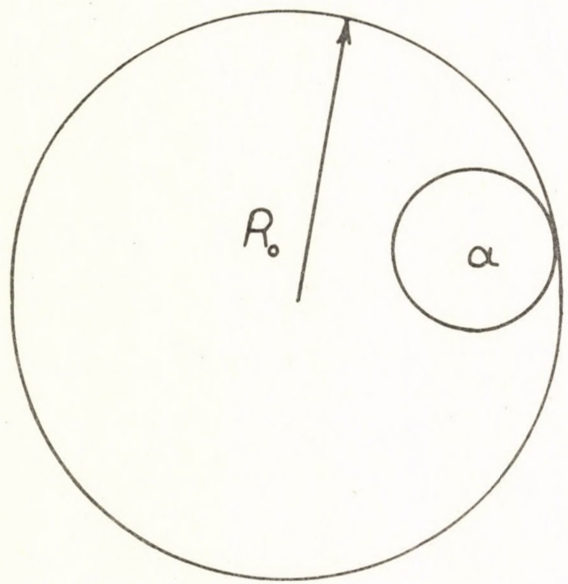

FIG. 2a

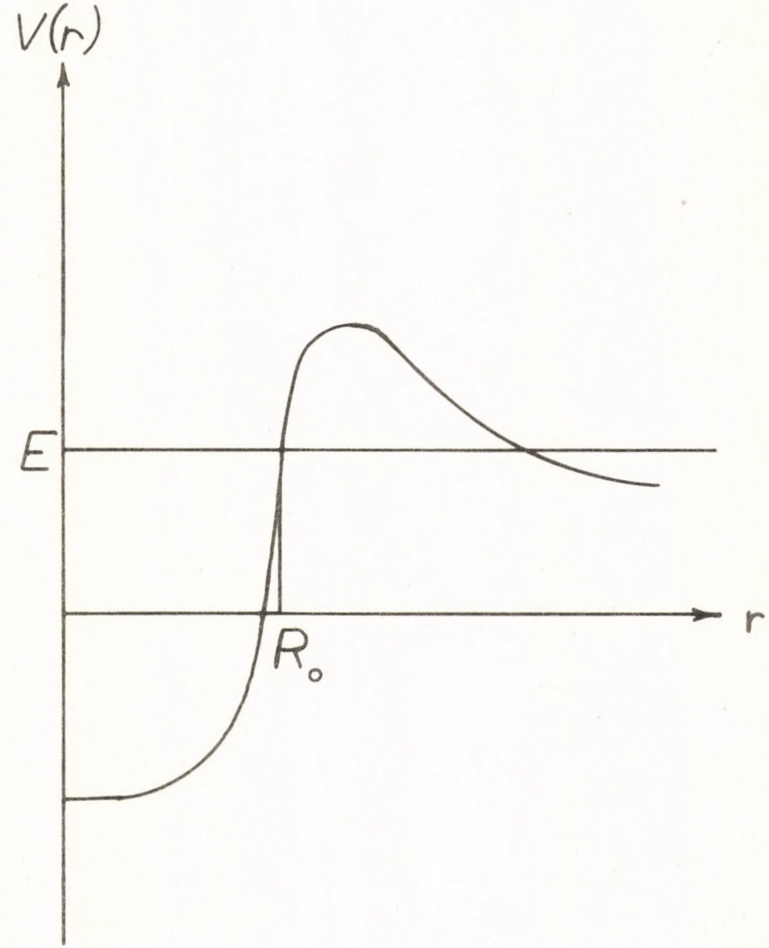

FIG. 2b

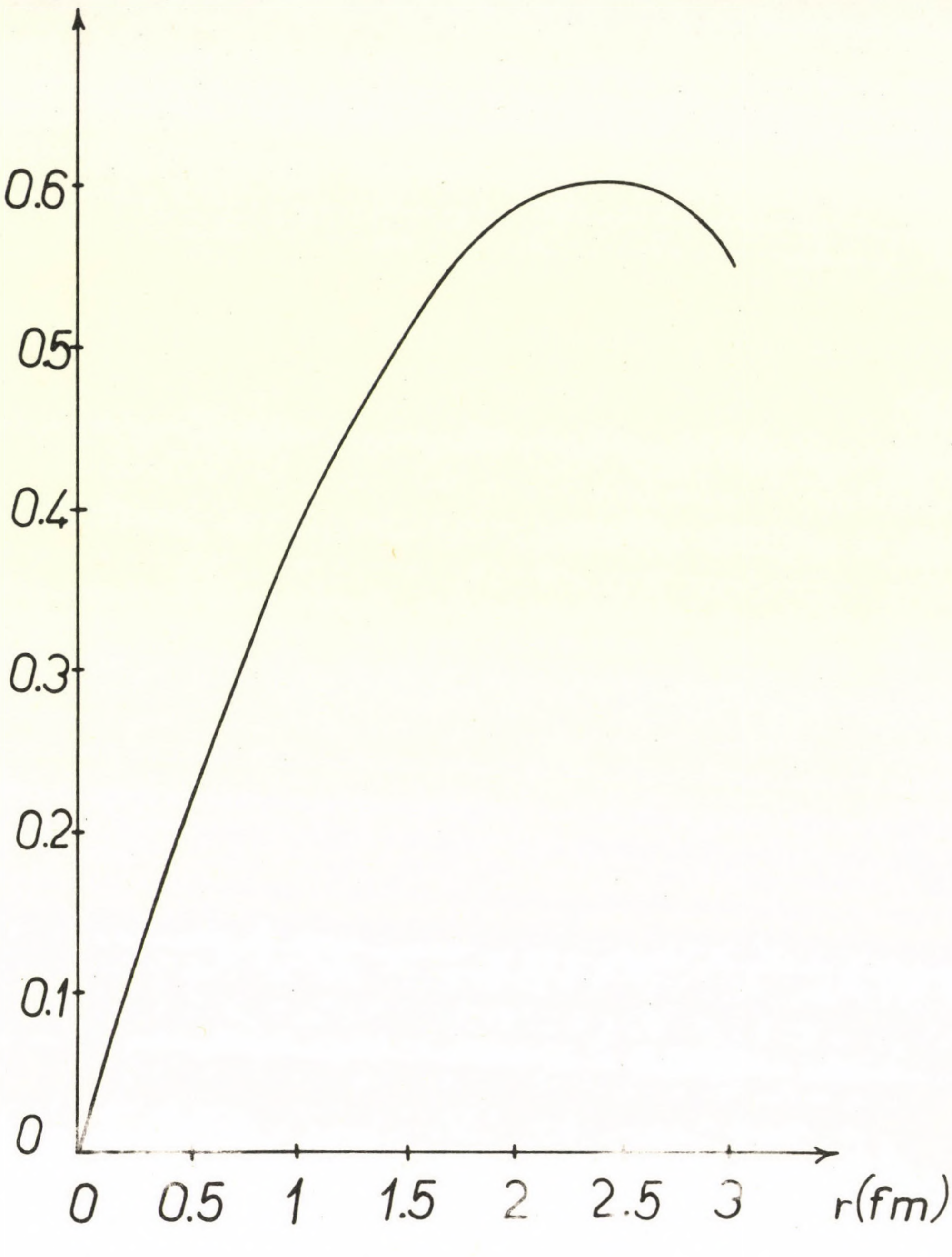

FIG. 3.

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THE NEUTRON ELECTRIC DIPOLE MOMENT AS A TEST OF THE SUPERWEAK INTERACTION THEORY

L. WOLFENSTEIN*)

CERN - Geneva

Theoretical calculations of the neutron electric dipole moment D_n are reviewed for various theories of CP violation. It is showm that for the superweak interaction theory D_n is less than 10^{-29} e-cm in contrast to values of 10^{-23} to 10^{-24} predicted by many but not all milliweak theories. It is concluded that prospective measurements of D_n may provide decisive evidence against or significant evidence in favour of the superweak theory.

1. Introduction

In the ten years since the discovery of CP violation little progress has been made in determining where this violation arises in the elementary particle interactions. It follows from the assumption of CPT invarience and from a detailed analysis[1] of K^0 decay experiments even without this assumption that the CP violating interaction is also non-invariant under time reversal. As a consequence it is possible that elementary particles may have non-vanishing electric dipole moments. So far experimental searches have not revealed such moments, but it has been possible to obtain an upper limit for the electric dipole moment D_n of the neutron as low as 1.0×10^{-23} e-cm [2]. It now seems possible [3],[4] to perform much more sensitive experimental measurement of D_n. In this note we review theoretical calculations of D_n to show how the problem of CP violation can be elucidated by such measurements and, in particular, stress the importance of such measurements as a test of the superweak interaction theory.

*) John Simon Guggenheim Foundation Fellow on sabbatical leave from Carnegie-Mellon University, Pittsburgh, Pa.

Theories of CP violation may be divided into four general groups.

Superweak interaction theory[5]

Here it is assumed that a new interaction exists which violates CP and is characterized by a coupling constant of the order 10^{-9} times the usual weak coupling G. There are two kinds of predictions that follow from the superweak theory:

Class A. All observable CP violating effects in K^0 decays may be explained in terms of a single parameter[6] which describes the mixing of the two CP eigenstates K_1 and K_2. Thus given one CP violating effect, such as the original observed rate of $K_L \to \pi^+ \pi^-$, al other CP violating observables are predictable. Within present accuracy such predictions have been borne out by experiment[7].

Class B. All CP violating and T violating effects in known processes other than those in K^0 decays are extremely small, of the order 10^{-9} or less*).

Milliweak theories**)

We call "milliweak" all theories in which CP violation is incorporated into the effective weak Hamiltonian in such a way that the consequences generally show up as a 10^{-3} correction to weak amplitudes. Many of these theories give the same class A predictons as the superweak and sometimes they are even labelled as having "superweak CP violation"[11] in the sense that CP violation in K^0 decays is entirely due to a single mixing parameter. However, these milliweak theories in general predict a CP violating effect in some known process at a level of 10^{-3} and so disagree with the class B predictions of the superweak theory. Unfortunately present experimental searches for CP and T violation seldom reach the level of 10^{-3} and so do not distinguish between milliweak and superweak theories.

**) For a discussion of examples of older milliweak theories see Refs. [8], and [9]. More recently theories of this sort have been developed [10],[11],[12], using the gauge formalism.

*) The superweak theory allows for CP violating effects of the order 10^{-3} in $\Delta S=2$ transition such as $\Xi \to N+\pi$, but these are so strongly forbidden that they have never been observed. If the superweak interaction is mediated by a special boson then processes involving real bosons may have large CP violation, but these bosons are so weakly coupled that their production is unlikely in the foreseeable future. Thus the limit of the order of 10^{-9} holds for "known processes"; that is processes that have been observed or are likely to be observed in the near future.

Electromagnetic CP violation

It has been suggested[13] that the electromagnetic interaction of hadrons is intrinsically CP violating but parity conserving. Thus weak amplitudes involving hadrons (such as those of parity violating or strangeness changing processes) obtain via radiative corrections a CP violating piece of the order $\alpha/\pi \sim 10^{-3}$. An alternative hypothesis[14],[15], which has similar consequences for weak amplitudes and for the neutron electric dipole moment, is that there exists an effective weak vertex for emitting photons that is CP violating and P violating and of order e times the usual weak coupling constant.

Millistrong CP violation

It is even conceivable that there exists in the strong interactions an effective coupling of the order 10^{-3} that violates CP but is P invariant[6],[16]. This idea has not been developed in great detail; for amplitudes other than those of strong interaction processes, this theory is equivalent to a milliweak theory. For these reasons we shall not discuss this class of theories further.

For orientation purposes we review the standard rough estimates of the electric dipole moment D_n of the neutron for the first three classes of theories[17]. The general approach is based on the assumption that D_n would be equal to the neutron magnetic moment or to the dimensional estimate e/M where M is the nucleon mass if it was not inhibited by the P and T selection rules. Thus in the case of electromagnetic CP violation there is no problem with T violation since we are dealing with an electromagnetic process, but we must pay with a factor G, the weak interaction coupling, for P violation. Thus we find*)

$$D_n \sim (e/M)(G/4\pi)M_x^2$$

where M_x is some characteristic mass required for dimensional reasons. In hadron physics characteristic masses are generally of the order M so that

$$D_n/e \sim (G/4\pi)M \sim 10^{-20}\text{cm} \qquad (1.E)$$

For milliweak theories we have an effective coupling that violates P and T of order 10^{-3}G so that

*) Throughout D_n signifies the magnitude of the electric dipole moment and we do not consider the sign.

$$D_n \sim (e/M)(G/4\pi)10^{-3}M_x^2$$

and setting $M_x=M$

$$D_n/e \sim 10^{-3}(G/4\pi)M \sim 10^{-23}\text{cm} \qquad (1.M)$$

Finally for the superweak theory, assuming the superweak interaction violates P as well as CP, the factor 10^{-3} is replaced by 10^{-9} and

$$D_n/e \sim 10^{-9}(G/4\pi)M \sim 10^{-29}\text{cm} \qquad (1.S)$$

These rough estimates are listed in the Table together with the experimental upper limit. These suggest three conclusion (1) that electromagnetic CP violation does not work (2) that present and prospective experiments are at the level to test milliweak theories, and (3) that it will be practically impossible to measure a non-vanishing D_n if the superweak theory is correct. More detailed calculations, all still very crude, are discussed in section 2 and their results are also shown in Table. While these modify and qualify the rough estimates, they generally confirm the conclusions.

Since the proton and neutron are linked by strong interactions one expects their electric dipole moments to be of the same order of magnitude. Occasionally calculations suggest a large difference between them, but these do not involve a realistic theory of the physical nucleon and therefore should not be taken seriously. Since experiments on the neutron are far more accurate than those on the proton we shall not discuss the proton further.

Similar considerations could be made for the electric dipole moment D_e of the electron and model calculations of D_e for milliweak gauge models are included in the paper by Pais and Primack[18]. Within each class of theories there is much more uncertainty in the estimate of D_e than there is for D_n. There are two reasons for this: (1) the mass scale may be defined either by the electron mass or by the masses of the hadrons or heavy leptons coupled to the electrons; (2) it is possible that CP violation is limited to the hadron sector in which case the electron dipole moment involves higher order weak interactions than if CP violation occurs directly for leptons. Thus the electron dipole moment provides a less clear distinction between the classes of interactions than does that of the neutron.

2. Calculations of the neutron electric dipole moment

There are two kinds of calculations of D_n. The first of these may be labelled "model independent" in the sense that for one of the three general classes of theories we are considering, the calculation is independent of any details of the model. The procedure is to consider a very limited class of intermediate states for the neutron, and to use the class of theory to determine the strength of the CP violating coupling of the neutron to those states. For the electromagnetic and milliweak cases this has been done by Barton and White[19], using a sidewise dispersion relation that has proved succesful in calculating the neutron anomalous magnetic moment. For the superweak interaction, we shall present a simple model independent calculation. The second kind of calculation is a detailed calculation using a specific model. Even for a given specific model it is not clear that this kind of calculation is better than the model independent one because the results are often rather ambiguous particularly since the models typically involve quarks rather than physical nucleons.

