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INDEX

Zum Geburtszentenarium von Josef Geszti 1

Tarcsay, Gy.—Ádám, f.: Accuracy of satellite platform location techniques: A study of error propagation 3

Asszonyi, Cs.: Plastic state and deterioration of rocks. III. Boundary conditions of rock deterioration 25

Pethő, Sz.: Analysis of separating efficiency functions for flotation plants 43

Pethő, Sz.—Patvaros, J.—Benke, L.: Partitioning of probabilistic distributions and the parameters of the partitioned distribution 65

Halmos, F.—Szádeczky-Kardoss, Gy.: Computation of geodesics from chord-lengths 95

Zambó, J.: Das Kostenminimum bei ebenem Sammeln 107

Treumann, R.: Stochastic model of magnetotellurics for an inhomogeneous Earth 115

Verő-Heténvi, M.: Contribution to the adjustment of free networks 127

Csernyák, L.—Steiner, F.: General properties of gravitational w-functions 131

Halmai, E.: Feldkomparator für Invarband-Nivellierlatten 144

Kovtun, A. A.—Porokhova, L. N.—Chicherina, N. D.: Effectivity of the solution of the problem based on magnetotelluric data from the North-Western part of the Russian Plate 153

Halmos, F.—Kádár, I.: The notion system of geodetical informations and the calculation of geodetical networks 165

Alpár, Gy.: A straight presentation of the adjustment of so-called free geodetic networks 195

Korvin, G.: Correlation properties of source generated seismic noise, scattered on velocity inhomogeneities 201

Korvin, G.: The hierarchy of velocity formulae: Generalized mean value theorems 211

Bencze, P.: Ionospheric sporadic E stratospheric warmings 223

Cz. Miletits, J.—Holló, L.—Tátrallyay, M.—Verő, J.: Experimental results with the characterization of geomagnetic micropulsions. VI. Summary of results, recommendations 231

Afanasyeva, L. T.: Space-time distribution of geomagnetic pulsations and the dependence on the geomagnetic activity 239

Tárczy-Hornoch, A.: Genauigkeitsvegleich der am Anfang orientierten gestreckten gleichseitigen offenen Theodolitpolygonzüge mit günstiger Gewichtsverteilung und der gestrecken Kreiselpolygonzüge 273

Zambó, J.: Theoretische Grundlagen zur Auswahl des Schachtansatzpunktes 287

Recensio

Means, W. D.: Stress and Strain. Basic Concepts of Continuum-Mechanics — *R. Richter* 301

Karl Rinner—65—jahre—f. Somogyi 303

Halmos, F.: Methodische Fragen der geodätischen Anwendung von Dopplerschen Satelliten-Beobachtungen 305

Lakatos, I. J.: The role of wettability in polymer flooding 325

Hristow, W. K.: Allgemeiner Fall der gemeinsamen Ausgleichung der Höhenwerte und Geschwindigkeiten eines Nivellements 337

Debreczeni, E.—Tarján, I.: Dimensionierung und Untersuchung der Funktionsweise von Seitendüsenejektorpumpen 347

Steiner, F.: Contribution to the determination of the gravitational effect of rectangular prisms 375

Zambó, J.: Das Ausstecken des Schachtansatzpunktes außerhalb des produktiven Gebietes 383

Roller, B.: Numerische Untersuchung des Zustandes von Gesteinskomplexen 395

Cz. Miletits, J.: A study of geomagnetic pulsation periods 425

Berktold, A.: On the distribution of the electrical conductivity in the area between the Rhinegraben, the Bohemian Massif and the Central Alps 437

Tátrallyay, M.: On the numerical modelling of two-dimensional conductivity structures in cylindrical and Cartesian co-ordinates 441

Iliceto, V.—Santarato, G.: Proposals for the standardization of magnetotelluric digital tapes 455

Hazay, I.: Adjustment of levellings for the determination of the vertical movements of the Earth's surface 463

Sátori, G.: Interpretation of simultaneous variations of galactic cosmic rays and of the level of atmospheric radio noise 475

Recenziones

Flügge, S. ed.: Handbuch der Physik — Encyclopedia of Physics — *P. Bencze* 487

Crustal and upper mantle structure in Europe — Monograph No. 1 — *E. Bisztricsány* 488

Schröder, W.: Entwicklungsphasen der Erforschung der Leuchtenden Nachtwolken — *A. Tárczy-Hornoch* 488

Piller, H.: Microscope Photometry — *M. Gerencsér* 489

Meissner, R.—Stegena, L.: Praxis der seismischen Feldmessungen und Auswertung — *O. Ádám* 490

Geoelectric and Geothermal Studies — KAPG Geophysical Monograph. Ed. in-chief: A. Ádám—*E. Takács* 491

ZUM GEBURTSZENTENARIUM VON JOSEF GESZTI

Am 1. Januar 1978 sind es genau hundert Jahre, daß der berühmte Polyhistor und Geophysiker Josef GESZTI in Budapest geboren wurde.

Josef GESZTI studierte ursprünglich Maschinenbauwesen und als diplomierter Ingenieur war er ein erfolgreicher Konstrukteur. So kam er allmählich auf den Gedanken, die Entwicklungsgeschichte der Erde nach den Gesetzen der Physik und der Festigkeitslehre zu prüfen, wobei er sich auf die wichtigen Feststellungen von Th. KÁRMÁN und Á. NÁDAI stützte, wonach die Gesteine unter hohem Druck denselben Gesetzen der plastischen Deformation unterworfen sind, wie die Metalle. J. GESZTI veröffentlichte über seine Untersuchungen in den Gerlands Beiträgen zur Geophysik zwischen 1929 und 1937 fünf umfangreiche Arbeiten.

In seiner ersten Arbeit aus 1929 »Zusammenschub der Erdrinde« wurde die Kräfteverteilung bei tangentieller Beanspruchung der Erdkruste untersucht und gezeigt, daß die tangentiellen Druckkräfte ihren extremen Wert in der Fließscheide erreichen, von welcher die bildsame Formänderung ausgeht. Die mechanische Umformung der Erdoberfläche wird sehr sinnreich mit den Gesetzen der bildsamen Verformung erklärt.

In seiner im Jahre 1929 erschienenen zweiten Arbeit »Zur Frage der Entstehung der Kontinente und Ozeane« wird die Entstehung der Kontinente und Ozeane von einem Anfangszustand hergeleitet, in dem das Wasser der Ozeane noch als Dampf gleichmäßig auf der ganzen, bereits festen Oberfläche der Erde verteilt, lastete. Die gleichmäßige Verteilung dieses Ausgangsdruckes wurde durch die Erkaltung, durch die Kondensation des Dampfes gestört; es entstand an den Tiefstellen eine Ansammlung von Wasser, hierdurch eine Mehrbelastung und eine Tieferlegung der im Entstehen begriffenen Becken der späteren Ozeane. An den höher liegenden Flächen, auf den zukünftigen Kontinenten, trat eine Entlastung ein und damit eine sich isostatisch vollziehende weitere Hebung dieser Gebiete. Die Kontinente werden nicht nur vom subkrustalen Material, sondern gleichzeitig vom Wasser der Meere, dem Archimedischen Schwimmprinzip gemäß getragen.

»Die Entstehung der Kontinente« aus 1930 weist darauf hin, daß im

Urzustand der Erde die geschmolzenen simischen Massen überall in gleicher Schichthöhe von leichteren geschmolzenen Sialmassen umhüllt waren. Diese konzentrische Materialverteilung wurde aber später durch thermodynamische Wirkung gestört und in der Gegenwart bestehen die Kontinente aus salischen, die Böden der Ozeane zumeist aus simischen Materialien. Die Entstehung der thermodynamischen Wirkung ist die Folge der Inhomogenität der salischen Massen, die verschiedene Beimischungen enthalten. In der, im Jahre 1931 erschienen Fortsetzung wird die Theorie der Silikatschmelzlösungen zur Erklärung der Entstehung einer thermisch isolierenden Schicht verwendet, welche, in das Stammagma abgesunken, die Erstarrung der darüber befindlichen Schmelze beschleunigt und eine Temperaturdifferenz an der Oberfläche der Erde verursacht, welche die Umlagerung der salischen Massen, das Entstehen der kontinentalen Blöcke einleitet und weiterführt.

In der letzten, im Jahre 1937 erschienenen größeren Abhandlung »Schichtungsvorgang in einem inhomogenen schweren Weltkörper hoher Temperatur« wird die Schwereschichtung auf zwei, oder noch mehr Stufen zurückgeführt, die die Faltung der Erdoberfläche und die Entstehung der Geosynklinalen erklären können und die Wahrscheinlichkeit für Pol- und starke Krustenbewegungen, sowie Klimaänderungen geben.

Die äußerst interessanten Untersuchungen von GESZTI sind mehr als vierzig Jahre alt. Die Erklärungen der Entstehung der Kontinente haben sich seitdem wesentlich vermehrt und erweitert. Aber viele Grundgedanken Geszti's sind darin erhalten geblieben und befruchteten die weiteren Forschungen. Es sei hier nur an die schöne Erinnerung von Prof. Beno GUTENBERG aus Pasadena in den Acta Technica Academiae Scientiarum Hungaricae, Bd. XIX (1957): IN MEMORIAM JOSEPH GESZTI hingewiesen.

J. GESZTI fand im Jahre 1944 den Märtyrertod. Er lebt aber in seinen Arbeiten unsterblich weiter. Auch die ungarischen Geophysiker gedenken am Zentenarium seiner Geburt mit ganz besonderer Huldigung Seiner, eines der größten ungarischer Geophysiker!

A. Tárczy-Hornoch

Acta Geodaet., Geophys. et Montanist. Acad. Sci. Hung. Tomus 13 (1—2), pp. 3—24 (1978)

ACCURACY OF SATELLITE PLATFORM LOCATION TECHNIQUES: A STUDY OF ERROR PROPAGATION

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[Manuscript received June 10, 1975]

The study aims to estimate the errors of platform locations produced by bias and random errors in range and Doppler data of range only, Doppler only and combined (range + Doppler) satellite platform location systems. After an outline of location equations, the results of a numerical error simulation are presented, which can be used for the estimation of probable location errors due to errors in the initial range and Doppler data. Further, for the range only method a formula of the platform position error is also given, which clearly demonstrates the effect of the geometry of location on the resulting position errors. The results obtained in this study are in good agreement with those obtained by others.

1. Introduction

Automatic platform location systems represent an integer part of satellite data collection systems when non-fixed, instrumented automatic platforms are involved. For platform positioning mainly three methods are used: range only, Doppler (or range-rate) only and their combination. Thus the physical quantities used for determining the platform location are distance (range) and Doppler shifts, measured in a one- or two-way communication link between a platform and a satellite.

The location accuracy is mainly affected by the following error sources:

- instrumental errors,
- uncompensated instantaneous motions and displacements of the platform between location measurements,
- tropospheric and ionospheric refraction errors,
- uncertainties in satellite position and velocity,
- error in the vertical component (height) of the platform position (which is generally supposed to be known).

In the present study we will not treat separately the effects of these error sources. Due to the fact that during data reduction and the subsequent computation of positions most of the possible errors are absorbed into the range and Doppler data, we formulate the problem of location accuracy in the following way: let be given a fixed Doppler or range error, what is the resulting error of the platform position.

The results of this study can be used, for example, for the specification of the technical parameters of a planned spacecraft location system, for the estimation of the accuracy required in satellite ephemeris or in the corrections for refraction effects etc.

The developed computer programs, with proper additions, are capable to solve the location problem in operative platform location systems.

2. Location equations

A geocentric rectangular coordinate system with the z axis directed to the North will be used. This system is more convenient than the use of spherical coordinates, which can be easily computed from the rectangular ones.

For the coordinates x, y, z of a platform located at the height h above the chosen reference ellipsoid the following equation can be written:

$$\frac{x^2}{(a+h)^2} + \frac{y^2}{(b+h)^2} + \frac{z^2}{(c+h)^2} = 1\,, \tag{1}$$

where a, b and c are the semi-axes of the ellipsoid. This is the first equation in all of the following equation systems. The platform height h is supposed to be known.

In the following we formulate the problem of location for the minimum number (i.e. two) of measurements which are necessary to get a position fix. Of course, the use of a greater number of measurements may improve the accuracy, and is very advisable.

In writing location equations for the different methods we consider the measured ranges and Doppler shifts as free from any disturbing effects (refraction, platform motion due to drifting and rotation of the Earth etc.).

2.1 *Range only method*

The position of a platform can be determined from at least two successive range measurements belonging to different satellite positions. This method was applied at the Interrogation, Recording and Location System (IRLS) on Nimbus 3 and 4 satellites and on the TIMATION satellites [1, 7]. Using two distances r_1 and r_2 between the platform and the satellite we can write for x, y, z a second order system of equations:

$$\frac{x^2}{(a+h)^2} + \frac{y^2}{(b+h)^2} + \frac{z^2}{(c+h)^2} = 1 \tag{2}$$

$$(x - x_{s1})^2 + (y - y_{s1})^2 + (z - z_{s1})^2 = r_1^2 \tag{3}$$

$$(x - x_{s2})^2 + (y - y_{s2})^2 + (z - z_{s2})^2 = r_2^2\,, \tag{4}$$

where x_{si}, y_{si} and z_{si} $(i = 1, 2)$ are known satellite coordinates at two successive positions. It can be seen that, as the intersection of an ellipsoid and two spheres, two solutions (locations) will be obtained. Based on different criteria the proper platform location can be selected in general.

2.2 *Doppler only method*

A single Doppler measurement locates the platform onto a cone of angle Θ around the instantaneous velocity vector $\mathbf{v}_s$ of the satellite, according to the relation

$$\Delta f = f - f_0 = -\frac{f_0}{c} v_s \cdot \cos \Theta \tag{5}$$

where f_0 and f are the transmitted and received frequencies, respectively, and c is the velocity of light in free space. If there is a transponder on the platform, the two-way Doppler shift will be very closely twice of that given by Eq. (5).

Determining Δf for at least two successive positions of the satellite we can write the following equations for the intersection of the ellipsoid and the two cones:

$$\frac{x^2}{(a+h)^2} + \frac{y^2}{(b+h)^2} + \frac{z^2}{(c+h)^2} = 1 \tag{6}$$

$$x'^2 + y'^2 = z'^2 \operatorname{tg}^2 \Theta_1 \tag{7}$$

$$x''^2 + y''^2 = z''^2 \operatorname{tg}^2 \Theta_2, \tag{8}$$

where the dashed coordinates are those of the platform in such coordinate systems which are centred at the two satellite positions, with z axes directed along the velocity vectors $\mathbf{v}_{s1}$ and $\mathbf{v}_{s2}$, respectively. After transforming the dashed coordinates into the geocentric system and performing lengthy manipulations, the following equation system can be obtained:

$$\frac{x^2}{(a+h)^2} + \frac{y^2}{(b+h)^2} + \frac{z^2}{(c+h)^2} = 1 \tag{9}$$

$$a_1 x^2 + a_2 y^2 + a_3 z^2 + a_4 xy + a_5 yz + a_6 xz + a_7 x + a_8 y + a_9 z = a_{10} \tag{10}$$

$$b_1 x^2 + b_2 y^2 + b_3 z^2 + b_4 xy + b_5 yz + b_6 xz + b_7 x + b_8 y + b_9 z = b_{10}, \tag{11}$$

where the coefficients a_i and b_i are complicated expressions of the coordinates and velocity components of the satellite and $\operatorname{tg} \Theta_1$ and $\operatorname{tg} \Theta_2$, taken at the first and second satellite positions, respectively. Similarly to the range only method,

the ambiguity arising from the two solutions here must also be resolved by some criterion.

The location procedure in case of operative Doppler only systems is more complicated, because in addition to the position of the platform it is advantageous (and useful) to determine also its drift velocity (two components) and the departure of the actual transmitter frequency f_0 from its nominal value [3].

A random access one-way Doppler location system has been implemented on Nimbus F and is planned on Tiros N and Itos H, I, J satellites [1, 2, 3, 4].

2.3 *Combined method (range + Doppler)*

If both the range and the Doppler shift are measured simultaneously at each position of the satellite, a position fix can be derived for each case by solving the following system:

$$\frac{x^2}{(a+h)^2} + \frac{y^2}{(b+h)^2} + \frac{z^2}{(c+h)^2} = 1 \tag{12}$$

$$(x - x_s)^2 + (y - y_s)^2 + (z - z_s)^2 = r^2 \tag{13}$$

$$a_1x^2 + a_2y^2 + a_3z^2 + a_4xy + a_5yz + a_6xz + a_7x + a_8y + a_9z = a_{10}\,. \tag{14}$$

The intersection of an ellipsoid, a sphere of radius r and a cone presents the same resolvable ambiguity as at the other two methods. Here the geometry of the solution is generally more favourable as in the other two methods, and the displacement of the platform between two location measurements is not disturbing. This method was used in the EOLE experiment [1, 5], and is planned for the GEOLE system [6].

3. Study of platform location accuracy by error simulation

3.1 *General remarks*

In this study we simulated numerically the whole location problem under rather realistic conditions. First the simulated Doppler and range measurements were computed together with all the necessary other data, then errors were added to the simulated Doppler and range data and the positions of the platforms were computed by using the equations presented in the previous section.

It is important to emphasize in advance that the propagation of measurement errors depends on the geometry of positioning, which, on the other hand, can be well described in terms of the azimuth and elevation angles of the satel-

lite measured from the platform. Therefore, *for the purpose of error simulation*, there is no need to vary the chosen orbital elements of the satellite or to involve a great number of fictive platforms in a complicated geometrical arrangement. Thus, a single passage on a nearly circular, polar orbit and a simple configuration of platforms have been used in the modelling. Due to the reason mentioned above, however, this does not impose any restrictions on the generality of the results obtained (the whole configuration can be rotated arbitrarily around the centre of the Earth).

The computer programs developed and used in this study permit the simulation of any planned system with arbitrary satellite orbits and platform positions.

3.2 *The model used*

a) The Keplerian elements of the satellite orbit for the given passage were:

$a = 7200$ km $\quad i = 90°$ $\quad \Omega = 5°$

$e = 0.01$ $\quad \omega = 50°$ $\quad \tau = 840$ sec

$\tau_{eq} = 0$ sec

The average height of the satellite above the Earth's surface was 750 km.

b) The 11 satellite positions used in this study followed each other at 1 minute intervals. The corresponding subsatellite points are shown in Fig. 1.

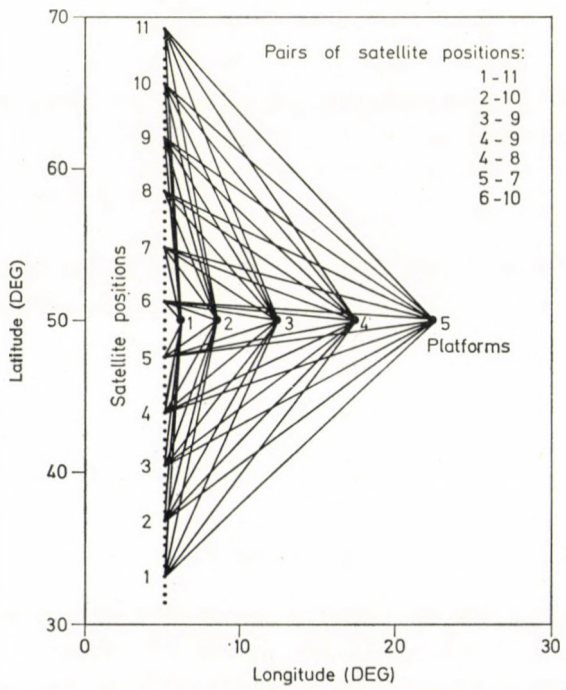

Fig. 1

Table I

Elevation and azimuth of the satellite measured from platforms Nos 1—5 at successive points along the orbit

	Elevation (Deg.)					Azimuth (Deg.)				
Sat. \ Plat.	1	2	3	4	5	1	2	3	4	5
1	12.1	11.1	10.7	8.5	5.9	182.9	190.0	200.9	213.3	223.9
2	18.8	18.4	16.7	13.5	9.9	183.5	192.1	205.0	218.9	230.1
3	28.4	27.5	24.5	19.4	14.3	184.5	195.6	211.4	227.6	238.4
4	43.2	41.1	34.8	26.1	18.7	186.8	202.9	223.0	239.4	249.8
5	67.3	60.6	46.4	32.3	22.5	195.6	225.0	246.8	258.4	264.9
6	77.2	67.1	49.9	34.4	24.1	207.3	240.1	252.9	283.5	282.9
7	50.1	47.7	40.4	30.7	22.7	353.2	337.6	319.5	307.2	301.1
8	32.6	31.8	29.0	24.1	19.0	356.4	347.8	335.6	324.1	316.4
9	21.7	21.3	20.0	17.6	14.6	357.8	352.8	343.9	335.0	328.1
10	14.1	14.0	13.3	12.0	10.2	358.8	354.7	348.9	342.2	336.6
11	8.5	8.4	8.0	7.2	6.2	359.0	356.3	352.1	347.2	342.8

For each position we computed the coordinates and velocity components of the satellite and other data.

c) The five platforms used in modelling were located along the same parallel of $\varphi_p = 50°$, at longitudes of $\lambda_p = 6°, 8.5°, 12.5°, 17.5°$ and $22.5°$ (Fig. 1).

Distance, Doppler shift (for $f_0 = 400$ MHz), cone-angle Θ (see Eq. (5), azimuth, elevation and other data were computed for each platform and for each satellite position. The azimuth (measured in clockwise direction) and elevation angles are listed in Table I.

d) For the Earth a sphere of radius $R_E = 6\,378\,160$ m was used instead of an ellipsoid, and the platform heights were taken to be zero ($h = 0$). The rotation of the Earth was neglected for the sake of simplicity.

3.3 *Error simulation*

From Section 2 it is clear that in the simplest case measurements at two satellite positions are needed to locate a platform by range or Doppler methods provided that its height is known. Therefore we have selected 7 pairs of satel-

Table II

Results of error simulation

Errors: Range 1 : 150 meters, Doppler 1 : 2×10^{-9}
Range 2 : 150 meters, Doppler 2 : 2×10^{-9}

Sat. Pos.	Plat. No.	Elev. 1	Elev. 2	Azidif	Error in position (meters)			
					Range	Doppler	Combined 1st Sat. Pos.	Combined 2nd Sat. Pos.
	1	12	9	176	4311	2546	4670	5559
	2	12	8	166	1272	1933	1402	1548
1—11	3	11	8	151	610	1374	710	755
	4	9	7	134	387	910	498	498
	5	6	6	119	297	675	429	402
	1	19	14	175	3509	1396	3337	3490
	2	18	14	163	1034	968	1003	1000
2—10	3	17	13	144	501	729	524	511
	4	14	12	123	324	530	385	363
	5	10	10	107	254	429	347	313
	1	28	22	173	2764	846	2358	1820
	2	27	21	157	815	487	715	554
3—9	3	25	20	133	403	380	390	324
	4	19	18	108	269	306	303	265
	5	14	15	90	218	275	284	251
	1	43	22	171	2297	1576	1780	1820
	2	41	21	150	682	524	546	554
4—9	3	35	20	121	347	324	309	324
	4	26	18	96	242	252	250	265
	5	19	15	78	202	230	242	251
	1	43	33	169	2100	559	1780	520
	2	41	32	145	624	248	546	243
4—8	3	35	29	113	319	195	309	212
	4	26	24	85	224	179	250	210
	5	19	19	66	190	183	242	215
	1	67	50	157	1587	403	1551	579
	2	61	48	113	479	144	475	242
5—7	3	46	40	72	258	110	269	199
	4	32	31	49	193	116	221	195
	5	23	23	36	171	136	217	202
	1	77	14	152	1238	4582	1269	3490
	2	67	14	115	393	1280	397	1000
6—10	3	50	13	99	238	613	238	511
	4	34	12	58	194	391	204	363
	5	24	10	54	178	305	204	313

Table III

Results of error simulation

Errors: Range 1 : 150 meters Doppler 1 : 5×10^{-9}
Range 2 : 150 meters Doppler 2 : 5×10^{-9}

Sat. Pos.	Plat. No.	Elev. 1	Elev. 2	Azidif	Error in position (meters)			
					Range	Doppler	Combined 1st Sat. Pos.	Combined 2nd Sat. Pos.
	1	12	9	176	4311	6420	10653	15491
	2	12	8	166	1272	4836	3308	4009
1—11	3	11	8	151	610	3435	1668	1915
	4	9	7	134	387	2274	1161	1227
	5	6	6	119	297	1688	999	960
	1	19	14	175	3509	3517	7317	9864
	2	18	14	163	1034	2422	2246	2673
2—10	3	17	13	144	501	1823	1163	1311
	4	14	12	123	387	2274	1161	1227
	5	10	10	107	297	1688	999	960
	1	28	22	173	2764	2130	4702	5622
	2	27	21	157	815	1219	1439	1594
3—9	3	25	20	133	403	951	778	823
	4	19	18	108	269	765	611	607
	5	14	15	90	218	688	590	544
	1	43	22	171	2297	4009	2915	5622
	2	41	21	150	682	1312	900	1594
4—9	3	35	20	121	347	810	517	823
	4	26	18	96	242	631	443	607
	5	19	15	78	202	575	460	544
	1	43	33	169	2100	1405	2915	2509
	2	41	32	145	624	619	900	775
4—8	3	35	29	113	319	488	517	475
	4	26	24	85	224	448	443	416
	5	19	19	66	190	459	460	421
	1	67	50	157	1587	1012	1910	403
	2	61	48	113	479	360	600	285
5—7	3	46	40	72	260	275	366	288
	4	32	31	49	193	291	342	315
	5	23	23	36	171	339	379	356
	1	77	14	152	1238	12113	1110	9864
	2	67	14	115	393	3211	385	2672
6—10	3	50	13	99	238	1535	285	1311
	4	34	12	58	194	978	299	879
	5	24	10	54	178	764	344	725

Table IV

Results of error simulation

Errors: Range 1 : 150 meters Doppler 1 : 1×10^{-8}
Range 2 : 150 meters Doppler 2 : 1×10^{-8}

Sat. Pos.	Plat. No.	Elev. 1	Elev. 2	Azidif	Error in position (meters)			
					Range	Doppler	Combined 1st Sat. Pos.	Combined 2nd Sat. Pos.
	1	12	9	176	4311	13046	19759	38077
	2	12	8	166	1272	9683	6460	8158
1—11	3	11	8	151	610	6872	3273	3866
	4	9	7	134	387	4549	2282	2461
	5	6	6	119	297	3376	1969	1914
	1	19	14	175	3509	7130	13541	22295
	2	18	14	163	1034	4849	4312	5496
2—10	3	17	13	144	501	3648	2244	2668
	4	14	12	123	324	2650	1648	1768
	5	10	10	107	254	2145	1501	1442
	1	28	22	173	2764	4311	8458	12511
	2	27	21	157	815	2440	2654	3360
3—9	3	25	20	133	403	1902	1449	1705
	4	19	18	108	269	1531	1154	1217
	5	14	15	90	218	1376	1130	1074
	1	43	22	171	2297	8268	4774	12511
	2	41	21	150	682	2630	1506	3360
4—9	3	35	20	121	347	1620	894	1705
	4	26	18	96	242	1263	802	1217
	5	19	15	78	202	1149	861	1074
	1	43	33	169	2100	2838	4774	5999
	2	41	32	145	624	1240	1506	1747
4—8	3	35	29	113	319	977	894	989
	4	26	24	85	224	897	802	818
	5	19	19	66	190	917	861	813
	1	67	50	157	1587	2039	2511	1797
	2	61	48	113	479	720	823	674
5—7	3	46	40	72	258	550	564	546
	4	32	31	49	193	582	589	582
	5	23	23	36	171	678	693	664
	1	77	14	152	1238	27439	877	22295
	2	67	14	115	393	6462	430	5496
6—10	3	50	13	99	238	3074	426	2668
	4	34	12	58	194	1957	512	1768
	5	24	10	54	178	1528	626	1442

Table V

Results of error simulation

Errors: Range 1 : 150 meters Doppler 1 : -1×10^{-8}
Range 2 : —150 meters Doppler 2 : 1×10^{-8}

Sat. Pos.	Plat. No.	Elev. 1	Elev. 2	Azidif	Error in position (meters)			
					Range	Doppler	Combined 1st Sat. Pos.	Combined 2nd Sat. Pos.
	1	12	9	176	155	—	19126	—
	2	12	8	166	154	7970	6242	8069
1—11	3	11	8	151	157	4006	3180	3914
	4	9	7	134	165	2739	2237	2495
	5	6	6	119	175	2353	1947	1935
	1	19	14	175	163	17616	12597	17563
	2	18	14	163	159	5738	4009	5627
2—10	3	17	13	144	163	2965	2117	2764
	4	14	12	123	175	2155	1588	1821
	5	10	10	107	190	1982	1469	1473
	1	28	22	173	183	12461	7105	11596
	2	27	21	157	170	3996	2251	3653
3—9	3	25	20	133	177	2169	1289	1843
	4	19	18	108	195	1734	1082	1285
	5	14	15	90	219	1758	1094	1109
	1	43	22	171	338	11881	2866	11693
	2	41	21	150	200	3815	991	3653
4—9	3	35	20	121	198	2090	715	1843
	4	26	18	96	219	1700	731	1284
	5	19	15	78	248	1767	826	1109
	1	43	33	169	247	8787	2866	11693
	2	41	32	145	201	2831	992	2179
4—8	3	35	29	113	213	1682	715	1161
	4	26	24	85	246	1557	731	891
	5	19	19	66	288	1795	826	846
	1	67	50	157	510	7918	388	3872
	2	61	48	113	327	2661	389	1211
5—7	3	46	40	72	350	1826	441	714
	4	32	31	49	426	2035	544	638
	5	23	23	36	522	2683	670	685
	1	77	14	152	1912	21322	2016	17563
	2	67	14	115	601	7100	659	5627
6—10	3	50	13	99	380	3715	476	2764
	4	34	12	58	342	2753	520	1821
	5	24	10	54	351	2584	625	1473

lite positions, then for each platform added errors to the exact ranges and Doppler shifts. With these simulated, inaccurate data we separately solved the equation systems (2—4), (9—11) and (12—14) to obtain platform coordinates by the three methods. Comparing these coordinates to the accurate values we could obtain the *errors in platform positions* (the magnitude of horizontal displacements measured from the exact positions). The combined method could be applied separately for each satellite positions, thus from each pair of satellite positions two independent error figures have been obtained for each platform (see Tables II—V).

The chosen pairs of satellite positions were: 1—11, 2—10, 3—9, 4—9, 4—8, 5—7 and 6—10 (Fig. 1).

The following four combinations of simulated measurement errors were used for each positioning:

Error in range (m)	*Error in Doppler* (relative error in the received carrier frequency)
150	2×10^{-9}
150	5×10^{-9}
150	1×10^{-8}
+ 150 and − 150	-1×10^{-8} and $+1\times10^{-8}$

It is to be noted that a range error of 150 m is equivalent to a timing error of 0.5 μsec in the *one-way* propagation time. An error of 1×10^{-8} in Doppler measurement is equivalent in this study to an absolute error of $10^{-8} f_0$ in the one-way Doppler shift, which corresponds to a radial velocity (i.e. $v_s \cos \Theta$) of −3 m/sec.

3.4 *Discussion of results*

a) The results are summarized in Tables II—V, each of which refers to a different combination of range and Doppler errors. In these tables Elev. 1 and 2 denote elevations to satellite at the two positions indicated in the first column and Azidif is the azimuth difference between the same positions.

A look at the first five columns of Tables II—V reveals the following feature of our error simulation study. For the range only and Doppler only methods the tables contain results for $7\times5 = 35$ different cases in the platform, first satellite position, second satellite position configuration. For the combined method the number of modelled geometrical configurations is $11\times5 = 55$. Therefore the presented results characterize rather well any geometry of location that may actually occur.

b) By comparing different tables it can be seen that the error propaga-

tion is nearly linear in the ranges of errors used. Therefore position error figures for other range and Doppler errors can be easily computed from those presented by a simple multiplication.

c) A closer examination of the results in Tables II—V shows that the obtained results can be directly used for the following purposes:

— estimation of the effect of *bias* (systematic) measurement errors in all three methods (from Tables II—IV),

— estimation of the effect of range and Doppler errors of given magnitude with *randomly* distributed signs in case of the *combined* method (from Tables II—V). This is justified by the fact that comparing the content of Table IV to that of Table V, which was computed with errors of opposite signs, it can be seen that the magnitudes of position errors for the combined method practically do not differ in the two tables (considering results in the last two columns together, of course).

d) On the basis of Tables IV and V, valuable inferences can be drawn on the *statistics of location errors*, in the following way.

Table VI

Percentage distribution of location errors for errors of ± 150 meters in range and $\pm 1\times10^{-8}$ (relative error) in Doppler measurements

Method	Per cent of cases between error limits (km)						
	0—0.5	0.5—1	1—2	2—3	3—4	4—5	>5
Range	75.6	10.0	7.1	4.3	1.5	1.5	0
Doppler	0	10.1	26.1	26.1	8.7	5.8	23.2
Combined	4.3	27.3	28.8	12.2	6.5	2.9	18.0

Table VI, which was constructed from Tables IV and V, contains a percentage distribution of location errors (for 70, 69 and 139 location errors in range, Doppler and combined methods, respectively). Let us consider the indicated errors of 150 m and 1×10^{-8} as standard (r.m.s.) errors of range and Doppler data, respectively. Then Table VI represents position errors corresponding to one sigma measurement errors. From Table VI we can get the more interesting Table VII, which shows results corresponding to *two sigma* confidence intervals for standard measurement errors of 150 m and 1×10^{-8}, i.e. the probability that the location errors will be less than the indicated error limits is $\sim$95 per cent (supposing that the measurement errors have a Gaussian distribution).

For example, there is a $\sim$95 per cent probability that for a one-way Doppler only system with standard errors of 1×10^{-8} about 64 per cent of location errors will exceed 4 km, and $\sim$71—77 per cent will remain below

Table VII

Percentage distribution of location errors below given limits at 95 per cent (two sigma) confidence level for standard errors of 150 meters in range and 1×10^{-8} (relative error) in Doppler measurements

Method	< 1 km	< 2 km	< 4 km	< 6 km	< 8 km	< 10 km
Range	75.6	85.6	92.7	97.0	98.5	100.0
Doppler	0	10.1	36.2	62.3	71.0	76.8
Combined	4.3	31.6	60.4	72.6	79.1	82.0

8—10 km. It directly follows that *the positioning of platforms to a location accuracy of better than ~4—5 km requires a one-way Doppler only system with a total r.m.s. error of* $\lesssim 5\times10^{-9}$ (or a two-way one with a total r.m.s. error of 1×10^{-8}). A similar location accuracy can be achieved by a *range only* system with standard errors of *6—700* m in the one-way range. Taking into account the remark made under point *c*, from Table II and III the same error characteristics can be derived for the combined method as those in the last line of Table VII. Then, summarizing the results for the *combined* method, it can be estimated at a ~95 per cent confidence level that for an r.m.s. error of 150 m in range and with standard errors of 2×10^{-9}, 5×10^{-9} and 1×10^{-8} in the Doppler data, about 73—77 per cent of the resulting location errors will remain below about 2, 3.5—4 and 6—7 km, respectively. Thus the accuracy of the combined system is somewhere between those of its range and Doppler subsystems.

The results presented here are in good agreement with those obtained actually or yielded by other considerations [1, 2, 7, 8]. For example, in the TWERLE experiment, where the planned location accuracy by a one-way Doppler system is 5 km (r.m.s.) [3, 4], "it is necessary to provide a (platform) oscillator with stability of a few parts in 10^{-9} averaged over one second intervals to provide the desired position location accuracy" [2, Annex C].

e) There is no place here to discuss in detail the optimum choice of satellite positions for obtaining maximum location accuracy in different methods. We only note that the Doppler and range methods generally give the best results for such pairs of satellite positions which are separated in azimuth by about 90°. The combined method tends to yield maximum accuracy for satellite positions near to the point of closest approach of satellite to platform (compare results for satellite positions Nos. 5, 6 and 7 with those for other positions). For the range method somewhat more details can be found in Section 4.

4. Location accuracy of a range only system — an analytical description

An analytical treatment gives generally a much better insight into the details of the problem under investigation than a numerical one. Therefore, based on simple geometrical considerations and plausible approximations, we derived the following formula for the error D of the platform position:

$$D = \frac{1}{\sin \Delta\alpha} \left(\frac{\Delta r_1^2}{\cos^2 \gamma_1} + \frac{\Delta r_2^2}{\cos^2 \gamma_2} - 2 \frac{\Delta r_1 \Delta r_2 \cos \Delta\alpha}{\cos \gamma_1 \cos \gamma_2} \right)^{1/2} \tag{15}$$

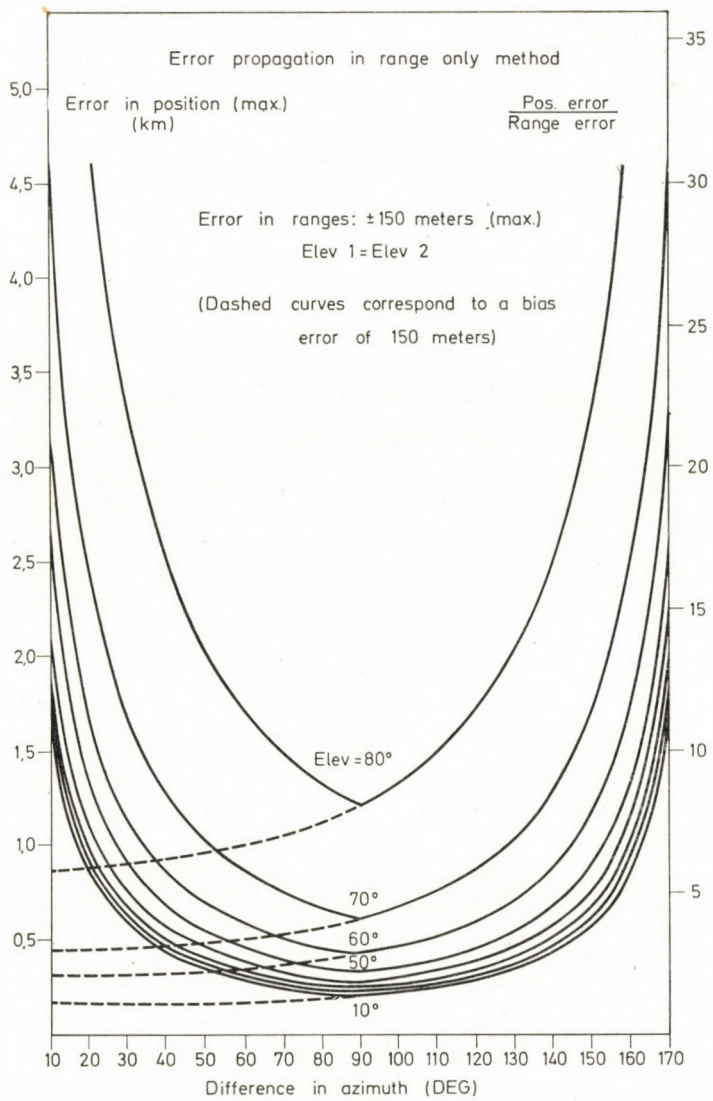

Fig. 2

where

$\Delta\alpha$ is the absolute value of the difference in azimuth between the two satellite positions,

Δr_1 and Δr_2 are errors in ranges r_1 and r_2, respectively,

γ_1 and γ_2 are elevations to first and second satellite positions, respectively.

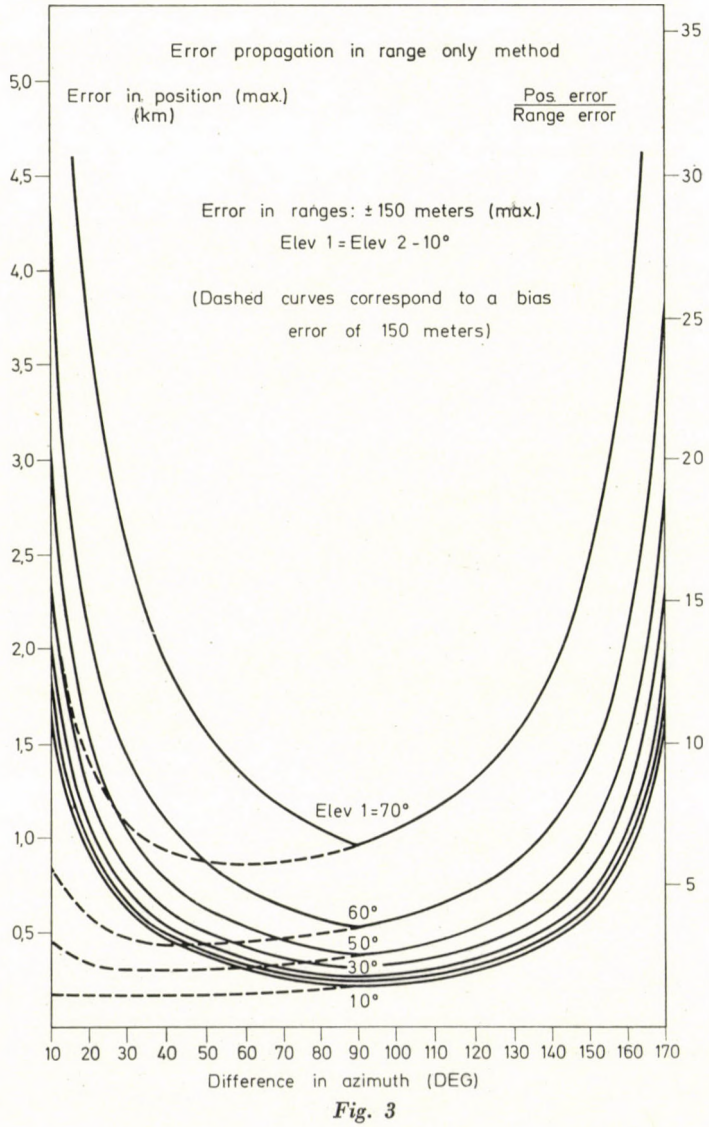

Fig. 3

From Eq. (15) the following basic features of location by range only method can be easily seen:

a) The position error will be the same for a given $\Delta\alpha$ and for equal signs of Δr_1 and Δr_2, than for an azimuth difference of $(\pi - \Delta\alpha)$ and opposite signs of Δr_1 and Δr_2.

b) With respect to the denominator $\sin \Delta\alpha$, it is advisable to combine range measurements at such positions of the satellite for which $\Delta\alpha$ is not too far from 90°.

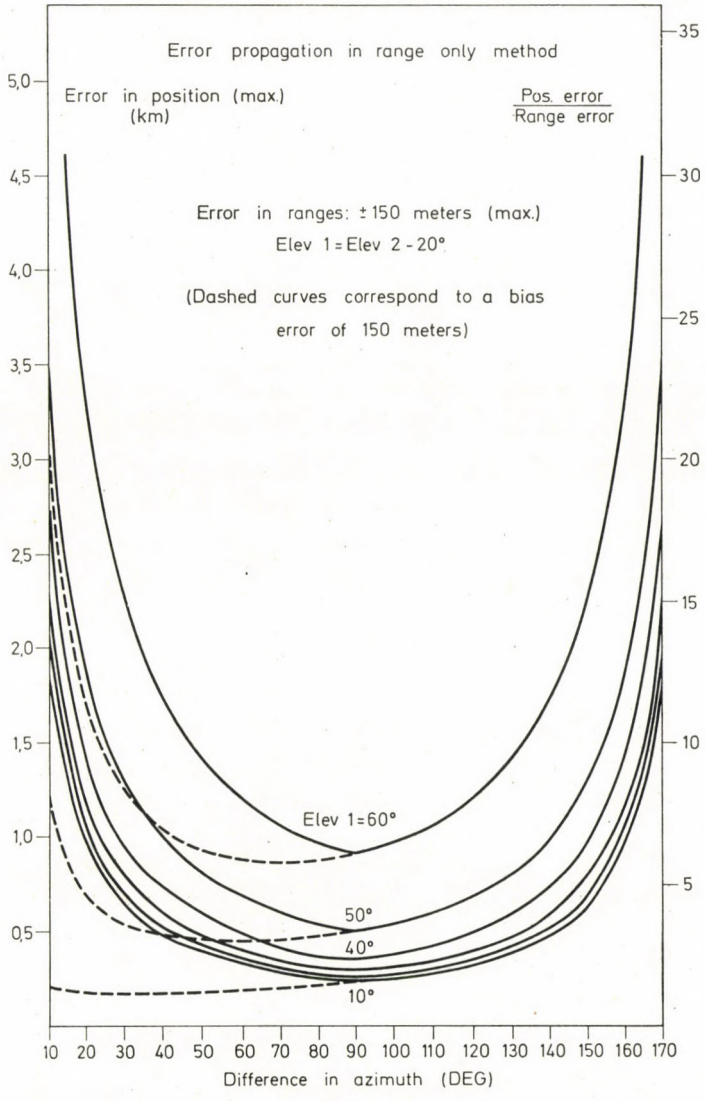

Fig. 4

c) The effect of ranging errors will be smaller for lower elevations (for greater values of cos γ_i).

Using Eq. (15) we computed D for a wide range of geometrical configurations that may occur in the following way. With given Δr_1 and Δr_2 values the position error was computed for $\Delta\alpha$ between 10–170° at steps of 10° with parameters γ_1 and γ_2. γ_1 was varied from 10° to 80° at steps of 10°, γ_2 was varied from γ_1 to 80° at steps of 10°, too.

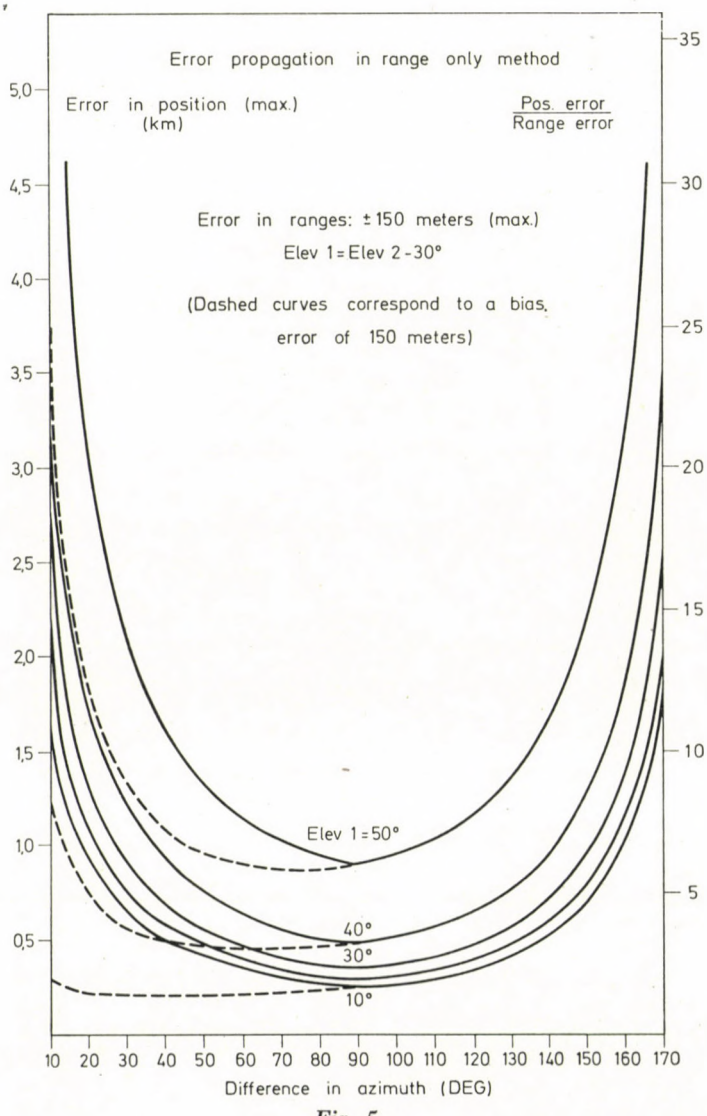

Fig. 5

The results for $|\Delta r_1| = |\Delta r_2| = 150$ m are plotted in Figs. 2 to 9. The full lines correspond to maximum location errors that may occur for ***random*** errors of ± 150 m in ranges. The dashed curves correspond to a ***bias*** error of $+150$ m. (For -150 m these curves are to be transposed through the line $\Delta\alpha = 90°$.)

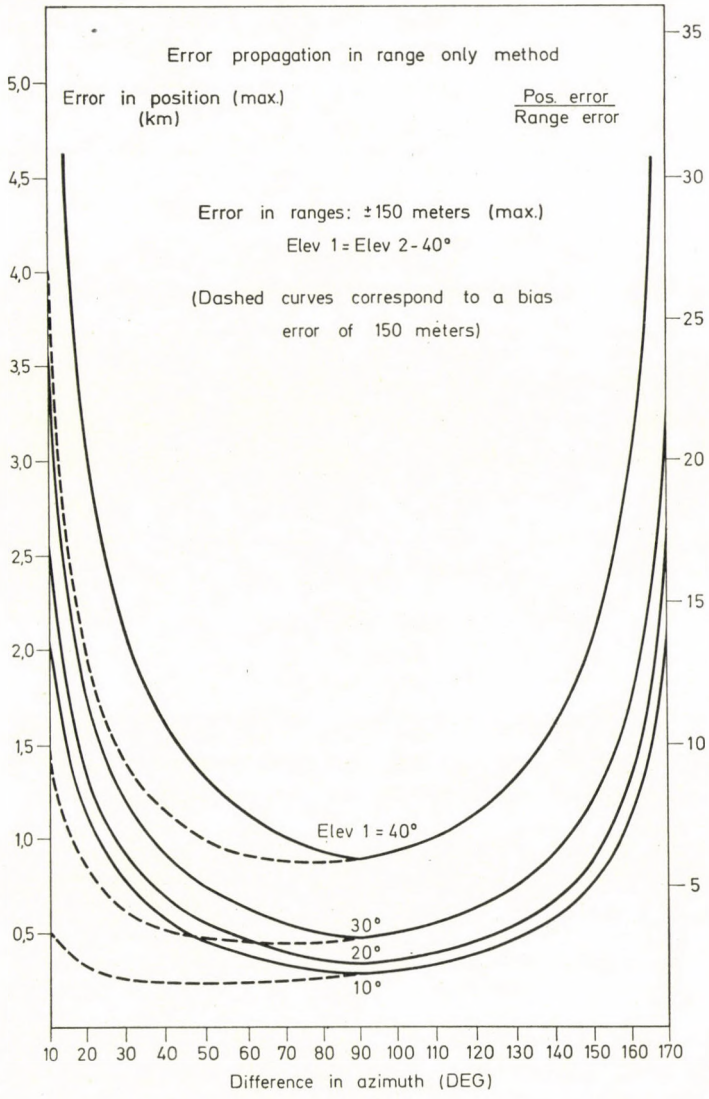

Fig. 6

The results obtained here are in complete agreement with those provided by error simulation (Section 3) for the range only method, and demonstrate clearly the basic role of geometry in the platform location problem.

Unfortunately, in case of the Doppler and combined methods the derivation of formulas similar to (15) seemed to be much more complex, hence it was not performed.

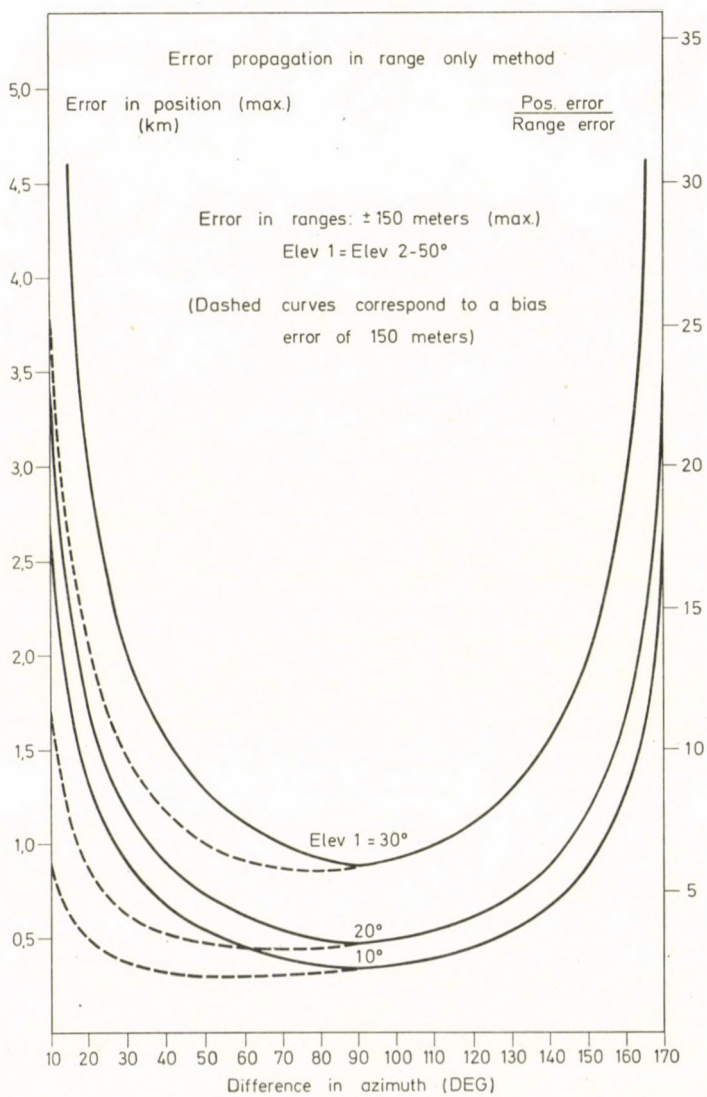

Fig. 7

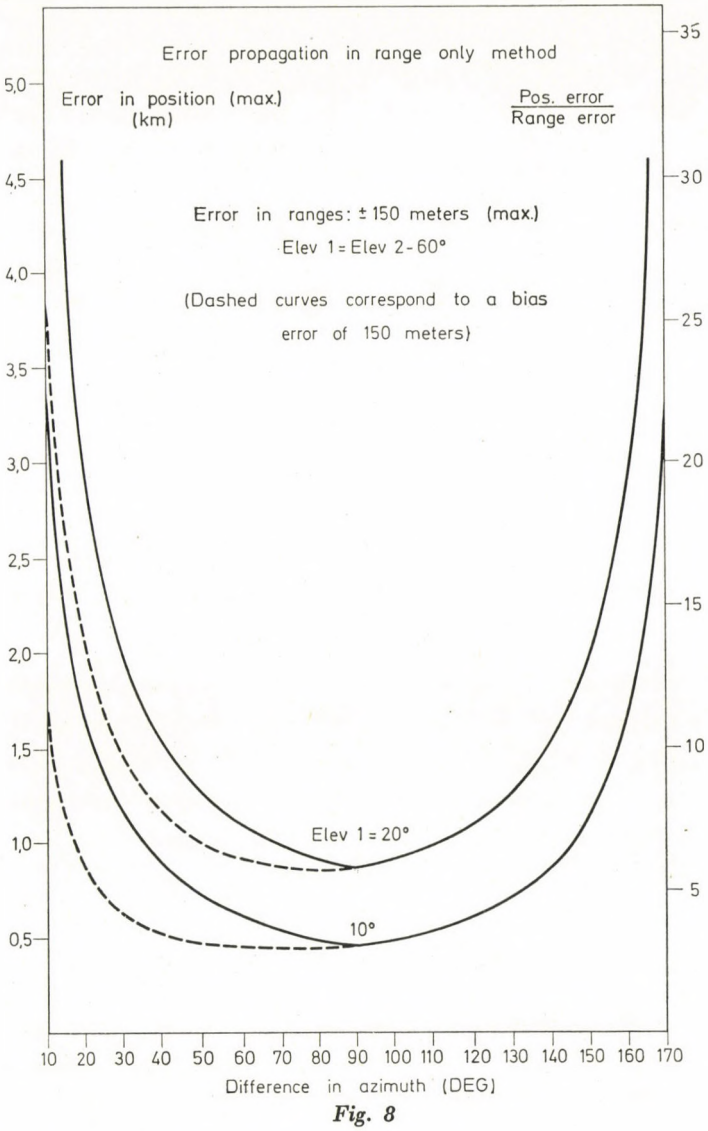

Fig. 8

5. Conclusions

On the basis of the numerical simulation of error propagation supported and checked by an analytical consideration for the range only method, we can estimate the probable location accuracy of the range only, Doppler only and combined location systems. Accordingly, there is about a 95 per cent probability that 71—77% of the errors in platform positions for a Doppler

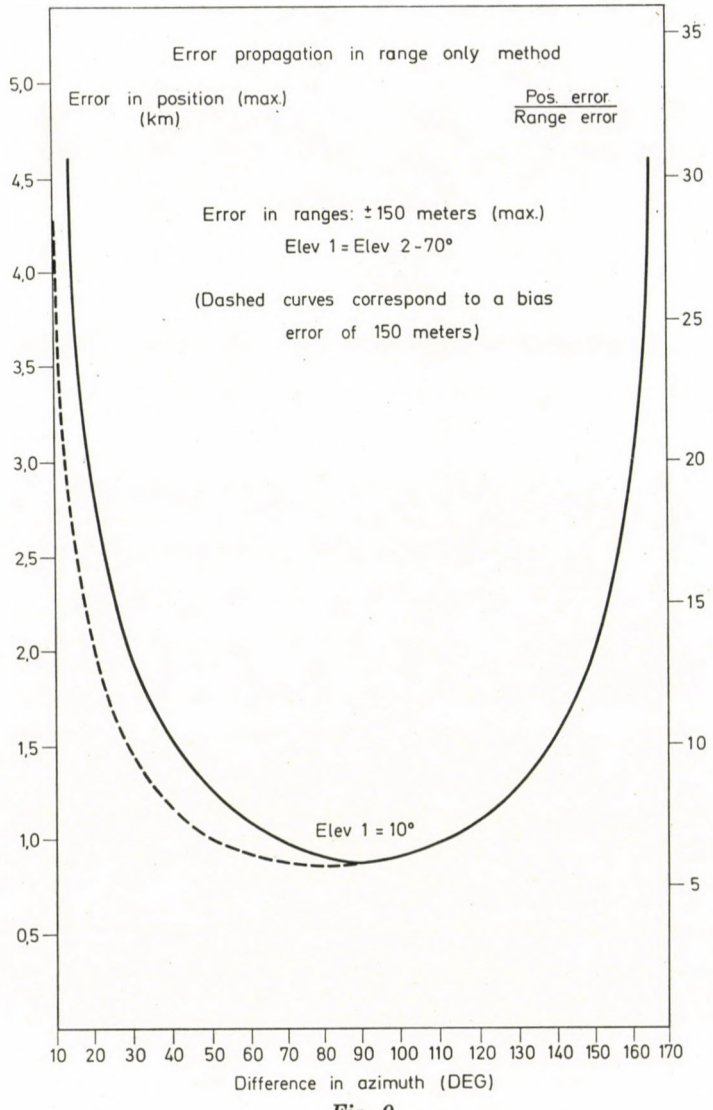

Fig. 9

only system will be less than about 4—5 km, provided that the one-way Doppler shifts are determined with an r.m.s. error of $\lesssim 5 \times 10^{-9}$ of the carrier frequency. A similar accuracy can be achieved by a range only system with standard errors of $\lesssim 6$—700 m in the one-way range. The location accuracy of a combined system will lay somewhere between those of its subsystems.

The use of redundant measurements may increase the accuracy of positioning, of course.

The bias errors in range and Doppler shift produce position errors of roughly the same magnitude as those caused by random errors.

REFERENCES

1. Morel, P.: Satellite techniques for automatic platforms location and data relay. *Space Research*, 13 (1973), 128—146, Akademie-Verlag, Berlin.
2. The Feasibility of the First GARP Global Experiment (FGGE) and the Criticality of Initiating Systems Planning, a Report to JOC, COSPAR Working Group 6, January 1971.
3. Carel, O.: Les systemes a Doppler Aleatoire: TWERLE, TIROS N. *La Recherche Spatiale*, 11, (1972), No. 3, 6—7.
4. Report on Active and Planned Spacecraft and Experiments, World Data Center A, Greenbelt, Maryland, January 1974.
5. Dargent, A.: Le traitement des donnees de l'experience EOLE. *La Recherche Spatiale*, 12, (1973), No. 1, 1—6.
6. Thieriet, D.: Le systeme GEOLE. *La Recherche Spatiale*, 11, (1972), No. 3, 8—9.
7. Ehrlich, E.: The Role of Time-Frequency in Satellite Position Determination Systems. Proc. of the IEEE. 60 (1972), 564—571.
8. Tarcsai, Gy.—Ferencz, Cs.: Platform location by satellites (in Hungarian). Manuscript, 10 pp., Budapest, 1974.

ТОЧНОСТЬ ТЕХНИК ОПРЕДЕЛЕНИЯ ПОЛОЖЕНИЯ АВТОМАТИЧЕСКОЙ ПЛАТФОРМЫ ПРИ ПОМОЩИ СПУТНИКОВ: ИЗУЧЕНИЕ РАСПРОСТРАНЕНИЯ ОШИБОК

ДЬ. ТАРЧАИ—Й. АДАМ

РЕЗЮМЕ

Настоящая работа имеет цель оценить ошибки положения платформы, вызваны систематическими и случайными ошибками измерений в дальномерных, допплеровских и комбинированных (расстояние + допплер) систем спутниковой локализации. После записи уравнений положения, сообращаются результаты численной симуляции ошибок, которые могут быть использованы для оценки вероятных ошибок положения, соответствующих ошибкам в измерениях расстояния и допплесровского сдвига Далее, задается также формула для ощибок положения в случае дальноизмерения, которая хорошо демонстрирует эффект геометрии локализации на полученных в результате ошибок положения. Полученные в этой статье результаты хорошо совпадают с результатами, полученными другими авторами.

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PLASTIC STATE AND DETERIORATION OF ROCKS

III. BOUNDARY CONDITIONS OF ROCK DETERIORATION

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In the rock mantle around mine openings the occurrence of deterioration or failure means an irreversible change which is connected with the amount of energy (deformation work) introduced into the system by the opening activity, by the reactions due to supporting, and by the care of covering.

The paper first reviews the classical boundary conditions of failure, then the possible energy conditions will be listed. It is deduced by means of elimination that, in accordance with the laws of irreversible thermodynamics,

— the principle of the maximum elastic distortion work, and

— the principle of maximum energy dissipation can be accepted as the failure theory of continua. Experimental data indicate the probability of the latter.

The difference between the energy introduced into the system (U) and the (elastic potential) energy (Φ) that can be stored in the system should continuously dissipate. The increase of the level (L) of the dissipation energy can reach a threshold value at which the material can only throw out the energy to be dissipated (i.e. to be transformed into heat) by ceasing the state of continuity, i.e. by deterioration. This energy level defines the boundary of failure.

The state and possible deterioration of mine openings in the crust is a question of distinguished importance with respect to the security of the miners working underground, but its role in the motion of the cover complex cannot be neglected either. For that very reason the investigation of the *boundary state of failure* or *deterioration* of rocks is a very important part of our article series. It seems necessary to outline the best-known failure theories to make more plausible our boundary condition for the deterioration of rocks.

KŁECZEK's experimental data [5], quoted so often in the foregoing part and forming the basis of our discussion, clearly show the dependence of rock deterioration on deformation velocity — similarly to the occurrence of plasticity effects. Hence, the boundary condition of deterioration can also be determined as a function of deformation work of the material or of the energy transport, respectively. Accordingly we will try to discuss also the classical theories from the energetical point of view (where this is possible at all). The review of the usual boundary conditions will closely follow CHOLNOKY's book on statics [4].

Classical theories of failure

Boundary state theories developed in course of the history of mechanics generally brought deterioration into connection with

— *the state of stress,*
— *the state of deformation, and*
— *the deformation work*

respectively. Since most of the classical theories assume a *linearly elastic medium*, the theories of boundary stress and boundary deformation can be transformed into each other. (This, of course, is also true for no matter how complicated material laws, only the transformation is not necessarily mutual unambiguous in such cases). In this way new possible theories could have been developed, this, however, would be rather aimless for the present level of continuum mechanics when there is no doubt the material state is determined by the volume of energy transport.

Theories of boundary conditions

Theory of maximum tensile stress

The first and simplest theory on the boundary state of failure has become known from GALILEI's scientific work at the beginning of the 17th century:

> *All stress states can be regarded as boundary states of deterioration, where the maximum normal (tensile) stress equals the uni-axial tensile strength of the material.* (1)

In rock mechanics, taking the pressure as positive, it follows from the inequality

$$\sigma_1 \geqq \sigma_2 \geqq \sigma_3$$

that σ_3 is the greatest tensile stress. Thus, denoting the tensile strength by σ_h, then — by GALILEI's law — stress states for which

$$\sigma_h \leqq \sigma_3 \tag{2}$$

cannot cause failure. This would mean a single characteristic property of the material (the tensile strength) and a single date of the stress state (the main stress σ_3) were enough to define the boundary state.

On the basis of MOHR's circles shown in Fig. 1 it is obvious that the theory cannot be maintained, for according to the assumption those stress states whose principal circle lies on the right hand side of the vertical straight line at value σ_h do not cause failure.

Four cases are distinguished in Fig. 1. The MOHR's circle (*a*) shows the boundary state of pure tension, when ($\sigma_1 = \sigma_2 = 0$). The principal-stress circle (*b*) represents the case of uni-axial pressure. Its diameter could be arbitrarily large, meaning that the material cannot be deteriorated by any linear pressure, what totally contradicts all practical experiences as well as our experiments.

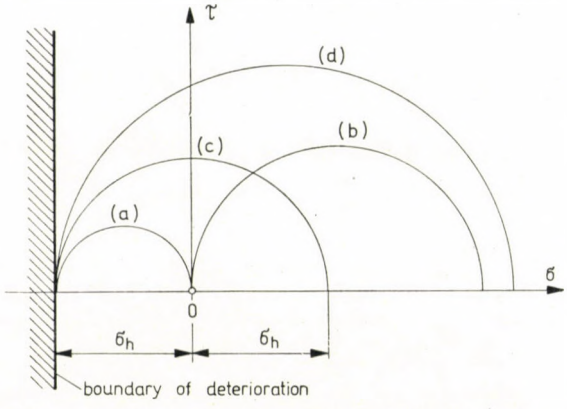

Fig. 1. MOHR's circles and deterioration boundary of GALILEI's boundary stress theory

The theory similarly leads to a false conclusion in case of pure shear, where $\sigma_1 = -\sigma_3$, $\sigma_2 = 0$. This state is represented by circle (*c*). According to the figure the maximal shear stress of the boundary state should be exactly equal to the tensile strength:

$$\tau_{max} = \sigma_h$$

whereas the experimental data known so far do not prove that. Circle (*d*) shows an even less probable case: namely that τ_{max} could increase at a specially chosen stress state up to infinity without deterioration.

The above deficiencies of GALILEI's theory not satisfying even the conventional concepts of statics lead to the development of new hypotheses.

Theory of limited main stresses

The most evident method to eliminate the deficiencies was the theory published by LAMÉ in 1831, according to which

> *to avoid failure besides the maximum tensile stress,*
> *also the maximum pressure stress must be limited.* (3)

According to this theory the condition of boundary stress is

$$\sigma_r \leq \sigma_3 \leq \sigma_1 \leq \sigma_{ny}, \tag{4}$$

where σ_{ny} is the so-called pressure strength determinable from uni-axial experiments. Thus two characteristic data of both stress state and material are needed.

MOHR's circles belonging to the stress state satisfying condition (4) are shown in Fig. 2. The principal-stress circles (*a*) and (*b*) represent the stress states of the uni-axial tension and uni-axial pressure. Circle (*c*) shows the

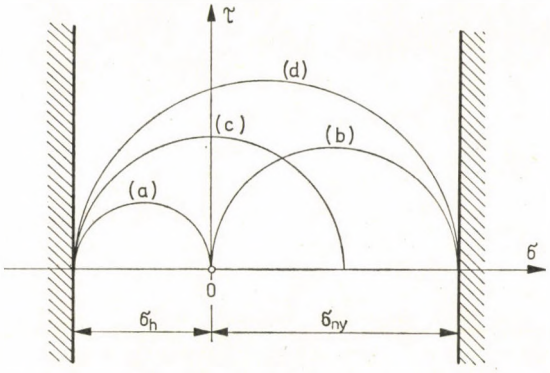

Fig. 2. MOHR's circles and deterioration boundary of LAMÉ's boundary stress theory

case of pure shear and is disputable as in case of the Galilean theory. The radius of circle (*d*) is the greatest; the permissible sliding stress τ_{max} not causing failure belongs to it.

Although this theory limits the permissible maximum τ value, eliminating a basic error of the former hypothesis, it cannot be accepted even on the basis of the conventional non-energetic concept for two reasons:

1. *For the reality of the boundary stress belonging to pure shear marked by the principal circle (c), the pressure strength should in absolute value be less than the tensile strength.*

2. *In case of a hydrostatic pressure greater than the uni-axial pressure strength deterioration ought to take place.*

Since both conclusions contradict practical experiences, new theories have been developed.

Theory of maximum shearing stress

In order to eliminate the two contradictions GUEST developed a theory in 1900, according to which:

> *the deterioration of the material takes place because the greatest shearing stress exceeds the (experimentally determinable) shearing strength τ_c of the material,* (5)

i.e. stress states below the deterioration boundary are characterized by

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} \leq \tau_c \,. \tag{6}$$

According to the theory one date of the material and two data of the stress state are necessary to decide whether the material is in a permissible state with respect to failure. Similarly to former theories the principal stress σ_2 has no influence on the deterioration boundary state.

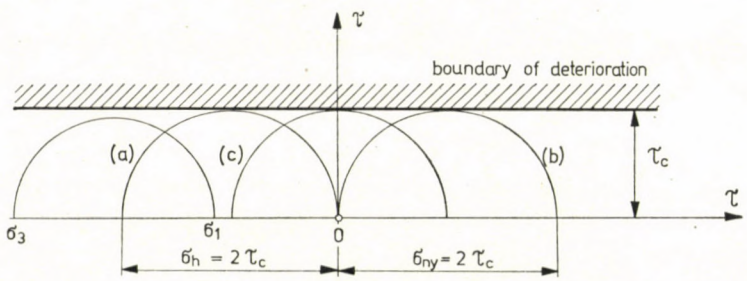

Fig. 3. GUEST's boundary stress theory

Figure 3 shows the straight line of the boundary stress according to GUEST's theory, (*a*), (*b*) and (*c*) are the principal stress (MOHR) circles of a uni-axial tension, of pressure and of pure shear. It can be seen from the figure that while the theory eliminates the contradictions of the former failure hypothesis, it has brought about at the same time new contradictions:

1. As seen in the figure condition (6) requires the simultaneous fulfilment of three different equations:

$$\tau_{\max} = \tau_c \,,$$

$$\tau_{\max} = \left| \frac{1}{2} \sigma_h \right| \,,$$

$$\tau_{\max} = \left| \frac{1}{2} \sigma_{ny} \right| \,,$$

that is the shearing ought to equal half the tensile and pressure strength (in absolute value), what is not at all in accordance with practical experiences.

2. The theory does not include limitations as regards hydrostatic stress states. This is undoubtedly correct in case of pressure. It does not seem acceptable, however, that the material could bear an unlimitedly great hydrostatic tension without failure.

3. The theory does not limit the magnitude of the normal stress. For example, even a much greater tensile stress than the boundary stress σ_h (visible at the left-hand side of the figure) can be permitted if the corresponding σ_1 is great enough to satisfy the condition $\tau_{max} \leq \tau_c$.

Coulomb's theory of boundary stress

The theories dealt with so far brought the boundary state in connection with either the normal or the shearing stresses alone. COULOMB constructed a theory in 1773, in which the boundary state was the function of both. This was the basis for the failure hypothesis of DUGUET in 1885, mainly applied in soil mechanics, but also used sometimes in rock mechanics. According to this theory

> *the deterioration of the material in a given point occurs, if on one of the surface elements laying across the point the shearing stress overcomes both cohesion and internal friction.* (7)

Since the theory is well known, we omit all deductions and only show the result (Fig. 4), that the boundary curve of deterioration is a straight line of equation:

$$\tau = c + \sigma \tan \varphi \ , \tag{8}$$

where c denotes cohesion, $\tan \varphi$ is the coefficient of internal friction expressed by the (internal) friction angle φ as usual in technical practice.

For the judgement of the failure condition two physical data — c and φ — of the material and also two data of the stress state are needed, since a straight line can be characterized by two parameters. The two necessary values of the stress state can be determined by any two independent experiments, since in the moment of deterioration the MOHR circle can be constructed

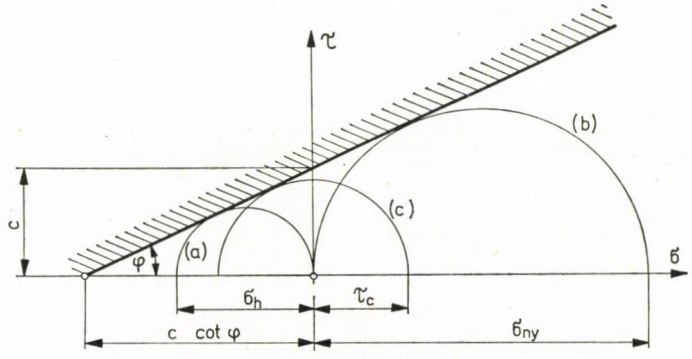

Fig. 4. Failure curve of the boundary stress theory of COULOMB and DUGUET

and the boundary curve calculated or constructed as the common tangent of the two MOHR circles.

In Fig. 4 circles (*a*), (*b*) and (*c*) are the MOHR circles of the uni-axial tension, pressure and pure shear.

Experiments carried out with a variety of materials have shown that the MOHR circles corresponding to these special cases are not arranged in such a way that a straight line touching each of them could have been drawn.

Mohr's theory of boundary stress

To eliminate the drawbacks of COULOMB's theory MOHR carried out a lot of experiments — presumably at equal stress change or deformation velocity — and found that an envelope touching all principal circles of the different boundary states can always be constructed. The basic idea of MOHR's theory developed in 1900 is that

> *there always exists an envelope corresponding to the principal circles of the different deterioration states and this is the boundary curve of failure of a given material.* (9)

In other words the boundary curve can be represented by an equation:

$$\tau = f(\sigma) \tag{10}$$

and a stress state causes deterioration if its principal circle is tangent to the boundary curve.

MOHR's theory has found several followers who addressed themselves to the determination of the explicit form of Eq. (10) corresponding to given kinds of materials. Equation (10) has been approximated by parabolas, hyperbolas and other curves of higher order to facilitate the incorporation of the boundary condition into the analytical treatments.

From the point of view of the boundary condition hypothesis MOHR's theory cannot be criticized. Its faults can be only proved from the side of energetics: it is not able, e.g., to describe the dependence of deterioration on deformation velocity. This defect can be eliminated, however, by generalizing the MOHR curve to a surface. The reason why we do not deal with this theory in detail is that it is a geometrical condition rather than a physical one, which only implicitly reflects the physical aspects of deterioration.

Theories of boundary form change

Theory of maximum extension

The theories discussed above brought the deterioration of the material into connection with stress state. MARIOTTE explained already at the end of the 17th century that

> *failure occurs in a point of the material if the greatest specific extension exceeds a certain value characteristic of the material quality.* (11)

In formula

$$\varepsilon_1 \leqq \varepsilon_c \tag{12}$$

ε_c is the greatest specific extension, which the material can bear.

Theory of de Saint-Venant

The above concept was further developed by DE SAINT-VENANT in the second half of the last century. What he constructed was in reality a boundary stress theory that had a leading role in technical sciences during many decades.

Supposing the validity of HOOKE's law Eq. (12) can be written as

$$\varepsilon_1 \equiv \frac{1}{E}\left(\sigma_1 - \frac{\sigma_2 + \sigma_3}{m}\right) \leqq \varepsilon_c ,$$

i.e. multiplying both sides by E we obtain:

$$\sigma_r \equiv \sigma_1 - \frac{\sigma_2 + \sigma_3}{m} \leqq \sigma_c , \tag{13}$$

where σ_r is the reduced main stress of the spatial stress state.

Other theories

Deformations can be represented in the $1/2\gamma$, ε plain — similarly to stresses — by MOHR's circles. Thus, physically meaningful boundary deformation theories can be deduced analogously to LAME's, GUEST's, COULOMB's or MOHR's theory.

Theories of failure work

The idea that not the stress state or the deformation state alone is characteristic to the failure of a material but both together already arose at the end of the last century. This idea founded the basis of the energetical theories.

Theory of maximum deformation work

This failure hypothesis is connected with the name of BELTRAMI and states that:

> *failure occurs in a point of the material if the specific deformation work exceeds in this point a certain value, which depends on material characteristics and can be determined experimentally.* (14)

In BELTRAMI's theory on linearly elastic materials the deformation work is equal to the potential energy and therefore BELTRAMI's theory can be expressed by the formula

$$U = \Phi \leq \Phi_{\max} \,. \tag{15}$$

This theory is usually handled in practice as a boundary condition theory.

In case of HOOKE's law the deformation work corresponding to a unit volume (elastic potential) can be written as:

$$U = \Phi = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \frac{2}{m}(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] =$$

$$= \frac{1}{2E}\left[\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \frac{2}{m}(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x)\right] +$$

$$+ \frac{1}{2G}(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) \,.$$

Introducing the concept of reduced stress in the form:

$$\sigma_r = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \frac{2}{m}(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)} \,,$$

we have

$$\Phi = \frac{\sigma_r^2}{2E} \,. \tag{16}$$

In case of a uni-axial stress state — as e.g. at pure shear — the potential corresponding to the deterioration is:

$$\Phi = \int_0^{\varepsilon} \sigma \, d\varepsilon = \int_0^{\sigma_{ny}} \sigma \, d\left(\frac{\sigma}{E}\right) = \left[\frac{\sigma^2}{2E}\right]_0^{\sigma_{ny}} = \frac{\sigma_{ny}^2}{2E} \leq \Phi_{\max} \tag{17}$$

Consequently, Eq. (15) can be written on the basis of Eqs. (16) and (17) in the form:

$$\sigma_r \leq \sigma_{ny} \,.$$

This is the usual form of BELTRAMI's theory as a boundary condition in technical practice.

A basic defect of BELTRAMI's theory in this form is that before deterioration the material is assumed as linearly elastic, whereas in reality it is already in the plastic state. A further defect is that according to it also a hydrostatic stress state can cause deterioration, the order of magnitude of this stress being the same for pressure and tension:

$$\left|\frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)\right| = |\sigma_0| \leq \sqrt{\frac{m}{3(m-2)}}\,\sigma_{ny}\,. \tag{18}$$

Theory of maximum distorting deformation work

The theory due to HUBER, MISES and HENCKY tried to eliminate the defects in BELTRAMI's failure theory by ordering only distorting deformation work (without volume change) to be the deterioration. Thus, according to the HUBER, MISES and HENCKY theory

> *the condition of the occurrence of deterioration is that the specific distorting deformation work should exceed a certain value, which depends on material characteristics and can be determined experimentally.* (19)

Since this theory was also developed on the basis of HOOKE's law, the deterioration condition has the form:

$$U' \leq U'_{\max}$$

$$\Phi \leq \Phi'_{\max}$$

In technical practice this condition is also applied in the form

$$\sigma_r \leq \sigma_{ny}$$

where the reduced stress is:

$$\sigma_r = \sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]} =$$

$$= \sqrt{\frac{1}{2}\left[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+3(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)\right]},$$

The only fault of the theory is that — besides not taking into account the plastic state before deterioration — it allows an arbitrarily large hydrostasic state, thus a hydrostatic tension of any magnitude, as well.

This theory cannot be criticized on the basis of the conventional theory, moreover, even when exceeding the domain of elastic deformations, it can be generalized to a real deterioration condition.

Boundary condition of rock deterioration

Review of the possible energetical conditions

Let us recall the energetical (or working) relationships used so far:

$$U = U' + U_0 = \Phi + L;$$

$$\Phi = \Phi' + \Phi_0;$$

$$U' = \Phi' + L;\ U_0 = \Phi_0 .$$

In order to determine the deterioration condition the limits corresponding to the following energy thresholds must be investigated:

$$\begin{aligned}
&a) && U \leq U_{\max} , \\
&b) && U' \leq U'_{\max} , \\
&c) && U_0 = \Phi_0 \leq \Phi_0^{\max} , \\
&d) && \Phi \leq \Phi_{\max} , \\
&e) && \Phi' \leq \Phi'_{\max} , \\
&f) && L \leq L_{\max} .
\end{aligned} \tag{20}$$

It can be assumed that the hydrostatic pressure does not cause deterioration and therefore (20c) cannot be a general boundary condition of failure. Consequently, from the rejection of condition (20c) it follows that, e.g., at the energy state

$$U' = 0$$

one has

$$U = U' + U_0 = U_0 = \Phi_0,$$

further, if

$$\Phi' = 0$$

then

$$\Phi = \Phi' + \Phi_0 = \Phi_0$$

implying that neither conditions (20a) and (20d) — meaning in the present case the generalization of BELTRAMI's theory — are characterizations of the energy threshold of failure. Thus, only the three remaining conditions need

a more thorough scrutiny, since they are independent of the variables eliminated and are only interrelated with each other:

$$L = U' - \Phi \ . \tag{21}$$

In the first article of the series it has been postulated — based on the conclusions drawn from the experiments in [5] — that for $\dot{\varepsilon} \to \infty$ (infinitely great deformation velocity) the plasticity and the deformation boundaries coincide. Due to the intensive energy input the material has not time to come

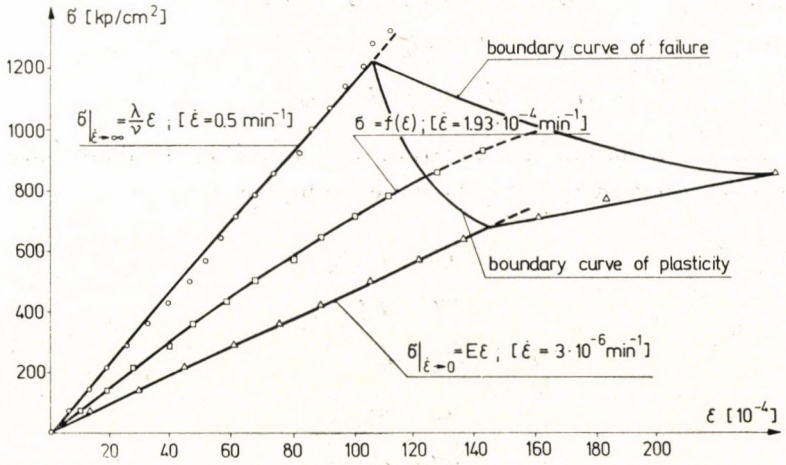

Fig. 5. Boundary curve of plasticity and deterioration (on the basis of KŁECZEK's experiments [5])

into the plastic state — this is shown by the fact that the curve $\sigma - \varepsilon$ in Fig. 5 approaching the case $\dot{\varepsilon} \to \infty$ is a straight line across the origin — i.e., if we would stop loading just before deterioration and would relieve with an $\dot{\varepsilon} \to -\infty$ velocity, we would get back to the origin along the same straight line. This would mean, however, because of the plasticity condition

$$U'|_{\dot{\varepsilon}\to\infty} = U'_k$$

deduced earlier in [1] that the deterioration condition were equal to the plasticity condition

$$U' = U'_{\max} = U'_k \ ,$$

what is an obvious contradiction, since at a deformation velocity $0 < \dot{\varepsilon}^{\pm 1} < \infty$ the distortion work further increases after exceeding the plasticity boundary.

This way possibility (20c) has also been eliminated and only one of the following two cases might come into consideration:

1. *The material can store only some definite quantity of elastic distortion potential per unit volume* ($\Phi' = \Phi'_{max}$) *without deterioration in the different aggregation degrees of its structural parts.* (The material behaves like a container which is demolished if the pressure of the liquid or gas stored in it exceeds a certain limit.)

2. *In course of deformation the difference between the energy* (U) *introduced into the system and the energy* (Φ) *that can be stored in the system must continuously dissipate in dependence of the deformation velocity.* (In case of infinitely slow loading only after exceeding the boundary of plasticity.) At fixed material properties the necessary dispersion velocity is determined by the velocity of energy input. *If the level of the dissipation energy reaches a certain threshold, the material can throw out the energy to be dissipated (transformed into heat) only by way of deterioration.*

It would seem logical to finish our investigation at this point. It follows namely from the laws of irreversible thermodynamics that only energetical principles can be accepted as boundary conditions of failure. A deductive argumentation, based on generally accepted experiences, helped us to select the only two possible failure conditions which could be applied as failure principles. It would be reasonable to propose particular experiments with different kinds of materials to decide which of the theories would be more adequate for the material in question.

In what follows, however, we proceed further and by evaluating KŁECZEK's experiment we will determine which of the conditions could be accepted for the Silesian coals investigated by him.

Conclusion drawn from uni-axial experiment

Theory of maximum elastic distortion work

In the present case this is a generalization of the theory of HUBER, MISES and HENCKY to a rheological medium of linear state equation.

Figure 6 shows the connections learned from the experiments carried out by KŁECZEK [5]; the amount of elastic distortion work belonging to the cases $\dot{\varepsilon} \to 0$ and $\dot{\varepsilon} \to \infty$ are marked by shadowed areas. It can be seen without any special calculation that this failure theory is not in accordance with the experimental data, since the condition

$$\Phi' |_{\dot{\varepsilon} \to \infty} = \Phi' |_{\dot{\varepsilon} \to 0}$$

is not met.

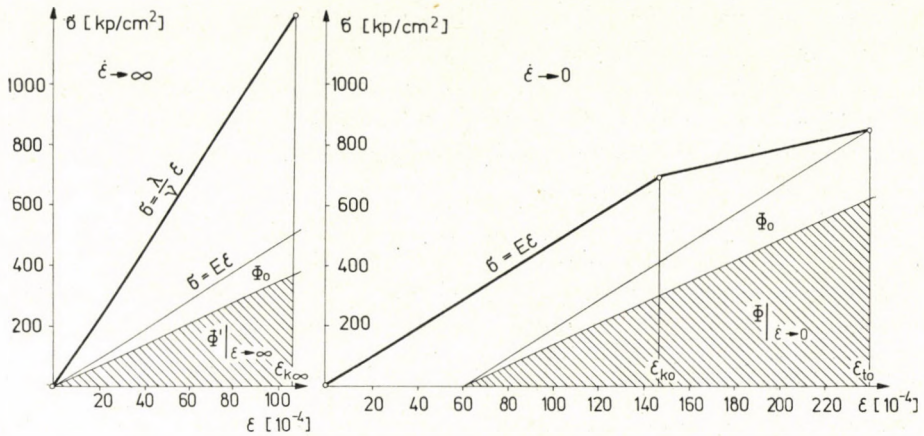

Fig. 6. Elastic distortion work in case of $\dot{\varepsilon} \to 0$ and $\dot{\varepsilon} \to \infty$

Theory of maximum dissipation work

This theory requires on the basis of the experimental data quoted the equality of the areas visible in Fig. 7, i.e. generally the fulfilment of the condition

$$L|_{\dot{\varepsilon}\to\infty} = L|_{\dot{\varepsilon}>0} = L|_{\dot{\varepsilon}\to 0} = L_{\max} \tag{22}$$

By using the numerical values obtained from the correlation analysis of the experimental data:

$$[L|_{\dot{\varepsilon}\to\infty}]_{\varepsilon_{k\infty}} = [U|_{\dot{\varepsilon}\to\infty} - \Phi|_{\dot{\varepsilon}\to\infty}]_{\varepsilon_{k\infty}} =$$

$$= \frac{1}{2}\left(\frac{\lambda}{\vartheta}\right)\varepsilon_{k\infty}^2 - \frac{1}{2}E\varepsilon_{k\infty}^2 = \frac{1}{2}\left(\frac{\lambda}{\vartheta} - E\right)\varepsilon_{k\infty}^2 \cong 357 \text{ cmkp/cm}^3,$$

$$[L|_{\dot{\varepsilon}\to\infty}]_{\varepsilon_{k0}} = (E - M)\,\varepsilon_{k0}\,(\varepsilon_{t0} - \varepsilon_{k0}) +$$

$$+ \frac{1}{2}M\left(1 - \frac{M}{E}\right)(\varepsilon_{t0} - \varepsilon_{k0})^2 \cong 351 \text{ cmkp/cm}^3,$$

where M is the linear plasticity module, being the directional tangent of the straight line in the plastic range corresponding to the case $\dot{\varepsilon} \to 0$. Consequently, the experiments repeated in [5] — not carried out by the author — show the likelihood of the *theory of the maximum dissipative work* as boundary condition of failure.

Making use of the results of the first paper of the series as well, the energetical relationships of the plastic state and deterioration are presented in Fig. 8.

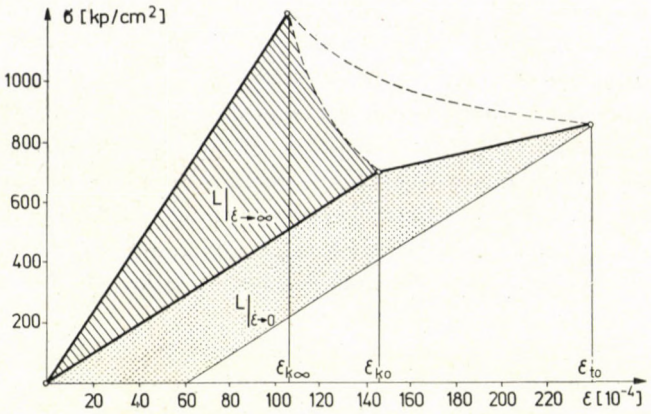

Fig. 7. Energy dissipation for KLECZEK's experiment

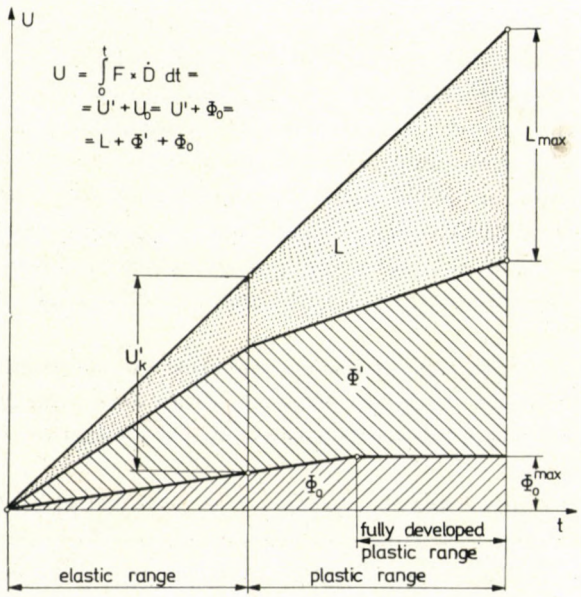

Fig. 8. Elastic and plastic ranges, and deterioration in case of continuous energy input

* * *

It was shown in Part I of this series of articles [1] that it is possible to develop a general phenomenological theory of rock mechanics, which has exact validity for any cases of material systems of different properties, for no matter small or great deformations, low or high deformation velocities are in question.

These investigations were motivated by the requirement to get rid off the restrictions of the hitherto conventional continuum-mechanical concept, which made impossible to treat the solution of rock mechanical problems on an identical basis: from the determination of the motion field of galleries with small deformations up to the definition of hewing phenomena, from the development of a sunken through on the surface up to the explanation of the propagation of pressure waves, etc. These questions could have been described hitherto only on the basis of different, relatively independent theories, which could not have been transformed into each other, although they all dealt with the behaviour of the very same medium.

The proved rheological character of the rock mass filling the part of the earth's crust utilized for mining purposes; the fact that among the material-mechanical properties of the rock mantle we can simultaneously observe some which separately are characteristic of solids, deformable mediums, pseudo-liquids, plastic bodies, etc., raised the necessity to give the basic equations of mechanics known since many centuries a new form which is equally valid for an arbitrary body. This seemed to be a demand of generalization at the time when the idea arose, but has become even more in course of its working out: namely, a kind of summarization of the real natural laws, whereby the basic equations of continuum mechanics have been cleaned from all defects, which resulted from incorrect generalizations of alien-to-nature hypotheses to basic laws of nature. The introduction of our general laws has not made the continuum-mechanical system more complicated, on the contrary, it made it much more clear, unambiguous and more directly applicable for a lot of practical problems.

One of the most important basic connections is the dynamical equation. This appears in classical statics in the form of an erroneous generalization of NEWTON's 2nd axiom and has led to the unnatural separation of the mechanics of solids and liquids.

The so-called geometrical equation was also ranged among the basic equations of mechanics so far; its role was to express the relationship between displacement and deformation. The geometrical equation is not a physical law, only an arbitrary definition, and there exist several different interpretations of it (CAUCHY, GREEN, ALMANSI, SWAINGER, HENCKY, etc.) even in up-to-date texts on mechanics. The author proved that this relationship caused the division of deformations into the ranges of "small" and "great" deformations and within them it made necessary to restrict the validity of the basic equations of mechanics to deformations approachable linearly. This connection must be excluded from among the basic equations, since its domain of validity does not follow from physical reasons, but depends on the validity limit of a groundless mathematical series expansion.

The third basic equation of continuum mechanics is the so-called physical

equation (material law). Its conventional interpretation was that it serves for describing the connection between stresses and deformations. It was shown [1] that there exist general laws from which the concrete form of this equation can be deduced and that it gives the connection between stresses and displacements (or the inhomogeneity of the latter). The material law expresses the basic natural law that every system changes into the direction of homogenization, of self-adjustment.

The investigations showed that all three equations appearing in the conventional theory in the form hitherto used contained more or less erroneous and inacceptable approximations: not those were the basic variables which were used as such, the content of the basic equations was not what had been thought to be, they did not express what had been read out of them.

It was possible by the formulation of the real basic equations of continuum mechanics to express these connections in the systems, which equally describe the mechanical changes at different material properties in arbitrary ranges of deformation magnitude and velocity. These basic equations, however, are not only basic equations of elasticity or plasticity, but are also valid from the occurrence of elastic deformations through plastic deformations till deterioration, i.e. up to the ceasing of continuity. Hence, they are the basic connections of continua of arbitrary state.

By means of the laws of thermodynamics it has been shown that the state of a system exposed to mechanical interactions can be unambiguously characterized by the balance equations of the so-called extensive variables (mass, momentum, moment of momentum, energy) as well as by the material law describing the connection between the characteristic extensive-intensive pair of variables (stress-deformation), i.e. the energy transport. The investigation of energy balance and energy transport, respectively, has shown that the occurrence of plastic state and failure is a function of the energy level of the system.

The change of the velocity of energy input also appears in apparent changes of material properties; in case of sudden dynamic load the rock seems to be of high strength and apt to rigid failure, while in case of a slow continuous increase of load the same material reveals flowing phenomena. Consequently such properties as rigidity, flowing boundary, plasticity, strength — held so far as characteristic — cannot be used for a correct classification, for the same material can show very different behaviour if the loading velocity is varied between broad boundaries.

If a solid is exposed to a mechanical effect, i.e. kinetic energy and momentum is introduced into it, momentum flux and energy flux will occur in it starting out from the point of input. If the energy input is of such a level, at which the material parts can conduct the momentum in a conductive way, we speak about elastic state. If the energy exceeds, however, a threshold

value, a convective momentum conduction (plastic flow) connected with macroscopic motion also occurs in the material, and we speak about plastic state.

Part of the energy introduced into the system will be stored as elastic potential energy, another will dissipate, i.e., it will be transformed into heat in consequence of overcoming the internal friction. Has the degree of the dissipation energy reached a threshold value, the material can produce the necessary energy dispersion only by means of ceasing to be continuous and then deterioration takes place.

The series of articles has laid the fundamentals of a uniform rock mechanical system tracing the mechanical changes of the material — rock mantle — from the elastic state through plastic state till deterioration. Thus, it created the possibility to design working methods and systems best fitting to *in situ* conditions, to find the optimum arrangement of mining openings, to choose the optimum supporting methods and the best technology to build them, that is, to influence the natural environment as required and wanted from the human point of view.

REFERENCES

1. Asszonyi, Cs.: On a new concept of rock mechanics. I, II, III. *Bányászati Lapok*, 1974, 145—150; 250—259; 325—332.
2. Asszonyi, Cs.—Richter, R.: Plastic state and deterioration of rocks. I. Boundary conditions of rock plasticity. *Acta Geod. Geoph. Mont. Hung.*, 10 (1975), 389—420.
3. Asszonyi, Cs.: On a new rock-mechanical theory. *Acta Geod. Geoph. Mont. Hung.*, 8 (1973), 313—322.
4. Cholnoky, T.: Mechanics. Vol. II. Statics. Tankönyvkiadó Budapest, 1966.
5. Kleczek, Z.: Wypln czasu na wytozymalocs skal. *Zeszyty Naukowe Akademii Górniczo-Hutniczej Górnictwo* Z-15 (1969), 67—73, Kraków.

СОСТОЯНИЕ ПЛАСТИЧНОСТИ И РАЗРУШЕНИЕ ГОРНЫХ ПОРОД

Ч. АССОНИ

РЕЗЮМЕ

Появление разрушения в породной мантии горных выработок означает такое необратимое изменение, наступление которого связано с уровнем внесенной энергии (работы на деформацию) — открытием шахты, предохранительными реакциями, способом ухода за кровлей выработки.

В статье кратко рассматриваются классические граничные условия разрушений, затем записываются все возможные условия для энергии. Методом исключения;— согласно законам необратимой термодинамики — выводится, что в качестве теории разрушения континумов можно принять

— принцип максимальной работы упругого искажения и
— принцип максимальной диссипации энергии.

Опытные данные делают вероятным последнее.

Разность энергий, внесенной в систему (U) и храняемой в системе (упругая потенциальная Φ) должна непрерывно диссипироваться. Повышение уровня энергии диссипации (L) может достигнуть такое пороговое значение, при котором разбрасываемая энергия может быть вытолкнута из вещества только путем устранения непрерывности, путем разрушения. Граница разрушения определяется этим уровнем энергии.

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ANALYSIS OF SEPARATING EFFICIENCY FUNCTIONS FOR FLOTATION PLANTS

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Since the geometrical distribution is a discrete analogon of the exponential distribution, by using it, various functions of separation, including values for constituents, outputs, product quality, separating efficiency can be derived. In the paper the extreme values of such functions are analysed and the laws governing their variations described.

1. Introduction

The relationship between continuous

$$k = 1 - e^{-\lambda t} \tag{1}$$

and discrete

$$k_\nu = q^{\nu-1} p, \quad q = 1 - p, \quad \nu = 1, 2, \ldots, N \tag{2}$$

flotation kinetic equations was discussed in [9]. For them, (1) is a function of exponential distribution and (2) the density function of geometric distribution. In order to determine the character of this relationship the constant λ in the exponential function, valid for tests conducted using a single flotation cell, will be defined. In the references λ is called flotability characteristic. $1/\lambda$ is the expected value of the exponential distribution, hence the reciprocal value of the flotability characteristic is the average residence time of grains within the cell if the time of flotation t is infinitely long [3, 9]. The above equations are related to mineral substances of identical flotation characteristics. Therefore, k and k_ν, respectively mean outputs and, in case of unit mass feed, also mass outputs.

When designing flotation plants the geometric distribution is to be used, it being a discrete analogon of the exponential distribution [4, 7]. As known, V cu.m/h is the volume of the fed slurry, while v cu.m that of the single cell. Their ratio T:

$$T = 60 \frac{v}{V} \tag{3}$$

gives the time of residence of the slurry within the single cell ($1/\lambda$ and T times

are, usually, given in minutes). The relationship for the two distributions can be expressed by the equation

$$p = 1 - e^{-\lambda T} \tag{4}$$

where p is the output of floating product for a single cell during T flotation time, q is the output of the tailings that underflow from the cell. In (2) q has as exponent $v - 1$, in which v is the number of cells connected in series, that may be $1, 2, \ldots N$.

Flotation plant feed consists of several minerals. On a flotability basis and also on a cell output each mineral can be divided in several fractions [1, 2, 5]. If there are n mineral constituents in the feed ($i = 1, 2, \ldots n$), the mass ratio of the fraction j for mineral i equals m_{ij}, hence the feed of unit mass can be characterized by

$$\sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} = \sum_{i=1}^{n} m_i = 1 \tag{5}$$

The cell output of the fraction of m_{ij} mass ratio is q_{ij}. Its whole range of values corresponds to the Tromp curve

$$q_{ij} : i = 1, 2, \ldots n \text{ and } j = 1, 2, \ldots l. \tag{6}$$

and of the operations involved in gravity concentration [8]. According to (6) each mineral is represented by a Tromp curve.

If N cells are connected in series and the froth product of each cell is led to the concentrates, further on, the underflow from the last cell N consists of tailings, then the mass output of concentrate m_h is

$$m_h = \sum_{v=1}^{N} \sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} q_{ij}^{v-1} p_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} (1 - q_{ij}^{N}) = 1 - \sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} q_{ij}^{N}. \tag{7}$$

If the feed is constituted of one mineral and each of its grains is of the same flotability characteristics q ($q_{ij} = q$) then the former mass output means, at the same time, also output of constituents

$$m_h = k_h = 1 - q^N. \tag{8}$$

This relationship is to be found in several references [11, 12, 13]. A precondition of its use [10] is that within the battery of flotation cells the residence time of the slurry in each cell should be, practically, constant.

In the followings, functions of separation for primary and secondary

flotation will be derived. By their use, the variations in mass output, constituent outputs and products quality will be examined and the loci of extreme values are given for different cell connections. In the last part of this paper, functions of separation for primary, secondary and purifying flotation will be presented. The methods used are suitable to derive functions of separation for any type of cell connection.

2. Functions of separation for primary and secondary flotation

In the top part of Fig. 1 an open flow-sheet for $N = N_1 + N_2 + N_3$ cells is presented. Grains in the feed of unit mass show the same flotability characteristics, and the cell output is q. With N_1 number of cells a concentrate of $1 - q^{N_1}$ mass and with N_2 number of cells, another of $q^{N_1} - q^{N_1+N_2}$ are

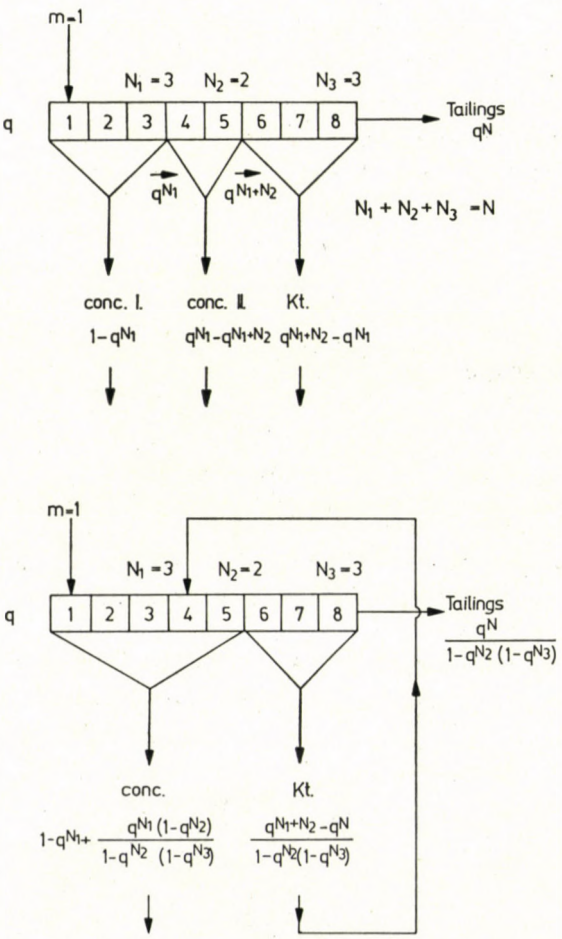


Fig. 1

produced, while the following N_3 cells yield middlings of $q^{N_1+N_2} - q^N$ mass. The mass of tailings is q^N. If middlings are recirculated to cell $N_1 + 1$ having q^{N_1} as feed, then for their mass Kt the following ratio may be written:

$$Kt = \frac{q^{N_1+N_2} - q^N}{1-q^{N_2}(1-q^{N_3})} . \tag{9}$$

For cell $N_1 + 1$ the total feed

$$q^{N_1} + Kt = \frac{q^{N_1}}{1-q^{N_2}(1-q^{N_3})} . \tag{10}$$

The mass output of N_2 cell concentrates:

$$\text{Conc. II} = \frac{q^{N_1}(1-q^{N_2})}{1-q^{N_2}(1-q^{N_3})} . \tag{11}$$

The total concentrates from cells $N_1 + N_2$
Conc. = Conc. I + Conc. II are of

$$\frac{1-q^{N_2}(1-q^{N_3}) - q^N}{1-q^{N_2}(1-q^{N_3})} \tag{12}$$

mass ratio.