As an example of a model calculation with electromagnetic violation we show in the Table the order of magnitude of the result of Salzman and Salzman[20]. In their model[14] the intermediate boson has an intrinsic electric dipole moment so that CP violation shows up in weak electromagnetic transitions; their result agrees with the rough estimate. Many authors*), however have made the point in various contexts that electromagnetic CP violation may be suppressed for the zero momentum photon involved in the static moment. In their model independent calculation Barton and White[19] obtain a value as low as 10^{-23}cm for D_n/e if they assume dynamical suppression at zero momentum. In case of a large suppression effects of virtual photons become important providing a term of order $(G/4\pi)e(\alpha/\pi)$, yielding as a rough estimate $D_n/e \sim 10^{-23}$ cm. We conclude then that the present experimental limit on D_n provides a significant argument against electromagnetic CP violation but does not completely rule it out.

We turn to the milliweak theories, limiting ourselves to models which are expressed in the form of renormalizable gauge theories**). While the marriage of CP violation and gauge theories appears in all cases to be a forced one, these theories have

*) For example see comments of T.D. Lee on p. 256 of Ref.[8] or similar suggestions in Ref. [21].

**) Some numerical values based on older milliweak models are listed in the experimental paper of Miller et al.,[22] and in Refs. [8] and [9].

the advantage that the higher order diagrams needed to calculate D_n are in principle calculable. However, the calculated results always depend on unknown masses and strong interaction effects and so for practical purposes are no more definite than results that depend on unknown cut-offs. In spite of these drawbacks these models probably illustrate the possibilities of CP violation in the weak interactions better than earlier theories.

Pais and Primack[18] have calculated D_n for two milliweak models. In the first of these[10] CP violation appears as a relative phase factor ϕ between the V and A parts of certain strangeness changing currents; the factor ϕ is an arbitrary parameter in the theory and is given a value of 10^{-4} in order to reduce the amount of CP violation as well as to suppress other undesired effects. Pais and Primack find in this theory that the electric dipole moment D_λ of the λ quark is

$$D_\lambda \sim eG\phi m_Q \sim e(m_Q/M)10^{-23}\text{cm}$$

where M_Q is a quark mass. However, they find for the non-strange quarks that the electric dipole moment vanishes in lowest order so that for the neutron quark q they find

$$D_q \sim eG\phi m_Q \alpha \sin\theta/\pi^3 \sim e(m_Q/M)10^{-27}\text{cm}$$

where θ is the Cabibbo angle. In tabulating their results they use this lower value as the value of D_n, while presumably the larger value D_λ would be given as the value of D for the physical Λ particle. We consider this unreasonable, however, since the strong interactions couple the physical nucleon to a $\Lambda\bar{K}$ state; therefore, taking the strong interactions into account there is a non-vanishing contribution to D_n of order $eG\phi$ and it is not necessary to go to higher order in the weak interaction. The necessity of going to strange particle intermediate states may give a reduction factor of the order of 10 so that we conclude*)

$$D_n/e \sim \frac{1}{10}G\phi m_Q \sim 10^{-24}(m_Q/M)\,\text{cm}$$

The presence of a quark mass in the answer indicates one of the problems with the calculation, but a conservative approach is to set $m_Q \sim M$ which gives the result shown in the Table.

*) Such a reduction factor of 10 is found for the contribution from strange particle intermediate states in a somewhat different context by Barton and White[19].

In the other model[11] considered by Pais and Primack CP violation is introduced by a relative phase factor of $\pi/2$ between the couplings of the muon current to two charged intermediate bosons W_1 and W_2. In general this phase factor has no observable consequences in the lowest order of weak interaction but can produce effects of order α/π in the next order. Thus this theory is effectively milliweak even though no small parameter of order 10^{-3} is introduced. In a purely hadronic process CP violation is typically induced by a virtual loop involving a muon and a heavy neutral lepton y_o. In particular, the CP violation in the K^o mass matrix is given by

$$\frac{\mathrm{Im}(\Delta m_K)}{\mathrm{Re}(\Delta m_K)} = 2\times10^{-3} = \frac{\alpha}{\pi}\left[\frac{m(y_o)}{m(W_1)}\right]^2 f_s \qquad (2)$$

where f_s is some ratio of quark mass differences. Pais and Primack find for the neutron electric dipole moment

$$\frac{D_n}{e} \sim G\frac{\alpha}{\pi^3}\left[\frac{m(y_o)}{m(W_1)}\right]^2 d^o \qquad (3)$$

where d^o is the mass difference between two neutral charmed quarks. Combining Eqs. (2) and (3) we find

$$D_n/e \sim 2\times10^{-3}(G/\pi^2)(d^o/f_s) \,. \qquad (4)$$

Assuming that the combination of mass differences given by (d^o/f_s) is between 0.1 and 1.0 GeV we find $D_n/e \sim 10^{-23}$cm.

In the theory of T.D. Lee[12] CP violation is associated with the exchange of spin zero bosons and can be introduced as a form of spontaneous symmetry breaking using the Higgs mechanism. The smallness of the CP violation again involves the introduction of an arbitrary small parameter, although this can be related to quark mass differences. An estimate of the neutron electric dipole moment in this theory has been given by T.D. Lee[23] and is included in the Table.

We see in the Table that the various calculations for the milliweak theory all give values of the order 10^{-23}cm to 10^{-24}cm for D_n/e. In truth these "calculations" are really just slightly refined rough estimates. It is to be remembered that the calculations give explicit results in terms of unknown masses, special choices of which might give much smaller values of D_n. Furthermore these calculations give directly quark dipole moments from which it is necessary to estimate the physical nucleon dipole moment. The model independent calculation of Barton and White has the advantage of treating the nucleon more realistically. In their calculation the milliweak CP violation is introduced through the assumption that the P violating CP violating nucleon pion ($NN\pi$) vertex is 10^{-3} times the usual (CP invariant)weak P violating $NN\pi$ vertex. Since the usual weak vertex is proportional to $\sin^2\theta$ as a result of CP invariance[24], they obtain a result proportional to $\sin^2\theta$ and of the order 10^{-24}cm. However, we consider this $\sin^2\theta$ suppression unreasonable for the CP violating vertex (and it does not occur in the models discussed above) since it entered in the first place as a result of CP invariance and so we quote their result as 10^{-23}cm.

In spite of the agreement among these calculations of D_n for milliweak theories it must be emphasized that it remains easy to construct milliweak theories in which D_n is suppressed by an extra factor as large as 10^5 or 10^6. All that is required is that the CP violating part of the effective weak Hamiltonian satisfies a selection rule that prevents it from contributing to the electric dipole moment of any hadron. One example is that of theories in which all CP violating terms in the Hamiltonian have $|\Delta S|=1$ [9]. It is then necessary to go to the next order in the weak interaction to obtain another term with $|\Delta S|=1$ in order that the effective operator connects states of the same strangeness. Thus the rough estimate of D_n in such theories is

$$D_n \sim (e/M)10^{-3}(G/4\pi)^2 M_x^4$$

The choice of M_x^4 depends on the way on which higher order weak interaction effects are made convergent, but if we set $M_x \simeq M$ as we did before

$$D_n/e \sim 10^{-3}(G/4\pi)^2 M^3 \sim 10^{-29}\text{cm} \qquad (5)$$

as in the superweak theory. Recently a gauge model with this feature has been proposed[25] by Frenkel and Ebel; they estimate

$$D_n/e \sim \sin(\Delta\alpha)\times 10^{-26}\text{cm}$$

where $\Delta\alpha$ is the CP violating phase introduced in the model. If we set $\Delta\alpha=10^{-3}$ so that the CP violation is milliweak*), we find

*) Frenkel and Ebel discuss the possibility that $\Delta\alpha$ could be of the order of unity for a special choice of their parameters. This choice would seem to be ruled out by the failure to observe large T violations in Λ decays.

$D_n/e \simeq 10^{-29}$cm in agreement with the estimate of Eq. (5).

Another example is that of theories in which all CP violating terms are parity conserving[26]. It is again necessary to go to the next order in the weak interaction to produce the parity violation required for a non-zero electric dipole moment. Thus the estimate of Eq. (5) would again be expected to hold. Such models have the added feature of exactly reproducing the class A predictions of the superweak theory. We do not know of a gauge theory of this type, but the demonstrated dexterity of professional gauge theory modelists suggests that they can construct such a model if it be needed.

For the case of the superweak interaction, we know of no published calculations of D_n. The basic idea of the superweak interaction is that the CP violating interaction can be very weak because it contains a $\Delta S=2$ part that causes a mixing of K^0 and $\bar{K}^0$ (or K_1 and K_2) in lowest order. The strength of the superweak interaction H_{SW} is empirically defined by the assumption that the matrix element producing the mixing between K_1 and K_2 is uninhibited and representative. The magnitude of the observed CP violation determines

$$\langle K_1|H_{sw}|K_2\rangle \simeq 3\times10^{-3}(m_L-m_S) \simeq 2\times10^{-17}m_K \tag{6}$$

where m_L, m_S, m_K are the masses of K_L, K_S and K, respectively.

To estimate the magnitude of D_n we use a simple non-relativistic perturbation formula

$$D_n = \sum_m \frac{\langle n|H_{sw}|m\rangle\langle m|D|n\rangle+\langle n|D|m\rangle\langle m|H_{sw}|n\rangle}{E_m-E_n} \tag{7}$$

where n is the neutron state, D is the electric dipole moment operator, and m are intermediate states, essentially excited spin $-\frac{1}{2}$ states of opposite parity from the nucleon. The T violating property of H_{SW} is required so that the sum does not vanish. We approximate the sum by replacing m by a single state (or group of states) n*. We may use dimensional arguments to estimate

$$\langle n^*|D|n\rangle = e/M \tag{8}$$

Alternatively we may use the sum rule that relates $\Sigma_n|\langle m|D|n\rangle|^2$ to the mean square radius of the neutron; since the latter is experimentally consistent with zero, the value given in Eq. (8) is not unreasonably small. The same result is obtained if n* is is identified as the 1550 S_{11} state[19]. In our calculation the superweak interaction serves once again to mix single particle states; in this case the neutron state with the excited state n^* of opposite parity. If we assume this mixing is not inhibited, as we assumed before in the case of K_1-K_2 mixing, then it is reasonable that

$$\langle n^* | H_{sw} | n \rangle \sim \langle K_2 | H_{sw} | K_1 \rangle \approx 2\times 10^{-17} m_K \qquad (9)$$

Combining Eqs. (7), (8), and (9) we have

$$D_n \approx 4\times 10^{-17} (e/M) [m_K/(E_{n^*} - E_n)]$$

Assuming $E_{n*}-E_n \approx m_K$

$$D_n/e \sim 4\times 10^{-17}/M \sim 10^{-30}\,\text{cm} \qquad (10)$$

Allowing for a factor of 10 uncertainty we may interpret this result as giving an upper limit for D_n in the superweak theory

$$D_n/e < 10^{-29}\,\text{cm} \qquad (11)$$

As a prediction of the superweak theory our result Eq.(10) depends most critically on the assumption that the matrix element of Eq. (9) does not vanish as a result of selection rules. In order that the matrix element in Eq. (6) be uninhibited it is required that a sizeable part of H_{SW} be parity conservating and allow $\Delta S=2$. In order that the matrix element of Eq. (9) be unhibited it is required that a sizeable part of H_{SW} satisfy the quite different requirements of being parity violating and allowing $\Delta S=0$. The superweak interaction theory is really a class of theories and it is easy to construct specific models in which this last condition is or is not satisfied. If it is not satisfied; that is, H_{SW} is either parity conserving or requires $\Delta S\neq 0$, then the superweak prediction for D_n would be 10^5 to 10^6 times lower than given by Eq. (10). We have already noted this same problem for milliweak theories as a class.

The inequality Eq. (11) is the most definite prediction of the superweak theory independent of further specifications of the theory. The approximation of a single intermediate state in Eq. (7) does not seem critical since if higher states are included their values of $\langle m|D|n\rangle$ **must** be approximately smaller to satisfy the sum rule previously mentioned. Thus our major assumption is that of Eq. (9); however, a much larger value for $<n^*|H_{sw}|n>$ would be contrary to the basic assumption of the superweak theory that the matrix element $<K_2|H_{sw}|K_1>$ is not inhibited.

The form of Eq. (7) implies that H_{sw} has no structure, such as that due to an intermediate boson, since if it does there could be contributions to D_n in which the photon comes from the structure. Indeed, in calculations of D_n in milliweak models, an important contribution sometimes comes from the current of the intermediate boson. Thus we offer a more general, though less detailed, derivation of our result. In calculating D_n in relativistic perturbation theory one draws all "self-energy graphs" that involve H_{sw} to lowest order and then appends the external photon line consecutively in all possible places. This is similar in principle to the calculation of the anomalous magnetic moment μ_a, in which H_{sw} is replaced by the strong interaction. Thus we expect

$$D_n/\mu_a \sim (\Delta m)_{sw}/(\Delta m)_{strong}$$

where $(\Delta m)_{sw}$ is the superweak self energy and $(\Delta m)_{strong}$ is a strong self-energy. From our previous dicussion

$$(\Delta m)_{sw} \approx 2\times10^{-17} m_K$$

setting $(\Delta m)_{strong} \sim m_\pi$ we have

$$D_n/e \sim 2\times10^{-17}(m_K/m_\pi)(\mu_{a/e}) \sim 10^{-30}\text{cm}$$

which is essentially the same as the previous result.

One might ask why this general argument cannot be used in the case of milliweak theories, particularly those with "superweak CP violation". The reason is that in these theories, the CP violating K^o self-energy, or mixing term does not appear in lowest order because of the selection rule $\Delta S=0,1$, so that the CP violating neutron electric dipole moment, which involves $\Delta S=0$, can appear in a lower order. Thus there is no simple relation between the CP violation in the K^o system and that which produces D_n. In contrast the basic assumption of the superweak theory is that $\Delta S=2$ occurs in lowest order.

3. Conclusion

The present experimental limit on D_n provides significant but not decisive evidence against electromagnetic CP violation. If further experiments lower this limit by an order of magnitude it would seem reasonable to rule out electromagnetic CP violation although models with special features might be found to fit within this limit.

For most of the milliweak theories considered here the calculations, admittedly very rough, give values of D_n/e between 10^{-23} and 10^{-24}cm. It is therefore likely, if the milliweak explanation is correct, that future experiments should find a non-vanishing value of D_n/e as they explore values between 10^{-23}cm, the present limit, and 10^{-26}cm. Such a non-vanishing value would decisively demonstrate that the superweak theory is incorrect, since we have argued that, independent of details, this theory must yield a value less than 10^{-29}cm. A discovery of such a non-vanishing value would provide a powerful impetus for searches for other CP violating effects predicted by milliweak theories.

If the electric dipole moment of the neutron continues to be consistent with zero as values of D_n/e are searched in the range 10^{-26} to 10^{-28}cm[4)], this would provide significant evidence in favour of the superweak theory. However, as we have emphasized, this would not be decisive because it is possible to devise milliweak theories with selection rules that could depress the value of D_n/e to 10^{-29}cm. Nevertheless more precise measurements of the neutron electric dipole moment seem to be the best test of a class B prediction of the superweak theory that is likely to be done in the near future. No experiment proposed using accelerators in the range of 10^9 to 10^{12} eV seems nearly so sensitive as the measurement of D_n using neutrons of 10^{-3} eV.

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TABLE

	Values of D_n/e in cm for various theories		
	Electromagnetic	Milliweak	Superweak
Rough estimate	10^{-20}	10^{-23}	10^{-29}
Model independent calcul.	10^{-21} to 10^{-23}	10^{-23}	10^{-30}
Salzman and Salzman	10^{-20}	-	-
T.D. Lee	-	10^{-23}	-
Mohapatra	-	10^{-24}	-
Pais	-	10^{-23}	-
Frenkel and Ebel	-	10^{-29}	-
Experimental result	$\leq 10^{-23}$		

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ATOMIC PHENOMENA DEPENDING ON WEAK INTERACTION AND NEUTRAL CURRENTS

Y. ZELDOVICH

Institute of Applied Mathematics, Moscow, USSR

Parity violation means that the state of a given particle or system can be a superposition of states with different parities.

For example, spin ½ state can be represented as $\alpha S_{\frac{1}{2}}+\beta P_{\frac{1}{2}}$ in the $\sigma_z=+\frac{1}{2}$ state, the $P_{\frac{1}{2}}$ itself is $\gamma(m_z=0,S_z=+\frac{1}{2})+\delta(m_z=0,S_z=-\frac{1}{2})$. It is the interference of S and P two substates wich is needed to explain the correlation of spin direction of initial particle and the direction of its decay products.

Is the particle stable, then one can still speculate on its virtual decay: so for example the electromagnetic properties of neutron are explained by diagrams with $(n,p\pi)$ and $(p,p\gamma)$ vertices, i.e. by virtual neutron decay.

The first question is, if there is a spin correlated virtual decay direction and whether it gives dipole moment if parity violation is taken into account.

The answer (in terms of xyz space function) is that real decay preferential direction depends on $i(\alpha\beta^{x}-\alpha^{x}\beta)$, but virtual decay dipole moment depends on $(\alpha\beta^{x}+\alpha^{x}\beta)$. The two expressions are different. T-parity -if conserved- prevents dipole moment. [Zeldovich JETP Russian 33 1488, (1957), Transl. JETP 6, 1148, (1958)].

The interference of $S_{\frac{1}{2}}$ and $P(m_z=1,S_z=-\frac{1}{2})$ components in the wave function of a stable particle corresponds to a peculiar toroidal magnetic field around the particle, which interacts with external current, $\Delta H=\vec{\sigma}\vec{j}\cdot$ const (in P-conserving theory it is obviously prohibited). The new constant is called anapole moment of the particle [Zeldovich JETP Russian 33, 1531, (1957) transl. JETP 6, 1184, (1958), a general discussion of electromagnetic properties, due to weak interaction is given by Zeldovich and Perelomov JETP Russian 39, 1115, (1960), transl. JETP 12, 777, (1961)].

Naively applying the isotopic spin rotation to the (pn)(e ν) the neutral current of the form [(pp)-(nn)][(ee)-(νν)] was predicted (Dirac operators being omitted.)

In Schrödinger approximation the parity violating term is $\vec{\sigma}$(proton)×$\vec{j}$(electron). Compared with the foregoing electromagnetic term $\vec{\sigma}\vec{j}$ it was predicted that direct weak interaction is ~137 times greater.

Further, it was pointed out that the 2S-2P states of hydrogen are the most sensitive to Coulomb degeneration which is removed only by Lamb shift [Zeldovich JETP Russ.]

This line of reasoning is now pursued by Moscalev, (Leningrad Institute of Nuclear Research named in honour of the late academician Konstantinov.)

He points out that the weak interaction mixes ~10^{-11} (amplitude) of 2P to the ~1 of 2S state.

The one-photon decay 2S→1S (by magnetic dipole) by interference with the electric dipole 2S→1S gives circular polarization, of the order of 10^{-4}, the enforcement being due to small matrix element of the magnetic dipole.

The trouble is, that 1-photon decay is improbable compared with 2-photon decay 2S→1S+2γ.

The use of muonic hydrogen-like atom is recommended. If all the other factors are the same, the effect is proportional to m^2, because it is Gm^2 which is dimensionless.

In reality, the Lamb shift of muonic atoms is not similar, one can choose better atoms. The circular polarization up to 20% is possible, the ratio of 1 photon to 2 photons is also greater.

The measurement of circular polarization can be substituted by measurement of correlation between photon emission and muon spin. Experiments are feasible because μ is always borne polarised and some polarization remains after capture on the orbit.

The worst known factor in muonic case is Auger effect leading to radiationless transition of 2S level.

Some figures are given with η = circular polarization = correlation of spin and photon direction. W are 2S→1S transition probabilities

	η[%]	$W_{1\gamma}[sec^{-1}]$	$W_{2\gamma}[sec^{-1}]$	E_γ[keV]
Li^6	10÷20	30	10^6	18
Be^9	10÷30	500	$6\cdot10^6$	32
C^{12}	1	$3\cdot10^4$	$7\cdot10^7$	70
O^{16}	0,5	$5\cdot10^5$	$4\cdot10^8$	120

The first Moskalev publication is in JETP Lett. Russ. 19 229, 1974.

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WEAK NEUTRAL CURRENTS IN ATOMIC PHYSICS

M.A. BOUCHIAT and C.C. BOUCHIAT

Laboratoire de Spectroscopie Hertzienne*, Ecole Normale Superieure, Paris, France

Laboratoire de Physique Theorique et Hautes Energies, Orsay, France*

Possible ways of detecting weak neutral currents in atomic physics through parity-violating effects in heavy atoms are analysed from a theoretical point of view.

Due to recent developments in the field theory of weak interactions [1] the problem of the possible existence of weak neutral currents has aroused considerable interest. In renormalizable models of weak interactions involving no other leptons than those already observed, the existence of leptonic and strangeness-conserving hadronic neutral weak currents has to be postulated. The neutral weak current involving neutrinos is now under vigorous investigation. A certain number of events which could be attributed to neutrino reactions on nuclei induced by neutral currents has been observed in the Gargamelle bubble chamber [2] and a N.A.L. spark-chamber experiment [3], but other explanations cannot be absolutely ruled out. In this letter, we would like to give the brief account of a theoretical work concerning effects associated with the existence of an electronic (and hadronic) component of weak neutral currents [4]. In the models discussed recently the coupling of the neutral vector boson Z_0, the source of the weak neutral currents, to the electron (or the muon) and the nucleons is parity-violating, as a consequence the electron nucleus potential will acquire a very short-range parity-violating component. The idea of looking for parity-violating effects in atomic physics as a way of investigating the possible existence of neutral weak currents is not a new one. Unfortunately, the theoretical investigation of Curtis Michel [5] was limited to the hydrogen atom where the expected effects are either very difficult to detect or discouragingly small. Stimulated both by the new theoretical ideas and

* Laboratoires associés au Centre National de la Recherche Scientifique.

by the tunable laser revolution in experimental atomic physics, we have decided to have a fresh look at this problem. A little to our surprise, we have found that there is a reasonable hope to detect - if they exist - the parity-violating effects induced by neutral weak currents in twice forbidden magnetic-dipole transitions provided the experimental search is conducted on heavy atoms ($Z>50$).

The parity-violating potential between an electron and a nucleus having Z protons and N nucleons associated with the exchange of the heavy vector boson Z_0 has the following form in the non-relativistic approximation and in the limit of infinite exchanged boson mass [5]:

$$V_{p.v.} = \frac{G_F}{4\sqrt{2}m_e}\left(\vec{\sigma}\cdot\vec{p}\,\delta^3(\vec{r})+\delta^3(\vec{r})\vec{\sigma}\cdot\vec{p}\right)Q_W(Z,N) \quad (1)$$

+ terms involving the nuclear spin

where m_e, $\frac{1}{2}\vec{\sigma}$, $\vec{p}$, $\vec{r}$ are respectively the mass, spin, momentum, and position of the electron. The part involving the nuclear spin plays a negligible role in the effects discussed in this paper. The effective weak charge Q_W depends on the choice of a particular model of weak interactions. The simplest hadronic versions of the "$SU_2 \otimes U_1$" model [6] without new heavy leptons give:

$$Q_W(Z,N) = -[(4\sin^2\theta_W-1)Z+N] \quad (2)$$

In models [7] where the neutral vector boson Z_0 is coupled only to charged leptons the weak charge $Q_W(Z,N)$ takes the simple form:

$$Q_W(Z,N) = +\sin^2\theta_W Z \quad (3)$$

In the above formulae, θ_W is an angular parameter characterictic of the model. If the Gargamelle and N.A.L. results previously mentioned are interpreted in the "$SU_2 \otimes U_1$" model, one finds that

$$\sin^2\theta_W = 0.35 \pm 0.05$$

Because of the Coulomb repulsion the weak parity-violating interaction between the electrons can be neglected.

In the independent particle model of the atoms, parity-mixing will occur only between S and P states because of the zero-range nature of the parity-violating potential. By an extension of a method used by Foldy [8] to put a formula of Fermi and Segre's giving the wave-functions of a valence electron at the nucleus on a more rigorous basis, we have derived the following expression for the matrix element of $V_{p.v.}$ between $nS_{\frac{1}{2}}$ and $n'P_{\frac{1}{2}}$ states:

$$\langle nS_{\frac{1}{2}}|V_{p.v.}|n'P_{\frac{1}{2}}\rangle = i\frac{G_F/\sqrt{2}Z^2Q_W(Z,N)[1+\delta_o(\varepsilon_n)][1+\delta_1(\varepsilon_{n'})]}{4\pi m_e a_o^4(\nu_n\nu_{n'})^{3/2}} \tag{4}$$

a_o is the Bohr radius; the quantities ν_n and $\nu_{n'}$ are the effective quantum numbers related to the binding energies ε_n and $\varepsilon_{n'}$ expressed in Rydberg unit energy by $\varepsilon_n=-1/\nu_n^2, \varepsilon_{n'}=-1/\nu_{n'}^2$. The explicit expression of $\delta_0(\varepsilon)$ and $\delta_1(\varepsilon)$ in terms of the interpolated quantum defects, together with a detailed derivation of the above formula, are given in Ref.[4]. In the case of interest (small binding energies $\varepsilon_n, \varepsilon_{n'} << 1$) $\delta_0(\varepsilon)$ and $\delta_1(\varepsilon)$ are small quantities of the order of a few per cents. In view of the success of the Fermi-Segre formula in explaining the hyperfine splitting of alkaline atoms [9], formula (4) is believed to be quite accurate. Before being used in actual computations, formula (4) has to be corrected for relativistic effects. They are expected to be quite important if one remembers that the Dirac $S_{\frac{1}{2}}$, $P_{\frac{1}{2}}$ wave-functions for an electron moving in the Coulomb field of a point-like charge diverge at the origin. Using a procedure suggested by β decay calculations where a similar difficulty arises, the relativistic correction factor K_r as a function of the nuclear charge and nuclear radius has been worked out in Ref. [4]. Here we shall only quote the results concerning cesium ($K_r \simeq 2.8$) and lead ($K_r \simeq 9.0$). The presence in the matrix element of $V_{p.v.}$ of the factor $Z^2Q_W(Z,N)K_r$ favours very strongly heavy atoms; going from hydrogen to cesium, one gets an enhancement of the order of 10^6!

A standard procedure to investigate parity mixings in atomic (or nuclear) wave-functions induced by a parity-violating time reversal invariant potential of type 1 is to look for it in radiative transitions between unpolarized atomic (nuclear) states for a dependence of the transition probability on the state of circular polarization of the emitted (absorbed) photons. For instance, a transition which would normally be a magnetic dipole one with an amplitude M_1, gets, through parity mixing with neighbouring states, a small electric dipole component E_1. The circular polarization of the photon emitted under initial conditions symmetrical with respect to space reflexion is given by:

$$P_c = 2\,\mathrm{Im}\left(\frac{E_1}{M_1^*}\right) \tag{5}$$

when time reversal is valid

$$\mathrm{Im}\left(\frac{E_1}{M_1^*}\right) = \pm\frac{|E_1|}{|M_1|} \qquad (6)$$

Assuming, for simplicity, that the mixing occurs only between the initial state ψ_i and some state ψ' of opposite parity separated by an energy interval ΔE, one gets for P_c:

$$|P_c| \simeq 2\frac{|E_1'|}{|M_1|}\cdot\frac{\langle\psi_i|V_{p.v.}|\psi'\rangle}{\Delta E} \qquad (7)$$

where E_1' is the allowed electric-dipole transition-amplitude. A violation of parity of nuclear forces which could be attributed to the weak interactions involving charged (and possibility neutral) currents seems to have been detected in nuclei [13]. It is instructive to compare atoms and nuclei from this respect. The parity-violating potential, being practically of zero-range nature, gives matrix elements proportional to the inverse of the volume of the atom (respectively the nucleus). The nucleus appears then to be strongly favoured, by a factor of 10^{12}; this factor is only partially compensated by the change of energy scale associated with ΔE and reduces to 10^6. But we have just seen that by working with heavy atoms one can get an enhancement of the order 10^6 while nothing similar is espected to happen in heavy nuclei, so that the situation of atoms versus nuclei is better than what one would naively expect.

A possible way to increase the circular polarization P_c is to work with forbidden transitions, but by doing so one has to face two problems:

i/ a technical one: the counting rate has to reach an acceptable level;

ii/ a physical one: one has to worry about "background" effects which may mask or even simulate the true parity-violating effects. The appearance of dye lasers with tunable frequency has opened a new area in atomic spectroscopy and one can comtemplate the possibility of exciting twice forbidden magnetic dipole transitions between S states. The excitation of the transition photon+$6S_{1/2}\to 7S_{1/2}$ in atomic cesium with a detection of fluorescent photons radiated from the $7S_{1/2}$ states via the 7S-6P allowed transitions, appears to be a good case for our purpose. This forbidden magnetic dipole transition is likely to be induced by relativistic effects which are also responsible for the difference between the gyromagnetic ratios of cesium and electron

$$\left(\frac{g_{Cs}-g_e}{g_e} = 1.1\cdot 10^{-4}\right).$$

Following an analysis performed by Feinberg and Sucher [10] in the case of the $2^3S \to 1^1S$ transition of helium, it is shown in Ref. [4] that M_1 defined as the matrix element μ_z of the z component of the magnetic dipole operator, is likely to be of the order of $10^{-4}\ \mu_B$, where μ_B is the Bohr magneton. In any case, the first step in an experiment trying to detect the effects discussed here would be the measurement of $|M_1|^2$. A detailed computation of the matrix element of the z component D_z of the electric dipole operator between the $7S_{\frac{1}{2}}$ and $6S_{\frac{1}{2}}$ states, induced by the parity violation potential $V_{p.v.}$ is given in Ref. [4]. The mixing with all possible states (including the continuum) is consiedered. The allowed electric dipole amplitudes are taken either from Hartree-Fock calculations [11] or from the Coulomb approximation by Bates and Damgaard [12], which both give when data are available, a fair agreement with experiment. Here we shall only quote the results relative to the choice

$Q_w = -[(4\sin^2\theta_w - 1)Z + N]$, with $\sin^2\theta_w = 0.35$:

$$E_1 = 1.23i.e\, a_o \frac{(G_F m_e^2) Z^2 \alpha^2 Q_w(Z,N) K_r}{2\pi\sqrt{2}}$$

$$= i1.82 \cdot 10^{-11}\ ea_o = i0.52 \cdot 10^{-8}\mu_B$$

Using the above estimate of M_1 ($M_1 = 10^{-4}\mu_B$), one gets for the circular polarization:

$$|P_c| \simeq 10^{-4}$$

Let us give now the counting rates which are to be expected under experimental circumstances which do not seem too unrealistic. As an example we shall consider a vapour of cesium under a pressure of one Torr at a temperature of 550°K, illuminated by a photon-beam of 3×10^{17} quanta per second corresponding to a power of 0.1 Watt. With an incident photon-beam having a frequency band-width smaller than the Doppler width Γ_D, the maximum cross section for unpolarized photons σ_{max} at the resonance frequency $\omega_o = 2\pi c/\lambda_o$ is given by:

$$\sigma_{max} = \frac{1}{4\sqrt{2\pi}} \frac{\Gamma_{M_1}}{\Gamma_D} \lambda_o^2 \qquad (8)$$

where Γ_{M_1} is the transition rate associated with the forbidden dipole transition and $\Gamma_D = \omega_o \sqrt{kT/M_{Cs}c^2} = 2.16 \cdot 10^9\ \sec^{-1}$.