The mass M of tailings is:

$$M = \frac{q^N}{1-q^{N_2}(1-q^{N_3})} . \tag{13}$$

The feed for the closed circuit flotation flow-sheet traced in Fig. 2 is determined by Eq. (5). This flow-sheet is identical to that presented in the lower part of Fig. 1: the number of primary flotation cells is $N_1 + N_2$, that of secondary one N_3, the total number of cells being $N = N_1 + N_2 + N_3$. Mass outputs m_h and m_m for concentrates and tailings, respectively in conformity with (12) and (13) [9]:

$$m_h = \sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} \frac{1-q_{ij}^{N_2}(1-q_{ij}^{N_3}) - q_{ij}^N}{1-q_{ij}^{N_2}(1-q_{ij}^{N_3})} \tag{14}$$

and

$$m_m = \sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} \frac{m_{ij} q_{ij}^N}{1-q_{ij}^{N_2}(1-q_{ij}^{N_3})} . \tag{15}$$

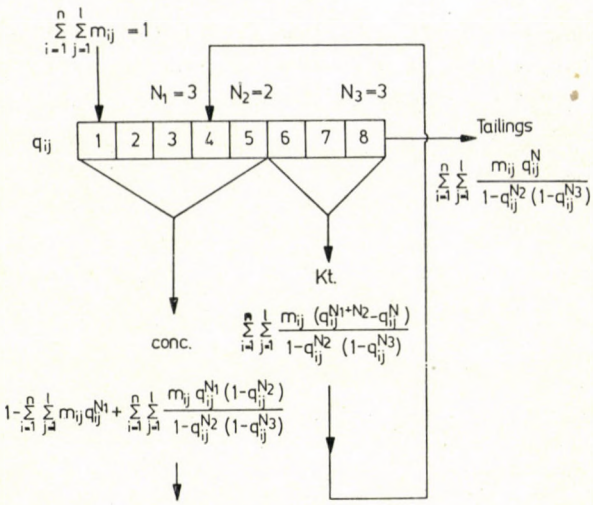


Fig. 2

Mineral contents b_i and c_i for concentrates and tailings [9] are:

$$b_i = \frac{\sum_{j=1}^{l} m_{ij} \dfrac{1-q_{ij}^{N_3}(1-q_{ij}^{N_3})-q_{ij}^{N}}{1-q_{ij}^{N_3}(1-q_{ij}^{N_3})}}{\sum_{i=1}^{n}\sum_{j=1}^{l} m_{ij} \dfrac{1-q_{ij}^{N_3}(1-q_{ij}^{N_3})-q_{ij}^{N}}{1-q_{ij}^{N_3}(1-q_{ij}^{N_3})}} \tag{16}$$

and

$$c_i = \frac{\sum_{j=1}^{l} \dfrac{m_{ij}\, q_{ij}}{1-q_{ij}^{N_3}(1-q_{ij}^{N_3})}}{\sum_{i=1}^{n}\sum_{j=1}^{l} \dfrac{m_{ij}\, q_{ij}^{N}}{1-q_{ij}^{N_3}(1-q_{ij}^{N_3})}} \tag{17}$$

k_i and K_i outputs of constituents:

$$k_i = \frac{1}{m_i} \sum_{j=1}^{l} m_{ij} \frac{1-q_{ij}^{N_3}(1-q_{ij}^{N_3})-q_{ij}^{N}}{1-q_{ij}^{N_3}(1-q_{ij}^{N_3})} \tag{18}$$

and

$$K_i = \frac{1}{m_{ij}} \sum_{j=1}^{l} \frac{m_{ij} q_{ij}^{N}}{1-q_{ij}^{N_3}(1-q_{ij}^{N_3})}. \tag{19}$$

The mass output of recirculated middlings m_{kt}

$$m_{kt} = \sum_{i=1}^{n}\sum_{j=1}^{l} m_{ij} \frac{q_{ij}^{N_1+N_2}-q_{ij}^{N}}{1-q_{ij}^{N_3}(1-q_{ij}^{N_3})}. \tag{20}$$

The minerals composition and constituents output for the middlings are to be written in accordance with the above equations.

The technical efficiency of separation is to be calculated by the equations:

$$\eta_i = \frac{k_i - m_h}{1 - m_i} \tag{21}$$

and

$$H_i = \frac{K_i - m_m}{m_i} . \tag{22}$$

Middlings are rather frequently recirculated to the first cell. In such cases $N_1 = 0$ and $N_2 + N_3 = N$. Mass outputs for concentrates and middlings:

$$m_h = \sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} \frac{1 - q_{ij}^{N_2}}{1 - q_{ij}^{N_2} + q_{ij}^{N}} \tag{23}$$

$$m_h = \sum_{i=1}^{n} \sum_{j=1}^{l} \frac{m_{ij} q_{ij}^{N}}{1 - q_{ij}^{N_2} - q_{ij}^{N}} . \tag{24}$$

3. Variations in the results of separation for primary and secondary flotations

Be the composition of a run-of-mine ore sample as follows [6, 9]: galenite content $m_1 = 0.02$, sphalerite content $m_2 = 0.04$, middlings content $m_3 = 0.94$. Average flotation times $1/\lambda_i$ ($i = 1, 2, 3$) = 0.83, 10 and 50 min. Slurry feeding rate = 1000 cu.m/h, volume of the sole cell = 10 cu.m. and residence time $T = 0.6$ min. Cell outputs p_i (Eq. (4)) are: 0.51325, 0.05824 and 0.001193 (hence the values for q_i are 0.48675, 0.94176 and 0.998807).

In Table I the results of separation for all the possible cell connections are listed. Its first column presents data on the method of connecting the flotation cells. In columns 3, 4 and 5 data on concentrate mass output, minerals content and minerals output are provided, while by the following three columns the same parameters for tailings are given. In columns 9 and 10 those on mass output and minerals content for middlings are shown. The last column includes efficiency data as calculated for galenite and sphalerite from Eq. (21) and for tailings from Eq. (22).

Using the data contained in Table I some laws can be formulated by which the results of primary and secondary flotations are governed, provided that the total number of cells (N) is given, the values m_{ij} and q_{ij} are known and the cell connections varied.

For assessing the extreme values of mass output it seems reasonable to transform Eq. (14) into the form:

$$m_h = 1 - \sum_{i=1}^{n} \sum_{j=1}^{l} \frac{m_{ij} q_{ij}^{N}}{1 - q_{ij}^{N_3} + q_{ij}^{N_3+N_3}} . \tag{25}$$

In such a case the value of m_h depends, in conformity with the above conditions, only on the denominator $1 - q_{ij}^{N_3} + q_{ij}^{N_3+N_3}$.

m_h has its maximum if the denominator takes the maximal value of 1:

$$1 - q_{ij}^{N_3} + q_{ij}^{N_3+N_3} = 1 \quad (i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, l) \tag{26}$$

This equality is only valid if $N_2 = N_3 = 0$ and $N = N_1$, which correspond to the case of an open flow-sheet. Thus the maximal mass output

$$m_{h\,\max} = 1 - \sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} q_{ij}^{N} , \tag{27}$$

that corresponds to the above derived Eq. (7).

The mass output is at minimum when the denominator reaches its lowest value. $1 - q_{ij}^{N_3} + q_{ij}^{N_3+N_3}$ is at minimum if the second member of negative sign is of the highest possible value, while the third member of positive sign is at its lowest possible one. The pattern of connection $N_2 = 0$, $N_3 = N$, $N_1 = 0$ is not possible to realize, however, the pattern $N_2 = 1$, $N_3 = N - 1$, $N_1 = 0$ is realizable. Therefore the least mass output

$$m_{h\,\min} = 1 - \sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} \frac{m_{ij} q_{ij}^{N}}{1 - q_{ij} + q_{ij}^{N}} . \tag{28}$$

The amount of recirculated middlings is, at the same time, according to

$$m_{kt\,\max} = \sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} \frac{q_{ij} - q_{ij}^{N}}{1 - q_{ij} + q_{ij}^{N}} \tag{29}$$

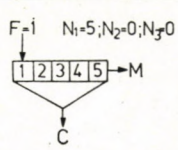
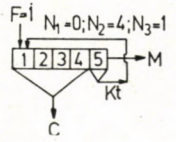
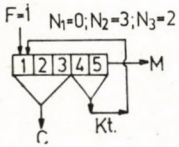
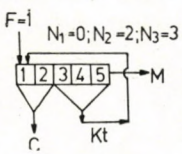
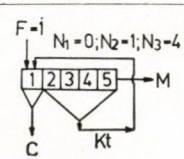
at its maximum.

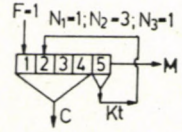
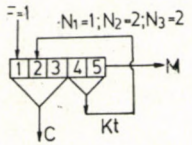
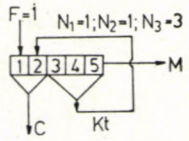
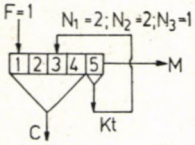
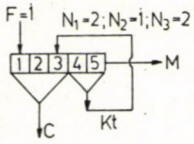
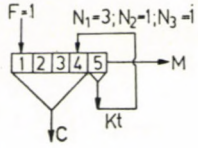
The recirculated middlings are of minimal mass when the foam product from the last cell is led back to the last but one cell. For such a cell connection $N_2 = 1$, $N_3 = 1$, $N_1 = N - 2$. The minimum amount of recirculated middlings:

$$m_{kt\,\min} = \sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} \frac{q_{ij}^{N-1} - q_{ij}^{N}}{1 - q_{ij} + q_{ij}^{2}} . \tag{30}$$

Table I

Separating results for primary and secondary flotation

Connection of flotation cells	Type of mineral	Concentrates			Tailings			Recirculated middlings		Efficiency η_i
		Mass output m_h	Mineral content b_i	Mineral output k_i	Mass output m_h	Mineral content c_i	Mineral output k_i	Mass output m_{kt}	Mineral content d_i	
1	2	3	4	5	6	7	8	9	10	11
	PbS	0.08456	0.23005	0.97268	0.91544	0.00060	0.02732			0.9062
	ZnS		0.12260	0.25918		0.03237	0.74082			0.1819
	SiO_2		0.64735	0.05824		0.96703	0.94176			0.0280
			1.00000			1.00000				
	PbS	0.07294	0.26647	0.97187	0.92706	0.00061	0.02813	0.01332	0.04453	0.9173
	ZnS		0.12263	0.22362		0.03381	0.77638		0.14413	0.1563
	SiO_2		0.61090	0.04741		0.96558	0.95259		0.81134	0.0272
			1.00000			1.00000			1.00000	
	PbS	0.06069	0.31965	0.97004	0.93931	0.00064	0.02996	0.02811	0.06866	0.9193
	ZnS		0.11989	0.18191		0.03484	0.81809		0.14843	0.1263
	SiO_2		0.56046	0.03619		0.96452	0.96381		0.78291	0.0261
			1.00000			1.00000			1.00000	
	PbS	0.04769	0.40484	0.96543	0.95231	0.00073	0.03457	0.04576	0.11591	0.9365
	ZnS		0.11106	0.13243		0.03654	0.86757		0.14957	0.0883
	SiO_2		0.48480	0.02456		0.96283	0.97544		0.73452	0.0246
			1.00000			1.00000			1.00000	
	PbS	0.03366	0.56412	0.94945	0.96634	0.00104	0.05055	0.07269	0.23381	0.9345
	ZnS		0.08661	0.07288		0.03838	0.92712		0.13837	0.0409
	SiO_2		0.34927	0.01251		0.96058	0.98749		0.62782	0.0225
			1.00000			1.00000			1.00000	

	PbS ZnS SiO_2	0.07271	0.26708 0.12175 0.61117 1.00000	0.97096 0.22130 0.04727	0.92729	0.00063 0.03359 0.96578 1.00000	0.02904 0.77870 0.95273	0.01335	0.04588 0.14427 0.80985 1.00000	0,9166 0.1548 0.0271
	PbS ZnS SiO_2	0.06016	0.32137 0.11743 0.56120 1.00000	0.96665 0.17660 0.03592	0.93984	0.00071 0.03504 0.96425 1.00000	0.03335 0.82340 0.96408	0.02836	0.07575 0.14807 0.77618 1.00000	0.9250 0.1213 0.0258
	PbS ZnS SiO_2	0.04666	0.40807 0.10558 0.48635 1.00000	0.95201 0.12315 0.02414	0.95334	0.00101 0.03679 0.96220 1.00000	0.04799 0.87685 0.97586	0.04790	0.15369 0.14440 0.70191 1.00000	0.9238 0.0797 0.0240
	PbS ZnS SiO_2	0.07244	0.26750 0.12083 0.61167 1.00000	0.96889 0.21883 0.04714	0.92756	0.00067 0.03369 0.96564 1.00000	0.03111 0.78117 0.95286	0.01340	0.04895 0.14418 0.80687 1.00000	0.9147 0.1556 0.0269
	PbS ZnS SiO_2	0.05947	0.32170 0.11495 0.56335 1.00000	0.95653 0.17089 0.03564	0.94053	0.00093 0.03526 0.96381 1.00000	0.04347 0.82911 0.96436	0.02905	0.09639 0.14557 0.75804 1.00000	0.9154 0.1161 0.0254
	PbS ZnS SiO_2	0.07210	0.26728 0.11994 0.61278 1.00000	0.96358 0.21619 0.04700	0.92790	0.00078 0.03379 0.96543 1.00000	0.03642 0.78381 0.95300	0.01352	0.05681 0.14338 0.79981 1.00000	0.9097 0.1500 0.0267

In this case the mass output of concentrates

$$m_h = 1 - \sum_{i=1}^{n} \sum_{j=1}^{l} \frac{m_{ij} q_{ij}^N}{1 - q_{ij} + q_{ij}^2} \tag{31}$$

reaches its highest value for all the possible cell connections of recirculation.

Table III helps to understand better the variations in mass output. In this table concentrate mass output values have been listed by considering all the possible values of N_2 and N_3. For this reason mass output values are to be found in it even for cell connections that are not enlisted in Table I. Mass output values included in Table I are underlined in Table II. By any of the table columns mass output values for identical N_3 connection, i.e. the

Table II

Variations of the mass output in function of N_2 and N_3

N_2 \ N_3	0	1	2	3	4	5
0	0.08456	0.07147	0.05752	0.04208	0.02381	0.—
1	0.08456	0.07210	0.05947	0.04666	0.03366	
2	0.08456	0.07244	0.06016	0.04769		
3	0.08456	0.07271	0.07294			
4	0.08456	0.07294				
5	0.08456					

Table III

Internal material balance according to Fig. 9

Pre-conc.:	$\dfrac{q^{N_1}(1-q^{N_2})}{q^{N_2+N_3}+(1-q^{N_2})(1-q_t^{N_t})} + \dfrac{(1-q^{N_1})\,[1-q^{N_2}(1-q^{N_3})]}{1-q^{N_2}(1-q^{N_3})-q_t^{N_t}(1-q^{N_2})}$
Kt I.:	$\dfrac{q^{N_1}(1-q^{N_2})\,q_t^{N_t}}{q^{N_2+N_3}+(1-q^{N_2})(1-q_t^{N_t})} + \dfrac{(1-q^{N_1}([1-q^{N_2}(1-q^{N_3})]\,q_t^{N_t}}{1-q^{N_2}(1-q^{N_3})-q_t^{N_t}(1-q^{N_2})}$
Kt II.:	$\dfrac{q^{N_1+N_2}-q^N}{q^{N_2+N_3}+(1-q^{N_2})(1-q_t^{N_t})} + \dfrac{(1-q^{N_1})\,q^{N_2}(1-q^{N_3})\,q_t^{N_t}}{1-q^{N_2}(1-q^{N_3})-q_t^{N_t}(1-q^{N_2})}$
Kt:	$\dfrac{q^{N_1}[q^{N_2}(1-q^{N_3})+(1-q^{N_2})\,q_t]}{q^{N_2+N_3}+(1-q^{N_2})(1-q_t^{N_t})} + \dfrac{(1-q^{N_1})\,q_t^N}{1-q^{N_2}(1-q^{N_3})-q_t^{N_t}(1-q^{N_2})}$

Feed for cell $(N+1)$: $Kt + q^{N_1}$

same number of open cells ($N_3 = N - (N_1 + N_2)$) are offered for varying N_2 cell number. While the connections $N_3 = 0$, $N_2 = 1, 2, 3, 4, 5$ cannot be realized, the values of mass output are, formally, to be calculated. They are identical to the mass output value of 0.08456 for $N_3 = 0$, $N_2 = 0$, $N_1 = N$ cell connections, i.e. for open ones, ana can be found in the first column of this table. $N_2 = 0$, $N_3 = 1$, $N_2 = 0$, $N_3 = 2$, $N_2 = 0$, $N_3 = 3$, $N_2 = 0$, $N_3 = 4$ connections correspond to open series consisting of $N = N_1 = 4, 3, 2, 1$ cells. Mass output data are included in the first row of this table. If mass values under each other or beside each other are subtracted from each other (the differences expressing only the variations in the number of cells $N_1 + N_2$ only for N_2 and the same N_2, respectively) then the validity of the general laws to which mass output data obey can be justified even numerically. (It is reasonable to prepare a similar table, by completing it with data on the quality and output of products, when flotation plants are designed, if there are changes in the raw material composition, in order to make it possible to provide cell connections offering the highest economics.)

The relationships valid for constituents output are very similar to those involving mass output data. Variations in cell outputs q_{ij} or cell numbers N_1, N_2 and N_3 result in altering both the constituent output values and mass output ones.

In agreement with what was discussed above the following general statements can be made.

In agreement with the preceding discussion it can be generally stated that the mass and constituents outputs of concentration are increasing proportionally with the total number of cells. For identical number of cells these parameters also increase with the number of primary flotation cells N_1 and that of secondary flotation cells N_2, they vary, however, in inverse proportion with the number of secondary flotation cells N_3. As regards the magnitude of these parameters it is decided, mainly, by the number of open cells ($N_1 + N_2 = N - N_3$) upon. If the total number of cells is unvaried, the value of the examined parameters is less affected by the points where the recirculation is effected (the number of cells N_2) provided that the number of open cells is constant.

By the recirculation of froth the quality of products is improved. For an open cell battery the output of each mineral is of maximal value. In the concentrate, the content of minerals with the highest flotability characteristics is the smallest possible, while that of minerals of the lowest flotability characteristic is the largest possible. A most valuable concentrate can be supplied by a cell connection yielding the smallest mass output ($N_1 = 0$, $N_2 = 1$, $N_3 = N - 1$). Minerals outputs in the concentrates are then at minimum and the quality of recirculated middlings is the best. As shown in Table I, the extreme values for separation efficiency are not yielded by cell connections

corresponding to the extreme values of mass output. The separating efficiency for galenite having good flotation properties is for open connection at minimum, while for the connection $N_1 = 0$, $N_2 = 2$, $N_3 = 3$, i.e. for a not minimum mass output, it is at maximum. The efficiency of tailings separation is for open connection at maximum, while its minimum is reached at the pattern $N_1 = 0$, $N_2 = 1$, $N_3 = 4$, in other words, at minimum mass output.

4. Separation functions for primary, secondary and purifying flotation

In the following discussion the flow-sheet presented in Fig. 3 will be examined [9]. Froth products, yielded by cells of N_1 and N_2 numbers, respectively, as obtained from primary and secondary flotation circuits will be conducted to the purifying cell battery consisting of N_t cells. Froth products from secondary flotation cells in number N_3 will be recirculated to cell $N_1 + 1$. Cell output from primary and secondary flotation is q, while that of purifying flotation q_t. The froth products of the purifying flotation are concentrates (Conc.), and the discharge products, middlings (Kt_t), while the tailings (M) constitute the discharge of secondary flotation. Mass output values for these products can be calculated by the method presented in Figs. 1 and 2, respectively:

$$\text{Conc.} = \frac{1 - q^{N_2}(1 - q^{N_3}) - q^N}{1 - q^{N_2}(1 - q^{N_3})} (1 - q_t^{N_t}) \tag{32}$$

$$Kt_t = \frac{1 - q^{N_2}(1 - q^{N_3}) - q^N}{1 - q^{N_2}(1 - q^{N_3})} q_t^{N_t}. \tag{33}$$

The mass output of tailings is the same as in (13).

If the minerals composition in the feed

$$\left(\sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} = 1 \right)$$

and cell outputs (q_{ij} and q_{tij}), respectively, are known, then the relationships covering mass outputs, constituents content and constituents outputs, respectively, can be derived in the above suggested manner.

If it is intended, for the flow-sheet presented in Fig. 3, to keep even the middlings Kt_t within the circuit, then for assessing the values of mass output and deriving the functions of separation, it seems reasonable to examine separately the problems arising from the variations in the material balances related to the continued flotation of froths from primary and secondary flotation, respectively.

In Fig. 4 the process of purifying the froth products $1 - q^{N_1}$ from primary flotation is shown, while the fraction of q^{N_1} mass ratio is discharged from the system. The fraction of mass ratio $(1 - q^{N_1})\, q_t^{N_t}$ along with the froth product of

$$\frac{(1 - q^{N_1})\, q^{N_2} (1 - q^{N_3})\, q_t^{N_t}}{1 - q^{N_2} (1 - q^{N_3})}$$

mass ratio (to be calculated also in the same way as described for Figs 1 and 2, respectively) obtained from secondary flotation cells, is the feed for $N_1 + 1$ cell. The volume of froth overflowing in cells of number N_2:

$$\frac{(1 - q^{N_1})\, (1 - q^{N_2}) q_t^{N_t}}{1 - q^{N_2} (1 - q^{N_3})}$$

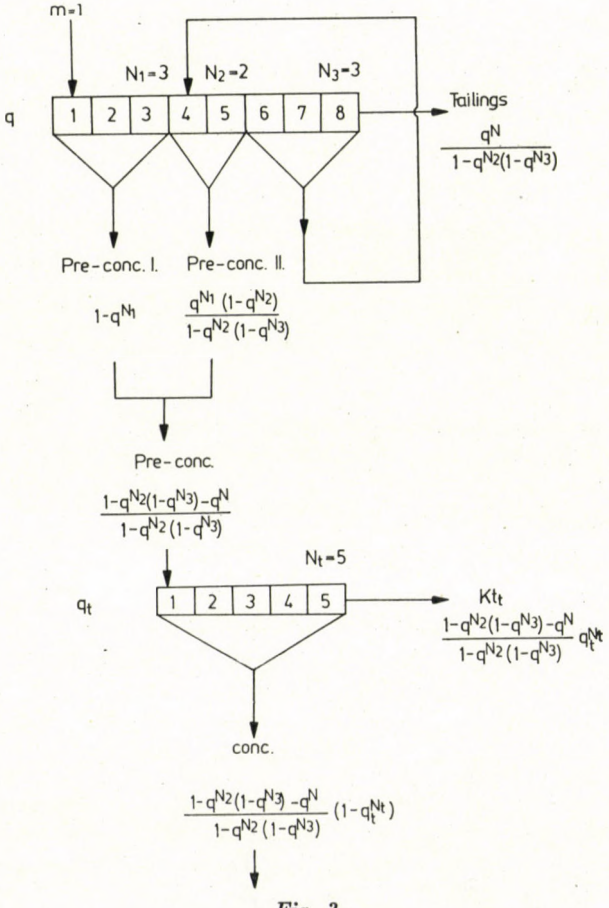


Fig. 3

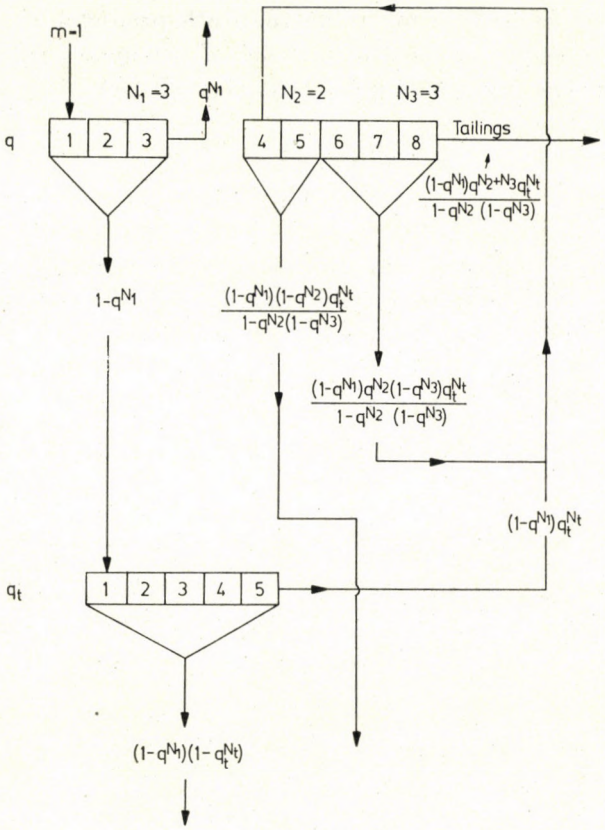


Fig. 4

while the amount of tailings from secondary flotation:

$$\frac{(1-q^{N_1})\,q^{N_2+N_3}\,q_t^{N_t}}{1-q^{N_2}(1-q^{N_3})}\,.$$

The pre-concentrate from cells of number N_2 (Fig. 5) (PC II is in the figure Pre-conc. II) will be recirculated to the purifying cell battery. In this case the ratio:

$$(1-q^{N_1})(1-q_t^{N_t})+\frac{(1-q^{N_1})\,q^{N_2+N_3}\,q_t^{N_t}}{1-q^{N_2}(1-q^{N_3})}:$$

$$:(1-q^{N_1})\,q_t^{N_t}+\frac{(1-q^{N_1})\,q^{N_2}(1-q^{N_3})\,q_t^{N_t}}{1-q^{N_2}(1-q^{N_3})}=1-q^{N_1}:\text{PC II}$$

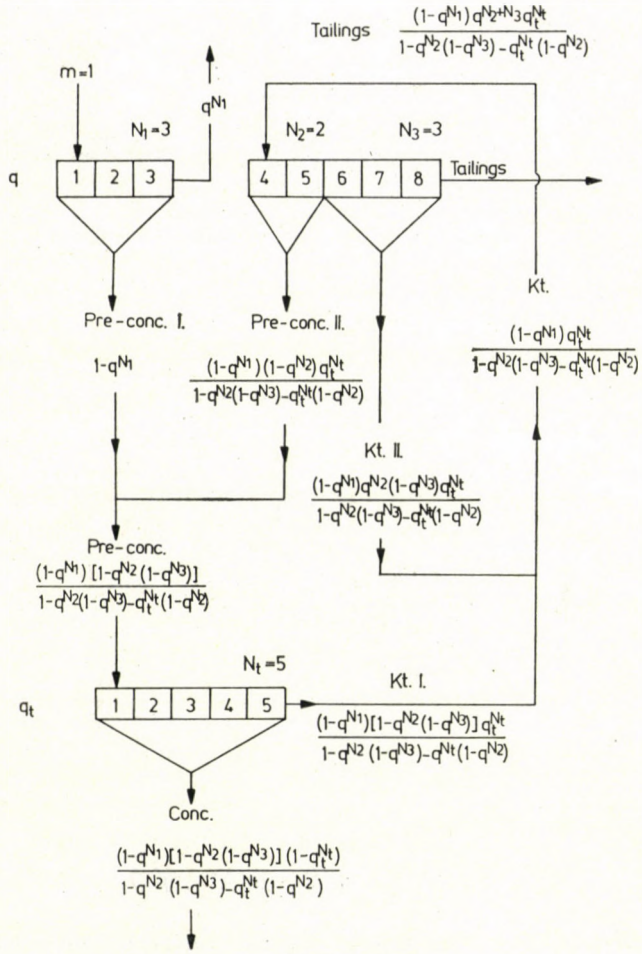


Fig. 5

is valid, from which the amount of pre-concentrate

$$\text{PC II} = \frac{(1-q^{N_1})\,q_t^{N_t}\,(1-q^{N_2})}{1-q^{N_2}(1-q^{N_3})-q_t^{N_t}(1-q^{N_2})}\,. \tag{34}$$

From this equation, following the same path, also other relationships, concerning the material balance affecting the flow-sheet shown in Fig. 5, can be deiived and, in fact, they are presented in it. The mass outputs for concentrates (Conc.) and tailings (T) are:

$$\text{Conc.} = \frac{(1-q^{N_1})[1-q^{N_2}(1-q^{N_3})](1-q_t^{N_t})}{1-q^{N_2}(1-q^{N_3})-q_t^{N_t}(1-q^{N_2})} \tag{35}$$

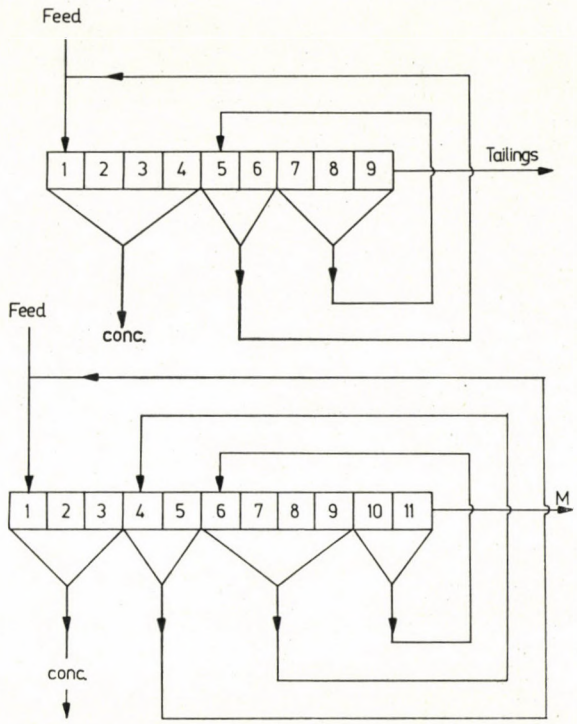


Fig. 6

and

$$T = \frac{(1 - q^{N_1}) q^{N_2+N_3} q_t^{N_t}}{1 - q^{N_2}(1 - q^{N_3}) - q_t^{N_t}(1 - q^{N_3})} . \tag{36}$$

By their sum the volume of feed $1 - q^{N_1}$ for the purifying cell battery is given

In the practice of flotation, flow-sheets similar to that traced in Fig. 6 are frequently encountered.

In the figure the process of repeated recirculation of froth products is demonstrated. For flow-sheets of this type the equation of separation are to be derived by the method presented in Figs 4 and 5.

Based on Figs 7 and 8 a case is discussed, in which the froth products from primary flotation cells of number N_1 will be discharged from the system by purifying only the froth products from secondary flotation. In Fig. 7 an open flow-sheet is to be seen. The mass of the purifying battery is q^{N_1}, that of the concentrates $q^{N_1}(1 - q^{N_2})(1 - q_t^{N_t})$, that of the two types of middlings $q^{N_1}(1 - q^{N_2})$ and $q^{N_1+N_2} - q^N$, finally that of tailings q^N. If the two types of middlings are recirculated in the known manner, then a ratio of

$$q^{N_1}(1 - q^{N_2})(1 - q_t^{N_t}) + q^N : q^{N_1}(1 - q^{N_2}) q_t^{N_t} + q^{N_1+N} - q^N = q^{N_1} : Kt$$

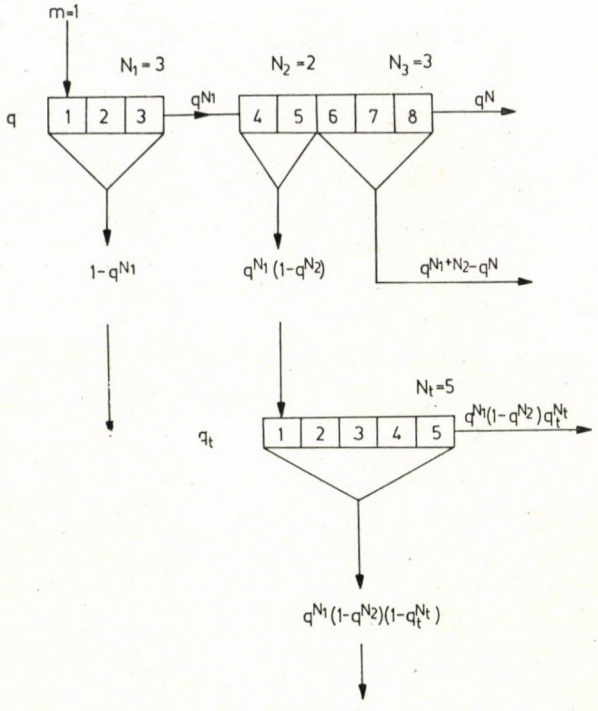


Fig. 7

is obtained. The mass ratio Kt of recirculated middlings (Fig. 8):

$$Kt = \frac{q^{N_1}[q^{N_3}(1-q^{N_3}) + (1-q^{N_3})\, q_t^{N_t}]}{q^{N_2+N_3} + (1-q^{N_3})(1-q_t^{N_t})} \,. \tag{37}$$

The concentrates obtained from purifying flotation cells are of

$$\text{Conc. II} = \frac{q^{N_1}(1-q^{N_3})(1-q_t^{N_t})}{q^{N_2+N_3} + (1-q^{N_3})(1-q_t^{N_t})} \tag{38}$$

mass ratio, while the mass of middlings

$$M = \frac{q^N}{q^{N_2+N_3} + (1-q^{N_3})(1-q_t^{N_t})} \,. \tag{39}$$

The open flow-sheet shown in Fig. 8 is used even in practice.

A closed version of the open flow-sheet presented in Fig. 3 is to be seen in Fig. 9. Here, the froth products obtained from both primary and secondary flotation are purified, while tailings from the purifying cell battery, as well as

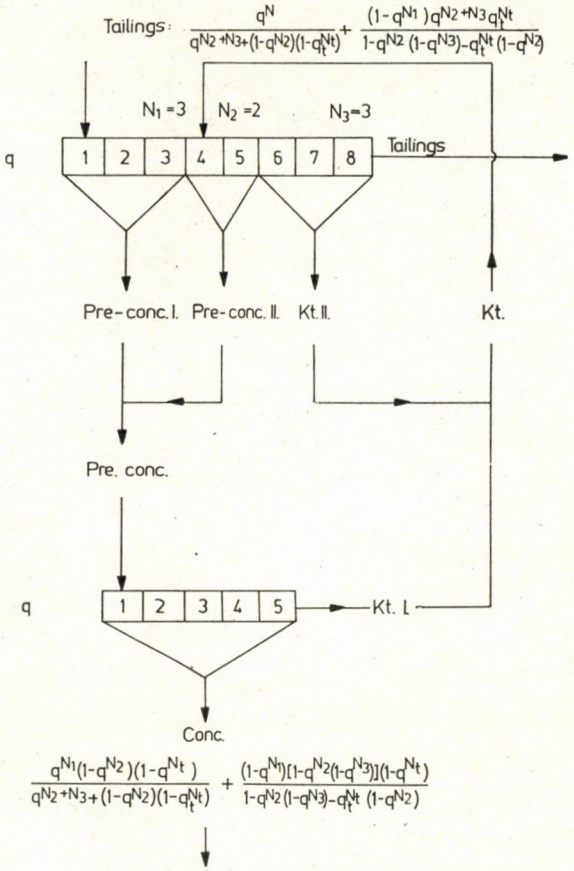


Fig. 8

the froth products from the last cell N_3 of the secondary flotation circuit are recirculated to cell $N_1 + 1$.

From totalizing the material flow values shown in Figs. 5 and 8, the corresponding mass ratios indicated by Fig. 9 result. In this figure only mass ratio data for concentrates and tailings are to be found, since the relationships for internal material flows are presented in Table III.

If the composition of feed (m_{ij}) and the flotability characteristics (q_{ij}) of its fraction to be separated are known, then the mass outputs for concentrates and tailings are

$$m_h = \sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} \left\{ \frac{q^{N_1}(1-q_{ij}^{N_2})(1-q_{tij}^{N_t})}{q_{ij}^{N_2+N_3}+(1-q_{ij}^{N_2})(1-q_{tij}^{N_t})} + \right.$$
$$\left. + \frac{(1-q_{ij}^{N_1})[1-q_{ij}^{N_2}(1-q_{ij}^{N_3})](1-q_{tij}^{N_t})}{1-q_{ij}^{N_2}(1-q_{ij}^{N_3})-q_{tij}^{N_t}(1-q_{ij}^{N_2})} \right\} \tag{40}$$

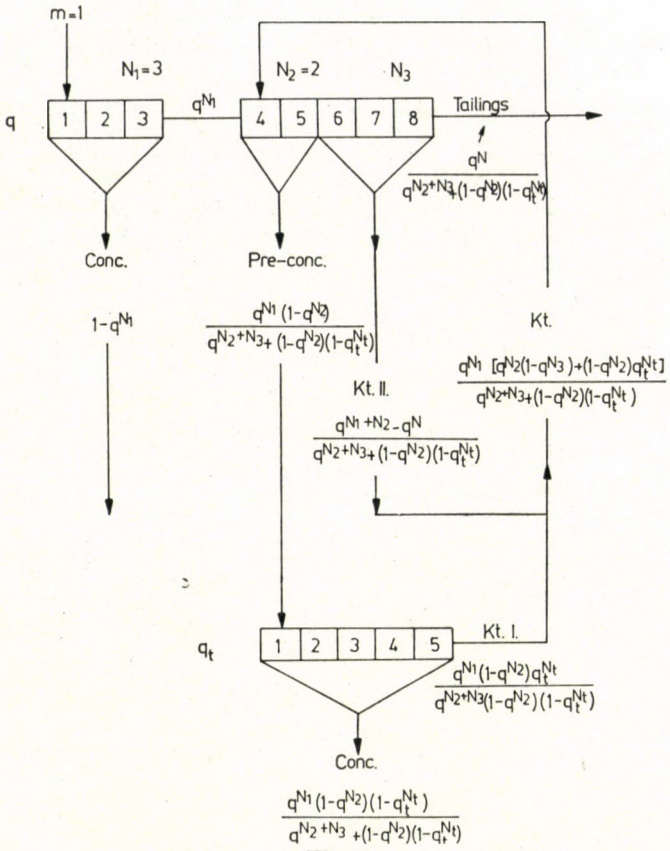


Fig. 9

and

$$m_h = \sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} \left\{ \frac{q_{ij}^{N}}{q_{ij}^{N_2+N_3} + (1-q_{ij}^{N_2})(1-q_{tij}^{N_t})} + \right.$$
$$\left. + \frac{(1-q_{ij}^{N_1})\, q_{ij}^{N_2+N_3} q_{tij}^{N}}{1-q_{ij}^{N_2}(1-q_{ij}^{N_3}) - q_{tij}^{N_t}(1-q_{ij}^{N_3})} \right\}. \tag{41}$$

Mineral contents b_i and c_i in the concentrates and tailings:

$$b_i = \frac{1}{m_h} \sum_{j=1}^{l} m_{ij} \left\{ \frac{q_{ij}^{N_1}(1-q_{ij}^{N_3})(1-q_{tij}^{N_t})}{q_{ij}^{N_2+N_3} + (1-q_{ij}^{N_3})(1-q_{tij}^{N_t})} + \right.$$
$$\left. + \frac{(1-q_{ij}^{N_1})\,[1-q_{ij}^{N_2}(1-q_{ij}^{N_3})]\,(1-q_{tij}^{N_t})}{1-q_{ij}^{N_2}(1-q_{ij}^{N_3}) - q_{tij}^{N_t}(1-q_{ij}^{N_3})} \right\} \tag{42}$$

and

$$c_i = \frac{1}{m_m} \sum_{j=1}^{l} m_{ij} \left\{ \frac{q_{ij}^{N}}{q_{ij}^{N_2+N_3} + (1-q_{ij}^{N_3})(1-q_{tij}^{N_t})} + \right.$$
$$\left. + \frac{(1-q_{ij}^{N_1})\, q_{ij}^{N_2+N_3} q_{tij}^{N_t}}{1-q_{ij}^{N_2}(1-q_{ij}^{N_3}) - q_{tij}^{N_t}(1-q_{ij}^{N_3})} \right\}. \tag{43}$$

Values for outputs from k_i and K_i ($k_i + K_i = 1$) constituents:

$$k_i = \frac{1}{m_i} \sum_{j=1}^{l} m_{ij} \left\{ \frac{q_{ij}^{N_1} (1 - q_{ij}^{N_2}) (1 - q_{tij}^{N_t})}{q_{ij}^{N_2+N_3} + (1 - q_{ij}^{N_2})(1 - q_{tij}^{N_t})} + \right.$$
$$\left. + \frac{(1 - q_{ij}^{N_1}) [1 - q_{ij}^{N_2} (1 - q_{ij}^{N_3})] (1 - q_{tij}^{N_t})}{1 - q_{ij}^{N_2} (1 - q_{ij}^{N_3}) - q_{tij}^{N_t} (1 - q_{ij}^{N_2})} \right\} \tag{44}$$

and

$$K_i = \frac{1}{m_i} \sum_{j=1}^{l} m_{ij} \left\{ \frac{q_{ij}^{N}}{q_{ij}^{N_2+N_3} + (1 - q_{ij}^{N_2})(1 - q_{tij}^{N_t})} + \right.$$
$$\left. + \frac{(1 - q_{ij}^{N_1}) q_{ij}^{N_2+N_3} q_{tij}^{N_t}}{1 - q_{ij}^{N_2} (1 - q_{ij}^{N_3}) - q_{tij}^{N_t} (1 - q_{ij}^{N_2})} \right\}. \tag{45}$$

Mass output m_{kt} of the recirculated middlings:

$$m_{kt} = \sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} \left\{ \frac{q_{ij}^{N_1} [q_{ij}^{N_2} (1 - q_{ij}^{N_3}) + (1 - q_{ij}^{N_3}) q_{tij}^{N_t}]}{q_{ij}^{N_2+N_3} + (1 - q_{ij}^{N_2})(1 - q_{tij}^{N_t})} + \right.$$
$$\left. + \frac{(1 - q_{ij}^{N_1}) q_{tij}^{N_t}}{1 - q_{ij}^{N_2}(1 - q_{ij}^{N_3}) - q_{tij}^{N_t} (1 - q_{ij}^{N_2})} \right\}. \tag{46}$$

The mass ratio m_{ft} of the feed material for the purifying cell battery

$$m_{ft} = \sum_{i=1}^{n} \sum_{j=1}^{l} m_{ij} \left\{ \frac{q_{ij}^{N_1} (1 - q_{ij}^{N_3})}{q_{ij}^{N_2+N_3} + (1 - q_{ij}^{N_2}) (1 - q_{tij}^{N})} + \right.$$
$$\left. + \frac{(1 - q_{ij}^{N_1}) [1 - q_{ij}^{N_2} (1 - q_{ij}^{N_3})]}{1 - q_{ij}^{N_2} (1 - q_{ij}^{N_3}) - q_{tij}^{N_t} (1 - q_{ij}^{N_2})} \right\}. \tag{47}$$

Functions of separation of the internal material balance are to be derived in conformity with the former discussion, from the equations presented in Table III.

REFERENCES

1. FUERSTENAU, D. W.: Froth Flotation 50th Anniversary Volume. The Am. Inst. of Min., Met., and Petr. Eng., Inc. New York, 1962.
2. KELSALL, D. F.: Application of probability in the assessment of flotation systems. Bulletin of The Institution of Mining and Metallurgy. January 1961, No. 650, pp. 191—204.
3. PETHŐ, SZ.: A flotálási folyamatokat leíró sebességi egyenletek vizsgálata (Analysis of the equations of velocity describing flotation processes). *A Magyar Tudományos Akadémia Műsz. Tud. Oszt. Közl.* 35 (1965), 99—106.
4. PRÉKOPA, A.: Valószínűségelmélet műszaki alkalmazásokkal (Probability theory with practical examples for its technical use). Műszaki Könyvkiadó, Budapest, 1962.
5. TARJÁN, G.: Adalék a széntelepek piritjeinek flotálhatóságára (Contribution to the study of the flotability characteristics of pyrites to be found in coal seams). *Bányászati Lapok*, 96 (1963), 577—579.

6. Tarján, G.: Ásványelőkészítés II (Mineral processing II). Manuscript, Tankönyvkiadó, Budapest, 1974.
7. Vincze, I.: Matematikai statisztika ipari alkalmazásokkal (Mathematical statistics with practical examples for their industrial use). Műszaki Könyvkiadó, Budapest, 1968.
8. Steiner, H. J.: Zur Frage einer Projektierung von Flotationsanlagen auf flotationskinetischer Grundlage. 2. Arbeitstagung des Fachausschusses für Erzaufbereitung der Gesellschaft Deutscher Metallhütten- und Bergleute e. V. 14. bis 16. Oktober 1971 in Hameln/Weser.
9. Pethő, Sz.—Tompos, E.—Bőhm, J.: Flotálóművek optimális tervezésének és folyamatirányításának függvényei. (Functions for the design and process control of flotation plants in optimal conditions). Manuscript.
10. Schubert, H.: Aufbereitung fester mineralischer Rohstoffe. Band II, VEB Deutscher Verlag für Grundstoffindustrie, Leipzig, 1967.
11. Yowett, A.—Ghosh, S. K.: Flotation Kinetics: Investigations lending to Process Optimization, VII. International Mineral Processing Congress, 1964. New York, 175—184.
12. Ek, C.: Cinétique de flotation (études en laboratories — essais industriels), IXe Congrès international de la préparation des minerais, Praha, 1970. 115—121 (Rudy).
13. Panu, H.: Beitrag betreffend Gleichungen und Methoden zur Berechnung der Flotationsergebnisse, IX. Internationaler Kongress für Aufbereitung der mineralischen Rohstoffe, Praha 1970, 123—132 (Rudy).
14. Inoue, T.—Imaizumi, T.: Some Aspects of the Flotation Kinetics, VIIIth International Mineral Processing Congress, Leningrad, 1968. 15.

ИССЛЕДОВАНИЕ ФУНКЦИЙ РАЗДЕЛЕНИЯ ФЛОТИРУЮЩИХ УСТРОСТВ

С. ПЕТЕ

РЕЗЮМЕ

Геометрическое распределение есть дискретный аналог экспоненциального распределения, поэтому с его помощью могут быть выведены функции разделения флотирующих устройств, как вынос масс и составных частей, качества продуктов, коэффициент полезно действия разделения. В статье описываются крайние значения функций, а также закономерности их формирования.

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PARTITIONING OF PROBABILISTIC DISTRIBUTIONS AND THE PARAMETERS OF THE PARTITIONED DISTRIBUTION

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[Manuscript received April 20, 1976]

It is a common problem in connection with distribution functions to determine the expectation and the variance of partitions including values greater or smaller as a limiting value of the random variable if this limiting value is changing continuously. The present paper contains the parameters for the general normal distribution, for the normal distribution with zero expectation and σ standard deviation, for the standardized normal distribution, for the lognormal, exponential, Weibull, and positive order gamma distributions. The physical and technical meaning of these parameters are discussed, too.

1. Introduction

In the practice the distribution and density functions, expectations and variances of random variables having values less or more than a certain limiting value of the variable, interpreted over a partition of the original domain are needed. The ore dressing functions give a classical example of the problem. From the distribution function of qualitative parameters as metal or ash content of the mineral raw materials the continuous change of the average qualitative parameters of concentrate and matrix are determined graphically or computationally in order to enable a decision on the parameters of selection.

1. *Definition.* Be $\Omega \mid \xi \mid [x_{\min}; x_{\max}]$ any random variable where Ω is domain, the range is $[x_{\min}; x_{\max}]$ the closed interval of real numbers.

Be further $x_{\min} < x_a < x_{\max}$ and $\Omega_1 = \langle \omega \in \Omega \mid \xi(\omega) \mid \leq x_a \rangle$ or $\Omega_1 = \{\omega \in \Omega \mid \xi(\omega) \mid > x_a\}$ respectively. Obviously $\Omega_1 \cup \Omega_2 \equiv \Omega$ and $\Omega_1 \cap \Omega_2 \equiv \emptyset$. In that case the random variables $\Omega_1 \mid \xi_1 \mid [x_{\min}; x_a]$ and $\Omega_2 \mid \xi_2 \mid [x_a; x_{\max}]$ for which $\xi_1(\omega) = \xi(\omega)$ if $\omega \in \Omega_1$ and $\xi_2\langle\omega\rangle = \xi(\omega)$, if $\omega \in \Omega_2$ is fulfilled, are called the x_a partitions of the random variable ξ.

2. *Definition.* Be $F(x) = P(\xi < x)$ the distribution function of ξ. In that case the functions

$$F_1(x) = \begin{cases} \dfrac{1}{F(x_a)} - F(x) & \text{if} \quad x \leq x_a \\ 1 & \text{if} \quad x > x_a \end{cases}$$

and

$$F_2(x) = \begin{cases} \dfrac{F(x) - F(x_a)}{1 - F(x_a)} & \text{if} \quad x > F(x_a) \\ 0 & \text{if} \quad x < F(x_a) \end{cases}$$

(the distribution functions of the random variable ξ_1 and ξ_2) are called the x_a partitions of the distribution function $F(x)$.

The continuous determination of the expectation and variance of distribution functions for partitions with given limiting value has an importance first of all in the theory of reliability. If the useful life is taken as the value of the random variable, many practical questions can unambiguously answered by using the expectations and variances of random variables formed from elements having longer or shorter lives than a given limit. Thus, e.g. producing firms can plan the expectation and variance of the number of failures in the guaranteed period. By fixing the expectation and the variance, a running time can be determined for the preventive maintenance when the apparatus or its parts must be substituted. Pension institutes can estimate the expected lives and the variance of people older than the pension limit.

The physical and technical content of the problem will be discussed later.

In what follows we give a general solution for the problem, afterwards the corresponding functions, expectations and variances of standardized normal, normal, lognormal, exponential, Weibull, gamma and positive order gamma distributions are presented.

2. General solution

Figure 1 shows the $f(x)$ frequency and $F(x)$ distribution function of a continuous random variable $\Omega \mid \xi \mid [x_{\min}; x_{\max}]$ [2, 3] with expectation $M(\xi)$ and variance $D^2(\xi)$. If the limiting value is x_a; $(x_{\min} \leq x_a \leq x_{\max})$ the expectation of the partitions $\Omega_1 \mid \xi_1 \mid [x_{\min}; x_a]$ and $\Omega_2 \mid \xi_2 \mid [x_a; x_{\max}]$ are the abscissae

$$M(\xi_1) = \frac{1}{F(x_a)} \int_{x_{\min}}^{x_a} f(x)\, dx \tag{1}$$

and

$$M(\xi_2) = \frac{1}{1 - F(x_a)} \int_{x_a}^{x_{\max}} f(x)\, x\, dx \tag{2}$$

of the gravity centres S_{I} and S_{II} of the areas (see Fig. 1) below the function $f(x)$ [1]:

$$\int_{x_{\min}}^{x_a} f(x)\, dx = F(x_a) \quad \text{and} \quad \int_{x_a}^{x_{\max}} f(x)\, dx = 1 - F(x_a)\,. \tag{3}$$

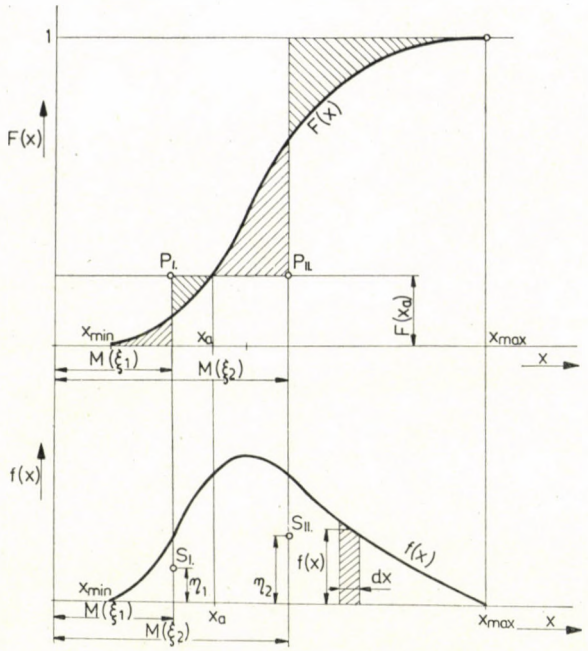

Fig. 1

The ordinates of the gravity centres of the partitions are random variables themselves $\pi_1 \mid \eta_1 \mid [0; f(x)]$ and $\pi_2 \mid \eta_2 \mid [0; f(x)]$ in the domain $x_a - x_{\min}$, and $x_{\max} - x_a$, respectively, with the expectances $M(\eta_1)$ and $M(\eta_2)$.

The ordinates of the gravity centres can generally be expressed in the following form:

$$M(\eta_1) = \frac{1}{2F(x_a)} \int_{x_{\min}}^{x_a} f^2(x)\, dx \tag{4}$$

and

$$M(\eta_2) = \frac{1}{2[1-F(x_a)]} \int_{x_a}^{x_{\max}} f^2(x)\, dx\,. \tag{5}$$

Figure 2 helps the determination of the corresponding variances. The density functions $f_1(x)$ and $f_2(x)$ of the random variables ξ_1 and ξ_2 are also represented here; they can be produced by making the area of the corresponding parts of $f(x)$ equal to unity.

$$f_1(x) = \int_{x_{\min}}^{x_a} \frac{f(x)}{F(x_a)} \cdot dx = 1 \quad \text{and} \quad f_2(x) = \int_{x_a}^{x_{\max}} \frac{f(x)}{1 - F(x_a)} \cdot dx = 1. \tag{6}$$

Therefore the variances of the partial populations $D^2(\xi_1)$ and $D^2(\xi_2)$ are the

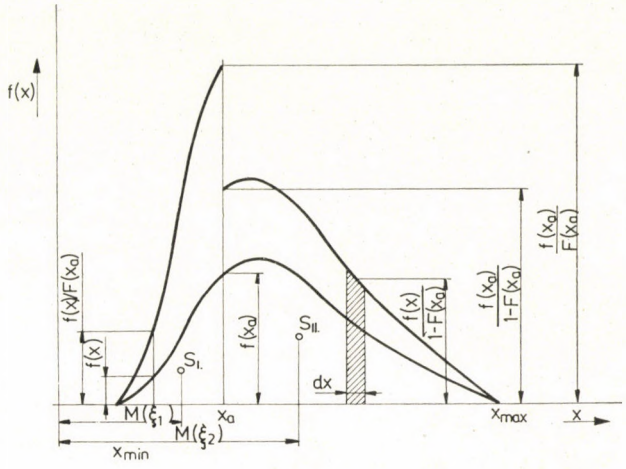

Fig. 2

following:

$$D^2(\xi_1) = \int_{x_{\min}}^{x_a} \frac{f(x)}{F(x_a)} \cdot [x - M(\xi_1)]^2 \cdot dx = \\ = \frac{1}{F(x_a)} \int_{x_{\min}}^{x_a} f(x) \cdot x^2 \cdot dx - M^2(\xi_1) \tag{7}$$

and

$$D^2(\xi_2) = \int_{x_a}^{x_{\max}} \frac{f(x)}{1 - F(x_a)} [x - M(\xi_2)]^2 \, dx = \\ = \frac{1}{1 - F(x_a)} \int_{x_a}^{x_{\max}} f(x) \cdot x^2 \cdot dx - M^2(\xi_2). \tag{8}$$

These results can also be simply deduced by deriving the distribution functions described in Definition 2.

3. Normal distribution with 0 expectation and standard deviation σ, and the standard normal distribution

The density function of the normal distribution with 0 expectation and standard deviation σ is:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} . \tag{9}$$

If $\varphi(x)$ and $\Phi(x)$ are the density and distribution functions of a normal distribution with expectation 0 and standard deviation 1, the expectations

of the partitions, and their variances will be:

$$M(\xi_1) = -\frac{\frac{\sigma}{\sqrt{2\pi}} e^{\frac{x_a^2}{2\sigma^2}}}{\Phi\left(\frac{x_a}{\sigma}\right)} = -\frac{-\sigma\varphi\left(\frac{x_a}{\sigma}\right)}{\Phi\left(\frac{x_a}{\sigma}\right)} \tag{10}$$

and

$$M(\xi_2) = \frac{\frac{\sigma}{\sqrt{2\pi}} e^{\frac{x_a^2}{2\sigma^2}}}{1-\Phi\left(\frac{x_a}{\sigma}\right)} = \frac{\sigma\varphi\left(\frac{x_a}{\sigma}\right)}{1-\Phi\left(\frac{x_a}{\sigma}\right)} \tag{11}$$

further

$$D^2(\xi_1) = -\sigma^2 - \frac{1}{\sqrt{2\pi}} \frac{\sigma x_a}{\Phi\left(\frac{x_a}{\sigma}\right)} e^{-\frac{x_a}{2\sigma^2}} - M^2(\xi_1) = \tag{12}$$

$$= \sigma^2 - \sigma x_a \frac{\varphi\left(\frac{x_a}{\sigma}\right)}{\Phi\left(\frac{x_a}{\sigma}\right)} - M^2(\xi_1)$$

and

$$D^2(\xi_2) = \sigma^2 + \frac{1}{\sqrt{2\pi}} \frac{\sigma x_a}{1-\Phi\left(\frac{x_a}{\sigma}\right)} e^{-\frac{x_a^2}{2\sigma^2}} - M^2(\xi_2) = \tag{13}$$

$$= \sigma^2 + \sigma x_a \frac{\varphi\left(\frac{x_a}{\sigma}\right)}{1-\Phi\left(\frac{x_a}{\sigma}\right)} - M^2(\xi_2).$$

The ordinates of both gravity centres are:

$$M(\eta_1) = \frac{\frac{1}{4\sigma\sqrt{\pi}} \Phi\left(\frac{x_a\sqrt{2}}{\sigma}\right)}{\Phi\left(\frac{x_a}{\sigma}\right)} \tag{14}$$

and

$$M(\eta_2) = \frac{\frac{1}{4\sigma\sqrt{\pi}} \left[1-\Phi\left(\frac{x_a\sqrt{2}}{\sigma}\right)\right]}{1-\Phi\left(\frac{x_a}{\sigma}\right)}. \tag{15}$$

By substituting $\sigma = 1$, the corresponding parameters of the normal distribution with expectation 0 and standard deviation 1 can be determined. From Eqs (10) and (11) one gets for the expectations:

$$M(\xi_1) = -\frac{\varphi(x_a)}{\Phi(x_a)} \tag{16}$$

and

$$M(\xi_2) = \frac{\varphi(x_a)}{1 - \Phi(x_a)} . \tag{17}$$

From Eqs (12) and (13) the corresponding variances of the standard normal distribution are:

$$D^2(\xi_1) = 1 - x_a \frac{\varphi(x_a)}{\Phi(x_a)} - \frac{\varphi^2(x_a)}{\Phi^2(x_a)} \tag{18}$$

and

$$D^2(\xi_2) = 1 + x_a \frac{\varphi(x_a)}{1 - \Phi(x_a)} - \frac{\varphi^2(x_a)}{[1 - \Phi(x_a)]^2} . \tag{19}$$

Figure 3 shows in addition to the density and distribution functions of the normal distribution with 0 expectation and 1 standard deviation, the variations of the expectations $M(\xi_1)$ and $M(\xi_2)$. The vertical from the limiting value x_a and the horizontal from the intersection of this vertical and the distribution function determine the corresponding values of $M(\xi_1)$ and $M(\xi_2)$. On the figure representing the density function, the coordinates of the gravity centre are also shown.

On the figure of the density function one gets the extremum of the function

$$M(\eta_1) = \frac{\Phi(x_a\sqrt{2})}{4\sqrt{\pi}\,\Phi(x_a)} \tag{20}$$

by substituting $\sigma = 1$ from the geometrical place of the gravity centres (Eq. (14)). This lies where the first derivate of Eq. (20) has the value zero. From $M(\eta_1) = 0$ one gets:

$$x_a = \sqrt{\ln 2 + 2 \ln \frac{\Phi(x_a)}{\Phi(x_a\sqrt{2})}} . \tag{21}$$

The x_a value yielding the maximum of $M(\eta_1)$ has to fulfil the transcendent equations (20) ($x_a = 0.7003$) and $M(\eta_1)_{\max} = 0.156094$.

The difference $\Delta\xi$ of $M(\xi_2)$ and $M(\xi_1)$ is also a function of x_a:

$$\Delta\xi = \frac{\varphi(x_a)}{\Phi(x_a)[1 - \Phi(x_a)]} . \tag{22}$$

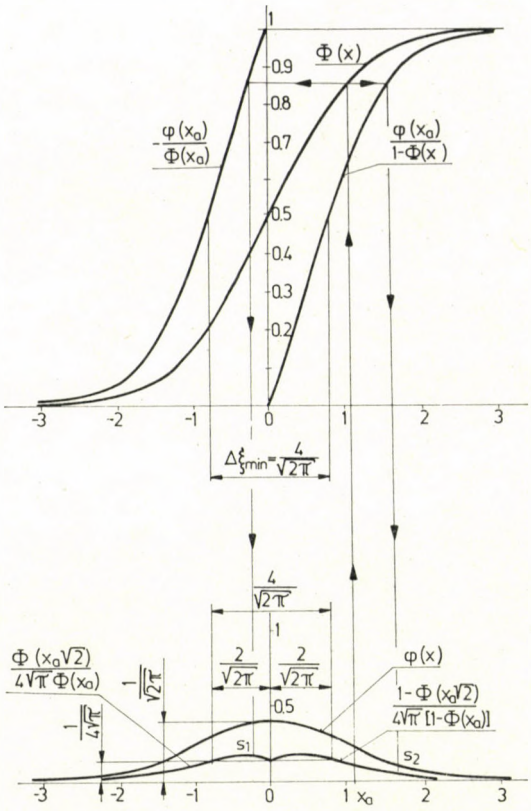

Fig. 3

Both the numerator and denominator of Eq. (22) have maxima at $x_a = 0$. Since the change of $\varphi(x_a)$ in function of x_a is less than the change of $\Phi(x_a)$ $[1 - \Phi(x_a)]$, Eq. (22) has a minimum at $x_a = 0$:

$$\Delta \xi_{\min} = \frac{4}{\sqrt{2\pi}} \tag{23}$$

and a maximum at $x_a \to \infty$ ($\Delta \xi_{\min} \to \infty$). As $x_a = 0$ the variances are equal:

$$D^2(\xi_1) = D^2(\xi_2) = 1 - \frac{2}{\pi} = 0.363380. \tag{24}$$

At $x_a \to \infty$ $D^2(\xi_1) \to 1$ and $D^2(\xi_2) \to 0$.

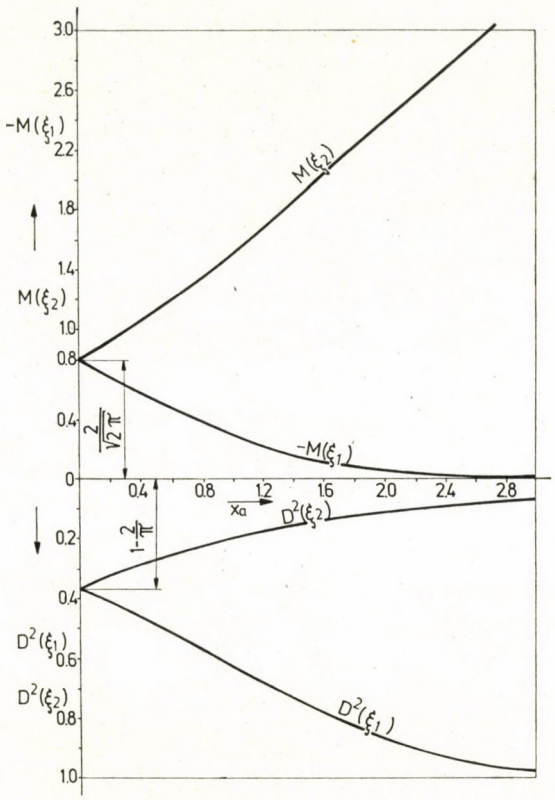

Fig. 4

Fig. 4 shows in function of the extreme value x_a for the normal distribution with the expectation 0 and standard deviation 1, the expectations $M(\xi_1)$ and $M(\xi_2)$, and the variations $D^2(\xi_1)$ and $D^2(\xi_2)$.

Table I contains the numerical values of the parameters of the standard normal distribution for the domain of x_a between 0 and 2.82. With the data of the table and using the corresponding connections the parameters treated here can be computed from any normal distribution.

4. Lognormal distribution

The density function of the lognormal distribution with the parameters m and σ [2, 3] are the following:

$$f(x) = \frac{1}{\sqrt{2\pi}\, x\sigma} \cdot \exp^{-\frac{(\ln x - m)^2}{2\sigma^2}} . \tag{25}$$

Table I

Parameters of the partitions of the standard normal distribution

x_a	$M(\xi_1)$	$M(\xi_1)$	$M(\eta_1)$	$M(\eta_2)$	$D^2(\xi_1)$	$D^2(\xi_2)$
0.00	0.797885	0.797885	0.141047	0.141047	0.363380	0.363380
0.01	0.791530	0.804261	0.141510	0.140578	0.365565	0.361207
0.02	0.785196	0.810660	0.141965	0.140100	0.367763	0.359044
0.03	0.778885	0.817080	0.142412	0.139616	0.369971	0.356893
0.04	0.772596	0.823522	0.142852	0.139124	0.372191	0.354753
0.05	0.766328	0.829986	0.143285	0.138624	0.374425	0.352623
0.06	0.760084	0.836470	0.143709	0.138118	0.376667	0.350507
0.07	0.753862	0.842975	0.144127	0.137604	0.378922	0.348401
0.08	0.747663	0.849501	0.144537	0.137083	0.381187	0.346308
0.09	0.741486	0.856049	0.144939	0.136555	0.383464	0.344225
0.10	0.735332	0.862618	0.145333	0.136020	0.385754	0.342152
0.11	0.729201	0.869206	0.145720	0.135477	0.388053	0.340094
0.12	0.723093	0.875815	0.146100	0.134928	0.390365	0.338046
0.13	0.717008	0.882446	0.146471	0.134372	0.392689	0.336007
0.14	0.710947	0.889095	0.146836	0.133809	0.395022	0.333983
0.15	0.704908	0.895766	0.147192	0.133239	0.397368	0.331968
0.16	0.698895	0.902455	0.147541	0.132662	0.399723	0.329968
0.17	0.692903	0.909166	0.147883	0.132079	0.402092	0.327975
0.18	0.686936	0.915897	0.148216	0.131489	0.404471	0.325994
0.19	0.680993	0.922645	0.148543	0.130892	0.406860	0.324028
0.20	0.675073	0.929416	0.148861	0.130290	0.409262	0.322068
0.21	0.669178	0.936205	0.149172	0.129680	0.411673	0.320124
0.22	0.663307	0.943012	0.149477	0.129064	0.414096	0.318190
0.23	0.657460	0.949841	0.149772	0.128442	0.416531	0.316266
0.24	0.651637	0.956688	0.150062	0.127814	0.418976	0.314353
0.25	0.645840	0.963553	0.150343	0.127179	0.421431	0.312454
0.26	0.640066	0.970439	0.150617	0.126539	0.423898	0.310563
0.27	0.634317	0.977343	0.150883	0.125893	0.426376	0.308683
0.28	0.628594	0.984265	0.151142	0.125240	0.428864	0.306817
0.29	0.622895	0.991207	0.151395	0.124582	0.431363	0.304959
0.30	0.617221	0.998165	0.151639	0.123919	0.433872	0.303116
0.31	0.611572	1.005145	0.151876	0.123250	0.436393	0.301278
0.32	0.605949	1.012141	0.152107	0.122575	0.438923	0.299456
0.33	0.600351	1.019155	0.152330	0.121895	0.441463	0.297645
0.34	0.594778	1.026188	0.152546	0.121209	0.444015	0.295842

Table I (contd)

x_a	$M(\xi_1)$	$M(\xi_2)$	$M(\eta_1)$	$M(\eta_2)$	$D^2(\xi_1)$	$D^2(\xi_2)$
0.35	0.589231	1.033239	0.152754	0.120519	0.446577	0.294051
0.36	0.583710	1.040305	0.152957	0.119822	0.449147	0.292275
0.37	0.578214	1.047393	0.153151	0.119122	0.451730	0.290503
0.38	0.572744	1.054495	0.153340	0.118416	0.454321	0.288748
0.39	0.567300	1.061618	0.153521	0.117705	0.456923	0.286998
0.40	0.561882	1.068757	0.153695	0.116990	0.459535	0.285261
0.41	0.556491	1.075912	0.153863	0.116269	0.462156	0.283537
0.42	0.551126	1.083085	0.154024	0.115546	0.464787	0.281823
0.43	0.545787	1.090275	0.154178	0.114817	0.467428	0.280118
0.44	0.540475	1.097482	0.154326	0.114084	0.470078	0.278426
0.45	0.535188	1.104708	0.154467	0.113347	0.472739	0.276739
0.46	0.529929	1.111949	0.154603	0.112605	0.475408	0.275067
0.47	0.524697	1.119204	0.154731	0.111860	0.478085	0.273408
0.48	0.519491	1.126479	0.154853	0.111111	0.480773	0.271755
0.49	0.514312	1.133771	0.154969	0.110359	0.483470	0.270112
0.50	0.509161	1.141076	0.155079	0.109602	0.486175	0.268483
0.51	0.504036	1.148400	0.155183	0.108842	0.488889	0.266861
0.52	0.498938	1.155740	0.155280	0.108080	0.491612	0.265250
0.53	0.493868	1.163096	0.155372	0.107312	0.494344	0.263649
0.54	0.488826	1.170466	0.155457	0.106544	0.497084	0.262062
0.55	0.483810	1.177854	0.155537	0.105771	0.499832	0.260480
0.56	0.478822	1.185257	0.155612	0.104996	0.502589	0.258910
0.57	0.473862	1.192676	0.155680	0.104218	0.505354	0.257349
0.58	0.468929	1.200112	0.155743	0.103437	0.508127	0.255796
0.59	0.464024	1.207562	0.155800	0.102655	0.510908	0.254255
0.60	0.459147	1.215026	0.155853	0.101869	0.513696	0.252725
0.61	0.454298	1.222506	0.155899	0.101081	0.516491	0.251208
0.62	0.449477	1.230001	0.155941	0.100291	0.519295	0.249698
0.63	0.444684	1.237513	0.155977	0.099499	0.522106	0.248194
0.64	0.439919	1.245039	0.156008	0.098706	0.524923	0.246703
0.65	0.435182	1.252579	0.156035	0.097909	0.527748	0.245223
0.66	0.430474	1.260133	0.156056	0.097113	0.530579	0.243753
0.67	0.425794	1.267702	0.156072	0.096314	0.533417	0.242291
0.68	0.421142	1.275288	0.156084	0.095514	0.536262	0.240836
0.69	0.416519	1.282887	0.156091	0.094713	0.539113	0.239394

Table I (contd)

x_a	$M(\xi_1)$	$M(\xi_2)$	$M(\eta_1)$	$M(\eta_2)$	$D^2(\xi_1)$	$D^2(\xi_2)$
0.70	0.411925	1.290497	0.156094	0.093909	0.541970	0.237964
0.71	0.407359	1.298127	0.156091	0.093108	0.544834	0.236536
0.72	0.402821	1.305771	0.156085	0.092303	0.547704	0.235117
0.73	0.398313	1.313425	0.156074	0.091498	0.550579	0.233715
0.74	0.393833	1.321094	0.156059	0.090693	0.553459	0.232320
0.75	0.389382	1.328780	0.156040	0.089885	0.556345	0.230929
0.76	0.384960	1.336477	0.156016	0.089080	0.559236	0.229551
0.77	0.380567	1.344187	0.155989	0.088273	0.562132	0.228186
0.78	0.376202	1.351915	0.155958	0.087467	0.565034	0.226820
0.79	0.371868	1.359650	0.155923	0.086659	0.567939	0.225476
0.80	0.367561	1.367405	0.155884	0.085852	0.570850	0.224128
0.81	0.363284	1.375168	0.155841	0.085046	0.573764	0.222799
0.82	0.359037	1.382947	0.155796	0.084238	0.576683	0.221475
0.83	0.354818	1.390741	0.155746	0.083434	0.579605	0.220155
0.84	0.350629	1.398544	0.155694	0.082629	0.582531	0.218851
0.85	0.346469	1.406358	0.155638	0.081824	0.585461	0.217562
0.86	0.342338	1.414188	0.155578	0.081021	0.588394	0.216273
0.87	0.338237	1.422037	0.155516	0.080217	0.591330	0.214983
0.88	0.334165	1.429890	0.155450	0.079417	0.594269	0.213719
0.89	0.330122	1.437761	0.155382	0.078616	0.597211	0.212451
0.90	0.326109	1.445644	0.155311	0.077818	0.600155	0.211193
0.91	0.322125	1.453539	0.155237	0.077020	0.603102	0.209944
0.92	0.318171	1.461447	0.155160	0.076223	0.606050	0.208703
0.93	0.314246	1.469359	0.155081	0.075430	0.609000	0.207487
0.94	0.310351	1.477293	0.154999	0.074635	0.611952	0.206262
0.95	0.306485	1.485239	0.154915	0.073846	0.614906	0.205042
0.96	0.302649	1.493190	0.154828	0.073056	0.617860	0.203846
0.97	0.298842	1.501163	0.154739	0.072269	0.620816	0.202635
0.98	0.295065	1.509141	0.154648	0.071485	0.623772	0.201451
0.99	0.291318	1.517133	0.154555	0.070701	0.626729	0.200270
1.00	0.287600	1.525138	0.154459	0.069923	0.629686	0.199093
1.01	0.283912	1.533147	0.154362	0.069145	0.632643	0.197939
1.02	0.280253	1.541179	0.154263	0.068370	0.635601	0.196770
1.03	0.276624	1.549215	0.154162	0.067598	0.638557	0.195625
1.04	0.273024	1.557264	0.154060	0.066827	0.641513	0.194484

Table I (contd)

x_a	$M(\xi_1)$	$M(\xi_2)$	$M(\eta_1)$	$M(\eta_2)$	$D^2(\xi_1)$	$D^2(\xi_2)$
1.05	0.269454	1.565326	0.153955	0.066063	0.644468	0.193348
1.06	0.265913	1.573400	0.153850	0.065297	0.647422	0.192216
1.07	0.262402	1.581477	0.153742	0.064538	0.650374	0.191112
1.08	0.258921	1.589576	0.153633	0.063780	0.653326	0.189990
1.09	0.255469	1.597676	0.153523	0.063025	0.656274	0.188899
1.10	0.252046	1.605798	0.153411	0.062276	0.659222	0.187791
1.11	0.248653	1.613919	0.153299	0.061529	0.662166	0.186716
1.12	0.245290	1.622062	0.153185	0.060784	0.665109	0.185625
1.13	0.241955	1.630214	0.153070	0.060043	0.668048	0.184545
1.14	0.238651	1.638374	0.152915	0.059572	0.670984	0.183477
1.15	0.235375	1.646542	0.152837	0.058574	0.673917	0.182423
1.16	0.232129	1.654729	0.152719	0.057844	0.676847	0.181357
1.17	0.228912	1.662922	0.152601	0.057120	0.679772	0.180310
1.18	0.225724	1.671119	0.152481	0.056398	0.682694	0.179282
1.19	0.222566	1.679332	0.152361	0.055682	0.685611	0.178249
1.20	0.219437	1.687547	0.152241	0.054968	0.688524	0.177241
1.21	0.216336	1.695792	0.142119	0.054259	0.691432	0.176199
1.22	0.213265	1.704034	0.151997	0.053556	0.694335	0.175189
1.23	0.210223	1.712274	0.151875	0.052854	0.697232	0.174215
1.24	0.207210	1.720539	0.151752	0.052159	0.700124	0.173214
1.25	0.204226	1.728813	0.151629	0.051468	0.703010	0.172222
1.26	0.201270	1.737094	0.151506	0.050781	0.705890	0.171243
1.27	0.198343	1.745397	0.151383	0.050099	0.708764	0.170243
1.28	0.195445	1.753687	0.151259	0.049419	0.711631	0.169302
1.29	0.192576	1.762012	0.151135	0.048749	0.714492	0.168309
1.30	0.189735	1.770337	0.151011	0.048080	0.717345	0.167346
1.31	0.186923	1.778657	0.150887	0.047417	0.720191	0.166419
1.32	0.184139	1.786990	0.150763	0.046759	0.723029	0.165493
1.33	0.181383	1.795352	0.150639	0.046104	0.725860	0.164529
1.34	0.178656	1.803702	0.150516	0.045457	0.728683	0.163619
1.35	0.175957	1.812077	0.150392	0.044812	0.731497	0.162680
1.36	0.173286	1.820454	0.150269	0.044174	0.734303	0.161764
1.37	0.170643	1.828852	0.150145	0.043541	0.737100	0.160828
1.38	0.168028	1.837245	0.150023	0.042912	0.739888	0.159928
1.39	0.165441	1.845653	0.149900	0.042289	0.742667	0.159023

Table I (contd)

x_a	$M(\xi_1)$	$M(\xi_2)$	$M(\eta_1)$	$M(\eta_2)$	$D^2(\xi_1)$	$D^2(\xi_2)$
1.40	0.162881	1.854049	0.149778	0.041669	0.745436	0.158170
1.41	0.160349	1.862476	0.149656	0.041058	0.748195	0.157273
1.42	0.157845	1.870908	0.149535	0.040450	0.750945	0.156392
1.43	0.155368	1.879340	0.149414	0.039848	0.753684	0.155537
1.44	0.152919	1.887794	0.149293	0.039251	0.756413	0.154657
1.45	0.150496	1.896266	0.149174	0.038658	0.759131	0.153760
1.46	0.148101	1.904727	0.149054	0.038073	0.761838	0.152916
1.47	0.145733	1.913198	0.148936	0.037492	0.764534	0.152075
1.48	0.143392	1.921674	0.148817	0.036917	0.767219	0.151246
1.49	0.141078	1.930180	0.148700	0.036346	0.769892	0.150373
1.50	0.138790	1.938683	0.148583	0.035782	0.772553	0.149433
1.51	0.136529	1.947177	0.148467	0.035223	0.775202	0.148738
1.52	0.134294	1.955718	0.148352	0.034669	0.777839	0.147859
1.53	0.132085	1.964239	0.148238	0.034121	0.780463	0.147050
1.54	0.129903	1.972767	0.148124	0.033579	0.783075	0.146252
1.55	0.127747	1.981295	0.148011	0.033041	0.785673	0.145478
1.56	0.125616	1.989850	0.147899	0.032511	0.788259	0.144663
1.57	0.123512	1.998394	0.147788	0.031982	0.790831	0.143900
1.58	0.121433	2.006990	0.147678	0.031465	0.793390	0.143035
1.59	0.119380	2.015562	0.147568	0.030948	0.795935	0.142252
1.60	0.117352	2.024140	0.147460	0.030441	0.798466	0.141482
1.61	0.115349	2.032715	0.147352	0.029937	0.800983	0.140741
1.62	0.113371	2.041320	0.147246	0.029438	0.803486	0.139952
1.63	0.111419	2.049908	0.147140	0.028946	0.805974	0.139226
1.64	0.109491	2.058513	0.147036	0.028456	0.808447	0.138485
1.65	0.107587	2.067169	0.146932	0.027978	0.810906	0.137641
1.66	0.105709	2.075786	0.146830	0.027502	0.813349	0.136917
1.67	0.103854	2.084397	0.146728	0.027029	0.815777	0.136233
1.68	0.102024	2.093037	0.146628	0.026566	0.818190	0.135498
1.69	0.100218	2.101700	0.146528	0.026104	0.820588	0.134729
1.70	0.098436	2.110380	0.146430	0.025654	0.822969	0.133943
1.71	0.096677	2.119018	0.146333	0.025203	0.825335	0.133282
1.72	0.094943	2.127703	0.146236	0.024761	0.827685	0.132528
1.73	0.093231	2.136377	0.146141	0.024325	0.830018	0.131824
1.74	0.091543	2.145030	0.146047	0.023892	0.832335	0.131199

Table I (contd)

x_a	$M(\xi_1)$	$M(\xi_2)$	$M(\eta_1)$	$M(\eta_2)$	$D^2(\xi_1)$	$D^2(\xi_2)$
1.75	0.089878	2.153756	0.145954	0.023467	0.834636	0.130408
1.76	0.088236	2.162442	0.145862	0.023043	0.836920	0.129743
1.77	0.086616	2.171129	0.145772	0.022631	0.839187	0.129098
1.78	0.085019	2.179865	0.145682	0.022220	0.841438	0.128348
1.79	0.083445	2.188584	0.145593	0.021813	0.843671	0.127667
1.80	0.081893	2.197333	0.145506	0.021416	0.845887	0.126928
1.81	0.080362	2.206040	0.145420	0.021020	0.848086	0.126320
1.82	0.078854	2.214755	0.145335	0.020632	0.850267	0.125715
1.83	0.077368	2.223532	0.145251	0.020248	0.852431	0.124969
1.84	0.075903	2.232296	0.145168	0.019868	0.854578	0.124280
1.85	0.074459	2.241032	0.145086	0.019495	0.856706	0.123684
1.86	0.073037	2.249798	0.145005	0.019126	0.858817	0.123034
1.87	0.071636	2.258581	0.144926	0.018766	0.860910	0.122357
1.88	0.070255	2.267371	0.144847	0.018407	0.862985	0.121686
1.89	0.068895	2.276153	0.144770	0.018056	0.865042	0.121057
1.90	0.067556	2.284912	0.144694	0.017708	0.867080	0.120510
1.91	0.066237	2.293714	0.144619	0.017364	0.869101	0.119870
1.92	0.064938	2.302547	0.144545	0.017027	0.871103	0.119168
1.93	0.063659	2.311399	0.144472	0.016695	0.873086	0.118435
1.94	0.062399	2.320167	0.144401	0.016367	0.875051	0.117950
1.95	0.061160	2.329010	0.144330	0.016044	0.876998	0.117282
1.96	0.059939	2.337825	0.144260	0.015726	0.878926	0.116712
1.97	0.058738	2.346689	0.144193	0.015412	0.880836	0.116029
1.98	0.057556	2.355490	0.144125	0.015102	0.882726	0.115538
1.99	0.056393	2.364409	0.144058	0.014801	0.884599	0.114745
2.00	0.055248	2.373229	0.143993	0.014499	0.886452	0.114241
2.02	0.053014	2.390908	0.143866	0.013917	0.890102	0.113193
2.04	0.050851	2.408710	0.143743	0.013351	0.893677	0.111883
2.06	0.048760	2.426498	0.143624	0.012804	0.897177	0.110695
2.08	0.046738	2.444229	0.143510	0.012275	0.900600	0.109740
2.10	0.044784	2.462136	0.143399	0.011762	0.903949	0.108372
2.12	0.042895	2.479922	0.143292	0.011269	0.907222	0.107423
2.14	0.041072	2.497840	0.143189	0.010791	0.910419	0.106174
2.16	0.039312	2.515719	0.143090	0.010328	0.913541	0.105110
2.18	0.037613	2.533523	0.142995	0.009881	0.916589	0.104341

Table I (contd)

x_a	$M(\xi_1)$	$M(\xi_2)$	$M(\eta_1)$	$M(\eta_2)$	$D^2(\xi_1)$	$D^2(\xi_2)$
2.20	0.035975	2.551578	0.142903	0.009446	0.919561	0.102920
2.22	0.034395	2.569518	0.142814	0.009037	0.922460	0.101907
2.24	0.032873	2.587506	0.142730	0.008633	0.925285	0.100825
2.26	0.031406	2.605317	0.142648	0.008245	0.928036	0.100339
2.28	0.029994	2.623371	0.142570	0.007877	0.930715	0.009211
2.30	0.028634	2.641462	0.142495	0.007518	0.933322	0.098041
2.32	0.027326	2.659597	0.142423	0.007174	0.935857	0.096809
2.34	0.026068	2.677512	0.142354	0.006835	0.938322	0.096306
2.36	0.024858	2.695772	0.142288	0.006525	0.940716	0.094835
2.38	0.023696	2.713838	0.142225	0.006222	0.943042	0.094016
2.40	0.022580	2.731707	0.142164	0.005920	0.945299	0.093875
2.42	0.021508	2.750092	0.142106	0.005641	0.947489	0.092216
2.44	0.020479	2.768022	0.142051	0.005365	0.949612	0.092027
2.46	0.019492	2.786278	0.141998	0.005115	0.951671	0.090897
2.48	0.018545	2.804584	0.141948	0.004863	0.953664	0.089678
2.50	0.017638	2.822593	0.141900	0.004624	0.955594	0.089452
2.62	0.016768	2.840849	0.141854	0.004383	0.957462	0.088517
2.54	0.015936	2.859026	0.141810	0.004166	0.959269	0.087898
2.56	0.015139	2.878267	0.141769	0.003956	0.961015	0.087138
2.58	0.014376	2.895771	0.141729	0.003759	0.962703	0.085599
2.60	0.013647	2.914175	0.141691	0.003577	0.964333	0.084439
2.62	0.012949	2.932695	0.141655	0.003398	0.965906	0.082962
2.64	0.012282	2.950911	0.141621	0.003219	0.967424	0.082529
2.66	0.011646	2.969065	0.141589	0.003049	0.968887	0.082367
2.68	0.011038	2.987486	0.141558	0.002873	0.970297	0.081389
2.70	0.010457	3.005750	0.141529	0.002739	0.971656	0.080992
2.72	0.009903	3.024251	0.141501	0.002593	0.972964	0.079870
2.74	0.009375	3.042526	0.141475	0.002434	0.974223	0.079558
2.76	0.008872	3.061057	0.141450	0.002311	0.975434	0.078448
2.78	0.008392	3.079356	0.141426	0.002172	0.976598	0.078177
2.80	0.007936	3.098024	0.141403	0.002045	0.977717	0.076714
2.82	0.007501	3.116565	0.141382	0.001950	0.978791	0.075736

Be the continuously changing limiting value once again x_a. This distributes the density function into parts with gravity centres S_{I} and S_{II}. The abscissae of the gravity centres $M(\xi_1)$ and $M(\xi_2)$ are the expectations of the two partial populations which can again be expressed with the distribution $N(0, 1)$ $[\Phi(x)]$:

$$M(\xi_1) = e^{m+(\sigma^2/2)} \frac{\Phi\left(\dfrac{\ln x_a - m - \sigma^2}{\sigma}\right)}{\Phi\left(\dfrac{\ln x_a - m}{\sigma}\right)} \tag{26}$$

and

$$M(\xi_2) = e^{m+(\sigma^2/2)} \frac{1 - \Phi\left(\dfrac{\ln x_a - m - \sigma^2}{\sigma}\right)}{1 - \Phi\left(\dfrac{\ln x_a - m}{\sigma}\right)}. \tag{27}$$

Both equations contain the expectation of the lognormal distribution $e^{m+(\sigma^2/2)}$.

The ordinates $M(\eta_1)$ and $M(\eta_2)$ of the gravity centres are the following:

$$M(\eta_1) = \frac{1}{4\sigma\sqrt{\pi}} e^{(\sigma^2/4)-m} \cdot \frac{\left(\Phi \dfrac{\sigma^2 - 2m + \ln x_a^2}{\sqrt{2}\,\sigma}\right)}{\Phi\left(\dfrac{\ln x_a - m}{\sigma}\right)} \tag{28}$$

and

$$M(\eta_2) = \frac{1}{4\sigma\sqrt{\pi}} e^{(\sigma^2/4)-m} \frac{1 - \Phi\left(\dfrac{\sigma^2 - 2m + \ln x_a^2}{\sqrt{2}\,\sigma}\right)}{1 - \Phi\left(\dfrac{\ln x_a - m}{\sigma}\right)}. \tag{29}$$

The variances for the two partial populations are the following:

$$D^2(\xi_1) = e^{2m+\sigma^2} \left\{ e^{\sigma^2} \frac{\Phi\left(\dfrac{\ln x_a - m - 2\sigma^2}{\sigma}\right)}{\Phi\left(\dfrac{\ln x_a - m}{\sigma}\right)} - \left[\frac{\Phi\left(\dfrac{\ln x_a - m - \sigma^2}{\sigma}\right)}{\Phi\left(\dfrac{\ln x_a - m}{\sigma}\right)} \right]^2 \right\} \tag{30}$$

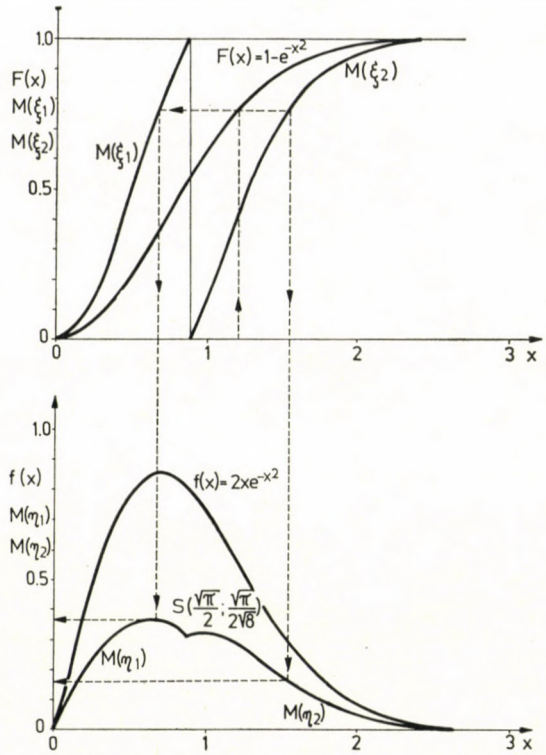

Fig. 5

and

$$D^2(\xi_2) = e^{2m+\sigma^2}\left\{e^{\sigma^2}\,\frac{1-\Phi\left(\dfrac{\ln x_a - m - 2\sigma^2}{\sigma}\right)}{1-\Phi\left(\dfrac{\ln x_a - m}{\sigma}\right)} - \left[\frac{1-\Phi\left(\dfrac{\ln x_a - m - \sigma^2}{\sigma}\right)}{\Phi\left(\dfrac{\ln x_a - m}{\sigma}\right)}\right]^2\right\}. \tag{31}$$

The corresponding limiting values of the last two equations correspond to the variances of the lognormal distribution.

Figure 5 shows the density and distribution function of the lognormal distribution with the parameters $m = 0$ and $\sigma = 0.4$, and the values of the expectations of the partial populations.

The gravity centre of the density function on the figure is denoted by S. The curve $M(\eta_1)$ representing the expectations of the partial populations reaches this point, and the curve $M(\eta_2)$ begins here. Both curves have maxima.

The use of the figure is explained by the arrows.

Figure 6 shows the variations of the variances of the former distribution. Both variances increase monotonously with x_a.

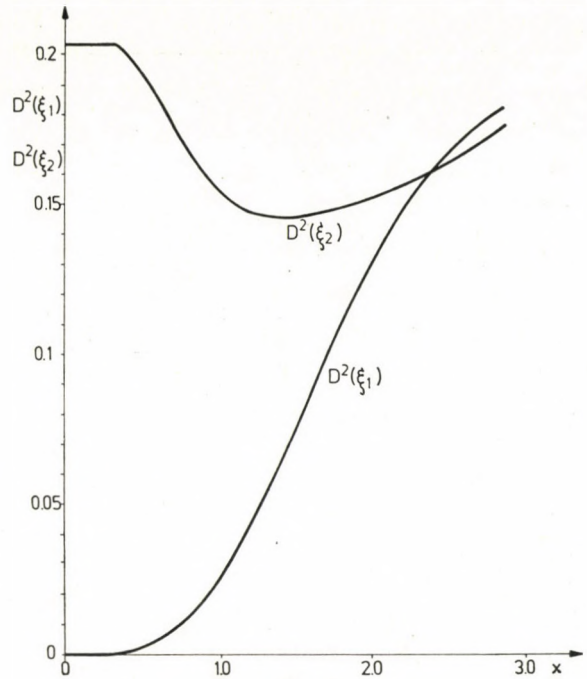

Fig. 6

5. Exponential distribution

The density function of the exponential distribution with the parameter λ is [2, 3]:

$$f(x) = \lambda\, e^{-\lambda x}\,. \tag{32}$$

If x_a is a continuously changing limiting value, the expectations of the partial populations and the ordinates of the gravity centres are the following:

$$M(\xi_1) = \frac{1}{\lambda} - \frac{x_a}{e^{\lambda x_a} - 1} \tag{33}$$

$$M(\xi_2) = \frac{1}{\lambda} + x_a \tag{34}$$

and

$$M(\eta_1) = \frac{\lambda}{4}\,(1 + e^{-\lambda x_a}) \tag{35}$$

$$M(\eta_2) = \frac{\lambda}{4}\, e^{-\lambda x_a} \tag{36}$$

The variances are:

$$D^2(\xi_1) = \frac{1}{\lambda^2} - \frac{x_a^2 \, e^{\lambda x_a}}{(e^{\lambda x_a} - 1)^2} \tag{37}$$

and

$$D^2(\xi_2) = \frac{1}{\lambda^2} \, . \tag{38}$$

The expectation $M(\xi_2)$ differs from the expectation $1/\lambda$ of the exponential distribution by x_a, and the variance $D^2(\xi_2)$ corresponds exactly the variance of the exponential distribution.

In Fig. 7 the distribution and density functions of the exponential distribution with the parameter $\lambda = 1$ can be seen. Both parts of the figure show the gravity centre functions, too. Figure 8 presents the two variances. Variance $D^2(\xi_1)$ is a monotonously increasing function, variance $D^2(\xi_2)$ has a constant value.

It is well known that the decay of radioactive atoms follows the exponential distribution. In case of the radium atom the expectation is $1/\lambda = 2279$

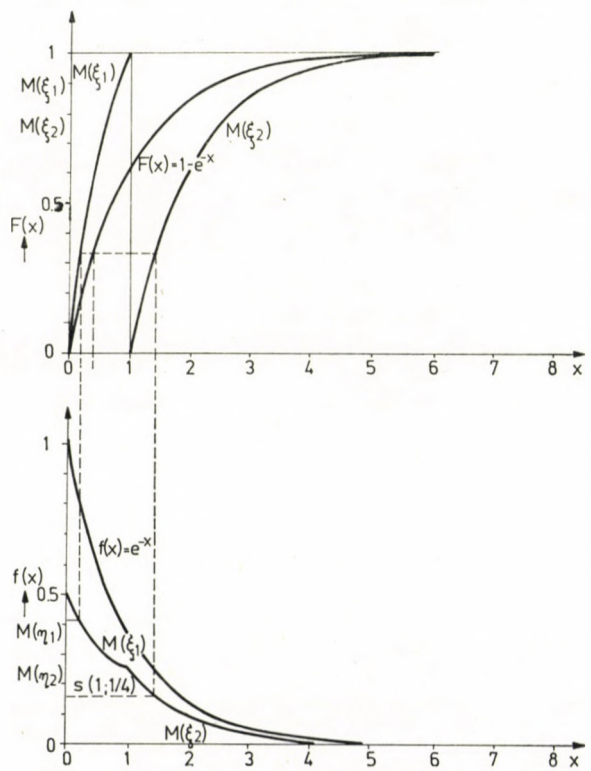

Fig. 7

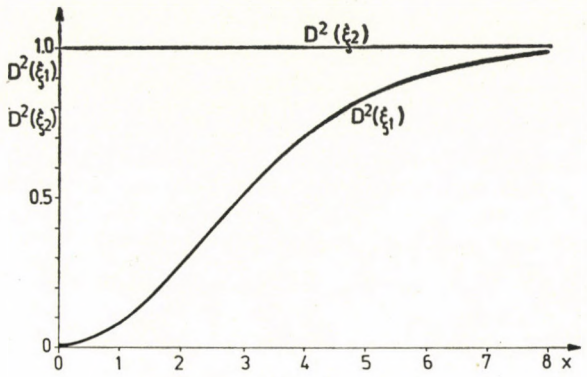

Fig. 8

years, the half-period 1580 years [3]. The expected life of atoms till the half-period ($x_a = \ln 2/\lambda$) is

$$\frac{1}{\lambda} - \frac{\ln 2}{\lambda} = 2279 - 1580 = 699 \text{ years}$$

the life of the not decayed atoms:

$$\frac{1}{\lambda} + \frac{\ln 2}{\lambda} = 2279 + 1580 = 3859 \text{ years.}$$

Their variances are:

$$\frac{1}{\lambda^2} - 2\left(\frac{\ln 2}{\lambda}\right)^2 = 2279^2 - 2 \cdot 1580^2 = 201041,$$

and

$$\frac{1}{\lambda^2} = 2279^2 = 5\,193\,841.$$

6. Weibull distribution

The useful life of very different products or that of their parts can be described by the Weibull distribution with the parameters α and λ [2]:

$$f(x) = \alpha\lambda x\, e^{-\lambda x^{\alpha}}. \tag{39}$$

If $\alpha = 2$ which is often met in the practice, the expectations of the partial

populations are:

$$M(\xi_1) = \frac{\frac{1}{2}\sqrt{\frac{\pi}{\lambda}} \cdot H(x_a\sqrt{\lambda}) - x_a\, e^{-\lambda x_a^2}}{1 - e^{-\lambda x_a^2}} \tag{40}$$

and

$$M(\xi_2) = \frac{\frac{1}{2}\sqrt{\frac{\pi}{\lambda}}\left[1 - H(x_a\sqrt{\lambda})\right] + x_a\, e^{-\lambda x_a^2}}{e^{-\lambda x_a^2}}. \tag{41}$$

Here the function $H(x)$ appears:

$$H(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt \tag{42}$$

its numerical values can be found in tables.

The corresponding ordinates are:

$$M(\eta_1) = \frac{\frac{1}{4}\sqrt{\frac{\lambda\pi}{2}}\, H(x_a\sqrt{2\lambda}) - \frac{\lambda x_a}{2}\, e^{-2\lambda x_a^2}}{1 - e^{-\lambda x_a^2}} \tag{43}$$

and

$$M(\eta_2) = \frac{\frac{1}{4}\sqrt{\frac{\lambda\pi}{2}}\left[1 - H\left(x_a\sqrt{\lambda}\right)\right] + \frac{\lambda x_a}{2}\, e^{-2\lambda x_a^2}}{e^{-\lambda x_a^2}}. \tag{44}$$

The variances of the partial populations are:

$$D^2(\xi_1) = \frac{1}{\lambda} + \frac{x_a^2}{1 - e^{\lambda x_a^2}} - M^2(\xi_1) \tag{45}$$

and

$$D^2(\xi_2) = \frac{1}{\lambda} + x_a^2 - M^2(\xi_2)\,. \tag{46}$$

The expectations and variances for $\lambda = 1$ can be found in Figs 9 and 10.

The Weibull distribution is also used for the description of grain size distributions of granulous materials under the name Rosin—Rammler distribution when separating the granulous material at a size x_a, $M(\xi_1)$ and $M(\xi_2)$ denote the average size of the coarse and fine section.

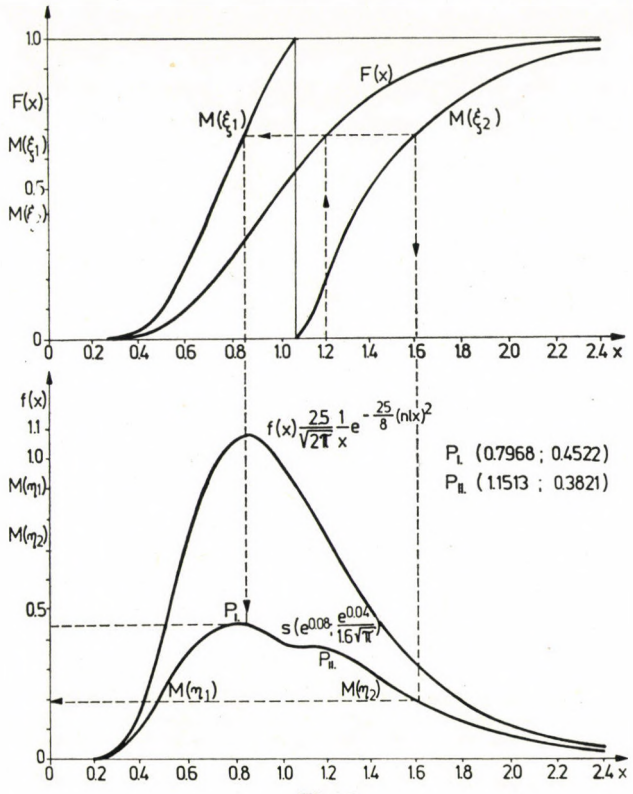

Fig. 9

7. The *n*-th order gamma distribution

The density function of the *n*-th order gamma distribution is [2, 3]:

$$f(x) = \frac{\lambda^n}{(n-1)!} x^{n-1} e^{-\lambda x} . \tag{47}$$

If x_a is the continuously changing limiting value, the expectations of the partial populations are:

$$M(\xi_1) = \frac{n}{\lambda} \frac{1 - e^{-\lambda x_a} \sum_{k=0}^{n} \frac{(\lambda x_a)^{n-k}}{(n-k)!}}{1 - e^{-\lambda x_a} \sum_{k=0}^{n-1} \frac{(\lambda x_a)^{n-k-1}}{(n-k-1)!}} \tag{48}$$

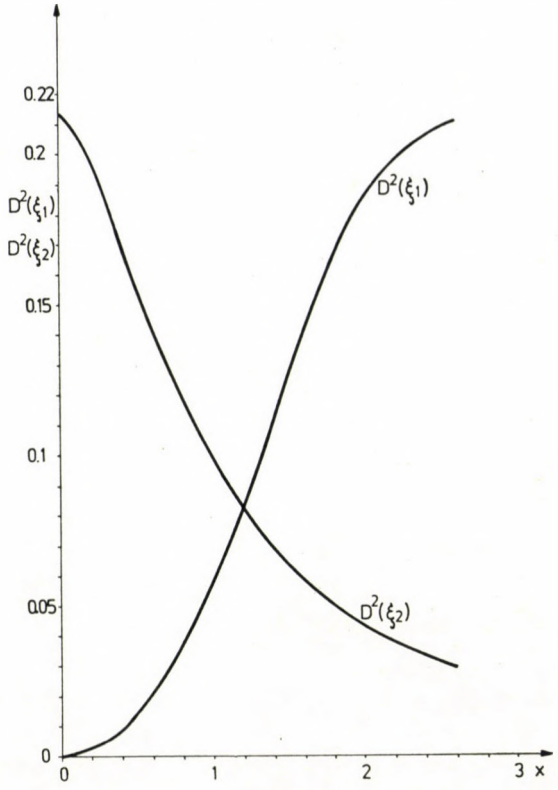

Fig. 10

and

$$M(\xi_2) = \frac{n}{\lambda} \frac{\sum_{k=0}^{n} \frac{(\lambda x_a)^{n-k}}{(n-k)!}}{\sum_{k=0}^{n-1} \frac{(\lambda x_a)^{n-k-1}}{(n-k-1)!}} . \tag{49}$$

The ordinates of the gravity centres are:

$$M(\eta_1) = \frac{\lambda}{4^n} \binom{2n-2}{n-1} \frac{1 - e^{-2\lambda x_a} \sum_{k=0}^{2n-2} \frac{(2\lambda x_a)^{2n-2-k}}{(2n-2-k)!}}{1 - e^{-\lambda x_a} \sum_{k=0}^{n-1} \frac{(\lambda x_a)^{n-1-k}}{(n-1-k)!}} \tag{50}$$

and

$$M(\eta_2) = \frac{\lambda}{4^2} \binom{2n-2}{n-1} e^{\lambda x_a} \frac{\sum_{k=0}^{2n-2} \frac{(\lambda x_a)^{2n-2-k}}{(2n-2-k)!}}{\sum_{k=0}^{n-1} \frac{(\lambda x_a)^{n-1-k}}{(n-1-k)!}} . \tag{51}$$

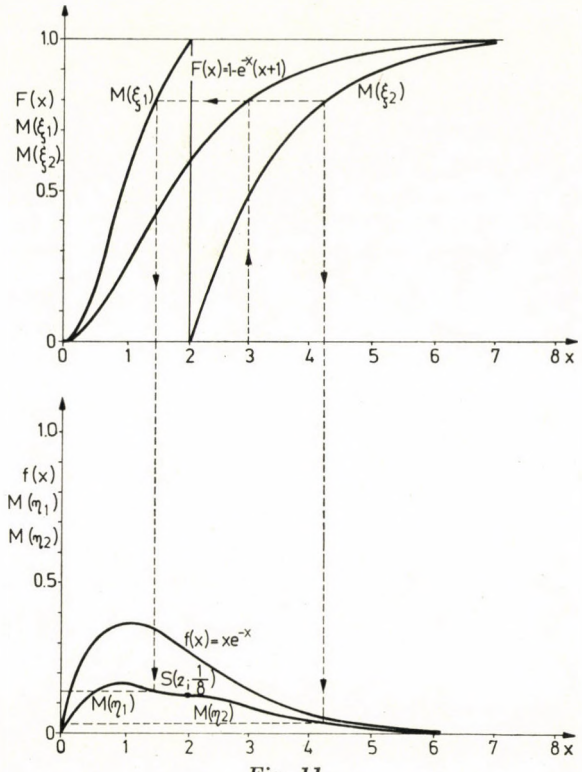

Fig. 11

The variances of the partial populations are:

$$D^2(\xi_1) = \frac{(n+1)n}{\lambda^2} \cdot \frac{1 - e^{-\lambda x_a} \sum_{k=0}^{n+1} \frac{(\lambda x_a)^{n+1-k}}{(n+1-k)!}}{1 - e^{-\lambda x_a} \sum_{k=0}^{n-1} \frac{(\lambda x_a)^{n-1-k}}{(n-1-k)!}} - M^2(\xi_1) \qquad (52)$$

further:

$$D^2(\xi_2) = \frac{(n+1)n}{\lambda^2} \cdot \frac{\sum_{k=0}^{n+1} \frac{(\lambda x_a)^{n+1-k}}{(n+1-k)!}}{\sum_{k=0}^{n-1} \frac{(\lambda x_a)^{n-1-k}}{(n-1-k)!}} - M^2(\xi_2)\,. \qquad (53)$$

The gamma distribution of $n = 1$ order corresponds to the exponential distribution, by substituting $n = 1$ one gets from the Eqs (48)–(53) the parameters of the exponential distribution, Eqs (33)–(37) and (14).

Figure 11 shows the density and distribution function of the gamma distribution $n = 3$ and $\lambda = 1$ and the expectations of the partial populations. In

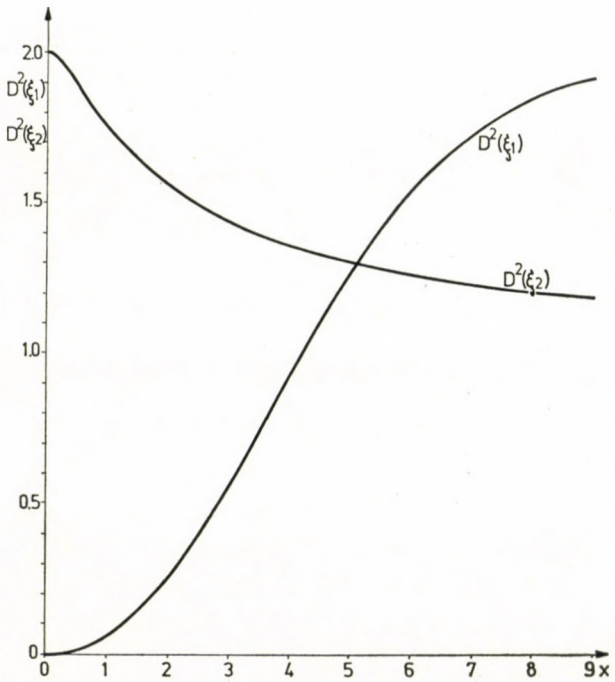

Fig. 12

Fig. 12 the variances for the same parameters are given. The expectations and variances of the gamma distribution with $n = 3$ and $\lambda = 1$ can be found in Figs 13 and 14.