Taking $M_1 = 10^{-4}\mu_B$, one gets

$$\sigma_{max} = 0.69 \cdot 10^{-24}\ cm^2 .$$

The yield N_f of fluorescent photons associated with the transition $7S_{\frac{1}{2}} \to 6P_{\frac{1}{2}}$ and $7S_{\frac{1}{2}} \to 6P_{3/2}$ for an illuminated length of 1cm, is given by[2]:

($F = 3 \to F = 4$ hyperfine component)

$$N_f = 2.4 \cdot 10^9 \text{ photon/sec}$$

Assuming the same total intensity of right (or left) polarized photons as before, one should observe a difference between the yield of fluorescent photons when the polarization of the incident beam is changed from right to left:

$$N_f^R - N_f^L = P_c(N_f^R + N_f^L) = 2P_cN_f \tag{9}$$

$$\simeq 4.8 \cdot 10^5 \text{ photons/sec, with } P_c = 10^{-4}$$

Other types of experiment involving the measurement of correlations between the electronic spin and the photon momentum are discussed in Ref. [4].

Symmetry considerations indicate that an external magnetic field can - in principle - give rise to effects similar to those associated with a parity violation. Indeed, through a mixing between absorption cross-sections for left and right photons of the order of $0.6 \cdot 10^{-4}$. In the type of experiment discussed here no magnetic field is needed, and to keep the residual field below the level of 10^{-2} Gauss does not seem to be a severe problem. Furthermore, the relative sign of the asymmetries associated with a true parity violation and a magnetic static field is changed by a reversal of the direction of propagation of light with respect to this field.

The effects we are going to discuss now briefly do not lead to any circular polarization but induces a $6S_{\frac{1}{2}} \to 7S_{\frac{1}{2}}$ transition amplitude not interfering with the magnetic dipole transition. If they cannot be subtracted away, the right-hand side of eq. 5 has to be multiplied by the factor $\Gamma_{M_1}/\Gamma_{M_1} + \Gamma_B$ where Γ_B is the rate associated with the processes in question. A static electric field mixes states of opposite parties but does not give rise to any circular polarization. However, it may contribute significantly to Γ_B. The electric dipole amplitude induced by a static field ε_o can easily be estimated

$$E_{1ind} = 0.16\cdot10^{-4}\ \frac{\mu_B}{c}\ \varepsilon_o(\mathrm{volt/cm}) \qquad (10)$$

We see that the static electric field has to be kept below the level of 1Volt/cm.

Multiphoton processes could also be a source difficulties. However, except for laser powers much higher than those considered here, they can be ignored. Much more troublesome are the processes associated with collisions between cesium atoms. The first one is the familiar pressure broadening of spectral lines; it has no effect on P_c but when the pressure width $\Delta\omega_c$ becomes much larger than the Doppler with Γ_D, the maximum cross-section (8) is suppressed by a factor of the order of $\Gamma_D/\Delta\omega_c$. This difficulty does not seem to appear at a pressure of one Torr where $\Delta\omega_c/\Gamma_D$ has been estimated to be of order of 0.17 [4]. Collisions can also induce electric dipole transitions when selection rules forbid them for free atoms. A theoretical analysis of this process, which by itself is of great interest, is given in Ref. [4]. Here, we shall only quote tha main results corresponding to a pressure of one Torr. The collision-induced electric dipole transition is roughly 2.7 times faster than the direct magnetic dipole transition at the resonance frequency. However, the frequency spectrum associated with collision-induced radiative transition has a width one hundred times larger than the Doppler width typical of the direct radiative transition, so that the line associated with the latter will appear as a peak sticking out from a broad background. Since the interference between the induced and direct transition amplitude averages to zero, the signal due to the forbidden magnetic transition could - in principle - be extracted unambiguously.

To conclude, we would like to point out that even if the theoretical model and more precisely the particular choice of weak charge $Q_W(Z,N)$ used in our numerical estimates appears to be in disagreement with subsequent neutrino experiments, an experimental program (now in progress in the E.N.S. laboratory in Paris) designed to detect the effects dicussed here, will not lose any of its interest since the existing limits on the strength of a parity-violating time-reversal invariant short-range potential of type (1) are still rather poor. To illustrate this point we shall quote the results of existing circular polarization measurements in magnetic dipole transitions performed on molecular oxygen [14] and atomic lead [15] together with order of magnitude estimates based on the theoretical considerations of this paper.

$(O_{16})_2$	$\lvert P_c^{exp}\rvert$	$\leq 6 \cdot 10^{-4}$	$\lvert P_c^{th}\rvert$	$\sim 10^{-10}$
Pb	$\lvert P_c^{exp}\rvert$	$\leq 5 \cdot 10^{-4}$	$\lvert P_c^{th}\rvert$	$\sim 10^{-7}$

The need for more sensitive experiments is evident. A publication on similar effects in mu-atoms is in preparation.

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PARITY VIOLATIONS BY NEUTRAL CURRENTS IN MUONIC ATOMS

J. BERNABEU*), T.E.O. ERICSON and C. JARLSKOG+)

CERN - Geneva

The 2S-1S transition in low Z mounic atoms is shown to be extremely sensitive to possible parity violation, such as may be introduced by neutral currents. The most striking effects of parity violation are found in the case of muonic Li and Be where they are estimated to be of the order of 10% on the basis of current models.

A central issue in weak interactions is the question of existence of neutral currents. On the theoretical side their existence would indicate the possibility of unifying electromagnetic and weak interactions using gauge theories[1] which have the attractive feature of being renormalizable.

The tangible evidence for neutral currents is presently controversial: a number of neutrino-like interactions[2] without muons or electrons as well as a few candidates[3] for the leptonic process $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$ have been observed in high-energy neutrino experiments.

If neutral currents exist they could imply a basic modification of electromagnetic phenomena, the most striking of which is the prediction of parity violation.

In this spirit Bouchiat-Bouchiat[4]*) have recently drawn attention to the possibility of observing parity violations and hence neutral currents by exciting certain states in heavy electronic atoms using intense polarized photon beams.

*) References to earlier work on parity violation in electronic atoms are given in this article.

*) University of Barcelona.
+) University of Lund and CERN.

The purpose of the present article is to show that under the right conditions the $2S_{\frac{1}{2}}-1S_{\frac{1}{2}}$ transition in muonic atoms is extremely sensitive to parity violation with the possibility of large effects in circular polarization and angular correlations.

Any hydrogenic atom is basically a system unusually sensitive to parity mixing in states of principal quantum number $n \geq 2$. The reason is that the nonrelativistic Bohr atoms has exactly degenerate states of opposite parity with the same n. In this case the slightest parity violation would cause complete mixing. In actual atoms the degeneracy is lifted, primarily by the effects of self mass, vacuum polarization and the finite size of the central nucleus **). Still, the levels are only slightly perturbed for light nuclei, as compared to the spacing of states of different n, so that the parity mixing is strongly enhanced.

In most models the parity mixing parameter is proportional to the weak interaction coupling constant G, but must be a dimensionless quantity. If only processes involving the same lepton as the one forming the atom are considered and finite nuclear size is neglected, the lepton mass is the only scale available and the mixing must be proportional to Gm_{ℓ}^2. Since the mass ratio $(m_{\mu}/m_e)^2 \approx 4 \times 10^4$ one immediately concludes that muonic atoms are much more sensitive to parity mixing than electronic atoms. This argument is invalidated by vacuum polarization which is largely due to virtual electron pairs both in ordinary and muonic atoms. Moreover, in heavy atoms the nuclear r.m.s. radius $\langle r^2 \rangle^{\frac{1}{2}}$ provides a new scale. Nevertheless, in light muonic atoms the essence of the argument above remains valid as seen from the actual parity effects given in Table 1.

Of all the spontaneous transitions in a hydrogenic atom, only the $2S_{\frac{1}{2}}-1S_{\frac{1}{2}}$ will show exceptional sensitivity to parity mixing with a reasonable branching ratio. All excited states in the atom with the exception of the $2S_{\frac{1}{2}}$ state decay principally via strong E1 transitions. Any parity violating effect in the amplitude for these E1 transitions is proportional to both the mixing parameter and a more forbidden transition, and will be quite small. For the $2S_{\frac{1}{2}}$ state, on the contrary, the E1 transition to the $2P_{\frac{1}{2}}$ state is either completely forbidden or has a small phase space. Its decay to the ground state is either by two-photon decay or by the M1 transition (see Fig.1). This M1 transition is strongly hindered and vanishes in the allowed non-relativistic approximation. The matrix element comes from relativistic effects and retardation with the rate for a massive nucleus given by

**) Other effects such as nuclear polarizability, higher-order vacuum polarization, weak interaction shifts, etc., are completely negligible for our purpose with the exception of some minor influence from the hyperfine splitting.

$$W^{\gamma}_{M1}(2S_{\frac{1}{2}}-1S_{\frac{1}{2}}) = 5.16\times10^{-4}Z^{10}\text{sec}^{-1} \tag{1}$$

For low Z this rate is very low and the dominant decay mode is via two-photon emission

$$W^{2\gamma}(2S_{\frac{1}{2}}-1S_{\frac{1}{2}}) = 1.70\times10^{3}Z^{6}\text{sec}^{-1} \tag{2}$$

Furthermore, the $2P_{\frac{1}{2}}$ level which is nearly degenerate with the $2S_{\frac{1}{2}}$ level decays rapidly to the ground state by E1 transition with the rate

$$W^{\gamma}_{E1}(2P_{\frac{1}{2}}-1S_{\frac{1}{2}}) = 1.29\times10^{11}Z^{4}\text{sec}^{-1} \tag{3}$$

Due to parity violation the $2P_{\frac{1}{2}}$ mixes with the $2S_{\frac{1}{2}}$ state so that the M1 transition acquires an E1 component: $M1\rightarrow M1+\eta E1$, where η is the mixing parameter. Therefore, the relative admixture amplitude in the matrix element is $\eta(E1/M1)$. Apart from the unusually strong admixture expected because the parity mixing states are nearly degenerate, there will therefore be an additional strong enhancement from the factor E1/M1 which for light elements is of the order 10^6.

Up to this point the arguments given are quite general. Optimal condition for parity mixing can be obtained by choosing Z so as to make the $2P_{\frac{1}{2}}$-$2S_{\frac{1}{2}}$ energy difference exceptionally favourable. In a light muonic atom the $2S_{\frac{1}{2}}$ is more bound than the $2P_{\frac{1}{2}}$ state due to the attractive vacuum polarization. With increasing nuclear size this attraction is rapidly compensated by the repulsive finite size effect. A cross-over will occur for a particular Z for which the $2P_{\frac{1}{2}}$-$2S_{\frac{1}{2}}$ energy difference is anomalously small.

The vacuum polarization correction to low Z atoms was approximately calculated two decades ago by Foldy and Eriksen[5]. Recent more accurate results[6-8] are available for some elements. For our purposes the agreement between these results is quite satisfactory.

The nuclear finite size correction shifts in practice only the $2S_{\frac{1}{2}}$ level. The contribution to the $2P_{\frac{1}{2}}$-$2S_{\frac{1}{2}}$ energy separation is given by

$$\Delta E_{size} = -\frac{1}{12}\alpha^4\mu^3Z^4\langle r^2\rangle \tag{4}$$

where μ is the reduced mass. Taking measured r.m.s. radii for ^4He [9], ^6Li, ^7Li [10], ^9Be and ^{12}C [11] we have computed the energy separation $\Delta E = E(2P_{\frac{1}{2}}) - E(2S_{\frac{1}{2}})$:

$\Delta E(H) = 0.2$ eV; $\Delta E(^4He) = 1.4$ eV; $\Delta E(^6Li) = 1.1$ eV

$\Delta E(^7Li) = 1.5$ eV; $\Delta E(^9Be) = -2.2$ eV; $\Delta E(^{12}C) = -33$ eV.

Manifestly, the finite size correction is of the same size as the vacuum polarization shift in Li and Be and it has completely overwhelmed vacuum polarization already in ^{12}C. It is hence to be expected that parity violation effects are particulary important in the Li, Be region. We will therefore concentrate the further discussion to the light elements altough much of the following can be immediately adapted also to heavy atoms.

We consider the single-photon emission in the $2S_{\frac{1}{2}}-1S_{\frac{1}{2}}$ transition. The normal hindered M1 amplitude and the allowed E1 amplitude which has been admixed interfere with each other.

The relevant M1 amplitude, denoted by A_{M1}, is conventionally expressed by quantizing along the photon direction

$$A_{M1} = e\sqrt{2}[\delta_{m_i,-\frac{1}{2}}\delta_{m_f,\frac{1}{2}}\varepsilon_- + \delta_{m_i,\frac{1}{2}}\delta_{m_f,-\frac{1}{2}}\varepsilon_+]I_{M1} \tag{5}$$

where $\varepsilon_\pm = \pm(1/\sqrt{2})(\varepsilon_x \pm i\varepsilon_y)$ are the photon polarization vectors; m_i and m_f refer to the component of the initial and final muon spin in the direction of the photon momentum. The quantity I_{M1} is a radial integral over the large and small components g and f of the Dirac wave function

$$I_{M1} = \int_0^\infty dr r^2 j_1(kr)[g_{1S}^*(r)f_{2S}(r) + f_{1S}^*(r)g_{2S}(r)] \tag{6}$$

Similary, the E1 amplitude A_{E1} is given by

$$A_{E1} = ie\sqrt{2}[\delta_{m_i,-\frac{1}{2}}\delta_{m_f,\frac{1}{2}}\varepsilon_- - \delta_{m_i,\frac{1}{2}}\delta_{m_f,-\frac{1}{2}}\varepsilon_+]I_{E1} \tag{7}$$

where

$$I_{E1} = \int_0^\infty dr r^2 j_0(kr)[g_{1S}^*(r)f_{2P}(r) + \frac{1}{3}f_{1S}^*(r)g_{2P}(r)] \tag{8}$$

To leading order for a point-like Coulomb field I_{M1} and I_{E1} are given by

$$I_{M1} = \frac{(\alpha Z)^4}{27\sqrt{2}}, \quad I_{E1} = \frac{-16}{81}\sqrt{\frac{2}{3}}\,(\alpha Z)$$

These lead to the M1 and E1 total transition rates (1) and (3) for muonic atoms and are in agreement with results previously quoted in the literature[12,13] for electronic atoms.

By parity mixing the initial state has the wave function $|2S_{\frac{1}{2}}\rangle+\eta|2P_{\frac{1}{2}}\rangle$ and the total transition amplitude $A_{M1}+\eta A_{E1}$ *). For unpolarized muons the circular polarization of the γ-ray is hence

$$a = \frac{W_+-W_-}{W_++W_-} = \frac{2\mathrm{Im}\eta\cdot I_{M1}I_{E1}}{I_{M1}^2+(\mathrm{Im}\eta)^2 I_{E1}^2} \qquad (9)$$

Substituting I_{M1} and I_{E1} yields for small admixture amplitude

$$a \simeq -\frac{64}{3\sqrt{3}(\alpha Z)^3}\,\mathrm{Im}\eta \qquad (10)$$

The experimental detection of the circular polarization seems difficult. A more convenient experiment would be the detection of the correlation of the emitted γ-ray with the muon polarization $\vec{P}$ given by

$$\frac{dW_{1\gamma}(\vec{K}_\gamma,\vec{P})}{d\Omega} = \frac{W_{1\gamma}}{4\pi}\,(1\pm a\hat{P}\cdot\hat{K}_\gamma)\ . \qquad (11)$$

Here the signs refer to the case of initial state polarization (+) and final state polarization (-) of the muon with the other state unpolarized.

If one chooses a case where the initial muon is unpolarized, the angular correlation of the photon and the electron from the decay of the final muon is proportional to the quantity

$$1+\frac{1}{3}a\ \cos_{e\gamma} \qquad (12)$$

We now calculate the mixing parameter in a model with a neutral current mediated via a heavy vector boson Z. In the external field approximation, the interaction Hamiltonian is given by

*) The amplitudes A_{M1} and A_{E1} are relatively imaginary. The parameter η is also imaginary if time-reversal invariance is assumed. Therefore, the relative phase of the E1 and M1 transitions is of importance.

$$H_{int}(\vec{x}) = j_\lambda(\vec{x})A^{\lambda,ext}(\vec{x})$$

where $j_\lambda(\bar{x})$ is the leptonic neutral current which contains the muonic part

$$J_\lambda^{mu}(x) = \bar{\psi}_\mu(x)(a+b\gamma_5)\gamma_\lambda\psi_\mu(x) \qquad (13)$$

Here a and b are constants. The external field is easily found to be

$$A^{\lambda,ext}(\bar{x}) = \frac{g(Z,N)}{M_Z^{\,2}} g^{\lambda o}\delta(\bar{x}) \qquad (14)$$

where M_Z is the mass of the heavy boson and $g(Z,N)$ denotes its coupling to the nucleus.

The interaction Hamiltonian can thus be expressed in terms of the effective coupling constants

$$g_1 = \frac{ag(Z,N)}{M_Z^{\,2}}, \quad g_2 = \frac{bg(Z,N)}{M_Z^{\,2}} \qquad (15)$$

in the form*)

$$H_{int}(\bar{x}) = \bar{\psi}_\mu(\bar{x})[g_1\gamma_o + g_2\gamma_5\gamma_o]\psi_\mu(\bar{x})\delta(\bar{x}) \simeq [g_1 - g_2\frac{\vec{\sigma}\vec{p}}{m_\mu}]\delta(\bar{x})$$

where the approximate right-hand side is the non-relativistic limit. We note that the assumption of vector boson exchange is mainly an artifice to emphasize that the interaction is short-ranged. In the form above with free effective coupling constants, the Hamiltonian has considerable generality. The effective coupling constants g_1 and g_2 should be considered as having a typical scale of AG, where A is the nucleon number.

The parity violating part of the interaction above connects the $2S_{\frac{1}{2}}$ and $2P_{\frac{1}{2}}$ **states** by the term $\gamma_o\gamma_5$. To leading order in $Z\alpha$ we have

*) The operator $\vec{\sigma}\cdot\vec{p}$ should be symmetrized to act equally on initial and final states.

$$<2P_{\frac{1}{2}},m_p|H_{int}|2S_{\frac{1}{2}},m_s> = -ig_2\frac{\sqrt{3}}{32\pi}\mu^3(\alpha Z)^4\delta_{m_p,m_s} \qquad (16)$$

and the mixing parameter η is this quantity divided by $E_{2S_{\frac{1}{2}}}-E_{2P_{\frac{1}{2}}}$. Substitution in Eq. (10) leads to the asymmetry parameter

$$a = \frac{2g_2}{3\pi}\frac{\mu^3(\alpha Z)}{E_{2S_{\frac{1}{2}}}-E_{2P_{\frac{1}{2}}}} \quad . \qquad (17)$$

A concrete model for neutral currents has been given by Weinberg[14]. In this model the effective parity violating coupling g_2 is given by

$$g_2^{\text{Weinberg}} = \frac{G}{\sqrt{2}}[\tfrac{1}{2}(Z-N)-2\sin^2\theta_W Z]$$

where θ_W is the Weinberg angle. So as to form an opinion about at which level the parity effects are expected to enter, we have tabulated the asymmetry parameter for the Weinberg coupling and the atoms mentioned above (see Table 1.). For illustration we have chosen the extreme cases $\sin\theta_W=0$ and 1 as well as the more realistic case $\sin^2\theta_W=0.4$.

A glance at the table reveals that expected effects are very large. We would like to emphasize that this feature does not originate from the specific model chosen, but is due to the inherent enhancement factors discussed before. Therefore, independent of the particular theory which may be wrong, even negative experiments would serve the important purpose of limiting future theories. It is also interesting to compare muonic and ordinary ions. For this reason we have calculated the ratio of the asymmetries for the two cases by using experimental results[15] for the Lamb shifts of the ordinary ions whenever available. One notes the smallness of the electronic effects.

Present experimental limits on the parity breaking interactions are very poor and would correspond to vastly enhanced rates of the $2S_{\frac{1}{2}}-1S_{\frac{1}{2}}$ transition. So for example, Poppe [16] finds an upper limit in Pb corresponding to 5×10^3 of the Weinberg coupling, which would correspond to $W\gamma(2S-1S)$ in Li of about 4×10^4 times stronger than the value we have used here. A similar upper limit is obtained from the one-photon transition in muonic helium by Placci et al.[17].

We now turn to the question of the backgrounds to the one-photon decay of the $2S_{\frac{1}{2}}$ state. An intrinsic background comes from the two-photon decay of this state with the configuration where one of the photons carries essentially all the available energy and the other nearly none. For low Z ions, the total rate for the two-photon decay, $W^{2\gamma}$, is larger than that for the one-photon decay by several orders of magnitude. Using the formulae given in Ref. [18], we have estimated the two-photon fractional branching ratio, $\delta W^{2\gamma}$, where the soft photon carries at most the fraction ε of the total available energy; we find

$$\delta W^{2\gamma} \approx 0.95\varepsilon^2 W^{2\gamma}.$$

To suppress this background to the level of the signal, given by the one-photon rate W^{γ}, requires $\varepsilon \simeq \sqrt{W^{\gamma}_{M1} / W^{2\gamma}}$.

For comparison, we have tabulated the one- and two-photon rates [$W(\gamma)$ and $W(2\gamma)$, respectively] and the quantity $\sqrt{W^{\gamma}_{M1}/W^{2\gamma}}$ (see Table 1). We observe that the level of accuracy discussed above requires a resolution for Li and Be of about 1% or better in the maximum energy of the X-rays which are in the region of about 20 keV. Another possible way of eliminating this background may be by the use of an appropriate absorber.

The practical problem of an actual experiment is associated with the following points: the probability of population of the 2S state, the branching ratio into the one-photon mode and the muon polarization. (We will not consider detection of circular polarization which we believe difficult experimentaly.)

The $2S_{\frac{1}{2}}$ state is expected to be populated by a fraction of the order of a few percent of all stopped muons, as has been shown directly in the case of He [17]. In light elements the lifetime of this state is very much larger than that of the $2P_{\frac{1}{2}}$ state and in practice of the order of the muon lifetime (nuclear absorption is negligible). An important part of the physical background would be the $2P_{\frac{1}{2}}$-$1S_{\frac{1}{2}}$ photons which have very closely the same energy. Simple time discrimination can be used to eliminate this background.

The lifetime of the $2S_{\frac{1}{2}}$ state exposes the atom to various molecular effects which depend on exact experimental conditions. First, the presence of electrons in the muonic atom leads to radiationless de-excitation with a rate of about 2×10^9 sec^{-1} [19]. This would reduce the one-photon branching ratio by several orders of magnitude. Normally all electrons are believed to get ejected from the light atoms during the muonic cascade. It is important to prevent recapture of electrons which strong militates against the use of metal targets which are rich in free electrons. Secondly, Stark mixing in collisions provides momentary parity breaking. While it will not simulate asymmetries or circular polarization, it can mix the $2S_{\frac{1}{2}}$ and $2P_{\frac{1}{2}}$ levels and produce de-excitation by E1 radiation. This effect is particularly important in the gaseous state, but it is reduced in a symmetric lattice.

In carrying out parity experiments of the type discussed above, it is most likely easier to detect a correlation between the photon and the polarization rather than the circular polarization. Muons stopped are usually strongly polarized; about 15% of this polarization remains after the muon cascades to the $2S_{\frac{1}{2}}$ state and can be used for correlation measurements. An alternative is to use unpolarized muons and measure the polarization of the final muon directly by its decay electron, i.e. an electron-photon correlation.

Finally, we note that the parity breaking effects are very much larger for light muonic atoms than for heavier muonic atoms. For heavy muonic atoms the finite nuclear size dominates the $2P_{\frac{1}{2}}$-$2S_{\frac{1}{2}}$ energy shift according to Eq. (4). As seen from the expression for the asymmetry parameter (17) the typical asymmetry is of order 10^{-3} to 10^{-4}. The smallness of the effect makes the question of systematic experimental errors more serious. Further, no time separation of the E1 versus M1 transition can be made in practice, though some possibilities for energy discrimination exist. The branching ratio for the M1 transition is rather poor, since the $2S_{\frac{1}{2}}$-$2P_{\frac{1}{2}}$ becomes physical and Auger de-excitation is a normally competing branch. Although we believe that the prospects are rather poor, this possibility should also be considered as an experimental alternative.

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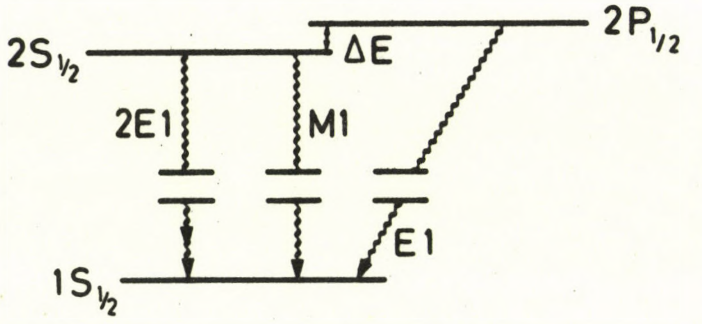
Table 1

Element	$a \times 10^2$ ($\sin \Theta_w = 0$)	$a \times 10^2$ ($\sin \Theta_w = 1$)	$a \times 10^2$ ($\sin^2 \Theta_w = 0.4$)	a_{el}/a_{mu}	W^{γ}_{M1}	$W^{2\gamma}$	$\sqrt{W^{\gamma}_{M1}/W^{2\gamma}}$
H	-2.6	+ 7.7	+1.5	-7×10^{-3}	4.6×10^{-4}	1.5×10^{3}	5×10^{-4}
$^{4}He^{(+)}$	0	+ 7.6	+3.0	-3×10^{-3}	5.1×10^{-1}	1.1×10^{5}	2×10^{-3}
$^{6}Li^{(++)}$	0	+22.2	+8.9	-0.5×10^{-3}	3.0×10	1.2×10^{6}	5×10^{-3}
$^{7}Li^{(++)}$	+1.4	+18.2	+8.1	-	3.0×10	1.2×10^{6}	5×10^{-3}
$^{9}Be^{(3+)}$	-1.3	-22.1	-9.6	-	5.3×10^{2}	6.9×10^{6}	9×10^{-3}
$^{12}C^{(5+)}$	0	- 3.1	-1.2	10^{-3}	3.1×10^{4}	7.9×10^{7}	2×10^{-2}



The one-and two-photon transitions between the levels n = 1 and 2

Fig. 1

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ENERGY DIFFERENCE OF MIRROR MOLECULES

EVA GAJZÁGÓ, GEORGE MARX

Department of Atomic Physics Eötvös University, Budapest

Several gauge theories predict a weak neutral current coupling between leptons and hadrons (e.g. the Weinberg model [1]) or at least between charged leptons and hadrons (e.g. the Lee-Prenthi-Zumino model [2]). This charge-retaining parity-asymmetric weak force is transmitted by the massive Z^o boson. An experimental indication of neutretto-nucleon and neutretto-electron forces seems to support this idea [3]. If the Hamiltonian of the atomic and molecular systems contains a term H', which anticommutes with parity, one may expect parity-impurities in atomic spectroscopy [4], [5], [6] and in quantum chemistry [7].

As a consequence of the large mass of the Z^o boson, the parity-asymmetric correction to the Coulomb force has a very short range. In electron-electron-interactions this contact potential is negligible because of the Pauli "repulsion"[5]. The weak odd-parity contribution to the electron-nucleus interaction turns out to be [5]

$$H' = -\frac{G}{4\sqrt{2}m_e c}\{\underline{\sigma}\underline{p}\delta^{(3)}(\underline{x}-\underline{x}_n)+\delta^{(3)}(\underline{x}-\underline{x}_n)\underline{\sigma}\underline{p}\}Q_n(N,Z) \qquad (1)$$

where in the Weinberg gauge model [1] the weak charge of the nucleus is given by the formula

$$Q_N(Z,N) = (4\sin^2\theta-1)Z+N \qquad (2)$$

(In the numerical estimations we shall use the value $\sin^2\theta=0.35$ compatible with the Gargamelle evidence [3] giving $Q_n \sim A-0.6Z$).

H' is of the first order in weak coupling constant G, and so are the parity impurities induced by this weak force. For simplicity, let us consider the $2s_{\frac{1}{2}}$ state of the H atom. As a rule, the nearest level produces the largest perturbation. In this case the separation of the $2p_{\frac{1}{2}}$ state is given by the Lamb shift:

$$\Delta E_L = E(2s_{\frac{1}{2}})-E(2p_{\frac{1}{2}}) = \tfrac{1}{2}\alpha^5 m_e c^2 = 0.5 \cdot 10^{-5}\mathrm{eV}. \qquad (3)$$

So the perturbed $2s_{\frac{1}{2}}$ state is described by the state vector

$$\psi(2s_{\frac{1}{2}}) = \phi(2s_{\frac{1}{2}})+\frac{\langle H'\rangle}{\Delta E_L}\phi(2p_{\frac{1}{2}}) \qquad (4)$$

The perturbing matrix element $\langle H'\rangle$ is very small compared to the atomic energies:

$$\langle H'\rangle = \langle 2p_{\frac{1}{2}}|H'|2s_{\frac{1}{2}}\rangle = -\frac{i}{32\pi\sqrt{3}\alpha}(Gm_e^2)\alpha^5(m_e c^2) = -i\{1.6 \cdot 10^{-16}\mathrm{eV}\} \qquad (5)$$

α is the innermost Bohr radius, its fourth power comes from the expression $\Psi_s(0)$, grad $\Psi_p(0)$. The combination $\varepsilon_o=(Gm_e^2)\alpha^4 m_e c^2=3\cdot 10^{-14}$e is the characteristic value of the weak contributions to the atomic and molecular energies. Due to the smallness of the Lamb splitting, the parity impurity in the $2s_{\frac{1}{2}}$ state turns out to be rather significar $\langle H'\rangle/\Delta E_L \sim 10^{-10}$ (This is an exceptionally favourable situation!)