8. Positive order gamma distribution

The distribution function of the positive order gamma distribution [2, 3] is:

$$f(x) = \frac{\lambda^p \cdot x^{p-1}}{\Gamma(p)} e^{-\lambda x}. \tag{54}$$

The gamma function $\Gamma(p)$ appearing in the denominator of the density function can be interpreted over the numbers $p > 0$ with the following integral [2]:

$$\Gamma(p) = \int_0^\infty x^{p-1} \cdot e^{-x} \cdot dx. \tag{55}$$

The abscissae of the gravity centres of the partitions less and more than

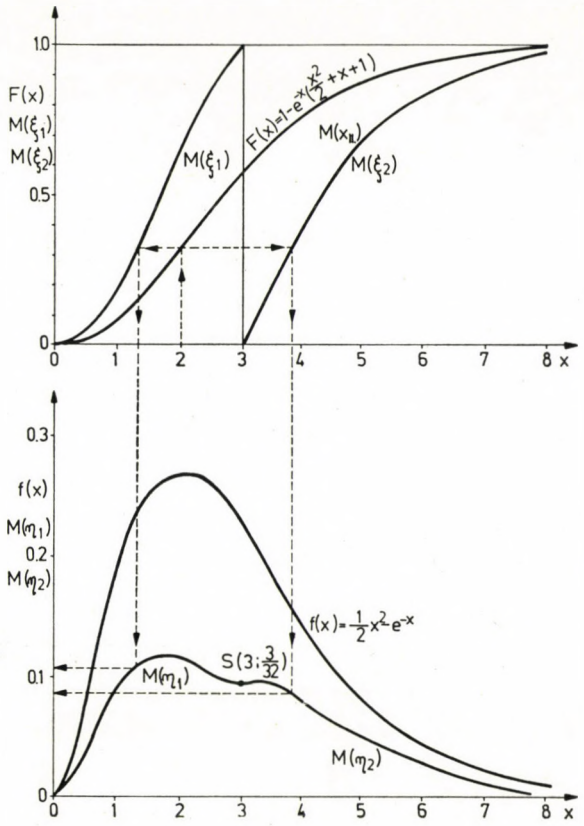
1.0
F(x)
M(ξ1)
M(ξ2)
0.5
0
M(ξ1)
F(x)=1-e^{-x}(½x²+x+1)
M(xR)
M(ξ2)
0 1 2 3 4 5 6 7 8 x
0.3
f(x)
M(η1)
0.2
M(η2)
f(x)=½x²e^{-x}
S(3;3/32)
0.1
M(η1)
M(η2)
0
0 1 2 3 4 5 6 7 8 x

Fig. 13

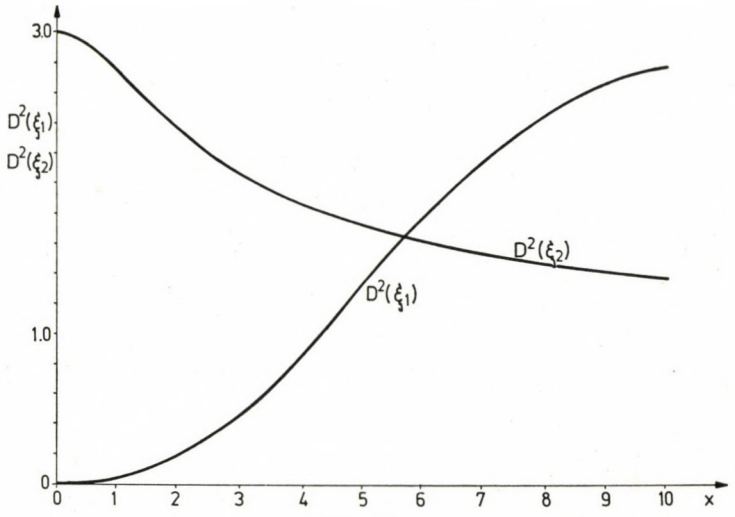
3.0
D²(ξ1)
D²(ξ2)
D²(ξ2)
D²(ξ1)
1.0
0
0 1 2 3 4 5 6 7 8 9 10 x

Fig. 14

x_a can be expressed as follows:

$$M(\xi_1) = \frac{3}{2\lambda} - \frac{\frac{2}{\sqrt{\pi}} \sqrt{\lambda x_a}\ e^{-\lambda x_a} - x_a}{H(\sqrt{\lambda x_a}) - \frac{2}{\pi} \sqrt{\lambda x_a}\ e^{-\lambda x_a}} \tag{56}$$

and

$$M(\xi_2) = \frac{3}{2\lambda} + \frac{x_a}{1 + \frac{\sqrt{\pi}}{2\sqrt{\lambda x_a}} e^{\lambda x_a}[1 - H(\sqrt{\lambda x_a})]}. \tag{57}$$

The ordinates of the gravity centres are:

$$M(\eta_1) = \frac{\lambda\left[1 - e^{-2\lambda x_a}(2\lambda x_a + 1)\right]}{2\pi\left[H(\sqrt{\lambda x_a}) - \frac{2}{\sqrt{\pi}} \sqrt{\lambda x_a}\, e^{-\lambda x_a}\right]} \tag{58}$$

and

$$M(\eta_2) = \frac{\frac{1}{2\pi} e^{-2\lambda x_a}(1 + 2\lambda x_a)}{\frac{2}{\sqrt{\pi}} \sqrt{\lambda x_a}\, e^{-\lambda x_a} + 1 - H(\sqrt{\lambda x_a})}. \tag{59}$$

The variances of partial populations are:

$$D^2(\xi_1) = \frac{\frac{15}{4\lambda^2} H(\sqrt{\lambda x_a}) - \frac{2}{\sqrt{\pi}} \sqrt{\lambda x_a}\, e^{-\lambda x_a} \cdot \left(x_a^2 + \frac{2.5 x_a}{\lambda} + \frac{15}{4\lambda^2}\right)}{H(\sqrt{\lambda x_a}) - \frac{2}{\sqrt{\pi}} \sqrt{\lambda x_a}\, e^{-\lambda x_a}} - M^2(\xi_1) \tag{60}$$

further:

$$D^2(\xi_2) = \frac{15}{4\lambda^2} + \frac{\frac{2}{\sqrt{\pi}} \sqrt{\lambda x_a}\, e^{-\lambda x_a}\left(x_a^2 + \frac{2.5\, x_a}{\lambda}\right)}{\frac{2}{\sqrt{\pi}} \sqrt{\lambda x_a}\, e^{-\lambda x_a} + 1 - H(\sqrt{\lambda x_a})} - M^2(\xi_2). \tag{61}$$

The positive order gamma distribution can be characterized for the case $x > 0$, if the coefficient $\lambda = 1$ and the exponent $p = 3/2$, by the following density function:

$$f(x) = \frac{2}{\sqrt{\pi}} \cdot \sqrt{x} \cdot e^{-x}. \tag{62}$$

The abscissae of the expectations of partial populations more or less of a prescribed value x_a can be written as follows:

$$M(\xi_1) = \frac{3}{2} - \frac{\frac{2}{\sqrt{\pi}} \sqrt{x_a} \cdot e^{-x_a} \cdot x_a}{H\left(\sqrt{x_a}\right) - \frac{2}{\sqrt{\pi}} \sqrt{x_a}\, e^{-x_a}} \tag{63}$$

and

$$M(\xi_2) = \frac{3}{2} + \frac{\frac{2}{\sqrt{\pi}} \sqrt{x_a} \cdot e^{-x_a} \cdot x_a}{1 + \frac{2}{\sqrt{\pi}} \cdot \sqrt{x_a}\, e^{-x_a} - H\left(\sqrt{x_a}\right)} \cdot \tag{64}$$

The ordinates of the expectations are:

$$M(\eta_1) = \frac{1 - e^{-2x_a}(2x_a + 1)}{2\pi\left[H\left(\sqrt{x_a}\right) - \frac{2}{\sqrt{\pi}} \sqrt{x_a} \cdot e^{-x_a}\right]} \tag{65}$$

and

$$M(\eta_2) = \frac{e^{-2x_a}(2x_a + 1)}{2\pi\left[1 + \frac{2}{\sqrt{\pi}} \sqrt{x_a}\, e^{-x_a} - H\left(\sqrt{x_a}\right)\right]} \cdot \tag{66}$$

The variances are:

$$D^2(\xi_1) = \frac{15}{4} - \frac{e^{-x_a}\sqrt{\frac{x_a}{\pi}}(2x_a^2 + 5x_a)}{H\left(\sqrt{x_a}\right) - \frac{2}{\sqrt{\pi}} \sqrt{x_a}\, e^{-x_a}} - M^2(\xi_1) \tag{67}$$

and

$$D^2(\xi_2) = \frac{15}{4} + \frac{e^{-x_a}\sqrt{\frac{x_a}{\pi}}(2x_a^2 + 5x_a)}{1 + \frac{2}{\sqrt{\pi}} e^{-x_a} - H\left(\sqrt{x_a}\right)} - M^2(\xi_2)\,. \tag{68}$$

The function $H(x)$ appears in these equations, too.

For the previous parameters ($\lambda = 1$, $p = 3/2$), the expectations are shown in Fig. 15, the variances in Fig. 16.

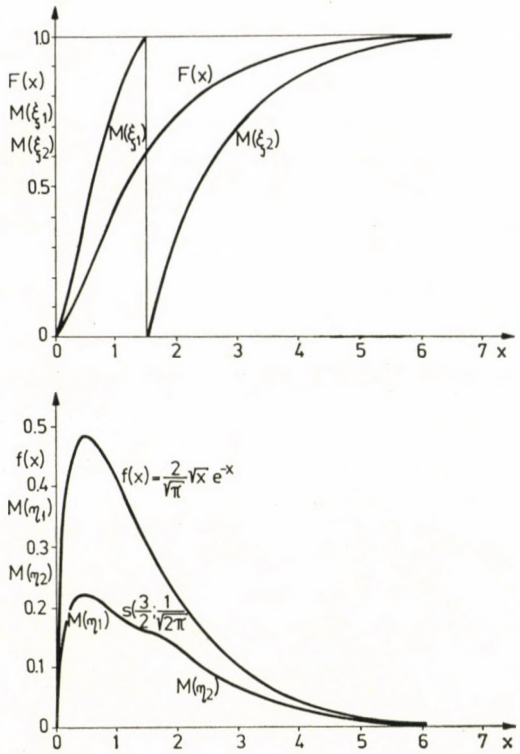
1.0
F(x)
M(ξ1)
M(ξ2)
0.5
0
F(x)
M(ξ1)
M(ξ2)
0 1 2 3 4 5 6 7 x
0.5
f(x)
0.4
M(η1)
0.3
M(η2)
0.2
0.1
0
f(x)=2/√π √x e^-x
M(η1)
s(3/2 · 1/√2π)
M(η2)
0 1 2 3 4 5 6 7 x

Fig. 15

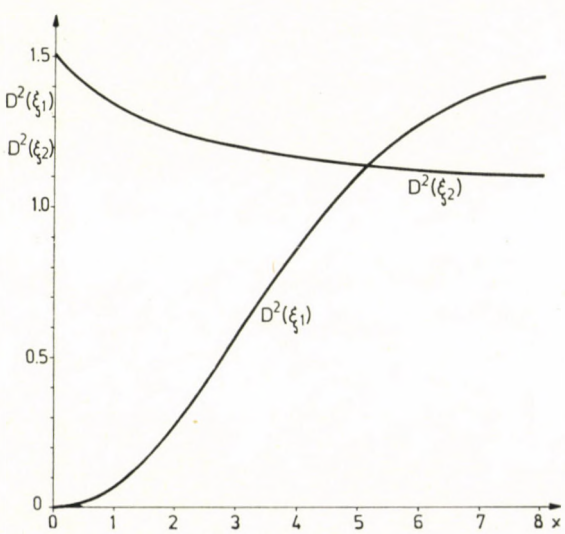
1.5
D²(ξ1)
D²(ξ2)
1.0
0.5
0
D²(ξ2)
D²(ξ1)
0 1 2 3 4 5 6 7 8 x

Fig. 16

9. Conclusion

The calculations presented in this paper yielded expectations and variances for the partitions of some simple distributions. In knowledge of the corresponding functions, the variations of the parameters have been graphically presented. These parameters can also be presented in tabular form with any small steps in order to facilitate their practical use (see Table I).

REFERENCES

1. Pethő, Sz.—Patvaros, J.: On the expectance and variances of partial populations. *Minőség és megbízhatóság*, No. II, 1975, 117—120 (in Hungarian).
2. Vincze, I.: Mathematical statistics with industrial applications. Műszaki Könyvkiadó, Budapest 1968 (in Hungarian).
3. Prékopa, A.: Theory of probability with technical applications. Műszaki Könyvkiadó, Budapest 1972 (in Hungarian).
4. Tarján, G.: Ore dressing I. (Manuscript) Tankönyvkiadó, Budapest 1969 (in Hungarian).
5. Raisz, I.: Report for the Mining Research Group of the Hungarian Academy of Sciences (Manuscript) Miskolc 1974 (in Hungarian).

ЧАСТНЫЕ МНОЖЕСТВА И ПАРАМЕТРЫ ФУНКЦИЙ РАСПРЕДЕЯЛЕНИ

С. ПЕТЕ—Й. ПАТВАРОШ—Л. БЕНКЕ

РЕЗЮМЕ

В связи с функциями распределения часто встает вопрос о познании математических ожиданий и дисперсий частных, заключающих в себе значения, меньшие или большие данного предела случайной величины, если предел меняется непрерывно. В связи с вставленной проблемой в статье можно найти параметры общего нормального, нормального с нулевым математическим ожиданием и дисперсией σ^2, стандартного нормального, логарифмически нормального, экспоненциального, Вейбуллского, гамма- и положительного порядка гамма-распределений. В статье указывается и на физическое и техническое содержание параметров.

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COMPUTATION OF GEODESICS FROM CHORD-LENGTHS

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[Manuscript received May 5 1976]

The authors give new equations (Eq. (37)) for computing the length of arc of a geodesic from the chord-length, the latitude of the point P_1, and the azimuth of the direction. In addition they give Eq. (40) which serves to calculate distances along normal sections. Finally some examples are worked out to demonstrate the theoretical results.

1. Introduction

In the field of satellite geodesy it is one of the main tasks to determine the chord-lengths of earth. This task plays a significant role, e.g. at the determination by VÄISÄLÄ's up-to-date method of the position of points on Earth. Distances usually encountered are in the range of 100 km. When determining the positions of points on the Earth via satellite measurements, however, the distances concerned are several thousands of kilometres long.

The density of the base network established through satellite geodesy can be increased by satellite observations making use of the Doppler-effect. The distance between the points to be determined is again several hundred kilometres long. The linking of existing geodetic networks (e.g. the geodetic networks of the socialist countries) is increasingly depending on the determination through satellite measurements of the directions and distances assuring the rigidity. All these problems make necessary to improve the accuracy of calculations of arc lengths from chord lengths.

In an earlier work [1] the connection was examined between calculations giving distances, which can be derived using satellite observations and geographical coordinates. One problem, however, remained unsolved: the determination of the length (s) of a geodesic appearing in the reverse problem of geodesy, from the length of chord (k) calculated from geodetic satellite measurements. The equations known from the literature do not have the accuracy necessary for our problem.

2. Connection between the length of arc of a geodesic and the length of chord

To determine the relation

$$s = f(k, \eta^2, \cos^2 A, t) \tag{1}$$

with sufficient accuracy it is necessary to derive some basic quantities with greater accuracy than that known in the literature. Splitting k from the power series form of the right-hand side of (1) we get

$$s = k\left\{1 + a\frac{k^2}{N^2} + b\frac{k^3}{N^3} + c\frac{k^4}{N^4} + d\frac{k^5}{N^5} + e\frac{k^6}{N^6} + \ldots\right\} \tag{2}$$

where s is the length of the geodesic between P_1 and P_2, A is the azimuth of the same geodesic, N is radius of curvature in prime vertical referring to point P_1, and t is the tangent of this point's latitude.

The term (k/N_1) can be approximated in the first order by a central angle σ. For example if k is equal to 3000 km $(k/N_1 \doteq 0.5)$ then $(k/N_1)^6 \doteq 0.016$. Later it will be seen that η_1^2 appears in (2) as a factor in the terms $b, d, \ldots$, but among the terms $a, c, e \ldots$ there are some which do not contain η_1^2. Consequently those terms which are of higher order than e, can be entirely neglected. In the expressions of d and e those terms containing η_1^4 will be omitted. In the case of c, however, all terms containing η_1^6, or some higher power of η_1^4, will be neglected. In the terms a and b nothing will be omitted. In order to determine the relation (2), some other quantities should be derived first.

2.1 *Supplementary relations describing the angle of depression*

To determine Eq. (2), the power series of the angle of depression has to be formulated. From the known relation

$$\tan(A_{12} - B_{12}) = \frac{\sin A \tan\mu \tan\delta}{1 + \cos A \tan\mu \tan\delta} \tag{3}$$

and using the equations

$$\begin{aligned} A_{12} - B_{12} = {} & \sin A \tan\mu \tan\delta - \sin A \cos A \tan^2\mu \tan^2\delta + \\ & + \sin A \tan^3\mu \tan^3\delta(\cos^2 A - \frac{1}{3}\sin^2 A) \end{aligned} \tag{4}$$

and

$$\tan\mu = \frac{1}{\sin\sigma}\left(\frac{N_1}{S_2} - \cos\sigma\right) \tag{5}$$

— where δ is the angle of the directions determined by P_1 and the points of intersection on the axis of rotation of the two normals N_1 and N_2 (K_1 and K_2, respectively) and S_2 is equal to the distance P_2K_1 it follows that

$$\tan\mu = \frac{1}{2}\sigma(1+\eta^2\cos^2 A) - \frac{t}{6}\eta^2\cos A\,\sigma^2\left[3+\eta^2\cos^2 A\,(1-12\,t^2)\right] + \\ + \frac{1}{24}\sigma^3\left[1+\eta^2(3t^2-2\cos^2 A)\right] + \frac{t}{24}\eta^2\cos A\,\sigma^4(1-3\cos^4 A) \tag{6}$$

where σ is equal to the angle at K_1 between the directions K_1P_1 and K_1P_2.

From formula (6) giving the value of $\tan\mu$ and with the help of the equation

$$\mu = \tan\mu - \frac{1}{3}(\tan\mu)^3 + \frac{1}{5}(\tan\mu)^5 \tag{7}$$

the power series for μ becomes

$$\mu = \frac{1}{2}\sigma(1+\eta^2\cos^2 A) - \frac{1}{6}t\eta^2\cos A\sigma^2\left[3+\eta^2\cos^2 A\,(1-12\,t^2\right] + \\ + \frac{1}{24}\sigma^3\eta^2(3t^2-5\cos^2 A) + \frac{1}{24}t\eta^2\cos A\sigma^4(4- \\ - 3\cos^4 A + 7\eta^2\cos^2 A) + \frac{1}{96}\sigma^5\eta^2(\cos^2 A - 3t^2)\,. \tag{8}$$

2.2 *Supplementary relations describing the central angle*

From the extended form of the equation in [2] (p. 21, (5)) the following relation can be derived:

$$\sigma = \frac{s}{N} + \frac{1}{6}\eta^2\cos^2 A(1-\eta^2\cos^2 A)\frac{s^3}{N^3} - \frac{1}{24}t\eta^2\cos A^2\left[3- \\ - \eta^2\cos^2 A\,(8+12\,t^2)\right]\frac{s^4}{N^4} + \frac{1}{120}\eta^2(3t^2-4\cos^2 A)\frac{s^5}{N^5} + \\ + \frac{1}{48}t\eta^2\cos A\,(1-\cos^4 A)\frac{s^6}{N^6}\,. \tag{9}$$

2.3 *Supplementary relations describing the difference of the central angle and the angle of depression*

From Eq. (8) the difference of the central angle and the angle of depression can be determined, and, using the equation

$$\sec(\sigma-\mu)=1+\frac{1}{2}(\sigma-\mu)^2+\frac{5}{24}(\sigma-\mu)^4+\ldots \tag{10}$$

we have

$$\begin{aligned}\sec(\sigma-\mu)=1&+\frac{1}{8}\sigma^2\left[1-\eta^2\cos^2 A(2-\eta^2\cos^2 A)\right]+\\&+\frac{1}{12}t\eta^2\cos A\,\sigma^3\left[3-2\eta^2\cos^2 A(1+6t^2)\right]+\\&+\frac{1}{384}\sigma^4\left[5+20\,\eta^2\cos^2 A-24t^2\eta^2+8\eta^4\cos^2 A(9t^2-\right.\\&\left.-5\cos^2 A)\right]+\frac{1}{96}t\eta^2\cos A\,\sigma^5(-3+6\cos^4 A)\,.\end{aligned} \tag{11}$$

2.4 *The length of chord as function of the central angle*

We start out from the equation ([2] p. 11, (17)):

$$\frac{k}{N}=\sin\sigma\cdot\sec(\sigma-\mu)\,. \tag{12}$$

Using this relation we get

$$\sin\sigma=\sigma-\frac{1}{6}\sigma^3+\frac{1}{120}\sigma^5-\ldots \tag{13}$$

Multiplying Eqs (11) and (13) terms by terms we get

$$\begin{aligned}\frac{k}{N}=\sigma\Big\{1&-\frac{1}{24}\sigma^2\left[1+3\eta^2\cos^2 A(2-\eta^2\cos^2 A)\right]+\\&+\frac{1}{12}t\eta^2\cos A\,\sigma^3\left[3-2\eta^2\cos^2 A(1+6t^2)\right]+\\&+\frac{1}{1920}\sigma^4\left[1+180\eta^2\cos^2 A-120t^2\eta^2+40\eta^4\cos^2 A(9t^2-\right.\\&\left.-6\cos^2 A)\right]-\frac{1}{96}t\sigma^5\eta^2\cos A\,(7-6\cos^2 A)\Big\}\,.\end{aligned} \tag{14}$$

To increase the range of validity of our equation to be derived we have to retain the term of sixth order appearing in braces in (14). As the series we had started with are not so accurate and the building up of the series by this method would be rather tiresome, we shall choose an interesting new possibility.

Let us consider the series (14) expanded to the order of σ^5. If we take a normal section, the azimuth of which is equal to 90° (that is, instead of the ellipse we consider a principal circle), all terms of an odd power will vanish. Designating the length of chord by k_k we get

$$k_k = \sigma\left\{1 - \frac{1}{24}\sigma^2 + \frac{1}{1920}\sigma^4\right\}. \tag{15}$$

This equation, however, can be more exactly formulated, if we use the equation of a chord belonging to a central angle σ of a circle of radius R:

$$\frac{k_k}{2} = R \sin\frac{\sigma}{2}. \tag{16}$$

The half length of the arc of the circle is equal to:

$$\frac{s_k}{2} = R\frac{\sigma}{2}. \tag{17}$$

We can get the more accurate series if, instead of $\sin(\sigma/2)$ we use the power series of this term. Then we get

$$\frac{k_k}{R} = 2\sin\frac{\sigma}{2} = \sigma\left\{1 - \frac{1}{24}\sigma^2 + \frac{1}{1920}\sigma^4 - \frac{1}{322560}\sigma^6\right\}. \tag{18}$$

Thus, we get a more accurate relation if in building Eq. (14) we take into account from Eq. (18) the last term

$$-\frac{1}{322560}\sigma^6 \tag{19}$$

too.

2.5 *The length of chord as a function of the geodesic*

The equation for the length of chord as a function of the geodesic can be derived in the following way: Let us substitute into (14) extended by (19)

the following powers of σ, using Eq. (9):

$$\sigma^2 = \frac{s^2}{N^2} + \frac{\eta^2}{3}\cos^2 A(1 - \eta^2\cos^2 A)\frac{s^4}{N^4} - \frac{1}{12}t\eta^2\cos A\left[3 - \right.$$
$$\left. - \eta^2\cos^2 A(8 + 12t^2)\right]\frac{s^5}{N^5} \tag{20}$$

$$\sigma^3 = \frac{s^3}{N^3} + \frac{1}{2}\eta^2\cos^2 A\frac{s^5}{N^5} \tag{21}$$

$$\sigma^4 = \frac{s^4}{N^4} \tag{22}$$

$$\sigma^5 = \frac{s^5}{N^5}\,. \tag{23}$$

Using Eq. (9) for σ, and separating the factor s/N we get

$$\frac{k}{N} = \frac{s}{N}\left\{1 + \frac{1}{6}\eta^2\cos^2 A(1 - \eta^2\cos^2 A)\frac{s^2}{N^2} - \frac{1}{24}t\eta^2\cos A\left[3 - \right.\right.$$
$$\left. - \eta^2\cos^2 A(8 + 12t^2)\right]\frac{s^3}{N^3} + \frac{\eta^2}{120}(3t^2 - 4\cos^2 A)\frac{s^4}{N^4} +$$
$$\left. + \frac{1}{48}t\eta^2\cos A(1 - \cos^4 A)\frac{s^6}{N^6}\right\}\left\{1 - \frac{1}{24}\left[1 + 3\eta^2\cos^2 A(2 - \right.\right. \tag{24}$$
$$\left. - \eta^2\cos^2 A)\right]\frac{s^2}{N^2} + \frac{1}{12}t\eta^2\cos A\left[3 - 2\eta^2\cos^2 A(1 + 6t^2)\right]\frac{s^3}{N^3} +$$
$$+ \frac{1}{5760}\left[3 + 460\eta^2\cos^2 A - 360\eta^2t^2 - 4\eta^4\cos^2 A(280\cos^2 A - \right.$$
$$\left.\left. - 27t^2)\right]\frac{s^4}{N^4} - \frac{1}{288}t\eta^2\cos A(17 - 18\cos^2 A)\frac{s^5}{N^5}\right\}.$$

After performing the necessary operations we get

$$k = s\left\{1 - \frac{1}{24}\left[1 + \eta^2\cos^2 A(2 + \eta^2\cos^2 A)\right]\frac{s^2}{N^2} + \frac{1}{8}t\eta^2\cos A(1 + \right.$$
$$+ 2\eta^2\cos^2 A)\frac{s^3}{N^3} + \frac{1}{1920}\left[1 - 72t^2\eta^2 + 76\eta^2\cos^2 A + \right.$$
$$\left. + 36\eta^4\cos^2 A(t^2 - 10\cos^2 A)\right]\frac{s^4}{N^4} - \frac{1}{576}\eta^2t\cos A(19 - \tag{25}$$
$$\left. - 36\cos^2 A + 12\cos^4 A)\frac{s^5}{N^5} - \frac{1}{322560}\frac{s^6}{N^6}\right\}.$$

As a reference, we quote the corresponding equation from [2] (p. 12, (20)):

$$k = s\left\{1|- \frac{1}{24}\left[1 - 2\eta^2 \cos^2 A\right]\frac{s^2}{N^2} + \frac{1}{8}t\eta^2 \cos A \frac{s^3}{N^3} + \frac{1}{1920}\frac{s^4}{N^4}\right\}. \quad (26)$$

2.6 *Relation between the geodesic and the length of chord*

Using the power series form of the equation

$$k = f(s, \eta^2, t, \cos^2 A) \quad (27)$$

let us formulate the power series form of the function

$$s = f(k, \eta^2, t, \cos^2 A) \quad (28)$$

using the equation

$$s = \frac{k}{1+a} = k(1 - a + a^2 - \ldots). \quad (29)$$

It can be seen that the value of a in (29) is equal to the quantity in the outermost braces of (25), if we omit the term "1". After performing in (29) the necessary operations we get

$$\frac{s}{N} = \frac{k}{N}\left(1 + c\frac{s^2}{N^2} + d\frac{s^3}{N^3} + e\frac{s^4}{N^4} + f\frac{s^5}{N^5} + g\frac{s^6}{N^6}\right). \quad (30)$$

The symbols used are

$$c = \frac{1}{24}\left[1 + \eta^2 \cos^2 A(2 + \eta^2 \cos^2 A)\right] \quad (31)$$

$$d = \frac{-1}{8}t\eta^2 \cos A(1 + 2\eta^2 \cos^2 A) \quad (32)$$

$$e = \frac{1}{5760}\left[7 + 216t^2\eta^2 - 188\eta^2 \cos^2 A + \right.$$
$$\left. + \eta^4 \cos^2 A(1100 \cos^2 A - 108t^2)\right] \quad (33)$$

$$f = \frac{1}{576}\eta^2 t \cos A(13 - 36 \cos^2 A + 12 \cos^4 A) \quad (34)$$

$$g = \frac{31}{967680}. \quad (35)$$

From this formula we have to derive a power series where, instead of the different powers of s, different powers of k appear. After a lengthy procedure, omitted here, using the symbols of Eq. (30) we get

$$\frac{s}{N} = \frac{k}{N} + c\frac{k^3}{N^3} + d\frac{k^4}{N^4} + (e + 2c^2)\frac{k^5}{N^5} + (f + 5cd)\frac{k^6}{N^6} + \\ + (5c^3 + 3d^2 + 6ce + g)\frac{k^7}{N^7} . \tag{36}$$

Substituting in (36) the respective expression of the symbols we get the relation between the geodesic and the length of chord:

$$s = k\left\{1 + \frac{1}{24}(1 + \eta^2\cos^2 A)^2\frac{k^2}{N^2} - \frac{1}{8}\eta^2 t\cos A(1 + \right. \\ + 2\eta^2\cos^2 A)\frac{k^3}{N^3} + \frac{1}{5760}[27 + 216t^2\eta^2 - 108\eta^2\cos^2 A + \\ + 4\eta^4\cos^2 A(275\cos^2 A - 27t^2)]\frac{k^4}{N^4} - \frac{1}{288}\eta^2 t\cos A(1 + \\ + 18\cos^2 A - 6\cos^4 A)\frac{k^5}{N^5} + \frac{1}{322560}[225 + \eta^2(3024t^2 - \\ \left. - 2366\cos^2 A)]\frac{k^6}{N^6}\right\} . \tag{37}$$

The above relation enables us to calculate from a given length of chord the length of geodesic. To use Eq. (37), of course, we have to know the length of chord k, the approximate latitude of the starting point (because of η^2 and t), and the approximate azimuth of the normal section A.

3. Relation between the length of arc in the normal section and the length of chord

In addition to the tasks given at the end of the Introduction we sketch here for sake of completeness the relation between the length of arc in the normal section and the length of chord.

Let us start with the equations [1] (63—66). Rearranging Eq. (63) we get

$$S_{12} = s\left\{1 + b_4\frac{s^4}{N^4} + b_5\frac{s^5}{N^5} + b_6\frac{s^6}{N^6}\right\} . \tag{38}$$

For the higher powers of s in the braces we may substitute the corresponding powers of k. We substitute the reciprocal of this equation

$$s = f(S_{12}, \eta^2, t, \cos A) \tag{39}$$

into the left-hand side of Eq. (37), and after solving for S this equation we have

$$\begin{aligned} S_{12} = k\Big\{1 + \frac{1}{24}(1 + \eta^2\cos^2 A)^2\frac{k^2}{N^2} - \frac{1}{8}\eta^2 t\cos A(1 + \\ + 2\eta^2\cos^2 A)\frac{k^3}{N^3} + \frac{1}{5760}\left[27 + 216t^2\eta^2 - 108\eta^2\cos^2 A + \right. \\ \left. + 4\eta^4\cos^2 A(106 + 259\cos^2 A - 27t^2)\right]\frac{k^4}{N^4} - \\ - \frac{1}{288}\eta^2 t\cos A(1 + 18\cos^2 A - 6\cos^4 A)\frac{k^5}{N^5} + \\ + \frac{1}{322560}\left[225 + \eta^2(3024t^2 - 2366\cos^2 A)\right]\frac{k^6}{N^6}\Big\}. \end{aligned} \tag{40}$$

An approximative solution for (40) can be found in [3] (p. 74) and in [4] (p. 54, (12)).

4. Example of calculation of arc-length from chord-length

To verify our theoretical procedures we solve here a problem in which a geodesic of length of 6000 km is considered ([5] (pp. 342—344), and [1] (p. 12)).

The two end-points are determined by the following data:

$$\begin{aligned} \varphi_1 &= 48^\circ\,50'\,11.0'' = 48.836\,388^\circ \\ \varphi_2 &= 40^\circ\,45'\,23.0'' = 40.756\,388^\circ \\ \Delta\lambda_{21} &= 76^\circ\,18'\,40.0'' = 76.311\,111^\circ \\ A &= 291^\circ\,51'\,20.749\,605'' = 291.855\,7637^\circ. \end{aligned} \tag{41}$$

From these data we get

$$\begin{aligned} k &= 5645\,982.2985\ \text{m} \\ \eta^2 &= 0.002\,932\,2578 \\ t &= +1.143\,755\,671 \\ \cos A &= +0.372\,271\,32 \end{aligned} \tag{42}$$

hence

$$\frac{s}{k} = 1.035\ 775\ 5829\ . \tag{43}$$

Transporting k to the left-hand side of Eq. (37) we have on the right-hand side the outer braces only, in which the values of the terms containing different powers of k/N are the following:

term of second order:	+0.032 549 2461
term of third order:	−0.000 107 7105
term of fourth order:	+0.002 938 8771
term of fifth order:	−0.000 007 8855
term of sixth order:	+0.000 347 4027

(44)

Increasing by one the sum of these terms (see Eq. (37)) we have the desired value of s/k:

$$\frac{s}{k} = 1.035\ 719\ 9299\ . \tag{45}$$

The deviation from the value of (43) is

$$\left(\frac{\Delta s}{k}\right) = 0.000\ 055\ 6530\ . \tag{46}$$

Multiplying by k the value of (45) we get the deviation of Eq. (37) from the exact value

$$\Delta s = 314.22\ \text{m}. \tag{47}$$

We will solve the problem for computation given here using another approximative method. Using for the points 1 and 2 Eq. (4) in THOMSON—VANIČEK's work [6] (p. 310), and then using Eq. (3) of the same work we calculate the mean radius, i.e. on the basis of the equation

$$s = 2R \arcsin\left(\frac{k_0}{2R}\right) \tag{48}$$

we get

$$s = 5848\ 391.18\ \text{m}. \tag{49}$$

The deviation from the exact value is

$$\Delta s = -420.57\ \text{m} \tag{50}$$

being somewhat greater than the value of (47).

If we take an example for a shorter length of arc, the quantity Δs will, of course, be significantly smaller. For example, using data of JORDAN—EGGERT [2] (p. 68, (11)) in case of a distance along normal section of $s = 1320$ km we get

$$\Delta s = 0.06 \text{ m}. \tag{51}$$

It can be seen from the given examples that at $s = 6000$ km the relative error 1 : 18 000 relating to (47) will at $s = 1320$ km be reduced to the relative error 1 : 23 000 000 relating to (51).

If in case of arc-lengths greater than 3000 km the accuracy of about 1 m is not sufficient then — instead of calculating higher terms of the related expression — the best method is to determine the exact solution of the reverse problem of geodesy.

The satellite technology of today using optical and laser instruments makes it possible to determine chord-lengths with an accuracy of less than about a meter, so it is justified to develop methods by which the problem can be solved with the accuracy mentioned. Research is now already being carried out to develop satellite laser technics the accuracy of which lays in the decimetre range.

REFERENCES

1. HALMOS, F.—SZÁDECZKY-KARDOSS, GY.: Azimut és távolságszámítás szimultán szatellita megfigyelésekből, földrajzi koordináták felhasználásával (Calculation of the azimuth and distance along normal sections from simultaneous satellite observations, using geographical coordinates). *Geod. és Kart.*, 27 (1975), 7—14.
2. JORDAN, W.—EGGERT, O.: Handbuch der Vermessungskunde. Bd. 3, Halbband 2, 9. edition, Metzlersche Verlag, Stuttgart, 1948.
3. BAESCHLIN, C. F.: Lehrbuch der Geodäsie. Orell Füsli Verlag, Zürich, 1948.
4. RINNER, K.: Über die Reduktion großer elektronisch gemessener Entfernungen. *Z. f. Vermessungswesen*, 81 (1956), 47—55.
5. SZÁDECZKY-KARDOSS, GY.: A felsőgeodézia számítási alapfeladatainak korszerűsítése (Making up-to-date computations of coordinates in geodesy). Dissertation, manuscript, Sopron, 1969.
6. THOMSON, D. B.—VANIČEK, P.: Note on the reduction of spatial distances to a reference ellipsoid. *Survey Review*, 22 (1974), 309—312.

ПЕРЕВЕДЕНИЕ ДЛИН ХОРД В ДЛИНЫ ДУГ ПО ГЕОДЕЗИЧЕСКОЙ ЛИНИИ

Ф. ХАЛМОШ—ДЬ. САДЕЦКИ-КАРДОШ

РЕЗЮМЕ

Авторы выводят новые формулы для переведения длин хорд в длины дуг по геодезической линии (формула 37), если заданы длины хорд и географические широты исходных точек, а также азимуты направлений к конечным точкам. В дальнейшем приводится формула (40) для вычисления длины дуги нормального сечения. Наконец, практическое применение теоретических исследований показывается на числовых примерах.

Acta Geodaet., Geophys. et Montanist. Acad. Sci. Hung. Tomus 13 (1—2), pp. 107—114 (1978)

DAS KOSTENMINIMUM BEI EBENEM SAMMELN

J. ZAMBÓ

O. MITGLIED DER UNGARISCHEN AKADEMIE DER WISSENSCHAFTEN

[Eingegangen am 12. Mai, 1976]

Der gestellten Aufgabe gemäß soll Material von verschiedenen Mengen aus gegebenen Punkten eines praktisch als eben zu betrachtenden Geländes in einem gemeinsamen Punkt so gesammelt werden, daß dabei die Transportkosten ein Minimum betragen. Nach Angabe der allgemeinen Lösung erörtert der Verfasser auch die Behandlung von Spezialfällen, und führt als Beispiele einfache Fälle aus der Praxis vor. Jede andere, diesen ähnliche Aufgabe kann sinngemäß auf einfache Weise gelöst werden.

In der Ebene sei eine Anzahl n von Punkten gegeben, und zu jedem dieser Punkte gehöre ein Gewicht (Abb. 1). Die Summe der zu einem beliebigen Punkt $P(x, y)$ gehörigen gewichteten Entfernungen ist dann

$$S = \sum_1^n Q_i s_i = \sum_1^n Q_i \sqrt{(x - x_i)^2 + (y - y_i)^2}$$

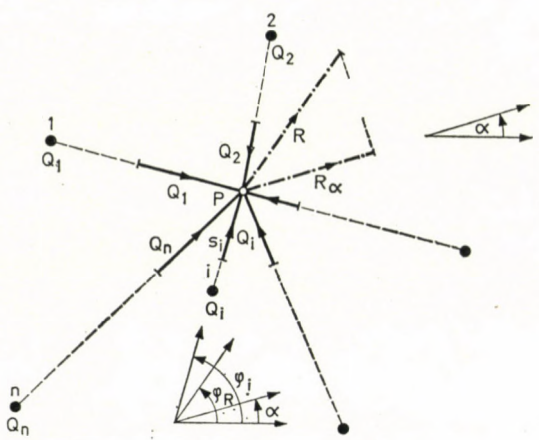


Abb. 1

und die beiden Derivaten lauten demzufolge:

$$\frac{\partial S}{\partial x} = \sum_1^n Q_i \cos \varphi_i$$

$$\frac{\partial S}{\partial y} = \sum_1^n Q_i \sin \varphi_i$$

Dies besagt, daß die beiden Derivaten die nach x bzw. y gerichteten Projektionen der Resultierenden R sind, wobei R die Resultierende des im Punkt P angreifenden Vektorsystems ist, wenn das Gewicht (Q_i) den skalaren Wert der Vektoren bezeichnet, die Vektorrichtung aber durch den gegebenen Punkt (i) und den beliebigen Punkt P gegeben ist, ihr Sinn aber zum Punkt P hinweist.

Wird nun das Koordinatensystem um einen beliebigen Winkel α verschwenkt, so kann einfach nachgewiesen werden, daß in der durch den beliebigen Winkel α bezeichneten Richtung die Derivate

$$R_\alpha = R \cos (\varphi_R - \alpha)$$

jene Projektion der Resultierenden ist, welche in die durch den Winkel α bezeichnete Richtung fällt.

Die Fläche, welche die Funktion

$$S = \sum_1^n Q_i s_i$$

graphisch darstellt, ist unten konvex. Die auf Punkt P bezogene Resultierende R ist die Tangente der durch den Punkt P verlaufenden Fallinie in Punkt P. R ist demnach in Punkt P zugleich die Normale zu der durch den Punkt P gelegten Isohypse (S = const.).

Der Punkt P bezeichnet dann das Minimum der Summe der gewichteten Entfernungen, wenn $R = 0$ wird. Ist $R \neq 0$, dann weist R seiner Tendenz nach von dem das Minimum bedeutenden Punkt 0 zum Punkt P hin. Hieraus folgt, daß man den Punkt 0 auf der dem Sinn (der Richtung) der Resultierenden R entgegengesetzten Seite suchen muß.

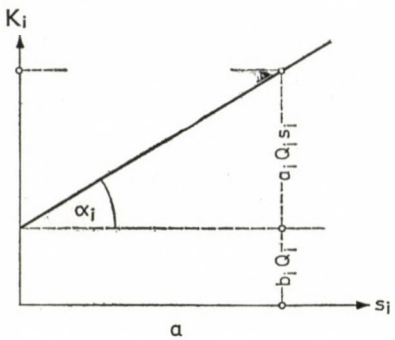
Diese einleitenden Überlegungen verhelfen zur Lösung der durch die Praxis aufgeworfenen Probleme. Die Praxis sucht jedoch nicht so sehr jenen Sammelpunkt, der das Minimum der Summe der gewichteten Entfernungen ergibt, als vielmehr jenen Punkt, der das Minimum der Sammelkosten bezeichnet.

Wenn entlang der geraden Strecke S_i in der Zeiteinheit die Last Q vom Punkt i zu dem gesuchten Punkt 0 bewegt werden soll, dann können die in der Zeiteinheit erwachsenden Bewegungskosten (K_i) auf Grund von Abb. 2 durch die Funktion

$$K_i = Q_i(a_i s_i + b_i)$$

ausgedrückt werden (Abb. 2*A*), worin $b_i Q_i$ die sog. fixen, $a_i Q_i s_i$ aber die variablen Kosten sind.

Wie die Praxis zeigt, kann die Funktion $K_i = f_i(s_i)$ nur für einen bestimmten Abschnitt als Beziehung akzeptiert werden; d. h. die Fixkosten



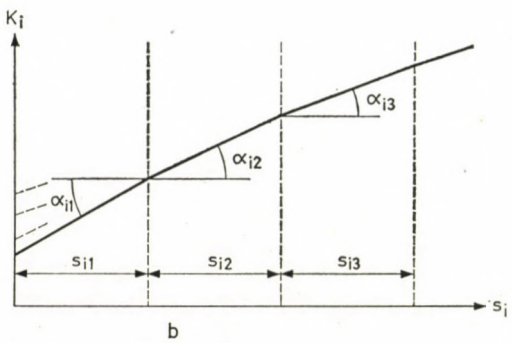


Abb. 2

ändern sich abschnittsweise, doch auch die Richtungstangenten der variablen Kosten ändern sich: erstere steigen, letztere sinken bei zunehmendem s_i.

Auf einen beliebigen Punkt bezogen sind die in der Zeiteinheit erwachsenden Gesamtkosten

$$\sum_1^n K_i = \sum_1^n Q_i (a_i s_i + b_i)$$

Jener Sammelpunkt bezeichnet das Kostenminimum, für welchen das folgende Gleichungssystem erfüllt wird:

$$\frac{\partial \sum_1^n K_i}{\partial x} = \sum_1^n a_i Q_i \cos \varphi_i = \sum_1^n Q_i' \cos \varphi_i = 0$$

$$\frac{\partial \sum_1^n K_i}{\partial y} = \sum_1^n a_i Q_i \sin \varphi_i = \sum_1^n Q_i' \sin \varphi_i = 0$$

Wie ersichtlich, ist die Lösung völlig analog jenem Ergebnis, das wir beim Suchen nach dem Minimum der Summe der gewichteten Entfernungen

fanden, aber mit dem Unterschied, daß anstelle von Q_i die Größe Q_i' getreten ist.

Es fragt sich nun, unter welchen Bedingungen ein gegebener Punkt k als Sammelpunkt das Kostenminimum bezeichnet.

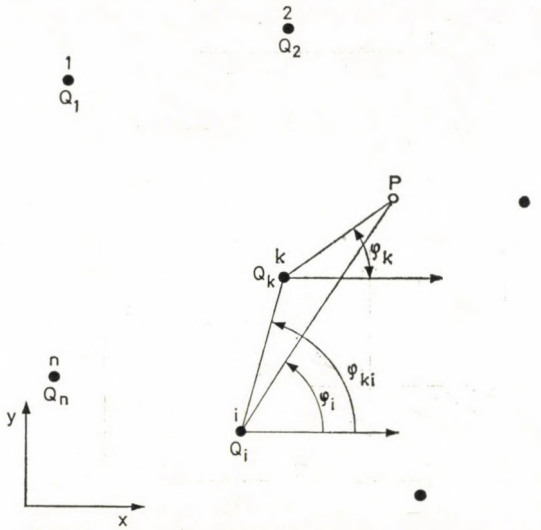


Abb. 3

Nach Abb. 3 gelten für einen beliebigen Punkt $P(x, y)$ die Beziehungen:

$$\frac{\partial \sum_1^n K_i}{\partial x} = \sum_1^{n-1} Q_i' \cos \varphi_i + Q_k' \cos \varphi_k$$

$$\frac{\partial \sum_1^n K_i}{\partial y} = \sum_1^{n-1} Q_i' \sin \varphi_i + Q_k' \sin \varphi_k$$

Nähert man den Punkt P entlang der Geraden $P-k$ dem Punkt k, so gelten für den Grenzfall, wenn P im Punkt k angelangt ist, die Beziehungen:

$$\frac{\partial \sum_1^n K_i}{\partial x} = \sum_1^{n-1} Q_i' \cos \varphi_{ki} + \lim_{\substack{x \to x_k \\ y \to y_k}} Q_i' \frac{x - x_k}{\sqrt{(x - x_k)^2 + (y - y_k)^2}}$$

$$\frac{\partial \sum_1^n K_i}{\partial x} = \sum_1^{n-1} Q_i' \sin \varphi_{ki} + \lim_{\substack{x \to x_k \\ y \to y_k}} Q_k' \frac{y - y_k}{\sqrt{(x - x_k)^2 + (y - y_k)^2}}.$$

Da aber

$$x = (y - b) \operatorname{ctg} \varphi_k$$

$$x_k = (y_k - b) \operatorname{ctg} \varphi_k$$

beziehungsweise

$$y = x \operatorname{tg} \varphi_k + b$$

$$y_k = x_k \operatorname{tg} \varphi_k + b$$

ist, erhält man den Zusammenhang

$$\lim_{\substack{x \to x_k \\ y \to y_k}} Q'_k \frac{x - x_k}{\sqrt{(x - x_k)^2 + (y - y_k)^2}} = \lim_{x \to x_k} Q'_k \frac{x - x_k}{x - x_k} \cos \varphi_k = Q'_k \cos \varphi_k$$

beziehungsweise

$$\lim_{\substack{x \to x_k \\ y \to y_k}} Q'_k \frac{y - y_k}{\sqrt{(x - x_k)^2 + (y - y_k)^2}} = \lim_{y \to y_k} Q_i \frac{y - y_k}{y - y_k} \sin \varphi_k = Q'_k \sin \varphi_k$$

Demzufolge lauten, auf den Punkt k bezogen, die Gleichungen

$$\frac{\partial \sum_1^n K_i}{\partial x} = \sum_1^{n-1} Q'_i \cos \varphi_{ki} + Q'_k \cos \varphi_k$$

$$\frac{\partial \sum_1^n K_i}{\partial y} = \sum_1^{n-1} Q'_i \sin \varphi_{ki} + Q'_k \sin \varphi_k$$

Mit R_k sei die Resultierende des im Punkt k angreifenden Vektorsystems bezeichnet, wobei die Skalarwerte der Vektoren $Q'_1, Q'_2, \ldots, Q'_{n-1}$ (also ausgenommen Q'_k) sind, ihre Richtung der Verbindungsgeraden zwischen dem gesuchten Punkt und Punkt k entspricht, ihr Sinn aber zum Punkt k hinweist. Aus obigem folgt nun, daß Punkt k dann den Sammelpunkt mit der kleinsten Kostensumme bedeutet, wenn

$$|R_k| \leq |Q'_k|$$

ist.

In Abb. 4 sind die vier Punkte (A, B, C, D) mit ihren Daten angegeben:

	A	B	C	D
Q_i [t/h]	140	215	87	192
a_i [F/tkm]	3,20	2,95	3,73	3,06
Q'_i [F/km, h]	448	634	325	588

Hierin bezeichnet F die Geldeinheit.

Angenommen sei ein Punkt P_1. Die Resultierende des in diesem Punkte angreifenden Vektorsystems sei R_1, für Punkt P_2 aber die Resultierende R_2. Werden die Resultierenden R_1 und R_2 im entgegengesetzten Sinn verlängert, so gelangt man zum Schnittpunkt O_1. Dabei ist die Resultierende des im

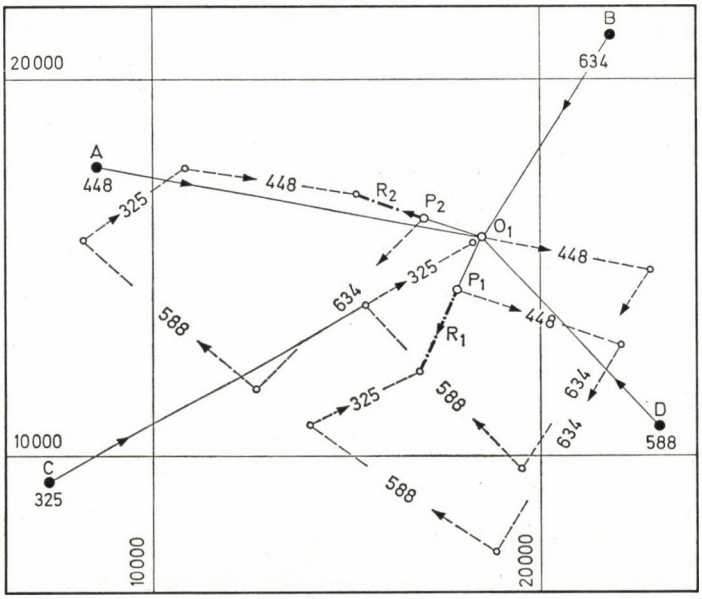


Abb. 4

Punkt O_1 angreifenden Vektorsystems schon wesentlich kleiner. Im vorliegenden Falle bedeutet O_1 praktisch bereits, wenigstens im ersten Schritt, die gesuchte Lösung. Der zweite Schritt wird erst dann erforderlich, wenn sich im Vergleich zu den Koeffizienten des ersten Schrittes die zu den Entfernungen $\bar{A}\bar{O}_1$, $\bar{B}\bar{O}_1$, $\bar{C}\bar{O}_1$, $\bar{D}\bar{O}_1$ gehörigen Koeffizienten a_i verändern.

Die Daten der fünf Punkte in Abb. 5 lauten wie folgt:

	A	*B*	*C*	*D*	*E*
Q_i	246	219	374	204	115
a_i	2,6	2,6	2,3	2,7	3,3
Q'_i	640	570	860	550	380

Die Resultierende des im Punkt B angreifenden Vektorsystems (A, C, D, E) beträgt $R_B = 520$. Da aber

$$|R_B| < |Q'_B|$$

ist, bezeichnet B den Punkt, der das Minimum an Sammelkosten aufweist. Selbstredend kann zur Kontrolle auch der zweite Schritt durchgeführt werden.

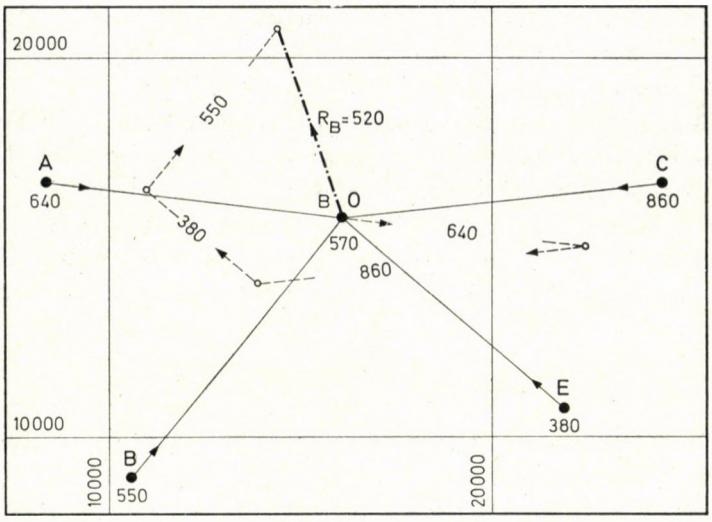

20000
550
R_B=520
A
640
B
O
C
860
380
570
640
860
E
380
10000
B
550
10000
20000

Abb. 5

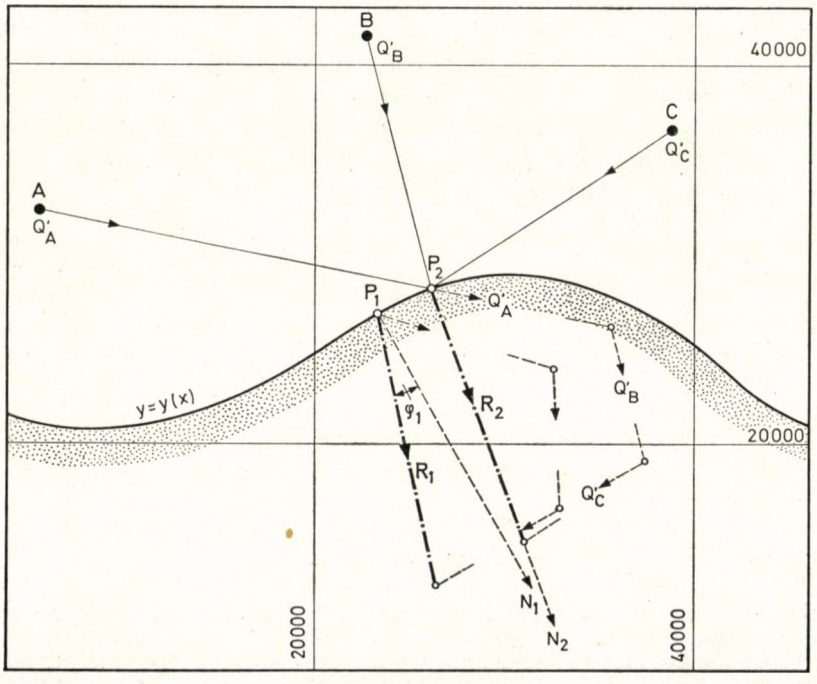

B
Q'_B
40000
C
Q'_C
A
Q'_A
P_2
P_1
Q'_A
Q'_B
y=y(x)
φ_1
R_2
20000
R_1
Q'_C
N_1
N_2
20000
40000

Abb. 6

Die abgeleiteten Beziehungen ermöglichen auch die Lösung anderer, als Sonderfälle zu betrachtender Aufgaben. Eine solche Spezialaufgabe ist in Abb. 6 zu sehen.

Die Kurve $y = y(x)$ bezeichnet eine Meeresküste. Die Gleichung der Küste ist selbstredend nicht bekannt, nur ihr Verlauf ist auf der Karte angegeben. Im Meere sind die drei Knotenpunkte (A, B, C) zu sehen, von denen aus in der Zeiteinheit die Materialmengen Q_A, Q_B, Q_C an Land geschafft werden sollen. Es wird jener Küstenort gesucht, der das Minimum an Transportkosten ergibt. Hierzu müssen, analog dem Vorherigen, die Größen

$Q_A = 480$; $Q_B = 370$; $Q_C = 380$ gebildet werden, d. h.
$Q_A : Q_B : Q_C = 1 : 0{,}771 : 0{,}792$.

Das Kostenminimum ergibt sich an der Küstenlinie in jenem Punkt, für den die Resultierende des in diesem Punkt angreifenden Vektorsystems am kleinsten ist, falls Q_A, Q_B, Q_C die Skalarwerte der Vektoren sind, die Vektorenrichtung aber durch den gesuchten und den gegebenen Punkt (A, B, C) festgelegt ist und ihr Sinn zum gegebenen Punkt hinweist. Die Resultierende des Vektorsystems kann dann am kleinsten sein, wenn die Resultierende auf die zum gesuchten Punkt gehörige Normale zur Küstenlinie entfällt. Von diesen muß jener Punkt ausgewählt werden, zu dem die kleinste Resultierende gehört.

Im vorliegenden Fall wird als erster Schritt ein Punkt P_1 angenommen; die dazugehörige Resultierende R_1 weicht um den Winkel φ_1 von der Normalen (N_1) ab. Fortschreitend in entgegengesetzter Richtung sei der Punkt P_2 angenommen. Die Resultierende R_2 entfällt praktisch auf die zu Punkt P_2 gehörige Normale (N_2). Wäre dies im zweiten Schritt nicht erfolgt, so würde noch ein weiterer Schritt erforderlich sein.

SCHRIFTTUM

1. ZAMBÓ, J.: Optimum Location of Mining Facilities. Publishing House of the Hungarian Academy of Sciences Budapest, 1968.

МИНИМУМ РАСХОДОВ СБОРА НА ПЛОСКОСТИ

Й. ЗАМБО

РЕЗЮМЕ

Задача: из разных точек практически плоской местности нужно собирать материал различного количества в одну общую точку таким образом, чтобы расходы на транспортировку были минимальными. Автор задает общее решение и останавливается на некоторых особенных случаях. Даются простые практические примеры. Любая подобная задача может быть решена по смыслу и просто.

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STOCHASTIC MODEL OF MAGNETOTELLURICS FOR AN INHOMOGENEOUS EARTH

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A statistical theory of magnetotellurics is developed, taking into account a two-dimensional dependence of the conductivity of the Earth $\sigma = \sigma(y, z)$. The problem is formulated for the E-polarization case. It is assumed that the conductivity can be partitioned to an overall slowly-varying part $\sigma_0(z)$, which can be determined from conventional magnetotellurics or by inverse methods, and a more quickly varying part $\sigma_1(y, z)$; that is $\sigma = \sigma_0 + \sigma_1$, where $\overline{\sigma} = \sigma_0$, $\overline{\sigma}_1 = 0$, the average taken over the horizontal scale length L of σ_1. Solving the linearized half-space problem and defining the impedance correlation function $\overline{Z_1 Z_1} = \zeta$, an integral equation is obtained for the modulus of the Fourier-transform of $\sigma_1(y)$. Solving the equation, $\overline{\sigma_1(y, z)^2}$, the lateral variation of the mean square horizontal deviation of the conductivity from its magnetotelluric value $\sigma_0(z)$ can be determined in terms of the impedance correlation function, which is a measurable quantity. Finally, the equivalence with the magnetovariational method is demonstrated.

I. Introduction

Magnetotellurics is governed by the equations

$$\Delta E + k^2 E = 0, \; k^2 = -i\omega\mu_0\sigma \tag{1}$$

where σ depends only on the vertical coordinate z. In that case Eq. (1) becomes one-dimensional: $\Delta = d^2/dz^2$. In more general cases of practical importance the experimenter is confronted with an Earth of at least two-dimensional dependence of σ on space coordinates, $\sigma = \sigma(y, z)$. However, it can often be assumed that the dependence on the horizontal coordinate is much weaker than on the vertical coordinate. In such a case magnetotellurics remains valid at least approximately, and the horizontal conductivity variation can be considered as a small disturbance of the global conductivity structure of the Earth. It is, however, of vital interest to determine this disturbance from magnetotelluric measurements. There exists an extended reservoir of methods which have been proposed to solve this problem (for a review cf. [1]). Most of these methods have been based on distinct and very special models of the lateral conductivity. Hence the problem is, to our knowledge, quite far from being solved. Moreover, none of these methods takes into account the statistical distribution of the magnetotelluric data material at the surface of the Earth,

i.e. the incorrectness of the boundary value problem under consideration [2]. The solution of the one-dimensional problem has been formulated — from our point of view — the most satisfactorily by WEIDELT [6], who solved the inverse problem of Eq. (1) under the assumption that $\sigma = \sigma(z)$ is a smooth function of z. In his approach the apparent resistivity curve of magnetotellurics is used as the experimental fact. Knowing this curve in all its details, he is in principle able to determine the dependence of the conductivity on the vertical coordinate below and in a distinct environment of the measuring point. This curve is, however, given as a function of the period of the variation and thus reflects the scattering of the measured values, due to the statistical behaviour of the inducing wave field, and to the small-scale statistical and non-smooth behaviour of the vertical conductivity dependence. This influence can be only taken into account within the framework of a statistical theory of magnetotellurics.

In some previous papers we have developed the idea of such a statistical approach using the models of an infinite Earth [3; 4] and thin sheets [5]. Here we try to extend this approach to the magnetotelluric problem. This extension will be given for the case of a two-dimensional dependence of the disturbance of the conductivity on space, in an otherwise laterally homogeneous conducting half-space. In Section 2 the half-space problem will be solved for the disturbed induction field produced by the conductivity disturbance. This solution will be used in Section 3 to derive an expression which makes it possible to calculate the mean horizontal deviation of the conductivity from its global (magnetotelluric) value by use of the correlation function of the surface impedance at the Earth's surface. Finally Section 4 compares this approach very briefly with the magneto-variational method.

II. The half-space problem

As it is well known, Eq. (1) describes for a two-dimensional spatial dependence the so called E-polarization case. To solve the complete problem, Eq. (1) has to be supplemented by a similar equation for the H-polarization. Since, however, the latter problem is in our case equivalent to the former, we shall restrict ourselves to the E-polarization case. Subsequently, the results will be generalized to H-polarization. If the conducting half-space is chosen as $z \geq 0$, $-\infty < y < \infty$, we have from Eq. (1) with $\sigma = 0$ for $z < 0$,

$$\begin{aligned} \Delta E &= 0, \; E \equiv E_x & z &< 0 \\ \Delta E + k^2 E &= 0, \; \Delta \equiv \partial^2/\partial z^2 + \partial^2/\partial y^2 & z &\geq 0 . \end{aligned} \tag{2}$$

Assume that the conductivity $\sigma(y, z)$ is the sum of a slowly varying component $\sigma_0(z)$, and a disturbance $\sigma_1(y, z)$:

$$\sigma(y, z) = \sigma_0(z) + \sigma_1(y, z) \tag{3}$$

where the z-dependence of σ_0 is considerably slower than the corresponding variation in σ_1. Substituting Eq. (3) into the second equation (2) and averaging with respect to the variation of σ_1, assuming thereby $\overline{\sigma_1(y, z)} = 0$, and, hence $\overline{\sigma(y, z)} = \sigma_0$, where the bar indicates averaging, we find

$$\Delta E_0 + k_0^2 E_0 + \overline{k_1^2 E_1} = 0, \qquad z \geq 0. \tag{4}$$

Here we have $k_0^2 = -i\omega\mu_0\sigma_0$, $(k^2)_1 = i\omega\mu_0\sigma_1$, and E_1 is the contribution to the electric field E resulting from the induction of the outer field in the conductivity structure σ_1: $E = \bar{E} + E_1 \equiv E_0 + E_1$. Since $\bar{\sigma}_1 = 0$, it is natural to assume $\bar{E}_1 = 0$. Equation (4) describes the variation of the global electric field within the Earth. Outside the Earth we have as usual $\Delta E_0 = 0$. However, inside the Earth the process is changed due to the contribution of the third term in Eq. (4), which describes the correlation between the disturbance of the conductivity and the electric field. Without any knowledge about this term resp. any further specifying assumption, Eq. (4) cannot be solved. It is, however, clear from a first glance at Eq. (4) that magnetotellurics must be changed whenever spatial fluctuations in the conductivity are present and since these fluctuations are not homogeneously distributed in the Earth, they can be replaced by an additional contribution to σ_0. On the other hand, assuming as a first step $\sigma_1 \ll \sigma_0$ everywhere, Eq. (4) reduces in the lowest approximation to $\Delta E_0 + k_0^2 E_0 = 0$ for the global field, and the equation for the first-order field

$$\Delta E_1 + k_0^2 E_1 + k_1^2 E_1 - \overline{k_1^2 E_1} = -k_1^2 E_0, \qquad z < 0 \tag{5}$$

which is obtained by subtracting (4) from (2), reduces under the above linearizing assumption to

$$\Delta E_1 + k_0^2 E_1 = -k_1^2 E_0, \quad z < 0. \tag{6}$$

In this approximation the problem is hence split into two problems: the solution of the inhomogeneous Helmholtz-equation (6) for the first-order field E_1, and the subsequent solution of Eq. (4) for the zero-order field. Of course, both equations have to be supplied with the usual boundary conditions at $z = 0$, i.e. the continuity of E and $H_y = (i/\omega\mu_0)\, \partial E/\partial z$ at the surface of the Earth.

When solving Eq. (6) for E_1, the right-hand side of (6) is assumed to be known, and $E_0 = \text{const}$, because of its considerably weaker spatial dependence as compared with E_1.

Equation (6) is most conveniently solved by using Green's theorem for the Helmholtz-equation. Choosing the representation of E_1 through the Green's function $G(r \mid r_0)$ and the derivation of E_1 in the direction of the outer normal at the surface, $-\partial E/\partial z$, we have

$$E_1(r) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dy_0 \frac{\partial E_1(y_0 \mid 0)}{\partial z_0} G(r \mid y_0, 0) + \int_0^{\infty} dz_0 \int_{-\infty}^{\infty} \varrho(r_0) G(r \mid r_0)\, dy_0\, dz_0 \quad (7)$$

where $r = (y, z)$, $r_0 = (y_0, z_0)$ are the space and source point coordinates, and $\varrho(r) = (-i\omega\mu_0/4\pi)\, E_0\sigma_1(r)$ the source density. The Green's function of the Neumann problem for the half-space and outgoing waves, which in our case have to be assumed since in the first-order problem outer sources of the field are absent, is given by

$$G(r \mid r_0) = \pi i[H_0^{(1)}(k_0 R) + H_0^{(1)}(k_0 R_0)] \quad (8)$$

where $H_0^{(1)}(x)$ is the Hankel function of zero order and first kind, and $R^2 = (y - y_0)^2 + (z - z_0)^2$, $R_0^2 = (y - y_0)^2 + (z + z_0)^2$. The branch of k_0 is choosen in such a way that Im $k < 0$:

$$k_0 = (1 - i)\delta_0^{-1}, \quad \delta_0^2 = 2/\omega\mu_0\sigma_0\,. \quad (9)$$

Here, δ_0 is the skin-depth in the undisturbed Earth. Equations (7) and (8) determine the solution of the first-order problem, if only the field-value at the surface is known. The latter can be found from the field in the nonconducting space above the Earth. Here, from the first of Eqs. (2) we have, because of the absence of field disturbances at $z = -\infty$,

$$E_1(r) = \int_{-\infty}^{\infty} C(s)\, e^{isy + |s|z}\, ds, \qquad z < 0. \quad (10)$$

The coefficient $C(s)$ is a so far unknown function of s, which has to be determined from the requirement of continuity of $E_1(r)$ at $z = 0$. Substituting E_1 from Eq. (10) for E_1 in Eq. (9), setting $z = 0$, taking advantage of the integral representation

$$H_0^{(1)}(k_0 R) = \frac{i}{\pi^2}\int_{-\infty}^{\infty} \frac{dK_z\, dK_y}{k_0^2 - K_z^2 - K_y^2} e^{iK_z(z - z_0) + iK_y(y - y_0)}$$

and the corresponding representation for $H_0^{(1)}(k_0 R_0)$, and using

$$\sigma_1(y, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \sigma_1(s, z)\, e^{isy}\, ds \quad (11)$$

we find after some simple calculations of a Fourier-transformation type an integral equation for $C(s)$:

$$\int_{-\infty}^{\infty} C(s)\left\{1 - |s| \int_{-\infty}^{\infty} \frac{dK_z}{k_0^2 - K_z^2 - s^2}\right\} e^{isy}\, ds = \tag{12}$$

$$= -\frac{i\omega\mu_0 E_0}{4\pi} \int_{-\infty}^{\infty} ds e^{isy} \left\{\int_{-\infty}^{\infty} \frac{dK_z}{k^2 - K_z^2 - s^2} \int_0^{\infty} dz_0\, \sigma_1(s, z_0)\, [e^{iK_z z_0} + e^{-iK_z z_0}]\right\}.$$

By inversion of the transform we get:

$$C(s) = -\frac{i\omega\mu_0 E_0}{4\pi(1 - \pi i|s|/\varkappa)} \int_{-\infty}^{\infty} \frac{dK_z}{\varkappa^2 - K_z^2} \int_0^{\infty} dz_0\, \sigma_1(s, z_0)(e^{iK_z z_0} + e^{-K_r z_0}). \tag{13}$$

Here, $\varkappa = +(k_0^2 - s^2)^{1/2} \equiv +i\, 2^{1/2}\delta\, (i + s^2\delta/2)^{1/2}$, $\operatorname{Im} \varkappa > 0$. Now, by use of the Laplace-transform

$$\sigma_1(s, z_0) = \frac{1}{2\pi} \int_{+ic-\infty}^{+ic+\infty} \sigma_1(s, \tau)\, e^{-i\tau z_0}\, d\tau, \quad \sigma_1(s, \tau) = \int_0^{\infty} \sigma_1(s, z_0)\, e^{i\tau z_0}\, dz_0 \qquad (\operatorname{Im} \tau > 0) \tag{14}$$

we can carry out the integration with respect to z_0 in Eq. (13). The result is

$$C(s) = -\frac{i\omega\mu_0 E_0}{4\pi(1 - \pi i|s|/\varkappa)} \int_{-\infty+ic}^{\infty+ic} \frac{\sigma_1(s, \tau)\, d\tau}{\varkappa^2 - \tau^2} \quad (\operatorname{Im} \varkappa > 0). \tag{15}$$

Inserting this expression into Eq. (10), and the latter on the right-hand side of (7), the full solution of the first-order problem is obtained. In magnetotellurics only the field values at the surface of the Earth are of interest. Therefore it suffices for our purposes to consider Eq. (10) and its derivative at $z = 0$:

$$E_1(y, 0) = -\frac{i\omega\mu_0 E_0}{4\pi} \int_{-\infty+ic}^{\infty+ic} d\tau \int_{-\infty}^{\infty} \frac{\sigma_1(s, \tau) \exp(isy)\, ds}{(\varkappa^2 - \tau^2)(1 - \pi i|s|/\varkappa)} \tag{16}$$

$$H_{y1}(y, 0) = \frac{E_0}{4\pi} \int_{-\infty+ic}^{\infty+ic} d\tau \int_{-\infty}^{\infty} \frac{|s|\sigma_1(s, \tau) \exp(isy)\, ds}{(\varkappa^2 - \tau^2)(1 - \pi i|s|/\varkappa)}. \tag{17}$$

Apart from this we have the additional expression

$$H_{z1}(y, 0) = -\frac{iE_0}{4\pi} \int_{-\infty+ic}^{\infty+ic} d\tau \int_{-\infty}^{\infty} \frac{s\sigma_1(s, \tau) \exp(isy)\, ds}{(\varkappa^2 - \tau^2)(1 - \pi i|s|/\varkappa)} \tag{18}$$

for the vertical component H_{z1} of the disturbed magnetic field. This equation is of minor importance in magnetotellurics and is only given here for sake of completeness.

In the last three formulae the parameters of the global problem E_0, σ_0 enter only as factors, resp. constants. It is therefore clear that the knowledge of the global model is supposed in the direct application of these expressions. On the other hand, it is obvious, that the second-order correlation term $\overline{k_1^2 E_1}$ in Eq. (4) can in principle be expressed through E_0, σ_0, and the conductivity fluctuation $\sigma_1(s, \tau)$ on the basis of Eq. (16). This produces an extremely complicated expression and leaves Eq. (4) almost intractable. It seems to be more advantageous to use the results of conventional magnetotellurics as a zero-order model for the Earth and to try to get some further informations about the conductivity fluctuation on the basis of the above derived results. This path will be followed in the next section.

Let us now briefly consider H-polarization. In that case we have

$$\Delta H_x + k^2 H_x = 0, \quad z \geq 0 \tag{19}$$

inside the Earth, $\Delta H_x = 0$ outside, and H_x is the only non-vanishing component of the magnetic field. Again the above used averaging procedure gives Eqs. (4) and (5), with the transformation $E \to H_x$, $H_x = H_{x0} + H_{x1}$. After linearization Eq. (5) becomes

$$\Delta H_{x1} + k_0^2 H_{x1} = -k_1^2 H_{x0} \tag{20}$$

where the right-hand side is assumed to be known, and the boundary conditions at $z = 0$ are the continuity of H_{x1} and E_{y1}. However, as can be easily verified from $\operatorname{div} E = 0$, $i\omega\mu_0 H_{x1} = \partial E_{y1}/\partial z - \partial E_{x1}/\partial y$, and $\Delta H_{x1} = 0$, the only possible solution outside the Earth is $H_{x1} \equiv H_{10} = \text{const}$, $E_{y1} \equiv E_{10} = \text{const}$, $E_{z1} \equiv 0$. Since there are no sources in the upper z half-space we have $H_{x1} = E_{y1} = E_{z1} = 0$ outside and at the surface of the Earth. Consequently, H-polarization does not contribute to the field at the surface, no matter what the solution of Eq. (20) is, and the whole problem is described by the solution of the E-polarization case.

III. The magnetotelluric approach

In magnetotellurics the impedance at the surface, $E/H_y = Z$, plays a vital role. We now turn to investigate this in the presence of a small-scale disturbance of the conductivity. This will be done in two steps. First it will be supposed that $\sigma_1 = \sigma_1(y)$ is a function of the horizontal coordinate only, and the vertical dependence is exactly described by the usual global magneto-

telluric assumption. In the second step, we shall consider vertical fluctuations of the conductivity as well.

In the first case Eq. (11) becomes

$$\sigma_1(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \sigma_1(s)\, e^{isy}\, ds \tag{21}$$

and Eqs (16)—(18) reduce to

$$\begin{aligned} E_1(y,0) &= -\frac{i\omega\mu_0 E_0}{2}\int_{-\infty}^{\infty} \frac{\sigma_1(s)\exp(isy)\,ds}{\varkappa^2(1-\pi i|s|/\varkappa)} \\ H_{y1}(y,0) &= \frac{E_0}{2}\int_{-\infty}^{\infty} \frac{|s|\,\sigma_1(s)\exp(isy)\,ds}{\varkappa^2(1-\pi i|s|/\varkappa)} \\ H_{z1}(y,0) &= -\frac{iE_0}{2}\int_{-\infty}^{\infty} \frac{s\sigma_1(s)\exp(isy)\,ds}{\varkappa^2(1-\pi i|s|/\varkappa)} \,. \end{aligned} \tag{22}$$

Averaging the impedance with respect to the horizontal coordinate y over the characteristic scale-length L_y of the conductivity fluctuation, it is easy to verify that $\bar{Z} \equiv Z_0 = E_0/H_{y0}$,

$$Z_1 \equiv Z - Z_0 = \frac{E_1}{H_{y0}} - Z_0\,\frac{H_{y1}}{H_{y0}} \tag{23}$$

up to first order in the fluctuation. Here, Z_1 denotes the disturbance of the surface impedance due to the disturbance of the conductivity, and this is, obviously, a function of y : $Z_1(y)$. This function can be taken as an additional information about the Earth, from which it is possible to determine the horizontal disturbance of the conductivity.

The correlation function $\zeta(\eta, Z_0) = \overline{Z_1 Z_1}$ at the Earth's surface can be obtained from Eq. (23) as

$$\begin{aligned} \zeta(\eta, Z_0) = H_{0y}^{-2}\big[&\overline{E_1(y)\,E_1(y+\eta)} - Z_0^2\,\overline{H_{1y}(y)\,H_{1y}(y+\eta)} \\ &- Z_0\{\overline{E_1(y)\,H_{1y}(y+\eta)} + \overline{H_{1y}(y)\,E_1(y+\eta)}\}\big] \,. \end{aligned} \tag{24}$$

Inserting the expressions (22) for the field components, carrying out the integration with respect to y, and using $\sigma_1(-s) = \sigma_1^*(s)$, what follows from Eq. (21) because of the requirement of reality of $\sigma_1(y)$, we get

$$\zeta(\eta, Z_0) = \frac{(\pi\omega\mu_0 Z_0)^2}{2\pi}\int_{-\infty}^{\infty} e^{is\eta}\, ds \left(1 + \frac{i|s|Z_0}{\omega\mu_0}\right)^2 \frac{|\sigma_1(s)|^2}{\varkappa^4(1-\pi i|s|/\varkappa)^2} \,. \tag{25}$$

This equation is of the form of an inverse Fourier-transform. Solving for $|\sigma_1(s)|^2$, we arrive at

$$|\sigma_1(s)|^2 = \left| \mathrm{Re} \left\{ \frac{\varkappa^4(1 - \pi i |s|/\varkappa)^2}{(\pi\omega\mu_0 Z_0)^2 (1 + i|s| Z_0/\omega\mu_0)^2} \zeta(s, Z_0) \right\} \right| \tag{26}$$

where

$$\zeta(s, Z_0) = \int_{-\infty}^{\infty} \zeta(\eta, Z_0)\, e^{is\eta}\, d\eta .$$

This is the solution of the problem. The right-hand side depends only on measurable quantities; the frequency ω, the global (usual magnetotelluric) impedance, and the horizontal correlation function of the impedance $\zeta(\eta, Z_0)$, which depends on the spacing η of the measuring points at the surface of the Earth, and, implicitly, on the assumption of a horizontally homogeneous large-scale conductivity structure, indicated by the dependence of ζ on Z_0. In practice, $\zeta(\eta)$ is easily found from a net of magnetotelluric stations and by analysing the horizontal deviation of the impedance curves.

Inverting Eq. (26), we have finally

$$\overline{\sigma_1^2} = \frac{1}{\pi} (\pi\omega\mu_0 Z_0)^{-2} \int_0^{\infty} ds \left| \mathrm{Re} \left\{ \frac{\varkappa^4(1 - \pi i |s|/\varkappa)^2}{(1 + i|s| Z_0/\omega\mu_0)^2} \zeta(s, Z_0) \right\} \right| \tag{27}$$

for the mean square horizontal deviation of the conductivity from the value σ_0, found from conventional magnetotellurics.

It is interesting to note that the expression (27) is a function of ω, the frequency of the induction field, which enters the calculation as a parameter. In magnetotellurics, ω is directly connected with the penetration depth (9) and, hence, with the depth of the layer under consideration, for which conventional magnetotellurics, or the solution of the inverse problem [6], give the global conductivity value σ_0. It is thus clear that the dependence of $\overline{\sigma_1(\omega)^2}$ on the depth is given through its dependence on δ_0 via ω in Eq. (27). In other words, the horizontal differences of the impedance curves for fixed ω enable us to calculate $\overline{\sigma_1(y)^2}$ for the fixed depth z, which in conventional magnetotellurics is prescribed to the corresponding period of the outer variation field.

Let us now consider the second case when the conductivity disturbance also manifests itself in the z-direction. In this case the problem is somewhat more difficult, since we cannot construct the correlation function of the surface impedance in the vertical direction. Of course, we could use measurements having been carried out in a mine. But mines are rare and usually not at a place where the measurements will be performed. Instead, we could use measurements of the impedance Z on some vertical profile in the nonconducting

air above the ground. The field components are simply

$$E_1(y,z) = -\frac{i\omega\mu_0 E_0}{4\pi}\int_{-\infty+ic}^{\infty+ic} d\tau \int_{-\infty}^{\infty} \frac{\sigma_1(s,\tau)\exp(isy+|s|\,z)\,ds}{(\varkappa^2-\tau^2)(1-\pi i\,|s|\,/\varkappa)} \qquad (z<0)$$

$$H_{y1}(y,z) = \frac{E_0}{4\pi}\int_{-\infty+ic}^{\infty+ic} d\tau \int_{-\infty}^{\infty} \frac{|s|\sigma_1(s,\tau)\exp(isy+|s|\,z)}{(\varkappa^2-\tau^2)(1-\pi i\,|s|/\varkappa)}\,ds \qquad (28)$$

$$H_{z1}(y,z) = -\frac{iE_0}{4\pi}\int_{-\infty+ic}^{\infty+ic} d\tau \int_{-\infty}^{\infty} \frac{s\sigma_1(s,\tau)\exp(isy+|s|\,z)}{(\varkappa^2-\tau^2)(1-\pi i\,|s|/\varkappa)}\,ds$$

and the impedance correlation function becomes

$$\begin{aligned}\tilde{\xi}(\eta,\xi,Z_0) = H_0^{-2}\big[&\overline{E_1(y,z)\,E_1(y+\eta,z+\xi)} - Z_0^2\,\overline{H_{1y}(y,z)\,H_{1y}(y+\eta,z+\xi)} \\ &- Z_0\{\overline{(E_1(y,z)\,H_{y1}(y+\eta,z+\xi)} + \overline{H_{1y}(y,z)\,E_1(y+\eta,z+\xi)}\}\big].\end{aligned} \qquad (29)$$

Comparing with (25) gives

$$\begin{aligned}\tilde{\xi}(\eta,\xi,Z_0) = \frac{-(\omega\mu_0 Z_0)^2}{16\pi}\int_{-\infty}^{\infty}\frac{e^{-is\eta-|s|\xi}\,ds}{|s|\,(1-\pi i\,|s|/\varkappa)^2}\left(1+\frac{i\,|s|\,Z_0}{\omega\mu_0}\right)^2 \times \\ \times\int_{-\infty+ic}^{\infty+ic}\frac{\sigma_1(s,\tau)\,\sigma_1^*(s,\tau')}{(\varkappa^2-\tau^2)(\varkappa^2-\tau'^2)}\,d\tau\,d\tau'\end{aligned} \qquad (30)$$

where $\xi > 0$. From symmetry considerations we have $\sigma_1(-s,\tau) = \sigma_1^*(s,\tau)$, however use of this relation is not permitted in (30), and thus Eq. (30) becomes worthless, since the integrals with respect to τ, τ' cannot be interpreted in a simple manner. Physically this is quite natural: the behaviour of the medium outside the Earth is inactive; there is no response in the air-space to any averaging procedure, the only expectable response comes from the interior of the Earth, and therefore any averaging has to be carried out with respect to the field within the Earth. This field, however, is indetectable apart from its surface value. Hence we are forced to think about another possibility of taking advantage of the surface impedance. But to this end the entire problem requires re-interpretation. Since this would go beyond the scope of the present contribution, we postpone it for a separate investigation.

IV. The magnetovariational approach

Complementary to magnetotellurics there exists the so-called magnetovariational approach to the induction problem which takes into account the magnetic field components ignoring the electric field and, hence, the imped-

ance. In our case, looking at Eqs (16)—(18) it is clear that we could have used $H_{z,1}(y, 0)$ as well instead of $E_1(y, 0)$. Following our earlier paper [3] we construct the surface field-correlation tensor

$$\overline{HH} = H_{y0}H_{y0}e_ye_y + \overline{H_{y1}H_{y1}}e_ye_y + \overline{H_{z1}H_{z1}}e_ye_z \qquad (31)$$
$$\overline{H_{z1}H_{y1}}e_ze_y + \overline{H_{y1}H_{y1}}e_ye_y\,.$$

Here we have, because of the symmetry of the problem,

$$\overline{H_{y1}H_{y1}} = \frac{\pi(Z_0H_{0y})^2}{2}\int_{-\infty}^{\infty}\frac{|s|^2\,|\sigma_1(s)|^2\;e^{-is\eta}\,ds}{\varkappa^4\,(1-\pi i\,|s|/\varkappa)^2} = -\,\overline{H_{z1}H_{z1}}$$
$$\overline{H_{y1}H_{z1}} = -\,\frac{i\pi(Z_0H_{y0})^2}{2}\int_{-\infty}^{\infty}\frac{|s|\;s\,|\sigma_1(s)|^2\;e^{-is\eta}\,ds}{\varkappa^4\,(1-\pi i\,|s|/\varkappa)^2} = \overline{H_{z1}H_{y1}}\;. \qquad (32)$$

We can, hence, use any one of these components to determine $|\sigma_1(s)|^2$. For convenience we choose $\overline{H_{z1}H_z}/H_{y0}^2$. Then, since $\overline{H_{z1}H_{z1}} = \overline{H_zH_z}$, we find

$$|\sigma_1(s)|^2 = (\pi Z_0)^{-2}\left|\,\mathrm{Re}\left\{\frac{\varkappa^4(1-\pi i\,|s|^5/\varkappa)^2}{s^2}\int_{-\infty}^{\infty}\frac{\overline{H_z(\eta)\,H_z(\eta)}}{H_{y0}^2}\,e^{is\eta}\,d\eta\right\}\right|. \qquad (33)$$

From this expression $\overline{\sigma_1(y)^2}$ is easily found. It is interesting to note that Z_0 appears as a coefficient in Eq. (33). In a certain sense, as it should be, the magnetotelluric and magnetovariational approaches are equivalent.

V. Conclusions

In this paper, we have developed a statistical method for the determination of the lateral conductivity dependence from magnetotelluric measurements. By assuming that the horizontal dependence of the conductivity is weak (an assumption which is natural in magnetotellurics) and that the conductivity disturbance is non-regularly distributed within the Earth, we were able to use standard statistical techniques to find a representation of the mean square deviation of the conductivity from its value determined from conventional magnetotellurics, in dependence on the horizontal coordinate (cf. Eq. (26)). This expression takes into account the insufficient knowledge of the distribution of the impedance on the Earth's surface and hence the incorrectness of the boundary condition imposed on the inverse magnetotelluric problem by means of a statistical quantity, the surface correlation function of the impedance. Since impedance measurements are given only at

distinctly distributed points, such an approach in which the averaging procedure is performed with respect to distances large in comparison with the measuring-point-spacing seems to be quite natural.

Equation (26) has the character of a direct inversion formula. From it the quantity $|\sigma_1(s)|^2$ can be directly calculated knowing the global magnetotelluric model, and the correlation function of the surface impedance, which is a pure measurable quantity. Hence, within the limitations of the present theory, the magnetotelluric problem can be considered as solved. These limitations are: two-dimensionality of the problem and of the outer inducing field, availability of a global magnetotelluric solution, weak horizontal dependence of the conductivity. In a certain sense this problem is, hence, complementary to that of the induction of the Earth, possessing a conductivity jump in horizontal direction. It is of interest to note that in this case H-polarization does not contribute to the solutions since there are no "outer" sources of the magnetotelluric disturbance field and no charges. Two-dimensionality seems to be, however, a not to serious restriction. It appears to be possible to solve the three-dimensional case as well, when $\sigma_1(r)$ depends on x, y and z, and the outer inducing field has three components. One needs only to know the Green's tensor of the half-space in such a case. What remains is a straightforward use of integration and Fourier-transforms. Then, however, the surface-impedance correlation becomes a tensor according to the two horizontal directions of the conductivity variation. The problem has, however, still not been solved totally. We were, at this stage, unable to give an equally simple expression as Eq. (26) for the deviation of the conductivity in the vertical direction from its global value determined by magnetotelluric methods. Since these methods make use of the dependence of the impedance on the frequency of the variation, and since frequency is through skin-depth equivalent to depth, one should take advantage of an averaging procedure with respect to frequency. In that case the problem should be reformulated. It is, however, obvious that, since the surface impedance is only known at distinct frequencies, the ill-posedness of the inverse magnetotelluric problem can only be overcome by a stochastic treatment of the kind proposed in the present paper.

Finally, we note that it seems straightforward to generalize the present method to anisotropic media, where the conductivity σ becomes a tensor σ. Examples of such media are the ionosphere and the magnetosphere. It would be worthwhile to think about the application of the statistical approach to those regions. Conductivity there plays a crucial role, too. Determination of the conductivity using direct measurements of the magnetic and electric fields in space by the help of satellite equipments would provide valuable additional information about space plasmas.

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REFERENCES

1. ÁDÁM, A. (ed.): Geoelectric and Geothermal Studies. Akadémiai Kiadó, Budapest, 1976.
2. PARKER, R. L.: The inverse problem of electrical conductivity in the mantle. *Geoph. J. Royal Astron. Soc.*, 22 (1971), 121—138.
3. SCHÄFER, K.—TREUMANN, R.: Statistical approach to the geomagnetic induction problem for the infinite medium. *Gerlands Beiträge zur Geophysik*, 84 (1975), 117—122.
4. TREUMANN, R.—SCHÄFER, K.: Statistical basis for geomagnetic induction theory. *Acta Geod. Geoph. Mont. Hung.*, 10 (1975), 287—297.
5. TREUMANN, R.—SCHÄFER, K.: Statistical theory of electromagnetic induction in thin sheets. *J. Geophys.*, 43 (1977) 521—533.
6. WEIDELT, P.: The inverse problem of geomagnetic induction. *Z. Geophys.* 38 (1972), 257—289.

СТОХАСТИЧЕСКАЯ МОДЕЛЬ МАГНИТОТЕЛЛУРИКИ В СЛУЧАЕ НЕОДНОРОДНОЙ ЗЕМЛИ

Р. ТРАЙМАНН

РЕЗЮМЕ

Стохастическая модель магнитотеллурики была образована с учетом двумерного распределения $\sigma = \sigma(y, z)$ проводимости Земли. Проблема была определена для случая E-поляризации. Предполагается, что проводимость можно разделить на медленно варьиру, ющую часть $\sigma_0(z)$, определяемую традиционной магнитотеллурикой или ;обратными методами, а также на одну быстро варирующую часть $\sigma_1(y, z)$ т. е. $\sigma = \sigma_0 + \sigma_1$ и $\overline{\sigma} = \sigma_0$ $\overline{\sigma_1} = 0$, проводя образование среднего для горизонтальной длины шкалы $\sigma_1 L$. Решая проблему линеаризованного полуполя и определяя корреляционную функцию $\overline{Z_1 Z_1} = \zeta$ импеданса, для модуля трансформированного по Фурье $\sigma_1(y)$ получится интегральное уравнение. Решив уравнение $\overline{\sigma_1(y, z)}$,² можно определитж среднее квадратическое отклонение значения проводимости в горизонтальном направлении от магнитотеллурической величины $\sigma_0(z)$ в виде корреляционной функции импеданса, что является уже измеряемой величиной. Наконец, показывается эквивалентность с магнитовариационным методом.

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CONTRIBUTION TO THE ADJUSTMENT OF FREE NETWORKS

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The adjustment of a levelling network as free network is discussed when several points have fixed heights. In this case, some additional constraints are to be used, in addition to the geometrical constraint, to the so-called pseudo-constraint [1].

As it has been shown in an earlier paper [1] the adjustment of free networks can be carried out (besides the generalized matrix inversion method which makes the matrix of the normal equations non-singular) by increasing the rank of the singular normal matrix with so-called *pseudo-constraints.*

In free adjusted networks the coordinates of each point get corrections that is in the correction equations corresponding to the measurements there will appear unknowns corresponding to each point. The normal matrix of these equations is singular, as its rank is lower than the number of unknowns. To eliminate the singularity as many constraints must be used as the rank of the matrix is less than the number of unknowns.

These constraints can be transformed so as to correspond to corrections of pseudo-observations; their zero closure can be ensured by attaching to them high enough weights.

In our previous paper [1] we have used the correction equations

$$\overline{\mathbf{v}} = \mathbf{A}'\overline{\mathbf{x}} - \overline{\mathbf{L}} \tag{1}$$

$$\overline{\mathbf{0}} = \mathbf{A}''\overline{\mathbf{x}} - \overline{\mathbf{0}} \,. \tag{1a}$$

For the case discussed in [1] Eq. (1*a*) expresses that the points of the network should be translation- and rotation-free.

The present paper will be concerned with a levelling network [2]. In this case the constraint refers to the height corrections of the points:

$$\sum \delta z = 0. \tag{2}$$

Obviously, Eq. (2) can also be written in the form of Eq. (1*a*), the unknown vector x will contain, however, corrections of heights.

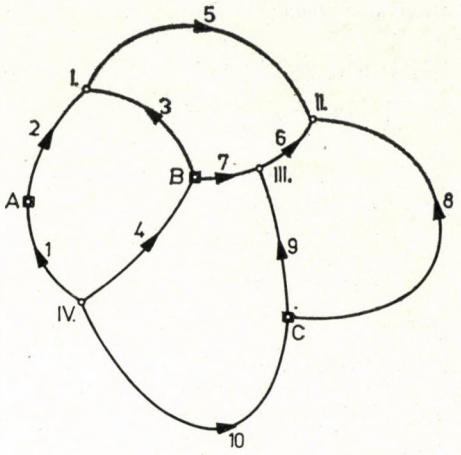

Fig. 1

The geometry of the case to be discussed is shown in Fig. 1. The net contains ten measured lines, and among the points four have unknown and three known heights.

It frequently occurs in practice that one uses more points with error-free heights than it would be necessary to eliminate undeterminedness (in that case, one). Similar cases occur in triangulation networks, when certain fixed directions or lengths are used.

Levelling networks have been adjusted up to now by the traditional methods of correction equations with weights or conditional equations.

As known, in the first method the additional points with fixed heights only appeared in the contradictions of the correction equations for the levelling lines. The heights of these points did not get corrections.

In the adjustment by the method of conditional equations, the conditional equations express that the sum of the corrections along closed levelling routes should be zero, further that the fixed heights of some of the points should remain unchanged. Any one condition might contain corrections for several lines.

If a levelling network like that in the present case (i.e. where the heights of several points are fixed) is to be adjusted as a free network then Eqs. (1) and (2) should be supplemented by some further correction equations. These imply constraints between the heights of points of fixed heights (A, B and C):

$$\begin{aligned} v_{11} &= \delta A - \delta B \\ v_{12} &= \delta C - \delta B \ . \end{aligned} \tag{3}$$

This can be written on the analogy of Eq. (1*a*) as

$$\bar{\mathbf{v}} = \mathbf{A}'''\bar{\mathbf{x}} - \bar{\mathbf{0}}. \tag{3a}$$

$$v_{\text{constr.}} \to 0 \qquad \text{if} \qquad p \to \infty$$

(Theoretically Eq. (1*a*) could be written in this form, too.)

To ensure zero closure of the equations of type 2, they must get very high weights (Table II shows that the computations were carried out even with weights as long as $p = 1000$ and $p = 10\,000$ for this levelling network).

Table I

Correction equations

	δI	δII	δIII	δIV	δA	δB	δC	l mm	p
v_1				−1	+1			0	3.333
v_2	+1				−1			0	3.000
v_3	+1					−1		− 7	2.727
v_4				−1		+1		+21	2.308
v_5	−1	+1						+23	1.500
v_6		+1	−1					0	5.000
v_7			+1			−1		0	6.000
v_8		+1					−1	+74	1.000
v_9			+1				−1	+50	2.500
v_{10}				−1			+1	0	1.111
v_{11}					+1	−1		0	10 000.0
v_{12}						−1	+1	0	10 000.0
v_{13}	+1	+1	+1	+1	+1	+1	+1	0	10 000.0

Table I shows the measurement results for the levelling network in Fig. 1 (the block inside the dotted line corresponds to the correction equations of the traditional adjustment). The zero corrections v_{11} and v_{12} are due to the constraint on height differences between points with fixed heights. The number of such constraints depends on the number of points with fixed heights, and in each equation only a single correction appears, unlike to the method of conditional equations. From the correction of the v_{13} type, corresponding to the geometrical constraint of the network, there is only one in levelling networks (their number is fixed for any type of networks). Such constraints are called *pseudo-constraints* to distinguish them from other types of constraints.

Table II contains the results of the adjustment of the levelling network according to the method of correction equations with different weights, and

Table II

Number of point	Number of measurements	Traditional adjustment		Adjustment as free network			
				$p = 1000$		$p = 10\,000$	
		δz (mm)	corrections v (mm)	δz (mm)	corrections v (mm)	δz (mm)	corrections v (mm)
I	1	1.827	−7.178	7.111	−7.185	7.132	−7.178
II	2	−26.921	1.827	−21.608	1.820	−21.615	1.827
III	3	−19.230	−5.172	−13.925	−5.149	−13.924	−5.169
IV	4	7.178	13.821	12.476	13.785	12.483	13.818
A	5	0	−5.748	5.291	−5.720	5.305	−5.748
B	6	0	−7.691	5.261	−7.683	5.302	−7.691
C	7	0	−19.230	5.393	−19.186	5.315	−19.226
	8		47.078		46.998		47.068
	9		30.769		30.681		30.760
	10		−7.178		−7.082		−7.168
					0.029		0.002
					0.131		0.013
					0.000		0.000

as a free network. The latter has been computed twice to see the changes of the closure of the last three correction equations for $p = 1000$ and $p = 10\,000$.

The advantage of the method described is that the computation work does not increase overduly even though the adjustment is made as for a free network, and many points of fixed height are used. From the constraints of both types correction equations are constructed by means of large weights (without measurement results). This results in an increased dimension of the form matrix as compared to a network with a single point of fixed height. Taking this into account, the adjustment can be carried out identically with the traditional adjustment, but its result will approach the result of the free network adjustment.

REFERENCES

1. V. HETÉNYI, M.: Free-network adjustment with constraints. *Acta Geod. Geoph. Mont. Hung.*, 1976 (11) 311—319.
2. HAZAY, I.: Adjustment methods. Tankönyvkiadó, Budapest 1968 (In Hungarian).

ДОПОЛНЕНИЕ К УРАВНИВАНИЮ СЕТЕЙ КАК СВОБОДНЫХ

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РЕЗЮМЕ

Описывается уравнивание нивелирной сети как свободной, где несколько пунктов имеют заданные высоты. В этом случае, помимо геометрического — т. н. псевдоусловия [1], должны быть использованы еще некоторые принудительные условия, соответствующие заданной расстановке.

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GENERAL PROPERTIES OF GRAVITATIONAL *w*-FUNCTIONS

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Gravitational and magnetic filters can be planned by using the demands for the weight functions w which have been introduced in [4] and which express the previously prescribed connection between geological structure and filtered data system (see also [5] and [6]). An arbitrary demand cannot be, however, fulfilled for the whole half-space $z > 0$, therefore it is necessary to study the general properties of the class of all possible w-functions. — The results of these investigations are presented in the first part of this paper in form of some general statements about gravitational w-functions. — The second part of the paper deals with a method for derivation of a filter whose w-function approximates an advantageous property the exact fulfilment of which is theoretically impossible. The degree of the approximation has been tested with the method described in [8]. This is at the same time an example to show that it is not sufficient to characterize the domain of realizable w-s by already unrealizable conditions; the possibilities must be enlarged by special, target-oriented solutions. The frame of the development is, however, set by general conclusions about w-functions, thus the methods of investigations in different directions and their success also depend on them.

1.

The increasing need for a quantitative interpretation of filtered data systems more and more demands the study of the connection between anomalous body and filtered data system, or even necessitates the planning of filters realizing prescribed properties of the weight function w, expressing thus connections between the geologic structure and the filtered data system. (For the definition of the function w see, e.g. Eq. 1 in [8].)

The results from [5] and [6] (mainly from the latter where conditions for the connection between anomalous body — filtered data system were fulfilled for a fixed dimension of the filter) indicate that many demands can be fulfilled with the w-functions, perhaps together with other conditions. This statement is generally true, but the following considerations show general properties of w-functions which limit the possibilities to plan filters based on prescribed w-s.

For the gravity interpretation it would be e.g. advantageous to compute filters for certain exploration tasks, the w-functions of which should have values ≥ 0 in a certain depth interval and a value 0 (or approximately 0) everywhere above and below this level. Such a depth-emphasizing filter cannot be, however, realized, as according to Section 1.1 the integral of the w-values

is in all depth levels equal for the whole horizontal plane; this quantity is only characteristic for the filter, but independent of depth.

Filters are often used for which the value of this depth-independent integral is zero; the domains of positive and negative w-values are counterbalanced (in sense of a surface-integral). Hence the thesis defined in Section 1.1 would yet allow that a filter should have w-values with high absolute values in a certain depth level, whereas in other levels the value of the w-function should be zero or approximately zero. For the depth interval above the explored level this cannot be realized, as according to Section 1.21 the integral of the square of w-values on a horizontal plane, i.e. the norm-square denoted by $n(z)$ decreases rigorously monotonously with the depth.

For the method to determine anomalous bodies described in [1] it would be advantageous to use filters characterized by very small variations of the w-values with depth around the average level of the density contrast. It can be very simply shown that if in some depth level $\partial w/\partial z \to 0$ is fulfilled, the expression of the connection between changes of the contrast surface and the filtered data system becomes more simple. — In Section 1.22 it will be shown that this condition cannot be exactly fulfilled; Part 2 of this paper gives an approximative solution for the problem.

Investigations described in [8] give direct informations about the values of the w-functions in the domain of their maximum; it is, however, often necessary to have a more complete information about the whole depth domain. The most important question is in which depth will be for any map transformation the integral $n(z)$ of the square of the w-values insignificant in respect of its value in an upper reference level $n(z_0)$. Such computations can be more easily performed using the connections deduced in Section 1.23 which reduce the computation of the norm-square to a Laplace-transformation.

At the end of Part 1 the question is raised whether the use of general quantitative methods with a comparatively high computational demand for the determination of an anomalous body could not be substituted by a filter reflecting proportionally or in accordance with any other simple function the changes of the contrast level $h(x, y)$ in the filtered values $r(x, y)$. Section 1.3 shows that no filter exists which could solve this problem even in case of small variations of the contrast level. Therefore it is unavoidable to develop general quantitative methods for the manipulation of filtered data systems because a very difficult development aiming at the correlation 1 between anomalous surface and filtered data system (which can be realized in certain limited accuracy limits) cannot reach the precision which is possible by using general quantitative methods.

1.1 In the following the notations of [4] will be used, thus we denote by $t(x, y)$ the map transformation, by p and q the frequency coordinates and by capitals Fourier transforms. Be further $\hat{p} = \sqrt{p^2+q^2}$.

The two-variable Fourier-transform of $w(x, y, z)$ corresponding to a fixed value of z is according to [4]:

$$W(p, q) = \text{const} \cdot e^{-z\hat{p}} \cdot T(p, q), \tag{1}$$

and the equation defining W is:

$$W(p, q) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w(x, y, z)\, e^{-j(px+qy)}\, dx\, dy\,. \tag{2}$$

From the latter the integral of $w(x, y, z)$ on a plane in the depth z_0 is equal with $W(0, 0)$, and this is according to (1), as $\hat{p} = 0$, equal with a constant $T(0, 0)$. In form of an equation one has:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w(x, y, z_0)\, dx\, dy = W(0, 0) = \text{const} \cdot T(0, 0)\,; \tag{3}$$

and the last is a quantity independent from the depth z_0.

1.2 In this section we shall investigate the properties of the norm-square

$$n(z) = \| w(x, y, z) \|^2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w^2(x, y, z)\, dx\, dy \tag{4}$$

excluding the trivial case $w(x, y, z) \equiv 0$ corresponding to $t(x, y) \equiv 0$.

1.21 At first we shall show that the function $n(z)$ is rigorously monotonously decreasing.

In order to prove this we shall use the well-known Parseval-formula stating

$$n(z) = \| w(x, y, z) \|^2 = \text{const} \| W(p, q) \|^2. \tag{5}$$

Taking (1) into account:

$$n(z) = c \| e^{-z\hat{p}} \cdot T(p, q) \|^2 = c \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2z\hat{p}}\, T^2(p, q)\, dp\, dq, \tag{6}$$

where c denotes a positive constant.

After derivation, Eq. (6) yields the following:

$$n'(z) = -2c \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{p}\, e^{-2z\hat{p}}\, T^2(p, q)\, dp\, dq < 0, \tag{7}$$

as the integrand is always positive. Thus the statement is proved.

1.211 Similarly it can be shown that

$$n''(z) = 4\,c \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{p}^2\, e^{-2z\hat{p}}\, T^2(p, q)\, dp\, dq > 0. \tag{8}$$

The function $n(z)$ is convex, that means that the increase with decreasing z is in every interval ever greater.

It should be remarked that the k-th derivate of $n(z)$ has changing sign in k, as it can be shown similarly to the previous case.

1.22 The next question is whether the derivate of $w(x, y, z)$ after z could be zero at a plane z_0. This must be answered in the negative as if at a plane z_0 would be

$$\left.\frac{\partial w(x, y, z)}{\partial z}\right|_{z=z_0} \equiv 0 \tag{9}$$

the equation

$$\begin{aligned} 0 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\left.\frac{\partial w(x, y, z)}{\partial z}\right|_{z=z_0}\right]^2 dx\, dy = \\ &= \frac{\partial}{\partial z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[w(x, y, z)\right]^2 dx\, dy\,|_{z=z_0} = n'(z_0) \end{aligned} \tag{10}$$

would hold, and this is in contradiction with the statement about $n'(z)$ in Section 1.2 (see Eq. 7).

1.23 The character of $n(z)$ in case of a map transformation given by its transfer $T(p, q) = T(\hat{p})$ can be often simply deduced by using the following remark.

Let $u(z)$ be the Laplace-transform of the function $\hat{p}T^2(\hat{p})$, i.e.

$$u(z) = \mathfrak{L}\,(\hat{p}T^2(\hat{p})\,. \tag{11}$$

In that case

$$\begin{aligned} n(z) &= \text{const} \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2z\sqrt{p^2+q^2}}\, T^2(p, q)\, dp\, dq = \\ &= \nu \int_0^{\infty} \hat{p}\, e^{-2z\hat{p}}\, T^2(\hat{p})\, d\hat{p} = \nu \cdot u\,(2z)\,, \end{aligned} \tag{12}$$

where ν is a positive constant.

As practical examples geophysically important cases will be shown in the following.

1.231 The analytical continuation of g to a level in the height h.

In that case

$$T(\hat{p}) = e^{-h\hat{p}}, \tag{13}$$

when (see, e.g. [2], p. 144):

$$u(z) = \mathfrak{L}\{\hat{p}T^2(\hat{p})\} = \mathfrak{L}\{\hat{p}\, e^{-2h\hat{p}}\} = \frac{1}{(z+2h)^2}\,. \tag{14}$$

From these:

$$n(z) = vu(2z) = \frac{v}{4}\,\frac{1}{(z+h)^2}\,. \tag{15}$$

Figure 1 shows the numerical values of $n(z)/n(z_0)$ for a value of z_0 equal to $2s = 1000$ m.

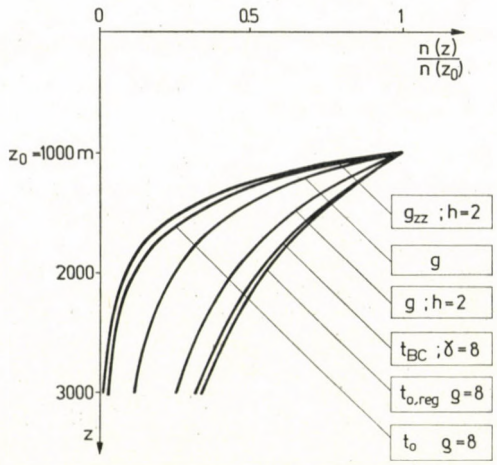

Fig. 1. The depth changes of the norm square ($n(z)$) of $w(x, y, z)$ on horizontal planes for some map transformations

1.232 The analytical upward continuation of the k-th derivate after z of g.

As it is known (see [7]), it is true for the k-th derivate continued to the height h:

$$T(\hat{p}) = \hat{p}^k e^{-h\hat{p}}, \tag{16}$$

from which follows that:

$$u(z) = \mathfrak{L}\{\hat{p}T^2(\hat{p})\} = \mathfrak{L}\{\hat{p}^{2k+1}\, e^{-2h\hat{p}}\} = \frac{(2k+1)\,!}{(z+2h)^{2k+2}}\,. \tag{17}$$

That means

$$n(z) = v \cdot u(2z) = \frac{v \cdot (2k+1)\,!}{2^{2k+2}}\,\frac{1}{(z+h)^{2k+2}}\,. \tag{18}$$

Figure 1 shows for the practically important case $k = 2$ the values of $n(z)/n(z_0)$ ($z_0 = 1000$ m).