To observe this, one must look for a first order effect, e.g. for interference phenomena. In the one-photon decay of this physical state a forbidden magnetic dipole transition $2s\to 1s(M)$ is mixed with an allowed electric dipole transition $2p\to 1s(E)$

$$\frac{|E|^2}{|M|^2} = \frac{\Gamma(2p\to 1s)}{\Gamma(2s\to 1s)} = \frac{9 \cdot 10^8\mathrm{sec}^{-1}}{10^{-7}\mathrm{sec}^{-1}} \cong 10^{16} \qquad (6)$$

The interference of the electric and magnetic radiation produces a partial circular polarization:

$$C = 2\frac{\langle H'\rangle}{\Delta E_L}\frac{|E|}{|M|} = 2\frac{\langle H'\rangle}{\Delta E_L}\frac{\Gamma(2p\to 1s)^{\frac{1}{2}}}{\Gamma(2s\to 1s)^{\frac{1}{2}}} \sim 10^{-3}. \qquad (7)$$

This is a surprisingly high value, smaller effects have been observed already in nuclear gamma transitions. Unluckily, the $2s \to 1s$ transition for the H spectrum is inconvenient for practical observation, because of the disturbing two-photon decays. M.A. Bouchiat prepares an experiment with the $7s \to 6s$ transition of the Cs atom at the Ecole Normale Superiore [7]. Here the larger value of Q_n (and the sharper increase of the Dirac wave function at the nucleus) compensates the larger energy denominator, so the expected circular polarization is $\sim 10^{-4}$.

A more favourable situation may be found in the case of the muonic atoms, where the larger mass produces a smaller Bohr radius α, consequently the muonic wave function concentrates stronger onto the nucleus. The weak matrix element $\langle H' \rangle$ will be larger by a factor $(m_\mu/m_e)^3$, than that for the electron, i.e. $\langle H' \rangle \sim 10^{-10}$ eV. Unluckily, the energy denominator increases, too, proportional to (m_μ/m_e), or even stronger, due to vacuum polarisation effects, so the expected parity impurity increases less, than $(m_\mu/m_e)^2$, compared to the normal H atom. The experimental possibilities are still remarkable, as stressed by J.Bernabeu, T.E.O. Ericson and C. Jarskog [10] . The energy splitting due to vacuum polarization and that due to the nuclear size have opposite effects and they make the energy denominator to be the smallest around the ^{6}Li muonic atom (~1 eV). If we look at the correlation between photon momentum and muon spin ($\underline{p}_\gamma \underline{\sigma}_\mu$), a few percent effect is predicted. The muon polarization is made observable by the momentum of the decay electron. In the most favourable case of the one photon decay in the muonic ^{6}Li atom the $\underline{p}_\gamma \underline{P}_e$ asymmetry may reach 9%. (The real experiment is made more difficult by the disturbing two-photon decays, which do not show this large parity violation.)

<u>These examples show, that the weak parity impurities may result in a detectable effect in atomic transitions.</u>

Another interesting domain to look for the consequences of the non-commutativity of the molecular Hamiltonian and parity is the chemistry of reflection-asymmetric molecules. The ground state energies of mirror molecules will not be equal necessarily [11],[12]. The essential aim of this report is to start the exploration of this phenomenon, which may be interesting also from the biochemical point of view.

It is evident from the form of the weak perturbation (1), that the contribution of an electron pair with compensated spin will cancel. Let us denote the orbital wave function of an odd electron (possessing an uncompensated spin within the molecule) with $\psi(\underline{x})$. In first approximation the perturbation (1) will produce an energy shift

$$\int \psi_e^*(\underline{x}) H' \psi_e(\underline{x}) d^3x \, . \qquad (7)$$

After a simple manipulation this gives the following energy difference between mirror molecules:

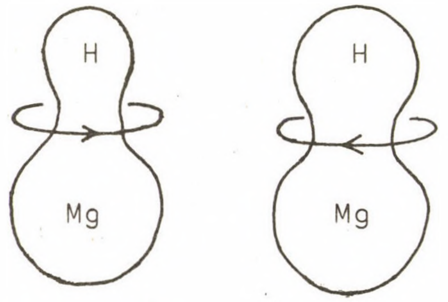
$$\Delta E = \frac{G}{\sqrt{2}} \sum_n \sum_e \underline{\sigma}_e \underline{j}_e(\underline{x}_n) Q_n(Z,N) \tag{8}$$

The summations are extended over all electrons (e) with uncompensated spin and over all nuclei (n) of the molecule. Here $\underline{j}_e(\underline{x})$ is the conventional Schrödinger current.

$$\underline{j}_e(\underline{x}) = \frac{i\hbar}{2m_e}\{\underline{\nabla}\psi_e^*(\underline{x})\psi_e(\underline{x}) - \psi_e^*(\underline{x})\underline{\nabla}\psi_e(\underline{x})\} \tag{9}$$

to be taken at the nuclei. $\underline{\sigma}_e$ is an axial vector, $\underline{j}_e$ is a polar vector, so the weak energy shift will have opposite sign in mirror nuclei, if it is non-vanishing at all.

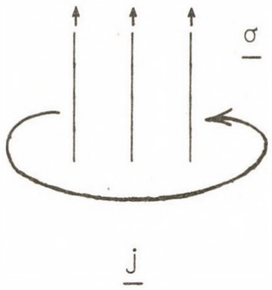
The simplest examples of asymmetric molecules are the two atomic molecules, made of different atoms and having an additional magnetic field, e.g. MgH. Here the three valence electrons feel the electric field of Mg^{++} and H^+ and will have bottleshaped wave functions (superposition of atomic s wave functions into bonding and non-bonding molecular orbitals). The electron spin will be influenced by the proton magnetic moment. The molecular axis $\underline{x}(Mg \to H)$ and the electron spin $\underline{\sigma}_e$ may give a pseudoscalar term $\underline{x}\cdot\underline{\sigma}_e$. Such a term, appears, however, only in a Hamiltonian which is asymmetric also from the point of view of time reversal. Our weak Hamiltonian (1,8) is symmetric from the point of view of time reversal, so it does not give any mass splitting between the two mirror MgH molecules



This negative conclusion has a wider validity. Let us assume, that the molecular Hamiltonian contains only kinetic energy and Coulomb energy terms. This Hamiltonian is completely real. Its ground state eigenfunction will be a real expression. This statement is not modified by the Pauli principle. (Even the degenerate excited states may be de described by real eigenfunctions.) In the case of real functions the expression (9) vanishes. There is no steady electron current at the place of the nucleus, so the weak contribution (8) to the molecular energy vanishes exactly.

If we take only the Coulomb interactions into account in shaping the orbital wave functions, the P-odd, T-even weak perturbation (1,8) will give no energy difference between mirror molecules in the first order of the perturbation theory.

In the case of molecules with paramagnetic properties the situation will be different. If the electrons with uncompensated spin build up a state with maximum spin multiplicity, this spin magnetism will be coupled to the orbital magnetic momentum of the electrons. In this case not the standing wave, but the running wave solutions will be favourable even for the ground state: one may expect a nonvanishing value of $\underline{j}_e(\underline{x})$ in certain regions. So the appearence of a weak energy shift (8) is not excluded. Formally speaking: the Pauli spin-energy-term and the spin-orbit coupling destroy the real character of the Hamiltonian: the complex eigenfunctions may give a non-vanishing value for the current (9).



To be more specific, the second approximation of perturbation theory will give an energy shift even in the case of the real ground state eigenfunction, as a consequence of an interplay between the weak Hamiltonian [1] and of the spin-orbit coupling. If one starts e.g. from an sp_x hybrid state with spin pointing in z direction, one will get a weak matrix element, similar to the formula (5), a spin-orbit matrix element of the order $\alpha^4 m_e c^2$ and an energy denominator of the order $\alpha^2 m_e c^2$. This means, that the energy shift is in general by a factor α^2 smaller, than the weak transition matrix element (5):

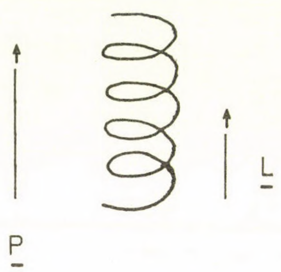
$$\Delta E \sim (G m_e^2)\alpha_Z^6 Q_n \cdot Z m_e c^2 .$$

Here

$$\alpha_Z = \frac{a}{\hbar/mc} = \frac{Z_{eff} e^2}{\hbar c n_{eff}^2} = \frac{Z_{eff}}{137 \cdot n_{eff}^2}$$

is the effective fine structure constant of the unpaired electrons. Z is the number of these electrons, taking part in producing this weak energy shift. (One can win something, if there is a nearby perturbing molecular energy level $E_n - E_m \lesssim 1 eV$. On the other hand, the Π bonding states (being the most sensitive for the molecular shape) give a small electron density but large gradient at the nucleus. This may produce an extra factor R/a if R is the nuclear radius. Putting everything together, an estimate $\Delta E \sim 10^{-14}$eV is rather optimistic for the energy splitting between the mirror molecules.

A realization of this situation is offered by the example, which has been investigated by P.Hrasko from a different point of view [13]. Let us consider a helix-shaped molecule with delocalized molecular orbitals. (A closed helix may eliminate the boundary value problems.) If an electron moves along a conducting right-handed screw, it has an orbital momentum component parallel to its linear momentum. The spin-orbit coupling will direct its spin into a position, which is antiparallel to the orbital momentum. The electrons with opposite spin and momentum will form "helical electron gas". The negative helicity may give a weak energy term, due to the Hamiltonian (1). The effect will be made smaller by the Π-bonding character of the delocalized molecular orbitals. The important point is, however, that we expect a difference in the ground state energy of right-handed screw-shaped and left-handed screw-shaped molecules [12]. The mirror molecules, showing weak energy differences, are expected to be rather complicated ones.



There is a very tiny hope, that this small energy difference may be observable directly. If we drive a tunable laser with the light of a right-handed molecule, it will radiate a frequency ν_R. We can beat this ν_R frequency with the optical frequency ν_L of the left-handed molecule, a beating may be produced. By measuring the beating frequency ω one finds the frequency difference $\nu_L - \nu_R$ [14]. Principally, one can go down to $\Delta\nu/\nu = 10^{-12}$ or even beyond this ratio by this metod, but a few orders of magnitude are still missing. Practical difficulties (e.g. line width) probably do not allow realization of this experiment.

W.Thiemann and K.Wagener reported about observing a small difference in the crystallization speed of enantiomorphic compounds [15]. They interpreted this effect as an indication for a difference in the lattice energies:

$$\Delta E/E \sim 10^{-5}$$

This value is definitely larger, than anything in our theoretical estimations. So we propose, that the observed effect - if real - should not be explained by energy differences. It is still possible, however, that transition probabilities are more sensitive to the weak correction (1). We have seen on the example of the hydrogen atom, that H' can lead to considerable value of a transition matrix element $\langle b|H'|a\rangle$, but it gives only small energy shift $\langle a|H'|a\rangle$ in molecules (if any at all.). The barrier penetration probability may be very sensitive for weak shape-dependent corrections. It is still worth to think about the possibility, that the amazing asymmetries of life are amplified manifestations of the weak asymmetry of fundamental forces, but these weak asymmetrics may work through a chain of transitions, not in steady states [16], [17].

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ESTIMATE OF ENERGY DIFFERENCES BETWEEN OPTICAL ISOMERS CAUSED BY PARITY NON CONSERVING INTERACTIONS

D. REIN

III. Physikalisches Institut Technische
Hochschule Aachen, Aachen, W-Germany

Introduction

Organic macromolecules as proteins being the constituents of living cells are composed from amino acids which are able to rotate the polarization plane of light. Each optically active amino acid molecule can exist in two enantiomeric forms, the so-called L-form and the D-form. Both forms are supposed to be exact mirror images of each other and they are normally synthesized in a racemic (i.e. 50:50) mixture in laboratory. It is, however, observed that exclusively one particular form does occur in living material which is the L-form in the case of amino acids.

It is not clear at the present why only one optical isomer of an amino acid molecule is accepted as building block of proteins and why it is just the L-form. One may ask, therefore, whether perhaps intrinsic differences between an enantiometric pair of molecules could have acted towards a preference of one particular enantiometric form during the long period of prebiotic evolution.

Now, since L-and D-form of an optical isomer are mirror images of each other the physical and chemical properties of them (as melting points, solubilities etc.) should be identical as long as the fundamental interactions are space reflection invariant. Parity non invariant interactions, however, can provide for small energetic differences between L- and D-molecules.

This talk is intended to give an estimate on the relative magnitude of such parity violating (pv) effects in molecules making use of the existing literature. It turns out that pv corrections to molecular binding energies are rather tiny, due to both the small coupling constant and the short range of the weak interaction which does violate parity.

Differences between mirror molecules due to parity non conserving interaction

Let us consider a molecule of spatially asymmetric shape. It will be described by a state vector $|\psi\rangle=|\psi^{+}+\psi^{-}\rangle$ which is not an eigenstate of parity P. The mirror molecule has state vector $|\psi^{P}\rangle=|\psi^{+}-\psi^{-}\rangle$ in obvious notation. As long as the Hamiltonian H is parity conserving, $P^{-1}H\,P=H$, both molecules have the same spectrum:

$$\langle\psi^{P}|H|\psi^{P}\rangle = \langle\psi|P^{-1}\ H\ P|\psi\rangle = \langle\psi|H|\psi\rangle \tag{1}$$

If there is in addition a parity violating part V^{pv} with $P^{-1}V^{pv}P=-V^{pv}$ then the spectra of both molecules differ by the appropriate expectation values of this potential V^{pv}:

$$\begin{aligned}\langle\psi^{P}|H+V^{pv}|\psi^{P}\rangle &= \langle\psi^{P}|H|\psi^{P}\rangle+\langle\psi^{P}|V^{pv}|\psi^{P}\rangle = \\ &= \langle\psi|H|\psi\rangle-\langle\psi|V^{pv}|\psi\rangle = \\ &= \langle\psi|H-V^{pv}|\psi\rangle\end{aligned} \tag{2}$$

While H connects wave functions with the same parity the (hermitean) parity violating potential gives only a contribution between eigenstates of opposite parity

$$\Delta E^{pv} = \langle\psi^{P}|V^{pv}|\psi^{P}\rangle-\langle\psi|V^{pv}|\psi\rangle \equiv -4\mathrm{Re}\langle\psi^{+}|V^{pv}|\psi^{-}\rangle \tag{3}$$

Apparently there is an energy difference between optical isomers provided a pv potential can be constructed from known interactions. Now atomic and molecular phenomena are governed by electromagnetic interactions which are assumed to be parity conserving. This is, however, not on equally firm grounds as for strong interactions, and a small admixture of a neutral conserved axial vector current may well be admitted in electromagnetic interactions (1). In addition parity violating weak interaction is always present and it is well known how to construct pv potentials out of weak scattering amplitudes (2). Such potentials are expected to be a factor Gm^2 smaller than the leading electrostatic potential where m denotes some average hadron mass. The pv potentials are also of short range extending at most over nuclear distances (3). Hence the actual pv effects can provide only for very small corrections to molecular binding energies or line shifts. This will be pursued more quantitatively in the following sections.