1.233 The Bullard—Cooper smoothing.
In this case the Fourier-transform is ([3], p. 219):

$$T(\hat{p}) = e^{-\gamma p^2}. \tag{19}$$

Therefore one has ([2], p. 146):

$$\begin{aligned} u(z) &= \mathfrak{L}\{p \cdot e^{-2\gamma\hat{p}^2}\} = \mathfrak{L}\left\{pe^{-\frac{\hat{p}^2}{4\cdot\frac{1}{8\gamma}}}\right\} = \\ &= 2\frac{1}{8\gamma} - 2\sqrt{\pi}\left(\frac{1}{8\gamma}\right)^{3/2} z e^{z^2/8\gamma}\,\mathrm{Erfc}\left(\frac{z}{\sqrt{8\gamma}}\right) = \\ &= \frac{1}{4\gamma} - \frac{1}{8}\sqrt{\frac{\pi}{2}}\frac{z}{\gamma^{3/2}} e^{z^2/8\gamma}\frac{2}{\sqrt{\pi}}\int_{z/\sqrt{8\gamma}}^{\infty} e^{-t^2}\,dt = \frac{1}{4\gamma} - \\ &- \frac{1}{8}\sqrt{\frac{\pi}{2}}\frac{z}{\gamma^{3/2}} e^{z^2/8\gamma}\frac{2}{\sqrt{\pi}}\left[\int_0^{\infty} e^{-t^2}\,dt - \int_0^{z/\sqrt{8\gamma}} e^{-t^2}\,dt\right] = \\ &= \frac{1}{4\gamma} - \frac{1}{8}\sqrt{\frac{\pi}{2}}\frac{z}{\gamma^{3/2}} e^{z^2/8\gamma}\left[1 - \Phi\left(\frac{z}{\sqrt{8\gamma}}\right)\right]. \end{aligned} \tag{20}$$

Now according to Eq. (12) we get:

$$\begin{aligned} n(z) &= \frac{\nu}{4\gamma} - \frac{\nu}{4}\sqrt{\frac{\pi}{2}}\frac{z}{\gamma^{3/2}} e^{z^2/2\gamma}\left[1 - \Phi\left(\frac{z}{\sqrt{2\gamma}}\right)\right] = \\ &= \frac{\nu}{4\gamma}\left\{1 - \sqrt{\frac{\pi}{2\gamma}}\, z\, e^{z^2/2\gamma}\left[1 - \Phi\left(\frac{z}{\sqrt{2\gamma}}\right)\right]\right\}. \end{aligned} \tag{21}$$

Figure 1 shows the curve of the smoothing belonging to the value $\gamma = 8$ ($z_0 = 1000$ m).

1.234 The simplest residual transformation denoted by t_0 means a subtraction of the average of g values on surface of a concentric circle with the radius ϱ from the g-value of the reference point. In that case the transfer function has the following form:

$$T(\hat{p}) = 1 - \frac{2}{\varrho\hat{p}} \cdot J_1(\varrho\hat{p}). \tag{22}$$

According to this one has further ([2], pp. 182—183):

$$
\begin{aligned}
u(z) &= \mathfrak{L}\{\hat{p}T^2(\hat{p})\} = \mathfrak{L}\left\{\hat{p} - \frac{4}{\varrho}J_1(\varrho\hat{p}) + \frac{4}{\varrho^2\hat{p}}J_1(\varrho\hat{p})\right\} = \\
&= \frac{1}{z^2} - \frac{4}{z\sqrt{z^2+\varrho^2}+z^2+\varrho^2} - \frac{4}{\varrho^2}\frac{d}{dz}\mathfrak{L}\left\{\frac{J_1^2(\varrho\hat{p})}{\hat{p}^2}\right\} = \\
&= \frac{1}{z^2} - \frac{4}{z\sqrt{z^2+\varrho^2}+z^2+\varrho^2} - \\
&- \frac{2}{\pi\varrho^2}\frac{d}{dz}\int_0^\pi \left[\sqrt{z^2+2\varrho^2-2\varrho^2\cos\varphi} - z\right](1+\cos\varphi)\,d\varphi = \\
&= \frac{1}{z^2} - \frac{4}{z\sqrt{z^2+\varrho^2}+z^2+\varrho^2} - \\
&- \frac{2}{\pi\varrho^2}\int_0^\pi \left(\frac{z}{\sqrt{z^2+2\varrho^2-2\varrho^2\cos\varphi}} - 1\right)(1+\cos\varphi)\,d\varphi = \\
&= \frac{1}{z^2} + \frac{2}{\varrho^2} - \frac{4}{z\sqrt{z^2+\varrho^2}+z^2+\varrho^2} - \\
&- \frac{2z}{\pi\varrho^2}\int_0^\pi \frac{1+\cos\varphi}{\sqrt{z^2+2\varrho^2-2\varrho^2\cos\varphi}}\,d\varphi .
\end{aligned}
\tag{23}
$$

From this one gets:

$$
\begin{aligned}
n(z) = \nu u(2z) = \nu\Bigg[&\frac{1}{4z^2} + \frac{2}{\varrho^2} - \frac{4}{2z\sqrt{4z^2+\varrho^2}+4z^2+\varrho^2} - \\
&- \frac{4z}{\pi\varrho^2}\int_0^\pi \frac{1+\cos\varphi}{\sqrt{4z^2+2\varrho^2-2\varrho^2\cos\varphi}}\,d\varphi\Bigg] .
\end{aligned}
\tag{24}
$$

It can be easily seen that the $n(z)$ values of the regional map transformations of the type $\varrho(x, y) - t_0(x, y)$ can be computed from the following formula:

$$
n(z) = \nu\left[\frac{2}{\varrho^2} - \frac{4z}{\pi\varrho^2}\int_0^\pi \frac{1+\cos\varphi}{\sqrt{4z^2+2\varrho^2(1-\cos\varphi)}}\,d\varphi\right] . \tag{25}
$$

The curves $n(z)/n(z_0)$ of the residual and regional filters corresponding to $\varrho = 8$ can also be seen on Fig. 1 (z_0 is here 1000 m, too).

1.3 Be $h(x, y)$ the height of the density contrast surface above the average level of this surface z_0. No such $t(x, y)$ filter can be found at the use of which

between the filtered values $r(x, y)$ and $h(x, y)$ the relation:

$$r(x, y) = f(h(x, y)) \tag{26}$$

would hold (here $f(x)$ is a monotonous function). Eq. (26) cannot be fulfilled even in case of small changes of the depth of the contrast surface.

One has in case of small depth changes:

$$\sigma(x, y) = \Delta\sigma\, h(x, y), \tag{27}$$

and the filtered value can be written as (see [4], Eq. 8):

$$r(x, y) = f(h(x, y)) = \Delta\sigma \cdot h(x, y) * w(x, y). \tag{28}$$

The form of $f(x)$ that would be allowed by Eq. (28) itself can be derived immediately. In case of superposition we have:

$$\begin{aligned} f(h_1(x, y) + h_2(x, y)) &= \Delta\sigma(h_1(x, y) + h_2(x, y)) * w(x, y) = \\ &= \Delta\sigma[h_1(x, y) * w(x, y) + h_2(x, y) * w(x, y)] = \\ &= f(h_1(x, y)) + f(h_2(x, y)) \end{aligned} \tag{29}$$

i.e. $f(x)$ could be only of the form $c \cdot x$, where c is a constant. In such a case, however, one has according to Eq. (28) in the spatial domain:

$$c \cdot h(x, y) = \Delta\sigma h(x, y) * w(x, y)\,, \tag{30}$$

and in the frequency domain:

$$c \cdot H(p, q) = \Delta\sigma\, H(p, q) \cdot W(p, q); \tag{31}$$

and taking Eq. (1) into account:

$$W(p, q) = c = \text{const} \cdot e^{-z_0\sqrt{p^2+q^2}}\, T(p, q)\,. \tag{32}$$

This cannot be realized with the usual convolution technics, as the integral yielding the elements of the filter matrix $t(x, y)$ of the W according to Eq. (32):

$$t(x, y) = \frac{1}{2\pi}\int_0^\infty \hat{p}\; e^{z_0\hat{p}}\; J_0\left(\hat{p}\,\sqrt{x^2 + y^2}\right) d\hat{p} \tag{33}$$

is divergent.

The practical importance of the theorem has been already mentioned in the introduction; the difficulties of the development of general quantitative methods cannot be avoided by a single filtering procedure.

2.

It is known that the function $w(x, y, z)$ can be computed as the filtered effect of a point-like mass, i.e. it can be expressed with the following integral:

$$w(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{z}{[z^2 + (y - \bar{y})^2 + (x - \bar{x})^2]^{3/2}} t(\bar{x}, \bar{y})\, d\bar{x}\, d\bar{y}\,. \tag{34}$$

As supposed the function $t(x, y)$ has circular symmetry, hence it is advantageous to use cylindrical coordinates x, z, ϑ where the last coordinate does not appear in the expression of w due to the cylindrical symmetry

$$\begin{aligned} w(x,z) &= \int_{-\infty}^{\infty} t(\bar{x}) \left[\bar{x} \frac{z}{(z^2 + \bar{x}^2 + x^2 - 2x\bar{x}\cos\varphi)^{3/2}} d\varphi \right] d\bar{x} = \\ &= \int_0^{\infty} t(\bar{x}) \Phi(x, z, \bar{x})\, d\bar{x}\,. \end{aligned} \tag{35}$$

Due to the causes mentioned in the introduction of Part 1, i.e. in order to enable the possibly most effective use of the method described in [1], it is demanded from the $w(x, y)$ that:

a) the value of $\partial w(x, z)/\partial z$ should be possibly near to zero for a given depth range z_0;

b) in the same depth the values should be near to a given series $f(x_i, z_0)$ $(i = 1, 2, \ldots, m)$.

Let us define Ψ similarly to Φ in Eq. (35):

$$\begin{aligned} \frac{\partial w(x, z)}{\partial z} &= \int_0^{\infty} t(\bar{x}) \left[\bar{x} \int_0^{2\pi} \left(\frac{1}{(z^2 + \bar{x}^2 + x^2 - 2x\bar{x}\cos\varphi)^{3/2}} - \right. \right. \\ &\left. \left. - \frac{3z^2}{(z^2 + \bar{x}^2 + x^2 - 2x\bar{x}\cos\varphi)^{5/2}} \right) d\varphi \right] d\bar{x} = \\ &= \int_0^{\infty} t(\bar{x})\, \Psi(x, z, \bar{x})\, d\bar{x}\,. \end{aligned} \tag{36}$$

It can be seen that the expressions for $w(x, z)$ and for $\partial w(x, z)/\partial z$ have similar character, therefore the demands a and b refer to a product-integral of the function $t(x)$ to be determined; consequently it is advantageous to produce the functions appearing in the product-integral by expansion into series after some orthonormed function system.

Bearing in mind that the filter matrix should have practicable dimensions, the orthonormed system should be the parabolic cylindrical function

system:

$$\varphi_k(x) = \frac{e^{-x^2/2} H_k(x)}{\sqrt{k!\, 2^k \sqrt{\pi}}} \quad (k = 0, 1, \ldots) \tag{37}$$

($H_k(x)$ is a Hermitian polynomial). As the functions Φ and Ψ must be known only for values $\bar{x} \geq 0$, the series expansion must be made for even order cylindrical functions.

Thus let be:

$$\left.\begin{aligned} \Phi(x, z, \bar{x}) &= \sum_{i=0}^{\infty} \Phi_{2i}(x, z)\, \varphi_{2i}(\bar{x}) \,, \\ \Psi(x, z, \bar{x}) &= \sum_{i=0}^{\infty} \Psi_{2i}(x, z) \varphi_{2i}(\bar{x}) \,, \end{aligned}\right\} \tag{38}$$

where:

$$\Phi_{2i}(x, z) = 2\int_0^{\infty} \Phi(x, z, \bar{x})\, \varphi_{2i}(\bar{x})\, d\bar{x}; \quad \Psi_{2i}(x, z) = 2\int_0^{\infty} \Psi(x, z, \bar{x})\, \varphi_{2i}(\bar{x})\, d\bar{x} \,,$$

and the function $t(x)$ is looked for in a form

$$t(x) = \sum_{i=0}^{n} c_{2i}\, \varphi_{2i}(x). \tag{39}$$

Then taking use of the orthonormed character of the system $\langle \varphi_k(x) \rangle$ one has:

and

$$\left.\begin{aligned} w(x, z) &= \sum_{i=0}^{n} c_{2i} \Phi_{2i}(x, z) \\ \frac{\partial w(x, z)}{\partial z} &= \sum_{i=0}^{n} c_{2i} \Psi_{2i}(x, z) \end{aligned}\right\}. \tag{40}$$

We shall use the least squares method, i.e. we want to approximate the exactly not fulfilled condition a and b as well as possible:

$$\left.\begin{aligned} \left.\frac{\partial w(x_i, z)}{\partial z}\right|_{z=z_0} &= \sum_{i=0}^{n} c_{2i} \Psi_{2i}(x_i, z_0) = 0 \\ & \qquad (i = 1, \ldots, m) \\ w(x_i, z_0) &= \sum_{i=0}^{n} c_{2i} \Phi_{2i}(x_i, z_0) = f(x_i, z_0) \end{aligned}\right\}. \tag{41}$$

To compute $n + 1$ coefficients c_k we have $2m$ equations. The different conditions can get different weights by multiplicating the corresponding equations.

The condition

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} t(x, y)\, dx\, dy = 0 \tag{42}$$

can also be easily fulfilled, meaning an equation for the coefficients c_{2i}:

$$\begin{aligned}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} t(x,y)\, dx dy = 2\pi\int_0^{\infty} x\, t(x)\, dx = 2\pi\int_0^{\infty}\left[x\sum_{i=0}^{n} c_{2i}\varphi_{2i}(x)\right]dx = \\ = 2\pi\sum_{i=0}^{n} c_{2i}\int_0^{\infty} x\varphi_{2i}(x)\, dx = 2\pi\sum_{i=0}^{n} c_{2i}\, a_{2i} = 0\end{aligned} \tag{43}$$

in what follows we shall use this in the more comfortable form

$$\frac{1}{2}\pi^{-3/4}\sum_{i=0}^{n} c_{2i}\, a_{2i} = 0\,. \tag{43a}$$

The integrals a_{2i} appearing in the summation are the linear combinations of integrals:

$$\int_0^{\infty} x^{2k+1}\, e^{-x^2/2}\, dx = 2^k k! \tag{44}$$

resulting from the form of $\varphi_{2i}(x)$.

The numerical values of the constants a_{2i} in Eq. (43a) can be determined from the Eqs (37) and (44), some of them have the following values:

$$a_0 = 1; \qquad a_2 = \frac{3}{\sqrt{2}}; \qquad a_4 = \frac{11}{2\sqrt{6}};$$

$$a_6 = \frac{23}{4\sqrt{5}}; \qquad a_8 = \frac{179}{8\sqrt{70}}; \qquad a_{10} = \frac{365}{48\sqrt{7}};$$

If $n = 5$, i.e. the parabolic cylindric functions are up to $\varphi_{10}(x)$ taken into account for the construction of $t(x, y)$ according to Eq. (39), then by substituting these numerical values into the conditional Eqs (43a), one of the c_{2i}-s (advantageously c_0) can be expressed by the other five c_{2i}-s, and the procedure of the adjustment contains only five unknowns: c_2, c_4, c_6, c_8, c_{10}.

It is clear that different weighting of Eqs (41) yields filters $t(x, y)$ which can be characterized by different $w(x, y, z)$, the properties of these filters can be most easily studied by using the method described in [8], i.e. by w-isolines.

The $t(x, y)$-s computed from Eq. (41) are at first normed to 1 in order to make the error of the filtered values equal to the known error of the initial gravity data system.

Figure 2 shows the w-isolines of the filter resulting from equal weight equations of the system (41), if the conditions are taken for the depth level of 1500 m. The demands for the w-system are hardly fulfilled by this transformation but it yields for $\partial w/\partial z$ very small values in respect of the usual transformations. The computational possibilities and the geological data at disposal determine the importance of the small side-effect at cost of a moderate increase of the value. This means nothing other than the last m equations of the system (41) get higher weights. The effective use of the algorithm described

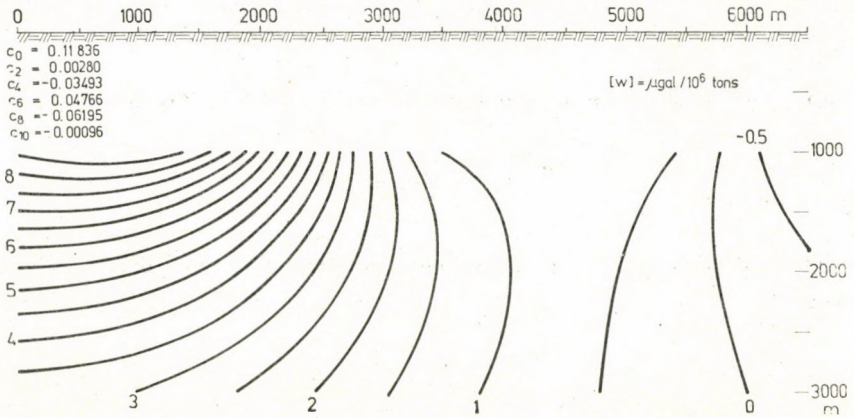

Fig. 2. The w-isolines of the map transformation approximating $\partial w/\partial z = 0$ at a horizontal plane (the elements of the filter matrix $t(x, y)$ can be computed with the coefficients c_{2i} given in the text)

in [1] needs that the memory of the computer at disposal and its other data, as well as the small side-effect and the condition should be realized together in order to attain the optimum.

A general property of the w-function has been described in Part 2, too: by demanding several physical properties (w-properties) from the transformation the fulfillment of the demands can be improved only at each other's cost. There are, however, no obstacle to look for best fitted transformations with special methods in order to solve structural problems.

REFERENCES

1. CSERNYÁK, L.—STEINER, F.: Noniterative method for determination of the undulations of density contrast surfaces on ground of filtered maps. Manuscript, Miskolc, 1976.
2. ERDÉLYI, A.: Tables of Integral Transforms. Vol. I. McGraw-Hill, New York, 1954.
3. GRANT, F. S.—WEST, G. F.: Interpretation Theory in Applied Geophysics. McGraw-Hill, 1965.
4. STEINER, F.: Characterization and derivation of map transformations on the basis of the anomalous body–map relationship. *Acta Geod. Geoph. Mont. Hung.*, 8 (1973), 71—84.
5. STEINER, F.: Derivation of map-transformations with small lateral effect. *Acta Geod. Geoph. Mont. Hung.*, 9 (1974), 255—260.

6. STEINER, F.: Derivation of gravity and magnetic filters of given physical effect and of given size. Proceedings of the 20th Geophysical Symposium Budapest-Szentendre. OMKDK-Technoinform, Budapest, 1976.
7. STEINER, F.: The computation of the vertical derivatives of the upward continued values of g from a gravimetric data system. *A Nehézipari Műszaki Egyetem Közleményei*, Miskolc, 34 N° 1—2 (1977).
8. STEINER, F.—ZILAHI-SEBESS, L.: The study of gravitational weight functions. *Magyar Geofizika*, 18 (1977), 10—17.

ОБЩИЕ СВОЙСТВА ГРАВИТАЦИОННЫХ w-ФУНКЦИЙ

Л. ЧЕРНЯК—Ф. ШТЕЙНЕР

РЕЗЮМЕ

Фильтры гравитационных и магнитных карт могут быть запланированы на основе требований, определенных для введенных в [4] весовых функций w, т. е. реализующих заданную заранее связь тектонической структуры с профильтрованной системой величин [5, 6]. Но для всего полуполя $z > 0$ произвольное требование не может;быть выполнено, поэтому обосновано изучать общие свойства класса всех возможных функций $w(x, y, z)$. — Результаты относящихся сюда исследований заключаются в первой части статьи в некоторых общих определениях о гравитационных w-функций. — Во второй части статьи показывается метод приближенного осуществления w-свойства, не выполняемого теоретически строго, но выгодного для алгорифма определения действующей; мера приближения была контролирована методом, описанным в [8]. Этим за одно показывается пример, что область осуществляемых w не достаточно характеризовать невыполнимыми уже условиями целе7риентированными решениями. Но рамки дальнейшего развития, так и методы и ожидаемую результативность в отдельных направлениях определяют обиие постановления для w-функций.

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FELDKOMPARATOR FÜR INVARBAND-NIVELLIERLATTEN

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[Eingegangen am 31. Mai 1976]

Es wird ein Komparator beschrieben, der mit eingebauter Halbmeter-Basis auch bei Feldarbeiten zur Komparierung der Invarband-Nivellierlatten gut geeignet ist.

Die Konstruktionsprinzipien und die Wirkung der einzelnen Elemente des Gerätes werden diskutiert. Durch ein Zahlenbeispiel wird eine Meß- und Auswertemethode beschrieben, und eine neue Zeiss-sche Latte mit einer alten, abgenutzten Rost-schen Latte verglichen.

Die Frage wo und in welchen Abständen der Latte die Beobachtungen durchzuführen sind, wird ebenfalls untersucht.

Präzisionsnivellements und die Messungen der langperiodischen vertikalen Erdkrustenbewegungen erfordern nicht nur leistungsfähige Nivellierinstrumente, sondern auch präzise Nivellierlatten, deren Länge zu jeder Zeit genau bekannt sein muß. Bei hohen Genauigkeitsanforderungen sollen deswegen nicht nur die Instrumente, sondern auch die Nivellierlatten von Zeit zu Zeit geprüft werden. In disem Sinne wurde im Geodätischen und Geophysikalischen Forschungsinstitut der Ungarischen Akademie der Wissenschaften zu Sopron ein Lattenkomparator entwickelt, um das mittlere Lattenmeter und den mittleren Teilstrichfehler einer mit Invarband ausgestatteten Nivellierlatte festzustellen. Das Gerät ist, infolge seiner kleinen Abmessungen und seines geringen Gewichtes, auch für Feldarbeiten gut geeignet. Der Komparator wurde nach den Konstruktionsplänen von D. Beskó in der Werkstatt des Geodätischen und Geophysikalischen Forschungsinstitutes der UAdW unter der Leitung von A. Bummer verfertigt.

Konstruktionsprinzip und Aufbau

Der Komparator eignet sich zur Ermittlung der Entfernungen zweier voneinander 50 cm weit liegender Teilstriche am Invarband. Das Gerät funktioniert in horizontaler Lage, demzufolge muß auch die Latte mit dem Invarband horizontal liegen [3].

Die zwei Pentaprismen (Abb. 1) projektieren die Bilder der zwei Teilstriche des Bandes durch die zweigliedrigen Objektive und durch die Mikrometerscheibe bzw. Korrektionsplatte auf die Trennkante eines Prismensystems, wo diese in Koinzidenz-Anordnung durch das Okular betrachtet werden kön-

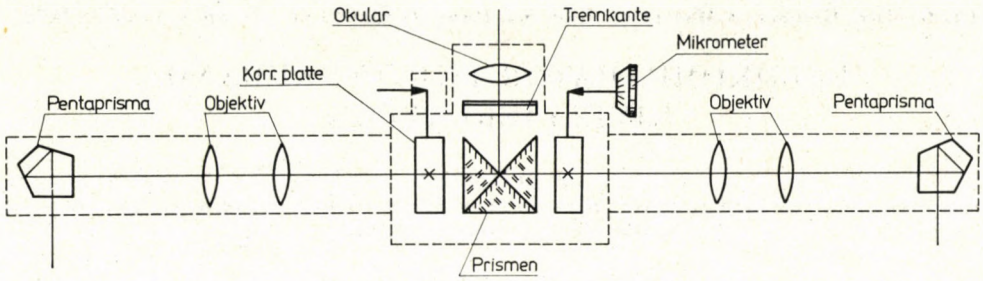

Abb. 1

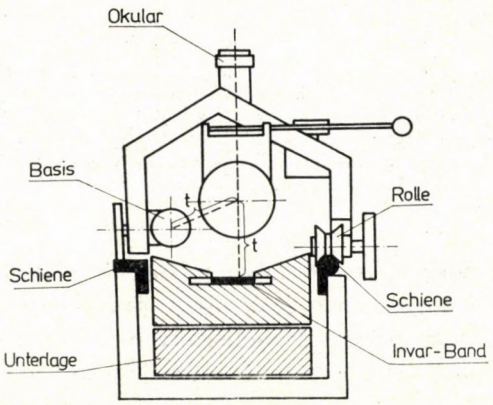

Abb. 2

nen. Durch Drehen des optischen Mikrometers werden die Teilstriche im Gesichtsfeld des Okulars in Koinzidenz gebracht, wodurch die Abweichung der Teilstrichentfernung von 500 mm festgestellt wird.

Das optische System wurde in Invar-Röhre, die anderen Teile in eine Aluminiumlegierung mit kleinem Ausdehnungskoeffizienten montiert.

Das 0,5 m lange Halbmeter-Etalon (Längenbasis) wurde im Komparator so angebracht, daß bei Kippen des optischen Systems die Objektenentfernungen gleich bleiben sollen (Prisma-Basis gleich Prisma-Lattenstrich). Abb. 2 zeigt den Querschnitt der Anordnung.

Die Längenbasis wurde aus einem speziellen keramischen Material mit sehr kleinem thermischem Ausdehnungskoeffizienten α hergestellt, mit Endmarken versehen und in ein Invar-Rohr mit elastischer Fassung eingebaut.

Prüfung und Eichung dieser Basis wurde im Amt für Meßwesen in Budapest durchgeführt. Daraus ergab sich die Länge der Basis als:

$$l = 500 \text{ mm} + 0{,}062 \text{ mm} \pm 0{,}002 \text{ mm}$$

und der Skalenwert des Mikrometers: 0,004 65 mm/Trommelteilung.

Eine Temperaturempfindlichkeit ist in der Größenordnung dieser Meßgenauigkeit nicht feststellbar, daher $\alpha = 0$.

Das Gerät rollt an einer Schienenbahn und berührt sie nur an drei Stellen (Abb. 2 und 3).

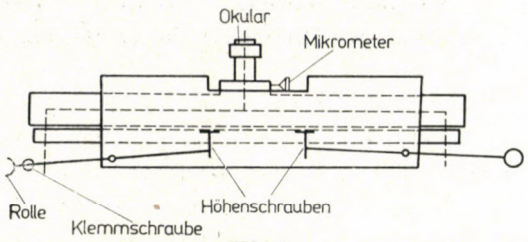

Abb. 3

Durchführung der Messungen

Die zu untersuchende Invarband-Nivellierlatte wird in die, mit der Rollbahn versehenen Kiste gelegt und mit Unterlagen in die Nähe der entsprechenden Entfernung vom optischen System gebracht (Abb. 2). Die Scharfstellung der Teilstriche geschieht durch heben oder senken des optischen Systems mit Hilfe der beiderseits angeordneten Rollen (Abb. 3).

Die Latte wird in der Kiste quer zur Längsrichtung so verschoben, daß entweder das eine oder das andere Teilstrichsystem der Doppelteilung des Invarbandes im Sehfeld des Komparators erscheinen soll. Um Kreuzungsfehler zu vermeiden, soll die Lattenteilung mit der optischen Längsachse des Komparators parallel sein. Mit Hilfe der Rollbahn kann man den Komparator über jeden Teilstrich des Lattenbandes stellen. Eine Klemm-Schraube am linken Rad dient zur Festhaltung des Komparators auf einer beliebigen Stelle.

Nach Einstellung und Scharfstellung der beiden Lattenstriche, richtet man das optische System auf die Längenbasis, stellt die zu 500,062 mm gehörige Ablesung am Mikrometer ein und richtet das optische System wieder in die Richtung der Lattenteilstriche. Nun wird ein Mikrometerwert abgelesen. Aus der Differenz der beiden Mikrometerwerte ΔK erhält man den Lattenhalbmeter:

$$L = B + \Delta K$$

wo $B = 500{,}062$ mm ist.

Das Lattenmeter

Unter dem Begriff Lattenmeter verstehen wir die im internationalen Meter ausgedrückte Entfernung zweier Teilstriche am Invarband, die nominell 1 m weit voneinander vermarkt wurden.

Das Lattenmeter weist an beliebigen Stellen des Invarbandes wegen mangelnder Maßbeständigkeit und Teilstrichgenauigkeit geringe Unterschiede auf. Laut der technischen Vorschriften sind die Fehlergrenzen für ein beliebiges Lattenintervall unter 0,03 mm zu halten. Während einer Lattenprüfung werden viele Lattenmeter gemessen; der Mittelwert dieser Messungen ergibt das mittlere Lattenmeter [2, 6].

Das Invarband ist gegen Temperaturschwankungen und Feuchtigkeitsl änderungen beinahe unempfindlich, es weist jedoch als eine aus 36% Nicke- und 64% Stahl bestehende Legierung eine labile Molekularstruktur auf und neigt infolge Erschütterungen oder Alterung zur Dehnung. Wird das Band während einer Messung oder beim Transport beschädigt, so erfolgt mit höchster Wahrscheinlichkeit eine plötzliche Änderung in der Lage des Lattenmeters zwischen zwei Meßstellen. Deshalb soll das Band während der Feldarbeit öfters geprüft bzw. ein neues Lattenmeter abgeleitet werden.

Auswertung der Meßergebnisse

Der Komparator ist mit einer 500,062 mm langen Basis ausgestattet, d. h. wir können damit ein mit 5 mm Doppelteilung versehenes 3 m langes Invarband an rund 1200 Strichen (an 1000 Strichpaaren) untersuchen. Es ist von praktischem Gesichtspunkt aus empfohlen das Lattenmeter nicht an allen möglichen Stellen, sondern nur z. B. dezimeterweise festzustellen und daraus einen mittleren Wert des Lattenmeters zu bilden. Es ist weiterhin ratsam, zur Längenbasis öfters einen Anschluß zu bilden.

Anschließend bringen wir noch ein Zahlenbeispiel der Beobachtung und der Ermittlung des mittleren Lattenmeters. An Abb. 4 sind die Mittelwerte graphisch dargestellt.

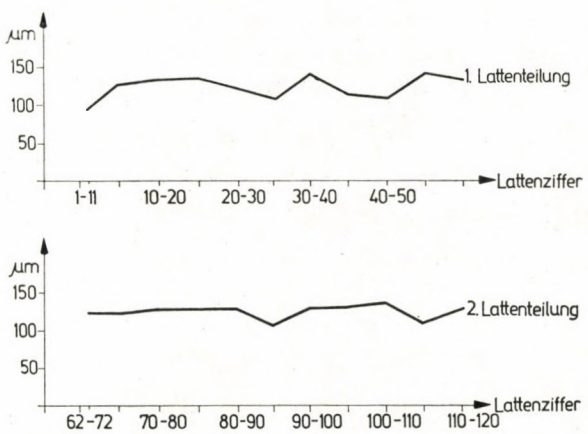

Abb. 4

Lattenmeterprüfung

Latte: Zeiss 30968 Datum: Sopron, 1.04.1976
Temperatur: 23,2 °C
Gleichung des Normalhalbmeters: $l = 500{,}062 \pm 0{,}002$ mm
Skalenwert des Mikrometers: 0,004 65 mm
Schätzwert des Mikrometers: 0,000 465 mm

Mikrometerablesungen vor und nach der Lattenprüfung:

	vor	nach
bei der 1. Lattenteilung	34,8	34,4
	34,6	34,8
	34,6	34,6
bei der 2. Lattenteilung	33,6	34,0
	33,4	33,7
	33,9	34,1
Mittelwert	34,2	
Red. 0,062 : 0,004 65	−13,3	
B_{500}:	20,9	

Für den mittleren Einstellfehler einer Doppelmessung ergibt sich:

$$m_D = \pm \frac{1}{2}\sqrt{\frac{[dd]}{n}} = \pm \frac{1}{2}\sqrt{\frac{16{,}33}{22}} \pm 0{,}43 \text{ Trommelwert, d. h.}$$

$$\pm 0{,}43 \cdot 0{,}004\,65 = \pm 0{,}002 \text{ mm}.$$

Für den Mittelwert der Lattenablesungen erhält man:

$$X_L = \frac{[M]}{n} = \frac{547{,}3}{22} = 24{,}9 \text{ (Tr)}.$$

Der Fehlerbetrag der einzelnen M-Werte beträgt:

$$m_L = \pm\sqrt{\frac{[vv]}{n-1}} = \pm\sqrt{\frac{112{,}27}{21}} = \pm 2{,}3 \text{ (Tr), d. h. } \pm$$

$$\pm 2{,}3 \cdot 0{,}004\,65 = \pm 0{,}011 \text{ mm}.$$

Dieser Wert darf aber nur als Bewertungszahl betrachtet werden, denn

Tabelle I

Beobachter: E. HALMAI *Protokol:* T. PLUZSIK

Prüfbereich	Hin-	Rück-	*d*	*dd*	Mittel	*v*	*vv*
1. Teilung	messung						
1—11	19,3	20,2	0,9	0,81	19,7	5,2	27,04
5—15	24,8	26,0	1,2	1,44	25,4	0,5	0,25
10—20	27,5	26,6	0,9	0,81	27,0	2,1	4,41
15—25	28,0	26,8	1,2	1,44	27,4	2,5	6,25
20—30	24,0	24,3	0,3	0,09	24,1	0,8	0,64
25—35	21,5	22,0	0,5	0,25	21,8	3,1	9,61
30—40	27,3	28,8	1,5	2,25	28,0	3,1	9,61
35—45	23,8	22,2	1,6	2,56	23,0	1,9	3,61
40—50	22,0	21,8	0,2	0,04	21,9	3,0	9,00
45—55	28,0	28,0	0,0	0,00	28,0	3,1	9,61
50—60	26,0	26,5	0,5	0,25	26,3	1,4	1,96
2. Teilung							
62—72	25,0	25,0	0,0	0,00	25,0	0,1	0,01
65—75	25,0	24,2	0,8	0,64	24,6	0,3	0,09
70—80	25,3	26,2	0,9	0,81	25,8	0,9	0,81
75—85	25,4	24,8	0,6	0,36	25,1	0,2	0,04
80—90	25,0	26,0	1,0	1,00	25,5	0,6	0,36
85—95	21,0	22,0	1,0	1,00	21,5	3,4	11,56
90—100	25,0	25,6	0,6	0,36	25,3	0,4	0,16
95—105	26,6	26,5	0,1	0,01	26,5	1,6	2,56
100—110	27,0	28,0	1,0	1,00	27,5	2,6	6,76
105—115	22,2	22,2	0,0	0,00	22,2	2,7	7,29
110—120	26,3	25,2	1,1	1,21	25,7	0,8	0,64
[ ]				16,33	547,3		112,27

ein Fischer-Test der Werte m_D und m_L weist auch darauf hin, daß im letzteren maßgebende, regelmäßige Fehleranteile vorhanden sind.

Der mittlere Fehler des Mittelwertes beträgt:

$$m_{XL} = \pm \frac{m_L}{\sqrt{n}} = \pm \frac{2{,}3}{\sqrt{22}} = \pm 0{,}49 \text{ (Tr), d. h.}$$

$$m_{XL} = \pm 0{,}49 \cdot 0{,}004\,65 = \pm 0{,}002 \text{ mm},$$

der Mittelwert des Lattenhalbmeters:

$$l_{1/2} = 500 \text{ mm} + 0{,}004\,65(X_L - B_{500}) \pm m_{XL} =$$
$$= 500 \text{ mm} + 0{,}019 \text{ mm} = \pm 0{,}002 \text{ mm}\,.$$

Die graphische Darstellung der Mittelwerte von Hin- und Rückmessungen der beiden Lattenteilungen enthält Abb. 4.

Weitere Bemerkungen

Der Einstellwert des Mikrometers auf die Basisendmarken, abgeleitet aus 30 Einstellungen ergab ± 4 Skalenschätzwerte, d. h. $\pm 0{,}002$ mm.

Unter idealen Verhältnissen (wie z. B. gleichbreite Strichbilder mit gutem Konstrast, gute und gleichmäßige Belichtung, 21,5 °C Lufttemperatur), ergab sich für die Einstellgenauigkeit auf ein ideales Strichpaar der Latte aus 30 Beobachtungen der Wert von 0,0025 mm.

Mit dem Komparator wurden weitere Untersuchungen durchgeführt, um die Beobachtungen an einer beliebigen Stelle der Latte mit den Beobachtungen an den nachbarlichen Strichpaaren zu vergleichen. Eine abgenutzte Invarband-Nivellierlatte wurde an allen Teilstrichen der 1. Lattenteilung untersucht. Die von den zufälligen Fehlern befreiten Ergebnisse wurden in Abb. 5 graphisch dargestellt. In Tabelle II wurden die Ergebnisse verschieden-

Tabelle II

Beobachtungen	Mittelwert (Trommelabl.)	Mittlerer Fehler des Mittelwertes (μm)
0.5 cm-weise	21,6	25
dm-weise aus 25 Beobachtungen	21,5	28
dm-weise, aber 5 cm rechts verschoben	22,0	20
2 dm-weise aus 13 Beobachtungen	22,0	25
2 dm-weise, aber 10 cm rechts verschoben	22,1	30
4 dm-weise aus 7 Beobachtungen	22,2	22
4 dm-weise, aber 10 cm rechts verschoben	19,9	33
20 cm rechts verschoben	21,8	31
30 cm rechts verschoben	20,4	29
40 cm rechts verschoben aus 6 Beobachtungen	20,9	37

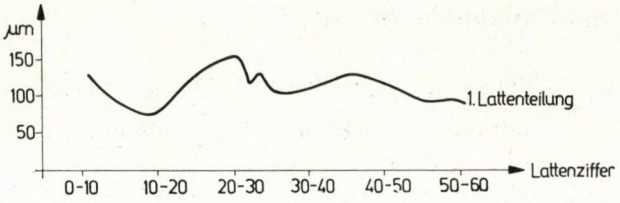

Abb. 5

artig gruppiert und gewertet. Es ist eindeutig, daß man aus 12—13 Beobachtungen, die entlang der Latte gleichmäßig verteilt sind, einen zuverlässigen Mittelwert berechnen kann. Entsprechend dieser Überlegungen soll bereits eine 2-dezimeterweise ausgeführte Beobachtungsreihe unseren Anforderungen entsprechen.

SCHRIFTTUM

1. Alpár, Gy.: Gedanken über die Komparierung. *Acta Geod. Geoph. Mont. Hung.*, 4 (1969), 33—40.
2. Csatkai, D.: Steigerung der Genauigkeit bei der Bestimmung der Komparationsverbesserung in der Nivellierung. *Acta Techn. Ser. Geod. Geoph.*, 3 (1961), 357—374.
3. Peschel, H.: Komparator für Invarband-Nivellierlatten. *Vermessungstechnik*, 1963, 283—289.
4. Regőczi, E.: Új szintezőléckomparátor (Ein neuer Nivellierlattenkomparator). *Geodézia és Kartográfia*, 1957, 1, 52—54.
5. Schäfer, W.: Moderne Verfahren zur Überprüfung geodätischer Instrumente und Geräte. *Mitteilungen aus dem Markscheidewesen*, 1971, 3. 111—126.
6. Jordan–Eggert–Kneissl: Handbuch der Vermessungskunde. III. Band, Stuttgart, Metzlerscher Verlag, 1956.

ПОЛЕВОЙ КОМПАРАТОР ДЛЯ ИНВАРНЫХ НИВЕЛИРНЫХ РЕЕК

Э. ХАЛМАИ

РЕЗЮМЕ

Описывается компаратор, который со встроенным полметровым базисом очень пригоден для компарирования инварных нивелирных реек и в полевых условиях.

Рассматриваются конструкционные принципы и действие отдельных элементов прибора. С помощью числового примера рассматривается один метод для измерения и обработки, а также сопоставляется одна старая ростская рейка с одной новой цейссовской рейкой.

Также исследуется вопрос, где и в каких промежутках рейки нужно провести исследования.

Acta Geodaet., Geophys. et Montanist. Acad. Sci. Hung. Tomus 13 (1—2), pp. 153—163 (1978)

EFFECTIVITY OF THE SOLUTION OF THE INVERSE PROBLEM BASED ON MAGNETOTELLURIC DATA FROM THE NORTH-WESTERN PART OF THE RUSSIAN PLATFORM

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[Manuscript received July 15, 1976]

Algorithms to solve the magnetotelluric reverse problem by means of a statistical method were applied for deep soundings in the North-Western part of the Russian Plate. As the result of this solution, electric parameters were obtained along the Liepaya-Gorki profile supposing an uniform inducing field and horizontal layering. The efficiency of the interpretation of the amplitude and phase curves was analysed. Ways are shown to increase the efficiency of the interpretation.

For a number of years the Department of the Physics of the Earth, Leningrad State University, has been carrying out studies on the conductivity of the crust and upper mantle in the North-Western part of the Russian platform by means of the magnetotelluric method. Some results obtained from these studies have already been published [1—7].

The present report aims at evaluating the efficiency of magnetotelluric methods for studies of the deep distribution of electrical conductivity. Observation points are shown in Fig. 1. Five components of the field have been registered in the intervals of periods from 1 to 10^5 sec along the geomagnetic

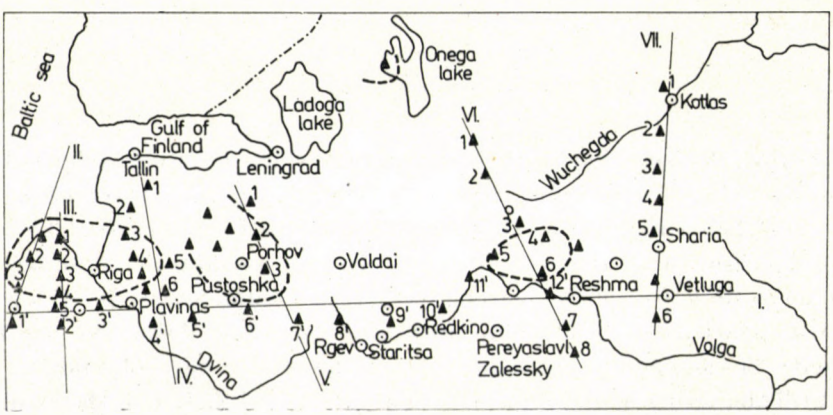

Fig. 1. Location of the magnetotelluric stations on the North-Western part of the Russian Platform. I, II, III, . . ., VII — numbers of profiles, ▲ — points of magnetotelluric sounding, ⊙ bore-holes, which hit the basement, ---- boundaries of regions of decreased resistivity of the basement

latitude Y and longitude X. According to bore-hole data the thickness of the high resistivity basement varies from 50 meters in the North-Western part to 3—3.5 km in the East and West on the territory investigated. The change of the relief of the high-resistivity basement along the latitudinal Profile I Liepaya-Jaroslavl, crossing the Baltic and Middle Russian depression is shown in Fig. 2.

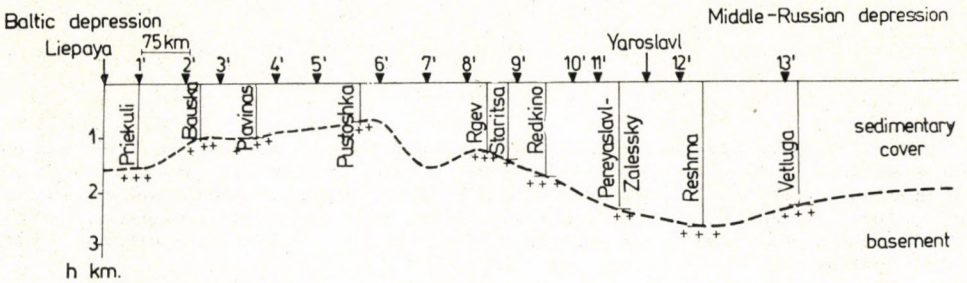

Fig. 2. Change of the relief of the high resistivity basement along Profile I, according to bore hole data

The influence of the non-uniform sedimentary layer has been studied by the technique presented in [7—8]. Here, let us only point out that the Baltic shield affects strongly the behaviour of the impedance diagrams.

The polar impedance diagrams are elongated as a rule along the depth contour lines of the basement. According to theoretical and model works the less distorted are those sounding curves plotted according to the magnitude of impedance along the major axis of the polar diagram. These curves are close to those plotted according to the horizontal field vectors when the impedance is:

$$Z = \sqrt{\frac{E_x^2 + E_y^2}{H_x^2 + H_y^2}}$$

This testifies the fact that in those regions the electric field is polarized along the major axes of impedance diagrams. Sounding curves observed at some points are shown in Fig. 3. These curves used to be interpreted by means of a comparison with theoretical curves and by asymptotic formulae. As a result an estimation of the parameters of the sedimentary layer has been obtained, which is in good conformity with bore-hole data. The depth and resistivity of the conducting mantle [1—7] have also been obtained.

Still the errors of the interpretation and the reliability of the estimation have remained an open question.

In order to simplify the interpretation and also to study the efficiency of the magneto-telluric methods applied for the study of the electrical conduc-

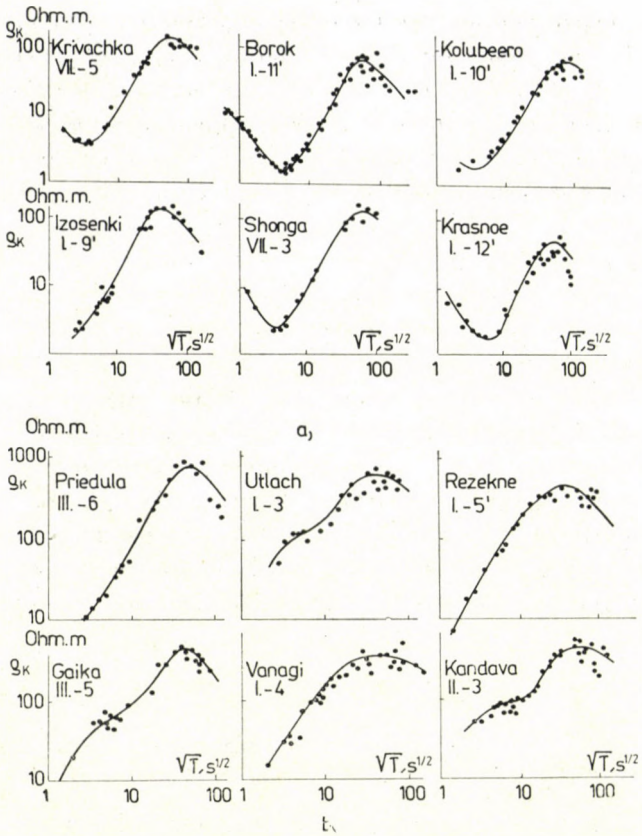

Fig. 3. Magnetotelluric curves obtained on the Eastern (a) and Western (b) parts of the investigated territory

tivity of the Earth's crust and mantle the inverse problem technique was applied, having been developed on the basis of statistical information theory.

Up to now some computerized algorithms of interpretation of some magneto-telluric parameters, viz. real and imaginary components of impedance, module and phases of the apparent resistivity have been elaborated for a flat, layered-uniform Earth.

The present report deals with the interpretation of apparent resistivity curves $\varrho_T = T/2\pi\mu \mid Z \mid^2$. The dispersion of values which have been obtained by means of various selections is significant (Fig. 3).

The principal reason of errors is the assumption about the uniformity of the primary field, besides the errors of treatment. The interpretation is carried out on the assumption that the random errors n_k are normally distributed, uncorrelated, centralized and of dispersion σ_k^2. Thus the mathematical model of the experimental material can be reprsented as $\varrho_T^k = f_k(\vec{\Theta}, N) + n_k$,

where ϱ_T^k is the experimental selection of the apparent resistivity, $k = 1, 2, \ldots K$; K is the number of points on the sounding curve; $f_k(\vec{\Theta}, N)$, the theoretical value (magnitude of the apparent resistivity), $\vec{\Theta}$, a vector characterizing the geoelectric section, its components being thicknesses $h_1, h_2, \ldots h_{N-1}$ and resistivities $\varrho_1, \varrho_2 \ldots \varrho_N$; N, the number of layers is generally also unknown. For the combined determination of the $2N - 1$ unknown parameters of the section $\vec{\Theta}_s$ $(s = 1, 2, \ldots 2N - 1)$ — provided the number of layers N is known — the Bayes criterion is used. If the volume of the experimental material is sufficiently large this leads within the frame of the model chosen to an asymptotically non-biased and normal estimate of the parameters. For this estimation the *a posteriori* density distribution $P_{\vec{\varrho}_T}(\vec{\Theta})$ of the parameters $\vec{\Theta}$ should be constructed for the given selection of the experimental material ϱ_T^k. For the solution of the problem those parameters are taken for which $P_{\vec{\varrho}_T}(\vec{\Theta})$ attains the maxmium value. The search for extrema implies the solution of a non-linear equation system. Without dwelling upon the intricacies of this non-linear problem [8—9] we would only mention that by solving it we shall obtain the parameters $\hat{\Theta}_s$ and the error matrix characterizing the efficiency of solution the inverse problem. The matrix elements are computed according to the formula

$$\left\| A_{ss'} \right\| = \left\| - \frac{\partial^2 \ln P_{\vec{\varrho}_T}(\vec{\Theta})}{\partial \Theta_s \cdot \partial \Theta_{s'}} \right\|_{\vec{\Theta}=\hat{\vec{\Theta}}}^{-1} ; \; s, s' = 1, 2, \ldots, 2N-1$$

The error matrix is symmetrical, its diagonal elements A_{ss} give the dispersion of the evaluation of $\hat{\Theta}_s$. The relative error γ_{Θ_s} is calculated as

$$\gamma_{\Theta_s} = \sqrt{A_{ss}}/\Theta_s . \tag{1}$$

Using the error matrix one can define the covariance matrix $A_{ss'}$ which characterizes the degree of dependence of the parameters:

$$a_{ss'} = \frac{A_{ss'}}{\sqrt{A_{s's'} \cdot A_{ss}}} \tag{2}$$

If $a_{ss'} \ll 1$, the corresponding parameters can be defined separately, with a negligible error. If $a_{ss'} \approx 1$, the parameters can not be separated. The reverse matrix $|| A_{ss'} ||^{-1} = || A^{ss'} ||$ is termed the informant. The diagonal components of this matrix are calculated by means of the formula

$$A^{ss} = \frac{1 + 2\sigma^2}{\sigma^2} \frac{1}{\Theta_s^2} \sum_{k=1}^{K} \alpha_k^s, \; \alpha_k^s = \left(\frac{\partial f_k(\vec{\Theta}, N)}{\partial \Theta_s} \cdot \frac{\Theta_s}{f_k(\vec{\Theta}, N)} \right)_{\vec{\Theta}=\hat{\vec{\Theta}}}^2 \tag{3}$$

These components characterize the degree of dependence of the *a posteriori* density $P_{\vec{\varrho}_T}(\vec{\Theta})$ on Θ_s for the given selection $\vec{\varrho}_T$. The above-described apparatus has been applied for the interpretation of those sounding curves, which were the least distorted (according to the analyses carried out) by the horizontal heterogeneity of the medium. The model according to which the interpretation of the curves has proceeded was fitted to the sounding curves, and to the *a priori* data of the geological and geophysical studies of the region.

The interpretation of the majority of the sounding curves observed in the central part of the Middle-Russian trough was carried out according to a four-layer model (model I): thickness of sedimentary cover $0.5 \leq h_{\text{sed}} \leq 3.5$ km; $h_{\text{sed}} = h_{1\text{sed}} + h_{2\text{sed}}$; the upper sediment layer resistivity $30 \leq \varrho_{1\text{sed}} \leq 300$ ohm.m; lower sediment layer resistivity $0.5 \leq \varrho_{2\text{sed}} \leq 20$ ohm.m; for the high-resistivity basement $400 \leq \varrho_{\text{bas}} \leq 5000$ ohm.m; basement thickness $100 < h_{\text{bas}} < 1000$ km; resistivity of the conducting mantle $\varrho_{\text{cond.m}} < 300$ ohm.m. In the boundary parts of the Middle-Russian depression a three-layer model was applied because the resistivity of the sedimentary layers in that region is only slightly differentiated. While interpreting the data it was necessary to take into account the complex nature of sounding curves pointing to the existence of an intermediate conducting layer at some 10—70 km depth, of resistivity 10—200 ohm.m. (These regions are shown by dotted lines in Fig. 1.) The interpretation was carried out by means of models having 5 or 6 layers (model 2). The results of interpretation for some of the curves are given in Table I. The sounding curves were plotted according to the parameters found and are shown by the solid lines in Fig. 3. Despite the fact that the curves are in good conformity with the experimental points, the evaluation of the section parameters within the framework of the models chosen had great errors. Even though the thickness of the high-resistivity basement was defined best of all, the average error was some 50% (Table I). The greatest errors occurred in the determination of the resistivity of the basement. The parameters of the sedimentary cover are also burdened with great errors.

The errors γ_{Θ_s} increased still more when the curves were interpreted within model 2. To understand the reasons leading to such great errors let us consider a theoretical example. According to the interpretations the section parameters in the central part of the Middle-Russian trough are as follows: $h_{1\text{ sed}} = 1$ km; $\varrho_{1\text{ sed}} = 50$ ohm.m; $h_{2\text{ sed}} = 2.15$ km; $\varrho_{2\text{ sed}} = 1.5$ ohm.m $h_{3\text{ bas}} = 300$ km; $\varrho_{3\text{ bas}} = 1000$ ohm.m; $\varrho_{4\text{ cond.m}} = 50$ ohm.m. The sounding curve plotted according to these data is shown in Fig. 4a. To study the efficiency of interpretation the set ϱ_T^k was chosen at first in the period interval $0.01 \leq T \leq 10^8$ sec. The dots along the curve are located equidistantly in the logarithmic scale, the step being $\Delta \ln \sqrt{T} = 0.1$. The covariance matrix for this case is shown in Table II. The correlations are small between the parameters but for $h_{2\text{ sed}}$ and $\varrho_{2\text{ sed}}$ where $a_{h_2 \varrho_2} = 0.949$. This large correlation

Table I

Results of the interprettaion

Points of soundings (See Fig. 1)		Evaluation of the parameters				
		Thickness, km		Resistivity, ohm.m		
		Sedimentary cover	Basement	Sedimentary cover	Basement	Conducting mantle
Shipitsino VII-I	A	2.6	259	2.68	4871	8.8
	B	2.8	296	2.65	5000	15.5
Shonga VII-3	A	1.05; 0.87	272	52; 0.78	2830	22.3
	B	0.98; 0.88	300	300; 0.85	3000	600
Krivachka VII-5	A	1.22; 1.17	251	19; 1.29	4877	13.8
	B	0.39; 3.5	291	20; 3.52	5000	10.8
Bazan I-13	A	3.14	176	3.72	4894	12.6
	B	2.85	240	3.66	5000	10.9
Joint interpretation of curves, Profile VII. (B)		— —	182	—	3482	57.5
Kolubeyevo I-10	A	1.28; 1.12	211	2.57; 1.06	4913	12.2
	B	1.61; 1.20	300	2.42; 1.22	5000	9.0
Borok, $k = 44$ I-11	A	0.76; 1.57	184	51; 0.74	4928	0.5
	B	0.74; 1.26	247	472; 0.83	5000	9.4
Borok, $k = 63$ I-11	B	0.70; 1.42	245	50; 0.77	4140	0.86
Kandava II-3	B	1.13	1.35; 56.7; 270	13.7	1000; 150; 5000	220
Gaika III-5	B	1.63	6.4; 64; 396	10.5	1000; 196; 2000	6.1

of the parameters of the lower-resistivity sedimentary layer is the well-known S' — equivalency in magneto-telluric practice. Table III summarizes the errors of determination of the section parameters γ_{Θ_s}, calculated on the assumption that the error of ϱ_T^k is equal to 20%. As it would be expected from analysing the covariance matrix the errors γ_{ϱ_2} and γ_{h_2} are great, also great is the error γ_{ϱ_3}. Such a great error can be accounted for by the insignificant influence of parameter $\varrho_{3\,\mathrm{bas}}$ on the observed field. To make the point clear let us consider the behaviour of the function characterizing the influence of parameters on the field:

$$\alpha_k^s = \left(\frac{\partial f_k(\vec{\Theta}, N)}{\partial \Theta_s} \cdot \frac{\Theta_s}{f_k(\vec{\Theta}, N)} \right)^2_{\vec{\Theta}=\hat{\vec{\Theta}}}$$

Figure 4 shows the function α_k^s at various T_k plotted for each parameter. It is the largest in a certain period range (let us call this range *informative* in relation to the parameter). Still, the value $\alpha_k^{\varrho_3}$ is small even at its maximum, hence the

of magnetotelluric curves

		Errors of the evaluation of γ_{Θ_s} in %				
		$\gamma_{h\ \text{sed}}$ sediment. cover	$\gamma_{h\ \text{bas}}$ basement	$\gamma_{\varrho\ \text{sed}}$ sediment. cover	$\gamma_{\varrho\ \text{bas}}$ basement	$\gamma_{\varrho\text{condm}}$ conducting mantle
Shipitsino VII-I	A	16.6	9.2	16	13	80
	B	45	16	27	1250	430
Shonga VII-3	A	21; 32	7.0	20; 8.0	36	26
	B	139; 36	50	140; 26	172	250
Krivachka VII-5	A	13; 26	7.0	34; 26	34	23
	B	10^5; 10^4	20	10^6; 10^6	985	10^4
Bazan I-13	A	10.3	7.0	25	34	9.3
	B	474	70	550	1975	138
Joint interpretation of curves, Profile VII. (B)		—	10.3	—	58	18.9
Kolubeyevo 1-10	A	21; 23	8.2	18; 23	34	26
	B	64; 10^5	51	500; 10^5	4081	238
Borok I-11	A	7.6; 22	9.1	97; 21	101	267
	B	78; 85	16.5	10^6; 62	1220	503
Borok, k = 63 I-11	B	47; 76	12	1055; 59	192	52
Kandava II-3	B	10^7	10^7; 10^4; 10^7	10^7	10^5; 10^4; 10^3	7.10^3
Gaika III-5	B	10^6	10^7; 10^4; 10^3	10^6	10^5; 10^4; 10^3	10^3

value of informant $A^{\varrho_3\,\varrho_3}$ calculated by means of formula (3) is small whereas the error γ_{ϱ_3} is large. The result obtained is known as *H-equivalence.*

Thus with the full section ϱ_T^k it was possible to evaluate the parameters $\varrho_{1\ \text{sed}}$, $\varrho_{4\ \text{cond.m}}$, $h_{1\ \text{sed}}$ and $h_{3\ \text{bas}}$ with errors 6—12%, the errors of the initial material being equal to 20%; parameters $\varrho_{2\ \text{sed}}$, $h_{2\ \text{sed}}$ with some 27—32% errors and $\varrho_{3\ \text{bas}}$ with 400% error. The low-quality interpretation of parameters $\varrho_{2\ \text{sed}}$, $h_{2\ \text{sed}}$ and $\varrho_{3\ \text{bas}}$ is caused by the particular type of the geoelectric section.

If the values ϱ_T^k are obtained in the field in a much narrower period interval ($10 < T < 10^5$ sec) this causes a great loss of information and at the same time an increase in the correlation between parameters and increased errors in their determination (see Table III). In such a case in fact all the parameters of the section are defined with impossibly great errors. To improve interpretation one has to pick out those parameters which are strongly correlated in the interval chosen and have negligible influence on the field. In our case the parameters are those of the sedimentary cover h_1, ϱ_1, h_2, ϱ_2 and basement resistivity ϱ_3.

Table II

Covariance matrices for the intervals of T *in sec:*

I : $10^{-2} \leq T \leq 10^{8}$; II : $1 \leq T \leq 10^{6}$; III : $10 \leq T \leq 10^{5}$

	$\varrho_{1\text{sed}}$	$\varrho_{2\text{sed}}$	$\varrho_{3\text{ bas}}$	$\varrho_{4\text{cond.m}}$	$h_{1\text{ sed}}$	$h_{2\text{ sed}}$	$h_{3\text{ bas}}$
I		0.088	0.014	0.001	0.150	0.087	0.008
II $\varrho_{1\text{ sed}}$	1	0.914	0.312	0.054	0.990	0.929	0.213
III		0.999	0.954	0.446	0.999	0.999	0.994
I			0.232	0.021	0.766	0.949	0.131
II $\varrho_{2\text{ sed}}$		1	0.410	0.073	0.954	0.989	0.281
III			0.957	0.449	0.999	0.999	0.995
I				0.121	0.124	0.507	0.607
II $\varrho_{3\text{ bas}}$			1	0.227	0.339	0.510	0.721
III				0.545	0.955	0.955	0.978
I					0.011	0.045	0.414
II $\varrho_{4\text{cond. m}}$				1	0.059	0.091	0.596
III					0.447	0.448	0.513
I						0.711	0.070
II $h_{1\text{ sed}}$					1	0.961	0.232
III						0.999	0.994
I							0.272
II $h_{2\text{ sed}}$						1	0.344
III							0.994
I							
II $h_{3\text{ bas}}$							1
III							

While interpreting the experimental curves one should give up the practice of dividing the sedimentary cover into two layers and should substitute it by a single equivalent one or, by using bore-hole data, one should fix the parameters h_1, ϱ_1, h_2 and only search for ϱ_2, h_3, ϱ_4. By this strategy the interpretation error may become ten times less.

Another and more successful way of improving the interpretation efficiency is to increase the period range in which the electro-magnetic field is recorded. An example of the improvement of interpretation quality due to the increase of the period range is provided by observation point Borok. Here the range of ϱ_T^k was enlarged from $T = 10^4$ sec to $T = 10^5$ sec, and the interpretation considerably improved (see Table I).

Table III

Errors of determination of the parameters, if the experimental errors are equal to 20%

Experimental data in the interval T in sec	Sediment. cover		High resistivity basement	Conducting mantle	Sediment. cover		High resistivity basement	Notice
	γ_{ϱ_1}	γ_{ϱ_2}	γ_{ϱ_3}	γ_{ϱ_4}	γ_{h_1}	γ_{h_2}	γ_{h_3}	
10^{-2}—10^{8}	7.8	27.7	435	5.9	8.1	32	12	Parameters computed by a four-layer model
1—10^{6}	1470	76	1070	12	105	104	18	
10—10^{5}	10^{7}	10^{5}	10^{4}	72	10^{6}	10^{5}	650	
10—10^{5}	—	18	1400	57	—	—	41	The parameters of the sedimentary layers ϱ_{1sed}, h_{1sed}, h_{2sed} are fixed
10—10^{5}	16		10t	39	62		54	The sedimentary cover is substituted by a single layer
10—10^{5}	8		250	29	—	—	24	The sedimentary cover is substituted by a single layer, h_{sed} is fixed

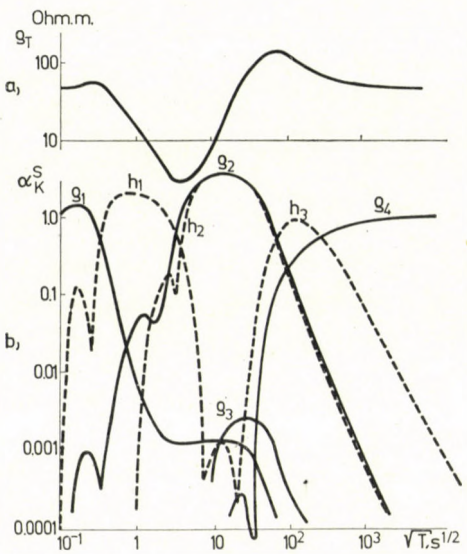

Fig. 4. The behaviour of the magnetotelluric curves (a) and the function (b), characterizing the influence of the parameters of the field at various periods T_k in case of the four-layer model:

$\varrho_1 = 50$ ohm.m $\quad h_1 = 1$ km
$\varrho_2 = 1.5$ ohm.m $\quad h_2 = 2.15$ km
$\varrho_3 = 1000$ ohm.m $\quad h_3 = 300$ km
$\varrho_4 = 50$ ohm.m

A further, but less reliable way of increasing the interpretation efficiency is to use *a priori* data obtained from previous geological and geophysical studies carried out in that region. The *a priori* data concerning the model are normally distributed with mean value

$$m_s = \frac{\Theta'_s - \Theta''_s}{2} \quad \text{and dispersion} \quad D_s = \left[\frac{(\Theta'_s - \Theta''_s) - m_s}{3}\right]^2,$$

where Θ'_s, Θ''_s are the boundaries of the given interval of the *a priori* parameters, they are introduced as multipliers into the *a posteriori* probability density $P_{\vec{v}_T}(\vec{\Theta})$.

Magnetotelluric curves observed in the Middle-Russian depression were interpreted in this way. The model I data were taken as the *a priori* ones. The interpretation quality (see Table I) has considerably improved, the interpretation errors of the strongly correlated parameters $h_{2\,\text{sed}}$, $\varrho_{2\,\text{sed}}$ and $\varrho_{3\,\text{bas}}$ decreased tenfold. It is needless to say the method works better for the parameters of the sedimentary cover then for the deep ones because the *a priori* data concerning the latter are rather vague.

The most reasonable way of increasing the interpretation efficiency is to determine the deep parameters by a joint interpretation of curves obtained in remote regions. It is natural to expect that parameters such as resistivity of the upper mantle are not likely to change within a range of some 300—400 km. Thus, these parameters can be determined by using all the data taken together which have been obtained on the territory. Parameters of the upper layers should be taken equal to those obtained by the individual interpretation of the curve. As an example let us cite the joint interpretation of the curves along Profile VII (see Table I). Since only a search of three depth parameters was carried out while the information of the material had increased, the errors of their determination greatly decreased as compared to the individual interpretation. The main geophysical result of joint interpretation was a more convincing evaluation of the depth of the conducting mantle.

All in all, it can be concluded that the statistical approach gives a convenient tool to evaluate more critically the results of depth-sounding and to envisage ways of increasing the efficiency of magnetotelluric methods.

REFERENCES

1. Janovski, B. M.—Brunelli, B. E.—Kovtun, A. A.—Kuznetsov, G. S.—Raspopov, O. M.—Chicherina, N. D.: Magnetotelluric sounding in the Middle-Russian trough. *Izv. ANSSSR, ser. Geof.* No. 7, (1964), 999—1006.
2. Janovski, B. M.—Kovtun, A. A.—Raspopov, O. M.—Chicherina, N. D.: The depth structure of the Middle-Russian trough according to data of magneto-telluric sounding. *Vopr. Geof.*, Uchenye Zapiski Leningrad. State Univ., No 329, issue 16 (1960), 3—9.

3. KOVTUN, A. A.—CHICHERINA, N. D.—LIPATOV, A. A.—POROKHOVA, L. N.: Magnetotelluric sounding in the Eastern part of the Middle-Russian trough. *Vopr. Geof.*, Uchenye Zapiski Leningrad. State Univ., No. 333, issue 17 (1967), 227—231.
4. KOVTUN, A. A.—CHICHERINA, N. D.—LIPATOV, A. A.—POROKHOVA, L. N.: The electrical parameters of the Earth's crust and upper mantle in Western Letland. *Vopr. Geof.* Uchenye Zapiski Leningrad. State Univ., No. 340, issue 18 (1968), 73—78.
5. KOVTUN, A. A.—CHICHERINA, N. D.: Results of the magnetotelluric studies of the Middle-Russian trough. Magnetotelluric Techniques of Studying the Structure of the Earth's Crust and Upper Mantle, *Nauka*, Moscow No. 4 (1969), 195—199.
6. KOVTUN, A. A.—CHICHERINA, N. D.: On the anomalous behaviour of the magnetotelluric sounding curves in the North-Western part of the Russian platform. *Vopr. Geof.*, Uchenye Zapiski Leningrad. State Univ., No. 366, issue 22 (1972), 185—193.
7. BERDICHEVSKIY, M. N.: Electrical Exploration with the Technique of Magneto-Telluric Profiling. *Nedra*, Moscow, 1968, pp 250.
8. GOLTSMAN, F. M.—KOVTUN, A. A.—POROKHOVA, L. N.: Questions of computerized interpretation of M. T. S. curves. *Izv. ANSSSR Fiz. Zemli*, No. 4 (1969), 96—99.
9. POROKHOVA, L. N.: Statistical methods applied for solving the inverse problem of magnetotelluric sounding. Cand. Sci. Thesis Leningrad. State Univ., 1971.
10. KOVTUN, A. A.—POROKHOVA, L. N.: Evaluation of the efficiency of magnetotelluric sounding in the North-Western part of the Russian platform. *Vopr. Geof.* Uchenye Zapiski Leningrad. State Univ., Issue 24 (1974), 262—291.

ЭФФЕКТИВНОСТЬ РЕШЕНИЯ ОБРАТНОЙ ЗАДАЧИ НА ОСНОВЕ МАГНИТОТЕЛЛУРИЧЕСКИХ ДАННЫХ СЗ-НОЙ ЧАСТИ РУССКОЙ ПЛАТФОРМЫ

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РЕЗЮМЕ

Алгоритмы решения магнитотеллурической обратной задачи статистическим методом были использованы при магнитотеллурических глубинных зондированиях, проведенных в СЗ-ной части Русской Платформы. В результате этого решения были определены электрические параметры по профилю Лиепая — Горький с предположением униформного индуцирующего поля и горизонтальной слоистости. Была проанализирована эффективность интерпретации амплитудных и фазовых кривых. Показываются способы для повышения эффективности интерпретации.

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THE NOTION SYSTEM OF GEODETIC INFORMATIONS AND THE CALCULATION OF GEODETIC NETWORKS*

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Physics investigates the connections and laws of material objects in real space, while geometry describes the connection between geometric objects in signal space. If we intend to review the two branches of geodesy, physical geodesy and geometric geodesy from a common point of view, then the physical and geometric spaces are to be substituted by an event space which includes, as special cases, both of them.

We looked for connections which can be used for the solution of problems of both physical and geometrical geodesy, and the connection between them has been examined, respectively. The algebra of events, the theory of probabilities and recently the information theory are being transformed in many respects into a physical theory (e.g. information physics). That means that a connection is to be found between the physical signal space occurring in geodesy and the geometrical signal space. Our investigations lead to a general model which interprets the information flow in geodetical networks as an exact physical model, independently from the network, from its measures, from the number and quality of points (distance angle or gravity measurements, measurements of g etc.). It can be shown that normal equation systems and their covariance matrices (generalized and pseudo-inverses) have unambiguous content: The investigations show also such characteristics in local adjustments.

1. Subject of geodesy—geodetic information

The subject of geodesy has been several times outlined, from among them HELMERT's words are most quoted. About the most general subject of geodesy, however, definitions can be scarcely found in handbooks and papers. One of the simplest and most proper definitions has been given lately by MEISSL [10]. It says that the task of geodesy is to transmit "informations", especially informations concerning the shape of the Earth's surface and of the potential levels of gravity.

As all definitions, this one has also an incompleteness as it does not define distinctly what should be understood on information, namely as under geodetic information further how the quantity of these informations can be determined. The aim of the present paper is to investigate these very problems, i.e. to present an exact mathematical, respectively physical information model

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by using of which all the elements of the geodetical information process can be explained with adequate strictness, and even the quantity of the flowing information can be followed in each moment.

2. The notion-system of geodetic information

2.1 *The role of geodetical networks*

At the beginning of the geodetic information activity data stored from earlier geodetic determinations can be found which can be at disposal — depending on the type of storage — in graphical, photographical, numerical or other forms. The end of the information process is a satisfaction of the information demand which may appear continuously or intermittently. The connection between initial and final data is ensured by geodetic measurements and computations, respectively. An often used notion is also in geodesy the *information flow* or *information process* that means implicitly that the information content of given points or measurements is transformed anyway into the quantities to be determined. It is, however, seldom mentioned which conditions regulate quantitatively and qualitatively this information flow. In an earlier paper [6, 7] we have treated this problem and reported on the results of certain experiments. Now we put the question more generally: *which definitions of the information notions and the use of which information models ensure that the information flow should fulfil similarly to all other physical processes every law of the physics* of flows?

In the following, initial data to be used for the satisfaction of different information demands be called *information sources*, independently from the form of their storage, be it any usual or unusual method. It is true that the primary information source is the field itself, but already at the lowest levels of abstraction one has to prescind from the natural appearance forms of informations, and their *information content* is mapped by measurements or without them into elements of a *signal-space*. The former activity is called *information collection*, the latter, *information recording*. The information content of the initial data and of different absolute or relative measurements is transformed by *information processing* into a form prescribed by the users of the geodetical informations. If this processing is realized without any loss of informations, then it is called *information transformation*, in other case *information filtering*. The quantities, the values of which are the real final products of geodetical activities are called *information sinks*.

At last we should fix the notion of *geodetic network*. In a wider sense, geodetical networks are *the ensemble of technical, economical and organizatory measures which together ensure the way of geodetical informations from the infor-*

mation sources to information sinks. In a restricted sense, the geodetic network is a *rather physically than mathematically organized system of measurement results, initial data, further of quantities to be determined which ensure together a channel system for the relatization of the information flow.*

2.2 *The measurement of geodetic information*

The smallest unit of geodetic networks is the point with known or unknown co-ordinates. For sake of simplicity we shall consider in the following always elevation networks where only a single co-ordinate belongs to each point, but the discussions can be extended to any geodetical network.

The information content of the co-ordinates of a given point or of a newly determined point (the so-called position information) can be measured in different scales and even in a given scale by choosing the origin of the information-measuring scale in different positions. Our requirements are best answered by a solution, where the so-called *rest-entropies* are investigated, i.e. we did not take the information gain, but the information lack into account. In order to make the latter quantity always positive, it is advantageous to choose the level of requirements unavoidably necessary for the definition of the rest-entropy (the information demand) by one or two orders of magnitude stricter than the actual one.

The rest-entropy of a point in the plane in the directions $H(X)$, $H(Y)$, and in X, Y, are given by REZA [11] as follows:

$$\begin{aligned} H(X) &= \ln \sqrt{2\pi e}\,\sigma_x \\ H(Y) &= \ln \sqrt{2\pi e}\,\sigma_y, \end{aligned} \tag{1}$$

where σ_x and σ_y are the dispersion of the co-ordinates X and Y, resp., of the point, ln is the natural logarithm with the basic e. The whole rest-entropy of the radius vector $\mathbf{R}$ of a point in the plane is, due to the additivity of the values, the algebraic sum of these two quantities, i.e.:

$$H(R) = H(X) + H(Y). \tag{2}$$

In the time when the measure of the entropy and of the information content, respectively, has been accepted in the logarithmic form of Eq. (1) following SHANNON's recommendation, two main aspects have been considered as significant:

1. It is simpler to sum information values than to multiply them.

2. It is simpler to use information values of smaller absolute value than those of higher absolute value.

In the last quarter of our century both aspects have lost their importance, as at present multiplications can be carried out even with pocket calcu-

lators as quickly as summations, therefore none of the additive and multiplicative measures hase xquisite role. On the other side, the floating point and the scientific notation of numbers faded the boundaries between great and small numbers.

In addition to the above two practical considerations, there is a third one being of theoretical character. Neither the possibilities of the filter theory necessary for the up-to-date computation of geodetic networks, nor the real structure of the physical models to be explained can be clearly shown if we adhere to the additive information measures corresponding to the mentioned practicistic points of view. Therefore we shall use, instead of Eq. (1), the following formulas for the determination of the one-dimensional rest-entropies:

$$\begin{aligned} M(X) &= \sigma_x \\ M(Y) &= \sigma_y \end{aligned} \qquad (1)'$$

and for the determination of the two-dimensional rest-entropy, instead of Eq. (2), the formula

$$M(R) = M(X) \cdot M(Y) = \sigma_x \cdot \sigma_y. \qquad (2)'$$

The first two quantities are equivalent to the mean square coordinate errors, the last with the point error. The difference is that the measure ot the point error is here not the trace of the variance-covariance matrix (the first invariant), but its determinant (the second invariant) [5].

If the formulas (1)' and (2)' are accepted as the measures of the coordinate-information, then — and only then — the two laws of Kirchhoff will be fulfilled, i.e. only in that case becomes it clear that the information as any other "flowing material" strictly follows the basic laws of physics.

In the following we shall use the notation of *useful* (pragmatic) *information*. This notion differs from Shannon's information not only by being a *multiplicative measure*, but also by being a *relative measure*, depending strongly or less strongly on different conditions, i.e. it can be used only for the comparison of information contents. One of the most important conditions strongly influencing the quantity of useful informations is to which new point, the given point (information source) or the measurement results (channel) considered carries information. In case of near information sources the relative measure of the useful information can be significantly higher than in case of distant information consumers, i.e. far lying new points.

3. Information models which can be fitted into the uniform physical world picture

Material and mind seemed for a long time to be opposite ideas and it could hardly be imagined that some theory or physical laws should exist to which both of them would obey. Only in most recent times began an integration process in natural sciences which tries to re-unite physics, chemistry and biology being at present very much separated from each other into a single general field of science. Therefore it shall not be perhaps surprising if we shall try to suggest for the qualitative and quantitative description of information-flow and information transformation in geodetic networks physical models which have been used up to now for definitely material-like elements (liquids, gases, electric currents etc.).

Though dynamical models in most cases prove to be more advantageous and of greater capacity, at the beginning we shall confine ourselves to a few simple static information models. Later, as a development, more complicated dynamic models shall be also described in detail.

3.1 *Some static information models*

3.11 *Potential of a mass point in continuous and in discrete space*

The physical geodesy deals with the physical space, it investigates one of its important characteristics, the potential function of the gravity field. The geometric geodesy deals with the geometric space (signal space), it records and investigates events in it. It is therefore not accidental that processes in these two kinds of spaces follow basically the same laws and they differ in unsignificant features only.

Let us compare these two kinds of spaces — the physical and the geometrical one — on an "atomistic" or molecular level. By taking a single "molecule" of the physical space as a mass point, the potential function of the force field can be given which is induced by the point around it according to Newton's law:

$$U = k^2 \frac{m_0}{r}, \tag{3}$$

where the effect of the molecula with the mass m_0 on a point of unit mass is considered.

If a given geodetic point is considered as the molecule of the geometric space, and its mass m_0 is identified with the quantity of the position information of the coordinates of the point, then a formula similar to Eq. (3) can be

written ([4] (Fig. 2a)):

$$U = \frac{1}{4\pi}\,\frac{1}{r} + \frac{1}{32\pi}\,\frac{1}{r^3}\left(5\,\frac{X^4+Y^4+Z^4}{r^4} - 3\right). \tag{4}$$

This formula differs in many aspects from Eq. (3) but it expresses substantially the same content.

The first difference is that Eq. (3) refers to a discrete space where the space itself is not continuous, i.e. it is present only in quanta being at certain distances from each other. One of the simplest of such a space is a three-dimensional geodetic network the points of which correspond to the points of an orthogonal regular space grid with unit grid constant and the measurement results are the height differences of neighbouring points. Eq. (4) corresponds to such a network.

The second difference is that Eq. (3) is a precise formula, while Eq. (4) is an asymptotic approximation, i.e. it becomes precise only at $r \to \infty$. In spite of the fact that for the discrete geometric space a closed formula similar to Eq. (4) cannot be given, its physical content is nevertheless clear. Namely a physical sub-system can be formed which can perfectly explain all moments of the flow of geodetic information. The subject of the present paper is this sub-system.

Due to the lack of the closed formula, the creators of the discrete potential theory [DUFFIN—SHAFFER, 1958] published, in addition to the approximative formula (4), numerical tables from numerical integration which characterize the *information force field* in the immediate vicinity of the molecules of the geometric space. The numerical values published here are valid for points inside of the grid domain $(10\times10\times10)$ bordered by the points $X = \pm5$, $Y = \pm5$, $Z = \pm5$, where Eq. (4) cannot be used anyway. If the constant of the space grid (the distance of neighboring points) is diminished or the distance of the point investigated is increased, Eqs (3) or (4) will give equivalent result of any prescribed accuracy.

Figure 1 shows the potential distribution of a single point numerically, too. The data originate partly from the already mentioned computer-solution for the $(10\times10\times10)$ grid domain, partly from the asymptotic formula (4).

The third difference between Eqs (3) and (4) is that, instead of the mass attraction constant k^2 valid for the physical space, the reciprocal value of the surface of the unit radius sphere, $\pi/4$ appears in the geometric space. The cause of the fact that instead of the mass m_0 in Eq. (4) only unity appears is that the formula of DUFFIN and SHAFFER has been deduced supposing $m_0 = 1$, but as we shall see it is also valid in a more general case. In such cases m_0 is to be substituted by the information content of the coordinates of the geodetic point creating the force field (its p weight or its rest-entropy σ).

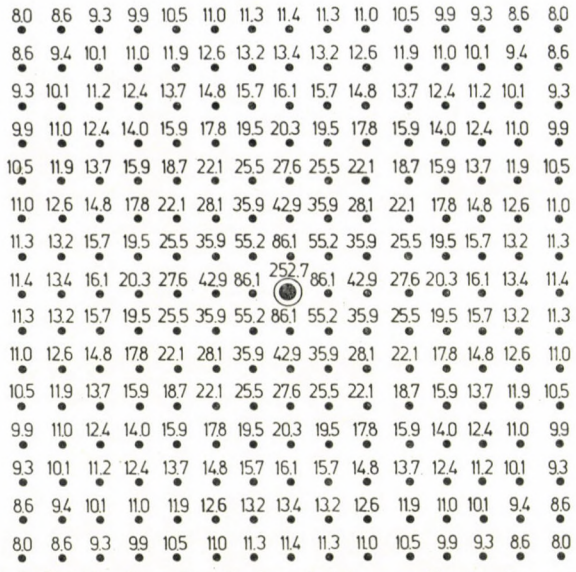


Fig. 1. The potential distribution of a single point in coordinate-planes of the discrete space

The common features of the potential functions of both kinds of force fields treated are:

1. They satisfy the Laplacian second order differential equation:

$$\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} = 0 \tag{5}$$

showing that both of them are harmonic functions.

2. The value of the potential attains its maximum at the surface of the molecula inducing (producing) it.

3. The potential in infinitely far points has zero value.

4. The force field has a point symmetry, i.e. its equipotential surface — disregarding the innermost domain of the discrete space — are spherical surfaces.

5. The potential varies — at sufficiently great distances from the molecule producing the force field — inversely proportionally with the distance in both cases.

In the following, we shall develop our model in two directions. On the one hand, we shall allow, instead of a single molecule, the presence of any number of active points being able to induce their own force fields. On the other hand, we shall prove, by taking the partial derivates of the potential function in different directions and of different order, that all features of the force field in the physical space can be found in the discrete potential function of the geometric field.

3.12 *Gradient components of the force field of a mass point*

By taking the partial derivates of the potential function of the elementary force field in the physical space (Eq. 3), the components of the gradient g can be obtained (g_x, g_y, g_z):

$$\frac{\partial U}{\partial X} = -k^2 m_0 \frac{X}{r^3}\ ;\quad \frac{\partial U}{\partial Y} = -k^2 m_0 \frac{Y}{r^3}\ ;\quad \frac{\partial U}{\partial Z} = -k^2 m_0 \frac{Z}{r^3} \tag{6a}$$

$$g = -k^2 \frac{m_0}{r^2}\ . \tag{6b}$$

If one tries to obtain the gradient components also in case of the potential function of Eq. (4) being defined in the geometric space, then the derivation used above cannot be applied. In a field containing mere singular points no continuous function can be defined, therefore the first condition of derivation — the continuity — is not valid for the function described by Eq. (4). A gradient function can be, however, defined in the geometiic space discussed resulting from the forming of the differences of neighbouring values of the function U in the direction X, Y and Z. In order to prove that this notion is asymptotically similar to the gradient of the physical space, let us consider Fig. 2a. If point P_1 lies at a sufficiently great distance from the point O inducing the force field, then the change of the potential $\Delta U = g_x$ between two grid

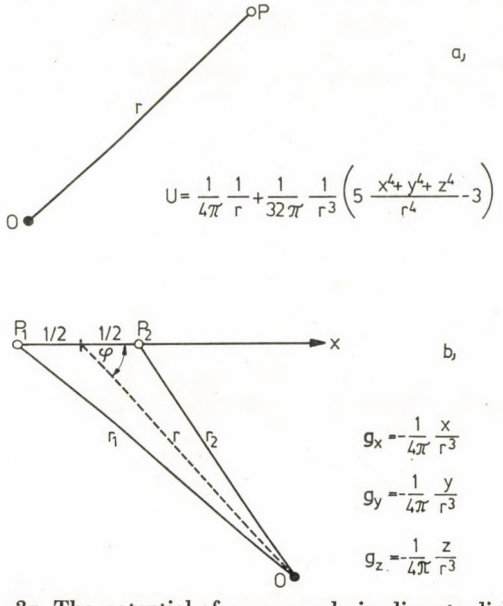


Fig. 2a. The potential of a monopole in discrete dielectric
Fig. 2b. The gradients of a monopole in discrete dielectric

points P_1 and P_2 in unit distance, can be computed more simply than by using Eq. (4) twice:

$$g_x = \Delta U_{12} = U_2 - U_1 = \frac{1}{4\pi}\frac{1}{r_2} - \frac{1}{4\pi}\frac{1}{r_1} = \\ = \frac{1}{4\pi}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{1}{4\pi}\left(\frac{1}{r - \frac{1}{2}\cos\varphi} - \frac{1}{r + \frac{1}{2}\cos\varphi}\right). \tag{7}$$

As

$$\frac{1}{r+\Delta r} = \frac{\frac{1}{r}}{1+\frac{\Delta r}{r}} = \frac{1}{r}\left(1+\frac{\Delta r}{r}\right)^{-1} = \frac{1}{r}\left(1 - \frac{\Delta r}{r} + \frac{(\Delta r)^2}{r^2} - \ldots\right) \tag{8}$$

therefore

$$\frac{1}{r_2} = \frac{1}{r - \frac{1}{2}\cos\varphi} = \frac{1}{r}\left(1 - \frac{\frac{1}{2}\cos\varphi}{r} + \ldots\right)$$

$$\frac{1}{r_1} = \frac{1}{r + \frac{1}{2}\cos\varphi} = \frac{1}{r}\left(1 + \frac{\frac{1}{2}\cos\varphi}{r} - \ldots\right) \tag{9}$$

and thus

$$\frac{1}{r_2} - \frac{1}{r_1} = -\frac{1}{r}\frac{\cos\varphi}{r} = -\frac{X}{r^3}. \tag{10}$$

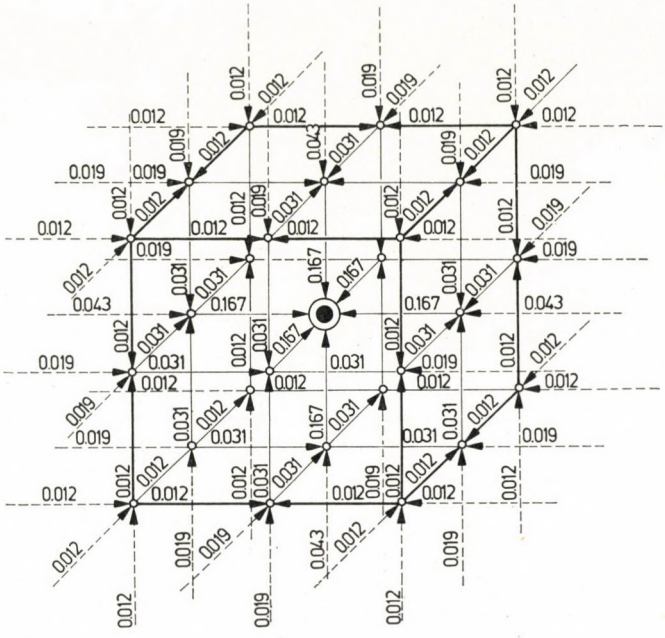
Accordingly, the value of the gradient in the direction X is, with good approximation,

$$g_x = \frac{1}{4\pi}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = -\frac{1}{4\pi}\frac{X}{r^3}. \tag{11}$$

It can be seen that this result is in complete accordance with Eq. (6*a*), showing that the continuous and the discrete force fields agree asymptotically in their numerical value, too.

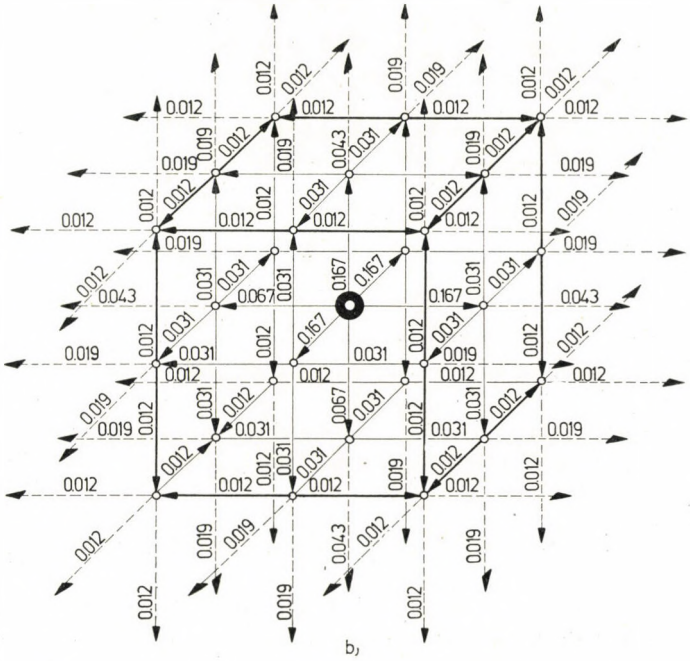
Figure 3a illustrates the value of the gradients in the immediate vicinity of point O.

Eqation (3) belonging to the physical force field has been interpreted hitherto as the Newtonian mass attraction potential of a mass point m_0. In order to be able to develop our model for the geometric field, the potential function



a)

Fig. 3a



b)

Fig. 3b

corresponding to the physical field must be somewhat generalized. Let us consider therefore Eq. (3) as the electrostatic potential of a material point with the charge Q; then Eqs (6*a*) and (6*b*) yield the components of the electric field X, Y and Z in the directions E_x, E_y and E_z, and the absolute value of the electric force E, respectively. Now essentially only the notations must be changed and one gets the formulas of the electrostatic field instead of the gravity field:

$$U = \frac{1}{4\pi\varepsilon} \frac{Q}{r} \tag{12}$$

$$\begin{aligned} E_x &= -\frac{Q}{4\pi\varepsilon} \frac{X}{r^3} \\ E_y &= -\frac{Q}{4\pi\varepsilon} \frac{Y}{r^3}; \quad E = -\frac{Q}{4\pi\varepsilon} \frac{1}{r^2} \\ E_z &= -\frac{Q}{4\pi\varepsilon} \frac{Z}{r^3}. \end{aligned} \tag{13}$$

It can be seen that the place of the mass attraction contant k^2 is occupied here by the coefficient $\frac{1}{4}\pi\varepsilon$, where ε is the so-called dielectric constant.

3.13 *The dielectric constant as one of the most important characteristics of homogeneous geodetic networks*

In continuous physical fields the dielectric constant depends on the quality of the medium (on its physical parameters) filling the space. In case of geodetic signal spaces the dielectric constant is a function of the structure of the network. This problem has not been sufficiently studied yet in the three-dimensional grids, therefore we deal only with some known results concerning planar networks, as illustrative examples.

In case of the planar analogon of the spatial grid, in the square grid, the value of the dielectric constant ε is $1/\pi$, if a determination of the elevation similar to the areal levelling between the grid points is supposed. This dielectric constant does even change if instead of height measurements, simultaneous azimuth and distance measurements are supposed. If the planar square grid is substituted by a regular triangular grid, on gets in certain cases a "stronger" network, in other cases a "weaker" one.

Let us take the dielectric constant for the square grid $\varepsilon_0 = 1/\pi$ as a reference value similar to the ε_0 value corresponding to vacuum in continuous fields. In case of differing networks, their constants can be expressed in form of the product ε_0, ε_r, where ε_r is the *relative dielectric constant* of the network

related to the dielectric constant of the square grid. According to the definition, the ε_r value of square grids in equal to unity. If one considers a regular triangular network of purely distance measurements, then its relative dielectric constant is $8/3\sqrt{3}$, significantly higher than unity. Greater dielectric constants characterize weaker, smaller constants, stronger networks. For example, if in addition to the distance measuremens, also the azimuths of each sides of the regular triangular network are measured, then a relative dielectric constant considerably smaller than unity, $1/\sqrt{3}$ is obtained (also given by BORRE—MEISSL [2]). It would be worth of determining, in addition to the dielectric constants of planar networks, the dielectric constants of spatial networks too, where a dielectric constant relative to the orthogonal simple space grid could be used. The first possible question is how does the value of ε_r diminish if all possible surface diagonals are also measured. The network continues to get stronger, i.e. the value of ε_r diminishes if, in addition to the surface diagonals, the body diagonals are taken as belonging to the network. In the course of this network development, the initial vicinity connection of six points increases at first to 18, then to 26 points, showing the increase of the quality of the network.

3.14 *The Laplacian and Poissonian differential equation system valid for discrete networks*

Equation (5) of Section 3.11 takes in case of space grids consisting of discrete points the following more simple form:

$$\sum_{j=1}^{s} U_{ij} - sU_i = 0\,. \tag{14}$$

This can be written in the following form, too:

$$(U_{i_1} - U_i) + (U_{i_2} - U_i) + \ldots + (U_{i_s} - U_i) = 0 \tag{15}$$

where U_{ij} means the potential value corresponding to the j-th neighbour of the i-th point. This formula can be interpreted as a Kirchhoff's node point law, giving a new interpretation for the Laplacian differential equations used hitherto only in case of continuous fields. We shall discuss this later in the chapter on dynamic models.

Equations (14) and (15) are formulas valid for every point of the network with the exception of a single singular point called in Fig. 3a a "sink", in Fig. 3b a "source". Here the Laplacian differential equation goes over into the more general Poissonian differential equation where on the right-hand side a value differing from zero appears. The scales of our figures are chosen so

that the "information flow" reaching in Fig. 3a the sink and leaving in Fig. 3b the source should be exactly unity. Therefore the Poisson-equation for these singular points takes the following form:

$$\sum_{j=1}^{s} Q_{ij} - sQ_i = \begin{Bmatrix} + \\ - \end{Bmatrix} 1 \tag{16}$$

where the negative sign should be chosen for a sink, the positive for a source.

The practical importance of the Laplace and the Poisson equations is that in knowledge of the potentials of all neighbours of a point, the potential of the point itself can be determined as a simple mathematical average. According to Eq. (14), it can be found after some easy mathematics:

$$U_i = \frac{\sum_{j=1}^{s} U_{ij}}{s} . \tag{17}$$

Another application of the Laplace equation is the determination of the potential U_{i_s} of the last, s-th neighbour point in case of sequential expansion of a network in knowledge of the potential U_i of an arbitrary point i and of the potential values of $(s-1)$ neighbouring points. The corresponding formula is in that case

$$U_{i_s} = sU_i - \sum_{j=1}^{s-1} U_{ij} . \tag{18}$$

The further role and importance of the formulas 14 and 15 will be treated in connection with the discussion of the Kirchhoff's stream-laws in the chapter about dynamic models.

3.15 *Connection of the mathematical and physical notions concerning networks*

The two most important characteristics of geodetic networks, the coefficient matrix (normal matrix) **N** and the weight coefficient matrix (variance-covariance matrix) **Q** are connected by the following well-known formula:

$$\mathbf{NQ} = \mathbf{E}. \tag{19}$$

Let us consider the rule for the determination of an arbitrary of the matrix **N**, and then let us investigate the physical content of Eq. (19):

$$\begin{array}{ccccccccccccc} P_{i_1} & & P_{i_2} & & P_i & & P_{i_3} & & P_{i_4} & & P_{i_5} & & P_{i_6} \\ \ldots & -1 & \ldots & -1 & \ldots & 6 & \ldots & -1 & \ldots & -1 & \ldots & -1 & \ldots & -1 & \ldots \end{array} \tag{20}$$

This normal equation row occurs in case of the orthogonal spatial grid of Fig. 3 and it characterizes all its points P_{i1} In that case there are mere 6-s in the main diagonal of the coefficient matrix of the normal equation system, then in all columns which belong to any neighbour of the point P_i ($j = 1, 2 \ldots$ $\ldots 6$), a further coefficient -1 can be found. Therefore all rows and columns of the matrix **N** contain 7 elements differing from 0, it is therefore a rather sparse matrix. From the point of view of topology or of the graph theory the matrix **N** is a matrix of vertex neighbourhood, while the coefficient matrix **A** of the observation equation system is an edge-vertex neighbourhood matrix. The form 20 is only valid if the measurements are all of unit weight. In the opposite case

$$\ldots -P_{i_1} \ldots -P_{i_2} \ldots [P_i] \ldots -P_{i_3} \ldots -P_{i_4} \ldots -P_{i_5} \ldots . -P_{i_6} \ldots \quad (21)$$

where P_{i1} is the weight of the measurement from point P_i to its j-th neighbour. In such cases the matrix **N** can be taken as a so-called admittance-matrix, multiplying which by the variance-covariance matrix **Q**, playing the role of a so-called impedance matrix, a transformation corresponding to Ohm's law is carried out. Each element of the matrix **N** contains namely a weight (branch conductivity), and in all cases can be found an element in **N**, which corresponds to the former with opposite sign. The product of these two elements with corresponding elements of the matrix **Q** yields the form

$$p\mathbf{Q}_i - p\mathbf{Q}_{i_j} = p(\mathbf{Q}_i - \mathbf{Q}_{i_j}) , \quad (22)$$

where the first term is conductivity, the second potential difference, therefore the product has the form of a current. Thus the sum of the elementary currents in Eq. (22) determines node-point Kirchhoff's laws.

The physical interpretation of the unit matrix on the right-hand side of Eq. (19) is a current matrix, more precisely the matrix of $m \times m$ pieces of nodal point currents. Let us consider its i-th column. It contains, besides $(m - 1)$ zeros, a digit 1 in the i-th row. Let us consider how this column of the unit matrix comes into being. An immediate consequence of the definition of matrix multiplication is that for production the second term only the i-th column of the matrix **Q** can be used, whereas the first term can be any row of the matrix, or even all rows of the matrix **N** are needed in order to get all elements of the i-th column of the matrix **E**. The i-th row of the matrix **N** has a very special characteristics in relation to all other rows. If this row is multiplied with the i-th column of matrix **Q**, then the result is a unit current. In all other cases the nodal point current has the value zero.

On the basis of those explained above, it is no difficult task to produce a simple isomorphous (mutually unambiguous) correspondence between all

rows of the matrix **N**, the i-th column of the **Q** matrix and similarly the i-th column of the matrix **E**, further the geometric and physical notions represented in Fig. 3a. The point shown in the figure as a sink is the i-th point of the network and the i-th row of the normal equation system corresponds to it. The elementary branch currents expressed generally by Eq. (22) are the numbers denoted at the edges in Fig. 3a. Taking into account that here we have measurements of unit weight, these values exactly correspond to the differences of the number pairs in the i-th column of the matrix **Q**. The most interesting and curious thing is that the linearly orderer elements in the i-th column of the matrix **Q** (its elements in a single vertical column) are the values pertaining to one point each of the three-dimensional space grid represented in Fig. 1 as a cross-section. Figure 1 illustrates even these potential values denoted earlier as potential U. Finally the i-th column of the matrix **E** contains $m - 1$ pieces of Kirchhoff's laws (Laplace-equations) closing to zero and one Kirchhoff's law (Poisson-equation) yielding direct current. In Figs 3a and 3b the directions of the branches are chosen so that the values of the elementary branch currents are always positive, meaning that the fulfilment of the nodal point Kirchhoff's law can be controlled simply in all points.

3.151 *The microstructure of the coefficient matrix of the normal equations expressing the nodal point law and belonging to the adjustment of observation equations*

The nodal point law, Eq. (19), can be written in an other form. As

$$\mathbf{N} = \mathbf{A}^T \mathbf{P} \mathbf{A} \tag{23}$$

and the coefficient matrix **A** can be decomposed into a diagonal matrix in the main diagonal of which projector matrices $\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_n$ depending on the dimension value of the network appear and into an edge vertex neighbourhood matrix (**EC**) where the scalars differing from zero can be only -1-s and $+1$-s, Eq. (19) can also be written as:

$$(\mathbf{EC})^T \mathbf{CPC} (\mathbf{EC}) \mathbf{Q} = \mathbf{E} . \tag{24}$$

The matrix (**EC**) transforms the vertex potential matrix **Q** into the branch potential matrix $\Delta \mathbf{Q}$. The matrix **C** transforms this matrix $\Delta \mathbf{Q}$ into transformed branch potentials where the branch potentials do not refer to the axes X and Y of the network coordinate system, but to the axis X' and Y' of the local coordinate system fixed by the measurement results. Denoting them by $\Delta \mathbf{Q}'$, we can write the following:

$$\Delta \mathbf{Q}' = \mathbf{C}(\mathbf{EC}) \mathbf{Q} . \tag{25}$$

A further multiplication with the weight matrix **P** as with a conductivity matrix produces from these transformed branch potentials $\Delta \mathbf{Q}'$ transformed branch currents, being left unchanged by the multiplication with the matrix occurring for the second time in Eq. (24). The characteristic feature of the projector matrix **C** is that its square and all its higher powers are equal with the original matrix **C**. This feature called in mathematics *idempotency* has been compared by [1] to the well-known feature of the unit matrix, and he calls such a matrix a *generalized unit matrix*. The matrix $(\mathbf{EC})^T$ appearing on the left-hand side of Eq. (24) selects the transformed branch potentials already computed and being at disposal according to nodal points and it summarizes them algebraically. Therefore it is called together with the matrix $\mathbf{A}^T$ appearing in Eq. (23) a *nodal point Kirchhoff's operator*.

3.152 *The microstructure of the coefficient matrix of the normal equations expressing the loop law and belonging to the method of conditional adjustment*

In Eq. (19) nodal point potentials $\mathbf{Q}_i$, appear. This matrix equation is identical with $m \times m$ scalar equations being sufficient for the determination of the elements of $\mathbf{Q}_i$, in knowledge of the matrix **N**. If the branch potentials $\Delta \mathbf{Q}_{ij} = (\mathbf{Q}_j - \mathbf{Q}_i)$ are taken as unknowns, then Eq. (19) is not sufficient for their determination. As we have $n \times m$ unknowns and only $m \times m$ equations, therefore some more connections must be found between the elements of the matrix $\Delta \mathbf{Q}$ yielding $n \times (n - m)$ supplementary equations. For this purpose, *Kirchhoff's loop laws* seem to be adequate, which can be deduced from the normal equation system of the adjustment with the method of conditional equations:

$$\mathbf{B}\mathbf{P}^{-1}\mathbf{B}^T\mathbf{k} = \omega \,. \tag{26}$$

The vector $\mathbf{P}^{-1}\mathbf{B}^T\mathbf{k}$ appearing on the left-hand side of this equation is the residual vector **v** belonging to the measurements expressed from the so-called correlate-equation, and which can be expressed from the observation equation system of the coordinate adjustment:

$$\mathbf{v} = \mathbf{P}^{-1}\mathbf{B}^T\mathbf{k} = \mathbf{A}\mathbf{x} + \mathbf{l} \,. \tag{27}$$

Therefore in Eq. (26) one can write $\mathbf{Ax} + \mathbf{l}$ instead of $\mathbf{P}^{-1}\mathbf{B}^T\mathbf{k}$. Further we can write again **Q** instead of **x** similarly to the substitution in Eq. (19) of the vector **x** of the unknowns by the matrix **Q**:

$$\mathbf{B}(\mathbf{A}\mathbf{Q} + \mathbf{l}) = \omega \,. \tag{28}$$

As $\omega = \mathbf{Bl}$, therefore

$$\mathbf{BAQ} + \mathbf{Bl} = \mathbf{Bl} \tag{29}$$

or

$$\mathbf{BAQ} = \mathbf{0}\,. \tag{30}$$

Taking the microstructure of **B** and **A** into account, this can be written also as

$$(\mathbf{HE})\mathbf{CC}(\mathbf{EC})\mathbf{Q} = \mathbf{Q}\,. \tag{31}$$

The multiplication with the matrix (**EC**) yields the untransformed branch potentials $\Delta\mathbf{Q}$. The multiplication with **C** projects these onto the local coordinate axis X' and Y'. The second matrix **C** expresses the redundancy of the matrix **B** which can be left fully out of consideration, and finally the matrix (**HE**) gives a command to carry out the polygon closing from the branch potentials $\Delta\mathbf{Q}'$ in form of matrix multiplication. Therefore it can be called together with the matrix **B** appearing in Eq. (26) a *loop-forming Kirchhoff's operator.*

3.153 *The determination of the unknowns by filtering the different source informations*

When the adjustment problem written in the standard form

$$\mathbf{Ax} = \mathbf{l} + \mathbf{v} \text{ or } \mathbf{Bv} = \omega \tag{32}$$

is solved using the generalized inverse matrices $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$ respectively,

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{l}, \text{ respectively } \mathbf{v} = \mathbf{B}^{-1}\omega \tag{33}$$

then properly speaking a linear filtering is carried out from the values of the same measured (source-) information given in two different forms. In case of the two filterings, however, not only the input vectors **l** and ω differ from each other, but the output vectors **x** and **v** too. The freedom in choosing the unknowns allows to use the special case $\mathbf{x} = \mathbf{v}$, or even it allows to filter informations referring to the unknowns from the contradiction-vector ω. Therefore the differences of left-hand sides of the Eqs (33) are notsignificant, but the difference of the input-vectors is very impotant. When the source informations are at disposal in the form of a free-term vector of the observation equations one speaks from *edge-filtering*, when the free-term vector of the condition equations is used, from *loop-filtering*. There is a third basic method of filtering, the so-called *vertex filtering*, which uses the free-term vector **L** of the system of normal equations:

$$\mathbf{x} = \mathbf{QL}\,. \tag{34}$$

An intermediate position between vertex filter and edge filter is occupied by the so-called *vectorial edge filter* resulting from a partial partitioning of the vertex filter. Namely, if the information content of the free-term vector **L** of the normal equation is partitioned according to the measurement results, then one gets elementary vectors whose components are the well-known values pal, pbl . . . These vectors contain, in case of planar networks, two-three elements, — in case of spatial networks, at least 3 elements. If these measurement result vectors are shortly denoted by $\mathfrak{L}$, then a filtering according to the equation

$$\mathbf{x} = \Delta \mathbf{Q} \mathfrak{L} \tag{35}$$

is carried out.