The parity violating potential

For application in atomic and molecular physics we have to discuss the parity violating electron-nucleon and electron-electron scattering amplitudes. Let us first turn to weak interaction effects:

In conventional models of weak interaction without neutral currents the lowest order to which parity violating eN- and ee-scattering can occur, is $G\alpha$, where G denotes the Fermi coupling constant and α is the fine structure constant. This is indicated in fig. 1.

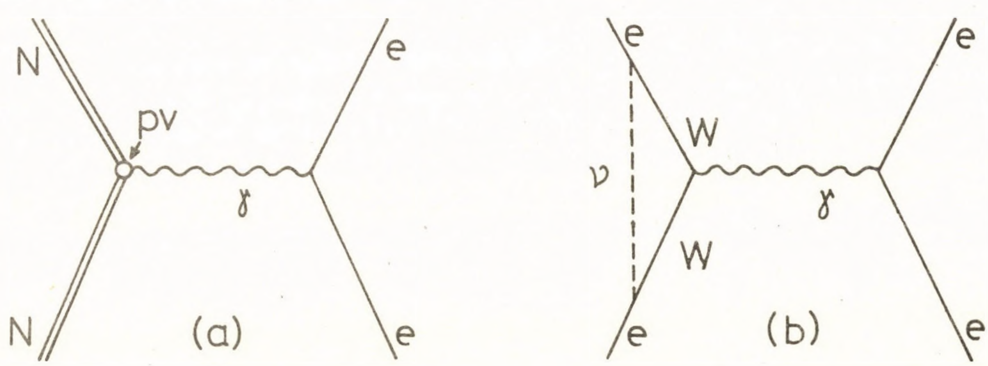

fig. 1.

Feynman diagrams for pv scattering processes of order $G\alpha$

In general a parity violating contribution to eN-scattering of order $G\alpha$ (fig.1.a) is expected due to the nonleptonic weak Hamiltonian at the nucleon vertex (4). It is, however, not clear at the moment, whether the actual pv-coupling of the photon to a nucleon via a neutral vector meson is zero or not. While $SU(6)_W$ - considerations (5) and calculations in the framework of a quark model (6) predict the existence of at least one nonvanishing weak NNV^o-coupling, current fild identity (CFI) together with the Cabibbo form of the non leptonic weak interaction seems to forbid a weak NNV^o-coupling (7). Anyway, the pv potential arising from diagrams (a) and (b) of fig. 1. has to be of short range since gauge invariance of the electromagnetic interaction forces the pv part of the eN-scattering amplitude to be of the form:

$$M^{pv} \sim e^2 \Gamma_\mu^{pv} \cdot \frac{g_{\mu\nu}}{q^2} \ell_\nu \sim -e^2 \bar{u}_N (\gamma_\mu q^2 - 2m_N q_\mu) \gamma_5 \tau_3 u_N \frac{h_A(q^2)}{q^2} \bar{u}_e \gamma_\mu u_e \sim$$

$$\sim -e^2 h_A(q^2) \bar{u}_N \gamma_\mu \gamma_5 \tau_3 u_N \bar{u}_e \gamma_\mu u_e . \qquad (4)$$

The factor $e^2 h_A(0)$ turns out ot be of order $\alpha G m_\rho^2 g_A \; 2\pi / f_\rho^2 \approx 10^{-6} \alpha$ in a simple factorization model. The apparent vanishing of the nucleon axial charge leads to a potential with a range determined by the q^2-dependence of the axial vector form factor $h_A(q^2)$ which is probably meson pole dominated. There is, however, one possible exception to the short range condition for the potential. To the extent that CP is violated in weak interaction, or that the nucleon of fig.1.a is off mass shell, a third axial vector form factor $\tilde{h}_A(q^2)$ may occur in the pv part of the vertex function

$$\Gamma_\mu^{pv} \sim -\bar{u} \{ (\gamma_\mu - \frac{2m_N}{q^2}) q^2 h_A(q^2) + i \sigma_{\mu\nu} q^\nu \varepsilon \tilde{h}_A(q^2) \} \gamma_5 u \qquad (5)$$

ε being typically of order 10^{-3}. That form factor does contribute to M^{PV} and is not constraint to vanish at $q^2=0$ because of gauge invariance.

A similar reasoning applies to the diagram of fig.1.b, which is, unfortunately, highly divergent and cannot be calculated reliably within the framework of conventional weak interaction theory.

Turning now to the possibility that the electromagnetic (em) interaction itself violates parity described by the inclusion of a neutral conserved hadronic axial vector current k_μ

$$H_{em} = e A_\mu (j^\mu_{em} + f k^\mu) . \qquad (6)$$

A short range pv force between an electron and a nucleon would then arise from the amplitude

$$\frac{4\pi\alpha f}{m_V^2} h(q^2)\,\bar{u}_N\gamma_\mu\gamma_5 u_N \bar{u}_e\gamma_\mu u_e \qquad (7)$$

where h(0) is now assumed to be of order unity (1). The dimensionless coupling constant f must not exceed 10^{-2} in order to be compatible with experimental data from nuclear pv transitions. More likely, f should be assumed somewhat smaller, of order 10^{-4}. But even using this smaller value the pv electron nucleon scattering amplitude turns out to be of order G, appreciably larger than the amplitude obtained from conventional weak interaction alone (fig.1.a).

Effectively the same can be inferred if weak neutral currents exist (8). In fact recent experiments of the Gargamelle Collaboration give strong evidence for the existence of both leptonic (9) and hadronic (10) neutral currents. Neutral currents can provide for the direct pv interaction between electron and nucleon as well as among electrons alone as illustrated in fig.2.

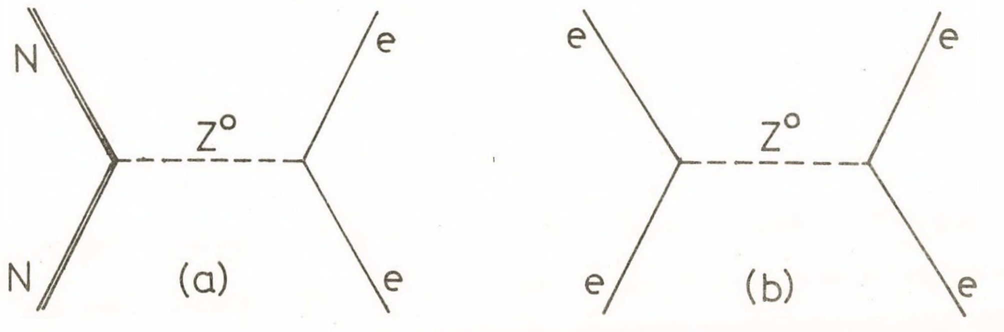

fig.2.

Feynman diagrams for pv scattering processes of order G.

Here the leptonic and semileptonic part of the weak interaction Hamiltonian is involved in contrast to the diagram of fig.1.a where the nonleptonic weak interaction plays a role.

According to the renormalizable theory of Weinberg and Salam the neutral current interaction is mediated by a neutral heavy vector boson Z_o. This makes the interaction extremely short range so that it can be treated as pointlike (which is common to every potential with a range much smaller than Bohr's orbit a_o). Since electrons repel electromagnetically a very short range interaction is practically not seen by them. Thus we neglect the diagram of fig.2.b for the rest of the discussion.

The scattering amplitudes corresponding to the diagrams of fig.2. is evaluated with the aid of the Weinberg model. The effective weak interaction produced by exchange of the neutral vector boson Z_o between an electron and a nucleon is given by (11)

$$L'_c = \frac{G}{\sqrt{2}}[(1-4\sin^2\Theta_W)\bar{e}\gamma_\lambda e+\bar{e}\gamma_\lambda\gamma_5 e]J^Z_\lambda \qquad (8)$$

where $J^Z_\lambda = J^3_\lambda - 2\sin^2\Theta_W J^{em}_\lambda$

and $J^3_\lambda = \bar{N}\gamma_\lambda(1+\gamma_5)\frac{\tau_3}{2}N$

$J^{em}_\lambda = \bar{N}\gamma_\lambda \frac{1+\tau_3}{2} N$

+ meson terms giving rise to form factors in the nucleon matrix elements

From this Lagrangian the lowest order parity violating eN-scattering amplitude is easily constructed. Applying the standard procedure for evaluating the effective potential leads to

$$V^{pv}_{eff} = \frac{G}{4m_e\sqrt{2}}\{\tau^3[(\vec{\sigma}_e-\vec{\sigma}_N)\{\vec{p}_e,\delta(\vec{x})\}-(\vec{\sigma}_e-2\vec{\sigma}_N)\{\vec{p}_e,\delta(\vec{x})\}\cdot 2\sin^2\Theta_W$$
$$+ i(\vec{\sigma}_e\times\vec{\sigma}_N)[\vec{p}_e,\delta(\vec{x})](1-4\sin^2\Theta_W)] \qquad (9)$$
$$- \vec{\sigma}_e\{\vec{p}_e,\delta(\vec{x})\}\cdot 2\sin^2\Theta_W\}$$

This has earlier been obtained by Michel (8) for the case of $\sin^2\Theta_W=0$ and has also been used (in the approximation that nuclear spin effects are neglected) by Bouchiat (12).

The effects that can be produced by such a pv potential are rather tiny if one considers light atoms. To illustrate this point let us quote (8) the "expectation value" of V^{pv}_{eff} between the almost degenerate states ^3S_o and ^3P_o of hydrogen ($<\tau_3>=1$).

$$<2\ ^1S_o|V^{pv}_{eff}|2\ ^3P_o> \simeq -i\frac{\sqrt{6}}{32\pi}6m_e^3\alpha^4(1-4\sin^2\theta_w) \sim i\cdot 10^{-16}eV$$

$$\text{for } \sin^2\theta_w \approx 0.5 \qquad (10)$$

Thus the parity impurity of hydrogen states is determined to be at most of order

$$-iF = \frac{<2\ ^1S_o|V^{pv}|2\ ^3P_o>}{|E_P-E_S|} = \frac{10^{-16}eV}{0.67\cdot 10^{-6}eV} \sim 1.5\cdot 10^{-10} \qquad (11)$$

It has, however, been argued by Bouchiat that an enhancement by a factor of roughly Z^2 (Z being the proton number of the atom) could be obtained for heavy atoms if the S- and P-wave functions are taken correctly into account. Since also the potential develops a factor of Z ($<\tau_3>=Z-N$, $<1>=Z+N$) an enhancement factor of 10^6 for heavy atoms ($Z>50$) is easily obtained (relativistic wave function corrections also being substantial) (12).

Application to mirror molecules

Optical active molecules like amino acids have at least one so called asymmetric (carbon-) atom and their spatial shape is quite unsymmetric. Their electronic ground state wave function (molecular orbital) will not even be an eigenstate of the parity operator since space reflection leads to a new molecule, the mirror isomer. Let us illustrate this in the case of alanine which is an optically active amino acid.

L-alanin D-alanin

fig.3.

Example of optical isomers (enantiomeric pair)

Due to the short range character of the pv electron-nucleon force the molecular orbitals are needed only at the location of the nuclei. In order to estimate the pv contribution to the electronic energy content of an enantiomeric molecule it may be sufficient to use the appropriate atomic orbitals instead of the true molecular orbital which may in turn be obtained by a certain linear combination of atomic orbitals LCAO procedure). For estimating ΔE^{pv} the matrix element of V^{pv}_{eff} between an atomics s orbital and a P orbital is certainly most important and has to be picked out from the relevant Slater determinants of the molecule orbital. With this in mind we may take over a formula for the "expectation value" of V^{pv}_{eff} given by Bouchiat (12):

$$|\langle ns_{\frac{1}{2}}|V^{pv}_{eff}|n'P_{\frac{1}{2}}\rangle| \sim \frac{G}{4m_e\sqrt{2}} Z^2 \frac{\{(4\sin^2\theta_w - 1)Z + N\}}{\pi a_0^4(\nu_n \nu_{n'})^{3/2}} * \qquad (12)$$

$$* \ (1 + \text{smaller terms})$$

with ν_n, $\nu_{n'}$ denoting the effective quantum numbers of the atom considered. This formula represents (up to a weight factor supposed to be of the order of unity) the weak energy correction to an enantiomeric molecula arising from one of its atoms. Summing up all atomic contributions will then give total amount of correction which is attributed to the pv interaction. For the case of alanine (see fig.3.) we therefore propose a result of the order of

$$\sum_{i \approx 3C+N+2O} 1.5 \cdot Z_i^2 \cdot 10^{-16} eV \sim 10^{-13} eV \qquad (13)$$

which may be uncertain by a factor of 10 due to the approximation involved.

We therefore conlcude that possible differences between the binding energies (electronic energy content) of optical isomers should be expected of the order of $\sim 10^{-13}$eV, too small to be detected at present.

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MOLECULAR ASYMMETRY AND WEAK INTERACTION

P. HRASKO, A. GARAY*, L. KESZTHELYI

Central Research Intitute os Physics, Budapest
* Biophysical Institute, Biological Centre
of the Hungarian Academy of Sciences, Szeged

Beside weak interaction parity is apparently strongly violated also in biology, because the cells of organisms contain most of the important molecules in the form of a pure optical isomer. It is natural to try to connect this one-handedness to weak interaction asymmetry. According to A. Garay electrons from beta-decay may interact differently with the stereoisomers of a given molecule and one of the isomers may happen to be more stable against beta-decay background, than the other. This might lead to a small initial difference between the concentrations of the stereoisomers, which has been amplified in the process of self-reproduction up to complete optical purity. Experiment [1] indicates, that this might indeed be the case. Racemic mixture was bombarded with beta-radiation, and after 18 months of irradiation the small prevalence of the isomer, existing in organisms, was found.

This experiment seems to demonstrate, that beta-electrons, which are longitudinally polarized, interact differently with the two isomers. It follows then that electrons, polarized against or along the motion, interact differently with the same stereoisomer, and this may lead to some pecularities of the electron distribution in a pure isomer [2].

Let us imagine an electron moving through a macroscopically isotropic homogeneous medium which is a pure optical isomer. In general the electron polarizes the medium and the motion of the polarization charges has two components: one is parallel to the motion of the electron, the other is orthogonal to it. The second component is a circular motion, which may be right- or left-handed with respect to the velocity of the electron. In a medium not possessing optical activity right- and left-handed motions compensate each other. However, in an optically active material helical structures of a given sense are always present, and the circular motion of the polarization charges may have also definite sense. The magnetic field, induced by this motion at the trajectory of the electron will be parallel (or antiparallel) to the velocity. Therefore, the spin of an electron, polarized perpendiculary to the motion, will be rotated in the induced field with a frequency proportional to the velocity. Since the helical structures in the two isomers are of opposite sense, the magnetic field and, therefore, the rotation of the spin with respect to the velocity will be directed differently in the two isomers.

There exists a very simple single-particle model, which reflects the main features of the propagation of an electron in an infinitely large optically active medium. The model is characterized by the single-particle Hamiltonian

$$H = \frac{1}{2m}p^{2} - w(\underline{\sigma p})$$

According to this Hamiltonian the rate of precession of the spin is proportional to w times the velocity, therefore, the effect of the induced magnetic field is incorporated into the constant w.