From the point of view of the construction of filters and of carrying out of the filtering, the significance of the fluid-mechanical models is that they draw attention to the physical content behind mathematical models become abstract not to be neglected. This content is concentrated mainly in the notion of the *information collecting area* and it is expressed by its always limited amount. As in case of all flows, in case of the information flow the *law of locality* is valid [6] which must be taken into account in case of all network investigations. The importance of such mathematical models supplemented by physical interpretations and its role have been treated by KRON [9] and SINGER [12].

3.2 *Some simple dynamic models of the information propagation*

3.21 *The simplest geodetical network — the dipole*

The static models described in 3.1 (the interpretation of the potential and gradient of a mass point and of the force field of an electrostatic charge in discrete field) gave physical explanation for networks containing a single active point (source *or* sink). Figures 3a and 3b demonstrate the two possibilities yielded by a single given point (source) and by a single new point (sink), respectively on a "potential" level. Namely for a real flow at least one given point as information source and a new point as information sink are needed. Figure 4 shows the potential distribution of such a real flow network in case of height-difference determinations between the grid points of a three-dimensional space grid. A network (grid) consisting of one single given and one single new point (in addition naturally a finite or infinite number of so-called transit points) is called *dipole*, which can be constructed from the monopole of Fig. 1 by superposing on it a monopole of exactly the same distribution and equal in absolute values, but of opposite sign with unit translation downwards. That means that on the potential kernel of +252.7 a value of −86.1 is super-

posed resulting in a value of +166.7 (Fig. 4). Similarly for the kernel of the negative monopole a value of −166.7 is obtained. If the potential values corresponding to an arbitrary point given by the grid coordinates p, q of the original positive monopole are denoted by $\mathbf{K}(p, q)$, then the potential value corresponding to the same point of the dipole can be computed by the formula

$$\mathbf{Q}(p, q) = \mathbf{K}(p, q) - \mathbf{K}(p, q + 1) \tag{36}$$

0.6	0.7	0.8	1.0	1.1	1.3	1.4	1.4	1.4	1.3	1.1	1.0	0.8	0.7	0.6	
0.6	0.7	0.9	1.2	1.4	1.7	1.9	1.9	1.9	1.7	1.4	1.2	0.9	0.7	0.6	
0.6	0.8	1.1	1.4	1.8	2.2	2.6	2.7	2.6	2.2	1.8	1.4	1.1	0.8	0.6	
0.6	0.9	1.2	1.6	2.2	3.0	3.8	4.1	3.8	3.0	2.2	1.6	1.2	0.9	0.6	
0.6	0.8	1.2	1.9	2.8	4.2	6.0	7.1	6.0	4.2	2.8	1.9	1.2	0.8	0.6	
0.5	0.8	1.2	2.0	3.4	5.9	10.4	15.3	10.4	5.9	3.4	2.0	1.2	0.8	0.5	
0.3	0.5	1.0	1.7	3.4	7.9	19.3	43.2	19.3	7.9	3.4	1.7	1.0	0.5	0.3	
0.1	0.2	0.3	0.7	2.1	7.0	30.9	166.667	30.9	7.0	2.1	0.7	0.3	0.2	0.1	
-0.1	-0.2	-0.3	-0.7	-2.1	-7.0	-30.9		-30.9	-7.0	-2.1	-0.7	-0.3	-0.2	-0.1	
-0.3	-0.5	-1.0	-1.7	-3.4	-7.9	-19.3	-43.2	-19.3	-7.9	-3.4	-1.7	-1.0	-0.5	-0.3	
-0.5	-0.8	-1.2	-2.0	-3.4	-5.9	-10.4	-15.3	-10.4	-5.9	-3.4	-2.0	-1.2	-0.8	-0.5	
-0.6	-0.8	-1.2	-1.9	-2.8	-4.2	-6.0	-7.1	-6.0	-4.2	-2.8	-1.9	-1.2	-0.8	-0.6	
-0.6	-0.9	-1.2	-1.6	-2.2	-3.0	-3.8	-4.1	-3.8	-3.0	-2.2	-1.6	-1.2	-0.9	-0.6	
-0.6	-0.8	-1.1	-1.4	-1.8	-2.2	-2.6	-2.7	-2.6	-2.2	-1.8	-1.4	-1.1	-0.8	-0.6	
-0.6	-0.7	-0.9	-1.2	-1.4	-1.7	-1.9	-1.9	-1.9	-1.7	-1.4	-1.2	-0.9	-0.7	-0.6	
-0.6	-0.7	-0.8	-1.0	-1.1	-1.3	-1.4	-1.4	-1.4	-1.3	-1.1	-1.0	-0.8	-0.7	-0.6	

Fig. 4. The potential distribution of a dipole in the coordinate planes XZ and UZ of the discrete space

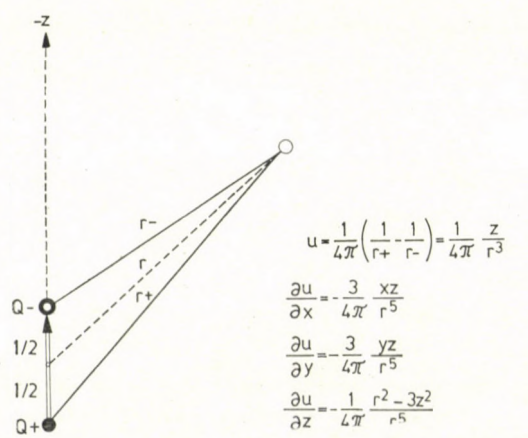


Fig. 5

where $\mathbf{K}(p, q)$ refers to the sink and $\mathbf{K}(p, q+1)$ to the source. Figure 5 shows the immediate formula for the determination of this dipole potential formally coinciding with the computation formulas of the gradient components of the monopole. The formulas for the determination of the gradients of the dipole are also given. The gradient values are demonstrated in Fig. 6.

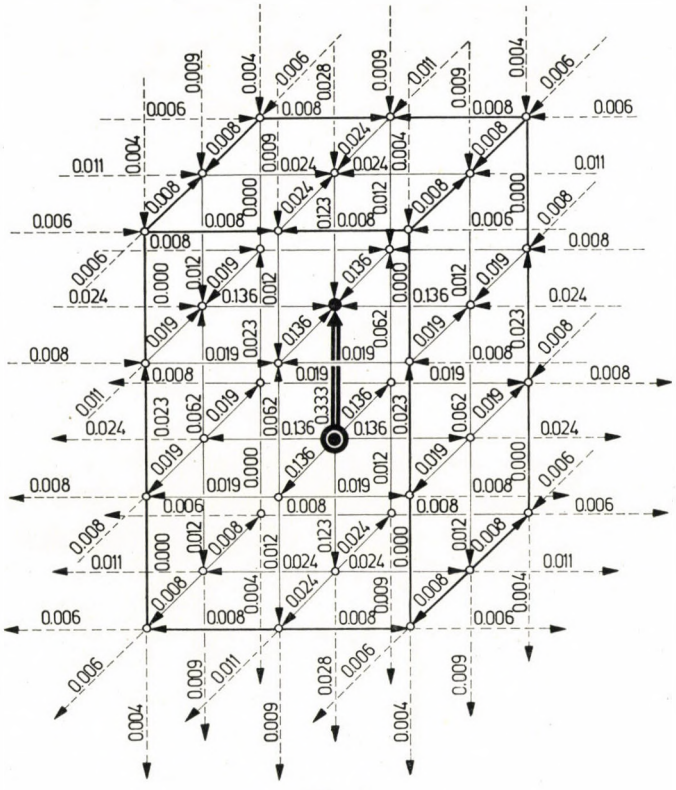


Fig. 6

The potential distribution $\mathbf{Q}(p, q)$ contains the elements of a row of matrix $\mathbf{Q}$ appearing in case of a coordinate adjustment, where the leading member belongs to the sink point. To a given point, many sink points can be chosen, when by taking the points of the spatial grid one after the other as sinks, all rows of the matrix $\mathbf{Q}$ can be physically interpreted. In spite of the fact that the notion of the dipole has been obtained from purely *static* considerations by superposing a positive and a negative charge, the model obtained completely coincides with the potential and gradient field of the flow force field between a given source and the sink chosen, i.e. this is in reality a *dynamic* model.

In a general case a source point is given with its coordinates p_0, q_0, r_0. Let us compute the spatial potential distribution yield the potential distribution of the dipole in an arbitrary point P in function of its coordinates p, q, r. For this purpose we have at disposal the monopole matrix **K**, in each row of which an infinite number of potential values can be found corresponding to a given spatial point, as positive monopole (Fig. 7). The function **F** gener-

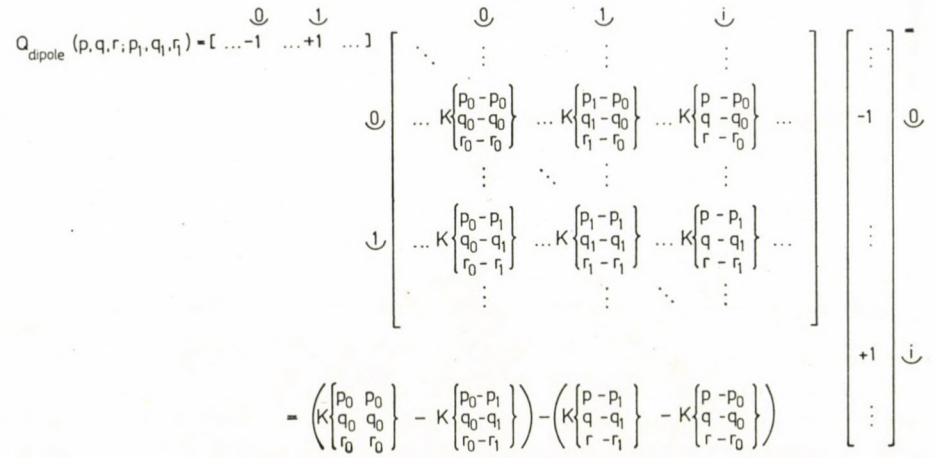


Fig. 7

ating the dipole from this monopole matrix contains only one element $+1$ and -1 each, where the element -1 belongs to the source with the coordinates p_0, q_0, r_0, the element $+1$ to the considered point with the coordinates p, q, r. The variance-covariance matrix has, according to Fig. 7, four terms. The first two terms are, however, constant values and express the demand that the potential of the dipole should be zero in the source point with the coordinates p_0, q_0, r_0. For sake of simplicity these two first terms can be neglected. Equation (36) and Fig. 4 are constructed with this simplified version having the advantaget hat the dipole is distributed into two obliquely symmetric parts. From the simplified formula

$$\mathbf{Q}_{\text{dipole}}(p, q, r; p_1, q_1, r_1) = \mathbf{K}\begin{Bmatrix} p - p_1 \\ q - q_1 \\ r - r_1 \end{Bmatrix} - \mathbf{K}\begin{Bmatrix} p - p_0 \\ q - q_0 \\ r - r_0 \end{Bmatrix} \tag{37}$$

containing no additive constant it can be clearly seen that the values of **Q** in the source point P_0 and in the sink point P_1 are in absolute value equal, but the signs are opposite.

If we look for the potential difference between two neighbouring points P_i and P_j of the dipole, then using the formula in Fig. 7 and Eq. (37) yields

the same results. This result is also equal with the branch current — supposing measurements with equal weights — flowing between the points $P_i\,(p_i, q_i, r_i)$ and $P_j\,(p_j, q_j, r_j)$.

$$\Delta \mathbf{Q}_{i,j} = \mathbf{I}_{i,j} = \mathbf{K}\begin{Bmatrix} p_j - p_1 \\ q_j - q_1 \\ r_j - r_1 \end{Bmatrix} - \mathbf{K}\begin{Bmatrix} p_j - p_0 \\ q_j - q_0 \\ r_j - r_0 \end{Bmatrix} + \mathbf{K}\begin{Bmatrix} p_i - p_0 \\ q_i - q_0 \\ r_i - r_0 \end{Bmatrix} - \mathbf{K}\begin{Bmatrix} p_i - p_1 \\ q_i - q_1 \\ r_i - r_1 \end{Bmatrix} = \tag{38}$$

$$= \mathbf{Q}(p_j, q_j, r_j;\, p_1, q_1, r_1) - \mathbf{Q}(p_i, q_i, r_i;\, p_1, q_1, r_1)\,.$$

Figure 5 showed an asymptotic approximation of this latter formula, yielding, in greater distances from the poles practically the same result as the exact formula presented as Eq. (38).

3.22 *Development of networks with the superposition of elementary networks*

Similarly to the construction of a dipole from two monopoles, described in the previous chapter, arbitrary multipoles can be constructed from more than two monopoles enabling the modeling of geodetic networks consisting of an arbitrary number of given points in an arbitrary distribution. For this theoretically only one positive monopole should be ordered to each source point p_{0k} $(k = 1, 2, \ldots n]$ as pole, then supplementarily a negative monopole in order to the sink point p_1 considered. This force field resulting from the superposition of $(n + 1)$ independent monopoles, yields a general dynamic model to the fixed sink of which information flows from n source points.

The linear combination

$$\mathbf{Q}_{\text{multipole}}(p, q, r; p_1, q_1, r_1) = \mathbf{a}_0 + \mathbf{K}\begin{Bmatrix} p - p_1 \\ q - q_1 \\ r - r_1 \end{Bmatrix} - \mathbf{a}_1\,\mathbf{K}\begin{Bmatrix} p - p_{01} \\ q - q_{01} \\ r - r_{01} \end{Bmatrix} - \tag{39}$$

$$- \mathbf{a}_2\,\mathbf{K}\begin{Bmatrix} p - p_{02} \\ q - q_{02} \\ r - r_{02} \end{Bmatrix} - \ldots - \mathbf{a}_n\,\mathbf{K}\begin{Bmatrix} p - p_{0n} \\ q - q_{0n} \\ r - r_{0n} \end{Bmatrix}$$

enables the superposition necessary for the determination of the resulting potential distribution, where the coefficient weights $a_0, a_1, \ldots, a_n$ are in the moment yet unknown. We know, however, that in case of $a_i = 1$ exactly unit information is flowing from the given point P_i in case of $a_i = 0.5$ its half and so on, therefore if we wish that exactly unit information should reach the single sink P_1 from 11 given point together, then the condition

$$a_1 + a_2 + \ldots + a_n = 1 \tag{40}$$

should be fulfilled. We know further that in the weight coefficient matrix $\mathbf{Q}$ of geodetic networks the given points do not appear, i.e. the values computed from Eq. (39) are in case of all points equal zero. That means that n further conditions of the form

$$\mathbf{Q}(p_{0k}, q_{0k}, r_{0k}; p_1, q_1, r_1) = \mathbf{a}_0 + \mathbf{K}\begin{Bmatrix} p_{0k} - p_1 \\ q_{0k} - q_1 \\ r_{0k} - r_1 \end{Bmatrix} - \\ - \mathbf{a}_1 \mathbf{K}\begin{Bmatrix} p_{0k} - p_{01} \\ q_{0k} - q_{01} \\ r_{0k} - r_{01} \end{Bmatrix} - \ldots - \mathbf{a}_n \mathbf{K}\begin{Bmatrix} p_{0k} - p_{0n} \\ q_{0k} - q_{0n} \\ r_{0k} - r_{0n} \end{Bmatrix} \tag{41}$$

can be written ($k = 1, 2, \ldots n$). Therefore together with the condition (40), ($n + 1$) equations are at our disposal for the determination of the unknown coefficients $a_0, a_1, a_2, \ldots, a_n$: these coefficients can be taken as the *useful information content of the information sources* (pragmatic information).

The realization of Eq. (39) bears no greater practical difficulties even in case of very great number of given points (sinks). Then for the determination of the potential change

$$\mathbf{Q}^{(k)} - \mathbf{Q}^{(k-1)} = \Delta \mathbf{Q}_{k-1,k} = \mathbf{b}_0 + \mathbf{b}_1 \left(\mathbf{Q}^{(k-1)} - \mathbf{K}\begin{Bmatrix} p - p_1 \\ q - q_1 \\ r - r_1 \end{Bmatrix} \right) - \\ - \mathbf{b}_2 \mathbf{K}\begin{Bmatrix} p - p_{0k} \\ q - q_{0k} \\ r - r_{0k} \end{Bmatrix} \tag{42}$$

only three parameters (b_0, b_1, b_2) are to be determined in each case. The latter recursive formula demonstrates the case when the potential function $\mathbf{Q}^{(k-1)}$ belonging to the $k - 1$ sinks is already at disposal and the effect $\Delta \mathbf{Q}$ of a new k-th sink is looked for on the matrix $\mathbf{Q}^{(k-1)}$ in order to produce the matrix $\mathbf{Q}^{(k)}$ of the network containing n sources. For the determination of the three parameters appearing in Eq. (42) the first condition is yielded by the connection:

$$b_1 + b_2 = 1 \tag{43}$$

while the other two conditions prescribe similarly to Eq. (41) that the value of $\mathbf{Q}$ should be in the newly considered k-th sink zero similarly to all other ($k - 1$) sinks. The latter demand can be fulfilled by taking a change Δb of the additive constant b_0 into account, that means, instead of ($k - 1$) conditions, practically a single condition.

3.23 *Application of the flow-mechanical models for spatial and planar networks*

The investigations carried out hitherto refer nearly exclusively to regular spatial grids with infinite dimensions, supposing that the measurements are levelled height differences, and the unknowns are the heights of grid points, It is true that this supposition can be generalized, and instead of the measurement of height differences the results of direction and distance measurements with unit weights can be used, but the regularity and infinite dimension of the network is far from the usual conditions in geodesy.

In the following we shall outline the algorithm for the development of the network which can be applied in the planning of planar and spatial geodetic networks, in the process control of measurement data collection and in processing of the results. The aim is a universal one, but it can be used flexibly and comprehensively for the selection of *network construction elements*, for their mathematical and physical modelling, then for the *construction of different technologies* which enable one to synthetize geodetic networks, fulfilling arbitrary demands, from these network construction elements.

The first and perhaps most important moment is the calculation of all existing possibilities, and then the choice by taking the instantaneous demands into account.

Among the *possibilities*, at first the channel types being able to be constructed between network points are to be taken into account since they are unavoidable elements of the information transfer. In addition to some channels which are of basic types, their arbitrary combinations can also be used, as, e.g. many combined channels can also produced from the basic types of the direction and distance measurements (e.g. direction and distance measurement in space, direction measurements there and back in plane, simple direction measurement with a single orientation unknown, spatial direction measurement with three orientation parameters etc.). The situation is further complicated by the dependence of the channel capacity on the spatial position of the end points of the channel, in addition to its so-called "physical" weight, — a situation which emphasizes the necessity of a systematic grouping and classification. Therefore one can speak from a so-called "geometric" weight, which expresses that side of the channel capacity which cannot be expressed by the physical weight. The "third side" of the channel capacity is the circumstance that the capacity of the channel depends also on the choice of the coordinate system and of the network parameters and unknowns. Thus, e.g. a very different information flow is found in the same channel network if the planar coordinates are taken as unknowns and another if the unknowns are the coordinate differences of neighboring points.

Similarly the great freedom in selecting the points and point pairs of

the network belong to the possibilities, among which geodetic measurements of any kind are to be carried out. This "configuration problem" is one of the most fruitful fields of network planning. The most labourious procedures in network planning are exactly the consideration of the configuration. The algorithm ot network construction to be presented aims just at the simplification and mechanization of this problem.

Requirements are in the planning of geodetic networks the demands of the users more or less well drafted for the position and accuracy of the points to be determined. In addition to the demands of the users, the field itself poses sometimes severe claims during the planning of geodetic networks, but this problem is reversible, as it can also be investigated, which are the view between points and other requirements for different types of networks and configurations, respectively. A problem of no less importance is nowadays in planning of geodetic networks the computer-capacity necessary in addition of up-to-date instrumentation, as it is supposed that this increases exponentially with the increase of the dimensions of the network. In the following chapter we should like to call attention to some computerized solution possibilities which call forth a possible re-evaluation of existing conceptions.

3.24 *Extension, reduction, modification of networks by a limitation to the area of useful information gathering*

In this chapter we shall deal with a computer program written for a Hungarian-made computer TPA/i of small capacity (operative memory 4K) for the realization of the problem outlined in the heading.

The program is written so that it could be used universally or in an elastically modified form in the possible differing sequences and in case of different network configurations. As elementary task, the program carries out the change due to an $(i + 1)$-th measurement result in the matrix $\mathbf{Q}$, in the unknowns $\mathbf{x}$ and in the values $\mathbf{v}^T\mathbf{P}\mathbf{v}$ of a network already adjusted and consisting of n points and of i measurements.

The requirements against the program were during its planning the following:

— it should be applicable even in case of $i = 0$, i.e. with no measurements in the network;

— it should be applicable for the removal of j-th measurement results already taken into account in the network containing measurements $(1 \leq \leq i \leq j)$;

— it should be applicable to decide which partial set of unknowns influences that the effect of the result of a certain measurement influence.

The sequential (recursive) formula for the determination of the modified value $\mathbf{Q}_{i+1}$ of the matrix $\mathbf{Q}$ from the original values $\mathbf{Q}_i$ can be written in the

following form [14]:

$$\mathbf{Q}_{i+1} = \mathbf{Q}_i - \frac{\mathbf{Q}_i a_{i+1} a_{i+1}^T \mathbf{Q}_i}{\frac{1}{p_{i+1}} + a_{i+1}^T \mathbf{Q}_i a_{i+1}} = \mathbf{Q}_i - \Delta \mathbf{Q}_{i+1}. \tag{44}$$

The vector a_{i+1}^T is here the coefficient vector of the observation equation of the $(i + 1)$-th measurement result, P_{i+1} is the weight of the same measurement.

The other sequential formula producing the modified value x_{i+1} from the original value x_i of the vector $\mathbf{x}$ can be written in the following form:

$$x_{i+1} = x_i + \Delta \mathbf{Q}_{i+1} b(x_i) = x_i + \Delta x_{i+1}, \tag{45}$$

where $b(x_i)$ is the free-term vector of the normal equation corresponding to the values x_i, $\Delta \mathbf{Q}_{i+1}$ is according to Eq. (44).

In order to make the program applicable even in case of $i < n$, and even if $i = 0$, the following way must be chosen. As preparation, for the initial matrix $\mathbf{Q}_0$ a unit matrix $\mathbf{E}_n$ of the order n is to be generated by the computer, what can be taken as the matrix $\mathbf{Q}$ of n fictitious coordinate measurements of unit weight. One begins to modify it, with the first $i = 1$-th measurements, then after the input of n measurement results (or later) the matrix $\mathbf{N}_0 = \mathbf{E}_n$ symbolizing the fictitious measurements can be removed from the network. Afterwards the result of further measurements (those of superfluous measurements) can be investigated one by one or in groups.

In order to permit that the program should indicate the effect of the removal of a real or of a fictitious measurement, the weight of the measurement result considered is to be made negative. Otherwise everything is similar as in case of measurements with positive weights. The removal of the unit matrices $\mathbf{N}_0$ or $\mathbf{Q}_0$ were made in such a way, when in a cycle consisting of n steps mere fictitious coordinate measurements with the weights $P = -1$ were introduced into the computer.

The program can print out as requested any of the following lists:

— the change-matrix $\Delta \mathbf{Q}_{i+1}$
— the modified matrix $\mathbf{Q}_{i+1}$
— the change vector $\Delta \mathbf{x}_{i+1}$
— the modified vector x_{i+1}
— the initial value $l^T \mathbf{P} l$
— instantaneous actual value of $\mathbf{v}^T \mathbf{P} \mathbf{v}$.

The first variant of the program makes pointwise "information-collection" meaning that it considers only a single point at once. Therefore only

three rows of the matrix **Q** are necessary to be present at once in the operative memory which correspond to the coordinate-changes *dx*, *dy* of the point investigated and to its orientation unknown *dz* as to leading terms. The matrix **Q** itself is on the disc of the background-memory.

An interactive data processing method has been without the necessity given on this small computer for the solution of the problem that to take all measurement results for the adjustment of a new point into account. The solution suggested by BÄHR in a dissertation [3] for the localization of the area of information collection is namely too high a task for a small computer, both from the point of view of the length of the program and of the loading of the memory. Our solution is practically a semiautomatic method which leaves to the operator to end the adjustment there, then and using as many data as necessary in the given situation, by considering the given conditions.

According to practical experiences it has been found that the *practical information collecting area* of the points of a fourth-order city network with direction and distance measurements lies on an area covered by 7 to 14 measurement results in the immediate vicinity of the point. The collective effect of the other several hundred measurements did not reach ± 1 mm. Similar results have been obtained when the adjustment was carried out not with real but with fictitious measurements. The theoretical *information collecting area* coincides on the whole with the practical information collecting area mentioned.

The second variant of the program has been chosen with input-orientation instead of the output-oriented structure of the first variant, meaning that only a single measurement is processed and its "information spreading" effect is investigated locally. In such a case the information content of the measurement is "spread" to new points to be determined in the immediate vicinity of the measurement and of the given point, respectively, and thus one gets more and more reliable, but always only partially adjusted coordinates. The law of locality [6] presented itself also here, but as the matrix **Q** is here less practicable, only the first variant of the program — local adjustment with information collection — has been built up until the very end.

The programs described can be applied equally for

— the adjustment of new networks with the rigorous method or with a fixed accuracy;

— extension, densification of existing networks, for the supplementation of destroyed points;

— for the planning of networks before *(a priori)* or after *(a posteriori)* the measurements are made, and for the modification of plans with the testing of several variants;

— for the process control of network measurements, i.e. for computer aided information collection.

— for the theoretical investigation of the useful information content of measurement results or of other initial data with a numerical method.

*

In this paper we have tried to prove the physical nature of the object of geodesy, i.e. of the geodetic information. We have shown by theoretical explanations and by practical examples that the information flow is subject to the Kirchhoff's laws, and the information flow fulfils the Laplace- and Poisson-equations valid in discrete fields. Therefore it can be shown very generally that gravimetric, electrostatic and electrodinamic field models have analogies with geodetic networks. Using this fact it can be proven that in geodetic networks the principle of the conservation of information, the law of locality and other similar physical laws are also valid. A practical application of these theoretical considerations is a computer program which carries out the local adjustment of any geodetic network of arbitrary dimensions on a small computer in a semiautomatical interactive manner effectively, and which can be excellently used for the planning of networks, and for the process control of network measurements, too.

By using different (mechanical, optical, electronic) analogies of the adjustment, results equivalent with those using the method of the least squares [8, 13] have been obtained. From among these, the use of electric analogies should be emphasized, which permits the planning and adjustment of networks by using the laws of locality and information propagation on small computers in a simple way.

REFERENCES

1. BJERHAMMAR, A.: Theory of errors and generalized matrix inverses. Elsevier Scientific Publ. Co. Amsterdam, London, New York, 1973.
2. BORRE, K.—MEISSL, P.: Strength analysis of leveling-type networks. An application of random walk theory. Geodetisk Institut Meddelelse No. 50, 1974.
3. BÄHR, A. G.: Zu Ausgleichungsverfahren bei der Ergänzung bestehender geodätischen Netze. Deutsche Geodätische Kommission, Reihe C, No. 187. München, 1973.
4. DUFFIN, R. I.: Discrete potential theory. *Duke Mathematical Journal*, 1953, 233—251.
5. GRAFAREND, E.: Die Genauigkeit eines Punktes im mehrdimensionalen euklidischer Raum. Deutsche Geodätische Kommission, Reihe C, No. 153, München, 1971.
6. HALMOS, F.—KÁDÁR, I.—KARSAY, F.: Local adjustment by least squares filtering. IAG Symposium „Computational Methods in Geometrical Geodesy", Oxford, 1973. 09. 2—8.
7. HALMOS, F.—KÁDÁR, I.—KARSAY, F.: Local adjustment by least squares filtering. Bulletin Géodésique, No. 111, Paris, 1974, 25—51.
8. HAZAY, I.: Kiegyenlítő számítások. Tankönyvkiadó, Budapest, 1968. Adjusting calculation in surveying. Akadémiai Kiadó, Budapest (1970).
9. KRON, G.: Diakoptics. Mac Millan Publ. London, 1961.
10. MEISSL, P.: Aufgaben der mathematischen und numerischen Geodäsie. *Öst. Zeitschrift für Vermessungswesen und Photogrammetrie*, 1973.
11. REZA, F. M.: Bevezetés az információelméletbe (An introduction to information theory). Műszaki Könyvkiadó, Budapest, 1966.
12. SINGER, D.: Nagy hálózatok megoldási módszereiről (Methods for solution of large networks). Műszaki Tudomány, No. 1—2. 1974. Akadémiai Kiadó, Budapest.

13. Wolf, H.: Ausgleichungsrechnung nach der Methode der kleinsten Quadrate. Dümmler Verlag, Bonn, 1968.
14. Zielke, G.: Numerische Berechnung von benachbarten Inversen Matrizen und linearen Gleichungssystemen. Friedrich Vieweg, Braunschweig, 1970.

ПОЛНАЯ СИСТЕМА ПОНЯТИЙ ГЕОДЕЗИЧЕСКОЙ ИНФОРМАЦИИ И ВЫЧИСЛЕНИЕ ГЕОДЕЗИЧЕСКИХ СЕТЕЙ

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РЕЗЮМЕ

Физика изучает зависимости и законы взаимодействий материальных объектов в истинном пространстве, в геометрия описывает зависимости геометрических объектов, интерпретированных в сигнальном пространстве. Если две отрасли геодезии, физическую геодезию и геометрическую геодезию, рассматреть с единой точки зрения, то указанные выше физическое и геометрическое пространства нужно заменить пространством событий, заключающим в себе оба пространства как специальные случаи. Изучением наиболее общих зависимостей относительно простанства событий занимается множество отраслей наук, как, например, алгебра событий, теория вероятностей и, в последнее время, информационная, теория. Эта последняя отрасль из математической теории ныне перешла в физическую, что способствовало образованию информационной физики.

В статье показывается общая модель, с помощью которой происходящее в геодезических сетях течение информации может быть интерпретируема как физический процесс, независимо от вида и размера сети, а также от числа и качества точек и результатов измерений. Показывается, что система нормальных уравнений и весовая матрица, а также их современные варианты, обощенные или псевдообратные матрицы приобретают ясное и однозначное физическое содержание.

Главное внимание посвящается трехмерным пространственным сетям, используя все возможности для того чтобы такие сети или заполненное такими сетями пространсво сопоставить с соответствующими понятиями и силовыми полями физической геодезии. В результате исследования показывается физическая интерпретация теории фильтров, дается предложение для некоторого видоизменения меры информации по Шаннону для более гучшего выяснения физического понятия информации.

Теоретические рассуждения обосновываются экспериментальными примерами геодезических сетей, вычисленных при помощи точных и приближенных формул.

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A STRAIGHT PRESENTATION OF THE ADJUSTMENT OF SO-CALLED FREE GEODETIC NETWORKS

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By solving least squares adjustment problems the quadratic forms $\mathbf{v}^T\mathbf{P}\mathbf{v} = \min$ and $\mathbf{x}^T\mathbf{x} = \min$ can be determined separately too. In this case the computation of the $\mathbf{x}^T\mathbf{x} = \min$ leads to the solution of linear inhomogeneous equations. On the basis of the general solution of such a system of equations the characteristic properties of the free parametric adjustment methods are presented in this paper.

Introduction

For several years the adjustment of the so called free geodetic networks has aroused active interest in the associated algorithms and computing technics (e.g. computation of pseudo-inverses, etc.). This type of adjustment is based on the well-known least squares technique, which is also called for coordinate adjustment or more generally for parametric adjustment. Little attention was paid, however, to the fact, that the same $\mathbf{v}^T\mathbf{P}\mathbf{v} = \min$ can be computed by the similarly well-known Lagrange multiplier method (otherwise known as condition adjustment) avoiding any coordinate- (or parameter-) computation. In this way one can split the problem, but the subsequent computation of the coordinates by the adjusted (i.e. consistent) network elements gives rise to interesting possibilities and this is the real subject of the free parametric adjustment methods.

Computation of network coordinates

Let us suppose, that we have already performed the adjustment of our measurements (e.g. by the Lagrange multiplier method) and we are looking for a suitable algorithm for the computation of the network coordinates. It can be immediately realized that we have many different choices of the possible patterns in the network for trigonometric computation of the coordinates. It is also possible, however, to apply the well-known linear equations of the coordinate adjustment method to this purpose:

$$\underset{n,m}{\mathbf{A}}\ \underset{m,1}{\mathbf{x}} = \underset{n,1}{\mathbf{l}} + \underset{n,1}{\mathbf{v}} \qquad (n > m) \tag{1}$$

where $\mathbf{x}$ is the vector of the coordinate supplements (which are introduced in the linearization procedure), $\mathbf{l}$ stands for the vector of the measurements (actually its elements are residuals composed from preliminary values and measurements), vector $\mathbf{v}$ represents the corrections to the measurements and $\mathbf{A}$ is the coefficient matrix of the functional model (otherwise known as design matrix). With the corrected (i.e. adjusted) values of the residuals

$$\bar{\mathbf{l}} = \mathbf{l} + \mathbf{v}$$

the matrix equation (1) constitutes a consistent system of inhomogeneous linear equations, the solution of which provides the required network coordinates. This solution depends, however, on the rank of $\mathbf{A}$, which is, in turn, determined by the freedom in location and orientation of the network. If the rank of $\mathbf{A}$ is less than the number of the coordinates to be determined, i.e. $r < m$ then from any m rows of the system (1) we get an infinite set of solutions. Denoting the coefficient matrix of the $m \times m$ sized subsets of the equation system (1) by $\underset{m,m}{\bar{\mathbf{A}}}$, then its so-called general solution is (see, e.g. [1]):

$$\underset{m,1}{\mathbf{x}} = \underset{m,1}{\mathbf{x}_0} + \underset{m,d}{\mathbf{U}}\ \underset{d,1}{\mathbf{y}} \quad (d = m - r), \tag{2}$$

where the matrix $\underset{m,n}{\mathbf{U}}$ represents the d linear independent solution vectors of the homogeneous system $\underset{m,m}{\mathbf{A}}\ \underset{m,1}{\mathbf{u}} = 0$, the vector $\underset{d,1}{\mathbf{y}}$ contains the so-called free parameters and $\mathbf{x}_0$ is the particular solution vector for $\underset{d,1}{\mathbf{y}}$.

This general solution can be computed without any special algorithm by using the well-known Gauss elimination technics, as it is shown, e.g. in [1]. In the course of the Gauss elimination procedure, however, a $d \times d$ sized upper triangular $\mathbf{0}$-matrix shall appear at the end, where for the sake of the general solution (2) the diagonal elements are replaced by number 1 and the last d unknowns are set to $\underset{d,1}{\mathbf{y}}$ during the back substitution.

In the matrix equation (2) the free parameters can take any real numbers and zeros, too. In this way Eq. (2) represents also the transformation formula between any two solution vectors, which are equally valid for a network.

Computation of coordinate variances and covariances

From the matrix equation (2) it is evident, that the variance-covariance matrix of the coordinates $\underset{m,1}{\mathbf{x}}$ is dependent on that of the free parameters $\underset{d,1}{\mathbf{y}}$. If we are looking for an optimum solution in the sense of the least squares technique, the free parameters $\underset{d,1}{\mathbf{y}}$ could be determined via the quadratic form

$\mathbf{x}^T\mathbf{x} = \min$. So from Eq. (2) we get

$$\underset{d,1}{\mathbf{y}} = -\underset{d,d}{(\mathbf{U}^T\mathbf{U})^{-1}}\ \underset{d,m}{\mathbf{U}^T}\ \underset{m,1}{\mathbf{x}_0} \tag{3}$$

and substituting Eq. (3) into (2) the general solution will be specialized to a unique one, with minimum variance:

$$\underset{m,1}{\bar{\mathbf{x}}} = [\underset{m}{\mathbf{I}} - \underset{m}{\mathbf{U}}(\mathbf{U}^T\mathbf{U})^{-1}\mathbf{U}^T]\underset{m,1}{\mathbf{x}_0}, \tag{4}$$

where $\mathbf{I}$ is a $m \times m$ sized unit matrix. Finally, from Eq. (4) we get the variance-covariance matrix of the $\bar{x}$ coordinates, as follows:

$$\mathbf{Q}_{\bar{x}\bar{x}} = [\mathbf{I} - \mathbf{U}(\mathbf{U}^T\mathbf{U})^{-1}\mathbf{U}^T]\,\mathbf{Q}_{x_0x_0}\,[\mathbf{I} - \mathbf{U}(\mathbf{U}^T\mathbf{U})^{-1}\mathbf{U}^T] \tag{5}$$

where $\mathbf{Q}_{x_0x_0}$ stands for the variance covariance matrix of $\mathbf{x}_0$.

The computation of the $\mathbf{x}^T\mathbf{x} = \min$ dealt here does not interfere with that of the $\mathbf{v}^T\mathbf{P}\mathbf{v} = \min$. That follows from Eq. (1), which is already a consistent one. In spite of the similarity of these two quadratic forms, however, the $\mathbf{x}^T\mathbf{x} = \min$ has no real stochastic background. It is simply a geometric choice with the traditional minimum property, which automatically follows from $\mathbf{v}^T\mathbf{P}\mathbf{v} = \min$ for arbitrary located and oriented systems in the form of a relative minimum.

In Eqs (3) and (4) we did not use a "weight matrix" for the $\mathbf{x}_0$ vector in the least squares procedure, but formally it is possible. With such a "pseudo-stochastic" assumption we can modify the computation of the x coordinates, but not the structure of the general solution (2).

A practical algorithm for the free parametric adjustment

In [2] and [3] a two-stage algorithm was presented for the coordinate adjustment of free geodetic networks, which was originated from F. R. HELMERT. Here we can use the same least squares procedure in both stages of the adjustment for the computation of $\mathbf{v}^T\mathbf{P}\mathbf{v} = \min$. resp. for $\mathbf{x}^T\mathbf{x} = \min$. For that purpose we partition equation (1) as follows:

$$\underset{n,r}{\mathbf{A}_1}\,|\,\underset{n,d}{\mathbf{A}_2}\begin{pmatrix}\underset{r,1}{\mathbf{x}_1}\\ \underset{d,1}{\mathbf{y}}\end{pmatrix} = \underset{n,1}{\mathbf{l}} + \underset{n,1}{\mathbf{v}} \qquad (r + d < n), \tag{1a}$$

where besides previous notations $\mathbf{x}_1$ denotes the first (major) part of the coor-

dinate unknowns, $\mathbf{A}_1$ and $\mathbf{A}_2$ are appropriate submatrices of $\mathbf{A}$. Now, in the first stage the least squares solution for $\mathbf{x}_1$ is

$$\underset{r,1}{\mathbf{x}_1} = -(\underset{r}{\mathbf{A}_1^T}\underset{r}{\mathbf{P}}\underset{r}{\mathbf{A}_1})^{-1}\,\underset{r}{\mathbf{A}_1^T}\underset{r}{\mathbf{P}}\underset{d}{\mathbf{A}_2}\,\underset{d,1}{\mathbf{y}} + (\underset{r}{\mathbf{A}_1^T}\underset{r}{\mathbf{P}}\underset{r}{\mathbf{A}_1})^{-1}\underset{r}{\mathbf{A}_1^T}\underset{n}{\mathbf{P}}\,\underset{n,1}{\mathbf{l}}\,, \tag{6}$$

where $\underset{n,n}{\mathbf{P}}$ is the weight matrix of the $\underset{n,1}{\mathbf{l}}$ measurements. It is striking, that the structure of Eq. (6) is similar to that of Eq. (2), but the number of the rows are still different. Looking for the total vector of coordinate unknown we can write:

$$\begin{pmatrix} \underset{r,1}{\mathbf{x}_1} \\ \underset{d,1}{\mathbf{y}} \end{pmatrix} = \begin{pmatrix} -(\underset{r}{\mathbf{A}_1^T}\underset{r}{\mathbf{P}}\underset{r}{\mathbf{A}_1})^{-1}\,\underset{r}{\mathbf{A}_1^T}\underset{d}{\mathbf{P}}\underset{d}{\mathbf{A}_2} \\ \underset{d,d}{\mathbf{I}} \end{pmatrix} \underset{d,1}{\mathbf{y}} + \begin{pmatrix} (\underset{r}{\mathbf{A}_1^T}\underset{r}{\mathbf{P}}\underset{r}{\mathbf{A}_1})^{-1}\underset{r}{\mathbf{A}_1^T}\underset{n}{\mathbf{P}} \\ \underset{d,n}{\mathbf{0}} \end{pmatrix} \underset{n,1}{\mathbf{l}}\,. \tag{2a}$$

The comparison of Eqs (2) and (2a) shows, that

$$\underset{m,d}{\mathbf{U}} = \begin{pmatrix} -(\mathbf{A}_1^T\mathbf{P}\mathbf{A}_1)^{-1}\,\mathbf{A}_1^T\mathbf{P}\mathbf{A}_2 \\ \mathbf{I} \end{pmatrix} \quad \text{and} \quad \underset{m,1}{\mathbf{x}_0} = \begin{pmatrix} (\mathbf{A}_1^T\mathbf{P}\mathbf{A}_1)^{-1}\,\mathbf{A}_1^T\mathbf{P} \\ \mathbf{0} \end{pmatrix} \mathbf{l}\,.$$

With these substitutions we can perform the free coordinate adjustment of a geodetic network according to the equations (3), (4) and (5), and this is Helmert's procedure. In the practical computation one can still benefit from the fact, that $\mathbf{x}_0$ and consequently also $\mathbf{Q}_{x_0x_0}$ are bordered by $\mathbf{0}$ submatrices.

Where and why to apply free parametric adjustment technics?

In connection with adjustment problems three important aspects should be mentioned. The starting point is certainly the concept of modern statistical estimation theory. Looking for appropriate algorithms, concepts of the modern linear algebra should also be useful. Finally, a profound knowledge of numerical analysis is also important in the computational implementation of the adjustment.

Free parametric adjustment technique do not interfere with the computation of $\mathbf{v}^T\mathbf{P}\mathbf{v} = \min$, i.e. the statistical concept is not modified by this method. From the linear algebraic point of view the free parametric adjustment method is certainly an interesting one, the merits of which are not yet fully exploited. If we are dealing with different sets of coordinates (or parameters) to be connected in a further adjustment stage for instance, careful examination is recommended concerning the appropriate variance-covariance matrices to be introduced. Finally, from the point of view of numerical analysis

it seems to be interesting to apply free parametric adjustment technics to ill-conditioned adjustment problems, where so-called "quasi free" $\underset{d,1}{\mathbf{y}}$ parameters are involved. In this connection "weighting" of the $\mathbf{x}_0$ vector promises useful possibilities.

REFERENCES

1. Zurmühl, F.: Matrizen und ihre technischen Anwendungen. Springer Verlag, Berlin/Göttingen/Heidelberg, 1950; 1958; 1961 and 1964.
2. Wolf, H.: Helmerts Lösung zum Problem der freien Netze mit singulärer Normalgleichungsmatrix. *Zeitschrift für Vermessungswesen*, 1972, p. 189.
3. Wolf, H.: Die Helmert-Inverse bei freien geodätischen Netzen. *Zeitschrift für Vermessungswesen*, 1973. p. 396.

ПРЕДСТАВЛЕНИЕ УРАВНИВАНИЯ ТАК НАЗЫВАЕМЫХ СВОБОДНЫХ ГЕОДЕЗИЧЕСКИХ СЕТЕЙ

ДЬ. АЛПАР

РЕЗЮМЕ

При решении проблем уравнивания по методу наименьших квадратов квадратические формы $\mathbf{v}^T\mathbf{P}\mathbf{v} = \min$ и $\mathbf{x}^T\mathbf{x} = \min$ могут быть определены отдельно. В этом случае вычисление $\mathbf{x}^T\mathbf{x} = \min$ приводит к решению неоднородной системы уравнений. На основе общего решения такой системы уравнений в статье показываются характерные особенности методов уравнивания со свободными параметрами.

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CORRELATION PROPERTIES OF SOURCE GENERATED SEISMIC NOISE, SCATTERED ON VELOCITY INHOMOGENEITIES

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The paper will be concerned with the spatial-temporal correlation properties of source-generated scattered seismic noise. As particular cases we shall consider seismic noise scattered on velocity inhomogeneities uniformly distributed in a near-surface thin layer, resp. in the infinite half space. We shall derive an expression for the coherency of the source-generated "inorganized" noise as a function of frequency, average velocity and geophone spacing. Connection with the optimal design of multiple-coverage field systems will be pointed out.

We start out from the following well-known statistical representation of the noisy parts of seismic traces [7]:

$$F(t) = \sum_k a_k s(t - t_k) \tag{1}$$

where the a_k-s are independent Gaussian variables of zero mean and variance a^2; the "*arrival times*" t_k obey a Poisson distribution of density λ, i.e. the probability that there are exactly N arrivals in the time-interval $(T_0, T_0 + T)$ is given by

$$e^{-\lambda T} \frac{(\lambda T)^N}{N!}\,; \tag{2}$$

the wavelet $s(t)$ is of zero mean and it is identically zero outside a limited interval, i.e.

$$\int_{-\infty}^{\infty} s(t)\,dt = 0$$

$$s(t) \equiv 0 \text{ if } t \notin [T_1, T_2].$$

(In seismic practice we generally have $T_1 = 0$.) The autocorrelation function of the series (1) is given, according to the theorem of CAMPBELL [13, 17, 18, 19][1] by

$$R_{FF}(\tau) = \langle F(t)F(t+\tau)\rangle = \lambda a^2 \int_{-\infty}^{\infty} s(t)\,s(t+\tau)\,dt\,. \tag{3}$$

[1] The notation $\langle \ldots \rangle$ will denote statistical mean values taken over all realizations of a stochastic process.

We shall need the following generalization of CAMPBELL's theorem, due to OLSHEVSKIY [17]:

Let

$$F_1(t) = \sum_i a_i s_1(t - t_i;\ \xi_i)$$

$$F_2(t) = \sum_j a_j s_2(t - t_j;\ \xi_j)$$

be two processes where s_1 and s_2 are different functions depending on a random parameter ξ as well. If the distribution function of ξ is $W(\xi)$ and the definition of λ and a^2 is as in Eq. (3), we have

$$\langle F_1(t) F_2(t + \tau)\rangle = \lambda a^2 \iint_\xi W(\xi) \int_{-\infty}^{\infty} s_1(t,\ \xi)\, s_2(t + \tau,\ \xi)\, dt\, d\xi\,. \tag{4}$$

Before proceeding further we note that the series representation (1) of the seismic noise can easily be brought into correspondence with the inhomogeneities of the medium (see the statistical reverberation theory of CARLETON [3]).

We start out from the wave equation

$$\nabla^2 p - \frac{1}{c^2}\,\frac{d^2 p}{dt^2} = 0\,, \tag{5}$$

where the inhomogeneous velocity c is approximated by $c = c_0/(1 + \varepsilon)$. $\langle\varepsilon\rangle = 0$, $\langle\varepsilon^2(x)\rangle = \varepsilon^2$ (this was termed as velocity model (25c) in KORVIN [10]), Neglecting multiple scattering, the solution to Eq. (5) is given by

$$F(t) = \frac{k_0 P_0}{2\pi} \int_V \varepsilon(x, y, z)\ \frac{s(t - 2r/c_0)}{r^2}\, dV\,, \tag{6}$$

where F is the backscattered signal detected at $(0, 0, 0)$, $P_0 s(t)$ ($s(t) \equiv 0$ if $t < 0$) is the wavelet generated by the source situated at the same point $(0, 0, 0)$, $r = \sqrt{x^2 + y^2 + z^2}$ is the distance of the inhomogeneity, V is a spatial domain containing the inhomogeneities and $k_0 = 2\pi f_0/c_0$, where f_0 is the dominant frequency of $s(t)$.

Assuming isolated local inhomogeneities ("diffracting points"):

$$\varepsilon\,(x, y, z) = \sum_i \varepsilon_i\, \delta(x - x_i,\ y - y_i,\ z - z_i)\,, \tag{7}$$

where (x_i, y_i, z_i) are the coordinates of the i-th diffracting point and δ is the

three-dimensional Dirac delta function. Inserting (7) to (6),

$$F(t) = \frac{k_0 P_0}{2\pi} \int_V \frac{s(t - 2r/c_0)}{r^2} \sum_i \varepsilon_i \, \delta(x - x_i, y - y_i, z - z_i) \, dV = \sum_i a_i s(t - t_i), \quad (8)$$

i.e. $F(t)$ is of the form (1) where we have put

$$a_i = \frac{k_0 P_0 \varepsilon_i}{2\pi r_i^2} \, ; \; t_i = \frac{2r_i}{c_0} \, . \quad (9)$$

The stationarity of the sequence a_i can be assured by digital or analog methods of amplitude regulation, while the Poisson distribution of the t_i-s follows from the evident independence, homogeneity and rarity of the diffraction points ([8], § 1).

We shall need a simple geometrical lemma:

Consider a tetrahedron $DD'P_1P_2$, where $DD'P_1$ and $DD'P_2$ are right angles (Fig. 1). Denote by O the midpoint of section $\overline{P_1P_2}$, introduce the notations

$$\overline{P_1O} = \overline{OP_2} = \varrho; \; \overline{P_1P_2} = r; \; \overline{P_1D} = R_1; \; \overline{P_2D} = R_2; \; \overline{OD} = R; \; \overline{DD'} = z;$$
$$P_1OD \measuredangle = \alpha'; \; \alpha = 90^\circ - \alpha'; \; D'OD \measuredangle = \vartheta;$$

and assume that $R_1 \gg \varrho$; $R_2 \gg \varrho$. Then

$$R_1 - R_2 \approx r \sin \alpha \cdot \cos \vartheta \quad (10)$$

Indeed, from the right triangle $DD'O$ we have $z = R \sin \vartheta$; $D'O = R \cos \vartheta$. For the triangle $D'P_1P_2$, by the cosine theorem

$$\overline{D'P_1^2} = \overline{D'O^2} + \varrho^2 - 2\varrho \cdot \overline{D'O} \cdot \cos \alpha'$$
$$\overline{D'P_2^2} = \overline{D'O^2} + \varrho^2 + 2\varrho \cdot \overline{D'O} \cdot \cos \alpha',$$

i.e.

$$R_1^2 = \overline{D'O^2} + \varrho^2 - 2\,\overline{D'O} \cdot \varrho \cdot \cos \alpha' + z^2$$
$$R_2^2 = \overline{D'O^2} + \varrho^2 + 2\,\overline{D'O} \cdot \varrho \cdot \cos \alpha' + z^2$$

from where

$$R_2^2 - R_1^2 = (R_2 + R_1)(R_2 - R_1) = 4\varrho \cos \alpha' \cdot \overline{D'O} = 4\varrho R \cos \alpha' \cos \vartheta;$$
$$R_2 - R_1 = 2\varrho \cos \alpha' \cos \vartheta + O\left(\frac{\varrho}{R}\right)$$

and the proof of Eq. (10) has been accomplished.

In what follows we shall investigate the spatial-temporal correlation of source-generated scattered seismic noise, i.e. the correlation between the noise amplitudes F_1 observed at the geophone G_1 at some time t_0, and F_2 at geophone G_2, a distance r apart and at the time instant $t_0 + \tau$. We assume (Fig. 2) that the measurements are performed in the (x, y) plane, the depth-coordinate z is oriented downwards, the geophones (resp. sources[2]) G_1 and G_2 are at points $(-r/2, 0, 0)$ and $(r/2, 0, 0)$, i.e. at a distance r from each other.

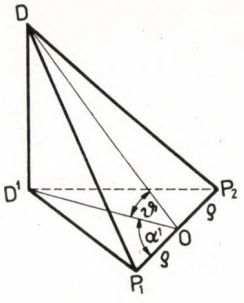

Fig. 1

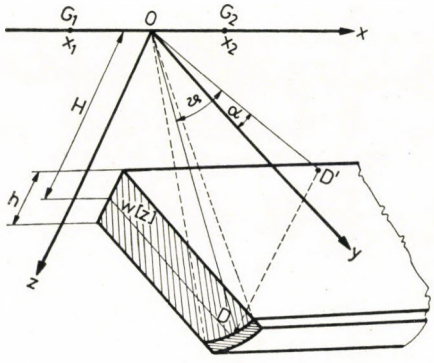

Fig. 2

Let us assume that the point-like velocity inhomogeneities are enclosed in a layer of thickness h and at depth H; if D is a diffracting point within the layer, let D' denote its projection on the (x, y) plane; let α denote the angle between the y axis and the line OD', let $\vartheta = D'OD\sphericalangle$. Let us denote by R the distance $\overline{OD}$ and assume that $R \gg r$. Angles corresponding to the i-th diffracting point D_i are denoted by α_i, resp. ϑ_i. With these notations the

[2] The assumption of the identical position of geophones and sources implies that our results only apply to *NMO-corrected* seismic sections.

noise-trains F_1, F_2 observed at geophones G_1 and G_2, respectively, are:

$$F_1(t) = \sum_i a_i s\left(t - t_i - \frac{\Delta t(\alpha_i, \vartheta_i)}{2}\right) \tag{11a}$$

$$F_2(t) = \sum_i a_j s\left(t - t_j + \frac{\Delta t(\alpha_j, \vartheta_j)}{2}\right). \tag{11b}$$

By Eq. (10)

$$\Delta t(\alpha, \vartheta) \approx \frac{r}{c_0} \cos\vartheta \sin\alpha\,. \tag{12}$$

If the distribution of the diffracting points D_i is circularly symmetric and their distribution with respect to depth is $W(z)$ then, putting $z = R \cdot \sin\vartheta$, we have

$$W(z)dz = W(R\sin\vartheta)R\cos\vartheta\ d\vartheta \tag{13}$$

and according to OLSHEVSKIY's theorem (4) the spatial-temporal correlation of the signals F_1 and F_2 is given by

$$A(r,\tau) = \langle F_1(t)\, F_2(t+\tau)\rangle = \lambda\langle a^2\rangle \frac{R}{2\pi}\int_0^{2\pi}\int_{\vartheta_1}^{\vartheta_2} W(R\sin\vartheta)\cdot$$

$$\cdot \int_{-\infty}^{\infty} s\left(t - \frac{\Delta t(\alpha,\vartheta)}{2}\right) s\left(t+\tau+\frac{\Delta t(\alpha,\vartheta)}{2}\right) dt\cos\vartheta\, d\vartheta\, d\alpha \approx$$

$$\approx \text{const}\cdot\lambda\cdot\langle a^2\rangle\frac{R}{2\pi}\int_0^{2\pi}\int_{\vartheta_1}^{\vartheta_2} W(R\sin\vartheta)\cdot\cos\left[\omega_0(\Delta t+\tau)\right]\cdot$$

$$\cdot\cos\vartheta\, d\vartheta\, d\alpha = \text{const}\cdot\lambda\cdot\langle a^2\rangle\frac{R}{2\pi}\int_0^{2\pi}\int_{\vartheta_1}^{\vartheta_2} W(R\sin\vartheta)\cdot$$

$$\cdot\cos\left[\omega_0\tau + k_0 r\cos\vartheta\sin\alpha\right]\cos\vartheta\, d\vartheta\, d\alpha.$$

In the derivation above

$$\vartheta_1 = \text{arc sin}\left[\left(H - \frac{h}{2}\right)\Big/R\right]$$

$$\vartheta_2 = \text{arc sin}\left[\left(H + \frac{h}{2}\right)\Big/R\right],$$

we assumed the *quasi-harmonicity* of the signal $s(t)$ and made use of the fact that for quasi-harmonic signals

$$\int_{-\infty}^{\infty} s(t)\, s(t+\tau)\, dt \approx \text{const}\cdot\cos\omega_0\tau \tag{14}$$

where ω_0 is the apparent circular frequency of the signal [17].

Let us now discuss some particular cases of the basic formula for the spatial-temporal correlation of scattered seismic noise

$$A(r,\tau) = \text{const} \cdot \lambda \cdot \langle a^2 \rangle \frac{R}{2\pi} \int_0^{2\pi} \int_{\vartheta_1}^{\vartheta_2} W(R \sin \vartheta) \cdot \cdot \cos [\omega_0 \tau + k_0 r \cos \vartheta \sin \alpha] \cos \vartheta \, d\vartheta \, d\alpha \,. \tag{15}$$

1. If $r = 0$,

$$A(0,\tau) = \text{const} \cdot \lambda \cdot \langle a^2 \rangle \frac{R}{2\pi} \int_0^{2\pi} \int_{\vartheta_1}^{\vartheta_2} W(R \sin \vartheta) \cos \vartheta \, d\vartheta \, d\alpha \cdot \cos \omega_0 \tau, \tag{16}$$

i.e. — by means of Eq. (14) — Equation (15) reduces to CAMPBELL's formula.

2. For $\tau = 0$ we obtain the *spatial* correlation of seismic noise observed at two geophones at a distance r from each other as a function of r.

2*a*. So, let $\tau = 0$, and suppose that the scatterers are *within a near-surface thin layer*. Then $H = 0$, $h/R \ll 1$, i.e.

$$\vartheta_1 = \arcsin\left(-\frac{h}{2R}\right) \approx -\frac{h}{2R};$$

$$\vartheta_2 \approx \frac{h}{2R}; \quad \cos \vartheta \approx 1$$

and, if we assume that ϑ is uniformly distributed in $(\vartheta_1, \vartheta_2)$, i.e. that

$$|W(\vartheta)| = \frac{1}{\vartheta_1 - \vartheta_2} \approx \frac{R}{h},$$

we obtain

$$A(r) = \frac{\text{const}}{2\pi} \int_0^{2\pi} \int_{-h/2R}^{h/2R} \frac{R}{h} \cos [k_0 r \sin \alpha] \, d\vartheta \, d\alpha = \\ = \frac{\text{const}}{2\pi} \int_0^{2\pi} \cos [k_0 r \sin \alpha] \, d\alpha = \text{const} \cdot J_0(k_0 r) \,. \tag{17}$$

In words: *the normalized spatial autocorrelation function of the source-generated noise scattered from a near-surface thin layer is given as a function of geophone-spacing r by*

$$B(r) = \frac{A(r)}{A(0)} = J_0 (k_0 r) \,, \tag{18}$$

where J_0 is the zeroth order Bessel function of first degree.

It should be noted that the theoretical autocorrelation function (18) fairly agrees with that found in the model-experiments of LEVIN and ROBINSON

[12] (Fig. 3, see also [4]). BACKUS et al. [1] and EDRINGTON [5] have derived a similar expression for the autocorrelation function of *ambient ground-noise* ([1] deals with noises of uniform azimuthal distribution, the case of the general distribution is treated in [5]).

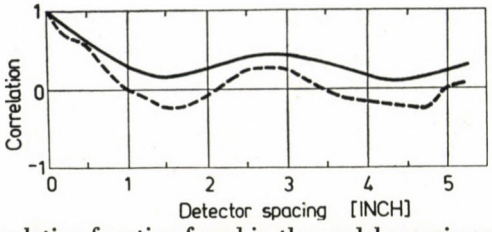

Fig. 3. Spatial autocorrelation function found in the model experiments of LEVIN and ROBINSON (after [12]). ———— wide-band noise, — — — filtered noise

2*b*. *Noises originating from the infinite half-space*

Again, $\tau = 0$; in Eq. (15) ϑ changes from $(-\pi/2)$ to 0, W is uniform, then

$$A(r, 0) = \frac{\text{const}}{\pi^2} \int_0^{2\pi} \int_{-\pi/2}^{0} \cos [k_0 r \cos \vartheta \sin \alpha] \cos \vartheta \, d\vartheta \, d\alpha =$$

$$= \frac{\text{const}}{\pi^2} \int_0^{\pi/2} \cos \vartheta \left[\int_0^{2\pi} \cos (k_0 r \cos \vartheta \sin \alpha) \, d\alpha \right] d\vartheta \, .$$

But

$$\frac{1}{2\pi} \int_0^{2\pi} \cos (k_0 r \cos \vartheta \sin \alpha) \, d\alpha = \mathcal{J}_0 (k_0 r \cos \vartheta)$$

i.e.

$$A(r, 0) = \text{const} \cdot \int_0^{\pi/2} \mathcal{J}_0 (k_0 r \cos \vartheta) \, d\vartheta \, .$$

Making use of the identity (cf. [6])

$$\int_0^{2\pi} \cos (2\mu x) \, \mathcal{J}_{2\nu} (2a \cos x) \, dx = \frac{\pi}{2} \mathcal{J}_{\nu-\mu}(a) \, \mathcal{J}_{\nu+\mu}(a) \tag{19}$$

we obtain the *basic result*

$$B(r) = \frac{A(r)}{A(0)} = \frac{\sin k_0 r}{k_0 r} \tag{20}$$

for the case of the infinite half-space.

As an application of Eq. (20) let us estimate the *coherence* between the noise-trains $F_1(t), \ldots, F_n(t)$ observed at the geophones $G_1, G_2, \ldots, G_n$, equally

spaced at distances r along a line. The coherence will be estimated as a function of geophone spacing r, of the dominant circular frequency ω_0 and of the average velocity c_0, for the case of the infinite half-space.

For wavelets of identical shape the coherence can be defined as

$$\varkappa = \frac{\left\langle \left(\sum_{i=1}^{n} F_i(t) \right)^2 \right\rangle}{n^2 \langle F^2(t) \rangle} = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} B(x_i - x_j) \tag{21}$$

[14, 16, 20], where x_i is the coordinate of geophone G_i, and

$$B(x_i - x_j) = \frac{\langle F_i(t) F_j(t) \rangle}{\langle F^2(t) \rangle} = \frac{A(x_i - x_j)}{A(0)} .$$

From Eq. (21), for completely correlating noises

$$B(x_i - x_j) \equiv 1, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, n$$

i.e. $\varkappa = 1$; for completely non-correlating noises

$$B(x_i - x_j) = \delta_{ij} = \begin{cases} 1 & \text{if} \quad i = j \\ 0 & \text{if} \quad i \neq j \end{cases}$$

and $\varkappa = 1/n$.

Substituting Eq. (20), i.e. the auto-correlation function of noises originating from the infinite half space, into Eq. (21), the coherence for $n = 24$ geophones becomes

$$\varkappa = \varkappa(f_0, c_0, r) = \frac{1}{24^2} \cdot \sum_{i=1}^{24} \sum_{j=1}^{24} \frac{\sin [(i-j) r k_0]}{(i-j) r k_0} . \tag{22}$$

The plot of the coherence function (22), for different geophone spacings and dominant frequencies, and for an average velocity of $c_0 = 4000$ m/s, is shown on Fig. 4.

We would like to point out *an application of the above results to the optimal design of multiple-coverage field systems*. The topic has been many times discussed in Hungarian literature: by BODOKY [2] from the point of view of the optimal suppression of multiples, and by KÉSMÁRKY [9] who considered the expected errors of velocity determination for the different systems. We think the suppression of the coherent noise background (i.e. ambient ground-noise, and source-generated scattered noise) or, more precisely, *the decrease of the coherence of these types of noise is an at least as significant aspect as those mentioned above*. The same opinion has been expressed by MOLNÁR et al. [15] who stated that: ". . .the design of a given multiple-coverage spread system

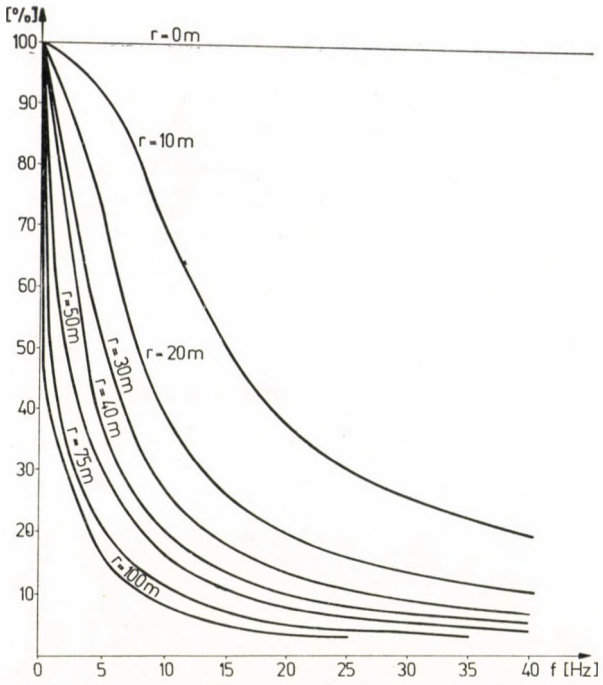

Fig. 4. Coherence of the "inorganized" scattered seismic noise observed on 24 geophones as function of frequency and geophone spacing, for 4000 m/s velocity

is by no means a simple optimization of the numerical values of the geophone spacing, offset and fold number using some reasonable parameters of the multiples and computed transfer characteristics. The basic rule in such investigations that one has to search an optimum which is most favourable from the point of view of regular and irregular noises" (*op. cit.* p. 45).

By applying Eqs (17) and (21), and taking into account the above-mentioned result of BACKUS et al. [1], we can determine the coherence of the *ambient ground noise* and of *near-surface scattered noise.* (According to LEVIN [11] these two types of noise are the most significant irregular noises.)

Similarly, by applying Eqs (20) and (21) we can investigate the coherence of the scattered noise in the deeper part of the sections.

The general theory outlined in the present paper might be useful

— *for the investigation of the effectiveness of certain linear procedures (as e.g. of stacking and migration)*;

— *for the selection of the significance level in the analysis of Velocity Spectra* [2];

— *for the detection of possible deeper reflections from the diffuse noise* ([21]) *reflected from the basin floor by means of optimal multi-channel filters* [1].

REFERENCES

1. Backus, M.—Burg, J.—Baldwin, D.—Bryan, E.: Wideband extraction of mantle P waves from ambient noise. *Geophysics*, 29 (1964), 672—692.
2. Bodoky, T.: Investigation of common depth point spread systems. C. Sc. Dissertation, Budapest; see also: *Magyar Geofizika*, 14 No. 3—4 (1973), 87—95 (In Hungarian).
3. Carleton, H. K.: Theoretical development of volume reverberation as of first-order scattering phenomenon. *J. Ac. Soc. Am.*, 33 (1961), 317—323.
4. Dunkin, J. W.: Scattering of a transient, spherical P wave by a randomly inhomogeneous, elastic half-space. *Geophysics*, 34 (1969), 357—382.
5. Edrington, T. S.: The covariance of external shot noise at a sensor array. *Geophysics*, 38 (1973), 957—958.
6. Gradshtein, I. S.—Ryzhik, I. M.: Tables of Integrals, Sums, Series and Products. Izd. Fiz. Mat. Lit. Moscow (In Russian), 1963.
7. Horton, C. W.: The structure of the noise background of a seismogram. I—II. *Geophysics*, 20 (1955), 565—584; 22 (1957), 261—267.
8. Jánossy, L.—Rényi, A.—Aczél, J.: On composed Poisson distributions, I. *Acta Math.* 1 (1950), 209—224.
9. Késmárky, I.: Investigation of seismic spread systems with regard to the quality of velocity estimation. Geof. Szimpózium, Szentendre, Sept. 15—19, 1975.
10. Korvin, G.: Certain problems of seismic and ultrasonic wave propagation in a medium with inhomogeneities of random distribution. *Geof. Közl.* 21 (1973), 5—34.
11. Levin, F. K.: Sources of noise on seismic records. *Applied Geophysics for Geologists.* University of Houston, Geol. Dept. May 21—June 1, 1973.
12. Levin, F. K.—Robinson, D. J.: Scattering by a random field of surface scatterers. *Geophysics*, 34 (1969), 170—179.
13. Makáry, E.: Gram–Charlier functions and their applications in seismic prospecting. M. Sc. Thesis, Budapest (In Hungarian), 1968.
14. Melton, B. S.—Bailey, L. F.: Multiple signal correlators. *Geophysics*, 22 (1957), 565—588.
15. Molnár, K.—Rumpler, J.—Sághy, Gy.—Zsitvay, Sz.: First experiences in Hungary with the common depth point seismic method in CH prospecting. *Magyar Geofizika*, 10 (1969), 43—55 (In Hungarian).
16. Neidell, N. S.—Taner, M. T.: Semblance and other coherency measures for multichannel data. *Geophysics*, 36 (1971), 482—497.
17. Olshevskiy, V. V.: Statistical Properties of Sea Reverberations. Nauka, Moscow (In Russian), 1966.
18. Rapoport, M. B.: Certain seismic applications of correlation theory. *Prikl. Geof.* 56 (1969), 21—30 (In Russian).
19. Rytov, S. M.: Introduction to Statistical Radiophysics. Nauka, Moscow (In Russian), 1966.
20. Simpson, S. M. Jr.: Similarity of output traces as a seismic operator criterion. *Geophysics*, 20 (1955), 254—269.
21. Szénás, Gy.—Ádám, O.: Seismo-geological conditions in SW-Hungary. *Geof. Közl.*, 2 (1953), 73—89 (In Hungarian).
22. Taner, M. T.—Koehler, F.: Velocity spectra: digital computer derivation and applications of velocity functions. *Geophysics*, 34 (1969), 859—881.

КОРРЕЛЯЦИОННЫЕ СВОЙСТВА РАЗБРОШЕННЫХ НА СКОРОСТНЫХ НЕОДНОРОДНОСТЯХ И ВЫЗЫВАЕМЫХ ВЗРЫВАМИ СЕЙСМИЧЕСКИХ ШУМОВ

Г. КОРВИН

РЕЗЮМЕ

В статье рассматриваются корреляционные свойства вызваемых взрывами шумов, разброшенных в пространстве и времени. Как специальный исследуется случай скоростных неоднородностей, равномерно распределенных в близповерхностном тонком слое или в бесконечном полуполе. Выводится когерентность вызываемых взрывами «неупорядоченных» шумов в зависимости от частоты, средней скорости и геофон-интервала и указывается возможность применения результатов для проектирования многократно покрываемы измерительных систем.

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THE HIERARCHY OF VELOCITY FORMULAE: GENERALIZED MEAN VALUE THEOREMS

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The paper presents a system-theoretical approach for the description of the macroscopic velocity of compressional waves in consolidated two-component rocks. Some physically plausible conditions will be described which must be met by the velocity*

$$\bar{v} = V(v_1, v_2, P, Q)$$

of any composite material consisting of constituents of v_1 resp. v_2 specific velocities and of respective volume fractions P and Q. The general form of the function $V(v_1, v_2, P, Q)$ will be derived on the basis of a standard result of the theory of the functional equations, and from the general solution a one-parameter family of formulae will be selected which will be shown to embrace most of the porosity-velocity rules used in practice. Generalizations to multicomponent materials and to the case of electric and thermal conductivity will also be touched upon.

In the last two decades a great number of publications have been devoted to the question of propagation velocity of compressional waves in porous rocks ([12, 13, 20, 25, 27] etc.). One formula which has found considerable acceptance in industry is that reported by WYLLIE et al. [32]:

$$\frac{1}{\bar{v}} = \frac{\Phi}{v_{\text{fluid}}} + \frac{1-\Phi}{v_{\text{matrix}}} . \tag{1}$$

A possible derivation of WYLLIE's equation is given in Part I of my paper [14].

In seismic practice or well-log interpretation we are frequently faced with Eq. (1) or with similar formulae. In some cases we are given measured transit times or interval velocities and have to determine the *porosity* or *component-ratio* (please refer to BERRY [5] about porosity, or TEGLAND [28] about sand-shale ratio determination). In seismic interpretation or modelling we might also encounter the inverse of this problem, i.e.: given the porosity we have to estimate the velocity of that formation (see any of the recent "bright-spot" studies or the classical work of BERZON et al. [6] on the seismic detection of water-bearing sandstones). No matter what our task might be, it should always be clear that WYLLIE's equation does not apply to all cases.

* From here on the term "velocity" of a formation will denote the macroscopic velocity of compressional waves propagating in the same formation.

MEESE and WALTHER [19] reported that for vugular carbonates WYLLIE's formula resulted in irreal fluid, resp. matrix specific velocities. A number of different modifying factors have been proposed (as e.g. that to compensate for the effect of pressure, in [5]). The standard text-book recipe (cf. [7] p. 184) is to use instead of (1) empirical formulae like

$$\frac{1}{\bar{v}} = A\Phi + B \quad \text{or} \quad \frac{1}{\bar{v}} = A\Phi^2 + B\Phi + C$$

where A, B, C have to be fitted to the given geological conditions by the method of least squares. There are exact theoretical formulae [8] which describe rigorously the effect of pressure or water-saturation, their use, however, is somewhat cumbersome for they assume the knowledge of the elastic parameters and are usually based on some simplified concept of the spatial configuration of grains. BERRY [5] is possibly right in stating that it is doubtful that any one relationship would completely describe *all* situations.

In what follows we shall derive a *set of physically plausible conditions* which must be satisfied by the velocity

$$\bar{v} = V(v_1, v_2, P, Q) \tag{2}$$

of any rock consisting of two components of specific velocities v_1 resp. v_2, the respective volume fractions of the components being P and Q. The theory of *functional equations* [2] will help us to find the *general form* of the function $V(v_1, v_2, P, Q)$ satisfying these conditions and, finally, a *one-parameter family of functions* will be picked out and shown to contain the most frequently used porosity–velocity relations.

Consider a composite material consisting of a fraction P of some material of specific velocity x, and of a fraction Q of another of velocity y and denote the velocity of the composite by $V(x, y, P, Q)$. The function $V(x, y, P, Q)$ is required to satisfy the following conditions:

1. *Reflexivity:*

$$V(x, x, P, Q) = x \text{ for all } P, Q \quad (P + Q > 0) \tag{3a}$$

2. *Idempotency* (cf. [8]):

$$V(x, y, P, 0) = x \text{ for all } P > 0 \tag{3b}$$

$$V(x, y, 0, Q) = y \text{ for all } Q > 0. \tag{3c}$$

3. *Homogeneity (of 0-th order) with respect to the volume fractions:*

The velocity of the composite does not depend on the actual values of the volume fractions, only on their ratio:

$$V(x, y, P, Q) = V(x, y, \lambda P, \lambda Q) \text{ for all } P, Q, \lambda \text{ such that } P + Q > 0, \lambda > 0. \quad (3d)$$

4. *Internity:*

The velocity of the composite lies between the specific velocities of the constituents; if $x < y$, say, then for $P + Q > 0$:

$$V(x, y, 1, 0) \leq V(x, y, P, Q) \leq V(x, y, 0, 1). \quad (3e)$$

5. *Bi-symmetry* (this concept is due to ACZÉL [1]): Given two composites, the first consisting of P_1 resp. Q_1 parts of materials of x_1 resp. y_1 velocity, the second of P_2 resp. Q_2 parts of materials of x_2 resp. y_2 velocity, the following two expressions for the velocity of the four-component aggregate must be equal:

$$\begin{aligned} &V(V(x_1, y_1, P_1, Q_1); V(x_2, y_2, P_2, Q_2); P_1 + Q_1; P_2 + Q_2) = \\ &\quad = V(V(x_1, x_2, P_1, P_2); V(y_1, y_2, Q_1, Q_2); P_1 + P_2; Q_1 + Q_2). \end{aligned} \quad (3f)$$

6. *Monotonicity with respect to the volume fractions:*

If $x < y$, say, $P + Q_1 > 0$, $Q_2 > Q_1$ then

$$V(x, y, P, Q_1) < V(x, y, P, Q_2). \quad (3g)$$

7. *Monotonicity with respect to the velocities:*

If $P + Q > 0$, $y < z$ then

$$V(x, y, P, Q) < V(x, z, P, Q). \quad (3h)$$

Some of the above conditions (3*a*, 3*b*, 3*c*, 3*e*) have already been introduced — in a slightly different form — in our previous work [15]. In connection with Condition (4) it should be noted that the inequality (3*e*) is not necessarily valid for very large ($\Phi > 0.6$) porosities. In this case, namely (see [11, 16, 21, 27]), the velocity of sound waves might be less than that measured in water. This restricts the validity of our conclusions to the (geophysically most important) porosity range of $\Phi < 0.6$.

According to a well-known theorem of the theory of functional equations (Theorem of FINETTI, KITAGAWA and ACZÉL [2], § 5.3.2.) *any function V satisfying the above conditions can be described in the general form*

$$V(x, y, P, Q) = f^{-1}\left[\frac{Pf(x) + Qf(y)}{P + Q}\right], \quad (4)$$

where f is some arbitrary, continuous, strictly monotone function (f is the so-called Kolmogorov—Nagumo function corresponding to the given mean value, cf. [23]).

Introducing the *porosity* Φ as

$$\Phi = \frac{P}{P+Q}, \tag{5}$$

Eq. (4) becomes

$$V(v_1, v_2, P, Q) = V(v_1, v_2, \Phi, 1-\Phi) = f^{-1}[\Phi f(v_1) + (1-\Phi)f(v_2)]. \tag{6}$$

Before proceeding further, let us recall some of the most widely accepted porosity-velocity formulae:

velocity-average equation ([5]):

$$\bar{v} = v_0 = \Phi v_1 + (1-\Phi)v_2; \tag{7}$$

time-average equation (WYLLIE et al. [32]):

$$\frac{1}{\bar{v}} = \frac{\Phi}{v_1} + \frac{1-\Phi}{v_2}; \tag{8}$$

generalized Wood equation (its derivation will be found in the Appendix, see also [14], Part II):

$$\frac{1}{\bar{v}^2} = \frac{\Phi}{v_1^2} + \frac{1-\Phi}{v_2^2}; \tag{9}$$

the "vugular carbonate" formula of MEESE and WALTHER [19]:

$$\bar{v} = v_1^{\Phi} v_2^{(1-\Phi)}. \tag{10}$$

It is immediately clear that all of the above formulae (7)—(10) are particular cases of the *generalized mean values* defined as follows:

The generalized t-th order mean value of the positive quantities v_1 and v_2, with respect to the weights Φ and $(1-\Phi)$, is given by

$$M_t(v_1, v_2, \Phi, 1-\Phi) = [\Phi v_1^t + (1-\Phi_1)v_2^t]^{1/t}. \tag{11}$$

In Eq. (11) t changes from $-\infty$ to $+\infty$. For $t = 0$ we have, by L'Hôspital's rule

$$M_0(v_1, v_2, \Phi, 1-\Phi) = \lim_{t\to 0} M_t = v_1^{\Phi} v_2^{1-\Phi}. \tag{12}$$

Next, setting $v^* = \max(v_1, v_2)$, if $t > 0$, $t \neq 0; 1$ then

$$\left.\begin{aligned} \Phi^{1/t} \cdot v^* \leq M_t(v_1, v_2, \Phi, 1-\Phi) \leq v^* \\ (1-\Phi)^{1/t} \cdot v^* \leq M_t(v_1, v_2, \Phi, 1-\Phi) \leq v^* \end{aligned}\right\} \tag{13}$$

i.e., letting $t \to \infty$ we have

$$M_\infty = \lim_{t\to\infty} M_t(v_1, v_2, \Phi, 1-\Phi) = v^* = \max(v_1, v_2). \tag{14}$$

Similarly, observing that for $t > 0$

$$M_t(v_1, v_2, \Phi, 1-\Phi) = \left[M_t\left(\frac{1}{v_1}, \frac{1}{v_2}, \Phi, 1-\Phi\right)\right]^{-1}$$

we have, for $t \to \infty$:

$$M_{-\infty}(v_1, v_2, \Phi, 1-\Phi) = \min(v_1, v_2). \tag{15}$$

From the point of view of geophysical applications the most important property of the generalized mean $M_t(v_1, v_2, \Phi, 1-\Phi)$ is expressed by the following theorem (this theorem easily follows from Jensen's inequality, see [4], § I. 16):

If v_1 and v_2 are positive, $\Phi \neq 0, \Phi \neq 1$ and $v_1 \neq v_2$ then $M_t(v_1, v_2, \Phi, 1-\Phi)$ is a strictly monotone function of t in $(-\infty, \infty)$.

Evidently, the generalized mean M_t is a special case of Eq. (6), the corresponding Kolmogorov—Nagumo function being

$$f(x) = x^t.$$

As particular cases the mean value M_t contains the Wood equation (9) (for $t = -2$), the time-average equation (8) of Wyllie ($t = -1$), the Meese–Walther equation (10) (for $t = 0$) and the velocity average equation (7) (for $t = 1$).

Because of the monotonicity of M_t:

$$v_{\text{Wood}} < v_{\text{Wyllie}} < v_{\text{Meese-Walther}} < v_0 \,. \tag{16}$$

For given specific velocities v_1 and v_2 we can construct a set of curves depending on the single parameter t, this set of curves will contain the familiar porosity-velocity formulae as well.

Sets of curves for sandstones and carbonates are shown in Figs 1 and 2.

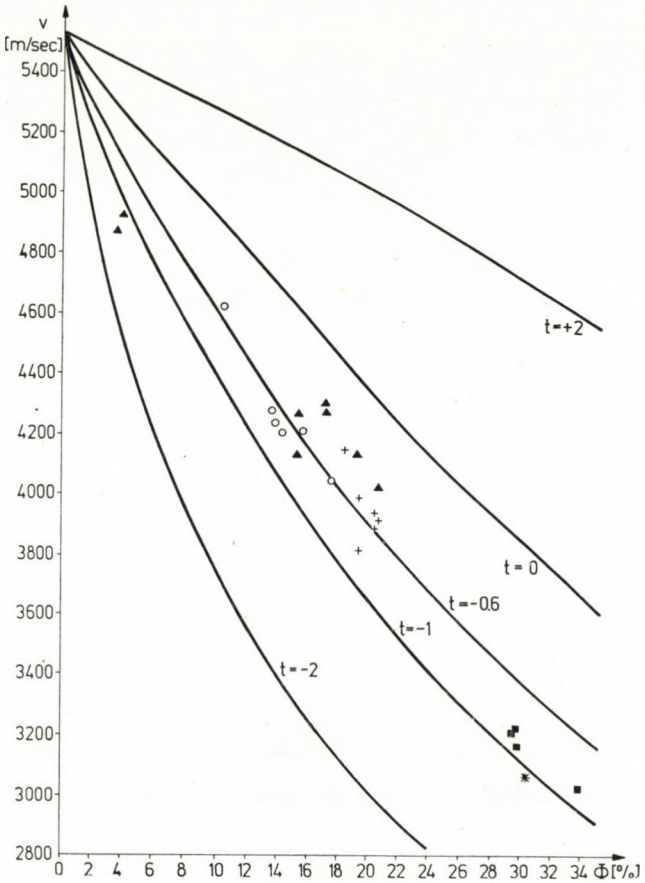

Fig. 1. Set of porosity-velocity curves for sandstones: + BEREA, ○ PAGE, ∗ MIOCENE ◢ TEXAS, ■ BOISE

The construction of the curves has been performed with the following parameters:

$v_{\text{fluid}} = 1545$ m/s (brine, [19])
$v_{\text{matrix}} = 5542$ m/s (for sandstones, [19])
$v_{\text{matrix}} = 6773$ m/s (for carbonates, [19])

Source of the measured data:

sandstones: BEREA, BOISE, MIOCENE, PAGE: [19]
TEXAS: [13]
carbonates: [19]

From among the sandstone curves that around $t = -0.6$ fits the best to the measured data; for very small ($\Phi < 5\%$) and very large ($\Phi \approx 30\%$) porosities, however, the $t = -1$ curve (i.e. WYLLIE's formula) also applies.

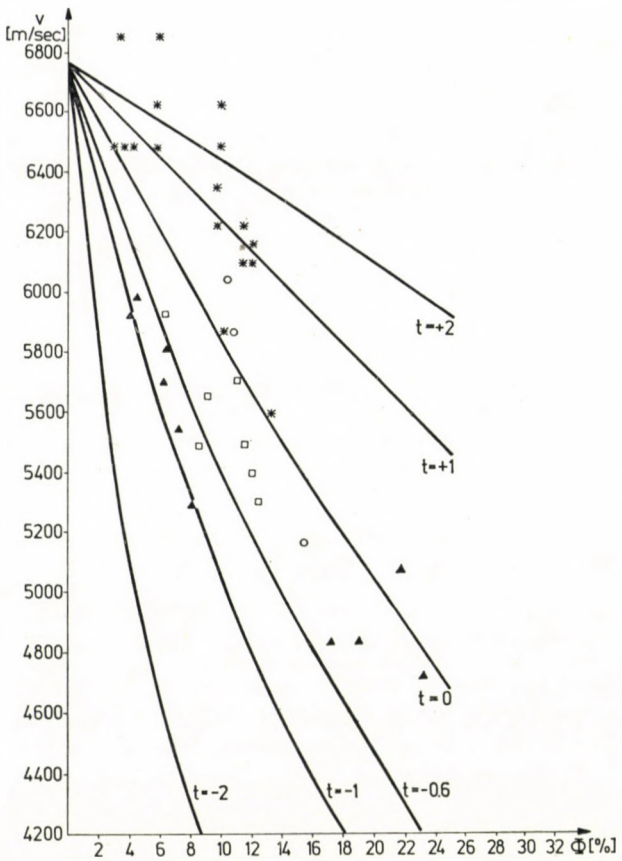

Fig. 2. Set of porosity-velocity curves for carbonates: ○ CADDO limestone, ◢ LANSING K. C. limestone, □ PARADOX limestone, ∗ ARBUCKLE dolomite

The measured *carbonate* data show a considerable larger scatter, they lie between the curves $t = -1$ and $t = +2$. Contrary to the assumption of MEESE and WALTHER [19] the $t = 0$ curve is not always the best to describe the porosity-velocity relation for carbonates. For the ARBUCKLE dolomite, e.g. a linear velocity rule ($t = 1$), for the PARADOX limestone a $t \approx 0.5$–0.6 rule seem to apply.

By using appropriate sets of curves we can avoid the application of the different kinds of modifying factors, of empirical formulae which do not always have a sound physical basis and we don't have to assume irreal specific velocities.

Owing to the monotonicity of the set of curves with respect to t it is expected that for any particular set of data the best-fitting curve (i.e. t) can be found by computerized optimum search. This can also be seen in Fig. 3. which illustrates the mean square deviation of the PAGE sandstone data [19] from the individual curves of the sandstone set.

The concept of the mean value M_t can be easily generalized for multi-component media, as well as for other physical properties; as e.g. for the estimation of the velocity [28] resp. seismic attenuation [17] in a sequence of alternating sand-shale layers; for the estimation of the thermal conductivity [3, 24, 26, 31] or electric conductivity [10, 22] of fluid-filled sedimentary rocks, etc.

In case of conductivities the formulae, corresponding to Eqs (7) and (8), have a clear-cut physical meaning. It can be also easily seen that the equation

$$\sigma = \sigma_1^{\Phi} \cdot \sigma_2^{1-\Phi} \tag{17}$$

corresponding to the Meese–Walther formula can be considered as an approximation of the well-known Maxwell equation

$$\sigma = \frac{2\sigma_2 + \sigma_1 + 2\Phi(\sigma_1 - \sigma_2)}{2\sigma_2 + \sigma_1 - \Phi(\sigma_1 - \sigma_2)} \cdot \sigma_2 \tag{18}$$

for small porosities ($\Phi \ll 1$) and small internal conductivity contrasts (i.e. $\sigma_2/\sigma_1 \approx 1$) (as it is well known, *Maxwell*'s formula refers to the case when a small fraction $\Phi \ll 1$ of a homogeneous medium of conductivity σ_2 is filled by small-sized, randomly distributed spheres of conductivity σ_1 [10, 18]).

Indeed (see [3]), introducing the notation

$$r = \frac{\sigma_2}{\sigma_1}, \tag{19}$$

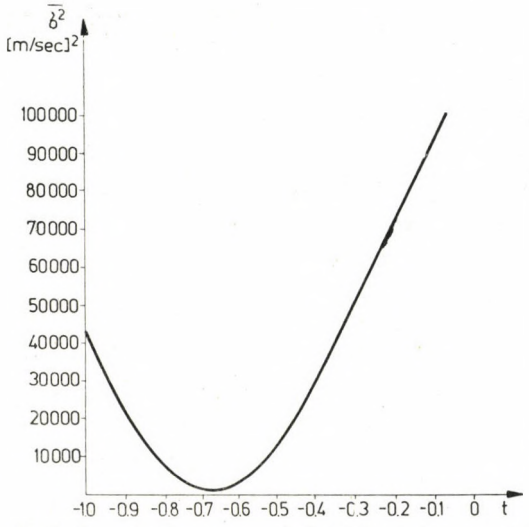

Fig. 3. Mean square deviation of the measured PAGE sandstone data from the individual curves of the sandstone set, as a function of t

Eqs (17) and (18) become, in turn:

$$\sigma = \sigma_2 \cdot r^{-\Phi} \tag{20}$$

resp.

$$\sigma = \frac{(2r+1) - 2\Phi(r-1)}{(2r+1) + \Phi(r-1)} \sigma_2 . \tag{21}$$

For $\Phi \ll 1$, $r \approx 1$ Eqs (20), (21) can be developed in series according to powers of $(r-1)$ and, neglecting higher-order terms, both expressions become

$$\sigma \approx \sigma_2[1 - \Phi(r-1)] . \tag{22}$$

It should be noted that in connection with thermal conductivity the equivalence of formulae (17) and (18) has been experimentally verified by WOODSIDE and MESSMER [31].

Appendix

Derivation of the Generalized Wood-equation (Eq. 9)

Given a composite material consisting of n components, let us denote the *velocity*, *density*, *compressibility* and *volume ratio* of the i-th constituent by c_i, ϱ_i, M_i and p_i, respectively. The velocity c_i can be expressed in terms of density ϱ_i and compressibility M_i as

$$\frac{1}{c_i^2} = \frac{\varrho_i}{M_i} \tag{23}$$

or, introducing *random variables:*

$$\frac{1}{c^2} = \frac{\varrho}{M} \tag{24}$$

where

$$p(\varrho = \varrho_i \quad \text{and} \quad M = M_i) = p_i; \; \sum_{i=1}^{n} p_i = 1. \tag{25}$$

Introducing average density and compressibility values by

$$\varrho_0 = \sum_{i=1}^{n} p_i \varrho_i ; \; M_0 = \sum_{i=1}^{n} p_i M_i \tag{26}$$

and denoting expected values with respect to the probability distribution

$\{p_i\}$ by $\langle \ldots \rangle$, we have:

$$\left\langle \frac{1}{c^2} \right\rangle = \sum_{i=1}^{n} p_i \frac{\varrho_i}{M_i} = \sum_{i=1}^{n} \frac{p_i(\varrho_i - \varrho_0)}{M_i} + \varrho_0 \sum_{i=1}^{n} \frac{p_i}{M_i} = \\ = \sum_{i=1}^{n} \frac{p_i(\varrho_i - \varrho_0)}{M_i} + \left(\sum_{i=1}^{n} p_i \varrho_i \right) \left(\sum_{i=1}^{n} \frac{p_i}{M_i} \right). \tag{27}$$

Developing the first term on the right-hand side of Eq. (27) into a power series:

$$\sum_{i=1}^{n} \frac{p_i(\varrho_i - \varrho_0)}{M_i} = \frac{1}{M_0} \sum_{i=1}^{n} \frac{p_i(\varrho_i - \varrho_0)}{1 + \dfrac{M_i - M_0}{M_0}} = \frac{\varrho_0}{M_0} \sum_{i=1}^{n} p_i \left(\frac{\varrho_i - \varrho_0}{\varrho_0} \right) - \\ - \frac{\varrho_0}{M_0} \sum_{i=1}^{n} p_i \frac{\varrho_i - \varrho_0}{\varrho_0} \cdot \frac{M_i - M_0}{M_0} + O\left[\left(\frac{M_i - M_0}{M_0} \right)^2 \right].$$

Taking into account that $\sum p_i(\varrho_i - \varrho_0) = 0$ and assuming that

$$\left| \frac{\varrho_i - \varrho_0}{\varrho_0} \right| \ll \left| \frac{M_i - M_0}{M_0} \right|$$

we have, finally, from Eq. (27):

$$\left\langle \frac{1}{c^2} \right\rangle = \left(\sum_{i=1}^{n} p_i \varrho_i \right) \cdot \left(\sum_{i=1}^{n} \frac{p_i}{M_i} \right) + O\left[\left(\frac{\Delta M}{M} \right)^2 \right] \tag{28}$$

if

$$\left| \frac{\Delta \varrho}{\varrho} \right| \ll \left| \frac{\Delta M}{M} \right| \tag{29}$$

which is a generalization of the classical Wood equation [30, 21]. (We recall that for two components the familiar form of Wood's equation is

$$c^2 = \frac{1}{(p\varrho_1 + q\varrho_2) \left(\dfrac{p}{M_1} + \dfrac{q}{M_2} \right)} \tag{30}$$

where $p = \Phi$, $q = 1 - p$). A similar derivation of the generalized Wood formula, on the basis of the low-frequency behaviour of the velocity in two-component media, will be found in Part II of my paper [14]. It should be noted that, starting out from considerations of multiple scattering, WATERMAN

and TRUELL [29] have also derived a generalization of the Wood formula of the form

$$\frac{1}{v^2} \approx \langle \varrho \rangle \cdot \left\langle \frac{1}{M} \right\rangle,$$

they, however, had to assume — instead of (29) — the more severe restriction

$$| \varrho_1 - \varrho_2 | \ll 1$$

(see [29], pp. 530—531).

REFERENCES

1. ACZÉL, J.: The notion of mean values. *Norske Vid. Selsk. Forh.*, 19 (1946), 83—86.
2. ACZÉL, J.: Vorlesungen über Funktionalgleichungen und ihre Anwendungen. VEB Deutscher Verlag der Wissenschaften. Berlin, 1961.
3. BECK, A. E.: An improved method of computing the thermal conductivity of fluid-filled sedimentary rocks. *Geophysics*, 41 (1976), 133—144.
4. BECKENBACH, E. F.—BELLMAN, R.: Inequalities. Springer Verlag. Berlin—Göttingen—Heidelberg, 1961.
5. BERRY, J. E.: Acoustic velocity in porous media. *J. Petr. Techn.*, 11 (1959), 262—270.
6. BERZON, I. S.—VASILYEV, YU. I.—STARODUBROVSKAYA, S. P.: On refracted waves corresponding to aquiferous sands. II. *Izv. ANSSSR Ser. Geof.*, No. 2 (1959), 177—182. (In Russian).
7. CSÓKÁS, J.: Geophysics III. Well Logging. Tankönyvkiadó, Budapest, 1969. (In Hungarian)
8. FUCHS, L.: On mean systems. *Acta Math.*, 1 (1950), 303—320.
9. GARDNER, G. H. F.—HARRIS, M. H.: Velocity and attenuation of elastic waves in sands. SPWLA 9th Symp. 1968.
10. GRANT, F. S.—WEST, G. F.: Interpretation Theory in Applied Geophysics. McGraw-Hill Book Co. New York, 1965.
11. HAMILTON, E. L.: Low sound velocities in high-porosity sediments. *J. Ac. Soc. Am.*, 28 (1956), 16.
12. HAMILTON, E. L.: Sound velocity and related properties of marine sediments, North Pacific. *J. Geoph. Res.*, 75 (1970), 4423—4446.
13. HICKS, W. G.—BERRY, J. E.: Application of continuous velocity logs to determination of fluid saturation of reservoir rocks. *Geophysics*, 21 (1956), 739—754.
14. KORVIN, G.: Certain problems of seismic and ultrasonic wave propagation in a medium with inhomogeneities of random distribution. *Geof. Közl.*, 21 (1973), 5—34. Part II: *Geof. Közl.*, 24 (1977), Suppl. 2., 3—38.
15. KORVIN, G.—LUX, I.: An analysis of the propagation of sound waves in porous media by means of the Monte Carlo method. *Geof. Közl.*, 20 (1971), 91—99.
16. LEVIN, F. K.: The seismic properties of Lake Maracaibo. *Geophysics* 27 (1962), 35—47.
17. MATEKER, E. J. Jr.: Lithologic predictions from seismic reflections. *Oil and Gas J.* Nov. 8. 1971, 96—100.
18. MAXWELL, C.: A Treatise on Electricity and Magnetism. 3rd ed. Clarendon, Oxford, 1892.
19. MEESE, A. D.—WALTHER, H. C.: An investigation of sonic velocities in vugular carbonates. 8th SPWLA Symp. 1967.
20. NAFE, J. E.—DRAKE, C. L.: Variation with depth in shallow and deep water marine sediments of porosity, density and the velocities of compressional and shear waves. *Geophysics*, 22 (1957), 523—552.
21. OFFICER, C. B.: Introduction to the Theory of Sound Transmission. McGraw-Hill Book Co. Inc. New York—Toronto—London, 1958.
22. PEARCE, D. C.—HULSE, W. H. Jr.—WALKER, J. K.: The application of the theory of heterogeneous dielectrics to low surface area soil systems. *IEEE Trans. Geosci. Electronics*, GE-11 (1973), 167—170.

23. RÉNYI, A.: Some basic questions of information theory. *MTA III. Oszt. Közl.* X. (1960), 251—282. (In Hungarian)
24. RZHEVSKY, V.—NOVIK, G.: The Physics of Rocks. Mir Publishers, Moscow, 1971.
25. SARMIENTO, R.: Geological factors influencing porosity estimates from velocity logs. *Bull. AAPG.*, 45 (1961), 633—644.
26. SCHÖN, J.: Ein Beitrag zur Klassifizierung und Systematik petrophysikalischer Parameter. *Neue Bergbautechnik*, 1 (1971), 2—10.
27. SHUMWAY, G.: Sound speed and absorption studies of marine sediments by a resonance method — Part II. *Geophysics*, 25 (1960), 659—682.
28. TEGLAND, E. R.: Sand-Shale ratio determination from seismic interval velocity. *23rd Ann. Midwestern Mtg. of SEG and AAPG, Dallas*, 1970.
29. WATERMAN, P. C.—TRUELL, R.: Multiple scattering of waves. *J. Math. Phys.*, 2 (1961), 512—537.
30. WOOD, A. B.: A Textbook of Sound. The Macmillan Co., New York, 1955.
31. WOODSIDE, W.—MESSMER, J. H.: Thermal conductivity of porous media. II. Consolidated rocks. *J. Appl. Phys.*, 32 (1961), 1696—1706.
32. WYLLIE, M. R. J.—GREGORY, A. R.—GARDNER, L. W.: Elastic wave velocities in heterogeneous and porous media. *Geophysics*, 21 (1956), 41—70.

ИЕРАРХИЯ СКОРОСТНЫХ ФУНКЦИЙ: ОБОБЩЕНИЕ ТЕОРЕМЫ О СРЕДНЕМ

Г. КОРВИН

РЕЗЮМЕ

В статье выводятся некоторые физически преимущественные условия, которые должны удовлетворять скорости $\bar{V} = V(v_1, v_2, P, Q)$ породы, составленной из компонент со специфическими скоростями v_1 и v_2 при объемной пропорции $P : Q$. Методом функциональных уравнений составляется общий вид функций $V(v_1, v_2, P, Q)$, удовлетворяющих условиям, из решений функционального уравнения выбирается однопараметровое множество функций, содержащее наиболее часто использованные зависимости пористости от скорости.

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IONOSPHERIC SPORADIC E AND STRATOSPHERIC WARMINGS

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The daily average departures of the sporadic E parameters *fbEs* and *h'Es* from the corresponding monthly median have been found to decrease during winter-time circulation disturbances connected with stratospheric warmings. Computed convergence of the ion drift velocity from these quantities on the basis of wind shear theory shows that the decrease of the sporadic E ionization (*fbEs*) may be attributed to the weakening of wind shear. The latter is presumably due to the unfavourable propagation conditions of atmospheric gravity waves during such circulation disturbances.

Introduction

It has been reported [1] that the parameters of the sporadic E layer show a strong scatter during winter and early spring months. This feature is similar to the behaviour of several characteristics of the lower ionosphere, among others to that of the ionospheric absorption. These conditions deserve a more detailed investigation. One of the most important phenomena, characteristic of this period and producing violent changes in the atmosphere, are stratospheric warmings. Numerous investigations have been devoted to this problem. The results have shown on the one hand that the consequences of the stratospheric warmings can be detected as high as the lower part of the E layer, on the other hand they revealed the complexity of the phenomenon. In winter the zonal component of the prevailing wind is normally eastwards (westerly) in the stratosphere and mesosphere, westwards above about 100 km [2]. Wind observations show that during stratospheric warmings the eastward wind is reduced, and even a reversed flow may occur due to the reversal of the polar vortex [3]. The latter leads to the strengthening of the westward flow above 100 km. These changes may affect the propagation of atmospheric gravity waves, resp. the intensity of wind shear, produced by the former and in this way control the production of sporadic E layers at middle latitudes. Thus, the investigation of the winter-time sporadic E ionization may contribute to our knowledge of the dynamical conditions of the upper atmosphere in winter.

Method of analysis and results

The sporadic E layer has been characterized by the parameters $fbEs$ and $h'Es$. h type sporadic E layers have been excluded from the investigations. The blanketing frequency is generally the best measure of the maximum electron density of the layer.

It is important to note that stratospheric warmings are phenomena occurring at latitudes above about 40°. Thus the appropriate data, measured at the ionospheric stations Békéscsaba (46°40′N, 21°10′E) [4] and Juliusruh (54°38′N, 13°23′E) [5] have been used to trace the stratospheric warming effect

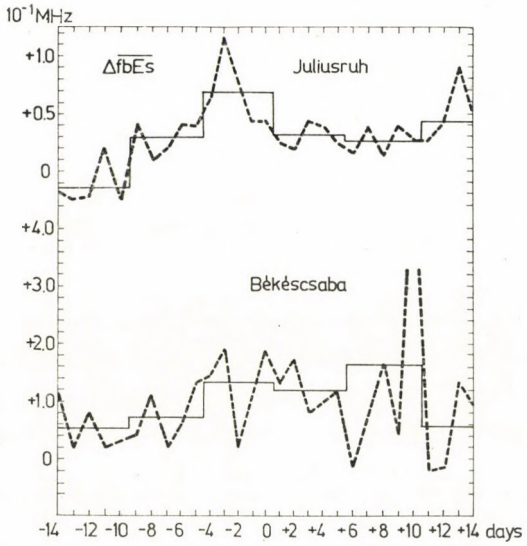

Fig. 1. Variation of the daily average departures of the blanketing frequency of the *Es* layer from the corresponding monthly median during stratospheric warmings determined with the superposed epoch method and taking the warming maximum as key day

in sporadic E ionization. For this analysis the daily average departures of $fbEs$ $(\overline{\Delta fbEs})$ respectively that of $h'Es$ $(\overline{\Delta h'Es})$ from the corresponding monthly median were computed. The characteristics of stratospheric warmings were taken from a Table, distributed at the XVIth Plenary Meeting of COSPAR in Konstanz, 1973. This Table contains the most important characteristics of stratospheric warmings, observed from 1956 to 1973.

Using the above mentioned data a superposed epoch analysis was carried out. The date of the warming maximum was taken as key day. Figs 1 and 2 show the results of the superposed epoch analysis carried out with these data for ten stratospheric warmings observed from 1962 to 1973.

The $\overline{\Delta fbEs}$ values show a definite maximum before the key day in case of Juliusruh (Fig. 1) and a decrease afterwards. An increase of $\overline{\Delta fbEs}$ can be

find in Békéscsaba too, but it is not convincing because of the strong scattering of the data. The trend is indicated only by the curve constructed with five-day averages. The $\Delta h'Es$ values show a minimum 6 days before the key day in case of Juliusruh (Fig. 2). Since the curve, constructed from the data from Békéscsaba does not indicate any definite change, the investigations have been continued with the data from Juliusruh alone, situated at a higher latitude than Békéscsaba.

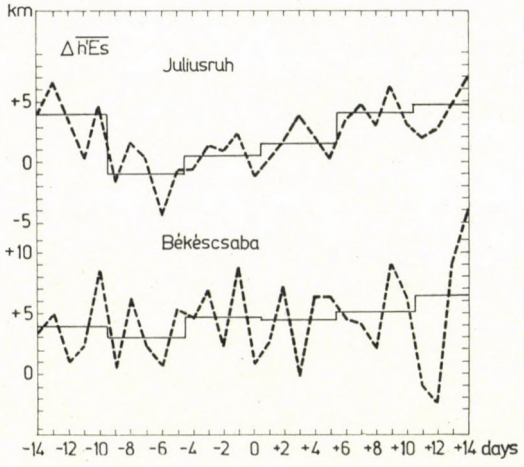

Fig. 2. Variation of the daily average departures of the virtual height of the *Es* layer from the corresponding monthly median during stratospheric warmings, determined with the superposed epoch method and taking the warming maximum as key day

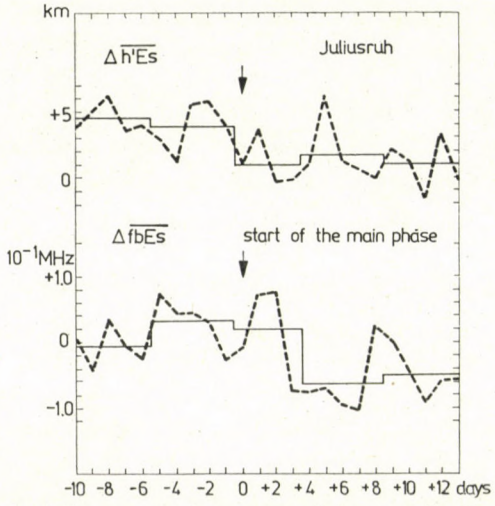

Fig. 3. Variation of the daily average departures of the sporadic *E* parameters *h'Es*, *fbEs* from the corresponding monthly median, determined with the superposed epoch method and taking the start of the main phase of the circulation disturbance as key day

Time and characteristics of a stratospheric warming are determined on the basis of stratospheric measurements which do not contain information about the effects of stratospheric warmings in the lower thermosphere, i.e. at the height range of sporadic E ionization. Therefore data referring to the winter time conditions of altitudes, nearer to this height range were considered.

Stratospheric warmings are generally accompanied by circulation disturbances in the lower thermosphere, as mentioned in the introduction. SPRENGER et al. [6] compiled a catalogue of winter-time circulation disturbances at an altitude of 90—95 km, on the basis of night-time ionospheric drift measurements. The disturbances are divided by the authors into three, sometimes four phases and the characteristics of these phases (duration, direction and speed of the zonal component) are also indicated. The actual disturbance, consisting of a reversal of the wind at this altitude, takes place during the main phase. In the case of an other superposed epoch analysis including eight winter-time circulation disturbances the start of the main phase has been used as key day. Fig. 3 shows that after the key day both $\overline{\Delta h'Es}$ and $\overline{\Delta fbEs}$ decrease.

Discussion

The results of both superposed epoch analysis show that the virtual height of the sporadic E layer, $h'Es$ decreases during stratospheric warmings.

As regards the maximum electron density of the sporadic E layer, it increases during stratospheric warmings according to the first analysis (key day: date of warming maximum), while the second analysis (key day: start of the main phase of the circulation disturbance) indicates a decrease of $fbEs$. The difference between the two results may be due to the different choice of key days, the change of the parameters is, however, better correlated with the event, determining the key day, in case of the second analysis. This suggests that the variation of the sporadic E layer parameters might be related to circulation disturbances in the lower thermosphere. The latter would mean that the decrease of $fbEs$ (the maximum electron density of the layer) is related to the decrease of wind shear, if the wind shear theory of sporadic E ionization is accepted. To check this assumption the wind shear, resp. the convergence of the ion drift velocity have been computed.

Interpretation of the results

The interpretation of the results has been based on the wind shear theory of the mid-latitude *Es* layer. The basic equation is [7]

$$\frac{d}{dz}\left(-D\frac{dN_e}{dz}+N_e v_{i,z}\right)=q-\alpha' N_e^2, \tag{1}$$

where D is the coefficient of ambipolar diffusion, N_e is the electron density $v_{i,z}$ the vertical component of ion velocity, q and α' are the rate of ion production, resp. the effective recombination coefficient within the *Es* layer. Assuming a parabolic electron density distribution in the neighbourhood of the maximum, after some modifications the relation

$$\alpha' N_{em}^2+\left(\frac{dv_{i,z}}{dz}-\frac{0{,}8D}{d_{0,9}^2}\right)N_{em}=\alpha_0' N_e^2\,, \tag{2}$$

can be derived. Here N_{em} is the maximum electron density of the *Es* layer, the parameters α_0' and N_e are the effective recombination coefficient, resp. the electron density corresponding to the background, i.e. to the state without wind shear. α_0' is replaced by its value outside of the layer. $d_{0,9}$ gives the thickness of the layer between the points, where the electron density decreases to 0.9 times of its maximum value. Equation (2) provides a relation between the maximum electron density of the *Es* layer, N_{em} and the background electron density N_e, which can be determined for various values of $-dv_{i,z}/dz$, the convergence of the ion drift velocity and α'. Positive convergence of the ion drift velocity (the negative divergence) is characteristic to those heights, where the electron density is increased due to wind shear. Therefore it is a direct measure of the effectiveness of wind shear, producing sporadic *E*. The convergence of the ion drift velocity differs from the effective wind shear, used by REDDY and MATSUSHITA [8] only in the factor $\nu_i/\omega_i \cos I$, where ν_i and ω_i are the collision frequency, resp. gyrofrequency of the ions, and I is the magnetic inclination. Thus, using the convergence of ion drift velocity the uncertainty connected with the introduction of the collision frequency is eliminated. Substituting reasonable values into the last equation, one finds that the second term in the brackets on the left-hand side is negligible small, as compared to the other terms. Hence, holding the parameters D, $d_{0.9}$ and α_0' fixed and varying the values of $dv_{i,z}/dz$ and α' a set of curves can be obtained, which enables the interpretation of the electron density variations (Fig. 4).

However, in the present case we are faced with the inverse problem: we have to determine the convergence of the ion drift velocity from the maximum electron density of the *Es* layer and from the background electron den-

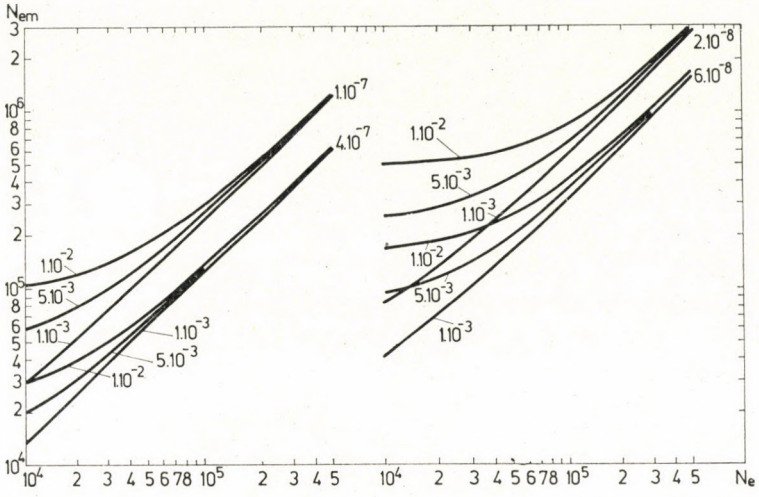

Fig. 4. Set of curves illustrating the relation between maximum electron density of the *Es* layer and the background electron density as a function of wind shear and effective recombination coefficient within the layer

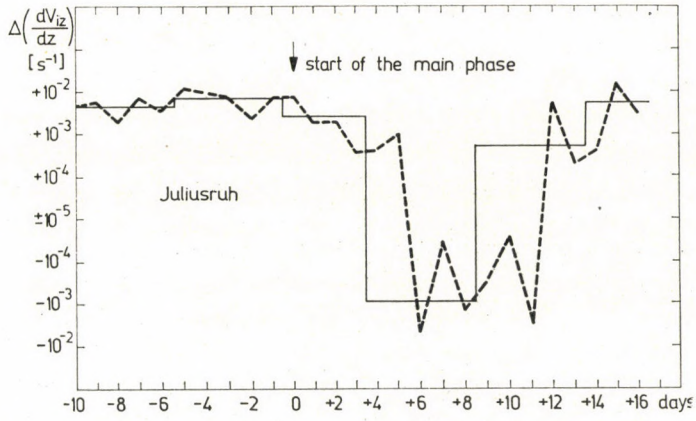

Fig. 5. Variation of the daily average departures of the computed convergence of the ion drift velocity from the corresponding hourly mean, determined with the superposed epoch method and taking the start of the main phase of the circulation disturbance as key day

sity, which can be deduced from experimental data. The maximum electron density of the *Es* layer is represented by *fbEs*, background electron density was computed on the basis of the International Reference Ionosphere 1972 [9] making use of *foE* and *h'Es*. It has been supposed that *h'Es* does not differ too much from the real height. The electron density for given *foE* and *h'Es* has been computed by means of the polynomial approximations, given in the Tables. Thus, the height variation of the sporadic *E* ionization could also be taken into account. $-dv_{i,z}/dz$ has been computed by giving to D and $d_{0.9}$ the same constant values again ($D = 50$ m^2 s^{-1}, $d_{0.9} = 1000$ m). As a first

approximation, it is further assumed that α_0' and α' are constants and have the same value ($\alpha_0' = \alpha' = 6 \cdot 10^{-7}$ cm^3 s^{-1}). This simplification is justified by the circumstance that since only convergence changes are studied, a joint variation of α_0' and α' may be expected.

The convergence of the ion velocity has been computed for day-time hours of seven winter-time circulation disturbances from 1966 to 1974, for which the parameters *fbEs*, *h'Es* and *foE* had been available. The computed values have been grouped according to time and averages were taken for various hours of the day. Then, deviations of the individual convergence values from the corresponding mean, $\Delta(dv_{iz}/dz)$ were computed to eliminate the daily variation. Finally, using the departures obtained for different hours of the same day, the daily average differences were determined. The results of a superposed epoch analysis, carried out with these data taking as key day the start of the main phase of the circulation disturbance are shown in Fig. 5. The curve indicates a sharp decrease of the ion velocity convergence from the fifth to the twelfth day after the key day.

Conclusions

The results of the interpretation imply that the decrease of the sporadic E ionization during winter time circulation disturbances, shown by the blanketing frequency (*fbEs*), may be attributed to the weakening of the wind shear. The latter is presumably connected with the unfavourable propagation conditions of atmospheric gravity waves due to the change of interaction between the different components of the wind (prevailing wind, atmospheric tides and gravity waves).

The reliability of the results can be increased by the application of improved E layer models and by the determination of the real height of sporadic E ionization. Further, the variation of the effective recombination coefficient in the background ionosphere should be considered as well.

REFERENCES

1. Bencze, P.: Physical processes in the lower ionosphere. *MTA X. Osztályának közleményei*, 3 (1970), 85 (in Hungarian).
2. Gregory, J. B.—Manson, A. H.: Winds and wave motions to 110 km at mid-latitudes II. Mean winds at 52°N, 1969—73. *J. Atmosph. Sci.*, 32 (1975), 1607.
3. Sprenger, K.—Schminder, R.: On the significance of ionospheric drift measurements in the LF range. *J. Atmosph. Terr. Phys.*, 30 (1968), 693.
4. Ionospheric Data Békéscsaba, Hungary, 1967—1974. Central Institute of Meteorology, Budapest.
5. HHI Geophysical Data, 1962—1974, Akademie der Wissenschaften der DDR, Zentralinstitut für solar-terrestrische Physik (HHI), Berlin.

6. SPRENGER, K.—SCHMINDER, R.—GREISIGER, K. M.—KÜRSCHNER, D.—SCHÄNING, B. Die Windsysteme in der oberen Mesopausenregion mittlerer Breiten nach Ionosphärendriftmessungen im Langwellenbereich. Akademie der Wissenschaften der DDR, Zentralinstitut für solar-terrestrische Physik (HHI), Berlin, 1974.
7. ЧАВДАРОВ, С. С.—ЧАСОВИТИН, Ю. К.—ЧЕРНЫШЕВА, С. П.—ШЕФТЕЛЬ, В. М. Среднеширотный спорадический слой Е ионосферы. Изд. «Наука», Москва, 1975.
8. REDDY, C. A.—MATSUSHITA, S.: The variations of neutral wind shears in the *E*-region as deduced from blanketing Es. *J. Atmosph. Terr. Phys.*, 30 (1968), 747.
9. RAWER, K.—RAMA KRISHNAN, S.: Tentative Tables of Electron Density and Excess Electron Temperature for Temperate Latitude. Arbeitsgruppe f. Physik. Weltraumforschung, Freiburg, 1972.

ИОНОСФЕРНАЯ СПОРАДИЧЕСКАЯ Е-ИОНИЗАЦИЯ И НАГРЕВЫ В СТРАТОСФЕРЕ

П. БЕНЦЕ

РЕЗЮМЕ

Автор находил, что отклонения параметров *fbEs* и *h'Es* спорадического *E*-слоя от соответствующего месячного медиана уменьшаются во время возмущений зимней циркуляции, связанных с нагревами в стратосфере. Вычислив сходимость скорости ионного дрейфа из этих величин на основе теории ветрового сдвига, кажется, что уменьшение спорадической *E*-ионизации (*fbEs*) можно приписать за счет ослабления ветрового сдвига. Это последнее, вероятно, можно приписать за счет обстоятельств, невыгодных с точки эрения распространения атмосферных гравитационных волн, имеющих место во время циркуляционных возмущений.

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EXPERIMENTAL RESULTS WITH THE CHARACTERIZATION OF GEOMAGNETIC MICROPULSATIONS

VI. SUMMARY OF RESULTS, RECOMMENDATIONS

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The paper summarizes advantages and problems connected with the geomagnetic pulsation catalogue and daily indices used in the Nagycenk Observatory, presents several examples for the use of these data and proposes to introduce similar world-wide indices.

1. This series of papers has been started several years ago to report about experiments for the characterization of geomagnetic pulsations. Part I [1] gave a short survey of the methods used earlier at the Nagycenk Observatory and elsewhere. Part IV [4] contained considerations on energetic (amplitude) indices. Part II, III and V [2, 3, 5] are devoted to certain results achieved by the use of these indices and to more generally results obtained in the processing of pulsation data at the Nagycenk Observatory. Some additional results can be found in [6, 7].

The results of the experiments will be summarized in the last part of this paper again, from the point of view of "production" of pulsation indices, of the catalogue of pulsation data, as well as of the spatial extension of correlated pulsation fields, the latter being an important factor if world-wide pulsation indices would be introduced. There are well-founded hopes that such indices would promote the use of pulsations for magnetospheric diagnosis, as using data of a single observatory (i.e. Nagycenk) quite sufficient results could be reached, e.g. in the determination of solar wind velocity [6].

2. The problems to be treated in this paper are the following:

a) A general survey of the method to determine pulsation indices, including comparisons of amplitude and frequency indices; necessary work in case of manual and computerized processing, number of bands to be used etc.

b) Possibitilies for the use of the intermediary data (pulsation catalogue). Unlike *a)*, this is bound to the processing method of the Nagycenk data, with readings taken at intervals of 15 or 30 min.

c) Comparison of pulsation indices at different observatories in order to study the possibility of the introduction of world-wide indices (bands in which this would be possible).

d) A summary of some results achieved using the pulsation indices or the catalogue to show their versatility for different purposes.

In addition, results are presented on comparisons with different type processings and argued that they can also be used for the determination of pulsation indices. The resolving power of other known processing methods is, however, not as high and, e.g. the periods of Pc 3 or Pc 4 pulsations can be seldom determined. We argue that the use of at least 12 period bands is necessary, or a method similar to the Niemegk method (computerized determination of average periods inside of bands) is recommended.

3. At present (in 1976) the following quantity of the different type pulsation indices (and catalogues) is available:

Frequency of occurence (spectral) indices

20 mm/min records (about three days (WD-s) in each month); 1957—1975, 18 years

6 mm/min records (theoretically 27 days from each month, except "World Days" till December, 1975; from 1976, all days); 1966—1976, 10 years

Amplitude (energetic) indices

6 mm/min records (similarly as for the previous case); 1971—1976, 5 years

That means that about 700 spectral indices from 20 mm/min records and 3000 from 6 mm/min records, as well as 1500 energetic indices are computed, in each case 12 indices for every day. Spectral indices are regularly published in the Observatory Reports.

It is quite obvious that a distinction of spectral and energetic indices of the pulsation activity as proposed by TROITSKAYA et al. [8] yields surplus information. It is, however, necessary to control very carefully the different indices. In the present case, Fig. 1 shows (from the material of the year 1972) the correlations between energetic (A), spectral (G) indices, the difference $A - G$ (Δ), and a local earth-current activity index (T, with linear scale). It is evident that energetic and spectral indices are very closely correlated (0.7 to 0.95), therefore their information is not independent. The correlation pattern $T - G$ (earth current activity — spectral index) reflects well known facts about the appearance of different period pulsations at different geomagnetic activities. The curve of the correlation $T - A$ is rather similar, only shifted at most periods towards algebraically higher correlations. The curve $T - \Delta$ has a quite different shape, showing that not A and G, but more G and Δ are independent indices. The meaning of Δ is rather simple: as A depends on the daily amplitude sum, G on the number of occurrences, therefore $A - G = \Delta$ is proportional with the average amplitude during appearance of the band. In this respect it is to be noted that this problem has been already mentioned by ANGENHEISTER [9].

At present there is a major obstacle in using Δ as a pulsation index: namely that its value is in many cases 0, as energetic and spectral indices are very often equal. In the future this possibility deserves special attention.

Though at present digital pulsation recording is not continuous at the Nagycenk Observatory, experiments with the digital determination of the catalogue (and indices) have been carried out. The idea of the program is that at first the band is chosen in which the amplitude reaches its maximum rela-

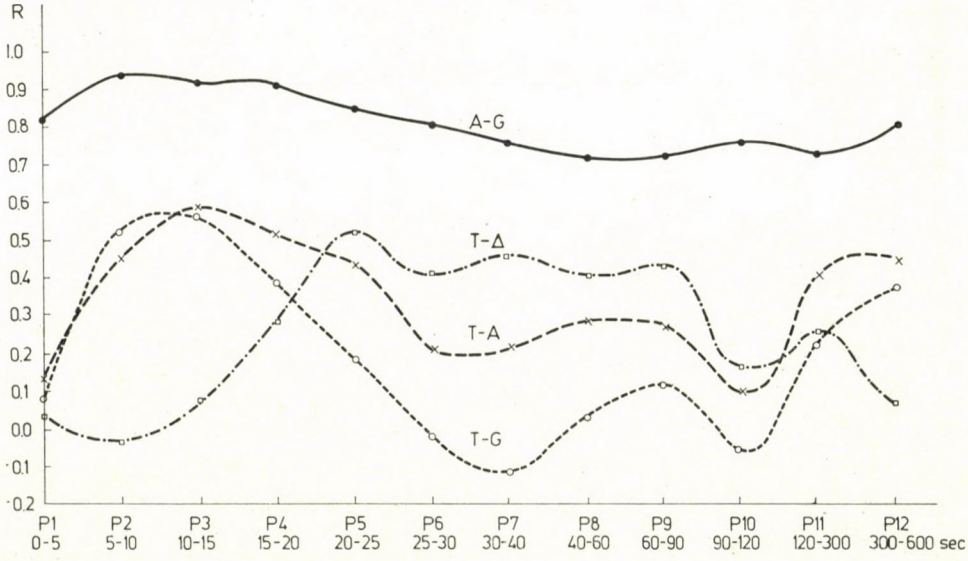

Fig. 1. The correlations of the energetic (A) and spectral type pulsation indices with each other ($A-G$), with the telluric activity index T (A and G), and the correlation of T with $\Delta = A - G$

tive to a line $a = c \cdot P^n$ (a is the amplitude, P, the period, c, a constant to be determined; the constant exponent n was in these experiments 1). Bands are then characterized active, when the amplitude reaches in them the value $a' = d \cdot c \cdot P^n$ (d is less than 1, e.g. 0.5). Amplitude values are attributed finally to these bands.

The common problem of manual and computer processing is the number of period bands to be used. In the present case the number of bands is 12. This enables the determination of the momentaneous peak period in the Pc 3—4 regime; a further subdivision at short periods seems possible, if adequate recordings are available. Concerning longer periods, the indices K1—K5 should be mentioned [10]; they have been used for some 20 years in the Nagycenk Observatory (longest period 60 min). A reduction of the number of the bands would decrease the resolving power of the processing.

In case of the manual processing, the necessary time is about 5—6 working days for a month of pulsation records with experienced persons.

In connection with a computerized determination it should be also mentioned that according to our experience it is better to use about 10—20 waves for the determination of the spectrum than the whole (30 min or any other) interval. This is naturally impossible at longer periods; therefore short periods are overestimated (as amplitude maxima are chosen). This can be compensated by using an exponent $n < 1$ when looking for active bands.

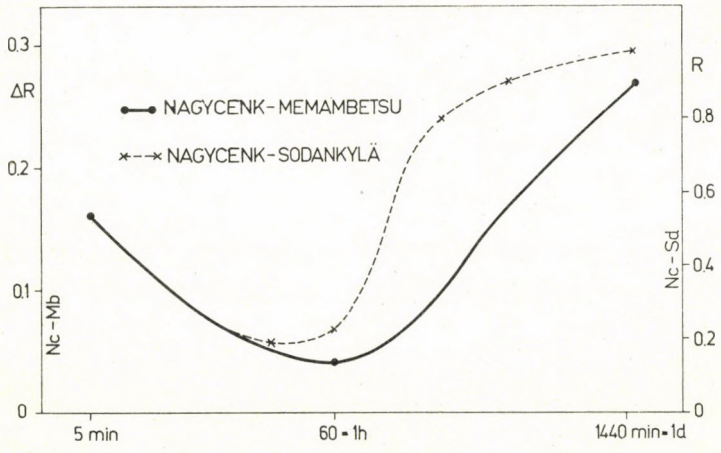

Fig 2. The change of correlation factor of geomagnetic pulsations in distant observatories for different long averaging intervals. The two lines were found from completely different material: observatories and methods, therefore the scales are also different (see also references)

4. Pulsation indices could be determined from the data of a single observatory for whole days. For shorter intervals, e.g. for hours, such a procedure is only possible by using a number of well distributed observatories. It is, however, obvious that the pulsation catalogue which is the basis for the determination of the indices and contains in case of the Nagycenk material data for each 30 min (or 15 min from 20 mm/min records), can also be used for correlation studies bearing in mind that night- and daytime data must be carefully split.

On the basis of our experiences gathered hitherto it seems to be advantageous, if such a pulsation catalogue is used for correlation studies. Original spectra contain too many data; 12 bands (and perhaps some additional ones in the short period-range) seem to be sufficient for most purposes even inside of the Pc 3 band (e.g. for the determination of the peak period).

It should be mentioned that for an arbitrary, well-defined condition (e.g. a certain value and direction of the interplanetary magnetic field) about

20—30 data already yield rather stable spectra for a 3-hour interval of the day, or about 100 data an acceptable daily mean value.

5. The final aim, i.e. the determination of world-wide pulsation indices, involves two problems. First, the time interval for which the index refers should be determined. Figure 2 shows the distribution of the correlation coefficients between distant observatories in function of the time interval to which the original data refer. This figure has been constructed from data in [5, 19], and it has only an explanatory character. For long intervals (days) the correlation between pulsation amplitudes in distant observatories is rather high (up to 0.9) since it reflects global increases and decreases in the pulsation activity. For intervals of about 1 h, the correlation is poor, in consequence of the daily variation of the pulsation amplitudes which are in different phases at different longitudes. At even shorter intervals (some minutes), correlations increase again, as pulsational events (like pse in [11]) have greater effect.

For these reasons, pulsation indices can be determined from a small number of observatories (at least two on opposite hemispheres) either for complete days, or for short intervals (e.g. 5 min). For 3-hour indices, more well-distributed observatories are needed, similarly to the determination of *Kp*-indices.

Pc 3 indices can be most easily determined. In case of the observatories Nagycenk, Niemegk, Sodankylä, correlation coefficients have values of about 0.4—0.5, and similar correlations were found between Nagycenk and Fürstenfeldbruck, where the processing methods were different [12].

Pc 1, Pc 2 and Pi 1 indices, or indices for periods corresponding to these bands can also be quite easily determined if records of suitable frequency response are at disposal. It must be, however, reminded that most observatories have problems in case of short periods with artificial disturbances. The Nagycenk pulsation indices for the bands 1—15 sec (P1, P2, P3) are less reliable than others, but mainly due to the parameters of recording (paper speed 6 mm/min).

Pc 5 is also a phenomenon with restricted geographical extension. This is most obvious from older maps showing distributions of *pg*-events. Not any correlation coefficient for this type is in case of the mentioned three observatories greater than about 0.3. In spite of this, there are no major obstacles to a determination of *daily* Pc 5 indices.

The situation with Pc 4 is rather different, e.g. the correlation coefficients for this band do not reach the 0.2 limit for the three observatories. This is the more surprising, as the Pi 2 events, which are of nearly global extension [13, 15], belong to this period band. Similar results were earlier achieved for the observatory pairs Nagycenk—Budkov [14] and Nagycenk—Fürstenfeldbruck [12]. In order to explain this phenomenon, Figure 1 should be stu-

died: the correlation between occurrence frequency and amplitudes is here the lowest. In geomagnetically disturbed periods, in a small number of intervals Pc 4 occurs with high amplitude, whereas in quiet times, Pc 4 appears for very long times but with comparatively small amplitudes. Depending on the choice of criterium for Pc 4 activity, rather great differences can occur. Even if subjective elements are eliminated, i.e. in case of computer processing, these differences cannot be eliminated. The situation is further complicated by the simultaneous appearance of Pc 3 and Pc 4, sometimes in different components. It is therefore very difficult to determine unambiguous indices for the Pc 4 bands.

A somewhat similar situation occurs also for Pc 3 and Pc 5—6. There are some negative correlations for these types, e.g. in Nagycenk, too [5]. This can have two causes: either it has some physical background (e.g. the disappearance of Pc 3 after geomagnetic impulses [16]), or it is artificially caused: when pulsations have great amplitudes, the long-period pulsations can be overseen (also by computers, depending on the choice-system of active bands).

6. During the last ten years, the pulsation indices of the Nagycenk Observatory have been used for a number of investigations. In the following we summarize some results and experiences with their use.

a) One of the first experiments — based only on a rather limited material from international world-days — has been carried out for a comparison of ionospheric storms and pulsation data [17]. It could be shown that positive ionospheric storms have a rather significant influence on pulsations. The results of this study initiated the continuous determination of pulsation indices. Here daily indices were used.

b) Having been introduced the continuous determination of pulsation indices, they have been checked by determining the influence of the geomagnetic activity on pulsation spectra. The results [2, 3] were generally in accordance with the previously known distribution.

c) The after-effects of geomagnetic disturbances have been studied by using cross-correlation functions between daily indices [2], by investigating the non-Markovian property of the daily index-series [7], and already earlier, by comparing spectra of similarly disturbed days following days with different activity. Here it must be borne in mind that due to the subjective method of the determination, the indices can be compared on a short run, that means that, e.g. in case of the study of the non-Markovian character, yearly series were investigated, and the results then summarized in order to avoid the effect of long-period trends.

d) Recently the effect of several interplanetary parameters have been studied, so among others that of the velocity of the solar wind [6], of the dimensions of the magnetosphere [18], and of the direction and value of the

vector of the interplanetary magnetic field [20]. In the first two cases, daily indices were used, in the third, the pulsation catalogue.

The general experience with the Nagycenk-type pulsation indices and other data was that in most cases rather stable results could be achieved from not very big quantities of data (e.g. the material of one year yielded stable spectra for 150 different situations of the interplanetary field and geomagnetic activity). If, however, amplitude data are used, it is absolutely necessary to take great care of very active days ($Kp > 6$) when the pulsation amplitudes increase considerably and the percentage of such days can influence the results (of course this problem does not emerge, if indices are used instead of the catalogue).

7. Summarizing we think that our pulsation catalogue and indices are an effective tool in studying the interconnections of pulsations, and their connections with other data, respectively. We hope also that their computerized determination would improve their quality, and would also allow long-period comparisons. An international co-operation for their determination on a world-wide basis would yield useful data for the diagnosis of the magnetosphere and interplanetary space.

REFERENCES

1. Holló, L.—Tátrallyay, M.—Verő, J.: Experimental results with the characterization of geomagnetic micropulsations. I. The methods of characterization used in the investigations. *Acta Geod. Geoph. Mont. Hung.*, 7 (1972), 155.
2. Holló, L.—Verő, J.: Experimental results with the characterization of geomagnetic micropulsations. II. Micropulsation index. Auto- and cross-correlation functions of micropulsations and geomagnetic activity. *Acta Geod. Geoph. Mont. Hung.*, 7 (1972), 167.
3. Verő, J.: Experimental results with the characterization of geomagnetic micropulsations. III. Effect of the geomagnetic activity on pc 3—4 and pi-2 type geomagnetic micropulsations. *Acta Geod. Geoph. Mont. Hung.*, 7 (1972), 177.
4. Tátrallyay, M.—Verő, J.: Experimental results with the characterization of geomagnetic micropulsations. IV. Comparison of occurrence frequency and amplitude spectra from different type records. *Acta Geod. Geoph. Mont. Hung.*, 8 (1973), 217.
5. Takács, L.: Experimental results with the characterization of geomagnetic micropulsations. V. Comparison of the geomagnetic pulsation activity at the observatories Nagycenk, Niemegk and Sodankylä. *Acta Geod. Geph. Mont. Hung.*, 10 (1975), 307.
6. Verő, J.: Determination of the solar wind velocity from pulsation indices. *J. Atm. Terr. Phys.*, 37 (1975), 561.
7. Verő, J.: Pulsation activity as a non-Markovian process. *Acta Geod. Geoph. Mont. Hung.*, 9 (1974), 291.
8. Troitskaya, V. A.—Gul'elmi, A. V.—Bolshakova, O. V.—Matveeva, E. T.—Schepetnov, R. O.: Indices of geomagnetic pulsations. *Planet. Space. Sci.*, 20 (1972), 849.
9. Angenheister, J.: Registrierungen erdmagnetischer Pulsationen, Göttingen, 1952—53. *Gerlands Beitr. Geoph.*, 64 (1955), 108.
10. Ádám, A.—Verő, J.: Das Erdstromobservatorium bei Nagycenk (Ungarn). *Geof. Pura e Appl.*, 39 (1958), 126.
11. Voelker, H.: Zur Breitenabhängigkeit erdmagnetischer Pulsationen. *Mitt. Max-Planck Inst. f. Aeronomie*, 11 (1963).
12. Holló, L.—Korschunow, A.: A comparison of activity character figures of geomagnetic pulsations at the observatories Fürstenfeldbruck and Nagycenk. *Acta Geod. Geoph. Mont. Hung.*, 9 (1974), 53.

13. STUART, W. F.—BOOTH, D. C.: Pc enhancement and stimulus. *J. Atm. Terr. Phys.*, 35 (1972), 2101.
14. CZUCZOR-MILETITS, J.: Vergleichende Untersuchung von elektromagnetischen Registrierungen der Observatorien von Nagycenk und Budkov. *Studia Geoph. Geod.*, 12 (1968), 74.
15. HOLLÓ, L.—VERŐ, J.: Pi-triggered Pc-pulsations. 15 years. Geod. Geoph. Res. Lab. of Hung. Ac. Sci., 1970, 53.
16. TÁTRALLYAY, M.—VERŐ, J.: Changes of geomagnetic micropulsations following sudden impulses. *J. Atm. Terr. Phys.*, 35 (1973), 1505.
17. HOLLÓ, L.—VERŐ, J.: Zur Charakterisierung der Pulsationstätigkeit des erdmagnetischen Feldes für einzelne Tage und deren Anwendung. *Acta Geod. Geoph. Mont. Hung.*, 2 (1967), 335.
18. VERŐ, J.: Determination of the position of the magnetopause from geomagnetic pulsations indices. *Acta Geod. Geoph. Mont. Hung.*, 10 (1975), 459.
19. CZ. MILETITS, J.—VERŐ, J.: Correlated Pc-type micropulsations observed at separate places. *J. Atm. Terr. Phys.*, 33 (1971), 967.
20. ВЕРЁ, Й.: Межпланетное магнитное поле и геомагнитные пульсации в 1972-ом году. Симпозиум КАПГ по солнечно-земной физике (Тбилиси, сентябрь 1976 г.) Тезисы докладов. часть III. Москва, *Наука*, 1976. 104.

ОПЫТНЫЕ РЕЗУЛЬТАТЫ, СВЯЗАННЫЕ С ОДНИМ НОВЫМ МИКРОПУЛЬСАЦИОННЫМ ИНДЕКСОМ

VI. ПОДЫТОЖИВАНИЕ РЕЗУЛЬТАТОВ, ПРЕДЛОЖЕНИЯ

Ю. Ц. МИЛЕТИЧ – Л. ХОЛЛО – М. ТАТРАЛЯИ – Й. ВЕРЁ

РЕЗЮМЕ

В статье подытоживаются преимущества и проблемы каталогизации геомагнитных пульсаций и суточных индексов, применяемых в обсерватории Надьценк, вместе с некоторыми примерами их использования. Предлагается ввести подобные распространенным в мире индексы.

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SPACE-TIME DISTRIBUTION OF GEOMAGNETIC PULSATIONS AND ITS DEPENDENCE ON THE GEOMAGNETIC ACTIVITY

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Morphological features of geomagnetic pulsations are discussed, including diurnal variation of the occurrence frequency, of amplitudes and periods, latitude dependence of the occurrence frequency, of amplitudes and periods, and their variations with the *Ko*-index.

Introduction

There is a wide range of plasma oscillations in the magnetosphere; a part of them is transmitted through the ionosphere and can be recorded on the Earth's surface. In the frequency range from some mHz to several Hz these variations are called pulsations. Pulsations have been studied on the Earth's surface for about hundred years. Though many features of the pulsations are not well understood yet, the tendency to increase the number of geomagnetic observatories is a clear sign of the efforts to understand the space-time characteristics of the pulsations. Only in the last years appeared the possibility to set up a qualitative theory of pulsations.

In the cosmic space around the Earth, pulsations have been observed for about 12 years, but their features have not yet been studied enough due to several difficulties. These difficulties are due partly to the information transmitting apparatus which limits the frequency characteristics and to the unavoidable compromise between the wide dynamic range of pulsations and the possibilities of the magnetometers. The study of pulsations has great importance from the point of view of the dynamics of particles. Satellite observations of waves in the frequency range of pulsations show that this type of waves controls the level of geomagnetic activity in the magnetosphere [1]. This connection gives further impetus to the study of pulsations. A new direction of investigation has been initiated, namely the diagnosis of the magnetosphere and of the interplanetary space using pulsation data.

At present the pulsations are described using a classification accepted by the XIIIth General Assembly (August 1963, Berkeley, California). This classification is based on the zonal and morphological characteristics of the pulsations; the most important among them being the periods, amplitudes, occurrence time and localization.

Table I

Continuous type	Period, sec	Irregular type	Period, sec
Pc 1	0.2—5	Pi 1	1—40
Pc 2	5—10	Pi 2	40—150
Pc 3	10—45	Pi 3[1)]	150—
Pc 4	45—150		
Pc 5	150—600		
Pc 6[1]	600—		

[1] The Pi 3 and Pc 6 types have been accepted as such at the Symposium on Geomagnetic Pulsations during the IAGA Assembly in Kyoto, 1974. Pi 3 includes pulsation types like Pip and Ps 6. These pulsation types will not be treated in the present article due to the insufficiency of the available material on their space-time distribution.

This classification splits the pulsations into two classes according to their regular or continuous character. Each class is subdivided according to its period.

In the Pi 1 range investigations carried out in recent years revealed many different types of events which are of different physical nature: PiB (Pi burst), or sip (short irregular pulsations), PiC (Pi const.) or AA (auroral agitation), IPDP (irregular pulsations with diminishing periods). Between Pc 1 and Pc 2, there is a transitional sub-group of the pulsations with periods $T = 4 - 6$ sec with features differing from both Pc 1 and Pc 2. The Pc 3 type can be subdivided into polar and low-latitude types, Pc 5 into local and global ones, etc.

In the present study of the space-time characteristics of geomagnetic pulsations only the generally accepted international classification will be used. The basis of the present work are two reviews writen by T. SAITO [2, 3] supplemented with some recent Soviet and other results.

Continuous pulsations

This type of pulsations is characterized by the regularity of the oscillations of quasi-sinusoidal form. A part of them has beating character and can last several hours.

1. *Time-variations of the main parameters of pulsations*

a) *Daily variation of the frequency of occurrence* $(N_{Pc}(t))$

(Pc 1) Figure 1 shows the daily variation of the occurrence frequency of Pc 1 for high-latitude stations and for stations in the auroral zone. In the polar cap the maximum occurrence frequency is around 12—14 h geomagnetic

time. At stations in the auroral zone the maximum occurrence frequency shifts to 15—16 h local geomagnetic time. The type of the daily variation of the occurrence frequency does not depend on the latitude of the station alone. The daily variation of two subclasses of Pc 1 ($T > 2$ sec and $T < 2$ sec) are shown on Fig. 1.

(Pc 2) The maximum frequency of occurrence of Pc 2 was found around 11—12 h LT (Fig. 2); this is a daytime type.

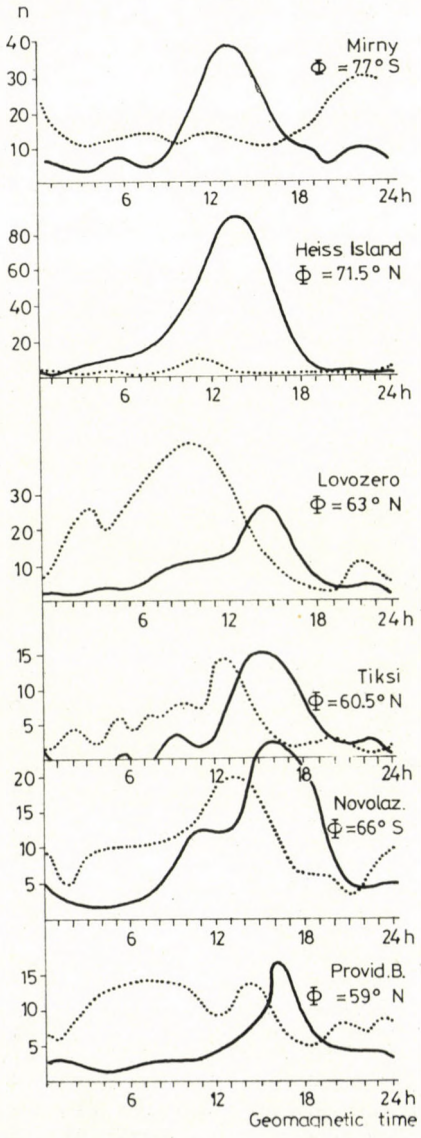


Fig. 1. Daily variation of Pc 1 with $T < 2$ sec (. . . .) and with $T > 2$ sec (— — —) for the latitude ranges 77—66°S and 63—59°N [4]

(Pc 3) The daily distribution of Pc 3 has been studied in many details from the material of a great number of stations. It is a daytime phenomenon with local-time controlled daily variation with maximum around 08—12 h LT [8]. The daily variation of the occurrence frequency has in Borok a wide maximum around 09—10 h LT (Fig. 2).

(Pc 4) This pulsation type appears almost at the very moment when the observation site reaches the enlighted hemisphere, the maximum is at local noon, and it disappears on the non-enlighted hemisphere [6]. Figure 2 shows the daily variation of the occurrence frequency of Pc 4 as well. An analysis of data from Dallas and other observatories [9] has shown that on magnetically quiet days Pc 4 as well as Pc 3 is a phenomenon of the local daytime with maximum around local noon, too.

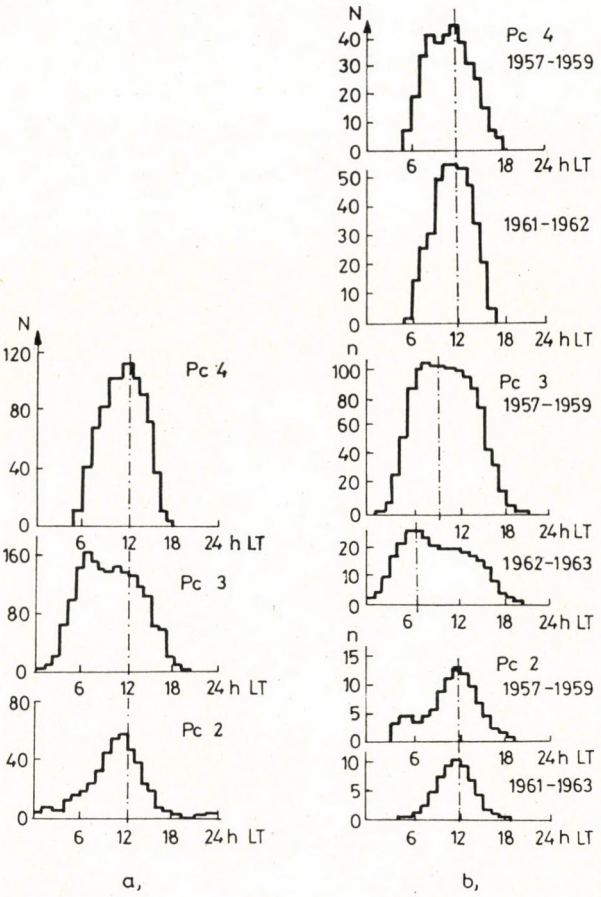


Fig. 2a. Daily variation of the occurrence frequencies of different Pc groups. *N* is the number of cases when the given Pc-type appeared in the interval 1957—64. The vertical dotted line shows the local noon of Borok [6] *b)* Daily variation of the occurrence frequencies of different Pc-groups in different phases of the solar cycle [6]

(Pc 5) The occurrence frequency of Pc 5 has two maxima during the day; the maxima occur in different observatories around 06—10 h and 14—16 h LT [11].

b) *Daily variation of the period* ($T_{Pc}(t)$)

(Pc 1) In case of Pc 1, there are two characteristic periods: T, the basic period of the phenomenon, and τ, the period of repetition of structural elements. T is nearly proportional to τ, and it can be expressed as $\tau = 86T$. The daily variation of T is shown on Fig. 3. The longest period T is observed around 12—16 h LT.

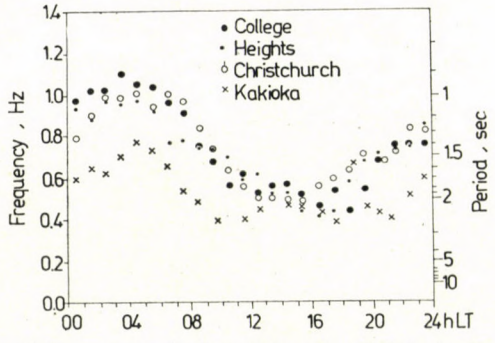


Fig. 3. Daily variation of the average Pc 1-period at the high-latitude stations College, Heights and Christchurch and at the middle latitude station Kakioka [3]

(Pc 2, 3) The daily variations of the Pc 2, 3 periods have different characters. In order to describe them, the terms "U-type" and "inverted U-type" variations have been introduced. These types are found on graphs expressing variations of the period (abscissae) with the time of the day (ordinates) (Fig. 4). Figure 4 summarizes the different daily variations of the periods: full lines indicate inverted U-type variations, dotted ones U-type variations. Fig. 4 shows that: 1) the inverted U-type includes periods between 6 and 21 sec; 2) the U-type occurs between periods 18 and 140 sec; 3) the change from the inverted U-type into the U-type takes place at periods between 18 and 21 sec.

(Pc 4) The daily variation $T_{Pc\,4}$ has a U-type form (Fig. 4). An analysis by GUPTA and STENING [12] has also confirmed a daily variation in the Pc 4-periods. They are shortest in the morning hours and longest in the evening hours at a latitude of 60° [12, 13].

For some days with nearly constant *Kp*-index, the daily variation of Pc 4 periods has been studied in [14] according to data of the Kerguelen observatory. The period is diminishing during daytime hours having a mini-

mum below the dawn sector of the magnetosphere, and a maximum below the dusk-night sector.

(Pc 5) The daily variation of the Pc 5 periods is shown on Fig. 6. The average period has a characteristic daily variation with its minimum in morning and maximum in evening hours.

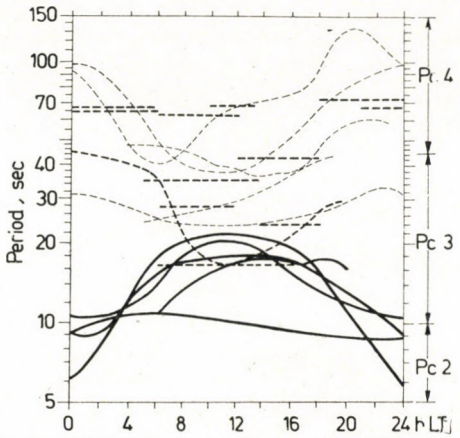


Fig. 4. Daily variation of Pc 2—4 periods compiled from several sources [3]

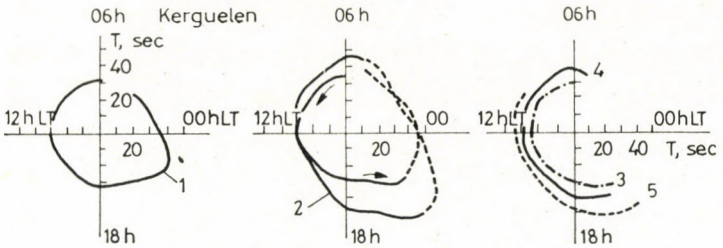


Fig. 5. Daily variation of Pc 3 periods at the Kerguelen observatory for days with approximately constant *Kp*-index. 1, June 27, 2, June 13—14, 3, June 10, 06—20 h LT; 4, June 22, 10—21 h LT, 5, July 25, 05—20 h LT [14]

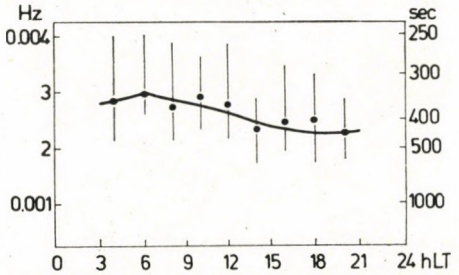


Fig. 6. Average daily variation of Pc 5 periods according to observations at 6 stations in the latitude interval 60—74° [13]

c) *Daily variation of the amplitudes* ($A_{Pc}(t)$)

(Pc 1) No distinct daily variation of Pc 1 amplitudes has been found. Some general trends of the amplitude variations can be seen on Fig. 7.

(Pc 2—5) For these types no clear daily variation of the amplitudes has been found. The daily variation of the average activity of each Pc-type is shown on Fig. 8 for the year 1958 [2], computed using the formula:

$$\frac{1}{365 - (N_{NR} + N_{ST})} \sum_{n=1}^{365} A_n(t)$$

where $A = a \cdot D$ is the activity,

a	amplitude in gamma,
D	duration of the pulsations,
N_{NR}	no records,
N_{ST}	disturbed intervals.

Figure 8 clearly shows that Pc 2, 3 and 4 are daytime phenomena and Pc 5 is a morning one.

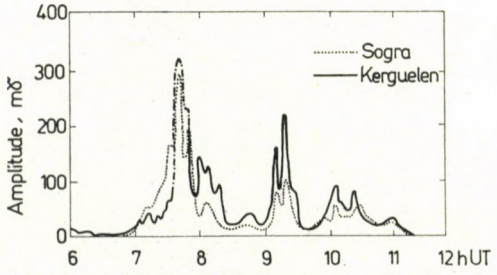


Fig. 7. Daily variation of Pc 1 amplitudes as observed at a pair of conjugated stations, June 20, 1964 [3]

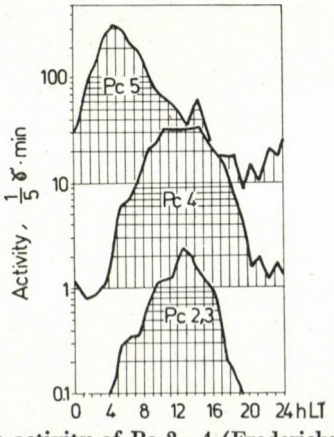


Fig. 8. Daily variation of the activity of Pc 2—4 (Fredericksburg), Pc 5 (Byrd), constructed from more than 5000 Pc series during 1958 [2]

2. *Space characteristics of geomagnetic pulsations*

a) *Latitude dependence of the occurrence frequency* ($N_{Pc}(\Phi)$)

(Pc 1) As it has been already noted, the type of the daily variation of the occurrence frequency depends on the latitude (cf. Sections 1a, 1b). Figure 9 summarizes daily variations of Pc 1 occurrence from different geomagnetic latitudes. The original curves have been normalized such as to make the maximum 100% in all observatories. It is clear the the maximum occurs in daytime hours at high latitudes and roughtly during night hours at lower latitudes. The time of the maximum occurrence of Pc 1 shifts continuously from daytime to nighttime hours beginning at latitudes around 60°.

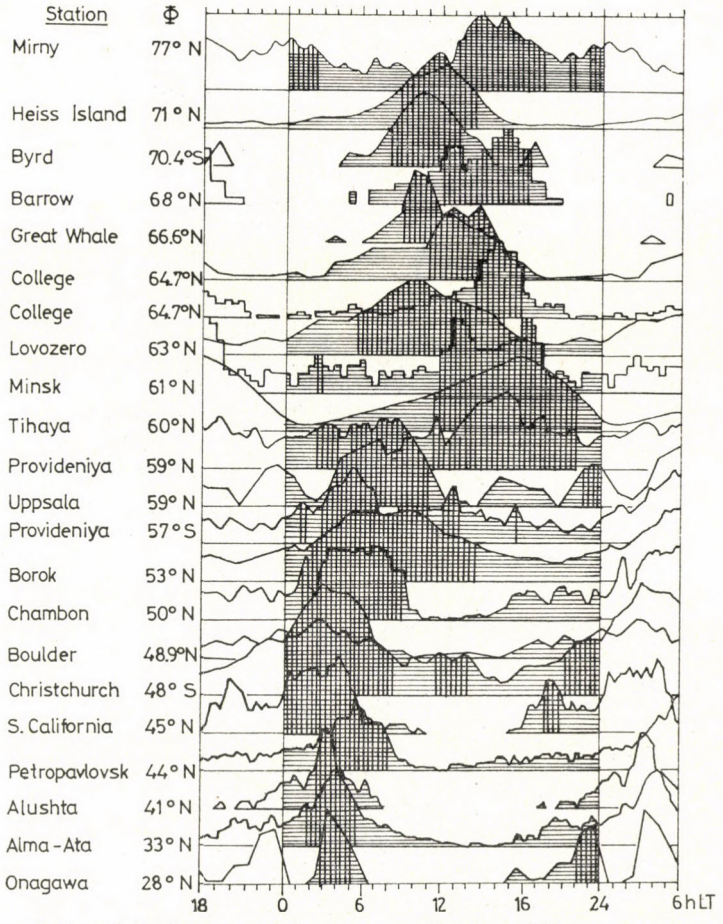


Fig. 9. Daily variation of the frequency of occurrence of Pc 1 at different geomagnetic latitudes. The original figures obtained by several authors have been normalized so that the maximum occurrence frequency should be 100 p. c. [3]

(Pc 2) Figure 10 shows the dependence of the occurrence frequency of Pc 2 from the *Kp*-index at the geomagnetic pole (Vostok observatory) and at the middle latitude observatory Borok. Pc 2 occurs more frequently at the pole than at lower latitudes. There are no other data about $N_{\mathrm{Pi}\ 2}(\Phi)$.

(Pc 3) Data of four stations from the latitudes 57—83° show (Fig. 11) that the activity of Pc 3 is higher during daytime than night and in the polar zone than in the auroral zone.

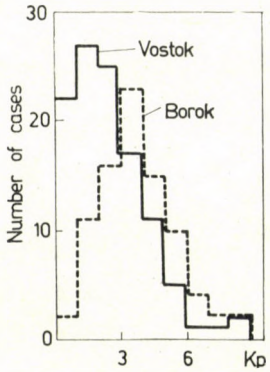


Fig. 10. Dependence of the Pc 2-occurrence frequency on the *Kp*-index at the geomagnetic pole (Vostok) and at the middle latitude station Borok [15]

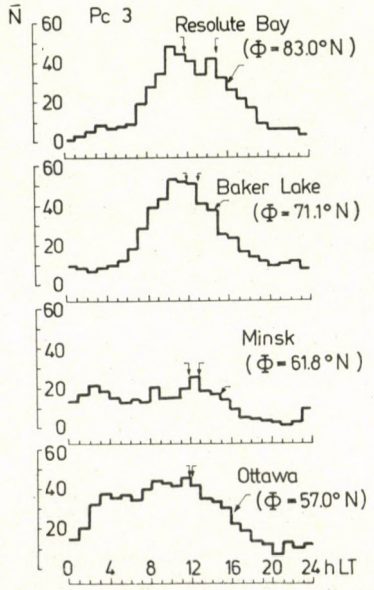


Fig. 11. Daily variation of the occurrence frequency of Pc 3 at different geomagnetic latitudes for the interval August 12 to 16, 1968 [18]

(Pc 4) No clear dependence $N_{Pc\,4}(\Phi)$ has been found. Data of four stations from latitudes 57—83° are shown on Fig. 12 [18]. It is, however, impossible to conclude from this figure any variation of the occurrence frequency with the latitude, as $N_{Pc\,4}(t)$ has here many maxima (23—24 h, 10—13 h, 16—17 h LT) which are not confirmed by other investigations (cf. e.g. [6, 9]). It is, however, mentioned in [19] that maximum occurrence of Pc 4 was found in the auroral zone.

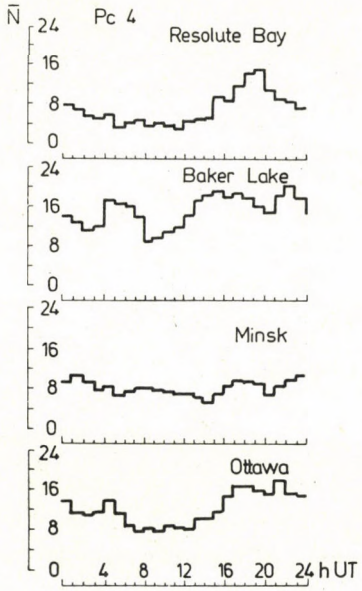


Fig. 12. Cf. Fig. 11, for Pc 4

(Pc 5) The region where Pc 5 can be most frequently found and where it has the most regular forms is limited by the latitudes 65 and 71°. This fact confirms that the occurrence frequency of Pc 5 depends strongly on the observation site [11, 13, 20].

At even higher latitudes, i.e. near the poleward boundary of the auroral zone, the maximum at 08 h is predominant, whereas at somewhat lower latitudes, i.e. at the equatorial boundary of the auroral zone, the 16 h-maximum increases, and that at 08 h decreases. With increasing latitudes the times of these maxima shift also: the 08 h maximum appears later, the 16 h-maximum earlier [11, 13, 20].

b) *Latitude dependence of the period* ($T_{Pc}(\Phi)$)

(Pc 1) The already mentioned splitting of Pc 1 in two types: 1) with periods $T_1 < 2$ sec and 2) $T_2 > 2$ sec has been deduced from middle latitude stations [21]. According to several morphological features, the boundary

between the two types of Pc 1 pulsations is at periods of 3 sec in the auroral zone [22]. The frequency spectrum of Pc 1 pulsations largely depends on the latitude. With increasing latitude, the frequency band shifts towards longer periods. The conjugacy of the Pc 1-activity has been confirmed using data of the station pairs Heiss—Mirny (1961) and Pyramida—Mirny (1958) [4].

For Pc 1 with T_1 the periods are identical for a given event everywhere, for Pc 1 with T_2 the scatter of the periods is significant. It seems that Pc 1 with T_1 has a global scale, whereas Pc 1 with T_2 are localized to small areas. It should be, however, mentioned that simultaneous appearance of Pc 1 has been observed at middle latitudes and at the equator, and also at both magnetic poles.

(Pc 2) No latitude dependence of Pc 2 periods has been found.

(Pc 3) Pc 3 periods do not change significantly with latitude and they are basically constant in any Pc 3-event for many oscillations [23]. Only in the subauroral zone could an increase of the periods with increasing latitude be confirmed [9]. A comparison of the records from the conjugate stations Sogra—Kerguelen has shown that the scatter of periods for conjugate pairs is of the same order of magnitude than for stations in the studied latitude interval of the Northern hemisphere [23].

(Pc 4) An increase of the Pc 4 periods with increasing latitudes has been found in [9]. The main peaks of the spectral density of Pc 4 are identical at conjugate stations [27, 28].

(Pc 5) The latitude distribution of Pc 5 periods is shown on Fig. 13. Generally the period decreases with decreasing latitude. A study of Pc 5

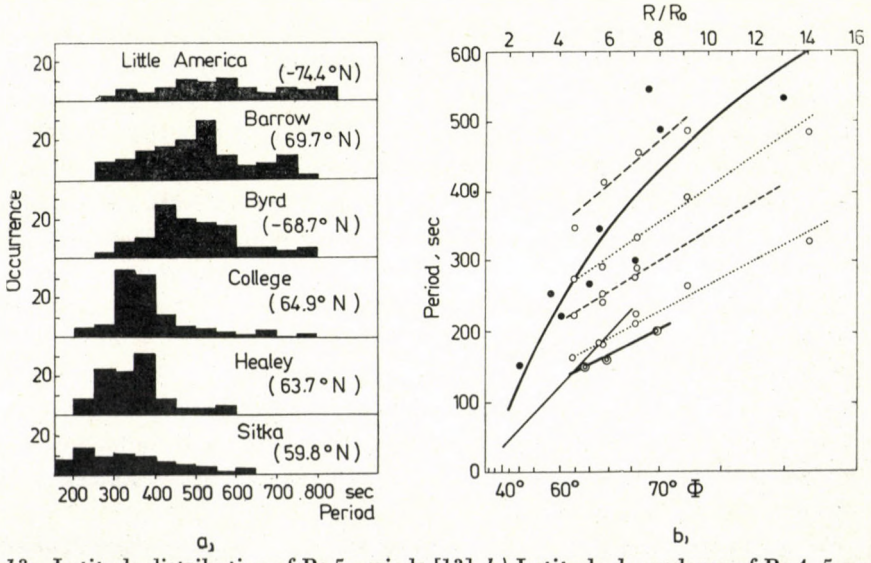


Fig. 13a. Latitude distribution of Pc 5 periods [13], *b)* Latitude dependence of Pc 4, 5 periods. The original figures have been redrawn in the common scale periods — geomagnetic latitude [3]

records from 8 stations at latitudes 64.9°N—79.6°S and 70.3—148°E has shown that Pc 5 spectra might have significant differences for similar latitudes at different longitudes of the same hemisphere, whereas the spectra at conjugate stations are identical even in small details [24, 25].

c) *Latitude dependence of the amplitudes* ($A_{Pc}(\Phi)$)

(Pc 1) Maximum amplitudes of Pc 1 appear in the subauroral and auroral zone [26]. The amplitude often reaches some hundred mililgammas at high latitudes, some tens at middle latitudes and about ten at the equator [27]. A more correct picture of the amplitude dependence is shown on Fig. 14 using

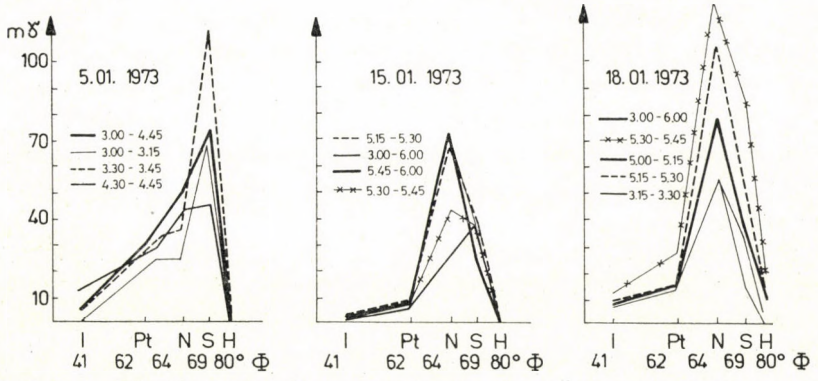


Fig. 14. Changes of Pc 1 intensity along a meridional profile for three days in January 1973 [28]

data of a meridional profile (41—80°). In that analysis of the latitude dependence of Pc 1 amplitudes, only Pc 1 series have been used which had very simila renvelopes, $h_r = (h^2_{x\,max} + h^2_{y\,max})^{1/2}$ at all stations, and also the periods and the forms of pulsations has been similar. Figure 14 shows that: 1) the maximum activity of Pc 1 is at 64° or at 65°; Pc 1 activity at the two hemispheres are very seldom similar. Therefore a narrow zone of reflecting source for the Pc 1 can be confirmed; 2) near to the reflecting source the damping of Pc 1 is different along the latitude profile, greater damping was found in the direction of the pole; 3) the zone of the reflecting source is at lower latitudes for Pc 1 with T_1. The main features of the time variations of Pc 1 amplitudes are similar at the conjugate pair Kerguelen—Sogra (cf. Fig. 7), though amplitudes are not always the same in conjugated stations, even the periods might be different. Figure 15 shows cases when the Pc 1 amplitude is by about 6—8 times more in Sogra than in Kerguelen (*a*) and vice versa (*b*), and in both cases the period T is the same.

(Pc 2) There have been no systematic studies of the latitude dependence of Pc 2 amplitudes. It has been mentioned in [15] that the amplitudes of Pc 2

have been the same at middle latitudes and at the pole at different *Kp*-values (cf. also 3c).

(Pc 3) The space distribution of Pc 3 activity (amplitudes) has been studied in [3, 8, 12, 29], see Figs 16a, b, c and d. Although from the statistical sense there are a rather small amount of data in each case, it is quite clear that amplitudes in the *H*-component increase with increasing geomagnetic latitude. According to GUDKOVA et al. [29], the maximum amplitude of Pc 3 can be found at 60°, and according to STENING and GUPTA [19], an amplitude maximum was found at latitudes 61.9—73.0°N during simultaneous measure-

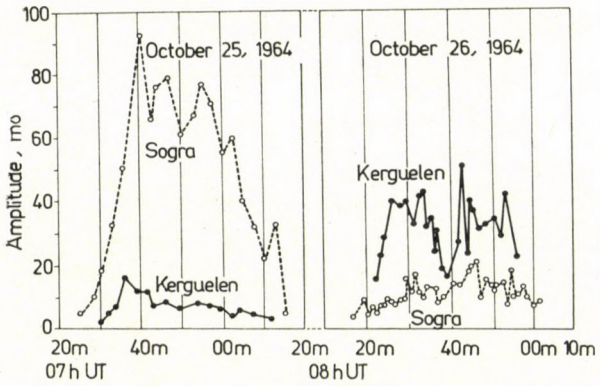


Fig. 15. An example of the changes of Pc 1 amplitudes at a pair of conjugated stations [3]

ments between 57 and 83°N. The amplitudes then decrease towards higher and lower latitudes nearly symmetrically. A more detailed analysis with a dense net of stations (*B*) has revealed the existence of two amplitude maxima (Fig. 16). The maximum intensity has been found in dawn hours at the latitude of Zhelaniya, and a secondary maximum has been found at the same time at the latitude of Sogra.

(Pc 4) The results of a comparison of pulsation periods and amplitudes at four Canadian observatories have been published in [19]. A significant change of the *A* has been reported in function of the latitude, with a maximum always in the auroral zone [29]. BARANSKY et al. [8] could not find such a distinct change of Pc 4 amplitudes along a meridional profile as for Pc 3. Figure 17a shows the dependence of H_i/H_A on Φ for 3 UT intervals. The main maximum of Pc 4 activity is till 10 h UT mostly at Heiss or Zhelaniya, and a secondary one is at Sogra or Borok. After 10 h UT the latitude of the most likely maximum shifts downwards to Lovozero. On the $H_i/H_A - \Phi$ graphs, a rather serious scatter of the experimental points can be seen, mainly in the high latitude part of the profile. The decrease of the amplitudes towards the equator is for Pc 4 less quick than for Pc 3.

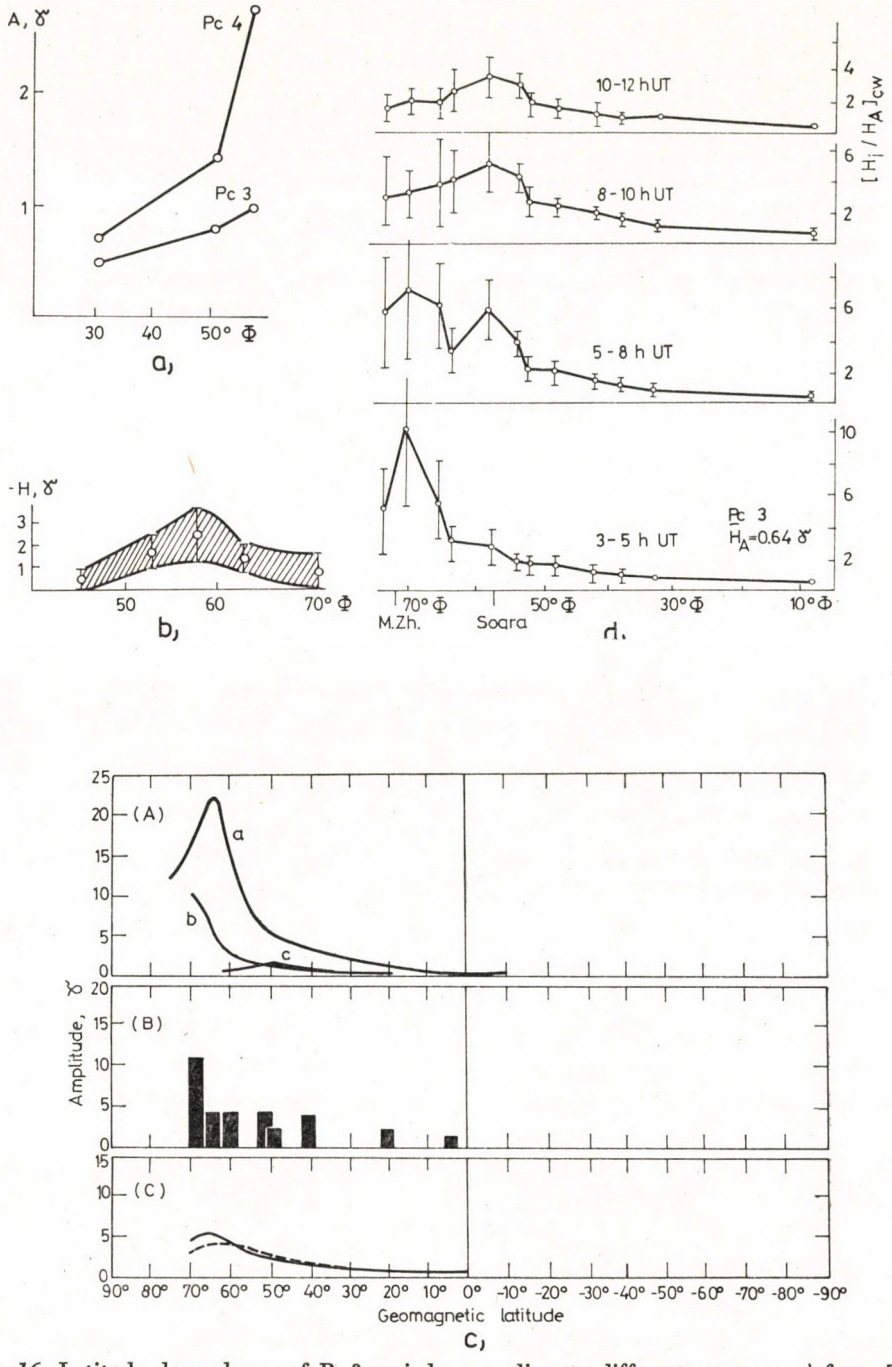


Fig. 16. Latitude dependence of Pc 3 periods according to different sources: *a)* from [23], *b)* from [29], *c)* from [3], *d)* average value of the normal intensity $(H_i/H_A)_{av}$ and the mean square error at each station in the interval 7—74° for the given UT-interval [8]

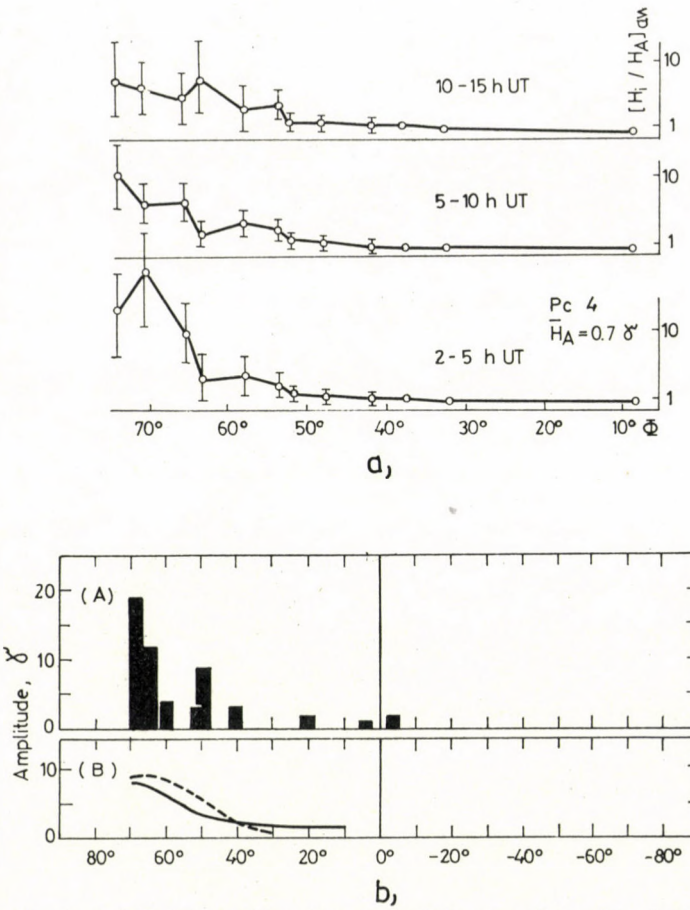


Fig. 17. Distribution of Pc 4 intensities along a meridional profile: *a)* 7—74° [8], *b)* 10—70 (*A* and *B* original graphs from [3])

(Pc 5) Maximum amplitudes of Pc 5 [30] can be found in a limited latitudinal zone around the auroral zone. The Pc 5 amplitudes decrease rather quickly towards higher and lower latitudes. At $\Phi > 75°$ and at $\Phi < 60°$, these pulsations do not occur [11, 13, 20, 32], see also Fig. 18.

3. *Connection between the space-time distribution of geomagnetic pulsations and the Kp-index*

a) *Dependence of the occurrence frequency* ($N_{Pc}(Kp)$)

(Pc 1) No direct connection between the occurrence frequency, amplitude variations and period of Pc 1 pulsations and *Kp* index could have been detected. A great part of Pc 1 events occur during the first week following

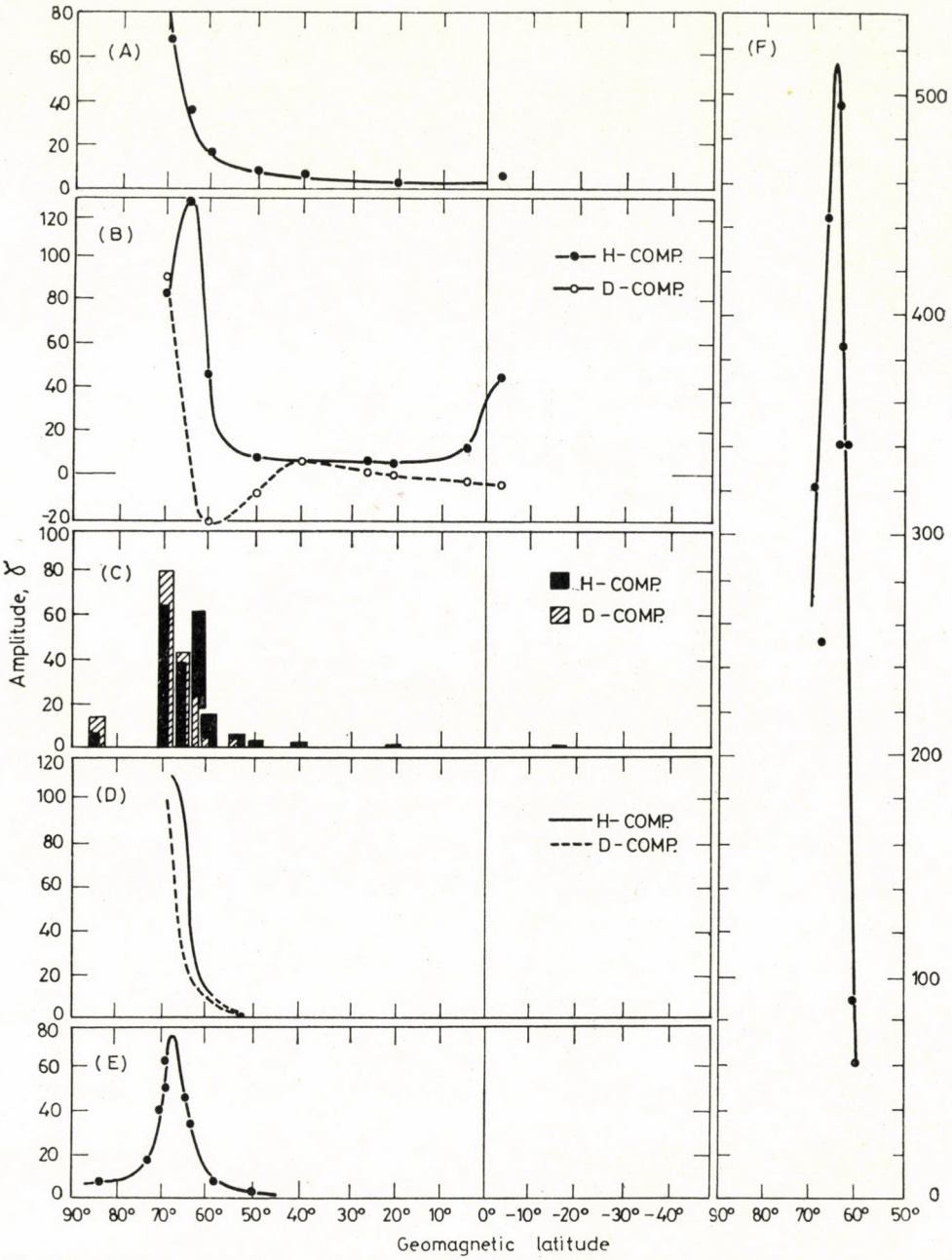


Fig. 18. Latitude dependence of Pc 5 amplitudes. The original figures have been redrawn in a general amplitude—geomagnetic latitude coordinate system [3]

geomagnetic storms. Sometimes Pc 1 appears repeatedly on continuous days, sometimes it disappears for months. No 27-day recurrence tendency could have been found [31]. The activity of Pc 1 has its maximum in the declining phase of the 11-year solar cycle (Fig. 19, [4, 5]).

(Pc 2) Figure 10 shows that Pc 2 occurs more often at the poles than at middle latitudes. During high *Kp*-values, Pc 2 can be observed as often in Borok as in Vostok, whereas at low *Kp*-values Pc 2 is very seldom in Borok; in Vostok, however, the maximum of the occurrence frequency lies at *Kp* 1—2.

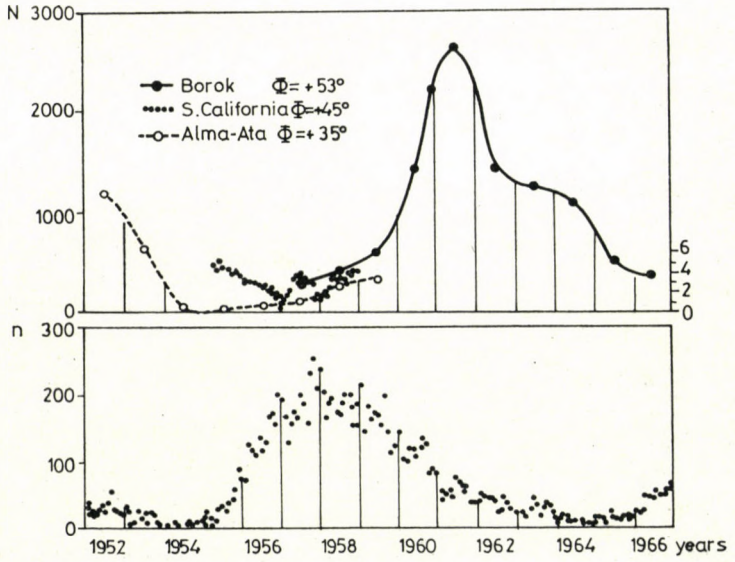


Fig. 19. Changes of the solar activity (*n*) and of the frequency of occurrence Pc 1 (N) [3]

(Pc 3) The frequency of occurrence of Pc 3 repeats the variations of $\sum Kp$ from day to day at all four stations studied (Fig. 20). The higher the $\sum Kp$, the more often Pc 3 is found.

On magnetically quiet days Pc 3 is a local daytime phenomenon with maximum occurrence at local noon. In solar minimum years one can observe a shift of the maximum occurrence of Pc 3 towards earlier hours in respect to solar maximum years (Fig. 2b).

(Pc 4) The time of the maximum occurrence of Pc 4 practically does not change in the solar cycle (cf. Fig. 2b). Therefore its dependence on the *Kp*-index has not been studied in [8], and also — according to [18] — Pc 4 has a very low correlation with $\sum Kp$. BOL'SHAKOVA concluded [7] that each class of the Pc pulsations (Pc 2, 3, 4) can be observed at a special value of the *Kp*-index which is the less the longer the period of the pulsation (Fig. 20b).

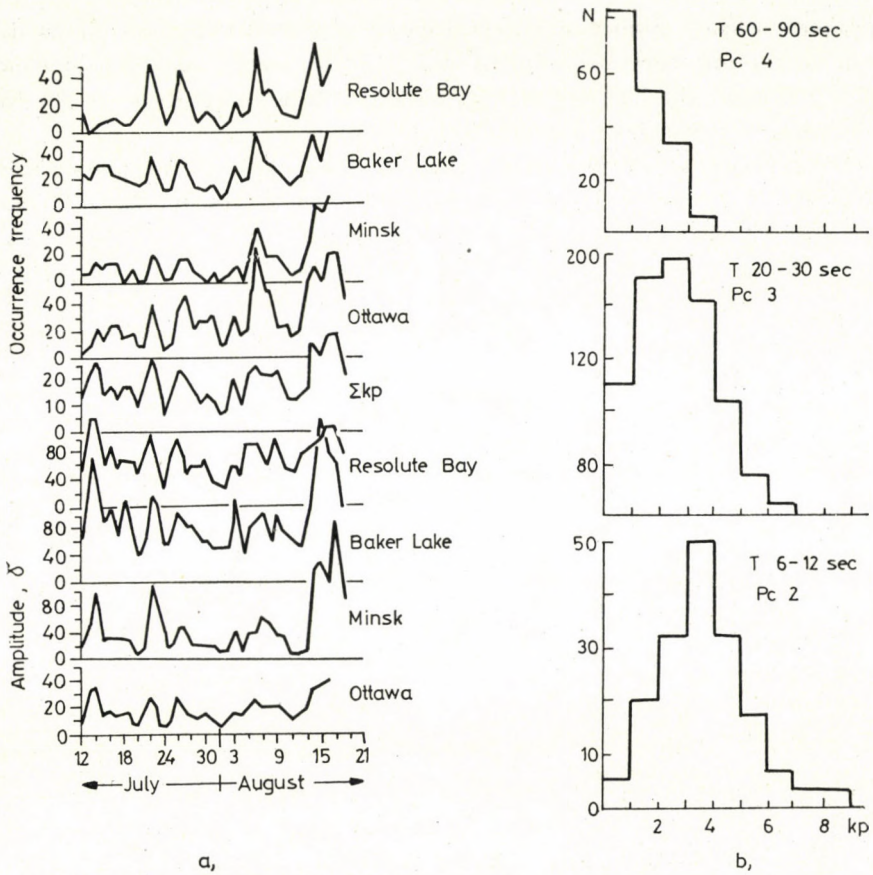
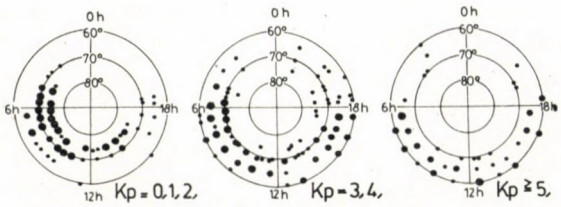
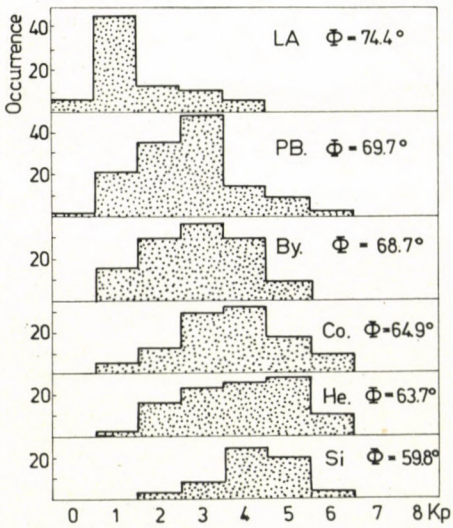


Fig. 20a. Changes of the Pc 3 occurrence frequency, *Kp* and daytime amplitudes of Pc 3 from day to day [18], *b)* $N_{Pc,\,2,3,4}$ (*Kp*); the maximum frequency of Pc 4 occurrence has been observed for *Kp* 0—1, for Pc 3 at *Kp* 2—3, for Pc 2 at *Kp* 3—5 [7]

(Pc 5) The latitude dependence of the occurrence frequency of Pc 5 is shown for different *Kp*-values on Fig. 21. At lower latitudes the maximum occurrence lies at higher *Kp*-values (Fig. 21a). If the geomagnetic field is very quiet (*Kp* 0, 1 or 2), Pc 5 can be observed at very high latitudes during morning hours. On the other hand, during moderately disturbed times (*Kp* 3, 4 or 5) it can be found in the centre of the auroral zone, ($\Phi \sim 70—64°$) with a secondary maximum in evening hours. During strong geomagnetic storms (*Kp* 6, 7, 8 or 9) Pc 5 can be observed mainly in the subauroral zone ($\Phi \sim 64—55°$). Similar conclusions were reported in [20, 25, 34], cf. also Fig. 25b.



a)



b)

Fig. 21a. Daily variation of Pc 5 occurrence frequency for different geomagnetic activities [13], *b)* Latitude dependence of Pc 5 occurrence frequency for different *Kp* values [13]

b) *Dependence of the period* (*T*(*Kp*))

(Pc 2) The dependence of Pc 2 periods on the magnetic activity is shown on Fig. 22. It shows that the period decreases with increasing *Kp* corresponding to the general rule of period decrease with increasing geomagnetic activity for all Pc-type pulsations [6].

(Pc 3) The period changes with latitude and geomagnetic activity are represented on Fig. 22. The shortening of Pc 3-periods with increasing *Kp*-indices has also been remarked in [19, 29]. The daily variation (in LT) of the correlation coefficient between average Pc 3 periods and *Kp* is shown on Fig. 23 [12]. For Pc 3, the correlation is higher in daytime than in nighttime, in agreement with its daytime character. At Baker Lake and Resolute Bay, the correlation is very poor and it is positive during night.

(Pc 4) The daily variation of the correlation coefficient between Pc 4 periods and *Kp* is also shown on Fig. 23. The correlation of $T_{\text{Pc 4}}$ is less obvious

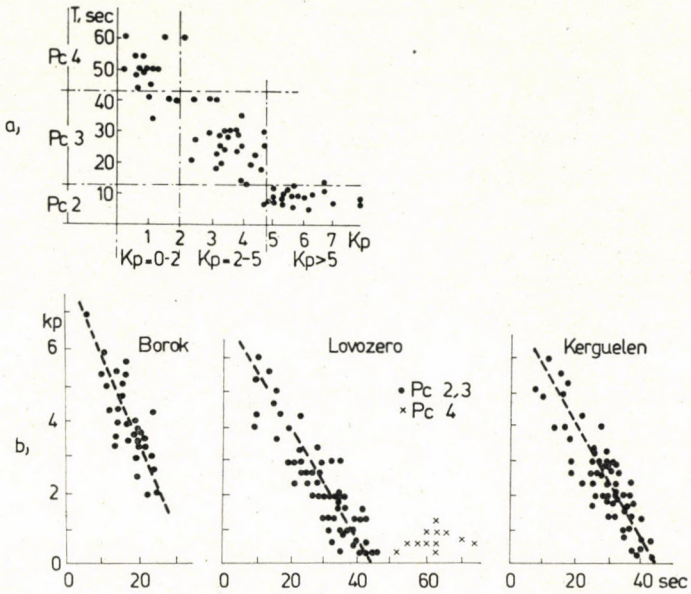


Fig. 22. Dependence of the period of different Pc groups on the *Kp*-index: *a)* for the station Borok (53°) [6], *b)* for the stations Borok, Lovozero (63°) and Kerguelen (58.5°S) [14]

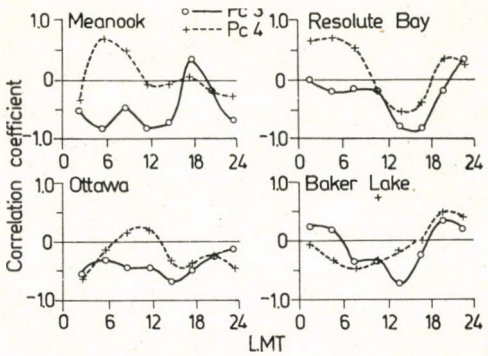


Fig. 23. Daily variation of the correlation between periods of Pc 3, 4 and the *Kp*-index. Full line, Pc 3, dotted line, Pc 4 [12]

than of $T_{Pc\,3}$, and in certain cases significant positive correlations occur as well. For the stations Ottawa and Meanook the correlation curves of Pc 3 and Pc 4 have little in common, whereas at Baker Lake and Resolute Bay they are rather similar.

(Pc 5) The changes of Pc 5 periods with solar activity have been treated in [13, 32]. Data from 13 years have shown that the periods of morning Pc 5 at the station Chelyuskin increase linearly with increasing Wolf-number. The correlation coefficient between T and W is very high: $r = +0.95$ [33].

c) *Amplitude dependence* ($A(Kp)$)

(Pc 2) In the few cases (cf. Fig. 10) when Pc 2 pulsations can be observed at small Kp values simultaneously at the poles and at middle latitudes, their amplitudes are at lower latitudes several times less than at the pole. At great Kp values the amplitudes at the pole and at middle latitudes are of the same order, but the amplitudes are not as prominent at the poles as in geomagnetically quiet times.

Thus the different dependence of Pc 2 on the latitude in case of different Kp values and its different amplitude distribution allow the conclusion that it is a phenomenon of the polar region [15—17].

(Pc 3) The dependence of the position of the maximum Pc 3 intensity along a meridional profile on the geomagnetic activity has been investigated only during daytime hours (middle latitude maximum). The level of the geomagnetic activity has been expressed by the maximum Kp_m value during the previous day. The absence of any dependence of the position of the morning maximum on the geomagnetic activity can be apparent, as the station network was very loose at high latitudes (for the stations Tambey, Zhelaniya, Heiss the L-parameter is 5.7, 8.6 and 13.8 respectively). On Fig. 24a one sees the dependence of the position of the daily maximum intensity of Pc 3 on Kp.

The average amplitude of Pc 3 also changes with the solar cycle, it decreases by about a factor two from the maximum to the minimum [6].

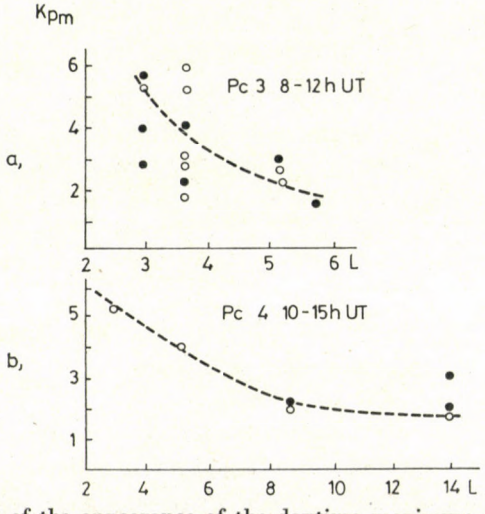


Fig. 24a. Dependence of the appearance of the daytime maximum of Pc 3 intensity on Kp, *b)* The position of the maximum of Pc 4 intensity along the meridional profile (7—74°) as a function of the geomagnetic activity. On the horizontal axis are the L-values which correspond to the situation of the maximum intensity of Pc 3, 4, the vertical axis shows the Kp-value. Thick points refer to cases when several experimental points coincide [8]

(Pc 4) Investigations about the position of the maximum intensity of Pc 4 along a meridional profile as a function of the geomagnetic activity yielded very similar results to those for Pc 3. This dependence is clear enough only in daytime hours (Fig. 24b).

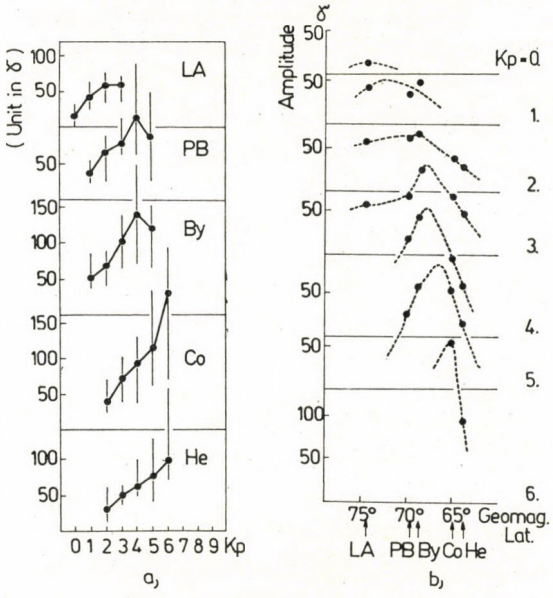


Fig. 25a. Dependence of the maximum amplitude of Pc 5 on the *Kp*-index for stations at different latitudes, *b)* Dependence of the middle latitude amplitude profile of Pc 5 on the *Kp*-value, based on the results also shown on Fig. 25a [13]

(Pc 5) The connection between Pc 5 amplitudes and *Kp* for some stations is shown on Fig. 25a. Pc 5 amplitudes increase with increasing *Kp* at high latitudes. If the *Kp* surmounts a certain limit characteristic for the station then the average amplitudes decrease very rapidly, and Pc 5 can be observed very seldom. The *Kp*-dependence of Pc 5 amplitudes along the meridional profile is shown on Fig. 25b.

Irregular pulsations

As already mentioned, there are two distinct forms of the Pi 1 activity which are characteristic for the auroral zone and clearly correlate with polar magnetic activity. They are generally investigated separately (e.g. [35—38]) even though there are no officially accepted subgroups yet. The first type has an impulsive and wide-band character, often with several resonant bands, this is the so-called Pi 1 (B). It is a characteristic part of the initial phase of the

polar magnetic substorms. The appearance of Pi 1 (B) takes place practically always simultaneously with Pi 2, it is the microstructure of Pi 2. During the initial phase of a substorm, Pi 1 (B) can be generated together with Pi 2 either in the form of singular wave trains, or as wave groups following each other in 10—15 minutes, while the duration of the whole series is not more than one hour [37]. Pi 2 with its sudden appearance characterizes almost always the final phase of the pulsation event, and it corresponds to a quick increase and then to a continuous decrease till the quiet state of the field [60]. So Pi 2 does not possess in the polar region the characteristic form known from middle latitudes. After the short Pi 2 + Pi 1 (B) event, continuous micropulsations follow with periods 5—10 sec (maximum at 6—8 sec, Pi 1 (C)) [39—41]. This second form of Pi 1 has a much narrower frequency band, and the upper frequency limit lies lower than at the first type and during its occurrence it increases continuously. The phenomenon can continue for several hours, while the first subgroup has a duration of only some minutes.

1. *Temporal variations of Pi-type pulsations*

a) *Daily variation of the occurrence frequency* ($N_{\mathrm{Pi}}(t)$)

Pi 2 and Pi 1 (B) have uniform daily variations of the occurrence frequency. Pi 1 (B) can be observed between 18 and 24 h LT, with a maximum between 21 and 22 h (Fig. 26). Fig. 27a shows the daily distribution of pulsation trains at the stations Oasis and Lovozero. The distribution is very similar at other stations [3, 42—44]. The daily variation has a single wave with maximum frequency around local midnight and minimum around local noon. Pi 2 is most common between 22 and 02 h LT, and between 07 and 15 h it can be found very seldom. SHERWOOD et al. [56] found that its appearance depends more clearly on geomagnetic time than on local time. The same maximum occurs between geomagnetic midnight and 01 h. Pi 1 appears more often in morning hours and it is characteristic for the increasing phase of the substorm (Fig. 26).

b) *Daily variation of the period* ($T_{\mathrm{Pi}}(t)$)

The daily variation of the Pi 1 (C) and Pi 1 (B) has not been studied yet. $T_{\mathrm{Pi}\,2}$ has some statistical daily variation: its periods are in the morning shorter than in the evening [3].

c) *Daily variation of the amplitudes* ($A_{\mathrm{Pi}}(t)$)

The daily variation of the Pi 1 (C) and Pi 1 (B) amplitudes has not been studied either. The most important feature of the Pi 2 intensity is the presence of a maximum at the midnight meridian [46, 52].

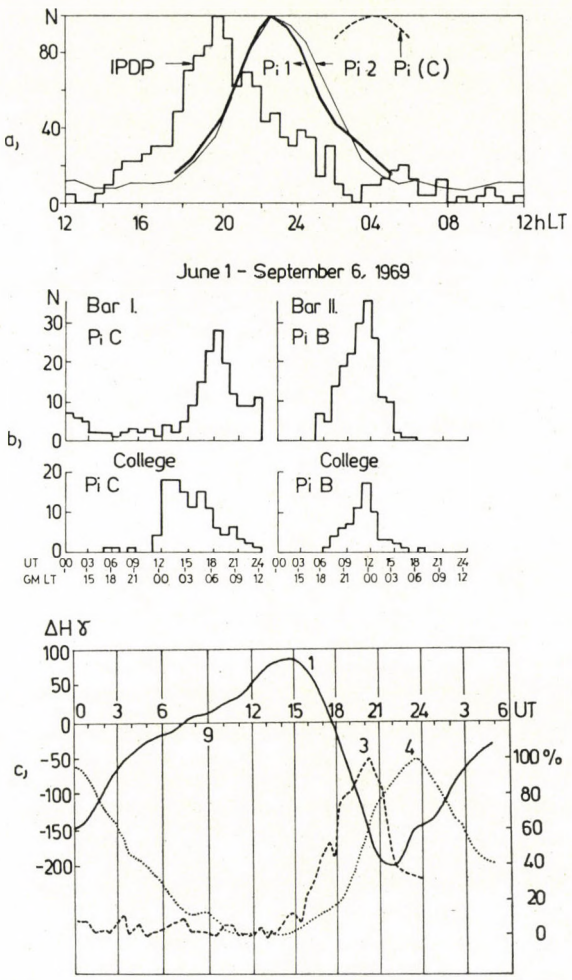


Fig. 26. Daily variations of the occurrence frequency: *a)* Bands with maximum intensity for the 4 Pi subgroups. Occurrence frequencies and bands with maximum intensity are normed so that their maximum values should be 100 (for more details refer to Fig. 54 in [3]); *b)* $N_{\mathrm{Pi}(B)}(t)$ and $N_{\mathrm{Pi}(C)}(t)$ for the stations Bar I (70.2°N) and College (64.6°N) with a longitude of 256.5° and 256.9°, resp. [39]; *c)* $N_{\mathrm{Pi}\,1(B)}(t)$ (3) and $N_{\mathrm{Pi}\,1(C)}(t)$ (4) in percents and the daily variation S_D (1) for the station Lovozero (63.6°) [40]

2. *Spatial characteristics of the irregular pulsations*

a) *Latitude dependence of the frequency of occurrence* ($N_{\mathrm{Pi}}(\Phi)$)

The spatial distribution of the appearance of Pi pulsations has been obtained in [47], Fig. 28. Pi 1 (C) can be observed mainly at the equatorial edge of the auroral zone, and it is quickly damped with the latitude [61]. In spite of this and of its localization, it can also be observed at middle lati-

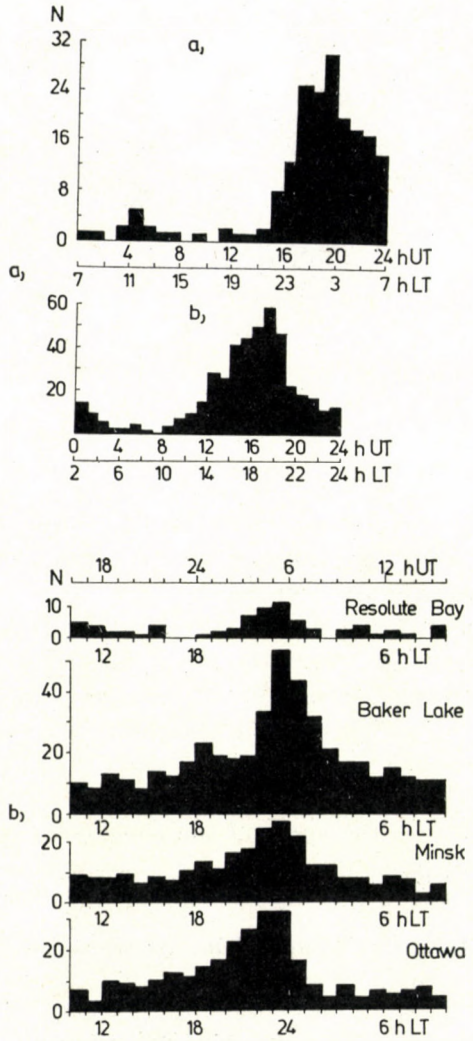


Fig. 27. Daily variation of Pi 2: *a)* at the stations Oasis (*a*) and Lovozero (*b*) for 1957—58 [45], *b)* at 4 Canadian stations for the interval July 13—August 16, 1968 [44]

tudes [45]. Using the f—t diagrams of Pi 1 (C) from 2 stations lying at different latitudes inside the auroral zone, it could be shown that Pi 1 (C) type pulsations appeared sometimes at the lower latitude station, whereas at higher latitudes they have been absent [47]. Further, the good conjugacy of Pi 1 (C) should be noted on the northern and southern hemispheres [51]. Studies of $N_{\text{Pi 1(B)}}(\Phi)$ and $N_{\text{Pi 2}}(\Phi)$ have shown (e.g. [34, 49]) that the main maximum of occurrence frequency can be observed at geomagnetic latitudes 60—70°.

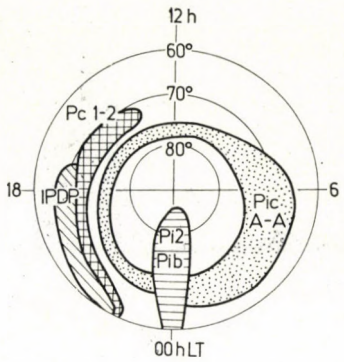


Fig. 28. Scheme of the spatial distribution of Pi generation [47]

b) *Latitude dependence of periods* ($T_{Pi}(\Phi)$)

An analysis of records from Borok and Lovozero has shown that the periods of these pulsations are about 1.5—2 times shorter in Borok than in Lovozero [48]. A visual scrutiny of records from the conjugate pair Sogra—Kerguelen has shown that periods are identical [47]. Figure 29 shows the distribution of Pi 2 as a function of periods. The periods of the pulsation trains are in polar regions mainly in the range 50—90 sec. The period dependence on the latitude is shown in Fig. 29b. In the investigated case the period of Pi 2 has been identical on the whole area from the polar to the equatorial region.

The frequency analysis of Pi 2 at stations with a latitude difference of 36° (16—52°) has proved that the spectral components do not change with the latitude of the station [62].

c) *Latitude dependence of the amplitudes* ($A_{Pi}(\Phi)$)

Earlier it has been stated [44, 54—58] that the main maximum of Pi 2 intensity can be observed at the same geomagnetic latitudes (60—70°) as the maximum of the occurrence frequency, and that there is a supplementary, weaker maximum at the latitude region 50—60°. Recent analyses [46, 52) of the spatial distribution of Pi 2 and Pi 1 (B) intensities are based on a special measurements series in 22 European-Asian stations for different values of the Q-index. The distributions found are shown on Figs 30a and b with norming to the station Borok. In both cases the maxima appear at the same place, but the decrease of Pi 1 (B) is quicker when the distance from the maximum increases than for Pi 2.

The latitude distribution of Pi 1 (B) also shows that these pulsations appear along the same field line as the simultaneous Pi 2. The latitude dependence of Pi 1 (C) amplitudes has been investigated in [57]. In order to obtain

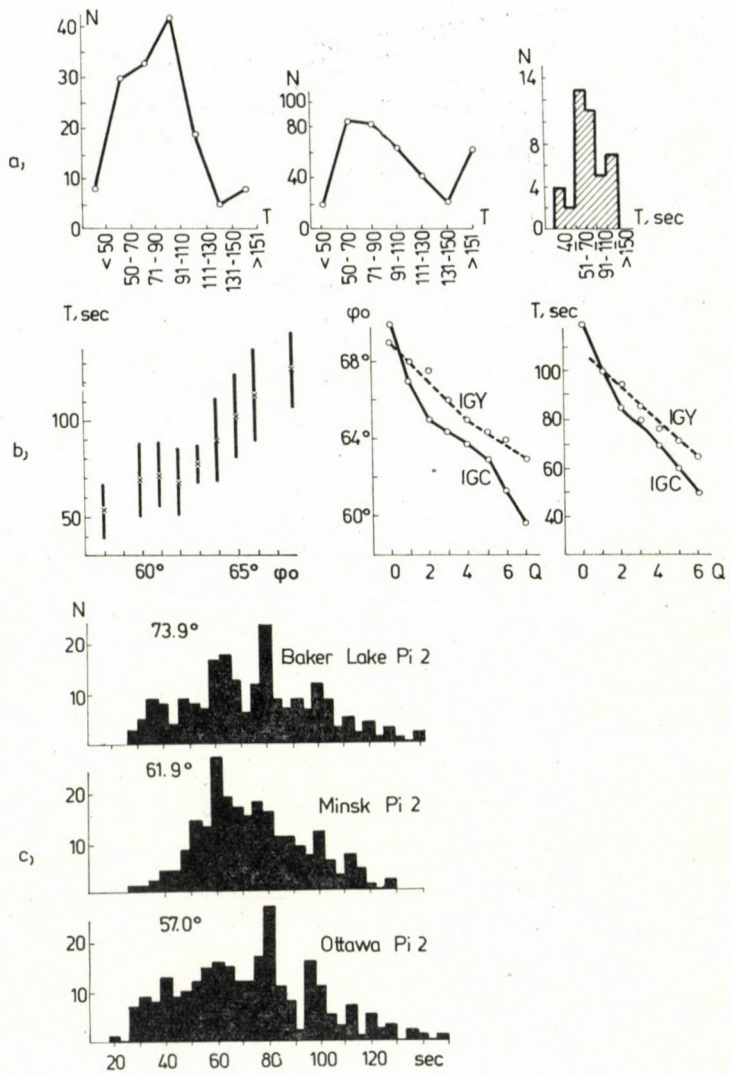


Fig. 29. Distribution of the periods of pulsation trains: *a)* at the stations Oasis (August 1957—June 1958), Lovozero (August 1957—June 1958), Chelyuskin (August—December 1957) [45], *b)* at latitudes 57—67° [52], *c)* at latitudes 57—73.5° [44]

an instantaneous distribution of the Pi 1 (C) intensity those cases were selected when pulsations could be recorded on the whole station network between 53.6 and 74.3° N. Figure 31 shows the latitude distribution of Pi 1 (C) amplitudes during relatively low and high activity level. The curve $A_{\mathrm{Pc}\,1(C)}(\Phi)$ has one maximum in the latitude range 56—68°, whose position depends on the geomagnetic activity. In quiet intervals the maximum intensity can be found at latitudes of about 64—68°, at higher activity this shifts to latitudes 58—60°.

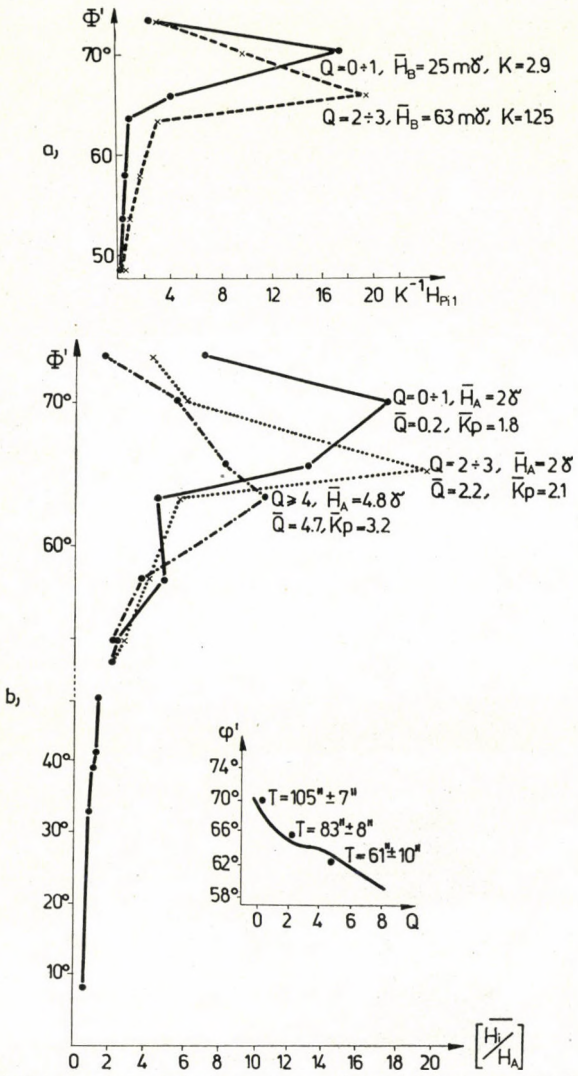


Fig. 30. Latitudinal distribution of the intensities: *a)* Pi 1(B) ($H_{\mathrm{Pi1\,(B)}}(\Phi)$) in dependence on the Q-index, *b)* Pi 2 for different Q-values [51]

3. *Connection of the space-time characteristics of irregular pulsations with the Kp-index*

a) *Dependence of the occurrence frequency* ($N_{\mathrm{Pi}}(Kp)$)

The changes of the characteristics of Pi 1 (B) in function of the *Kp*-index has been analysed in [51]. If the *Kp* value increases, the frequency of Pi 1 (B) remains unchanged, or decreases but the amplitude increases. The greatest number of Pi 2 can be observed according to [59] at low geomagnetic activity

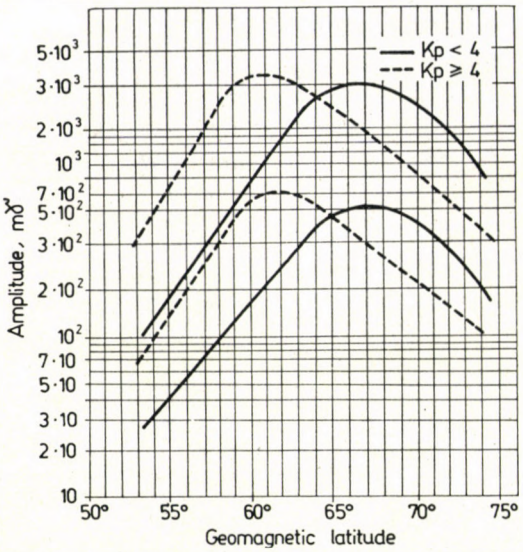


Fig. 31. Latitudinal profile of Pi 1(C) amplitudes at different levels of geomagnetic activity (the curves limit the area within which the instantaneous latitudinal amplitude distribution appears [56])

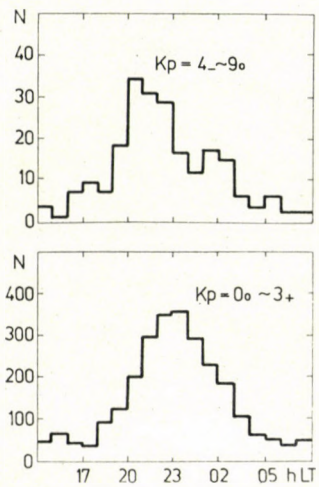


Fig. 32. Daily variation of Pi 2 occurrence frequency during high and low magnetic activity [51]

(Kp O_0—3_+), Fig. 32. There are, however, opinions (e.g. [62]), that the occurrence of Pi 2 increases and also the amplitudes are greater during polar magnetic disturbances (Kp 3—5). These different results might be due to the difficulties in the identification of quickly damped impulses during magnetic disturbances. The number of cases when Pi 2 appears in the polar caps depends on the level of solar activity and changes during the solar cycle [4]. The changes

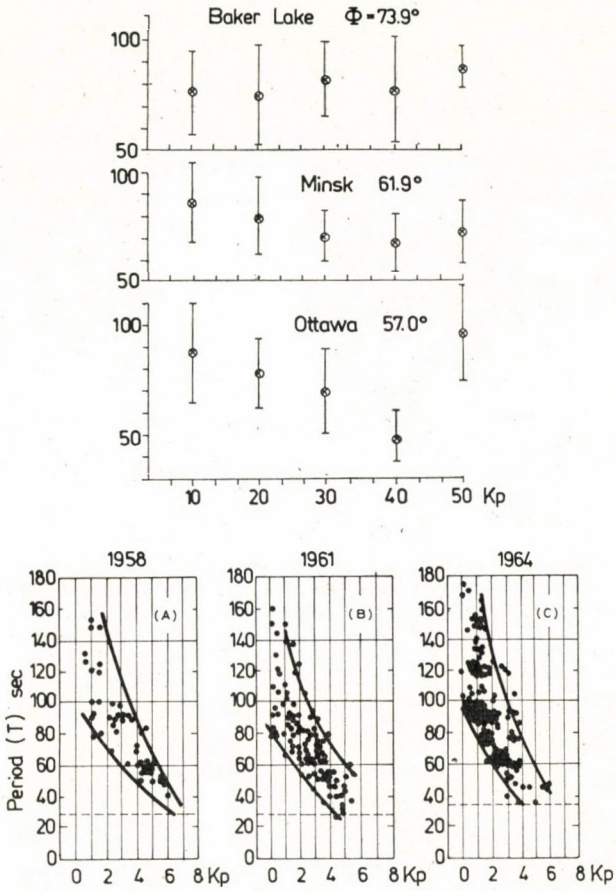


Fig. 33. Dependence of Pi 2 periods on the *Kp*-index: *a)* for the station Borok for several years [57], *b)* for latitudes 57°—73.9°N [44]

of $N_{\text{Pi 1(C)}}(t)$ at different K values have been studied in [62], we do not know of other informations about $N_{\text{Pi 1(C)}}(Kp)$. The greatest number of Pi 1 (C) cases can be observed at Kp 4—6.

b) *Dependence of the period on Kp* ($T_{\text{Pi}}(Kp)$)

According to [3], the period decreases in the following cases: 1) If the amplitude of the associated bay increases (M). The corresponding function is $M = a \cdot T_{\text{Pi 2}}{}^{-4.56}$ where a is a constant which depends on the phase of the solar cycle. 2) If the AE index increases. This connection has the form $AE = 1.0 \cdot 10^{10} \cdot T_{\text{Pi 2}}{}^{-4.08}$. 3) If the Kp value increases. In case of small Kp values around 1, the period is about 120 sec; when Kp is around 3, it decreases

to 60 sec or even less (Fig. 33a, [58]). Some more recent investigations [44] have confirmed this dependence only for stations with latitudes 57—62°, but for stations at 74° the period does not depend on *Kp* (Fig. 33b).

c) *Dependence of the amplitudes* ($A_{\mathrm{Pi}}(Kp)$)

The connection $A_{\mathrm{Pi}}(\Phi)$ for Pi 1(B), Pi 2 and Pi 1(C) shown on Figs 30 and 31 confirms the shift of the maximum $A_{\mathrm{Pi}\ 2}(\Phi)$ between the limits 70 to 63° when the *Q*-index increases. Such a shift has also been found for several individual cases, when Pi 1(B) and Pi 2 appeared several times one after the other with increasing geomagnetic activity. Without considering the great scatter of the individual distributions at high latitudes (reaching 50—60 p.c. in high latitudes) the latitude of the maximum is nearly unambiguously determined by the *Q*-index [46]. It has been shown [40] that $A_{\mathrm{Pi}\ 1(C)}$ is proportional to the amplitude of the bay in the *H*-component.