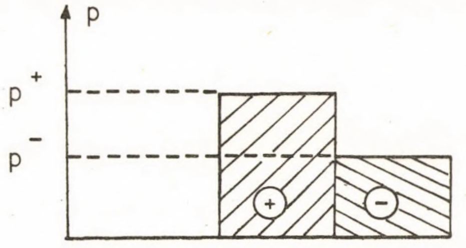
This Hamiltonian describes a kind of free electron gas, which will be called helical. The Fermi-distribution of this gas is characterized by a single chemical potential μ and by two Fermi momenta $p_{\pm}$, corresponding to electrons polarized along or opposite to the motion. Denoting the corresponding electron density by $n_{\pm}$, one finds, that

$$(n^{+}-n^{-}) \sim \begin{cases} w & \text{for } T \ll \mu \\ \frac{w}{\sqrt{T}} & \text{for } T \gg \mu \end{cases}$$

An attempt to observe this difference was made in [3]. Let us confine ourselves to a single spatial dimension and assume that the spin and the direction of motion are completely correlated. The sign of correlation (i.e. the sign of the difference $n^{+}-n^{-}$) is different in the case of the two isomers. Positrons from a radioactive source are polarized along their motion. Therefore, it can be expected, that positronium-annihilation characteristics are somewhat different for the two isomers. If one takes into account, that positronium formation depends on the relative velocity of the positron and electron and assumes that spins are not influenced by the process of capture, then it is easily seen that in one of the isomers predominantly triplet positroniums are formed while in the other most of the positroniums will be in singlet states. It seems that a difference of this kind was observed experimentally [3].

Several other consequences of the model can also be considered. One may calculate [2] the rate of beta-decay of a nucleus, embedded into a medium with w different from zero. This leads to a small change in the final state wave function linear in w, which makes the decay rate dependent on the sign of w. The reflection of an electron beam from the surface of the optical isomer may also lead to some information on the parameter w.

The helical gas model itself produces rotation of polarization of the light, because the ground state is not a parity eigenstate. At zero temperature the ground state can be symbolized by the diagram



Let us take an electron polarized along the motion with momentum from the interval (p_-, p_+), and turn its spin, leaving the momentum unchanged. This will be a zero momentum excitation with finite excitation energy and inner spin, equal to 1. In the long wave length limit this seems to give the important contribution to the optical rotation. The excitation is characterized also by the momentum $\underline{p}$, along which the projection of the spin is equal to one. This is, therefore, a symmetric top in the momentum space. States of definite total angular momentum can be obtained with the aid of the symmetric top wave functions. In the long wave length limit $J=1$ triplet state will be excited.

Assuming that electrons inside molecules can be considered as a helical electron gas, one gets for the angle of rotation per molecule the formula

$$\phi = \frac{64\pi^2}{3} e^2 \frac{Rw}{p_F} \frac{\nu^2}{\nu_g^2 - \nu^2}$$

where R is the radial dimension of the molecule and ν_g is the excitation energy of the triplet state.

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IS THERE ANY ENERGY DIFFERENCE BETWEEN ENANTIOMORPHIC MOLECULAR STRUCTURES?

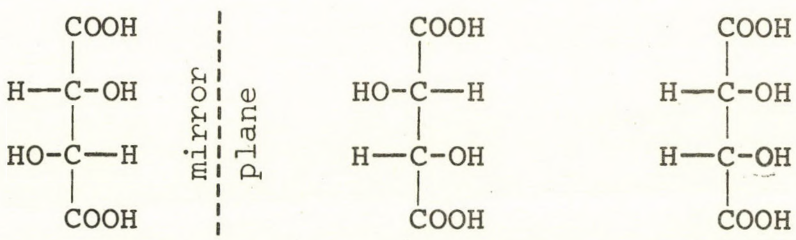
W. THIEMANN

Institut für physikalische Chemie, Kernforschungsanlage Jülich

Introduction

To question the energetic identity of enantiomorphyic molecules is deemed a very serious heresy among chemists. Since the days of Pasteur it has been generally accepted that enantiomeric molecules are the exact mirror images of each other, i.e. they have exactly equal binding energies, enthalpies of formation, vibration and rotation energy patterns, melting points, solubilities, etc. The problem is really the question of symmetry in chemistry, the idea of symmetry has proved so convenient that almost nobody is willing to sacrifice such a fine construction without an established and generally accepted substitute for it.

Let us see in a practical example how Pasteur and his successors came to the postulate of the enantiomorphic molecules as being the mirror images of each other. The phenomenon of molecular optical activity was discovered while studying the tartaric acid - and its various salts which are the products of neutralisation of the free acid with the corresponding bases -. The spatial arrangement of the sixteen atoms in the molecule allows the existence of three chemically distinguishable isomers of the tartaric acid, namely



D_s-tartaric L_s-tartaric meso-tartaric acid

(An equimolar mixture of L_s- and D_s-tartaric acid is called the racemic acid). Pasteur was surprised to find that in the living nature only the optically active D_s- as well as the mesotartaric acid exist in grapes and other fruits, while a chemical synthesis - e.g. through hydroxylation of fumaric or maleic acid - results in the optically inactive racemic and meso-tartaric acid. The deduction from this experience was at hand: The D- and L-forms are enantiomorphic structures, while the meso-form is chemically isomeric though not enantiomorphic to the first ones. This can be seen from the structural formulas, it follows that L_s- and D_s-tartaric acids would differ from each other in the same way as a left glove from a right one. Although their scalar physico-chemical properties (such as binding energies, enthalpies of formation, entropies, etc.) are identical, they are not superimposable. It is consistent with this model that laboratory synthesis always produces both enantiomers of an asymmetric molecular structure at the same concentration.

Problem

One may ask then why it is necessary to search for an energetic difference between enantiomorphic structures, if the energetic identity has been approved for such a long time without any doubt. The justification of questioning Pasteur's principle is based upon several independent observations:

1. The mentioned problem why biology prefers only one enantiomorphic form out of two possible ones has never been resolved in a satisfactory way by the old theory.
2. A systematic study of literature reveals that there are many examples which apparently violate the chemical symmetry. By far the most anomalies have been reported in crystallisation experiments.
3. There is a similar phenomenon in nuclear physics where parity is violated in weak interactions during beta-decay. One may wonder whether any properties of the nucleus - such as parity violation in weak interaction - influence the characteristic properties of the electrons of the atom in a higher order correction.

As to our knowledge it was Yamagata (1966), [16], who first discussed the possibility that the parity violation of weak interaction may cause a tiny parity violation of the electromagnetic interaction, which may bear some significance for the course of chemical evolution. Rein (1974), [9] calculated the magnitude of such an asymmetric perturbation of electromagnetic interaction due to the weak interaction of the nucleus. I do not wish to review the theoretical approaches to the problem, but I rather want to summarize the most important chemical experiments pointing into the same direction. This means that I like to draw the attention to those experiments that do not fit into the classical scheme and allow the interpretation on the basis of energetic difference of enantiomers. The only criterion for referring any work of this kind should be its reproducibility and reliability.

Experimental Evidence

Experiments which indicate possible non-symmetric behaviour of enantiomers can be classified mainly into three categories:

1. Crystallisation of organic compounds where optical activity results from molecular structure.
2. Crystallisation of inorganic compounds where optical activity results from crystal structure.
3. Polymerisation of organic compounds.

By far the largest number of experiments is presented in the first of these groups 1.. The principle may be seen from Fig.1. Starting from a racemic mixture in solution, precipitation results consistently in the enrichment of the one enantiomer over the other one. Examples are given by Pasteur himself (1852, sodiumhydrogenmalate [8]), Kipping and Pope (1903, sodiumammoniumtartrate [7]), Darmois (1953, adrenaline [4]), Havinga (1954, methylethylallylaniliniumiodite [6]), Harada (1970, sodiumammoniumtartrate [5]), Thiemann (1974, asparagine [10]), and Campbell and Garrow (1936, mandelic acid [2]). The authors most frequently make random processes responsible for their results. Their argument reads as follows: Solutions of organic substance have a tendency towards supersaturation, i.e. the thermodynamical equilibrium is not established as long as there are no seeds for initiating the crystallisation. Spontaneous seeding requires a certain activation energy. If this energy is not supplied, precipitation cannot proceed, the solution becomes supersaturated. As soon as one single crystal is introduced saturation will be achieved rather quickly. - Starting from the spontaneous seeding of only one enantiomer, the solution would become saturated in respect to the same one enantiomer; in respect to the other one cannot precipitate. Repetition of experiments together with a careful exclusion of dust particles should finally result in an equal yield of D and L enantiomers averaged over all the independently performed precipitations. The authors mostly were startled that this was not the case in the mentioned works.

The situation is very similar in experiments with crystallisation of inorganic compounds 2. Here the asymmetry comes into play at the level of the crystal structure, the asymmetric information is lost in solution and retained only in the solid crystals. Quite a number of interesting experiments is reported by Copaux (1912,[3]) and Wyrouboff (1896,[15]) on the sodium and potassium salts of heteropolyacids such as silicomolybdato-, silicowolframato-, phosphomolybdato-, and phosphowolframato-acids. A convincing argument for the fact that mostly only one (left or right) enantiomorphous sort of crystals was precipitated has not been given so far. The experimental results were further supported by kinetic studies of the precipitation or solution processes involved.

More recent works on optically active chemicals frequently center on chiral polymers 3., particulary because of their relevancy in relation to the biopolymers occuring in living organisms, such as the proteins, carbohydrates, nucleic acids, etc. Assuming again the validity of the mirror image model of enantiomeric molecules one is astonished to see that studies of the properties of polymers of amino acids as a function of enantiomer contents give often no asymmetric plots around the racemic point. Of peculiar interest are the works of Blout and Idelson (1956,[1]) and Wada (1961,[14])on the γ-benzyl-polyglutamic acids. The principle of these experiments may be seen from Fig.2. An extensive study has been reported by Thiemann and Darge (1974,[11]) on several racemic polyaminoacids, which yielded very tiny optical activity of the polymers.

A non-symmetric behaviour of the enantiomorphic molecules in consideration is apparent, and an interpretation seems to be permitted that questions the validity of the mirror image model.

Conclusion

Taking into account the recent calculations by Rein (1974, [9]) and others on the possible pv-nucleon-electron interaction it appears that the "asymmetry effects" encountered in chemical experiments are too large by many orders of magnitude as to be interpreted by parity violating electromagnetic interactions alone. Yet, among other independent interpretations one may think of an amplification process that would be responsible for the rather drastic "asymmetry effects" observed in the reported precipitation and polymerisation experiments, namely in the range of $\Delta E/E \approx 10^{-4}$ to 10^{-7}.

It has to be emphasized that we do by no means deduce from the chemical evidence that parity violation of weak (and possibly electromagnetic) interaction necessarily were the sole cause for the observed effects; the explanation can be quite different, such as interaction with polarized sunlight, with electromagnetic fields, with the surface of chiral crystals, or others.

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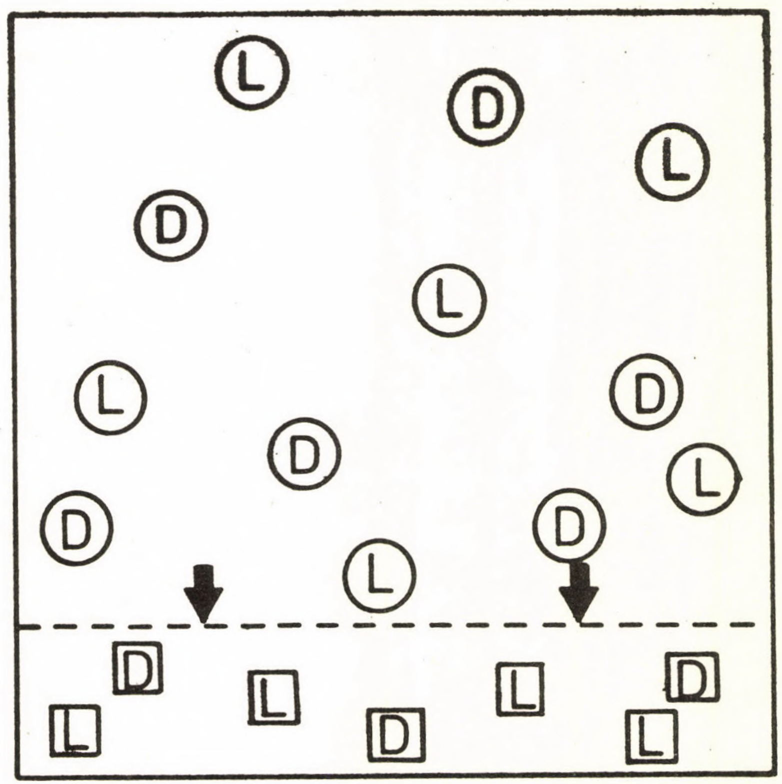
FIG. 1.



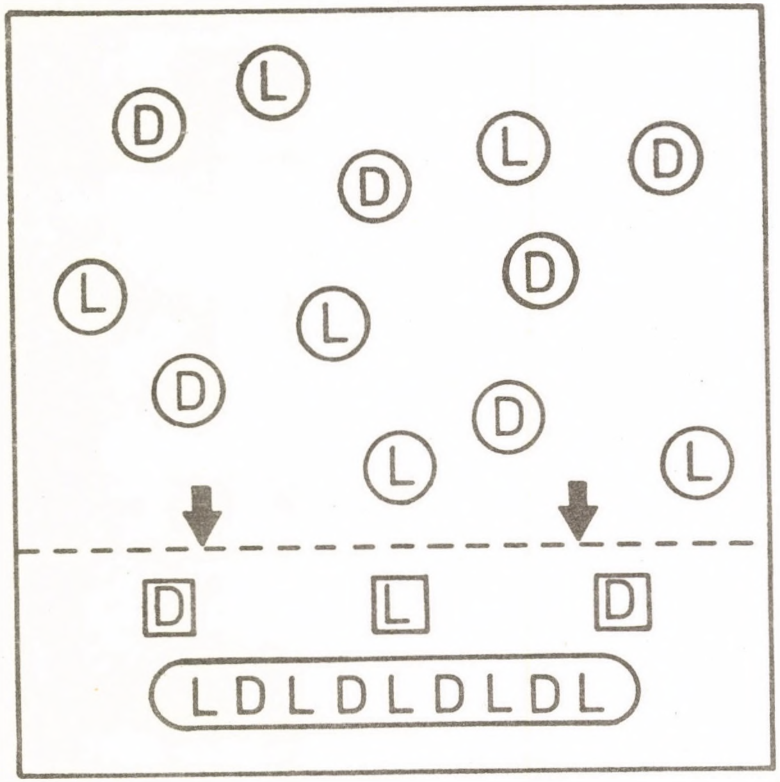
Numerical example
simplified and exaggerated
for illustration of principle:

Saturated
solution
(D/L = 6/6 = 1)

Solid phase
(D/L = 3/4 = 0.75)

Principle of "racemic precipitation"

FIG. 2.



Numerical example simplified and exaggerated for illustration of principle:

Monomers <u>before</u> polymerization (D/L = 6/6 = 1)

Unreacted monomers plus polymer <u>after</u> polymerization (D/L = 4/5 = 0.8)

Principle of "racemic polymerisation"

Az ATOMKI KÖZLEMÉNYEK évenként több számban jelenik meg. Tudományos intézeteknek cserepéldányképpen vagy kérésükre dijtalanul megküldjük, kötelezettség nélkül. Magánszemélyeknek esetenkénti kérésére 1-1 számot vagy különlenyomatot szivesen küldünk. Ilyen irányu kéréseket az intézet könyvtárszolgálatához kell irányitani. /ATOMKI, 4001 Debrecen, Postafiók: 51./

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D e b r e c e n

A kiadásért és szerkesztésért felelős
Szalay Sándor az Intézet igazgatója

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