REFERENCES

1. McPherron, R. L.—Russell, C. T.—Coleman, P. J. Jr.: Fluctuating magnetic fields in the magnetosphere. II. ULF waves. *Space Sci. Revs.*, 13 (1972), 411—454.
2. Saito, T.: Mechanisms of geomagnetic continuous pulsations and physical states of the exosphere. *J. Geomagnet. Geoelect.*, 16 (1964), 115—150.
3. Saito, T.: Geomagnetic pulsations. *Space Sci. Revs.*, 10 (1969), 319—412.
4. ТРОИЦКАЯ, В. А.: Короткопериодные колебания электромагнитного поля Земли в Арктике и Антарктике. Атлас Антарктики II, Изд.-во «Гидромет.», Л., 1969, 188—205.
5. МАТВЕЕВА, Э. Т.—ТРОИЦКАЯ, В. А.: Общие закономерности колебательного режима типа жемчужин. *Геомагн. аэрономия*, 5 (1965), 1078—1084.
6. БОЛЬШАКОВА, О. В.: Устойчивые геомагнитные микропульсации и солнечные корпускулярные потоки. *Геомагн. аэрономия*, 6 (1966), 1089—1093.
7. БОЛЬШАКОВА, О. В.: Микропульсации геомагнитного поля и динамика магнитосферы. *Геомагн. аэрономия*, 5 (1965), 868—873.
8. БАРАНСКИЙ, Л. Н.—ПЛЯСОВА-БАКУНИНА, Т. А.—РАМАНУДЖАГАРИ, К. Р.—СОБОЛЕВ, Л. В.—СЕЛИВАНОВ, В. З.: Распределения интенсивности колебаний Pc 3—4 вдоль геомагнитного меридиана. *Геомагн, аэрономия*, 13 (1973), 1092—1097.
9. Green, A. W.—Hoffman, A. A.: Transient magnetospheric current sheets as generators of Pc 3, Pc 4 and Pi 2 micropulsations. *An. Geophys.*, 26 (1970), 741—749.
10. КОПЫТЕНКО, Ю. А.—РАСПОПОВ, О. М.—ТРОИЦКАЯ, В. А.—ШЛИШ, Р.: Некоторые результаты анализа устойчивых геомагнитных пульсаций типа Pc 4 на сети станций. *Геомагнт. аэроном.* 12 (1972), 720—726.
11. ОЛЬ, А. И.: Долгопериодические гигантские пульсаци геомагнитного поля. Диссерция, Л-д, Арктич. Антарктич. н.—й. ин-т, 1964.
12. Gupta, J. C.—Stening, R. J.: On the diurnal variation of the periods of the Pc 3, 4 micropulsations. *J. Geomagn. and Geoelec.*, 23 (1971), 213—224.
13. Hirasawa, T.: Long-period geomagnetic pulsation (Pc 5) with typical sinusoidal wave forms. *Report of Ionosphere and Space Res. Japan*, 24 (1970), 66—79.
14. КОПЫТЕНКО, Ю. А.—РАСПОПОВ, О. М.—ФАМ ВАН, ЧИ.: Связь устойчивых пульсаций геомагнитного поля с параметрами магнитосферы. *Космическая исследования*, 7 (1969), 266—272.
15. БОЛЬШАКОВА, О. В.—ГУЛЬЕЛЬМИ, А. В.: Проникновение в полярную шапку гидромагнитных волн из хвоста магнитосферы. *Геомагнит. аэрономия*, 12 (1972), 569—571.
16. Troitskaya, V. A.—Bolshakova, C. V.—Hessler, V. P.: Preliminary results of micropulsation studies at magnetoconjugate points in the Arctic and Antarctic. *Ann. Geophys.*, 24 (1968), 741—746.

17. HEACOCK, R. R.—HESSLER, V. P.—OLSSEN, J. K.: The 2—0.1 Hz polar cap micropulsation activity. *Atmosph. Terr. Phys.*, 32 (1970), 129—138.
18. GUPTA, J. C.—STENING, R. J.—VAN BEEK, G.—JANSEN: Micropulsations in the Pc 3 to Pc 4 peroid range at four Canadian observatories, *J. Geoph. Res.*, 76 (1971), 933—946.
19. STENING, R. J.—GUPTA, J. C.: Amplitudes and periods of geomagnetic micropulsations in the Pc 3, 4 range at Canadian observatories. *Geophys. J. Roy. Astron. Soc.*, 23 (1971), 379—386.
20. GUPTA, J. C.: Occurrence studies of geomagnetic micropulsations Pc 5 at high latitudes. *J. Atmosph. and Terr. Phys.*, 35 (1973), 2217—2235.
21. МАТВЕЕВА, Э. Т.: Свойства колебаний Pc I в периоды максимума и минимума солнечной активности. Всб. *Геомагнитные исследования*, 1 (1969), 22—27, изд-во «Наука»,
22. ЛОГИНОВ, Г. А.: Временное распределение колебаний типа Pc I по данным Ловозеро. *Геомагнет. аэрономия*, 6 (1966), 799—801.
23. БАРАНСКИЙ, Л. Н.—БОЛЬШАКОВ, О. В.—ГЕЛЛЕР, Л. А.—КАЗАК, Б. Н.: О широтной зависимости периода и амплитуды устойчивых колебаний Pc 3—4. *Геомагнет, аэроном.*, 9 (1969), 1128—1129.
24. SAITO, T.: Some characteristics of the dynamic spectrum of long period geomagnetic pulsations. *J. Geophys. Res.*, 72 (1967), 3895—3904.
25. SAMSON, J. C.—ROSTOKER, G.: Latitude-dependent characteristics of high-latitude Pc 4 and Pc 5 micropulsations. *J. Geophys. Res.*, 77 (1972), 6133—6144.
26. KENNEY, J. F.—KNAFLICH, H. B.: A systematic study of structured micropulsations. *J. Geophys. Res.*, 72 (1967), 2857—2689.
27. HEACOCK, R. R.: Large amplitude Pc 1 events at College. *J. Geomagnet. Geoelect.*, 20 (1968), 263—269.
28. ВИНОГРАДОВА, В. Н.—ХАРЧЕНКО, И. П.: Изменение интенсивности Pc I на меридиональном профиле. В сб. «Исследов. по геомегнетизму, аэроном. и физ. Солнца», вып. 34, Иркутск, 1974, 74—76.
29. ГУДКОВА, В. А.—РАСПОПОВ, О. М.—ВАНЬЯН, Л. Л.—ГЕЛЛЕР, Л. А.—КОПЫТЕНКО, Ю. А.—ПРИВАЛОВ, В. В.: Закономерности распределения амплитуд геомагнитных пульсаций по меридиональному профилю. *Геомагн. аэрономия*, 11 (1971), 1123—1125.
30. ОБЕРЦ, П.: Исследоваие геомагнитных пульсаций типа Pc 5. *Учебные записки ЛГУ* 346 (1969), 107—126.
31. MCPHERRON, R. L.—WARD, S. H.: Correlation between occurrence of pearl pulsations and interplanetary magnetic field sector boundaries. *J. Geophys. Res.*, 72 (1967), 393—398.
32. ОЛЬ, А. И.: О физической природе долгопериодических пульсаций геомагнитного поля. *Тр. Аркт. и Антаркт. и-й. ин-та*, 280 (1966), 37—47.
33. ОЛЬ, А. И.: О зависимости частоты появления и периода геомагнитных пульсаций Pc 5 от солнечной активности. *Геомагн. исследования*, 11 (1971), 15—19.
34. SAMSON, J. C.—JACOBS, J. A.—ROSTOKER, G.: Latitude-dependent characteristics of long-period geomagnetic micropulsations. *J. Geophys. Res.*, 76 (1971), 3675—3683.
35. HEACOCK, R. R.: Geomagnetic micropulsations at the auroral zone. *Trans. Amer. Geophys. Union*, 67 (1966), 65.
36. HEACOCK, R. R.: Two subtypes of type Pi micropulsations. *J. Geophys. Res.*, 72 (1967), 3095—3917.
37. ТРОИЦКАЯ, В. А.—ГУЛЬЕЛЬМИ, А. В.—ШЕПЕТКОВ, Р. В.: Полярные возмущения и геомагнитные пульсации. В сб. «Солнечно-Земная физика», вып. I, М, 1969, 234—236.
38. БАХУР, Л. В.—ЛЯЦКИЙ, В. Б.: Кратковременные вспышки PP как особый тип геомагнитных пульсаций. *Изв.* АН СССР. *Физ. Земли* 3 (1967), 68—73.
39. HEACOCK, R. R.: Spatial and temporal relations between Pi bursts and IPDP micropulsation events. *J. Geophys. Res.*, 76 (1971), 4494—4504.
40. ЛОГИНОВ, Г. А.: Микроструктура возмущённой солнечно-суточной вариации. *Геомагн. аэрономия*, 7 (1967), 304—309.
41. ТРОИЦКАЯ, В. А.—МАТВЕЕВА, Э. Т.—КАЛИШЕР, А. Л.: Связь возбуждения геомагнитных типов Pi 1 и Pc 1 с магнитносферными суббурями. *Геомагн. аэрономия*, 13 (1973), 755—757.
42. SMITH, B. P.: On the occurrence of Pi 2 micropulsations. *Planet. Space Sci.*, 21 (1973), 831—837.
43. ISIKARA, A.: A study of geomagnetic micropulsations, Pi 2, at Istanbul. *Planet. Space Sci.*, 2 (1971), 137—142.
44. GUPTA, J. C.—STENING, R. J.: Some characteristics of irregular micropulsations Pi 2 at high latitude stations. *Can. J. Phys.*, 49 (1971), 2338—2349.

45. ТРОИЦКАЯ, В. А.: Устойчивые колебания и колебания типа цугов в Арктике и Антарктике. В сб. «Короткопериодические колебания электрического поля Земли» Изд-во АН СССР, М., 1961, 41—61.
46. БАРАНСКИЙ, Л. Н.—ЩЕПЕТНОВ, Р. В.—АФАНАСЬЕВА Л. Т. и др.: Пространственное распределение интенсивности и элементов поляризации пульсаций Pi1. В сб. «Суббури и возмущения в магнитосфере». Изд-во «Наука», Л., 1975, 226—236.
47. HESSLER, V. P.—HEACOCK, R. R.—OLSSEN, I.—SUCKSSDORFL, C.: Micropulsations associated with polar magnetic substorms. Rept. on IVth Gen. ASS, IAGA, Moscow, 1971.
48. МАЛЬЦЕВА, Н. Ф.: Диссертация. Москва, 1971.
49. SAITO, T.—SAKURAI, T.: Mechanism of geomagnetic Pi 2 pulsations in magnetically quiet condition. *Sci. Repts. Tohoku Univ.*, 20 (1970), 49—70.
50. КАЛИШЕР, А. Л.—ТРОИЦКАЯ, В. А.: Длительные иррегулярные пульсации Pi I (авроральнаяь ажитация). *Исследования по геомагнетизму, аэрономии, физике Солнца*, 34 (1974), 77—82.
51. РАСПОПОВ, О. М.—ТРОИЦКАЯ, В. А.—ЖАНДРЕН, Р.—ЛОРАН, Ж.: О поведении микропульсаций типа PiI в магнитосопряжённых точках. *Докл. АН СССР*, 179 (1968), 345—347.
52. БАРАНСКИЙ, Л. Н.—ЩЕПЕТНОВ, Р. В.—АФАНАСЬЕВА, Л. Т. и др.: Распределение интенсивности пульсаций Pi 2 вдоль геомагнитного меридиала и на ночной стороне Земли. *Геомагн. аэроном.*, 14 (1974), 871—875.
53. АФАНАСЬЕВА, Л. Т.—РАСПОПОВ, О. М.—ЩЕПЕТКОВ, Р. В. и др.: О связи геомагнитных пульсаций Pi 2 с параметрами авроральной зоны. *Геомагн. аэроном.*, 10 (1970), 756—758.
54. РАСПОПОВ, О. М.: О возможном механизме возбундения пульсаций геомагнитного поля типа Pi 2. *Геомагн. аэроном.*, 8 (1968), 325—329.
55. DOOBOV, A. L.: Latitude effects on the amplitude of Pi 2 micropulsations. *J. Atmosph. Terr. Phys.*, 35 (1972), 1945—1949.
56. SHERWOOD, V. E.—MACINTOSH, S. M.—STUART, W. F.: The geomagnetic time dependence of Pi 2's and bays. *J. Atmosph. Terr. Phys.*, 35 (1973), 1407—1413.
57. КАЛИШЕР, А. Л.—ПОПОВ, А. Н.—ЧЕРНОУС, С. Н.—СОЛОВЬЕВ, С. И.: Особенности широтного распределения геомагнитных пульсаций типа Pi 1 *Геомагн. аэрономия.* 14 (1974), 328—331.
58. TROITSKAYA, V. A.—GUL'ELMI, A. V.: Geomagnetic micropulsations and diagnostics of the magnetosphere, *Space Sci. Rev.*, 7 (1967), 689—768.
59. SMITH, B. P.: On the occurrence of Pi 2 micropulsations. *Planet. Space Sci.*, 21 (1973), 831—837.
60. STUART, W. F.: A special feature of impulsive pulsations (Pi 2). *J. Atmosph. and Terr. Phys.*, 34 (1972), 829—936.
61. СОЛОВЬЕВ, С. И.—БАРКОВА, Е. С.: Некоторые закономерности геомагнитных пульсаций Pi 1 по наблюдениями на станциях меридионального профиля. В сб. «Верхняя атмосфера высок. широт», вып. 1., Якутск, 1969, 178—183.
62. DOOBOV, A. L.—MAINSTONE, J. S.: Investigations of Pi 2 micropulsations. I. Frequency spectra and polarisation. *Planet. Space Sci.*, 21 (1973), 721—730.
62. АФАНАСЬЕВА, Л. Т.—СЕЛИВАНОВ, В. П.: Геомагнитные пульсации. В сб. «Авроральные явления 70-II», Апатиты, 1974, 38—41.
64. ANNEXSTAD, J. O.—WILSON, C. R.: Characteristics of Pg micropulsations at conjugate points. *J. Geophys. Res.*, 73 (1968), 1805—1818.

ПРОСТРАНСТВЕННО-ВРЕМЕННОЕ РАСПРЕДЕЛЕНИЕ ГЕОМАГНИТНЫХ ПУЛЬСАЦИЙ И ЕГО ЗАВИСИМОСТЬ ОТ ГЕОМАГНИТНОЙ АКТИВНОСТИ

Л. Т. АФАНАСЬЕВА

РЕЗЮМЕ

Изучаются морфологические особенности геомагнитных пульсаций, включая суточную вариацию частоты появления, амплитуды и периода, зависимость от широты частоты встречаемости, амплитуды и периода и их вариацию *Kp*-индексом.

Acta Geodaet., Geophys. et Montanist. Acad. Sci. Hung. Tomus 13 (1–2), pp. 273–285 (1978)

GENAUIGKEITSVERGLEICH DER AM ANFANG ORIENTIERTEN GESTRECKTEN GLEICHSEITIGEN OFFENEN THEODOLITPOLYGONZÜGE MIT GÜNSTIGSTER GEWICHTSVERTEILUNG UND DER GESTRECKTEN KREISELPOLYGONZÜGE

A. TÁRCZY-HORNOCH
MITGLIED DER UNGARISCHEN UND ANDERER AKADEMIEN DER WISSENSCHAFTEN

[Eingegangen am 20. Oktober 1976]

Durch die günstigste Verteilung der Winkelmessungen im Theodolitpolygonzug kann der mittlere Querfehler des Endpunktes bei der gleichen Meßarbeit herabgesetzt werden. Dadurch wird auch der Bereich der günstigeren Verwendung des Theodolitpolygonzuges im Vergleich zum Kreiselpolygonzug vergrößert. Die Grenze n der Punktzahl gibt in diesem Falle die Beziehung (10*a*) an. Die günstigsten Winkelwiederholungszahlen ergeben die Gl. (8).

Man kann die günstigste Gewichtsverteilung der Winkel auch auf solche Theodolitpolygonzüge anwenden, die als Kreiselpolygonzüge fortgesetzt werden. Der die Grenze des Theodolitpolygons angebende günstigste Punkt x kann durch Gl. (12) ermittelt werden, während die Wiederholungszahlen der Winkelmessungen und Kreiselorientierungen durch die Gl. (21*b*) bestimmt werden.

Zum Schluß werden einige Zahlenbeispiele angeführt. Aus diesen geht hervor, daß zur Erreichung eines bestimmten mittleren Querfehlers des Endpunktes ein Theodolitpolygonzug mit günstigster Gewichtsverteilung, der von der durch Gl. (12) bestimmten Stelle mit einem Kreiselpolygonzug fortgesetzt wird, die kleinste Meßarbeitszeit erfordert.

In einer früheren Studie [1] haben wir jene Grenzen untersucht, von wo angefangen die gestreckten offenen Kreiselpolygonzüge günstiger sind als jene offenen Theodolitpolygonzüge, bei denen die Meßzeit der wiederholten Messungen eines Winkels der Meßzeit einer Kreiselorientierung entspricht. Wenn wir den mittleren Fehler einer Winkelmessung mit $\pm m_w$, jenen der Kreiselorientierung mit m_k (beide im Bogenmaß), die durchschnittliche Seitenlänge mit s und das Verhältnis der Meßzeiten t_k und t_w mit f bezeichnen, so werden die mittleren Querfehler des Endpunktes n gleich, bzw. jene des Theodolitpolygonzuges größer, wenn:

$$\frac{|m_w|}{\sqrt{f}} s \sqrt{\frac{n^3}{3}} \geq |m_k| \, s \sqrt{n} . \tag{1}$$

Dabei bedeutet

$$m_{q,n,k} = \pm m_k \cdot s\sqrt{n} \tag{1a}$$

den mittleren Querfehler des Endpunktes n beim Kreiselpolygonzug.

Daraus wird:

$$n \geq \frac{|m_k|}{|m_w|} \sqrt{3f} \tag{2}$$

Von dem der Bedingung der Gleichheit in (2) entsprechenden n-Wert angefangen sind die Kreiselpolygonzüge bereits günstiger.

Nun kann man die Winkel des Theodolitpolygonzuges nicht nur gleich oft messen, sondern die Winkelmessungen auch am günstigsten verteilen. In diesem Falle wird bei gleicher Meßarbeit ein kleinerer mittlerer Querfehler des Endpunktes auftreten und der Theodolitpolygonzug bis zu einem größeren Wert n günstiger sein. Dies soll nun auch mathematisch nachgewiesen werden.

Wenn wir die Wiederholungszahlen der Winkelmessungen im Punkt A der Abb. 1 mit p_A, im Punkt 1 mit p_1 usw. bezeichnen, so wird das Quadrat des mittleren Querfehlers des Endpunktes $m^2_{q,n,t}$ des Theodolitpolygonzuges:

$$m^2_{q,n,t} = m^2_w s^2 \left(\frac{n^2}{p_A} + \frac{(n-1)^2}{p_1} + \ldots + \frac{1^2}{p_{n-1}} \right). \tag{3}$$

Wir bilden daraus in bekannter Weise die *Lagrange*'sche Funktion G, indem dazu $C^2([p] - S)$ addiert wird. Darin bedeutet S hier die Summe der Winkelmessungen und C eine Unbekannte, hier mit der Dimension der Länge. Da $([p] - S) = 0$ ist, folgt daraus (vgl. [2; S. 281—283]:

$$G = m^2_{q,n,t} = m^2_w s^2 \left(\frac{n^2}{p_A} + \frac{(n-1)^2}{p_1} + \ldots + \frac{1^2}{p_{n-1}} \right) + C^2([p] - S). \tag{3a}$$

Um das Minimum von $m^2_{q,n,t}$ als Funktion der Gewichte zu erhalten, differenzieren wir G nach $p_A, p_1 \ldots$ und setzen diese gleich Null, wobei nur die positiven Gewichte eine Bedeutung haben:

$$\begin{aligned} \frac{\partial G}{\partial p_A} &= m^2_w s^2 \left(-\frac{n^2}{p_A^2} \right) + C^2 \qquad \text{daraus:} \quad p_A = \frac{1}{C} |m_w| s \cdot n \\ \frac{\partial G}{\partial p_1} &= m^2_w s^2 \left(\frac{-(n-1)^2}{p_1^2} \right) + C^2 \quad \text{daraus:} \quad p_1 = \frac{1}{C} |m_w| s(n-1) \\ &\vdots \end{aligned} \tag{4}$$

und füglich:

$$p_{n-1} = \frac{1}{C} |m_w| s \cdot 1 .$$

Dann ist die Summe $S = [p]$:

$$S = \frac{1}{C} \left(|m_w| s(n + (n-1) + \ldots + 1) \right) = \frac{1}{C} |m_w| s \left(\frac{n+1}{2} n \right). \tag{5}$$

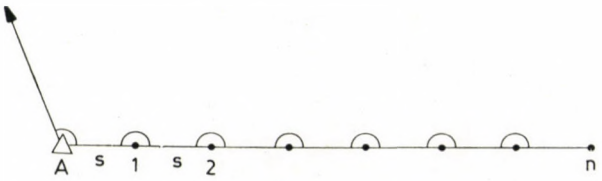


Abb. 1

Will man das herzuleitende Ergebnis mit jenem des Kreiselpolygonzuges mit je einer Kreiselmessung der Seiten vergleichen, so entsprechen n Kreiselmessungen in der Zeit $f \cdot n$ Winkelmessungen. Es ist mithin in Gl. (5)

$$S = f \cdot n \tag{6}$$

zu setzen. Aus Gl. (5) wird

$$f \cdot n = \frac{1}{C} \mid m_w \mid s \, \frac{(1+n)n}{2}$$

und daraus:

$$\frac{1}{C} = \frac{2f}{(1+n) \mid m_w \mid s} \, . \tag{7}$$

Mit diesem Wert werden die Gl. (4):

$$p_A = \frac{2f}{1+n} \, n, \; p_1 = \frac{2f}{1+n} \, (n-1) \ldots, \; p_{n-1} = \frac{2f}{(1+n)} \cdot 1 \, . \tag{8}$$

Durch Einführung dieser Werte in Gl. (3) erhalten wir:

$$m_{q,n,t}^2 = m_w^2 s^2 \, \frac{1+n}{2f} \, (n + (n-1) + \ldots + 1) = m_w^2 s^2 \, \frac{(1+n)^2 n}{4f} \tag{9}$$

und daraus:

$$m_{q,n,t} = \pm \, m_w s \, \frac{(1+n)}{2} \sqrt{\frac{n}{f}} \, . \tag{9a}$$

Wird nun dieser Wert an Stelle der linken Seite der Gl. (1a) eingesetzt, so erhalten wir daraus bei günstigster Gewichtsverteilung jene Punktzahl n des Theodolitpolygonzuges, von wo angefangen der Kreiselpolygonzug günstiger ist. Aus

$$\mid m_w \mid s \, \frac{(1+n)}{2} \sqrt{\frac{n}{f}} \geq \mid m_k \mid s \sqrt{n} \tag{10}$$

folgt:

$$n \geqq 2 \frac{|m_k|}{|m_w|} \sqrt{f} - 1 = \frac{|m_k|}{|m_w|} \sqrt{4f} - 1 . \tag{10a}$$

Ein Vergleich mit der Beziehung (2) zeigt, daß der wesentliche Unterschied zwischen beiden der Wert $\sqrt{4f} - \sqrt{3f}$ ist und daher etwa 16% beträgt. Wenn der nach Gl. (2) ermittelte n-Wert genügend groß ist, kann die Zunahme nach der Beziehung (10a) schon merklich, und die Anwendung der günstigsten Gewichtsverteilung nützlich sein. Dies wird später auch an einem Zahlenbeispiel gezeigt.

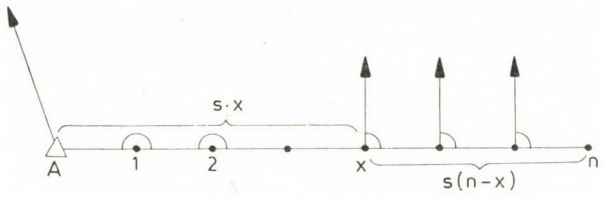


Abb. 2

Man kann die günstigste Gewichtsverteilung auch auf Polygonzüge anwenden, die mit einem Theodolitpolygonzug beginnen und als Kreiselpolygonzüge fortgesetzt werden. Hier ist zunächst zu bestimmen, bis wie weit die Führung des Theodolitpolygonzuges am günstigsten ist. Wenn wir diesen Punkt mit x bezeichnen, so wird nach Abb. 2 in diesem Falle das durch die Theodolitmessungen verursachte mittlere Querfehlerquadrat aus Gl. (9) hier:

$$m_{q,n,t}^2 = m_w^2 s^2 \frac{(1+x)^2 x}{4f},$$

während das durch die Kreiselmessungen verursachte mittlere Querfehlerquadrat des Endpunktes im Sinne der Gl. (1a):

$$m_{q,n,k}^2 = m_k^2 s^2 (n - x)$$

ist. Das mittlere Querfehlerquadrat $m_{q,n}^2$ des Endpunktes erhält man als Summe der beiden:

$$m_{q,n}^2 = s^2 \left(\frac{m_w^2}{4f} (1+x)^2 x + m_k^2 (n - x) \right). \tag{11}$$

Um den günstigsten Wert x zu erhalten, bilden wir den ersten Differentialquotienten von $m_{q,n}^2$ nach x und setzen diesen gleich Null:

$$\frac{dm_{q,n}^2}{dx} = s^2 \left(\frac{m_w^2}{4f} (1 + 4x + 3x^2) - m_k^2 \right) = 0.$$

Aus dem großen Klammerausdruck erhalten wir:

$$3m_w^2x^2 + 4m_w^2x + (m_w^2 - 4fm_k^2) = 0$$

und daraus — unter Berücksichtigung, daß hier nur ein positiver x-Wert einen Sinn hat:

$$x = \frac{-2m_w^2 + \sqrt{m_w^4 + 12fm_w^2m_k^2}}{3m_w^2} . \tag{12}$$

Daß hier ein Minimum vorliegt, zeigt der zweite Differentialquotient. Es sei hier festgestellt, daß x von der Anzahl n der Polygonpunkte unabhängig ist. Wenn der so berechnete x-Wert an Stelle von n in die Gl. (8) eingeführt wird, erhalten wir die einzelnen Wiederholungszahlen der Winkelmessungen im Theodolitpolygonzug-Teil, während im Teil der Kreiselmessungen eine jede Richtung in diesem Falle einmal zu messen ist. Je größer f und im Vergleich zu m_w^2 der Wert m_k^2 ist, um so größer wird x und um so mehr trägt die günstigste Gewichtsverteilung bei der gleichen Meßarbeitszeit zur Genauigkeitsteigerung des mittleren Querfehlers des Endpunktes bei.

Durch die Ergebnisse der bisherigen Untersuchungen können wir den der Meßzeit von $f \cdot n$ Winkelmessungen entsprechenden kleinsten mittleren Querfehler sichern.

Etwas anders wird die Lösung aber, wenn ein erforderlicher mittlerer Querfehler mit der kleinsten Meßzeit, bzw. Meßarbeit erreicht werden soll.

Wenn wir hierzu nur eine Art des Polygonzuges verwenden wollen, so ist zunächst im Sinne der Beziehungen (10) und (10a) zu entscheiden, ob der Kreiselpolygonzug oder der Theodolitpolygonzug günstiger ist.

Wenn der Kreiselpolygonzug der günstigere ist, so kann bei p-maliger Messung der einzelnen Kreiselrichtungen aus Gl. (1a) geschrieben werden:

$$m_{q,n,k}^2 = \frac{m_k^2}{p} s^2 n \tag{13}$$

und daraus die erforderliche Wiederholung p zur Sicherung von $\pm m_{q,n,k}$:

$$p = \frac{m_k^2}{m_{q,n,k}^2} s^2 n . \tag{14}$$

Wenn p keine ganze Zahl ist, so werden die Wiederholungen teils auf-, teils abgerundet, wobei der Betrag der Aufrundungen etwas mehr, als der der Abrundungen sein soll, worauf noch im Zahlenbeispiel hingewiesen wird.

Im Falle, daß der Theodolitpolygonzug der günstigere ist, können wir

von Gl. (3*a*) ausgehen, mit dem Unterschied, daß hier $m^2_{q,n,t}$ gegeben ist und S gesucht wird. Wenn wir die Bezeichnung:

$$|m_w| s \left(\frac{n+1}{2} n\right) = A \tag{15}$$

in Gl. (5) einführen, wobei A aus dem Polygonzug gegeben ist, so wird

$$S = \frac{A}{C}, \quad \text{bzw.} \quad \frac{1}{C} = \frac{S}{A}. \tag{16}$$

Andererseits erhalten wir aus Gl. (3) durch Einführung der Gewichte nach Gl. (4):

$$m^2_{q,n,t} = m^2_w s^2 \left(\frac{n^2 \cdot A}{S\,|\,m_w\,|\,sn} + \frac{(n-1)^2 \cdot A}{S\,|\,m_w\,|\,s(n-1)} + \ldots + \frac{1^2 \cdot A}{S\,|\,m_w\,|\,s \cdot 1}\right) =$$

$$= \frac{A}{S}\left(|\,m_w\,|\,s(n+(n-1)+\ldots+1)\right) = \frac{A}{S}\left(|\,m_w\,|\,s\,\frac{n+1}{2}\,n\right) = \frac{A^2}{S}.$$

Daraus ist bei einem einzuhaltenden mittleren Querfehlerquadrat $m^2_{q,n,t}$ die erforderliche Summe S der Meßarbeit:

$$S = \frac{A^2}{m^2_{q,n,t}}. \tag{17}$$

Damit sind auch nach Gl. (16) der Wert $1/C$ und mit diesem auch die erforderlichen Wiederholungszahlen p nach Gln. (4) berechenbar.

Wenn man einen bestimmten mittleren Querfehler des Endpunktes durch günstigste Gewichtsverteilung in einem, mit Theodolitpolygonzug beginnenden und mit Kreiselpolygonzug fortgesetzten Zug bestimmen will, so kann man wie folgt vorgehen. Nach Bestimmung von x durch Gl. (12) wird hier die *Lagrange*'sche Funktion mit Berücksichtigung, daß bei den Kreiselmessungen die Wiederholung bei allen Seiten dieselbe verkleinernde Wirkung und dasselbe Gewicht p_k hat, die folgende Form annehmen:

$$G = m^2_{q,n} = s^2\left\{m^2_w\left(\frac{x^2}{p_A} + \frac{(x-1)^2}{p_1} + \ldots + \frac{1^2}{p_{x-1}}\right) + (n-x)\frac{m_k^2}{p_k}\right\} + \\ + C^2(p_A + p_1 + \ldots + p_{x-1} + (n-x) \cdot f \cdot p_k - S). \tag{18}$$

Das Gewicht p_k wird deshalb mit f multipliziert, da die Zeit einer Kreiselorientierung f-mal größer ist, als die einer Winkelmessung. Es wird mithin

die Summe S der Meßzeiten in der Einheit einer Winkelmessung:

$$S = p_A + p_1 + \ldots + p_{x-1} + (n-x)f \cdot p_k \,. \tag{19}$$

Wir können noch auch m_k in mittleren Fehlern m_w der Winkelmessungen ausdrücken:

Wenn

$$\frac{m_w^2}{m_k^2} = q_k \text{ ist, so wird } m_k^2 = \frac{m_w^2}{q_k} \,. \tag{20}$$

Mithin erhalten wir aus Gl. (18):

$$\begin{aligned} G = m_{q,n}^2 = s^2 \left\{ m_w^2 \left(\frac{x^2}{p_A} + \frac{(x-1)^2}{p_1} + \ldots + \frac{1^2}{p_{x-1}} + \frac{n-x}{p_k q_k} \right) \right\} + \\ + C^2 (p_A + p_1 + \ldots + p_{x-1} + (n-x) f p_k - S) \,. \end{aligned} \tag{18a}$$

Es werden hier an Stelle der Gl. (4) sinngemäß:

$$\begin{aligned} \frac{\partial G}{\partial p_A} &= m_w^2 s^2 \left(-\frac{x^2}{p_A^2} \right) + C^2, \qquad \text{daraus:} \quad p_A = \frac{1}{C} |m_w| sx \\ \frac{\partial G}{\partial p_1} &= m_w^2 s^2 \left(-\frac{(x-1)^2}{p_1^2} \right) + C^2, \quad \text{daraus:} \quad p_1 = \frac{1}{C} |m_w| s(x-1) \\ &\vdots \\ \frac{\partial G}{\partial p_{x-1}} &= m_w^2 s^2 \left(-\frac{1}{p_{x-1}^2} \right) + C^2, \qquad \text{daraus:} \quad p_{x-1} = \frac{1}{C} |m_w| s \cdot 1 \\ \frac{\partial G}{\partial p_k} &= m_w^2 s^2 \left(-\frac{n-x}{q_k p_k^2} \right) + C^2 (n-x) f, \text{ daraus: } p_k = \frac{1}{C} |m_w| s \frac{1}{\sqrt{f q_k}} \,. \end{aligned} \tag{21}$$

Wenn wir diese Werte in die Gl. (19) einsetzen, so erhalten wir:

$$S = \frac{1}{C} \left\{ |m_w| s \left(x + (x-1) + \ldots + 1 + (n-x) \sqrt{\frac{f}{q_k}} \right) \right\} = \frac{A}{C} \,. \tag{21a}$$

Dabei bedeutet A eine in den geschlungenen Klammern stehende gegebene Größe. Also:

$$\begin{aligned} |m_w| s \left(x + (x-1) + \ldots + 1 + (n-x) \sqrt{\frac{f}{q_k}} \right) = \\ = |m_w| s \left(\frac{x+1}{2} x + (n-x) \sqrt{\frac{f}{q_k}} \right) = A \,. \end{aligned} \tag{22}$$

Daraus wird, wie in Gl. (16):

$$\frac{1}{C} = \frac{S}{A} .$$

Setzt man diesen Wert in die Gl. (21) ein, so werden die Wiederholungszahlen p:

$$p_A = \frac{S}{A} \mid m_w \mid sx , \quad p_1 = \frac{S}{A} \mid m_w \mid s(x-1) \ldots , p_k = \frac{S}{A} \mid m_w \mid s \frac{1}{\sqrt{f q_k}} . \qquad (21b)$$

Hier ist das mittlere Querfehlerquadrat des Endpunktes aus (vgl. Gl. (18a)):

$$m_{q,n}^2 = s^2 m_w^2 \left\{ \frac{x^2}{p_A} + \frac{(x-1)^2}{p_1} + \ldots + \frac{1^2}{p_{x-1}} + \frac{n-x}{p_k q_k} \right\} ,$$

nach Einsetzen der Gewichte nach Gl. (21b) aber:

$$m_{q,n}^2 = \frac{A}{S} \left\{ \mid m_w \mid s \left[x + (x-1) + \ldots + 1 + (n-x) \sqrt{\frac{f}{q_k}} \right] \right\} = \frac{A^2}{S} .$$

Daraus wird die letzte Unbekannte, ähnlich, wie in Gl. (17):

$$S = \frac{A^2}{m_{q,n}^2} . \qquad (17a)$$

Jetzt können alle Wiederholungszahlen nach Gl. (21b) berechnet werden. Infolge Umrechnung der Kreiselmessungen liefert p_k unmittelbar die Wiederholungszahlen der Kreiselorientierungen der einzelnen Seiten. Die nicht ganzzahligen Gewichte sind auch hier entsprechend auf- bzw. abzurunden.

Auf die Untersuchung weiterer Fragen wollen wir hier nicht eingehen.

*

Es sollen nun einige Zahlenbeispiele angeführt werden, wobei $s = 100$ m, $m_w = \pm 3''$, $m_k = \pm 12''$ und $f = 7$ seien. Im folgenden werden m_w und m_k Werte im Bogenmaß bedeuten. In diesem Falle ist

$$\frac{s}{\varrho} = \frac{100\,000}{206\,265} = 0{,}485 \text{ mm} .$$

Nach der Beziehung (10a) wird der Kreiselpolygonzug von

$$n = \frac{12''/\varrho}{3''/\varrho} \sqrt{28} - 1 = 20{,}16 \doteq 20$$

angefangen günstiger als der Theodolitpolygonzug mit günstigster Gewichtsverteilung. Es wird z. B. bei einem Theodolitpolygonzug mit $n = 25$ Seiten nach Gl. (9*a*):

$$m_{q,n,t} = \pm 35{,}75 \text{ mm}.$$

während bei einem Kreiselpolygonzug nach Gl. (1*a*)

$$m_{q,n,k} = 0{,}485 \cdot 12 \cdot 5 = \pm 29{,}10 \text{ mm}$$

ist; in beiden Fällen bei der gleichen Meßarbeitszeit! Bei $n = 20$ sind noch beide mittleren Fehler annähernd gleich: $m_{q,n,t} = \pm 25{,}83$ mm und $m_{q,n,k} = \pm 26{,}01$ mm. Der kleine Unterschied liegt darin, daß wir n ein wenig abgerundet haben.

Wenn wir beim Kreiselpolygonzug mit $n = 25$ einen mittleren Fehler des Endpunktes von ± 16 mm sichern wollen, so ist die Wiederholungszahl p der Kreiselmessungen:

$$p = \frac{0{,}485^2 \cdot 12^2 \cdot 25}{16^2} = 3{,}18 \doteq 3 .$$

Mit diesem Gewicht liefert Gl. (13):

$$m_{q,n,k} = \frac{\pm 0{,}485 \cdot 12 \cdot \sqrt{25}}{\sqrt{3}} = \pm 16{,}81 \text{ mm} .$$

Die kleine Vergrößerung von $m_{q,n,k}$ kommt von der Abrundung von p. Sollte dies stören, so können je nach dem Unterschied 1—2 Seiten viermal gemessen werden. Bei $p = 3$ würden diese Messungen $25 \cdot 7 \cdot 3 = 525$ Winkelmessungen entsprechen.

Im Falle, daß wir mit Theodolitpolygonzug bei günstigster Gewichtsverteilung den Wert $m_{q,n,t} = \pm 16$ mm sichern wollen, so werden aus Gl. (15):

$$A = 0{,}485 \cdot 3 \frac{26}{2} 25 = 472{,}87$$

und aus Gl. (17):

$$S = \frac{472{,}87^2}{16^2} \doteq 29{,}5^2 \doteq 870 .$$

also fast doppelt so viele Winkelmessungen. Bei Berechnung der einzelnen Wiederholungszahlen können durch Rundungen kleinere, unwesentliche Abweichungen entstehen.

Noch mehr Winkelmessungen sind bei gleich genauen Messungen der Polygonwinkel. Aus der Beziehung (Vgl. [3; S. 596]):

$$m_{q,n,t} = \pm m_w \cdot s \sqrt{\frac{n^3}{3}} = \pm 0{,}485 \cdot 3 \sqrt{\frac{25^3}{3}} = \pm 105{,}00 \text{ mm}.$$

Um den mittleren Fehler $m_{q,n,t} = \pm 16$ mm zu erhalten ist ein jeder Winkel

$$p = \frac{105^2}{16^2} = 6{,}5^2 = 42{,}25$$

Mal zu messen. Dies gibt zusammen $25 \cdot 42{,}25 = 1056$ Winkelmessungen!

Wenn der Polygonzug nur aus 18 Seiten besteht, so ist bereits der Theodolitpolygonzug mit günstigster Gewichtsverteilung vorteilhafter. Es wird in diesem Falle nach Gl. (15):

$$A = 0{,}485 \cdot 3 \frac{19}{2} 18 = 248{,}80$$

und aus Gl. (17):

$$S = \frac{248{,}8^2}{16^2} = 241{,}80 \doteq 242.$$

Bei gleich genauen Messungen der Winkel wird:

$$m_{q,n,t} = \pm 0{,}485 \cdot 3 \sqrt{\frac{18^3}{3}} = \pm 64{,}15 \text{ mm}$$

und somit

$$p = \frac{64{,}15^2}{16^2} = 16{,}08.$$

Bei dieser Wiederholungszahl der Winkelmessungen erhalten wir bei 18 Winkeln $16{,}08 \cdot 18 = 288{,}44 \doteq 288$ Winkelmessungen. Beim Kreiselpolygonzug wird p nach Gl. (14):

$$p = \frac{0{,}485^2 \cdot 12^2 \cdot 18}{16^2} = 2{,}38.$$

Bei 18 Seiten entsprechen diesem Wert $2{,}38 \cdot 18 \cdot 7 = 298{,}88 \doteq 299$ Winkelmessungen.

Wir wollen nun den Polygonzug mit $n = 25$ Punkten mit einem Theodolitpolygonzug beginnen und mit dem Kreiselpolygonzug fortsetzen. Nach

Gl. (12) wird mit guter Näherung:

$$x = \frac{-18 + \sqrt{81 + 108\,864}}{27} = = 11{,}55 \doteq 12.$$

d. h. wir führen den Theodolitpolygonzug bis zum Punkt 12. Nach Gl. (22) wird in diesem Falle A:

$$A = 0{,}485 \cdot 3 \left(\frac{12+1}{2} \cdot 12 + 13 \sqrt{7 \cdot 16} \right) = 313{,}64$$

und daher S nach Gl. (17a):

$$S = \frac{313{,}64^2}{16^2} = 384{,}16 \doteq 384.$$

Mit diesem Wert können die einzelnen Gewichte nach Gl. (21b) berechnet werden. Da hier

$$\frac{S}{A} = 1{,}225 \quad \text{und} \quad \frac{S}{A} \mid m_w \mid s = 1{,}782$$

ist, werden die Wiederholungszahlen der Winkelmessungen:

$$p_A = 1{,}782 \cdot 12 = 21{,}38 \doteq 21 \qquad p_1 = 1{,}782 \cdot 11 = 19{,}60 \doteq 20$$

$$p_2 = 17{,}82 \doteq 18, \qquad p_3 = 16{,}04 \doteq 16, \qquad p_4 = 14{,}26 \doteq 14,$$

$$p_5 = 12{,}47 \doteq 12, \qquad p_6 = 10{,}69 \doteq 11, \qquad p_7 = 8{,}91 \doteq 9,$$

$$p_8 = 7{,}13 \doteq 7, \qquad p_9 = 5{,}35 \doteq 5, \qquad p_{10} = 3{,}56 \doteq 4,$$

$$p_{11} = 1{,}78 \doteq 2 = p_{x-1},$$

zusammen 139 Winkelmessungen.

Alle mit dem Kreisel orientierten Seiten haben grundsätzlich dieselbe Widerholungszahl $p_k = 1{,}782 \sqrt{16/7} = 2{,}694$. Hier braucht man aber nicht alle aufwärts runden, sondern es können mit Rücksicht auf die günstigste Fehlerfortpflanzung einige auch abwärts gerundet werden. Dabei ist allerdings zu bemerken, daß die abgerundeten Werte mit einem kleineren p_k zu dividieren sind als die aufgerundeten, so daß die Aufrundungen größer sein sollen als die Abrundungen. Wir werden daher etwa 3 Seiten zweifach und 10 Seiten dreifach orientieren. Die zweifach orientierten drei Seiten erfordern die Meßzeit von $3 \cdot 2 \cdot 7 = 42$ Winkelmessungen, während die dreifache Orientierung von zehn Seiten die Zeit von $10 \cdot 3 \cdot 7 = 210$ Winkelmessungen braucht. Es ist

mithin infolge der Rundungen die Arbeitszeit von $139 + 42 + 210 = 391$ Winkelmessungen gegenüber dem theoretischen Wert von 384 erforderlich.

Zur Kontrolle berechnen wir im Sinne der Gl. (18) den Wert $\pm m_{q,n}$:

$$m_{q,n} = 0{,}485 \sqrt{3^2\left(\frac{12^2}{21}+\frac{11^2}{20}+\frac{10^2}{18}+\frac{9^2}{16}+\frac{8^2}{14}+\frac{7^2}{12}+\frac{6^2}{11}+\frac{5^2}{9}+\frac{4^2}{7}+\frac{3^2}{5}+\frac{2^2}{4}+\frac{1^2}{2}\right)+12^2\left(\frac{3}{2}+\frac{10}{3}\right)} \doteq$$

$$= 0{,}485\sqrt{9 \cdot 43{,}80 + 144 \cdot 4{,}83} = \pm 0{,}485 \cdot 33{,}01 \doteq 16{,}009 \text{ mm}.$$

Der angestrebte mittlere Querfehler $m_{q,n} = \pm 16$ mm wurde sehr genau erreicht.

Weil bei $n = 25$ dieser Wert die Bedingung (10*a*) erfüllt, ist die alleinige Verwendung des Kreiselpolygonzuges günstiger, als die alleinige Verwendung des Theodolitpolygonzuges. Es fragt sich nun, wieviel Winkelmessungen entsprechen den, den Wert $m_{q,n} = \pm 16$ mm sichernden Kreiselorientierungen. Nach Gl. (1*a*) wird $m_{q,n,k}$ bei einmaliger Kreiselorientierung der Seiten und $n = 25$ Seiten, wie dort angegeben, $\pm 29{,}1$ mm. Um diesen Wert auf ± 16 mm herabzusetzen, sind

$$p = \frac{29{,}1^2}{16^2} \doteq 3{,}31$$

Messungen der Orientierungen der einzelnen Seiten erforderlich. Bei 25 Seiten, und wenn eine Kreiselorientierung die Meßzeit von 7 Winkelmessungen braucht, entspricht dies $3{,}31 \cdot 25 \cdot 7 = 579{,}2 \doteq 579$ Winkelmessungen. Dieser Wert ist um 195 Winkelmessungen größer, als der vorherige Wert 384; die Verbindung des Theodolitpolygonzuges mit günstigster Gewichtsverteilung und des Kreiselpolygonzuges kann also zu wesentlicher Arbeitsersparnis führen. (Wir behielten $p = 3{,}31$ für die Berechnung; praktisch wird ein Teil ab-, ein Teil aufgerundet, wie bereits gezeigt wurde).

Bei gleichgenauen Winkelmessungen wird bei $x = 12$ und $n = 25$:

$$m_{q,n} = s\sqrt{\frac{m_w^2}{f}\frac{x^3}{3} + m_k^2(n-x)} =$$

$$= 0{,}485\sqrt{\frac{9}{7}\frac{1728}{3} + 144 \cdot 13} = \pm 24{,}79 \text{ mm}.$$

Um ± 16 mm zu erreichen, ist die Wiederholungszahl p:

$$p = \frac{24{,}79^2}{16^2} \doteq 2{,}40, \text{ mithin } S = 25 \cdot 7 \cdot 2{,}40 = 420.$$

Im Vergleich zu 384 Winkelmessungen bedeutet dies 36 Winkelmeßzeiten, also rund 10% mehr. Der Prozentsatz vergrößert sich mit der Verkleinerung von $n - x$; z. B. ist bei $x = 12$ und $n - x = 2$ die Ersparnis bei der günstigsten Gewichtsverteilung mehr als 14%, so daß diese ihre Berechtigung hat.

Wir ersehen jedenfalls, daß ein Theodolitpolygonzug mit günstigster Gewichtsverteilung, der mit einem Kreiselpolygonzug fortgesetzt wird, zur Erreichung eines mittleren Querfehlers des Endpunktes die kleinste Meßarbeit erfordert, vorausgesetzt, daß die Grenze zwischen beiden im Sinne der Gl. (12) gewählt wurde. Diese Kombination verdient daher Beachtung. Mit guter Näherung gilt dies auch für etwas gebogene Polygonzüge mit nicht ganz gleichen Seitenlängen, auf die wir hier nicht näher eingehen wollen.

SCHRIFTTUM

1. Tárczy-Hornoch, A.: Genauigkeitsvergleiche der am Anfang orientierten gestreckten gleichseitigen offenen Theodolitpolygonzüge und der Kreiselpolygonzüge bei gleicher Gesamtarbeitszeit und bei gleicher Wiederholungszahl der Winkelmessungen im Theodolitpolygonzug. *Österr. Zeitschr. f. Vermessungsw. u. Photogr.*, 64 (1977), 105—111.
2. Tárczy-Hornoch, A.: Kiegyenlítő számítás (Ausgleichungsrechnung, III. Aufl.), Budapest (1956).
3. Jordan—Eggert—Berroth: Handbuch der Vermessungskunde, Bd. II. (10. Aufl.) Stuttgart (1950).

СОПОСТАВЛЕНИЕ ТОЧНОСТИ РАВНОСТОРОННИХ ОТКРЫТЫХ ТЕОДОЛИТНЫХ ХОДОВ С ВЫГОДНЫМ РАСПРЕДЕЛЕНИЕМ ВЕСОВ И ОРИЕНТИРОВАННЫХ В НАЧАЛЬНОЙ ТОЧКЕ ХОДА С ТОЧНОСТЬЮ ХОДОВ, ОРИЕНТИРОВАННЫХ ГИРОТЕОДОЛИТАМИ

А. ТАРЦИ-ХОРНОХ.

РЕЗЮМЕ

Выгодным распределением угловых измерений в теодолитных ходах при одинаковых затратах измерительных работ может быть уменьшена средняя квадратическая поперечная ошибка конечной точки хода. Этим, по сравнению с гиротеодолитными ходами, увеличивается область выгодного применения теодолитных ходов. Предел количества точек n задается в этом случае через соотношение (10*a*). Выгоднейшее количество повторных угловых измерений можно получить из уравнения (8).

Наивыгоднейшее распределение весов для теодолитных ходов, продолжающихся как гиротеодолитные, определяется уравнениями (21*a*). Наивыгоднейшая длина сторон теодолитных ходов может быть вычислена при помощи формулы (12).

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THEORETISCHE GRUNDLAGEN ZUR AUSWAHL DES SCHACHTANSATZPUNKTES

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MITGLIED DER UNGARISCHEN AKADEMIE DER WISSENSCHAFTEN

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Der Ansatzpunkt von Schächten kann dann als optimal betrachtet werden, wenn die mit dem Ansatzpunkt zusammenhängenden Kosten ein Minimum ergeben. Mit dem Ansatzpunkt sind in erster Linie die Kosten für Förderung und Fahrung, sowie jener Wertverlust verbunden, der aus dem im Schachtsicherheitspfeiler gebundenen, nicht gewinnbaren Nutzmineralvorrat erwächst. Die praktische Lösung dieser Aufgaben erfordert das Kennen und Erkennen der grundlegenden theoretischen Beziehungen. Vom Verfasser werden die wichtigsten Beziehungen abgeleitet und die auf diesen beruhende praktische Lösung vorgezeigt.

Dem Anlegen neuer Bergwerksbetriebe geht im allgemeinen das Kennenlernen des Mineralvorkommens durch Tiefbohrungen voraus. Aus den Ergebnissen dieser Bohrungen kann die wahrscheinliche Menge, Qualität und räumliche Lagerung des Vorkommens festgestellt werden.

Beim Anlegen von Bergwerksbetrieben sind mehrere wichtige Parameter zu bestimmen. Ein solcher wichtiger Parameter ist bei Tiefbaugruben der Schachtansatzpunkt, oder Ansatzpunkte. Je nach dem Ansatzpunkt des Schachtes oder der Schächte ändert sich nämlich die beim Abbau des Vorkommens erwachsende Menge an Förderung und Fahrung. Unter »Förderung« ist hier das Bewegen der Nutzprodukte, Berge, sonstigen Betriebsmaterials, Luft, Wasser, u. dgl., unter »Fahrung« aber die Bewegung der Menschen in der Grube zu verstehen. Ändert sich die Menge an Förderung und Fahrung, so ändern sich damit auch die Kosten. Als Funktion des Schachtansatzpunktes verändern sich auch die Anlagekosten. Zum Ausgangspunkt sei angenommen, daß der Ansatzpunkt des Schachtes oder der Schächte dann optimal gewählt ist, wenn alle vom Ansatzpunkt abhängigen Kosten einen Mindestwert betragen.

Zunächst seien die wichtigsten theoretischen Zusammenhänge dargelegt.

Nach Abb. 1 sind in der Ebene eine Anzahl n von Punkten $(1, 2, \ldots \ldots, i, \ldots, n)$ gegeben, zu denen die Gewichte $(Q_1, Q_2, \ldots, Q_i, \ldots, Q_n)$ gehören. Diese Gewichte werden über gerade Strecken im Punkt P gesammelt, wobei die spezifischen Kosten der Förderung $(k_1, k_2, \ldots, k_i, \ldots, k_n)$ gegeben sind. Das Gewicht $\sum_{i=1}^{n} Q_i = Q$ soll aus dem Punkt P in den Punkt R bewegt werden,

wobei mit k_R auch die zu diesem Förderweg gehörigen spezifischen Förderkosten gegeben sind.

Somit kann die Beziehung für die ganzen Förderkosten wie folgt aufgeschrieben werden:

$$K = \sum_{i=1}^{n} Q_i s_i k_i + Q s_R k_R \,.$$

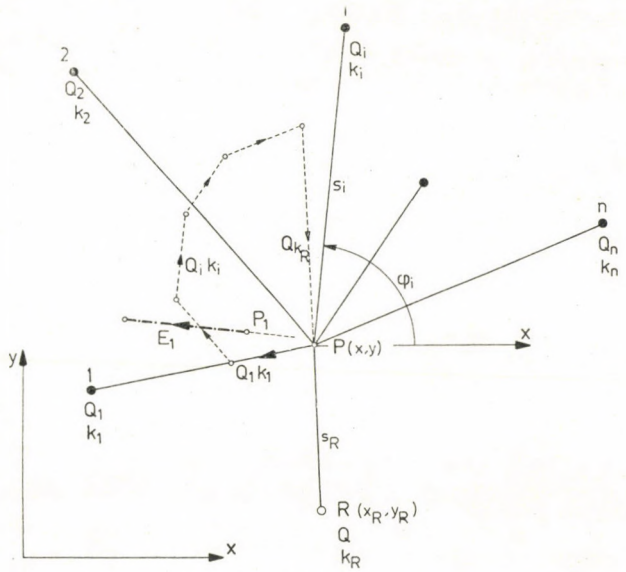

Abb. 1

Auf einfache Weise läßt sich nachweisen, daß der Punkt P den Ort des Kostenminimums bezeichnet, für den die folgenden Beziehungen gelten

$$\sum_{i=1}^{n} Q_i k_i \cos\varphi_i + Q k_R \cos\varphi_R = 0$$

$$\sum_{i=1}^{n} Q_i k_i \sin\varphi_i + Q k_R \sin\varphi_R = 0 \,.$$

Jener Punkt P bezeichnet das Kostenminimum, für den die Resultierende des in dem Punkt angreifenden Vektorsystems gleich Null ist, wobei die Skalarwerte der Vektoren $Q_1k_1, Q_2k_2, \ldots, Q_ik_i, \ldots, Q_nk_n$; Qk_R betragen; ferner die Richtung der Vektoren durch den gegebenen Punkt der Vektoren aber auf den gesuchten Punkt hinweist.

Es ist leicht einzusehen, daß der gesuchte Punkt P auf den Punkt R entfällt, wenn der Skalarwert der Resultierenden des im Punkt R angreifenden Vektorsystems $(Q_1k_1, Q_2k_2, \ldots, Q_ik_i, \ldots, Q_nk_n)$ gleich oder kleiner als Qk_R ist.

Ebenso ist es auch klar, daß der gesuchte Punkt auf irgendeinen Punkt (j) der n Punkte entfallen kann, wenn der Skalarwert der Resultierenden des in diesem Punkt angreifenden Vektorsystems ($Q_1k_1, Q_2k_2, \ldots, Q_ik_i, \ldots, Q_{n-1}k_{n-1}$; Qk_R) gleich oder kleiner als der zu diesem Punkt gehörige Skalarwert Q_jk_j ist.

Die Funktion $K = K(s_1, s_2, \ldots, s_n; s_k)$ ist nämlich eine Kontinuitätsfunktion, jedoch erscheinen an den einzelnen Punkten Spitzen auf der die Funktion darstellenden Fläche.

Klar ist ferner, daß der Punkt P nur mit dem Iterationsverfahren gesucht werden kann. Da es sich hier um ein Entwerfen handelt, kann das Iterationsverfahren zeichnerisch durchgeführt werden. Der erste Punkt des Iterationsverfahrens sei P_1; die dazugehörige Resultierende E_1. Der nächste Punkt wird in der Richtung von E_1 angenommen, doch in einem E_1 entgegengesetzten Sinne.

Die spezifischen Kosten wurden bisher als konstant betrachtet. Es kann jedoch sein, daß auch die spezifischen Kosten Funktionen der Entfernung sind, d. h. $k = k(s)$. Das Iterationsverfahren gibt die Möglichkeit, auch eine solche Abhängigkeit in Betracht zu ziehen.

Nach Abb. 2 ist in der Ebene eine gekrümmte Linie $y = y(x)$ gegeben, auf dieser eine Anzahl n von Punkten, wie auch die zu den Punkten gehörigen Gewichte.

Außerhalb der Kurve sind m Punkte samt den ihnen zugeordneten Gewichten gegeben. Die spezifischen Kosten seien auch hier mit k bezeichnet. Zu suchen ist jener Punkt P der Kurve, zu dem das Kostenminimum des

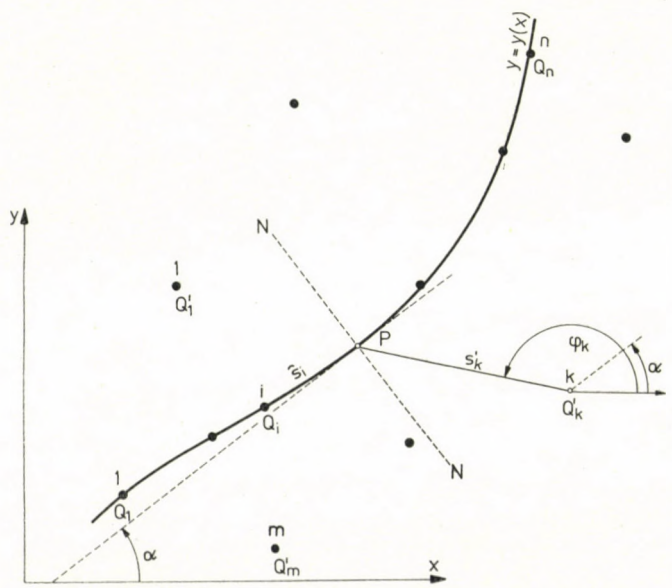

Abb. 2

Sammelns gehört, wenn die auf der Kurve befindlichen Gewichte sich auf der Kurve bewegen, die außengelegenen Gewichte aber über gerade Linien in den Punkt $P[x, y(x)]$ gelangen.

Die vollen Kosten erhält man aus:

$$K = \sum_{i=1}^{n} Q_i \, \widehat{s_i} \, k_i + \sum_{k=1}^{m} Q'_k \, s'_k \, k'_k ,$$

worin

$$\widehat{s_i} = \int_{x_i}^{x} \sqrt{1 + [y'(x)]^2 \, dx}$$

$$s'_k = \sqrt{(x - x_k)^2 + [y(x) - y_k]^2}$$

und

$$y'(x) = \operatorname{tg} \alpha$$

ist.

Nach erfolgter Substituierung kann nachgewiesen werden, daß K ein Minimum hat, wenn die Beziehung

$$\frac{1}{\cos \alpha} \left(\sum_{j=1}^{i} Q_j \, k_j - \sum_{l=i+1}^{n} Q_l \, k_l \right) + \sum_{k=1}^{m} Q'_k \, k'_k (\cos \varphi_k + \sin \varphi_k \operatorname{tg} \alpha) = 0$$

erfüllt ist, d. h. wenn

$$\sum_{i=1}^{n} Q_i \, k_i + \sum_{k=1}^{m} Q'_k \, k'_k \cos (\varphi_k - \alpha) = 0$$

ist, worin der Sinn von Q_k rechts und links von Punkt P entgegengesetzt ist.

Jener Kurvenpunkt P bezeichnet das Kostenminimum, bei dem die Resultierende des im Punkt P angreifenden Vektorsystems auf die zu Punkt P gehörige Normale ($N - N$) entfällt, wobei die Skalarwerte der Vektoren im allgemeinen $Q_i k_i$, bzw. $Q'_k k'_k$ sind, die Richtung der sich auf der Kurve bewegenden Vektoren immer gleich der Richtung der Tangenten ist; die Richtung der zu den außengelegenen Punkten gehörigen Vektoren immer von dem gegebenen und dem gesuchten Punkt bestimmt wird; der Sinn der Vektoren aber vom gegebenen Punkt zu dem gesuchten Punkt hinweist.

Wenn mehrere Punkte der Kurve die Bedingung erfüllen, so gehören die Minimalkosten zu jenem Punkt, bei dem die Resultierende am kleinsten ist.

Der gesuchte Punkt kann auch irgendein gegebener Punkt der Kurve sein, wenn die auf die Tangente entfallende Projektion der Resultierenden der übrigen, im Punkte angreifenden Vektoren skalarisch gleich oder kleiner als der Skalarwert des zum betreffenden Punkt gehörigen Vektors ist.

In Abb. 3 wird ein Zahlenbeispiel vorgezeigt. Auf der Kurve $y = y(x)$ sind drei Punkte, außerhalb der Kurve aber zwei Punkte gegeben. An den Punkten auf der Kurve ist $Q_1k_1 = 4{,}6$; $Q_2k_2 = 2{,}8$; $Q_3k_3 = 4{,}5$, während für die Punkte außerhalb der Kurve $Q_1'k_1' = 1{,}9$; $Q_2'k_2' = 3{,}9$ ist. Der Punkt P wurde dann richtig gewählt, wenn der einen Skalarwert $2{,}8 + 4{,}5 - 4{,}6 = 2{,}7$ besitzende, tangential wirkende Vektor, ferner die Resultierende (E) der nach Sinn und Richtung durch die beiden Außenpunkte und den Punkt P bestimmten Vektoren von 1,9 und 3,9 Skalarwert auf die zum Punkt P gehörige Normale $(N - N)$ entfällt. Der Punkt P kann nur mit Hilfe eines Iterationsverfahrens ermittelt werden.

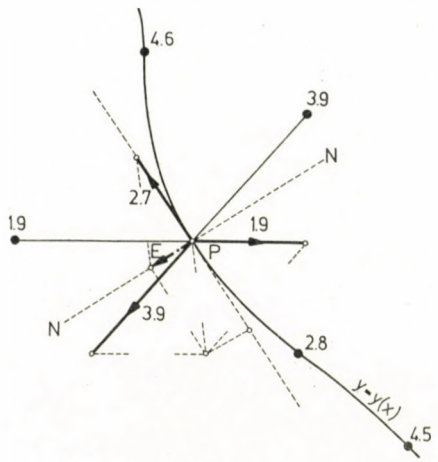

Abb. 3

Abb. 4 zeigt eine grundlegende Aufgabe. Gegeben ist in der Ebene eine gekrümmte Linie $y = y(x)$; zwischen zwei ihrer Punkte liegt im Bogenabschnitt $\widehat{AB}$ das Gewicht Q_0 an der Einheitsbogenlänge. Es sei $\widehat{AB} \cdot Q = Q$. Gleichfalls gegeben ist auch ein Punkt $R[x_R, y_R]$. Die Aufgabe lautet nun wie folgt: Die auf den Bogenabschnitt $\widehat{AB}$ gleichmäßig verteilte Gewichtsanhäufung soll durch Fortbewegung auf dem Bogenabschnitt selbst in einem Punkt $(P[x, y])$ gesammelt, und dann von Punkt P auf einer Geraden bis zum Punkt R gefördert werden, wobei die spezifischen Förderkosten auf dem Bogenabschnitt $\widehat{k}$, auf dem geraden Abschnitt $\overline{PR}$ aber k_R betragen.

Der Halbierungspunkt des Bogenabschnittes $\widehat{AB}$ sei $P_0[x_0, y_0]$, d. h. $\widehat{AP_0} = \widehat{P_0B} = \widehat{s}_0$. Die vollen Kosten der Förderung sind:

$$K = Q_0(\widehat{s}_0 + \widehat{s^2})\,\widehat{k} + Qs_Rk_R\,,$$

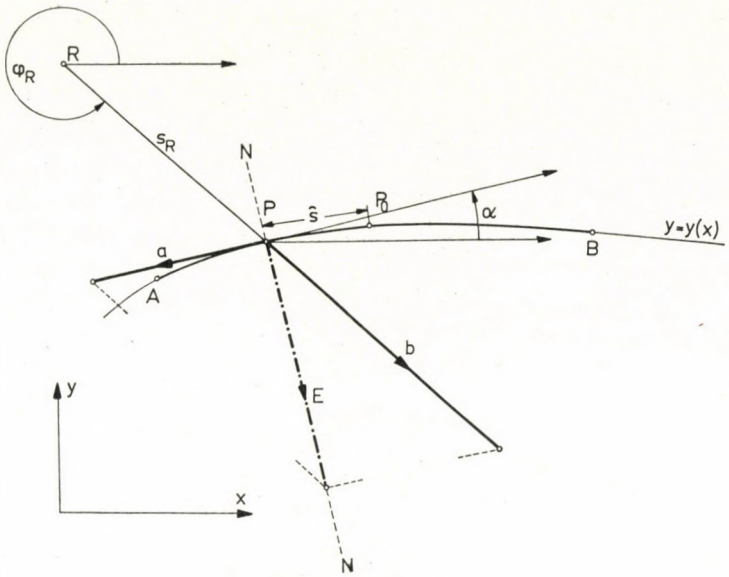

Abb. 4

worin

$$\hat{s} = \int_{x_0}^{x} \sqrt{1 + [y'(x)]^2}\, dx$$

$$s_R = \sqrt{(x_R - x)^2 + [y_R - y(x)]^2}$$

$$y'(x) = \operatorname{tg} \alpha \text{ ist.}$$

Wird nun $Q_0\hat{s}$ mit $Q_{\hat{s}}$ bezeichnet und für $Q_{\hat{s}}/Q = \lambda_{\hat{s}}$ gesetzt, so kann ähnlich wie bei der Aufgabe nach Abb. 2 bewiesen werden, daß K dann einen Extremwert annimmt, wenn

$$2\lambda_{\hat{s}}\hat{k} + k_R \cos(\varphi_R - \alpha) = 0$$

beziehungsweise

$$2\ \lambda_{\hat{s}} + \frac{k_R}{\hat{k}} \cos(\varphi_R - \alpha) = 0$$

oder kurz

$$a + b \cos(\varphi_R - \alpha) = 0 \text{ ist.}$$

Demzufolge kann das Kostenminimum bei einem solchen Punkt P liegen, bei dem die Resultierende (E) der zum Punkt P angreifenden Vektoren a und b auf die zum Punkt P gehörige Normale $(N - N)$ entfällt, wobei die Richtung des Vektors **a** der zum Punkt P gehörigen Tangenten, die Richtung des Vektors **b** der durch die Punkte R und P gelegten Geraden entspricht, der Sinn des

Vektors **a** von Punkt P_0 zu dem gesuchten Punkt P, der Sinn des Vektors **b** aber von Punkt R zum gesuchten Punkt P hinweist.

Die Festlegung des Punktes P ist nur mit dem Iterationsverfahren möglich, wobei auch die Möglichkeit besteht, daß die den einzelnen Schritten entsprechenden spezifischen Kosten in Erscheinung treten.

Grundsätzlich sind auch mehrere solcher Punkte möglich, die der obigen Bedingung entsprechen. In solchen Fällen ergibt jener Punkt das absolute Minimum, zu dem die kleinste Resultierende gehört.

Eine solche Aufgabe ist graphisch leicht zu lösen, so daß die Gleichung der Kurve $y = y(x)$ gar nicht nötig ist; praktisch ist dies auch gar nicht möglich. Es genügt, wenn die Kurve in der Zeichnung gegeben ist.

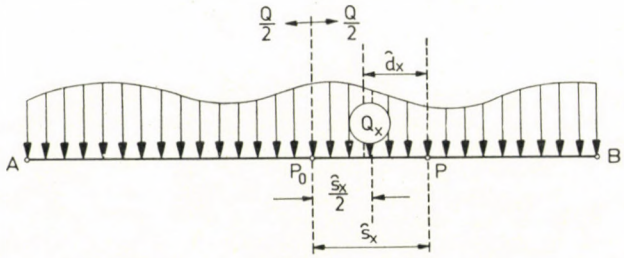

Abb. 5

Auf dem Bogen $\widehat{AB}$ der Kurve $y = y(x)$ wurde eine gleichmäßige Verteilung der Gewichte angenommen. In der Praxis kommt dies nur vereinzelt vor und die Verteilung ist mehr oder weniger gleichmäßig.

Der Bogen $\widehat{AB}$ werde nun gestreckt, die Gewichtsverteilung aber sei ungleichmäßig, d. h. Q_0 sei eine variable Größe (Abb. 5). Das zum Bogen $\widehat{AB}$ gehörige Gewicht sei auch jetzt Q. Q wird durch den Punkt P_0 halbiert, d. h. rechts und links vom Punkt liegt die Hälfte des vollen Gewichtes Q vor. Wird die entlang des Bogenstückes $\widehat{AB}$ verteilte Gewichtsanhäufung auf dem Bogen gesammelt, dann bezeichnet das Sammeln in Punkt P_0 das Minimum. Im allgemeinen ist also $\widehat{AO}_0 \neq \widehat{P_0B}$. Erfolgt das Sammeln des vollen Gewichtes in einem beliebigen Punkt P, so kann leicht nachgewiesen werden, daß die Arbeit des Sammelns durch die Beziehung

$$M = M_0 + 2Q_x\hat{d}_x$$

ausgedrückt wird, worin

M_0 Arbeit zum Sammeln in Punkt P

Q Gewichtsanhäufung am Bogenstück $\widehat{P_0P} = \hat{s}_x$

$\hat{d}_x$ Schwerpunktsabstand der Gewichtsanhäufung Q_x von Punkt P

bedeuten. Im allgemeinen ist $2\hat{d}_x \neq \hat{s}_x$. Ist die Abweichung beträchtlich, so

gebraucht man zweckmäßig eine Korrektion, am einfachsten dadurch, daß statt λ_x die Größe λ'_x gesetzt wird, womit man dann die Beziehung

$$\lambda'_x = \frac{2\widehat{d}_x}{\widehat{s}_x}\lambda_x = \frac{2\widehat{d}_x}{\widehat{s}_x}\frac{Q_x}{Q} .$$

erhält.

Nach der Darlegung der grundsäztlichen Zusammenhänge, sei nun ein wirkliches Vorkommen betrachtet.

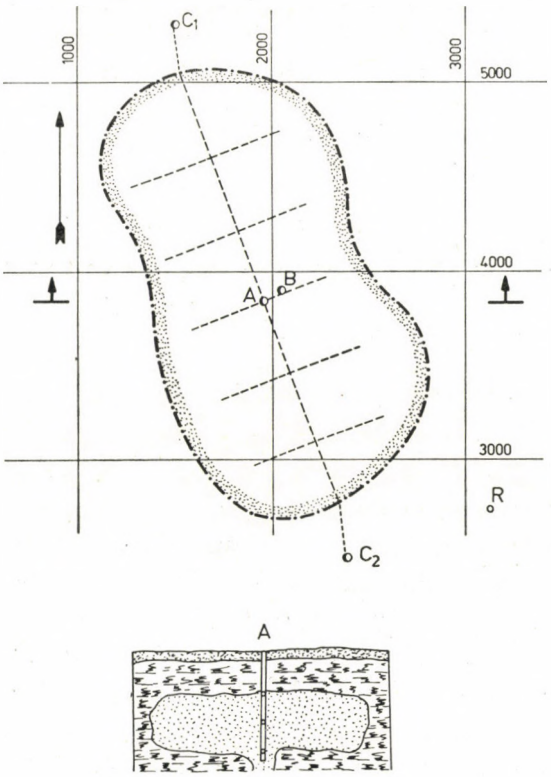

Abb. 6

Abb. 6 zeigt das Gebiet eines Erzvorkommens. Seine Grundfläche beträgt 2,2 km²; die gewinnbaren Erzworräte $Q = 106 \cdot 10^6$ t. Das vererzte Grubenfeld ist in einer Tiefe von 200—500 m gelagert. Nach den ersten Überlegungen müssen hier ein vertikaler Förderschacht (A), ein Hilfsschacht (B) und zwei Wetterschächte (C_1, C_2) vorgesehen werden.

Unter Tage können drei Sohlen geplant werden, von denen die beiden unteren als Fördersohle, die obere aber als Wettersohle dienen soll. Die Hauptförderstrecken untertage sind parallel zur Längsachse des Vorkommens, die Querschläge aber zu diesen normal ausgerichtet.

Das Fördergut soll zum Punkt R in das obertägige Aufbereitungswerk gefördert werden.

Abb. 7 gibt weitere detaillierte Informationen. Bei jedem der 16 Bohrlöcher wurde jene Teilfläche bezeichnet, bei der jeder ihrer Punkte dem Bohrloch, welches die Fläche und den dazugehörigen Mineralvorrat repräsentiert, am nächsten gelegen ist. So liegt z. B. innerhalb des zum Bohrloch 10 gehö-

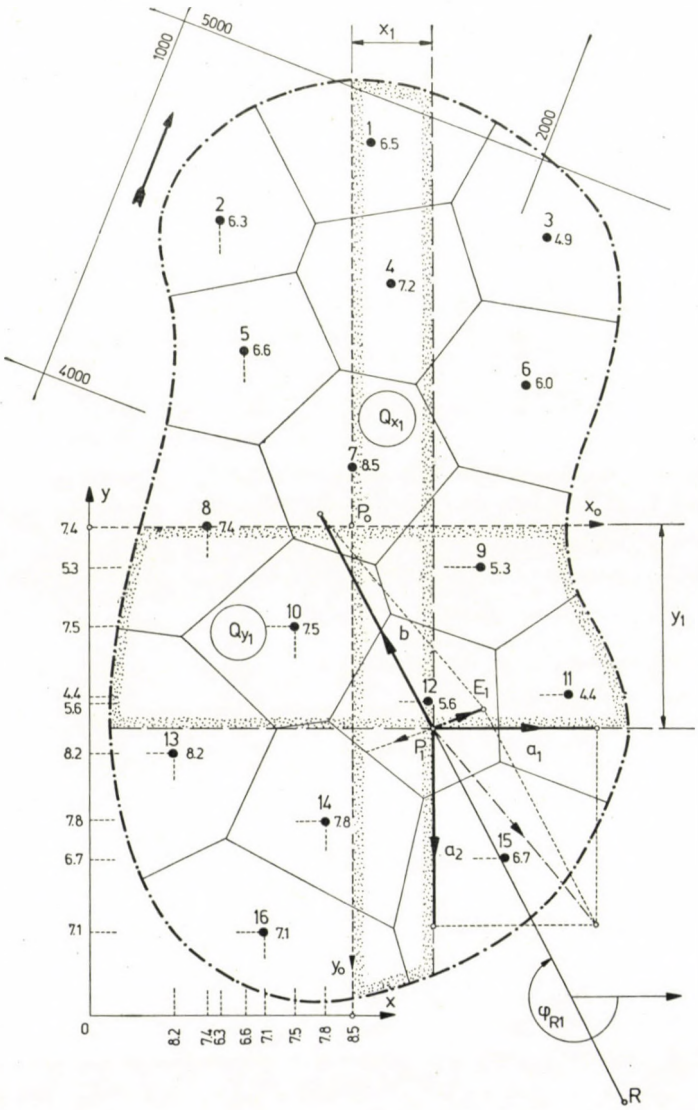

Abb. 7

rigen, 0,17 km² großen Sechsecks jeder Punkt dem Bohrloch 10 am nächsten, so daß Bohrloch 10 den Vorrat $7{,}5 \cdot 10^6$ t dieser Teilfläche repräsentiert.

Ein Achsenkreuz (x, y) sei nun herausgegriffen, dessen y-Achse parallel zur Längsachse des Vorkommens verläuft. Danach werden die zu den einzelnen Bohrlöchern gehörigen abbaubaren Mineralvorratsmengen auf die x- und y-Achse projektiert, und dann vom 0-Punkt ausgehend die aus den einzelnen Teilgebieten gewinnbaren Mineralvorräte summiert (8,2 + 7,4 + 6,3 . . . bzw. 7,1 + 6,7 + 7,8). Auf der x-Achse wird bei dem zu Bohrloch 7 gehörigen Wert 8,5 die Hälfte des ganzen gewinnbaren Mineralvorrates überschritten, nämlich den Betrag von $58 \cdot 10^6$ t.

Aufgrund des Vorhergesagten ist leicht einzusehen, daß die untertägigen Förderkosten in Querschlagsrichtung praktisch dann am kleinsten sein werden, wenn der Hauptförderweg durch das Bohrloch 7 verläuft.

Wird auch an der y-Achse die kumulative Summierung durchgeführt, so überschreitet man die Hälfte von Q beim Bohrloch 8, und gelangt somit zu Punkt P_0. Wird der Förderschacht in Punkt P_0 angelegt, dann werden die Kosten für untertägige Förderung praktisch am kleinsten sein.

Da jedoch das Fördergut zum Obertagspunkt R gebracht werden soll, so wird das Minimum der Gesamtförderkosten nicht durch den Punkt P_0 bezeichnet.

Wird nun die Hauptlinie (y-Achse) parallel auf die Entfernung x_1, die x_0-Achse aber auf die Entfernung y_1 verschoben, so erhält man den Punkt P_1. Der abbaubare Mineralvorrat im Streifen von der Breite x_1 beträgt $Q_{y1} = 21{,}4 \cdot 10^6$ t, der Vorrat des y_1 breiten Streifens aber $Q_{y1} = 26{,}1 \cdot 10^6$ t, d. h. $\lambda_{x1} = 0{,}202$ und $\lambda_{y1} = 0{,}246$.

Der angenommene Punkt P_1 bezeichnet dann das Kostenminimum, wenn dabei das folgende Gleichungssystem erfüllt wird:

$$2\lambda_{x_1} + \frac{k_{R_1}}{k_{x_1}} \cos \varphi_{R_1} = 0$$

$$2\lambda_{y_1} + \frac{k_{R_1}}{k_{y_1}} \sin \varphi_{R_1} = 0$$

oder kurz:

$$a_1' + b_1' \cos \varphi_{R_1} = 0$$

$$a_2' + b_2' \sin \varphi_{R_1} = 0$$

Es sei $k_{x_1} = k_{y_1} = k_{f_1}$, ferner $k_{R_1}/k_{f1} = 0{,}6$.

Dementsprechend ist

$$a_1 + b \cos \varphi_{R_1} = 0$$
$$a_2 + b \sin \varphi_{R_1} = 0 \ .$$

Nummerisch und abgerundet aber

$$0{,}4 + 0{,}6 \cos \varphi_{R_1} = 0$$
$$0{,}5 + 0{,}6 \sin \varphi_{R_1} = 0 .$$

Das Gleichungssystem des ersten Schrittes zeigt, daß das Vektorsystem nicht schließt; seine Resultierende weicht von Null ab, es ensteht die Resultierende E_1; selbstredend ist der Punkt P_1 schlecht gewählt. Im zweiten Schritt muß der Punkt P_2 in entgegengesetztem Sinn als die Richtung der Resultierenden E_1 angenommen werden. Da im vorliegenden Fall der Skalarwert von E_1 im Vergleich zum Skalarwert der beteiligten Vektoren klein ist, wird auch Punkt P_2 ziemlich nahe zu Punkt P_1 entfallen.

In bergbautechnischer Hinsicht kann es begründet sein, daß die zur Längsachse des Vorkommens parallelen Hauptstrecken auf ihrem ursprünglichen Platz verbleiben, d. h. durch den Punkt P_0 verlaufen. Der Sammelort ist also an die Achse y_0 gebunden (Abb. 8). Es ist leicht einzusehen, daß nunmehr die Bedingungsgleichung wie folgt lautet

$$2Q_y k_y + Q k_R \sin \varphi_R = 0$$

d. h.

$$2\lambda_y + \frac{k_R}{k_y} \sin \varphi_R = 0$$

bzw. kurz

$$a + b \sin \varphi_R = 0 .$$

Natürlich kann der die Bedingung erfüllende Punkt P auch hier nur durch Iteration bestimmt werden, wobei die Resultierende (E) auf die Normale entfällt.

Das vorgezeigte Beispiel ist ein Erzvorkommen; das Nebengestein hat also eine große Festigkeit (500—1000 kp/cm²), wobei das Erz in Stücken oder Nestern vorkommt. Von dem produktiven Raum werden nur etwa 6% abgebaut. Hieraus folgt, daß das im Schachtsicherheitspfeiler anstehende Erz im großen ganzen auch abgebaut werden kann. Im vorliegenden Fall kann also die Rolle des in den Schutzpfeilern blockierten Mineralvorrates vernachläßigt werden.

Ähnlich wie oben erläutert, können auch zahlreiche andere Probleme gelöst werden.

Aus gesagtem geht hervor, daß die ausschlaggebendste Frage die Bestimmung des gegenseitigen Verhältnisses der spezifischen Kosten ist. Da es sich hier um ein Verhältnis handelt, fällt die Rolle des Zeitfaktors eigentlich weg. Als Funktion der Zeit können sich nämlich die Absolutwerte der Kosten ändern, ihr Verhältnis zueinander aber bleibt im Wesen das gleiche.

Das Bilden, die Kalkulation der spezifischen Kosten ist keine einfache Aufgabe. Sie enthält die Betriebskosten und auch die Rücklaufkosten für Investitionen. Glücklicherweise stehen in den laufenden Betrieben in reichlicher Anzahl Erfahrungsangaben zur Verfügung. In der Praxis genügt es schon, wenn man die Kostendaten einer vorhandenen Förderstrecke benützen kann, die der geplanten Förderstrecke technisch am nächsten liegt.

Solche Aufgaben werden zeichnerisch, wie auch mit dem Iterationsverfahren gelöst. Um die Genauigkeit zu steigern, müssen beim zweiten Schritt principiell neue spezifische Kosten verwendet werden; in der Praxis jedoch nur dann, wenn vom ersten Schritt der zweite wesentlich abweicht, bzw. allgemein: wenn sich zwischen den aufeinander folgenden Schritten eine beträchtliche Differenz ergibt. Durch Erhöhung der Anzahl der Schritte kann die Genauigkeit erhöht werden. Ein Übertreiben aber hat keinen Sinn, da ja die spezifischen Kosten mit einem gewissen Fehler behaftet sind, ferner unbestreitbar ist, daß sich die vollen Kosten in der Nähe des Optimums nur

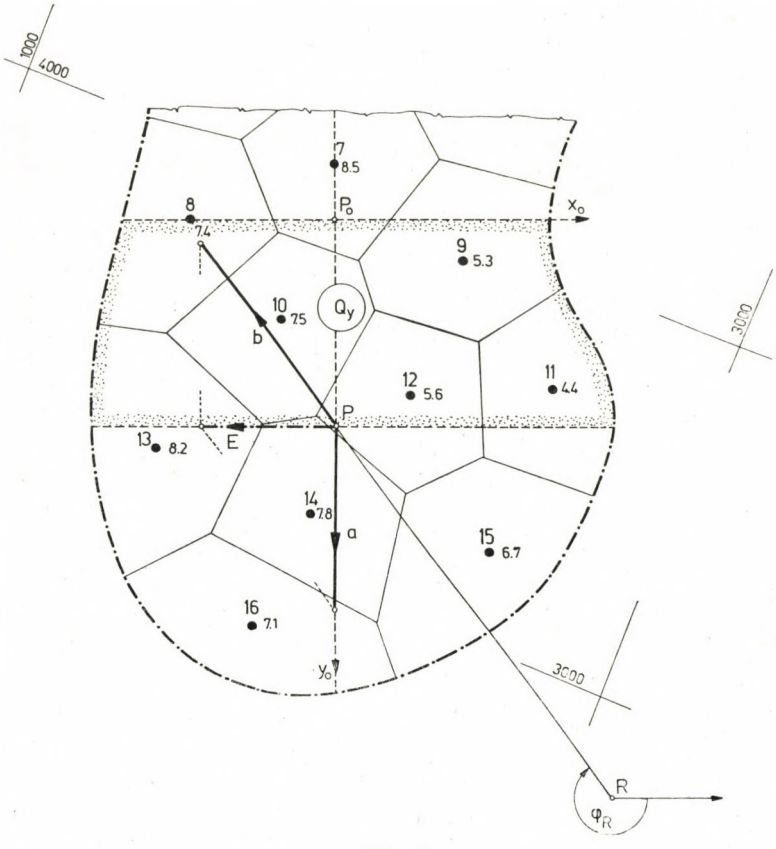

Abb. 8

mehr in geringem Maße ändern. Eine sehr wesentliche Kostenzunahme aber kann eintreten, wenn man vom optimalen Punkt erheblich entfernt ist. Es kann sogar vorkommen, daß bei einem groben Anlegefehler die Förderkosten auf das Zweifache ihres Minimalwertes steigen.

Ein Spezialproblem ist die Berücksichtigung des im Schutzpfeiler gebundenen Mineralvorrates. Hier kann ein Verfahren zum Ziel führen, bei dem der optimale Schachtansatzpunkt mit Außerachtlassung des Schutzpfeilers gesucht wird, danach aber auch mit Berücksichtigung des Pfeilers Alternativen geprüft werden. Hierbei können natürlich die Förderkosten der einzelnen Alternativen im Vergleich zu den optimalen Förderkosten zunehmen; zugleich aber vermindert sich der dem blockierten Mineralvorrat im Schutzpfeiler proportionale Wertverlust. Nach diesen Gesichtspunkten kann die beste Alternative leicht erkannt werden.

Der Wertverlust durch den im Pfeiler gebundenen, nichtgewinnbaren Mineralvorrat kann beispielsweise

$$K_P = Q_P \cdot e$$

betragen, worin Q_p den nicht gewinnbaren Vorrat im Pfeiler,
e aber den spezifischen reinen Nutzen bedeutet.

Selbstredend kann die Rolle des Pfeilers auch auf andere Weise gewertet werden. Die erörterte Methode scheint aber am einfachsten zu sein, da sie leicht kalkuliert werden kann und auch eine Verzinsung nicht nötig ist; denn man kann es auch so betrachten, daß sowohl die Förderkosten wie auch der entgangene Nutzen während der ganzen Lebensdauer des Betriebes von Jahr zu Jahr erscheinen werden. Eine Verzinsung ist nur dann erforderlich, wenn sich die zu vergleichenden Werte zu verschiedenen Zeiten melden. So müssen z. B. die Rücklaufssummen der Investitionskosten mit Anwendung des Zinsverfahrens bestimmt werden, da die Investitionen vor Betriebsbeginn erscheinen, der reine Nutzen aber erst bei bereits laufendem Betrieb.

Die erörterten theoretischen Zusammenhänge sind nur Hilfsmittel. Ihre sinngemäße Benützung erleichtert zwar das Projektieren von Schachtanlagen, ersetzt jedoch nicht die fachlichen Erfahrungen und den auf diesen beruhenden weiten Gesichtskreis.

ТЕОРЕТИЧЕСКИЕ ОСНОВЫ К ВЫБОРУ МЕСТА УСТАНОВКИ ШАХТЫ

Й. ЗАМБО

РЕЗЮМЕ

Место установки шахты можно рассмотреть как оптимальное, когда связанные с ним затраты приводят к минимуму. С местом установки шахты связаны, в первую очередь, затраты для выработки и транспорта, а также уменьшение стоймости, связанное с невырабатывамым полезным минеральным составом из-за страхующей опоры шахты. Практическое решения этой задачи требует знания и познания основных теоретических хоотношений. Авторы выводятся самые важные соотношения и показывается практическое решение.

RECENSIO

W. D. Means

Stress and Strain. Basic Concepts of Continuum Mechanics
Springer Verlag, Heidelberg—New York, 1976; 339 pages, 223 figures, US $ 14.80

The character of the book is precisely and clearly given in its short description: "This is a text for students and working geologists who wish to improve their understanding of mechanical topics relevant to geology. It provides an extensive introduction to the theories of stress and strain that is clear and simple enough for the typical student of structural geology and tectonics.

Written as a text for a semester lecture course or as a self-instruction manual, Stress and Strain is divided into four parts and twenty-seven short chapters, each followed by solved problems. The first section introduces such fundamental ideas as the distinction between instantaneous quantities like stress and two-state quantities like finite strain. Part Two discusses stress in detail. Part Three treats deformation and strain, introduces infinitesimal and finite strain tensors and discusses strain history. The relations between stress and strain for ideally elastic and ideally viscous materials are examined in Part Four".

This shows clearly that the books deals with a basic knowledge in continuum mechanics; it gives a detailed discussion but it neither aims at generalizations of the continuum mechanics by sophisticated theoretical considerations, nor tries to reveal or present new results. It supposes only minimum basic knowledge in mechanics or even its complete lack, and aims to help the instruction of university students or the self-instruction of practical geologists. Thus the question arises why should be such a textbook reviewed in these Acta?

In the course of the development of geological sciences concrete physical analyses gained more and more area in deciphering geological phenomena. The problems of tectonics, of structural geology and even those of deposits cannot be solved without the knowledge in continuum and complex continuum mechanics, therefore it is natural that their instruction became an integer part of university courses in geology, and as self-instruction, a need for practical geologists. From this point of view, this book fills a gap.

On the other hand — from a didactic-pedagogic point of view — Stress and Strain is an excellently written book. It is a surprising easiness with which the author leads the reader always hinting at geological examples to the stress and deformation tensors, to their fields (spatial distributions) and to the history of the stresses and deformations. The presentation includes in addition to HOOK's model, the Newtonian model, too, and gives an introduction to the analysis of deformation energies.

And at last it should be emphasized that the author never absorbs into mere mathematical formalism, he lays stress upon the physical-mechanical significance and elucidates it with well-chosen figures and exercises at the end of each chapter, giving also the solution of the exercises. Thus the reader is stimulated to independent work with the possibility to control the results.

These are the causes which justify a recension of the book here, and it can be added that the book is a delightful, easy text for those being experts in continuum mechanics.

For a more detailed information the table of content follows:

Part I. *Introductory*
1 Material constitution of rock
2 Mechanical state
3 Change in mechanical state
4 Mechanical significance of structure
Part II. *Forces in Rocks*
5 Classes of forges

6 Stress on a plane
7 The stress ellipsoid I
8 The stress ellipsoid II
9 Mohr circle for the stress
10 Tensor components of stress
11 Cauchy's formula, transformation of tensor components
12 Stress fields
13 Stress history
Part III. *Deformation of Rocks*
14 Distortion and deformation, measures of distortion
15 The strain ellipsoid
16 Mohr circle for infinitesimal strain
17 Mohr circle for finite strain
18 Displacement and deformation gradients
19 Tensor components of infinitesimal strain I
20 Tensor components of infinitesimal strain II
21 Tensor components of finite strain I
22 Tensor components of finite strain II
23 Strain fields
24 Strain history
Part IV. *Topics Involving Forces and Deformation*
25 Hookean behaviour
26 Newtonian behaviour
27 Energy consumed in deformation
Solution to Problems
Index

R. Richter

INDEX

Zum Geburtszentenarium von Josef Geszti 1

Tarcsai, Gy.—Ádám, J.: Accuracy of satellite platform location techniques: A study of error propagation 3

Asszonyi, Cs.: Plastic state and deterioration of rocks. III. Boundary conditions of rock deterioration 25

Pethő, Sz.: Analysis of separating efficiency functions for flotation plants 43

Pethő, Sz.—Patvaros, J.—Benke, L.: Partitioning of probabilistic distributions and the parameters of the partitioned distribution 65

Halmos, F.—Szádeczky-Kardoss, Gy.: Computation of geodesics from chord-lengths 95

Zambó, J.: Das Kostenminimum bei ebenem Sammeln 107

Treumann, R.: Stochastic model of magnetotellurics for an inhomogeneous Earth 115

Verő-Hetényi, M.: Contribution to the adjustment of free networks 127

Csernyák, L.—Steiner, F.: General properties of gravitational w-functions 131

Halmai, E.: Feldkomparator für Invarband-Nivellierlatten 145

Kovtun, A. A.—Porokhova, L. N.—Chicherina, N. D.: Effectivity of the solution of the inverse problem based on magnetotelluric data from the North-Western part of the Russian Platform 153

Halmos, F.—Kádár, I.: The notion system of geodetical informations and the calculation of geodetical networks 165

Alpár, Gy.: A straight presentation of the adjustment of so-called free geodetic networks 195

Korvin, G.: Correlation properties of source generated seismic noise, scattered on velocity inhomogeneities 201

Korvin, G.: The hierarchy of velocity formulae: Generalized mean value theorems 211

Bencze, P.: Ionospheric sporadic E stratospheric warmings 223

Cz. Miletits, J.—Holló, L.—Tátrallyay, M.—Verő, J.: Experimental results with the characterization of geomagnetic micropulsations. VI. Summary of results, recommendations 231

Afanasyeva, L. T.: Space-time distribution of geomagnetic pulsations and its dependence on the geomagnetic activity 239

Tárczy-Hornoch, A.: Genauigkeitsvergleich der am Anfang orientierten gestreckten gleichseitigen offenen Theodolitpolygonzüge mit günstiger Gewichtsverteilung und der gestreckten Kreiselpolygonzüge 273

Zambó, J.: Theoretische Grundlagen zur Auswahl des Schachtansatzpunktes 287

Recensio

Means, W. D.: Stress and Strain. Basic Concepts of Continuum-Mechanics — *R. Richter* 301

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Professor Dr. K. RINNER feierte am 4. Oktober 1977 seinen 65. Geburtstag. Die ungarischen Geodäten begrüßen ihn zu diesem Anlaß und wünschen aus ganzem Herzen, daß er in guter Gesundheit und Aktivität die Wissenschaft der Geodäsie noch sehr lange diene. Sie danken ihm eigens für seine erfolgreiche Tätigkeit, mit der er die guten Beziehungen zwischen den österreichischen und ungarischen Fachleuten förderte.

Professor RINNER wurde am 4. Oktober 1912 in Gratkorn geboren. Im Herbst 1930 begann er mit dem Studium von Mathematik und Darstellender Geometrie an der Universität Graz und belegte gleichzeitig an der Technischen Hochschule in Graz die Studienrichtung Vermessungswesen. Er hat sein Hochschulstudium im Jahre 1936 mit ausgezeichnetem Erfolg abgeschlossen. Wenige Monate danach erfolgte seine Promotion zum Doktor der Technischen Wissenschaften. 1953 habilitierte er sich an der Technischen Hochschule in Graz. Durch seine Arbeit wurde die Fachwelt auf ihn aufmerksam und 1957 wurde er als Direktor des Deutschen Geodätischen Forschungsinstitutes nach München berufen. Im Jahre 1959 wurde er an die Technische Hochschule in Graz berufen.

Karl RINNER führte nun intensive Forschungen auf dem Gebiet der Photogrammetrie, der Satellitengeodäsie, der Erdgezeitenmessung, der elektro-

nischen und elektrooptischen Entfernungsmessung, der Triangulierung usw. Ungefähr 160 wissenschaftliche Publikationen und mehrere Standardwerke über Geodäsie und Photogrammetrie sind das Ergebnis seiner wissenschaftlichen Tätigkeit. In den Studienjahren 1970/71 und 1971/72 wurde er zum Rektor gewählt. Er ist sehr aktiv in den internationalen Organisationen tätig. Seit 1969 ist er Präsident der Special Study Group 1/26 der IAG, seit 1972 Präsident der IAG Kommission »Education in Geodesy« der IAG, außerdem Mitglied verschiedener Studiengruppen und Kommissionen der IAG und der Internationalen Gesellschaft für Photogrammetrie. 1969 wurde ihm das Österreichische Ehrenkreuz für Wissenschaft und Kunst, 1. Klasse, 1972 das Große Goldene Ehrenzeichen des Landes Steiermark verliehen. Im Jahre 1974 wählte ihn die Österreichische Akademie der Wissenschaften zum korrespondierenden und 1975 zum ordentlichen Mitglied. Er ist seit 1976 Ehrenmitglied der Ungarischen Akademie der Wissenschaften.

Es sei ihm gegönnt, daß er noch viele Jahre mit seiner bekannten Tatkraft die österreichische und die internationale geodätische Wissenschaft zu weiteren Erfolgen führe!

J. Somogyi

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METHODISCHE FRAGEN DER GEODÄTISCHEN ANWENDUNG VON DOPPLERSCHEN SATELLITEN-BEOBACHTUNGEN*

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Zur Ausgestaltung von geodätischen Netzen großer Ausdehnung, zur Verbindung von vorhandenen Netzen, oder deren Versteifung ist die Radio-Dopplersche Beobachtung die wirksamste Methode sowohl vom Gesischtspunkt der Wirtschaftlichkeit, als auch hinsichtlich der Kosten und des Zeitaufwandes. Die Meßdaten können unmittelbar den Computern weitergegeben werden. Man kann, von der angewendeten Auswertungsmethode abhängig, einen mittleren Fehler von 2—3 m, bzw. 0,5—1 m erreichen. Bei der Bestimmung von einzelnen Punkten bzw. Punktpaaren kann die geometrische, bei der Ausgestaltung von zusammenhängenden Netzpunkten die mechanische Ausgleichungsmethode mit kurzem Bahnbogen angewendet werden. Die Genauigkeit des letzteren kann nach Bedarf mit der Inbetrachtnahme der bekannten Daten des Schwerefeldes erhöht werden. Bei der geometrischen Ausgleichung werden die Korrektionen (z. B. Refraktion, regelmäßiger Fehler der Zeitmessung, usw.) auf die rohen Meßergebnisse angewendet, bei der mechanischen Ausgleichung werden die Korrektionsglieder auch als Unbekannten in das Ausgleichsmodell eingebaut.

1. Einleitung

Nebst den bekannten photographischen und lasertechnischen Beobachtungen gewinnen die Radio-Dopplerschen Beobachtung immer mehr an Bedeutung. Die vom Satelliten ausgestrahlten Radio-Wellen können von Empfängern auf der Erde oder in Satelliten eingebauten Empfängern durch Messung der Doppler-Verschiebung unabhängig vom Wetter und von der Tageszeit sehr viele Meßdaten liefern. Zur Zeit sind bereits solche integrierte prozeßgeregelte Empfänger vorhanden die am Gelände oder vom Satelliten registrierte Daten gleich in einer zur Anwendung für Rechenanlagen geeigneten Form angeben und so die auf Grund von mehreren Satelliten-Durchgängen gewonnenen rechtwinkligen räumlichen Koordinaten der terrestrischen Punkte mit einer Genauigkeit von 1—5 m liefern [10]. In neuester Zeit werden für gewisse geodynamische Zwecke mittlere Fehler von ±2—4 cm erwartet [13]. Im Gebiete des Äquators können täglich 10—12 Durchgänge, bei dem 70° Breitengrad 35—40 Durchgänge registriert werden. Es werden bei einem jeden

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Durchgang ung. 100 – 200 solche genaue Daten registriert, die auch zur Anwendung von präzisen mathematischen Modellen geeignet sind. In der Studie [10] wurden die grundlegenden Fragen erörtert. In dieser Arbeit werden die vom Gesichtspunkt der geometrischen Geodäsie auftretenden Probleme besprochen.

2. Genauigkeitsfragen der Dopplerschen Satelliten-Beobachtungen

Die Genauigkeit der für Navigations- oder geodätische Zwecke durchgeführten Dopplerschen Satelliten-Beobachtungen wird bedeutend durch die Zahl der Durchgänge, die Zahl der je Durchgang beobachteten Intervalle, die Genauigkeit der Ephemeriden Prediktionen, die Verläßlichkeit der angegebenen atmosphärischen Reduktionen, sowie die Methode der Auswertung beeinflußt.

Es ist auch unmittelbar einzusehen, daß mit der Erhöhung der Zahl der Durchgänge, falls die Wirkung der regelmäßigen Fehler nicht in Betracht gezogen wird, der Fehler der Punktbestimmung sich annähernd mit der Wurzel ihrer Anzahl vermindert. Hier ist naturgemäß auch ein optimales Verhältnis zu finden, das laut den bisherigen Erfahrungen etwa bei 30 Durchgängen liegt.

Es wird meistens eine Bahnbestimmung von kurzem Bogen gewählt (ung. 1/6-tel der Umdrehung) bei welchem je 4 Sekunden die Doppler-Verschiebungen gemessen werden (beim JMR je 4,6 Sekunden). Dies bedeutet praktisch so viel, daß je Durchgang 100 – 200 Messungen zur Verfügung stehen. Um diese große Anzahl der Meßinformationen verwerten zu können, muß eine verfeinerte Auswerttechnik angewendet werden.

Die der Bahnbestimmung dienenden präzisen Ephemeriden der Amerikanischen Meeres-Navigations-Systeme (Navy Navigation Satellite System: NNSS) werden von 20 Erdstationen auf Grund von 48-stündigen Beobachtungen durch strenge Ausgleichung mit 2 – 3 m Genauigkeit angegeben. Die Bestimmung der von Sendern ausgestrahlten Ephemeriden erfolgt ähnlicherweise (in Richtung der Satellitenbewegung mit einer Genauigkeit von 24 m, darauf rechtwinkelig mit 17 m und radial mit 8 m Genauigkeit).

3. Doppler-Satelliten Beobachtungs-Methoden

Man kann grundsätzlich zwei verschiedene Methoden mit folgenden Beobachtungs-Techniken unterscheiden:

A) Ausgleichung ohne Bahnbestimmung

1. Einfache Punktbestimmung

2. Translokation, d. h. die relative Lagebestimmung von zwei oder mehreren Punkten

B) Ausgleichung mit Bahnbestimmung

1. Geodätische Lagebestimmung mit kurzem Bahnbogen

2. Translokation mit kurzem Bahnbogen.

Stehen uns von Radio-Sendern ausgestrahlte Ephemeriden kleiner Genauigkeit zur Verfügung, so muß unbedingt das Prinzip der Bahnausgleichung mit kurzem Bogen angewendet werden, damit eine größere Genauigkeit erzielt werden kann.

Die einfache Punktbestimmung ermöglicht sowohl beobachtungstechnisch als auch hinsichtlich der Ausgleichung den größten Freiheitsgrad. Die Reduktion erfolgt eigens für eine jede Station. Es wird vorausgesetzt, daß die genauen Ephemeriden bekannt sind [1]. Als Unbekannten erscheinen die X, Y, Z räumlichen Koordinaten des Punktes, sowie der Frequenzfehler. Da von Durchgang zu Durchgang der Frequenzfehler ausgeschaltet wird, bleiben nur die drei unbekannten Koordinaten. Somit ist von einer rechentechnisch sehr geringe Anforderung benötigenden Methode die Rede. Da bei den Ephemeriden das Erscheinen von Fehlern unvermeidlich ist, werden diese unmittelbar an die zu bestimmenden Unbekannten weitergeleitet. Mit der Anwendung der Daten von sehr vielen Durchgängen und infolge der zum größten Teil zufälligen Fehler der Ephemeriden wird ihre Wirkung in den berechneten Koordinaten in wesentlich verminderten Form auftreten. Wird z. B. angenommen, daß die Ephemeriden das Radiosenders mit Inbetrachtnahme von 100 Durchgängen in die Berechnungen einbezogenwerden, so kann ein mittlerer Fehler von $\pm$ 3 m erwartet werden. Die Gegebenheiten der Doppler-Satelliten-Beobachtungen in Betracht gezogen, kann eine so große Zahl von Durchgängen ung. in 6—8 Tagen gesichert werden. Bei den Punktlagen kann bei Anwendung der präzisen Ephemeriden und 30 Durchgängen ein mittlerer Fehler unter $\pm$ 0,5 m erwartet werden. Zu diesen ist ung. eine viertägige Beobachtung erforderlich.

Wie ersichtlich ist die einfache Punktbestimmungsmethode eine wirtschaftliche Lösung zur Verdichtung der Satellitengrundpunkte. Dies um so mehr, wenn in Betracht gezogen wird, daß der an das Beobachtungs-Instrument angeschlossene Klein-Computer unmittelbar am Gelände ausgeglichene Koordinaten liefert. Nachteil der Methode ist, daß sie nur bei der Anwendung von präzisen Ephemeriden bessere Ergebnisse liefert.

Die Translokationsmethode erfordert im Gegensatz zur vorherigen, an zwei Stellen einen gleichzeitigen Radio-Signal Empfang, es sind also mindestens zwei Doppler-Empfänger nötig. Die Bedingung des Empfanges der von Satelliten ausgestrahlten Radio-Signale ist die Simultanität, d. h. die Sicherung der Gleichzeitigkeit. Bei der Ausgleichung werden nur jene Daten reduziert, bei welchen die Bedingung der Simultanität gesichert ist. Ist die Zahl der

Stationen größer als 2, so kann die relative Translokations- Ortsbestimmung gewissermaßen ausgedehnt werden. Sind die Beobachtungsstationen in Bezug auf die Durchgangsbahn symmetrisch angeordnet (annähernd gleiche Entfernungen), so wird der aus den Ephemeriden-Fehlern stammende Lagefehler in sämtlichen relativen Lagebestimmungen gleich sein. Infolgedessen ist die relative Punktbestimmung (ΔX, ΔY, ΔZ) wesentlich genauer, als die bei der einfachen Punktbestimmung erhaltene absolute Lagebestimmung (X, Y, Z). Die Entfernung zwischen den Punkten darf nicht größer sein als $7-800$ km. Die Translokation ermöglicht also die Ausgestaltung eines Netzes großer innerer Genauigkeit.

Bei zwei Empfängern erhalten wir dann eine organisatorisch günstige Meßanordnung, wenn ein Instrument im Mittelpunkt an einem ständigen Punkt aufgestellt wird und das andere Instrument der Reihe nach an den zu bestimmenden Punkten arbeitet. Mit dieser Anordnung wird mit der punktweise erfolgten Beobachtung von etwa 30 Durchgängen ein mittlerer Fehler von $\pm\ 2-3$ m gesichert. Ihr Nachteil ist, daß die Ungenauigkeit der Simultanität die Verläßlichkeit der Ergebnisse wesentlich beeinflußt.

Diese sind also die mit den bekannten Bahndaten erfolgenden Punktbestimmungsmethoden. Man nennt sie kurz auch geometrische Methoden. Eine andere Lösung ist die Berechnung der Bahnparameter unmittelbar aus den Doppler-Beobachtungen. Hier können auch zwei Lösungen unterschieden werden.

Die geodätische Lagebestimmung mit kurzem Bahnbogen (short arc method) kann als die strengste Auswertungsmethode der Radio-Dopplerschen Beobachtungen betrachtet werden. Die Satelliten-Bahn kann man in der später noch zu erörternden Polynom-Form, oder mit den sechs Kepplerschen Elementen aufschreiben [3, 6, 15, 19]. Als Unbekannten werden die Koeffizienten der Polynome (Bahnparameter, von welchen man auf die Kepplerschen Elemente übergehen kann), oder die sechs Keppler-Bahnelemente, die Raumkoordinaten X, Y, Z der Beobachtungsstationen und vom mathematischen Modell abhängig $4-5$ Korrektions-Koeffizienten (der Fehler der Zeitmessung, troposphärische und ionosphärische Refraktions-Wirkung, Frequenz-Drift, usw.) betrachtet. Da die Bahnelemente zeitabhängig sind, muß man für einen jeden Durchgang neue Bahnelemente bzw. Parameter berechnen. Dies bedeutet, wie später erörtert, die Verminderung der Anzahl der Unbekannten in bezug auf die geometrischen Methoden. Um die Satelliten-Lagen und die Koordinaten der Stationspunkte auf dieselben terrestrischen Koordinaten zu beziehen, muß man die von der Polbewegung, der Präzession, der Nutation und der Erdrotation herrührenden Korrektionen genau kennen. Die Zahl der Unbekannten erhöht sich, falls man im Falle eines globalen Netzes die, das Schwerfeld der Erde beschreibenden harmonischen Koeffizienten bestimmen will. Die gleichzeitige Bestimmung sämtlicher Unbekannten mit Inbetrachtnahme der Daten von

mehreren tausend Satellitendurchgängen bedeutet ein ziemlich großes computertechnisches und rechentechnisches Problem. Dies ist die Ursache der auf Teile gegliederten Lösung.

Man bezeichnet mit der Benennung, Translokation mit kurzem Bahnbogen die Ausgleichung der von zwei oder mehreren Stationen stammenden synchronen Beobachtungen mit kurzem Bahnbogen, wodurch die wahrscheinlichsten Werte der, die Stationen verbindenden Vektoren erhalten werden [17]. Eine weitere Vereinfachung dieser Methode ist, wenn die Bahnparameter der nacheinander folgenden Durchgänge mit einfacher Verschiebung bestimmt werden; d. h. man sucht die Verbindungen der nacheinander folgenden Durchgänge. Bei der Auswertung werden die Daten der beiden oder mehreren Empfänger-Stationen mit der Methode der kurzen Bahnbögen (short arc method) gemeinsam ausgewertet. Letztere bedeutet wesentlich weniger Rechenarbeit; man erhält aber bei sämtlichen Gruppierung für denselben Bahnbogen andere Bahnparameter. So sind, z. B. bei 3 synchron arbeitenden Empfängern 3, bei 4 synchron arbeitenden Empfängern 6 verschiedene Kombinationen der Ausgleichung möglich.

Stehen uns präzise Ephemeriden zur Verfügung, so kann die Ausgleichung mit kurzem Bahnbogen keine bedeutende Genauigkeitssteigerung erbringen. In diesem Falle ist die einfache Punktbestimmung zweckmäßig, oder die gemeinsame Bestimmung von mehreren Punkten mit der Ausgleichung von Koordinaten und Korrektions-Koeffizienten. Bei der einzelweisen Punktbestimmung liefern die vom Radiosender ausgestrahlten Ephemeriden ein annähernd gleich genaues Ergebnis zur Punktbestimmung als die bei der Ausgleichung mit kurzem Bahnbogen (short arc method) erhaltenen Ergebnisse. Daraus folgt, daß die Vorteile der Lösung mit kurzem Bahnbogen in jenem Falle ausgenutzt werden können, wenn mehrere Stationen am Beobachtungsprogramm teilnehmen, unabhängig davon, ob die Messungen der Doppler-Verschiebung synchron sind oder nicht.

Die Methode mit kurzem Bahnbogen kann auch »mechanische« Methode genannt werden, da bei der Bahnbestimmung außer der geometrischen auch eine physikalische, mechanische Betrachtungsweise in Erscheinung tritt [6].

Die Gegebenheiten und Nachteile der verschiedenen Methoden zusammengefaßt, kann folgendes festgestellt werden:

Bei geometrischen Methoden ist keine genaue Zeitmessung nötig, die Lösung ist unabhängig von den geophysikalischen, dynamischen (Bahnbestimmungs-) Voraussetzungen und die Genauigkeit der Lagebestimmung hängt in hohem Maße von der Genauigkeit der Beobachtung ab. In einigen Fällen kann eine ungünstige Konfiguration entstehen [2, 7, 13], so z. B. wenn die Beobachtungs-Stationen an einer Kurve zweiter Ordnung (Ellipse, Hyperbel) angeordnet sind, oder die Beobachtungsstationen und die beobachteten Satellitenlagen einzelweise je eine eigene Fläche bilden. Da von einer großen

Zahl von Objekten die Rede ist, kommen diese Fälle selten vor. Man muß vor allem auf die vorteilhafte, optimale Anordnung der Beobachtungsstationen achten. Bedingung ist, daß mindestens vier Stationen gleichzeitig arbeiten. Die Sicherung der Simultanität ist eine ziemlich strenge Bedingung. Die Punktlagen ergeben sich aufeinander bezogen in relativem Sinne unabhängig von der Art der Auswertung.

Bei der mechanischen Methode (kurzer Bahnbogen) ist die erwähnte simultane Beobachtung keine Bedingung. Wie bekannt und später noch erörtert wird, werden anstatt der Bahnpunkte 6 Keppler-Elemente bzw. vorteilhafter 9 polynomiale Bahnparameter als Unbekannten im mathematischen Modell vorkommen. Die Bestimmung der Bahnparameter, die beste Anpassung an die Meßdaten basiert auf dem Prinzip der räumlichen Glättung, es ist kein Zwang vorhanden. Hingegen ist eine sehr genaue Zeitmessung erforderlich, und man muß auf die Bestimmung des Maßstabes achten (von den geometrischen und physikalischen Parametern abhängig).

4. Mathematisches Modell der Auswertung

Die Auswertung basiert auf der Einführung eines solchen räumlichen rechtwinkeligen Koordinatensystems (X, Y, Z), das auf den mittleren Pol und auf den Greenwicher Ausgangsmeridian orientiert ist. Die Zahl der Messungen ist immer größer als die der Unbekannten, so daß mit der gemeinsamen Befriedigung des Funktionsmodells (Beziehung zwischen Messungen und Unbekannten) und des stochastischen Modells (die die gegenseitige Beziehung der beobachteten Werte ausdrückende Varianz-Kovarianz Matrix) die zu den Meßwerten gehörigen Verbesserungen $\mathbf{v}$ und die unbekannten Parameter $\mathbf{x}$ mit der Methode der kleinsten Quadrate bestimmt werden können.

4.1. *Das mathematische Modell bei der geometrischen Methode*

Aus der vom Satelliten S ausgestrahlten, an den Beobachtungsstationen A gemessenen Frequenz-Verschiebung kann mit Hilfe der Anwendung von mit (Δf) integrierten Frequenzen die Entfernungsgleichung, bzw. die Entfernungsdifferenz-Gleichung zwischen der Station A und dem Satelliten S aufgeschrieben werden. Die Entfernungs-Gleichung lautet [8] (Abb. 1):

$$(\mathbf{s} + \mathbf{v}_S)^2 = (\mathbf{x}_S - \mathbf{x}_A)^2 + (\mathbf{y}_S - \mathbf{y}_A)^2 + (\mathbf{z}_S - \mathbf{z}_A)^2. \tag{1}$$

Da aus den Doppler-Verschiebungen nur Entfernungsdifferenzen abgeleitet werden können, werden im Ausgleichungsmodell die Zusammenhänge entweder für Entfernungsdifferenzen aufgeschrieben, oder die erste Entfernung wird

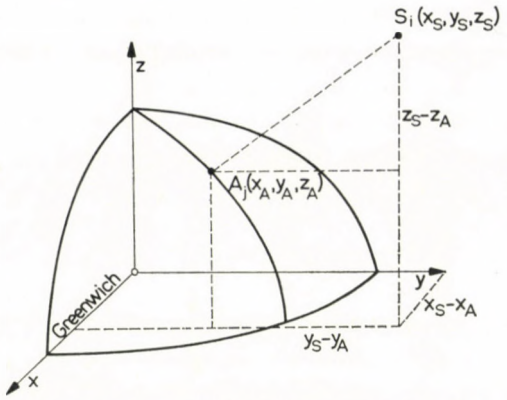

Abb. 1

als unbekannt aufgefaßt [8, 9]. Bei jedem Durchgang, brz. bei der Unterbrechung der Zählung eines jeden Radiosignals und beim neuen Anlauf muß immer eine neue Unbekannte eingeführt werden. Dies ist zwar neben den übrigen anderen Unbekannten nicht besonders nachteilig, erhöht aber bei der gemeinsamen Auswertung von mehreren Durchgängen die Zahl der Unbekannten beträchtlich. Bei der anderen Lösung werden die Zusammenhänge für die Entfernungsunterschiede $(s_2 - s_1)$ aufgeschrieben. In diesem Falle entsteht wegen der geringen Winkeldifferenz zwischen den eingeleiteten zwei Entfernungen zur Bestimmung der Unbekannten ein ziemlich ungünstiges geometrisches Verhältnis. Eine solche Lösung führt oft zu schlecht kondizionierten Gleichungssystemen und damit zu einer ungenauen Lösung. Der Vorteil ist aber, daß in den kurzen Zeitintervallen zwischen je zwei benachbarten Signalempfängen sich die Wirkung der Refraktions- und der Zeitmeß-Ungenauigkeit vermindert. Der allgemeinen Form der Verbesserungsgleichung entspricht in jenem Falle, wenn die erste Entfernung als Unbekannte in Matrix-Form eingeführt wird, auf Grund von (1):

$$\mathbf{v}_{S_i} = \delta\mathbf{s}_1 + \frac{\mathbf{x}_{S_i} - \mathbf{x}_A}{\mathbf{s}_i}(\delta\mathbf{x}_{S_i} - \delta\mathbf{x}_A) + \Sigma\delta\mathbf{k} + \mathbf{L}_i\,, \tag{2}$$

worin $\mathbf{v}_{S_i}$ die zur i-ten Messung gehörige Verbesserung, $\delta\mathbf{s}_1$ die unbekannte Entfernung der ersten Messung, $\mathbf{x}_{S_i}$ die dreidimensionalen Koordinaten des i-ten Satellitenpunktes, $\mathbf{x}_A$ die Koordinaten-Matrix der Beobachtungsstation, $\mathbf{s}_i$ die i-te Entfernung zwischen der Beobachtungsstation und dem i-ten Satellitenpunkt, $\delta\mathbf{x}_{S_i}$, $\delta\mathbf{x}_A$ die Koordinatenänderungen, $\delta\mathbf{k}$ die ionosphärischen und troposphärischen Refraktionsglieder, $\mathbf{L}_i$ aber den mit den Annäherungswerten und den gemessenen Daten berechneten Widerspruchsvektor bezeichnen.

Haben wir bei dem Durchgang 50 oder 100 solche beobachtete Lagen, so muß man für sämtliche einen der Gl. (2) entsprechenden Zusammenhang aufschreiben. Arbeiten z. B. 4 Beobachtungsstationen gleichzeitig und wurden je Station 125 Doppler-Verschiebungen bei jedem der Durchgänge gemessen, so ist die Zahl der Unbekannten, wenn die Zahl der Korrektionen für 5 angenommen wird:

$3 \times 4 = 12$ Stationskoordinaten
$3 \times 125 \times 100 = 37\,500$ Bahnkoordinaten
$4 \times 5 \times 100 = 2000$ Korrektions-Unbekannten
$4 \times 100 = 400$ unbekannte Anfangs-Entfernungen.

Wie ersichtlich ist die Zahl der Unbekannten ziemlich beträchlich. Bei der Lösung der Gleichung kann man mit Hilfe der Partitionierung gewisse Unbekannten im voraus ausmerzen, bis schließlich die Änderungen der gesuchten unbekannten Stations-Koordinaten erhalten werden. Dies erfordert aber eine beträchtliche Rechenarbeit.

Anders ist die Lage, wenn die genauen Ephemeriden der Satellitenpunkte bekannt sind. In diesem Falle entfällt ein wesentlicher Teil der Unbekannten, die Anzahl der übrig gebliebenen kann aber auch ziemlich groß sein.

4.2. *Mathematische Zusammenhänge zu halbdynamischen Modellen*

Die Satellitenbahn kann auf die Zeitsekunde »t« bezogen mit folgender Gleichung simuliert werden [13]:

$$\mathbf{x}_{S_t} = \int_{t_0}^{t} (t - \tau)\, \mathbf{F}(\tau) + \mathbf{d}\tau + (t - t_0)\, \dot{\mathbf{x}}\,(t_0) + \mathbf{x}\,(t_0) + \mathbf{x}\,(t_0)\,, \tag{4}$$

worin $\mathbf{x}_{S_t}$ die auf die Zeitsekunde t bezogene Satelliten-Koordinatenlage, die Integrationskonstanten die auf die t_0 Zeitsekunde bezogenen Lagevektoren, sowie $\mathbf{F}(\tau)$ die auf Zeitraum τ bezogene Kräftefunktion bezeichnen. Die Kräftenfunktion symbolisiert eigentlich die durch das Schwerefeld verursachten Bahnstörungen. Auf die Bestimmung der letzteren wollen wir hier nicht näher eigehen. Zweckmäßigerweise wird die Beziehungszeit t_0 im mittleren Punkt der Bahn angenommen. Den Zeitwert $t - t_0 = \bar{t}$ für die Lagevektoren eingeführt, erhält man laut (4) das folgende Polynom [3, 19, 6]:

$$\mathbf{x}_{S\bar{t}} = \mathbf{x}_{S_{\bar{0}}} + \alpha_1 \bar{t} + \alpha_2 \bar{t}^2 + \alpha_3 \bar{t}^3 + \ldots \tag{5}$$

worin $\mathbf{x}_{S_t}$ die auf Zeitsekunden $\bar{t}$ bezogenen Koordinaten der Satelliten und α das Symbol der Polynomkoeffizienten bezeichnen. Detailliert aufgeschrieben

und die Geschwindigkeit, bzw. Beschleunigungsvektoren $\dot{x}$, $\dot{y}$, $\dot{z}$, $\ddot{x}$, $\ddot{y}$, $\ddot{z}$ in Betracht gezogen:

$$\begin{bmatrix} x_S & \dot{x}_S & \ddot{x}_S \\ y_S & \dot{y}_S & \ddot{y}_S \\ z_S & \dot{z}_S & \ddot{z}_S \end{bmatrix} = \begin{bmatrix} x_{S_0} & \alpha_1 & \alpha_2 & \dots \\ y_{S_0} & \beta_1 & \beta_2 & \dots \\ z_{S_0} & \gamma_1 & \gamma_2 & \dots \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \bar{t} & 1 & 0 \\ \bar{t}^2 & 2\bar{t} & 2 \\ \vdots & \vdots & \vdots \\ \bar{t}^n & n\bar{t}^{n-1} & n(n-1)\bar{t}^{n-2} \end{bmatrix}. \tag{6}$$

Den Übergang auf das erdgebundene dreidimensionale Koordinatensystem sichern folgende bekannte Transformationsgleichungen:

$$\begin{bmatrix} X_S \\ Y_S \\ Z_S \\ \dot{X}_S \\ \dot{Y}_S \\ \dot{Z}_S \end{bmatrix} = \begin{bmatrix} R & O \\ \dot{R} & R \end{bmatrix} \begin{bmatrix} x_S \\ y_S \\ z_S \\ \dot{x}_S \\ \dot{y}_S \\ \dot{z}_S \end{bmatrix} \quad \text{worin} \quad R = \begin{bmatrix} \cos \omega t & \sin \omega t & 0 \\ -\sin \omega t & \cos \omega t & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \dot{R} = \begin{bmatrix} -\sin \omega t & \cos \omega t & 0 \\ -\cos \omega t & -\sin \omega t & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{7}$$

In der Gleichung bezeichnet ω die Rotationsgeschwindigkeit der Erde.
Wird der Polynom (6) näher untersucht, so besteht für die Geschwindigkeitskomponenten folgender Zusammenhang:

$$\begin{bmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{bmatrix} = \begin{bmatrix} \dot{x}_S \\ \dot{y}_S \\ \dot{z}_S \end{bmatrix}. \tag{8}$$

Für die Beschleunigungskomponenten kann folgender Zusammenhang aufgeschrieben werden:

$$\begin{bmatrix} \ddot{x}_S \\ \ddot{y}_S \\ \ddot{z}_S \end{bmatrix} = \begin{bmatrix} 2\alpha_2 \\ 2\beta_2 \\ 2\gamma_2 \end{bmatrix}. \tag{9}$$

In diesem Sinne bestehen zwei Möglichkeiten zur Bestimmung der Beschleunigungskomponenten; sie werden entweder als freie Werte mit Polynomausgleichung bestimmt, oder von den für das Schwerefeld bekannten Normalwerten mit einer, den hiesigen Zwecken entsprechenden Genauigkeit auf-

geschrieben [21]:

$$\alpha_2 = -\frac{1}{2}\frac{\partial w}{\partial x_S} = -\frac{fg_0}{2s^3}x_{S_0}$$

$$\beta_2 = -\frac{1}{2}\frac{\partial w}{\partial y_S} = -\frac{fg_0}{2s^3}y_{S_0} \tag{10}$$

$$\gamma_2 = -\frac{1}{2}\frac{\partial w}{\partial z_S} = -\frac{fg_0}{2s^3}\left(z_{S_0} - \frac{3\alpha}{g_0 s^2}\right)$$

und

$$\begin{bmatrix} \alpha_3 \\ \beta_3 \\ \gamma_3 \end{bmatrix} = \frac{f}{2s^6}\begin{bmatrix} wx_{S_0} - \frac{1}{3}s^2 x_{S_0}^2 \\ wy_{S_0} - \frac{1}{3}s^2 y_{S_0}^2 \\ wz_{S_0} - \frac{1}{3}s^2 z_{S_0}^2 \end{bmatrix}, \tag{11}$$

wobei neben den bereits erwähnten, f die Gravitationskonstante der Erde $\alpha = a_e^2 C_{2,0}$ (a_e = äquatorialer Radius der Erde, $C_{2,0}$ das harmonische Glied des Erdschwerefeldes zweiten Grades und nullter Ordnung) bezeichnen. Weiters ist:

$$s = (x_{S_0}^2 + y_{S_0}^2 + z_{S_0}^2)^{1/2}$$

$$w = x_{S_0}\dot{x}_{S_0} + y_{S_0}\dot{y}_{S_0} + z_{S_0}\dot{z}_S \tag{12}$$

$$g_0 = 1 - \frac{3\alpha}{2s^2}\left(1 - \frac{5z_{S_0}}{s^2}\right).$$

Da unser System mit der Erde rotiert, muß man die daraus herrührende Beschleunigung, sowie bei den x und y Werten die Coriolis Glieder in Betracht ziehen:

$$\text{im Falle von } x\text{: } \omega^2 x_S + \omega\left(\frac{\partial y}{\partial \bar{t}}\right)$$

$$\text{im Falle von } y\text{: } \omega^2 y_S + \omega\left(\frac{\partial x}{\partial \bar{t}}\right). \tag{13}$$

Mit dem Beweis des Zusammenhanges zwischen Polynom-Koeffizienten und Normalwerte des Gravitationsfeldes werden die Beziehungen zwischen rein geometrischer und dynamischer Satellitengeodäsie auch geklärt. Letztere werden von Gliedern höherer Ordnung der Polynome absorbiert (verschluckt), falls sie nicht in Betracht gezogen werden [6].

Um auf die Bestimmung mit kurzem Bahnbogen umzuschalten, muß man an Stelle der in Gl. (2) ursprünglich aufgeschriebenen x_S, y_S, z_S Koordina-

tenwerte die der Gl. (5) entsprechenden Polynome setzen. Dementsprechend treten an Stelle von mehreren hundert Unbekannten Satelliten-Koordinatenwerte von nur 9 Bahnparametern (Polynom-Koeffizienten). Dies soll auch an einem Beispiel veranschaulicht werden. Es werden an 4 Stationen bei 100 Durchgängen je Durchgang 125 Doppler-Verschiebungen gemessen. In diesem Falle ist die Zahl der Beobachtungsgleichungen $4 \times 100 \times 125 = 50\,000$. Gleichzeitig ist die Zahl der einzuführenden Unbekannten, wenn 9 Polynomwerte je Durchgang eingeführt werden:

$$\begin{aligned} 3 \times 4 &= 12 && \text{Stationskoordinaten} \\ 9 \times 100 &= 900 && \text{Bahnparameter} \\ 4 \times 5 \times 100 &= 2000 && \text{Korrektionsunbekannten} \\ 4 \times 100 &= 400 && \text{Ausgangsunbekannten.} \end{aligned} \tag{14}$$

Bei der rein geometrischen Methode war die Zahl der Unbekannten 39 912, bei der erörterten Methode aber, wenn keine simultanen Beobachtungen vorausgesetzt werden, ist die Anzahl der Unbekannten nur 3312. Naturgemäß ist die Lösung eines, so viele Unbekannten enthaltendes Systems nur mit Partitionierung zweckmäßig durchführbar. Mit den Einzelheiten der Frage wollen wir uns hier nicht näher befassen [9]. Es soll nur erwähnt werden, daß mit der an das Ende der Normalmatrix erfolgten Eingliederung der gemeinsamen Unbekannten (Stations-Koordinaten) die Koeffizienten Matrizes der übrigen Unbekannten um die Hauptdiagonale gruppiert werden können. So bedeutet die Partitionierung bei der Lösung eine wesentliche Vereinfachung. Man kann sogar auf Satellitenbeobachtungen verschiedenen Typs gleichförmige und sich wiederholende Normalmatrix-Teile einbauen (die Vereinfachung der Berechnungsarbeit) (Abb. 2).

Zur Bestimmung der, die Satellitenbahn annähernden Polynome sind 3 Lösungen zu finden. Das erste ist die Bestimmung der Polynom-Koeffizienten des Ausgleichungsmodells. Letztere kann vom Gesichtspunkt der Glieder höherer Ordnung wegen der aus der verschiedenen Größenordnung der Koeffizienten stammenden geringen Bestimmbarkeit ein gewisses Problem bedeuten. Da von einem kurzen Bahnbogen die Rede ist, könnte es zweckmäßig sein in gewissen Fällen ein orthogonales Polynom anzuwenden. Man muß aber feststellen, daß die Zahl der Polynomglieder bei kurzem Bahnbogen so klein ist (z. B. 9), daß dieser Fehler immer oder größtenteils vernachläßigt werden kann (zweite Lösung). Wegen des hier erwähnten Grundes ist es übrigens nicht zweckmäßig die Kepplerschen Bahnelemente als Unbekannte einzuführen (ein kurzer Bogen, wodurch das eine oder das andere Element ziemlich ungewiß wird). An deren Stelle ist es zweckmäßiger an den ausgeglichenen Bahnpolynomen die Kepplerschen Bahnelemente nachträglich zu bestimmen [21]. Die dritte Lösung wäre, daß bis zu dem zweitgradigen Glied des Bahnpolynoms

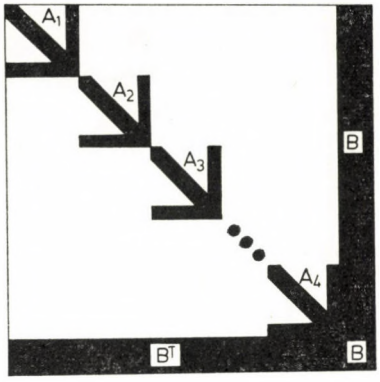

Abb. 2

die Koeffizienten auf Grund der Ausgleichung bestimmt werden, und die Glieder dritten Grades werden dann aus den bekannten Werten des Schwerefeldes auf Grund der bekannten Zusammenhänge bestimmt. Dies wird auch durch die in Tabelle I enthaltenen Koeffizienten bewiesen, die die Polynom-Koeffizienten des Pageos-Satellitendurchganges sind. Werden die Daten der Tabelle näher untersucht, so ist es ersichtlich, daß die Glieder dritter Ordnung der Polynombestimmung ziemlich verstreut sind (wenn man die Koeffizienten der verschiedenen Durchgänge nacheinander betrachtet). Zur gleichen Zeit liefern die aus den bekannten Daten des Schwerefeldes berechneten Koeffizienten eine relativ gute Übereinstimmung. Es sei bemerkt, daß bei den hier angeführten Werten nicht von nacheinander folgenden Satellitendurchgängen die Rede war. Die zwischen den zwei Berechnungsmethoden

Tabelle I

Polynom-Koeffizienten		Satelliten-Durchgang	Erster	Zweiter	Dritter	Vierter	Fünfter
aus einfachem Polynom		α_2	— 32,5	— 84,3	— 93,2	—106,3	—121,2
		β_2 (cm s^{-2})	59,7	19,2	63,6	13,4	48,1
		γ_2	—141,2	—131,6	—128,4	—132,8	—129,4
		α_3	0,0254	—0,3112	—0,0519	—0,3944	—0,0756
		β_3 (cm s^{-3})	—0,0552	—0,1365	0,0323	—0,1908	0,0517
		γ_3	0,0811	—0,6596	—0,0265	—0,7132	—0,0481
aus dem Schwerefeld		α_2	— 31,7	— 82,4	— 84,3	—105,5	—120,9
		β_2 (cm s^{-2})	54,2	20,5	57,2	15,4	42,3
		γ_2	—128,6	—117,8	—114,4	—131,7	—132,8
		α_3	0,0202	0,0215	0,0246	0,0247	0,0265
		β_3 (cm s^{-3})	—0,0114	—0,0116	—0,0062	—0,0154	—0,0176
		γ_3	—0,0131	—0,0133	—0,0159	—0,0212	—0,0208

auftretenden Geschwindigkeitskomponenten α_2, β_2, γ_2, sind von kleineren Abweichungen abgesehen ziemlich übereinstimmend. Demgegenüber ist bei den Beschleunigungskomponenten (α_3, β_3, γ_3) eine ziemlich große Abweichung vorhanden. Wie ersichtlich liefert die einfache Polynombestimmung für die letzteren immer einen größeren Wert. Es kann natürlich auch vorkommen, daß, da das Schwerefeld immer nahezu gleich ist, oder da beinahe ähnliche Elemente zur Bestimmung der Koeffizienten der verschiedenen Durchgänge angewendet werden, die Übereinstimmung deshalb so gut ist [5]. Dies ist aber bei der reinen Polynom-Berechnung nicht der Fall. In dieser Hinsicht ist die letztere Lösung unabhängiger. Man muß trotzdem feststellen, daß zwischen den Geschwindigkeits- und Beschleunigungsvektoren der nacheinander folgenden Durchgänge eine ziemlich große Korrelation besteht.

4.3 *Das Problem der Normalgleichungs-Lösungen bei der geometrischen und halbdynamischen Ausgleichung*

Die Stationspunkte auf der Erde und die Satellitenpunkte, oder die diese charakterisierenden Bahnparameter, sowie auf Grund der zwischen den Unbekannten aufschreibbaren Verbesserungsgleichungen erhaltenen Normalgleichungen liefern bei beiden Arten der Ausgleichung eine singuläre Normalmatrix. Die Frage der Orientierung und des Ausgangspunktes des Netzes (6 Unbekannten) bleibt offen. Ist von einer optischen Satellitenbeobachtung die Rede, so erhebt sich auch die Frage des Maßstabes (7 Unbekannten), aber in diesem Falle muß in Betracht gezogen werden, daß die drei Orientierungsparameter durch die Deklinations- und Rektaszensionswerte gegeben sind. Dies ist nicht der Fall bei Dopplerschen und lasertechnischen Beobachtungen, wo z. B. der Maßstab gegeben sein kann [19]. Die Frage kann auf zwei verschiedenen Wegen gelöst werden.

Es werden 6, zu drei Beobachtungsstationen gehörige Koordinaten fixiert, d. h. für bekannt angenommen.

x_1, y_1, z_1 vom ersten Punkt, x_2, y_2 vom zweiten Punkt, z_3 vom dritten Punkt. Man muß naturgemäß darauf achten, daß vom Gesichtspunkt der geometrischen Bestimmung die drei Punkte sich nicht an »gefährlichen« Orten, Flächen befinden. Werden mehr als 6 Koordinaten angenommen, z. B. dem Punkt 3 entsprechend 9, so kann man bei der Bestimmung der Zahl der Freiheitsgrade der Ausgleichung dies in Betracht ziehen. Von der räumlichen Anordnung der 6 Koordinaten abhängig, werden die Genauigkeitsverhältnisse des berechneten Netzes verschieden sein. Darauf wollen wir hier nicht näher eingehen; das Problem ist in seinen Grundsätzen von der Berechnung des Horizontalnetzes her bekannt.

Eine andere Lösung, die zur besten Beurteilung der inneren Genauigkeit des berechneten Netzes am geeignetesten ist, ist die zur Berechnung der

ebenen Netze bekannte, sog. freie Ausgleichung [11, 12, 16, 20]. Die Ausgleichung liefert mit dieser Methode praktisch ein übereinstimmendes Ergebnis, das in jenem Falle erhalten wird, wenn die Koordinaten-Ausgangspunkte im Schwerpunkt sämtlicher vorkommenden Koordinaten angenommen, und auf die übrigen drei die im weiteren beschriebenen Zwangsbedingungen angewendet werden. In diesem Falle wird man zur Bestimmung der Genauigkeit der Netzpunkte, die geometrische Konfiguration in Betracht gezogen, eine Varianz-Kovarianz Matrix von minimalem Maße erhalten. Werden für die ursprüglichen Matrix-Normalgleichungen die Bezeichnungen: N = Normal-Matrix, δ = Änderung der Unbekannten, L = reines Glied eingeführt, die also nunmehr nur die Änderungen der Stationskoordinaten enthalten, so gelangen wir zur Gleichung

$$\mathbf{N}\delta = -\mathbf{L} \tag{15}$$

(die übrigen Unbekannten wurden ausgemerzt). Werden zu diesen noch die weiteren sechs Bedingungen und die sich daraus ergebenden δ Unbekannten geschrieben, erhalten wir [14]:

$$\begin{bmatrix} \mathbf{N} & \mathbf{K}^T \\ (3n,\,3n) & (3n,6) \\ \mathbf{K} & \mathbf{O}^T \\ (6,3n) & (6,6) \end{bmatrix} \begin{bmatrix} \delta \\ \\ \bar{\delta} \\ \\ \end{bmatrix} = \begin{bmatrix} -\mathbf{L} \\ (3n,\,1), \\ \mathbf{O} \\ (6,1) \end{bmatrix}, \tag{16}$$

wobei n die Zahl der Stationspunkte bezeichnet; für die Matrix $\mathbf{K}$ kann aber aufgeschrieben werden:

$$\underset{(6,3n)}{\mathbf{K}^T} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & & 0 & 0 & 1 \\ 0 & z_1 & -y_1 & 0 & z_2 & -y_2 & & 0 & z_n & -y_n \\ -z_1 & 0 & x_1 & -z_2 & 0 & x_2 & & -z_n & 0 & x_n \\ y_1 & -x_1 & 0 & y_2 & -x_2 & 0 & & y_n & -x_n & 0 \end{bmatrix}, \tag{17}$$

worin x, y, z die vorherigen Näherungskoordinaten der in Betracht gezogenen Stationspunkte sind. Die Varianz-Kovarianz Matrix der $\mathbf{x}$ Matrix wird:

$$\underset{(3n,\,3n)}{\mathbf{Q}_x} = \left(\underset{3n,\,3n}{\mathbf{N}} + \underset{3n,6}{\mathbf{K}^T} \; \underset{6,3n}{(\mathbf{K}} \; \underset{3n,6}{\mathbf{K}^T)^{-1}} \; \underset{6,3n}{\mathbf{K}} \right)^{-1} \left(\underset{3n,\,3n}{\mathbf{E}} - \underset{3n,6}{\mathbf{K}^T} \; \underset{6,3n}{(\mathbf{K}} \; \underset{3n,6}{\mathbf{K}^T)^{-1}} \; \underset{6,3n}{\mathbf{K}} \right). \tag{18}$$

Mit dem erhaltenen Ergebnis wurde also ein solches Koordinatensystem fixiert, bei welchem es keine ausgezeichneten Stationskoordinaten gibt (Schwerpunkts-Koordinatensystem und Schwerpunktsorientierung) und das Netz

die maximale innere Genauigkeit liefert. Es sei bemerkt, daß die Lösung sowohl bei einem geometrischen, als auch bei einem mechanischen Modell angewendet werden kann. Es muß betont werden, daß im Falle, wenn die Dopplerschen Beobachtungen wegen Netzverdichtung durchgeführt werden, die Ausgangsdaten bekannt sind.

Das in den Gln. (18) verkommende $\mathbf{N}$ ist die Normalmatrix des mit gegebenem Orientierungs-Maßstab und Ausgangspunkt (Origo) versehenen Netzes. Das Netz wird meistens zuerst in dessen Kenntnis ausgeglichen, und die Varianz-Kovarianz Matrix $\mathbf{N}^{-1} = \mathbf{Q}_{\bar{x}}$ bestimmt (um es von den vorherigen zu unterscheiden, wurde es mit einem oberen Strich gekennzeichnet). So kann die Frage auch verkehrt aufgeworfen werden. Wie wird nun die innere Genauigkeit des Netzes bestimmt? In Kenntnis von $\bar{\mathbf{x}}$ und $\mathbf{Q}_{\bar{x}}$ kann das auf das Schwerpunktssystem oder auf das System ohne Ausgangsdaten bezogene $\mathbf{x}$ und $\mathbf{Q}_x$ auf folgendem Wege bestimmt werden. Zwischen den beiden Systemen bestehen folgende Transformationsgleichungen:

$$\mathbf{x} = \bar{\mathbf{x}} + \mathbf{K}\mathbf{t} = \mathbf{K}\mathbf{t} + \bar{\mathbf{x}}\,, \tag{19}$$

worin $\mathbf{t}$ die Transformationsparameter sind (diese sind in unserem Falle 3 Verschiebungen und 3 Verdrehungen). Bei $\mathbf{x}^T\mathbf{x} = \text{Min}$ [22] und mit der Einführung von guten Annäherungswerten wird:

$$\mathbf{t} = -(\mathbf{K}^T\mathbf{K})^{-1}\mathbf{K}^T\bar{\mathbf{x}}\,. \tag{20}$$

Auf Grund von Gl. 19 ist $\mathbf{x}$:

$$\mathbf{x} = \bar{\mathbf{x}} - \mathbf{K}(\mathbf{K}^T\mathbf{K})^{-1}\mathbf{K}^T\bar{\mathbf{x}} = (\mathbf{E} - \mathbf{K}(\mathbf{K}^T\mathbf{K})^{-1}\mathbf{K}^T)\bar{\mathbf{x}} = (\mathbf{E} - \mathbf{G})\bar{\mathbf{x}}\,, \tag{21}$$

worin $\mathbf{G} = \mathbf{K}(\mathbf{K}^T\mathbf{K})^{-1}\mathbf{K}^T$ ist. Nach Anwendung des Fehlerfortpflanzungsgesetzes wird:

$$\mathbf{Q}_x = (\mathbf{E} - \mathbf{G})\mathbf{Q}_{\bar{x}}(\mathbf{E} - \mathbf{G})^T = \mathbf{Q}_{\bar{x}} - (\mathbf{G}\mathbf{Q}_{\bar{x}} + \mathbf{Q}_{\bar{x}}\mathbf{G}^T - \mathbf{G}\mathbf{Q}_{\bar{x}}\mathbf{G}^T)\,. \tag{22}$$

Gl. (22) gibt also die Abweichungen der Varianz-Kovarianz-Matrizes des Netzes mit Zwang und mit freier Ausgleichung an. Die Durchführung der in Gl. (22) vorgemerkten Operationen bedeutet kein bedeutendes Problem, nicht einmal bei großen Netzen, und gibt eine gute Möglichkeit zur Beurteilung der inneren Genauigkeit des Netzes ohne den Ausgleichsprozeß noch einmal wiederholen zu müssen.

Bezüglich der Aufstellung der Normalgleichungen wollen wir hier nicht auf Einzelheiten eingehen [9]. Enthalten die Zeitmessungen in den einzelnen Gleichungen neben den Entfernungen und Entfernungsdifferenzen auch Ver-

besserungen, so können diese zu fiktiven Verbesserungen zusammengezogen werden und sokann die Aufgabe auf die reine vermittelnde Beobachtungen zurückgeführt werden. Mit der vorteilhaften Gruppierung der in der Gleichung vorkommenden Unbekannten, d. h. mit der Einordnung der zusätzlichen Unbekannten (Korrektionskoeffizienten, Bahnparameter, Ausgangsentfernung je Durchgang) zum Anfang der Normal-Gleichungsmatrix, sowie mit Aufschreibung der Hauptunbekannten (Änderungen der Stationskoordinaten) am Ende der Gleichungen, gelangen wir hinsichtlich der Lösung der Gleichungen zu wesentlichen Vorteilen. Die zusätzlichen Unbekannten ändern sich je Durchgang; so entstehen viele Null-Elemente am Anfang der ausgestalteten Normalmatrix. Die von Null abweichenden Elemente gruppieren sich um die Hauptdiagonale der Normalmatrix, und die die Hauptunbekannten beinhaltende Teilmatrix schließt sie mit einer vertikalen Spalte und mit einer horizontalen Reihe ab (Abb. 2). Ist von einer gemeinsamen Ausgleichung von Beobachtungen mehrerer Stationsgruppen die Rede, so wiederholt sich die hier erwähnte Anordnung mit zueinander gegenseitig verschobenen Unbekannten-Gruppen und die einzelnen partitionierten Matrix-Gruppen werden nur mit gemeinsamen Unbekannten verbunden. Die Rolle einer solchen verbindenden Unbekannten-Gruppe kann z. B. die Koordinaten-Änderung des Ausgangspunktes bilden (3 Unbekannten), oder die mit Gl. (16) bzw. Gl. (17) bezeichneten Gleichungen. Das hier kurz skizzierte Schema der Aufstellung der Normalmatrix kann auf die photographischen, Dopplerschen- und Laser-Satellitenbeobachtungen, sowie sogar zur deren Kombination angewendet werden [9].

5. Modelluntersuchungen und praktische Ergebnisse bei der Aufarbeitung von verschiedenen Beobachtungsmethoden

Bevor wir auf die Ergebnisse der Modelluntersuchungen eingehen, müssen wir noch kurz einige beobachtungstechnische Fragen klären:

A) Verfügen wir nur über ein einziges Empfangssystem, so muß man die Beobachtung punktweise durchführen, und die Berechnung der räumlichen Koordinaten der Beobachtungsstationen erfolgt in Kenntnis der Ephemeriden der Satellitenlagen mit einfacher Punktbestimmung.

B) Arbeiten wir mit zwei Empfängern, so ist es zweckmäßig mit einem Instrument immer in einem mittleren Basispunkt zu beobachten; das andere soll aber laufend nacheinander in den Beobachtungsstationen aufgestellt werden. In diesem Falle kann man bei der Auswertung die Translokation, oder die mit mechanischer Methode erfolgte Bahnparameter Bestimmung anwenden.

C) Bei vielen geodätischen Lagebestimmungen ist es zweckmäßig 4 Empfangsgeräte einzustellen. Damit kann man neben der, an dem im Mittel-

punkt aufgestellten Instrument erfolgten, auch an 3 weiteren äußeren Stationspunkten eine unseren Genauigkeitserfordernissen entsprechende Anzahl an Beobachtungen von Satellitendurchgängen durchführen. Die einzelnen Konfigurationen sind diagonal-viereckig, auf welche u. a. die bedingte Ausgleichung laut Quaternionen auch anwendbar wäre. Für die hier erwähnte Lösung werden einige praktischen Konfigurationsmöglichkeiten gezeigt (Abb. 3). An Abbildung

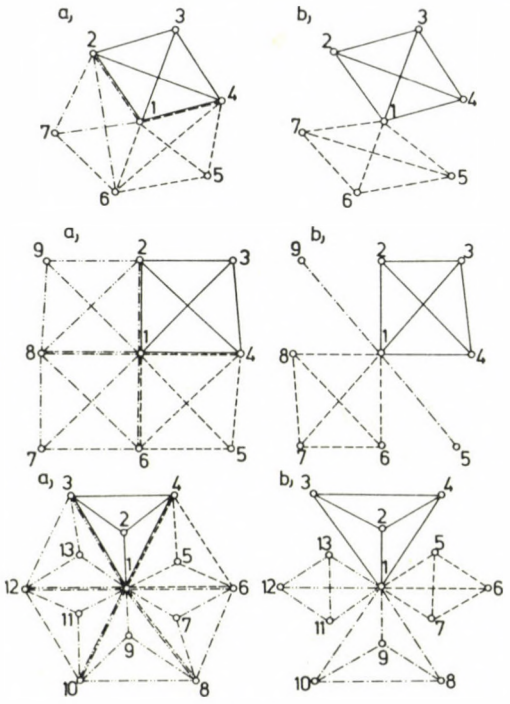

Abb. 3

3.1 kann man bei einem aus 7 unbekannten Punkten bestehenden Netz die mit 4 Doppler-Empfängern durchgeführte Satellitenbeobachtungs-Anordnung sehen. Bei der Lösung 3.1 registriert ein Instrument laufend im Mittelpunkt, die übrigen sind an je drei Stationen für den Zeitraum der Registrierung der erforderlichen Beobachtungen angeordnet. Bei der Bestimmung der Anschlußseiten der einzelnen Formationen wird die Bestimmung verdoppelt; gegenüber der Lösung 3.1b ist aber sein Vorteil, daß vom geodätischen Standpunkt aus gesehen die Versteifung stärker ist. Abb. 3.1b gibt die Anordnung der minimalen Beobachtungen; 3.1a sichert die optimale Genauigkeit. Abb. 3.2. zeigt die Anordnung mit 9 Punkten. Anordnung a und b entstand in ähnlichem Sinne wie bei Abb. 3.1. Abb. 3.3 zeigt die Anordnung mit 13 Punkten. Wie auf Grund der Untersuchungen ersichtlich, ist die Methode mit 2 bzw. 4 Emp-

Tabelle II

	Methode	Zahl der Instrumente	Zahl der Beobachtungstage	Zahl der beobachteten Durchgänge	Relative Verläßlichkeit der Punkte	Absolute Verläßlichkeit der Punkte
1	Einfache Punktbestimmung	1	26	25	5—6	5—6
2	Translokation (zweifache Punkte)	2	22	25	2—2,5	2—3
3	Translokation (Punktgruppe)	4	11	25	1,5—2	2—3
	Netz aus 7 Punkten mit anschließenden Viereckspaaren	4	13	25	0,3—0,8	1,5—2
4	Netz aus 7 Punkten mit unabhängigen Viereckspaaren	4	7	25	1—1,5	2—2,5
	Netz aus 9 Punkten mit anschließenden Viereckspaaren	4	16	25	0,5—0,8	1,5—2
5	Netz aus 9 Punkten mit unabhängigen Viereckspaaren	4 (3)	9	25	1—1,5	2—2,5
6	Netz aus 13 Punkten mit anschließenden Viereckspaaren	4	19	25	0,5—0,8	1,5—2
	Netz aus 13 Punkten mit unabhängigen Viereckspaaren	4	11	25	1—1,5	2—2,5

Tabelle III

Station	Zahl der Durchgänge	mit geometrischer Methode			mit mechanischer Methode		
		Abweichung (m) von den gegebenen geodätischen Koordinaten					
		Δx	Δy	Δz	Δx	Δy	Δz
1	35	—13,2	5,8	15,9	—0.1	0,1	0,1
2	17	—10,9	—8,1	0,8	0,7	—0,1	0,1
3	14	13,5	1,0	15,9	—1,9	1,4	—0,7
4	10	13,5	2,3	— 4,1	—1,3	0,2	1,4
5	20	— 3,9	—2,9	— 4,9	0,6	—0,1	—0,1
6	13	— 9,9	—2,3	— 2,6	—2,1	—1,0	—0,5
7	20	— 7,4	—3,6	— 5,5	1,5	—0,2	0,1
8	15	— 6,0	3,5	— 4,8	2,1	—0,4	—0,3
9	22	— 1,4	7,7	— 3,2	2,6	—0,5	—0,5

fängern hinsichtlich der optimalen Durchführung der Satellitenbeobachtungen und der Anwendung empfehlenswert. Es sei bemerkt, daß die Zahl der Beobachtungen der Variation b etwa Zweidrittel der Variation a ist.

Bezüglich der Auswertung wollen wir die geometrische Methode (einfache Punktbestimmung) und halbdynamische Methode mit kurzem Bahnbogen kurz vergleichen. Erfolgt die Auswertung punktweise, so gibt es bei der Aufarbeitung mit geometrischer Methode weniger Unbekannten bei der Ausgleichung (die Ephemeriden der Satellitenlagen sind gegeben, die Zahl der Refraktions-, der Zeitmeß- und übrigen Unbekannten ist relativ gering, bzw. sie können auf Grund von bekannten Näherungswerten als Korrektionen in Betracht gezogen werden). Sind die Satellitenlagen vorher nicht bekannt, so erhöht die Bestimmung ihrer unbekannten Lagen in einem Modell die Zahl der Unbekannten beträchtlich. Mit Einführung der Parameter mit kurzem Bahnbogen kann man die Zahl der Unbekannten wesentlich vermindern und die Genauigkeit der Bestimmung erhöht sich. Als Beispiel sei das in Abb. 3.2 veranschaulichte 9 Punkte Modell-Netz erwähnt. Die Daten wurden dem in Florida (USA) mit dem JMR Instrument durchgeführten Test entnommen [3]. Mam muß aber naturgemäß die wesentliche Erhöhung der Berechnungsarbeit in Betracht ziehen. Die mittleren Fehler der Koordinaten sind auch etwa um 30—40 % geringer bei der Ausgleichung mit kurzem Bahnbogen, und die Genauigkeit der Bestimmung ist nicht in Funktion mit der Höhenlage der Satelliten-Durchgänge. Eine weitere Genauigkeitssteigerung kann bei der halbdynamischen Methode mit kurzem Bahnbogen erwartet werden, wenn man bei der Herstellung der Funktionsreihe höheren Grades die bekannten Werte des Schwerefeldes auch in Betracht zieht. Anschließend genügt es die »ellipsoidischen« Kennzahlen des Schwerefeldes in Betracht zu ziehen.

Auf weitere Einzelheiten wollen wir hier nicht eingehen [10]. Bezüglich der erzielten praktischen Ergebnisse soll die Arbeit [Kouba-Peterson-Boal, 1974] hervorgehoben werden. Die durchschnittliche Abweichung zwischen den auf geodätischem Wege und mit Doppler-Satelliten Beobachtungen abgeleiteten Koordinaten beträgt 1,5—2 m, die maximale Abweichung liegt zwischen —11 und +6 m. Weitere nützliche Ergebnisse können in der Arbeit von [18] gefunden werden, in welcher von einer, auf Grund von Doppler-Beobachtungen abgeleiteten Punkt-Unsicherheit unter ±1 m die Rede ist.

SCHRIFTTUM

1. Ádám, J.—Fejes, I. :Új Doppler-vevők és geodéziai alkalmazásuk (Neue Doppler-Empfänger und ihre geodätische Anwendung). *Geodézia és Kartográfia*, 27 (1975), 327—334.
2. Blaha, G.: Investigations of critical configurations for fundamental range network. Ohio State Univ. Department of Geodetic Science No. 150, 1971.
3. Brown, D. C.—Trotter, G.: Precise determination of geodetic positions by the method of continuous traces. Melbourne, 1967.
4. Brown, D. C.: Doppler Surveying with the IMR—1 Receiver. *Bulletin Géodésique*, 50. (1976)

5. EHRENSPERGER, W.: Numerische Erfahrungen bei der Berechnung von Bahnparametern für räumliche Glättung bei Mini-Arcs. DGK Reihe A, Nr. 32, 1974.
6. HALMOS, F.—KÁDÁR, I.—KARSAY, F.: The evaluation of asynchron and synchron satellite observations by orthogonal polynomials. Observations of Artificial Satellites of the Earth. Bucharest, No. 10, 1970.
7. HALMOS, F.: Hosszú bázisú rádióinterferométeres megfigyelések geodéziai alkalmazása (Die geodätische Anwendung der radiointerferometrischen Messungen mit langen Basen). *Geodézia és Kartográfia*, 26 (1974), 338—346.
8. HALMOS, F.: A kombinált kozmikus geodéziai megfigyelési módszerek elemzése (Analyse der kombinierten kosmisch-geodätischen Beobachtungsmethoden). *Geodézia és Kartográfia* 27 (1975), 233—239 und Observations of Artificial Earth Satellites, No. 14, 1975.
9. HALMOS, F.: Matematikai modellek korszerű szatellita-megfigyelések geodéziai célú felhasználásához (Mathematische Modelle zur Anwendung zeitgemäßer Satellitenbeobachtungen in der Geodäsie). Manuscript, 1975.
10. HALMOS, F.—ALMÁR, I.—ÁDÁM, J.—FEJES, I.: Application of radio (Doppler) observational methods in the geometrical and dynamical satellite geodesy. *Acta Geod. Geoph. et Mont.*, 11 (1976), 237—263.
11. MEISSL, P.: Die innere Genauigkeit eines Punkthaufens. *Öst. Z. f. Vermessungswesen*, 50 (1962), 159—165; 186—194.
12. MITTERMAYER, E.: Eine Verallgemeinerung der Methode der kleinsten Quadrate zur Ausgleichung freier Netze. *Z. f. Vermessungswesen*, 96 (1971), 401—410.
13. MUELLER, I.—GELDER, W.—KUMAR, M.: Error analysis for the proposed close grid geodynamic satellite measurement system (GLOGEOS). Ohio State Univ. Department of Geodetic Science, Report No. 230, 1975.
14. RINNER, K.—KILLIAN, K.—MEISSL, P.: Beiträge zur Theorie der geodätischen Netze im Raum. Deutsche Geod. Kom. Reihe A, No. 61, 1969.
15. SCHWARZ, CH. R.: The use of short arc orbital constrainst in the adjusment of geodetic satellite data. Ohio State Univ. Department of Geodesic Science, Report No. 118, 1969.
16. VERŐNÉ, HETÉNYI M.: Az önálló háromszöghálózatok kiegyenlítéről. *(Über* die Ausgleichung von lokalen Dreiecksnetzen). *Geodézia és Kartográfia*, 25 (1973), 248—253.
17. WELLS, D. E.: Doppler satellite control. Univ. of New Brunswick Department of Surveying Engineering, Techn. Report No. 29, 1974.
18. WELLS, D. E.—KOUBA, J.: Semi-dynamical Dopplersatellite positioning. General Assembly of the International Association of Geodesy, Grenoble, 1975.
19. WOLF, H.: Computation of satellite triangulation by fitting of the orbit. Deutsche Geodätische Kommission, Reihe B. Nr. 153, 1967.
20. WOLF, H.: Helmerts Lösung zum Problem der freien Netze mit singulärer Normalgleichungsmatrix. *Z. f. Vermessungswesen*, 97 (1972), 189—192.
21. WOLF, H.: Rechenmodelle der geometrischen Satellitengeodäsie. Mitt. aus dem Institut für Theoretische Geodäsie der Univ. Bonn, No. 23, 1973.
22. WOLF, H.: Freie Ausgleichungen in klassischer Sicht. Vortrag im Geodätischen und Geophysikalischen Forschungsinstitut, Sopron, 1976.

МЕТОДИКО-ТЕХНИЧЕСКИЕ ВОПРОСЫ ПРИМЕНЕНИЯ В ГЕОДЕЗИИ ДОППЛЕРОВСКИХ СПУТНИКОВЫХ НАБЛЮДЕНИЙ

Ф. ХАЛМОШ

РЕЗЮМЕ

В статье анализируются вопросы техники различных радиодоппплеровских спутниковых наблюдений, их обработки и уравнивания. Рассматриваются возможности применения уровнивания геометрическим методом и механическим методом коротких дуг. Отдельно рассматривается учет возможностей, дистигаемых знанием гравитационного силового поля. Анализируются проблемы вычислительной техники, а также вопрос принятия системы координат. Наконец, показывается результаты модельных исследований и практических наблюдений.

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THE ROLE OF WETTABILITY IN POLYMER FLOODING

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The flow characteristics of polymer solutions and carrier phases (formation water, distilled water) are examined in water wet and oil wet porous systems. According to the experimental results, the aqueous fluids will move in oil wet sandstone core with their original mobility and viscosity and no apparent permeability reduction takes place. The cause of this phenomenon is attributed to the fact that the sorption loss of polyacrylamide on oil wet rock surface is zero, and/or the sweeping effect of the carrier phase is perfect.

A disadvantageous consequence of the lower resistance factor and of the apparent permeability reduction is that an identical displacement and areal sweep efficiency can be attained in an oil wet or intermediate reservoir at considerably higher costs (polymer amounts) only. The examinations have been carried out in a compatible polymer rock system where the mechanical entrapment is negligible.

Introduction

As early as in 1955 RICHARDSON et al. [1] showed that the effectiveness of water flooding depends to a great extent upon the wetting character of the reservoirs. The secondary and tertiary oil recovery methods that ensure surplus output by the injection of surfactants or alkalines, resulting in the reduction of the interfacial tension and the alteration of wettability, are partly founded on the recognition of the fact above. The effectiveness of the oil displacement by polymers, however, is attributed to the lower mobility of the displacing phase, so the wettability seems to have no share in the mechanism of the displacement process. But if we consider that the mobility of the pure aqueous phases following the injection of a polymer solution depends not only on the viscosity, on the dynamic and irreversible entrapment of polymers by the pores but also on the polymer adsorption which reduces the permeability to water, then it is evident that the shaping of the effectiveness may, and indeed is, influenced by the wetting character of the rocks.

Within the scope of the experiments to be described here we shall be concerned with the influence of the wettability which it exerts upon the polymer adsorption, and on the flow characteristics as well, using a compatible polymer rock system. In the course of our considerations a comparison will be made between the change in mobility, resistance factor, apparent permeability, viscosity, and viscosity ratio of the fluids in an absolute water wet and oil wet system.

The dependence of sorption losses of polyacrylamides on wettability

Flooding of any porous rock results in lesser or greater losses in the system, depending on the energy state of the solid surfaces, the quality of the fluids and solids. It has been shown in the case of polyacrylamides that, except for the mechanical entrapment, the sorption losses have chemical nature. It has already been determined earlier how the specific losses are dependent on the quality (structure, molecular weight, degree of hydrolization) of the rock and polymer concerned. However, little has been said about the quantity of the polymer sorbed being dependent to a great extent, on the energy state of the interfaces and wettability for a given rock. SMITH [2] has found that the wetting character and the presence of oil do not effect the chemisorption of the polymer. On the other hand, a decrease in the quantity of polymer retarded in the presence of oil was reported by MUNGAN et al. [3]. JENNINGS et al. [4] agree with this, although they are adding that the mobility reduction of the fluids is independent of the losses. Also SAREM's work [5] is worthy of attention who has suggested that on a low energy teflon surface the specific adsorption of acrylamides is much less than in reservoir rocks, for example.

The cause of the contradictions may be attributed to the fact that no differentiation is made by the authors as regards the type of loss (physical or chemical sorption) and, on the other hand, they are disposed to talk about a mechanical entrapment when, in fact, a sorption retardation is concerned. In order to elucidate the question, we have made detailed investigations to determine the role of the sorption phenomenon in general and that of the wetting character [6]. For the study of the latter, unconsolidated silica sands having different surface character were used. The dynamic loss for the polymer of the Separan NP-10 is shown in Table I, and, the so-called saturation curves serving as calculation basis are presented in Fig. 1.

Table I

Specific loss of a Separan NP-10 polyacrylamide on silica of different surface characteristics

Wetting character	Specific sorption loss, g/g
Oil wet (siliconized)	~ 0
Intermediate (siliconized to 20%)	$0.65 \cdot 10^{-5}$
Natural	$1.90 \cdot 10^{-5}$
Water wet (extracted)	$6.87 \cdot 10^{-5}$

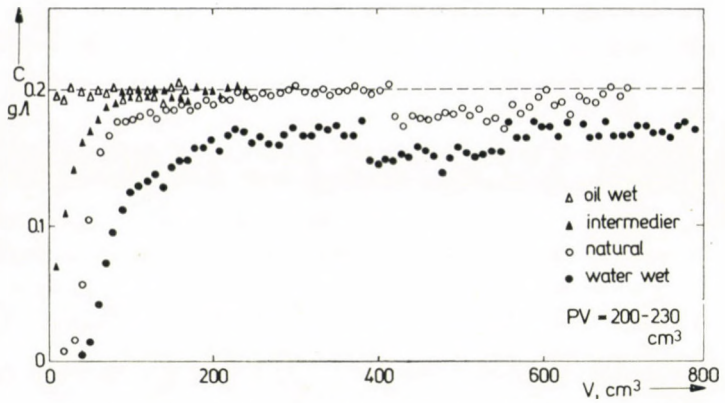

Fig. 1. Dynamic adsorption of Separan NP-10 on silica models with different surface characteristics

It may be seen from the data listed that in the liquid taken off the oil wet model the polymer appears directly in the initial (injection) concentration; thus, no sorption retardation can be observed. From this it follows that in an oil wet reservoir, or on an oil film, the sorption loss of polyacrylamides equals zero. On the other hand, the greatest loss in polymer occurs on the absolute water wet model and the obtained value is about three times as great as that measurable on a natural silica surface. On the silica bed, which contains the mixed absorbent (20 % of siliconized silica), the measured loss is situated, according to the expectation, between the specific loss obtained on the natural model and that obtained with the model having hydrophobic character.

As a final result, it has been proven that the sorption loss of polyacrylamides incurred in reservoir rocks depends to a great extent on the wettability. Between the adsorption processes of polyacrylamides and methylene blue on a rock surface, a certain parallelism can be observed. By this far-reaching analogy a new methodological possibility for semi-quantitative determination of the wetting character in reservoir rock, is afforded using the polymer adsorption similarly to the dye adsorption method of HOLBROOK and BERNARD [7].

Flow phenomena in water wet and oil wet porous media

The requirements for the presentation of the general flow characteristics of polymer solutions were best met by the Separan AP-30 of molecular weight $3.5 \cdot 10^6$ and a hydrolization degree of 40 %. Concentration of the polymer was 100 ppm, viscosity of the solution under laboratory conditions (24 °C) was 3.05 cP. Taking into consideration the rock's permeability (59 and 62 mD)

and the equivalent diameter of the gel coils (2600Å), the polymer-rock system may still just be regarded as a compatible one.

For the tests two sandstone cores of the same characteristics were used. Of these, one was extracted by benzene, then by acetone for 36 hours. The other core was wetted with silicon oil dissolved in toluene. Burning-in of the silicon film was performed at 350 °C.

In the course of the laboratory work comparison is made between the flow phenomena occurring in the porous cores considered to be water wet and oil wet ones, respectively. When determining the permeability by means of "connate water" (2 % sodium chloride solution), the injection sequence of the fluids was as follows: *a)* polymer solution, *b)* connate water, and *c)* distilled water.

The difference between the two systems has been determined for each flow characteristic. In our figures the curves characterizing the water wet cores are represented by solid lines, those characterizing the oil wet cores, by dotted ones.

The change in the mobility of the aqueus phases

In the water wet model the well-known trends may be observed (Fig. 2):

a) the mobility of the polymer solution and advancing connate water as well as that of the distilled water is greatly reduced; within this

b) the mobility of the polymer solution is the lowest one, and

c) that of the distilled water is inferior to the connate water's mobility. On the other hand, in the oil wet system:

a) a substantial change takes place only in the polymer solution; however

b) the mobility of the polymer solution does not reach the value measured in the water wet system, but rather exceeds it by an amount of $10 \cdot 10^{-3}$D/cP,

c) having reached the equilibrium value, the mobility of either the connate water or the distilled water does not exhibit any substantial difference as compared to the initial state, i. e., prior to the passage of the polymer ($52 \cdot 10^{-3}$ D/cP), and

d) the mobility of the distilled water is higher than that of the connate water.

A detailed discussion and interpretation of the results will also be made following the presentation of the experimental material as a whole. Here, attention is called only to the fact that practically both the connate and the distilled water pass with their respective mobility through the porous medium, and, on the other hand, the mobility of the distilled water is superior to that of the connate water. The cause of the latter phenomenon may probably be

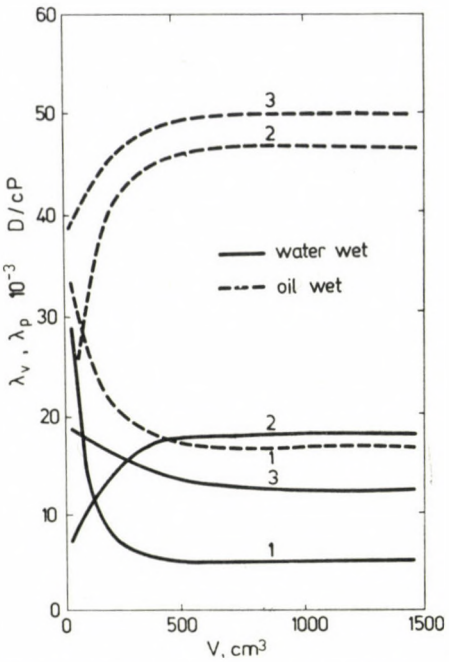

Fig. 2. Change in mobility in function of the volume of the fluid injected

found in that a rearrangement of the accumulated gels, which had triggered off the mechanical entrapment of negligible extent, takes place with the ceasing of flow then, with the flow of distilled water, a new sweep may again be observed. This, however, is less than the previous one. (After reaching a higher value, the mobility comes to equilibrium again.)

From the above said it may be seen that in an oil wet system, no expansion–contraction type transformation of the gel coils takes place as a result of their ionic strength. However, irrespective of the theoretical importance of the cause of the anomaly, it may be stated right now that in a porous system with oil wet character the greatest advantage of the application of polymers, namely that the mobility of the phases carrying the polymeric slug also decrease, is missing.

Change in the resistance factor

The difference between the two systems is likewise apparent from the change in the resistance factor (Fig. 3). In a water wet system, the polymer solution moves at a rate 10 times lower than that of the polymer-free water, the resistance factor being, on the other hand, only 3 in an oil wet core. The resistance factor of the carrier phases in the former lies at 3 – 4, reducing to

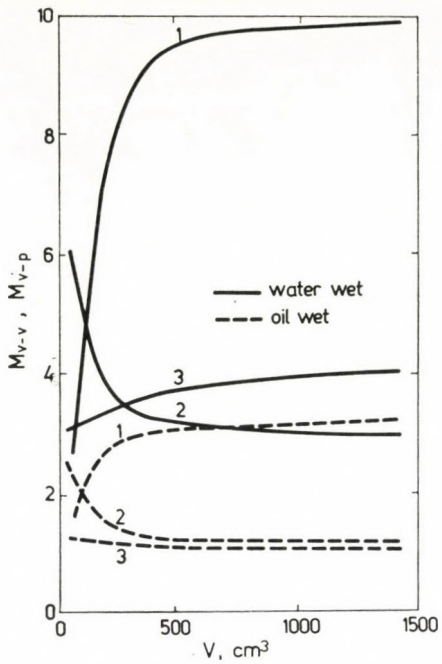

Fig. 3. Change in the resistance factor in function of the volume of the fluid injected

only 1.0 – 1.2 in the latter, i. e., there is no change in the mobility with respect to the latter.

The dynamic viscosity of the solution Separan AP-30 of the same concentration is nearly 3 times greater than that of the Separan NP-10 type. In spite of this, the mobility of the lower molecular weight, unhydrolized polymer as exhibited in the porous medium is 5 times greater than that of the polymer Separan AP-30. In connection with this, the mobility of the carrier phases also changes more advantageously following the injection of Separan NP-10. This surprising experimental finding is interpreted as that the greater the quantity of the polymer retarded in the system, the greater change takes place in the mobility, resistance factor, etc. of the flowing fluids, irrespective of the polymer type. According to the results obtained in studying the sorption losses, the specific loss of the Separan NP-10 on sandstone was $1.5 \cdot 10^{-7}$ g/cm², that of the Separan AP-30 polymer, $0.77 \cdot 10^{-7}$ g/cm². Likewise, there is a great difference between the sorption losses measured on limestone [6].

It may be seen from the laboratory results that

a) the quantity of the polymer sorbed plays a determining role concerning the permanent changes measurable with respect to the carrier phases, and

b) flow characteristics similar to those of the solution of high molecular weight, hydrolized polymers may be obtained with low molecular weight, unhydrolized polymers, even without the absolute need to change the concentration.

These statements are, of course, valid for a water wet and compatible system. No such phenomenon and possibility exists in systems, where mechanical entrapment plays an important role (systems of great heterogeneity or with the average pore size and gel size falling into the same order, for example). In this case, the higher the molecular weight and degree of hydrolization, the greater will be the reduction in mobility and the increase in the resistance factor observed in the porous medium.

The change in the apparent permeability

It is very easy to differentiate between the two cores with different wettability in Fig. 4. In an oil wet system, the decrease in the apparent permeability as compared to the polymer solution is only 10 mD, the decrease being four times that amount in the water wet case. The original 10 mD decrease in apparent permeability is certainly due to a mechanical entrapment of negligible extent, then the re-establishment of the initial condition is triggered off by the above mentioned rearrangement and the gradual sweeping out of the polymer. After all, it can be stated definitive that in an oil wet system no substantial change occurs in the permeability to either the polymer solution or to the carrier phase (connate water, distilled water).

The change in the apparent viscosity and the viscosity ratio

The results obtained up to the present are rendered unequivocal by Figs 5 and 6, in which changes in the apparent viscosity and viscosity ratio are plotted in function of the fluid volume injected.

On the basis of the two figures it may be observed that

a) in a water wet system the phenomena discussed earlier are the characteristic ones: the apparent viscosity of the polymer solution and carrier phases shown in porous media is 3 to 6 times higher then the viscosity value obtained in laboratory, whereas

b) the viscosities of fluids flowing in oil wet media are practically the same as measured in laboratory ($VR \sim 1$).

When using an oil wet rock, the deviation from unity of the viscosity ratio is due here also to the mechanical entrapment of negligible extent rather than to the non-Newtonian type of the polymer solution. In the opposite case,

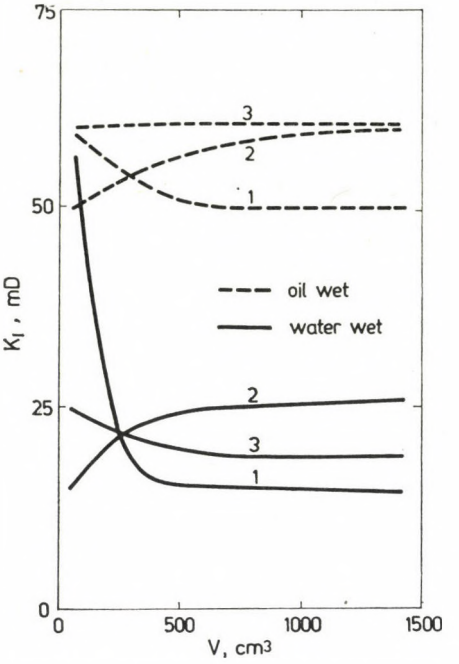

Fig. 4. Change in the apparent permeability in function of the fluid volume injected

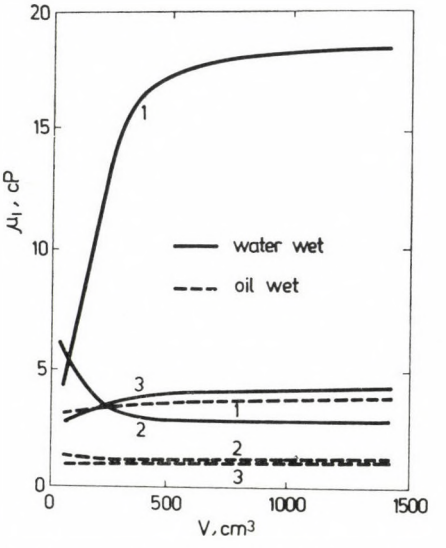

Fig. 5. Change in the apparent viscosity in function of the fluid volume injected

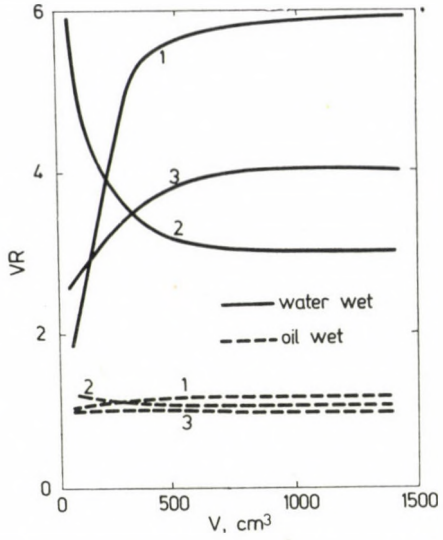

Fig. 6. Change in the viscosity ratio in function of the fluid volume injected

the curve obtained for the connate water with no polymer content would start from a value of $VR = 1.2$.

Other consideration

The experimental results summed up in Figs 2 – 6 are reviewed in Table II. It may be stated that in an oil wet system the flow of the polymer solution advancing connate water, and distilled water, respectively, takes place with its initial viscosity and mobility, further, that in the porous media no apparent permeability reduction occurs. In knowledge of the close relationship between the polymer sorbed on the rock surface and the flow characteristics, this phenomenon may be traced back to the absence of an irreversibly fixed gel layer. On an oil wet surface the quantity of polymer bound by chemisorption, regardless of the type, is zero, and thus the sweeping effect of the carrier phase is perfects.

As regards industrial application of the method, the phenomenon entails disadvantageous consequences. In an oil wet reservoir, an identical hydrodynamical effect can be approximated only if the apparent viscosity exhibited in the water wet medium is compensated by an increase in the laboratory viscosity of the solution employed. This actually requires the use of 3 – 6 times as much of polymer. Even then, the permanent decrease in the apparent permeability and the permanent improvement of the flow characteristics of the carrier phase cannot be guaranteed.

From the point of view of the flow characteristics of reservoir fluids, the wettability in the reservoir are of primary importance. It seems to be established, on the basis of the fundamental works of BROWN and FATT [9], BOBEK et al. [10], JOHANSEN and DUNNING [11], IVANKOV [12], SCHMIED [13], OWENS and ARCHER [14], MUNGAN [15], and SALATHIEL [16] that the wettability in a reservoir is of heterogeneous character, that is, the small pores and consolidation points are water wet, while the inner surface of the large pores belongs to the oil wet type. As a result of this, it may be expected that the sorption distribution of the polymer injected into the

Table II

Flow characteristics of the polymer solution and carrier phases in water wet and oil wet systems

Change in flow characteristic	Polymer solution		Connate water	
	Water wet	Oil wet	Water wet	Oil wet
	system		system	
Mobility	very great	moderate	great	little
Resistance factor	very great	moderate	great	none
Apparent permeability	very great	little	great	none
Apparent viscosity	very great	very little	great	none
Viscosity ratio	very great	very little	great	none

reservoir will likewise be characterized by heterogeneity. Thus, the features of the polymer solutions have to change forcibly between wide limits: in the large pores only the viscosity increasing effect will manifest itself, while the viscosity increasing effect and the decrease in the apparent permeability will operate jointly in the small pores. A somewhat different situation arises in the case of the carrier phases. After the injection of a certain amount of polymer-free fluid, the water passes through the large oil wet pores with its original mobility, while flow characteristics similar to those of the polymer solution occur in small, water wet pores only. In addition to this, the general flow pattern characteristic of the rock is also depending on whether the system has a continuous water wet channel or an oil wet one.

In a porous system, the flow patterns which characterize the polymer solution and carrier phases appear simultaneously. However, if we know the features characteristic for the absolute water wet, oil wet and natural state respectively, of a given rock, then useful information may be gained on the wettability or structural build-up existing in the natural sandstone, on the basis of the behaviour of the polymer solutions and carrier phases.

Thus, for example, if the flow characteristics of the carrier phases do approximate those of the polymer solutions, i. e., the permanent change in resistance of the system is very great, then the assumption may be made that

a) the rock is a strongly water wet one;

b) the proportion of small pores is greater than that of the large ones.

If, on the other hand, the change in resistance is small, then the probability exists of

a) the rock having an intermediate surface character, eventually an oil wet one;

b) the pore volume of the rock being made up mainly of large-sized pores exceeding by magnitudes the average size of the gel coils.

Finally, it will again be emphasized that the statements made in the present publication refer absolutely to compatible systems only. In the case of a mechanical polymer entrapment, other phenomena too become involved.

REFERENCES

1. Richardson, J. G.—Perkins, F. M.—Osoba, J. S.: Differences in behavior of fresh and aged East Texas Woodbine cores. *Trans. of AIME*, 204 (1955), 86.
2. Smith, F. W.: The behavior of partially hydrolyzed polyacrylamide solutions in porous media. *J. Pet. Techn.*, 1970, 148.
3. Mungan, N.—Smith, F. W.—Thompson, J. L.: Some aspects of polymer floods. *J. Pet. Techn.*, 1966, 1143.
4. Jennings, R. R.—Rogers, J. H.—West, T. J.: Factors influencing mobility control by polymer solutions. *J. Pet. Techn.*, 1971, 391.
5. Sarem, A. M.: On the theory of polymer solution flooding processes. SPE Preprint Ser., No. 3002.
6. Lakatos, I.—Lakatosné-Szabó, J.: Poliakrilamid oldatokkal történő kőolajkiszorítás mechanizmusának fizikai-kémiai vizsgálata, 2. rész (Examination of liquid–liquid and liquid–solid interfacial phenomena — Part 2). *Kőolaj és Földgáz*, 106 (1973), 336.
7. Holbrook, O. C.—Bernard, G. G.: Determination of wettability by dye adsorption. *Trans. of AIME*, 213 (1958), 261.
8. Lakatos, I.: Poliakrilamid oldatokkal történő kőolajkiszorítás mechanizmusának fizikai-kémiai vizsgálata 3. rész (Physico-chemical examination of the displacement mechanism using polyacrylamide solutions — Part 3). *Kőolaj és Földgáz*, 107 (1974), 309.
9. Brown, R. J. S.—Fatt, I.: Measurement of fractional wettability of oil field rocks by NMR methods. *J. Pet. Techn.*, 1956, 262.
10. Bobek, J. E.—Mattax, C. C.—Denekas, M. O.: Reservoir rock wettability — its significance and evaluation. *Trans. of AIME*, 213 (1958), 155.
11. Johansen, R. T.—Dunning, H. N.: Relative wetting tendencies of crude oils by the capillarimetric method. *Prod. Monthly*, 9 (1959), 20.
12. Iwankov, E. N.: A correlation of interistitial water saturation and heterogeneous wettability. *Prod. Monthly*, 10 (1960), 18.
13. Schmied, C.: Über den Benetzungszustand von Erdölspeichergesteinen und experimentelle Untersuchungen der Auswirkung von Veränderungen des Benetzungszustandes von Gesteinproben. *Erdöl und Kohle*, 8 (1964), 605.
14. Owens, W. W.—Archer, D. L.: The effect of rock wettability on oil-water relative permeability relationship. *J. Pet. Techn.*, 1971, 873.
15. Mungan, N.: Relative permeability measurements using reservoir fluids. *Soc. Pet. Eng. J.*, 5 (1972), 398.
16. Salathiel, R. A.: Oil recovery by surface film drainage in mixed wettability rocks. SPE Prepint Ser., No. 4104.

РОЛЬ СМАЧИВАЕМОСТИ ПРИ ПОЛИМЕРНОЙ ВЫТЕСНЕНИИ НЕФТИ

И. Й. ЛАКАТОШ

РЕЗЮМЕ

В статье показано различие особенностей течения полимерных растворов и несущих фаз (пластовой воды) в водовлажной и нефтевлажной пористой системе. По лабораторным результатам в нефтевлажной системе жидкости (полимерный раствор, пластовая вода, дестилированная вода) перемещаются оригинальной мобильностью и вязкостью, видимое уменьшение проводимости не замечается ни против полимерных растворов, ни против несущих фаз, свободных от полимеров. Прочину этого явления можно привести к тому, что сорбционная задержка полиакриламидов в нефтевлажной пористой системе равна нулю или влияние выноса несущих фаз полна.

Неблагоприятным последствием меньшего отношения мобильности и видимого уменьшения проводимости в нефтевлажной или промежуточной системе является то, что коэффициент полезного действия выжимки и затопления, характеризующий водовлажную систему может быть достигнут только большими затратами (количеством полимеров). Исследования и определения относятся к компатибильной системе полимер-горная порода, где механической задержкой полимеров можно пренебречь.

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ALLGEMEINER FALL DER GEMEINSAMEN AUSGLEICHUNG DER HÖHENWERTE UND GESCHWINDIGKEITEN EINES NIVELLEMENTS

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Die Ausgleichung von Nivellierungsnetzen, wo alle Nivellierungslinien, die zwei Knotenpunkte verbinden, geschlossene Polygone bilden und je zweimal beobachtet worden sind, wird verallgemeinert. Bei dieser Verallgemeinerung ist eine Verbindungslinie mehr als zweimal, oder eine Verbindungslinie weniger als zweimal beobachtet worden. Im zweiten Falle können besondere Bedingungen und Umstände auftreten.

Man ist der Ansicht, falls eine Karte mit Linien von gleichen Vertikalgeschwindigkeiten der Erdoberfläche vorliegt, und daraus längs der Nivellierungslinien die Geschwindigkeiten entnommen werden, so könnte man damit die beobachteten Höhenunterschiede auf eine und dieselbe Epoche reduzieren, worauf dann die Ausgleichung des Nivellements in *gewöhnlicher* Weise durchführbar wäre. Dies ist leider nicht möglich, da die Reduktionen auf die Epoche aus einer *vorhergehenden* Ausgleichung stammen und wir sollten daher die entsprechende *Korrelationsmatrix* kennen, die ist aber *unbekannt*. Deshalb ist es vorteilhafter, ja sogar notwendig, die Höhenwerte und Geschwindigkeiten — sei es nach vermittelnden Beobachtungen, oder nach Bedingungen — aus einer *gemeinsamen* Ausgleichung abzuleiten. Der Verfasser hat sich die Aufgabe gestellt, unter der Voraussetzung konstanter Geschwindigkeiten, sowohl nach vermittelnden Beobachtungen, wie auch nach Bedingungen sie zu lösen.

Die Bezeichnungen sind die folgenden:

T_0	— die Normalepoche, auf welche die Beobachtungen reduziert werden
i, j	— benachbarte markante oder Knotenpunkte
$T_p = T_0 + t_p$, $T_q = T_0 + t_q$	— Beobachtungsepochen
H_{0i}, H_{0j}	— vorläufige Höhenwerte
dH_{0i}, dH_{0j}	— Verbesserung der vorläufigen Höhenwerte für die Normalepoche
H_i, H_j	— ausgeglichene Höhenwerte für die Normalepoche
V_i, V_j	— ausgeglichene Vertikalgeschwindigkeiten
$h_{ij,p}, h_{ij,q}$	— beobachtete Höhenunterschiede
$v_{ij,p}, v_{ij,q}$	— Verbesserungen der beobachteten Höhenunterschiede.

Wir berufen uns auf die folgenden Arbeiten [1, 2, 3]. Laut Gl.(2) der Arbeit [1] sind die *Beobachtungsgleichungen* zwischen den Punkten i und j:

$$\begin{aligned} -1 \cdot dH_i + 1 \cdot dH_j - t_p \cdot V_i + t_p \cdot V_j &= [h_{ij,p} - (H_{0j} - H_{0i})] + v_{ij,p} \\ -1 \cdot dH_i + 1 \cdot dH_j - t_q \cdot V_i + t_q \cdot V_j &= [h_{ij,q} - (H_{0j} - H_{0i})] + v_{ij,q}. \end{aligned} \tag{1}$$

Sämtliche Verallgemeinerungen der Ausgleichung nach vermittelnden Beobachtungen sind ohne Schwierigkeiten, deshalb wollen wir uns damit weiter nicht befassen. Jedoch müssen die Fälle, wenn manche Nivellierungslinien ausfallen, in Betracht gezogen werden, denn sonst könnte man auf einen unreellen Fall stößen.

Jetzt behandeln wir die Ausgleichung nach *Bedingungen*. Aus (1) folgt durch Elimination

$$-V_i + V_j = \frac{h_{ij,q} - h_{ij,p}}{t_q - t_p} - \frac{v_{ij,p}}{t_q - t_p} + \frac{v_{ij,q}}{t_q - t_p} \tag{2}$$

$$\begin{aligned} -dH_i + dH_j = \frac{t_q [h_{ij,p} - (H_{0j} - H_{0i})]}{t_q - t_p} - \frac{t_p [h_{ij,q} - (H_{0j} - H_{0i})]}{t_q - t_p} + \\ + \frac{t_q \cdot v_{ij,p}}{t_q - t_p} - \frac{t_p \cdot v_{ij,q}}{t_q - t_p}. \end{aligned} \tag{3}$$

Wurde das Netz zweimal nivelliert, so werden die beiden Bedingungen: die Summe der ausgeglichenen relativen Geschwindigkeiten soll Null sein $\Sigma(-V_i + V_j) = 0$, die Summe der ausgeglichenen Höhenunterschiede soll Null sein $\Sigma(-dH_i + dH_j) = 0$. Auf diese Weise erhalten wir die gesuchten *zwei* Bedingungsgleichungen für ein geschlossenes Polygon:

$$-\Sigma \frac{v_{ij,p}}{t_q - t_p} + \Sigma \frac{v_{ij,q}}{t_q - t_p} + \left[\Sigma \frac{h_{ij,q} - h_{ij,p}}{t_q - t_p} \right] = 0 \tag{4}$$

$$\Sigma \frac{t_q \cdot v_{ij,p}}{t_q - t_p} - \Sigma \frac{t_p \cdot v_{ij,q}}{t_q - t_p} + \left[\Sigma \frac{t_q \cdot h_{ij,p} - t_p \cdot h_{ij,q}}{t_q - t_p} \right] = 0, \tag{5}$$

wobei die zweite Gleichung durch eine Kürzung aus (3) stammt.

Wir müssen mindestens eine Pegelstation haben. Wenn wir mehr als eine Pegelstation haben, so ist für jede zweite, dritte usw. Pegelstation für die gegebene Höhe und die gegebene Geschwindigkeit der entsprechenden Pegelstation je eine Bedingungsgleichung nötig.

Wir werden zuerst den einfachen Fall betrachten: es möge *eine* Nivellierungslinie *mehr* als zweimal, etwa dreimal, viermal usw. nivelliert worden sein; dazu kommen *noch* eine, zwei usw. Bedingungsgleichungen. Beispiels-

weise wenn die Linie i, j in drei Epochen p, q, u nivelliert worden ist, so muß (2) dreimal aufgeschrieben werden:

$$-V_i + V_j = \frac{h_{ij,q} - h_{ij,p}}{t_q - t_p} - \frac{v_{ii,p}}{t_q - t_p} + \frac{v_{ij,q}}{t_q - t_p} \tag{6a}$$

$$-V_i + V_j = \frac{h_{ij,u} - h_{ij,q}}{t_u - t_q} - \frac{v_{ij,q}}{t_u - t_q} + \frac{v_{ij,u}}{t_u - t_q} \tag{6b}$$

$$-V_i + V_j = \frac{h_{ij,u} - h_{ij,p}}{t_u - t_p} - \frac{v_{ij,p}}{t_u - t_p} + \frac{v_{ij,u}}{t_u - t_p} . \tag{6c}$$

Die Differenz von je zwei Gleichungen gibt die zusätzliche *neue* Bedingungsgleichung:

$$(6^b) - (6^a) = 0 \text{ oder } (6^c) - (6^b) = 0, \text{ oder } (6^c) - (6^a) = 0 . \tag{7}$$

Hier entstand tatsächlich *eine* Bedingungsgleichung, da sich alle drei Gleichungen (7) durch einen Multiplikationsfaktor unterscheiden. Wenn vier Verbindungslinien vorhanden sind, so haben wir der Gl. (7) ähnlich fünfzehn Gleichungen, von denen nur *zwei* voneinander unabhängig sind, die aber naturgemäß in verschiedener Form geschrieben werden können. Im allgemeinen Fall haben wir bei n beobachteten Verbindungslinien zwischen zwei Punkten $n-2$ Bedingungsgleichungen.

Der Fall mit Vergrößerung der Anzahl der Verbindungslinien zwischen zwei Knotenpunkten wird nicht von der Struktur des Nivellierungsnetzes beeinflußt, und demnach wird er mit mehr als zweimal nivellierter Verbindungslinie *allein* für sich behandelt.

Nun betrachten wir den umgekehrten, den viel schwierigeren Fall: wir *verringern* allmählich die Anzahl der Verbindungen zwischen zwei Knoten punkten. Es wird vom Grundfall in Abb. 1 ausgegangen, wobei *zwei* Bedingungen etwa (4) und (5) vorhanden sind. In einem *geschlossenen* Polygon mit *einer* einfach beobachteten Linie (Abb. 2) haben wir nur *eine* Bedingung die weder (4), noch (5) sein kann, und eigens zu bilden ist. Bei Abb. 3, in einem *geschlossenen* Polygon mit *zwei* einfach beobachteten Linien, oder in einem *offenen* Polygon (Abb. 4), nur mit lauter *zweifachen* Verbindungen, haben wir *keine* Bedingung und dennoch können die Höhenwerte und Geschwindigkeiten berechnet werden. Schließlich in Abb. 5, in einem *geschlossenen* Polygon mit *drei* einfach beobachteten Linien, oder in Abb. 6 in einem *offenen* Polygon

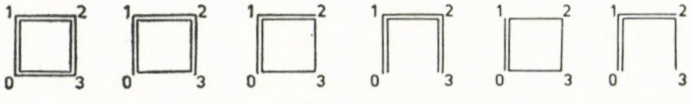

Abb. 1–6

mit einer einfach beobachteten Linie, können nicht alle Höhenwerte und Geschwindigkeiten berechnet werden.

Wir haben eine besondere Bedingung für den Fall in Abb. 2 zu bilden. Alle Verbindungslinien h_{mn} sind sowohl in der Epoche t_n wie auch in der Epoche t_s beobachtet worden und nur die Verbindungslinie h_{ij} ist allein in der Epoche p, aber *nicht* in der Epoche q beobachtet worden. Nehmen wir eine *fiktive* Verbindungslinie ij, q an und bilden:

$$t_p \times (4) + (5) = 0\,, \tag{8}$$

womit wir den Zusammenhang:

$$\begin{aligned} &-t_p \Sigma' \frac{v_{mn,r}}{t_s - t_r} - t_p \frac{v_{ij,p}}{t_q - t_p} + t_p \Sigma' \frac{v_{mn,s}}{t_s - t_r} + t_p \frac{v_{ij,q}}{t_q - t_p} + \\ &+ \Sigma' \frac{t_p h_{mn,s} - t_p h_{mn,r}}{t_s - t_r} + \frac{t_p h_{ij,q} - t_p h_{ij,p}}{t_q - t_p} + \\ &+ \Sigma' \frac{t_s v_{mn,r}}{t_s - t_r} + \frac{t_q v_{ij,p}}{t_q - t_p} - \Sigma' \frac{t_r v_{mn,s}}{t_s - t_r} - \frac{t_p v_{ij,q}}{t_q - t_p} + \\ &+ \Sigma' \frac{t_s h_{mn,r} - t_r h_{mn,s}}{t_s - t_r} + \frac{t_q h_{ij,p} - t_p h_{ij,q}}{t_q - t_p} = 0 \end{aligned} \tag{9}$$

erhalten, oder nach Reduktion:

$$\begin{aligned} &\Sigma' \left\{ (h_{mn,r} + v_{mn,r}) + \frac{t_p - t_r}{t_s - t_r} \left[(h_{mn,s} + v_{mn,s}) - (h_{mn,r} + v_{mn,r}) \right] \right\} + \\ &+ (h_{ij,p} + v_{ij,p}) = 0. \end{aligned} \tag{10}$$

Die *fiktive* Verbindungslinie $(h_{ij,q} + v_{ij,q})$ fällt also aus.

Die Bedingung ist, daß die Summe der ausgeglichenen gerade auf die Epoche $T_0 + t_p$ reduzierten Höhenunterschiede Null sein soll. Statt Gl. (10) kann auch der folgende Zusammenhang geschrieben werden:

$$\begin{aligned} &\Sigma' \frac{t_s - t_p}{t_s - t_r} \cdot v_{mn,r} + \Sigma' \frac{t_p - t_r}{t_s - t_r} \cdot v_{mn,s}, + v_{ij,p} + \\ &+ \Sigma' \frac{(t_s - t_p) h_{mn,r} + (t_p - t_r) h_{mn,s}}{t_s - t_r} + h_{ij,p} = 0\,. \end{aligned} \tag{11}$$

Zum Schluß betrachten wir Abb. 7. Obwohl sie ziemlich kompliziert ist, ist sie dennoch instruktiv. Das Polygon I gibt allein keine Bedingung, das Polygon II gibt allein 2 Bedingungen (eine kommt von der Linie 4.5), das Polygon III gibt allein keine Bedingung, und das Polygon IV gibt allein

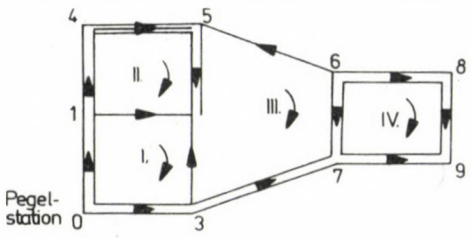

Abb. 7

2 Bedingungen. Insgesammt erhalten wir damit $0 + 2 + 0 + 2 = 4$ Bedingungen. Das ist jedoch nicht richtig, denn wir haben für das ganze Netz in Wirklichkeit $24 - 18 = 6$ Bedingungen. In der Tat geben beide Polygone I + II 3 Bedingungen, etwa 1 Bedingung für die Linie 4.5 in der Form (7), 1 Bedingung für das Polygon II in der Form (11) und 1 Bedingung für das Polygon I + II ebenfalls in der Form (11). Das Po'ygon III gibt zwar allein keine Bedingung, jedoch das Polygon I + II + III gibt noch 1 Bedingung in der Form (11). Schließlich das Polygon IV gibt die gewöhnlichen 2 Bedingungen (4) und (5), also erhalten wir tatsächlich 6 Bedingungen. Diese 6 Bedingungen können auf verschiedene Weise geschrieben werden.

Netze wie in Abb. 7 kommen dann vor, wenn wir Staatsnetze miteinander verbinden. In einem Staatsnetz können ja mehrfache Nivellierungslinien vorhanden sein, und an den Grenzen können aber auch einfache Verbindungslinien auftreten.

Zahlenbeispiel

Wir nehmen ein fiktives Beispiel in Zusammenhang mit den Polygonen I und II in Abb. 7. Der Einfachheit halber nehmen wir die Gewichte aller Nivellierungsstrecken für gleich, nämlich für 1 an. Mit diesem Beispiel soll die besondere Art der allgemeinen Ausgleichung nach Bedingungen gezeigt werden. Die fünfte Dezimalstelle ist die *letzte* Stelle.

Der *Ausgangspunkt* 0 soll

$$H_0 = 0 \qquad \text{und} \qquad V_0 = 0 \qquad \text{sein.}$$

Als genäherte Koordinaten setzen wir:

	m	*mm*
H_{01}	= 15.875.2	
H_{02}	= 31.542.7	
H_{03}	= 24.251.5	
H_{04}	= 23.152.2	
H_{05}	= 25.417.7 .	

Die beobachteten Höhenunterschiede mit den Beobachtungsepochen, wobei auch *die Reste nach* der Ausgleichung nach vermittelnden Beobachtungen hinzugefügt wurden, sind:

		m *mm*		*a*		*mm*
1.	$h_{01,p}$	15.869.75	p_{01}	−5.5 *r*	$v_{01,p}$	−0.87918
2.	$h_{01,q}$	15.880.40	q_{01}	+5.3 *s*	$v_{01,q}$	−0.01660
3.	$h_{12,p}$	15.655.15	p_{12}	−5.4	$v_{12,p}$	+0.03434
4.	$h_{32,p}$	7.279.80	p_{32}	−5.3	$v_{32,p}$	+0.89578
5.	$h_{03,p}$	24.243.05	p_{03}	−5.2 *r*	$v_{03,p}$	+0.90422
6.	$h_{03,q}$	24.260.10	q_{03}	+5.4 *s*	$v_{03,q}$	−0.00844
7.	$h_{14,p}$	7.267.70	p_{14}	−4.8 *r*	$v_{14,p}$	−0.97560
8.	$h_{14,q}$	7.289.60	q_{14}	+5.5 *s*	$v_{14,q}$	+0.04550
9.	$h_{45,p}$	2.258.05	p_{45}	−4.7 *r*	$v_{45,p}$	−0.26890
10.	$h_{45,q}$	2.260.20	q_{45}	−3.7	$v_{45,q}$	−0.79266
11.	$h_{45,u}$	2.274.40	u_{45}	+5.6 *s*	$v_{45,u}$	+0.13147
12.	$h_{52,p}$	6.132.90	p_{52}	−4.6 *r*	$v_{52,p}$	−0.99361
13.	$h_{52,q}$	6.115.80	q_{52}	+5.7 *s*	$v_{52,q}$	+0.06352.

Ausgleichung nach vermittelnden Beobachtungen

Die *Beobachtungsgleichungen* nach (1) lauten: (s. S. 343)
da wir für 5 Beobachtungspunkte je 2 Unbekannten, d. h. $5 \times 2 = 10$ Unbekannten haben.

Die *Lösung* der Normalgleichungen gibt die Verbesserungen der Höhenwerte — daraus die *ausgeglichenen Höhen* — und die *Geschwindigkeiten*:

		mm		*m* *mm*		*mm/a*
1.	dH_1	−0.46629,	H_1	15.874.37371	V_1	+1.06598
2.	dH_2	−0.39413,	H_2	31.542.30587	V_2	+3.36002
3.	dH_3	+0.37065,	H_3	24.251.87065	V_3	+1.52239
4.	dH_4	−0.06021,	H_4	23.152.13979	V_4	+3.29133
5.	dH_5	−0.13574,	H_5	25.417.56426	V_5	+4.91758.

Ausgleichung nach Bedingungen

Wir bilden die erste Bedingungsgleichung nach (6) und (7) für die Linie 4.5 — nämlich $(p, q) - (q, u)$ —, dann die zweite und die dritte Bedingungsgleichung nach (11) für das Polygon II bzw. I + II. In der Tat haben wir 13 Verbindungslinien, d.h. 13 Beobachtungen, und 5 Punkte zu bestimmen, *d. h.* 10 Unbekannten, also 13 − 10 = 3 Bedingungsgleichungen:

$$-1.00000\ v_{45,p} \quad +1.10753\ v_{45,q} \quad -0.10753\ v_{45,u} \quad +0.62312 = 0$$

$$+1.05825\ v_{14,p} \quad -0.05825\ v_{14,q} \quad +1.06796\ v_{45,p} \quad -0.06796\ v_{45,u}$$
$$+1.07767\ v_{52,p} \quad -0.07767\ v_{52,q} \quad -1.00000\ v_{12,p} \quad +2.44126 = 0$$

$$+0.98148\ v_{01,p} \quad +0.01852\ v_{01,q} \quad +1.04854\ v_{14,p} \quad -0.04854\ v_{14,q}$$
$$+1.05825\ v_{45,p} \quad -0.05825\ v_{45,u} \quad +1.06796\ v_{53,p} \quad -0.06796\ v_{52,q}$$
$$-1.00000\ v_{32,p} \quad -1.00943\ v_{03,p} \quad +0.00943\ v_{03,q} \quad +5.05467 = 0.$$

	dH_1	dH_2	dH_3	dH_4	dH_5	V_1	V_2	V_3	V_4	V_5	l	v
1.	+1					−5.5					= − 5.45	+ $v_{01,p}$
2.	+1					+5.3					= + 5.20	+ $v_{01,q}$
3.	−1	+1				+5.4	−5.4				= −12.35	+ $v_{12,p}$
4.		+1	−1				−5.3	+5.3			= −11.40	+ $v_{32,p}$
5.			+1					−5.2			= − 8.45	+ $v_{03,p}$
6.			+1					+5.4			= + 8.60	+ $v_{03,q}$
7.	−1			+1		+4.8			−4.8		= − 9.30	+ $v_{14,q}$
8.	−1			+1		−5.5			+5.5		= +12.60	+ $v_{14,q}$
9.				−1	+1				+4.7	−4.7	= − 7.45	+ $v_{45,p}$
10.				−1	+1				+3.7	−3.7	= − 5.30	+ $v_{45,q}$
11.				−1	+1				−5.6	+5.6	= + 8.90	+ $v_{45,u}$
12.		+1			−1		−4.6			+4.6	= + 7.90	+ $v_{52,p}$
13.		+1			−1		+5.7			−5.7	= − 9.20	+ $v_{52,q}$.

da wir 13 beobachtete Verbindungslinien haben.

Die *Normalgleichungen* sind:

	dH_1	dH_2	dH_3	dH_4	dH_5	V_1	V_2	V_3	V_4	V_5	
1.	+5.00	−1.00	0.00	−2.00	0.00	− 4.90	+ 5.40	0.00	− 0.70	0.00	= + 8.800
2.	−1.00	+4.00	−1.00	0.00	−2.00	+ 5.40	− 9.60	+ 5.30	0.00	− 1.10	= − 25.050
3.	0.00	−1.00	+3.00	0.00	0.00	0.00	+ 5.30	− 5.10	0.00	0.00	= + 11.550
4.	−2.00	0.00	0.00	+5.00	−3.00	− 0.70	0.00	0.00	− 2.10	+ 2.80	= + 7.150
5.	0.00	−2.00	0.00	−3.00	+5.00	0.00	− 1.10	0.00	+ 2.80	− 1.70	= − 2.550
6.	−4.90	+5.40	0.00	−0.70	0.00	+140.79	− 29.10	0.00	− 53.29	0.00	= −123.095
7.	+5.40	−9.60	+5.30	0.00	−1.10	− 29.16	+110.90	− 28.09	0.00	− 53.65	= + 38.330
8.	0.00	+5.30	−5.10	0.00	0.00	0.00	− 28.09	+ 84.29	0.00	0.00	= + 29.960
9.	−0.70	0.00	0.00	−2.10	+2.80	− 53.29	0.00	0.00	+120.43	− 67.14	= + 9.475
10.	0.00	−1.10	0.00	+2.80	−1.70	0.00	− 53.65	0.00	− 67.14	+120.79	= +193.245.

Daraus die *Normalgleichungen*:

k_1	k_2	k_3	
+2.23819	−1.06065	−1.05199	+0.62312 = 0
−1.06065	+4.43585	+3.40276	+2.44126 = 0
−1.05199	+3.40276	+6.35292	+5.05467 = 0.

Damit bekommen wir die *Korrelaten*:

$$k_1 = -0.71570, \qquad k_2 = -0.03433, \qquad k_3 = -0.89577.$$

Aus den Korrelatengleichungen errechnen wir die *Verbesserungen* und damit die *ausgeglichenen Höhenunterschiede*:

		mm		*m* *mm*		*a*
1.	$v_{01,p}$	−0.87918	$h_{01,p} + v_{01,p}$	15.868.87082	p_{01}	−5.5
2.	$v_{01,q}$	−0.01659	$h_{01,q} + v_{01,q}$	15.880.38341	q_{01}	+5.3
3.	$v_{12,p}$	+0.03433	$h_{12,p} + v_{12,p}$	15.655.18433	p_{12}	−5.4
4.	$v_{22,p}$	+0.89577	$h_{32,p} + v_{32,p}$	7.280.59577	p_{32}	−5.3
5.	$v_{03,p}$	+0.90422	$h_{03,p} + v_{03,p}$	24.243.95422	p_{03}	−5.2
6.	$v_{03,q}$	−0.00845	$h_{03,q} + v_{03,q}$	24.260.09155	q_{03}	+5.4
7.	$v_{14,p}$	−0.97558	$h_{14,p} + v_{14,p}$	7.266.72442	p_{14}	−4.8
8.	$v_{14,q}$	+0.04548	$h_{14,q} + v_{14,q}$	7.289.64548	q_{14}	+5.5
9.	$v_{45,p}$	−0.26891	$h_{45,p} + v_{45,p}$	2.257.78109	p_{45}	−4.7
10.	$v_{45,q}$	−0.79266	$h_{45,q} + v_{45,q}$	2.259.40734	q_{45}	−3.7
11.	$v_{45,u}$	+0.13147	$h_{45,u} + v_{45,u}$	2.274.53147	u_{45}	+5.6
12.	$v_{52,p}$	−0.99364	$h_{52,p} + v_{52,p}$	6.131.90636	p_{52}	−4.6
13.	$v_{52,q}$	+0.06354	$h_{52,q} + v_{52,q}$	6.115.86354	q_{52}	+5.7

So erhalten wir die ausgeglichenen *Geschwindigkeiten* und die ausgeglichenen *Höhenwerte*:

	mm/a	*m* *mm*
1.	+1.06598	15.874.37371
2.	+3.06002	31.542.30587
3.	+1.52239	24.251.87065
4.	+3.29133	23.152.13979
5.	+4.91758	25.417.56426.

Ein Vergleich mit dem entsprechenden Ergebnis aus der Ausgleichung nach vermittelnden Beobachtungen zeigt eine einwandfreie Übereinstimmung. Dieses Zahlenbeispiel bestätigt außer Zweifel, überzeugend, daß der allgemeine Fall der Ausgleichung von Höhenwerten und Geschwindigkeiten eines Nivellements unter der Voraussetzung konstanter Geschwindigkeiten, richtig gelöst ist.

SCHRIFTTUM

1. HRISTOW, W. K.: Über die Ableitung der Vertikalgeschwindigkeiten der Erdkruste aus zwei Nivellierungen nach Bedingungsgleichungen. *Acta Geod. Geoph. Mont. Hung.*, 11 (1976), 111–119

2. Hazay, I.: Die Ausgleichung von Nivellierungsnetzen für die Beobachtung der vertikalen Erdkrustenbewegung. Das Markscheidewesen in den sozialistischen Ländern, Miskolc, 3 (1969), 103—110.
3. Hristow, W. K.: Verbindung von zwei Nivellierungsnetzen zur Bestimmung von Vertikalgeschwindigkeiten. *Acta Geod., Geoph. et Mont. Hung., 10* (1925). 465—468.

ОБЩИЙ СЛУЧАЙ СОВМЕСТНОГО УРАВНИВАНИЯ ЗНАЧЕНИЙ ВЫСОТ И СКОРОСТЕЙ НИВЕЛИРОВАНИЯ

В. К. ХРИСТОВ

РЕЗЮМЕ

В статье обобщается уравнивание нивелирных сетей, где все нивелирные ходы заключены между двумя узловыми пунктами, образуют замкнутые полигоны и отнаблюдены каждый по два раза. При этом обобщении один ход измерен больше или меньше двух раз. Во втором случае могут возникать специальные условия и обстоятельства.

Acta Geodaet., Geophys. et Montanist. Acad. Sci. Hung. Tomus 13 (3—4), pp. 347—374 (1978)

DIMENSIONIERUNG UND UNTERSUCHUNG DER FUNKTIONSWEISE VON SEITENDÜSENEJEKTORPUMPEN

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Die Arbeit befaßt sich mit der Untersuchung des Arbeitsprinzips und der theoretischen Bestimmung der Kennlinie der Seitendüsenejektorpumpe, die einen Spezialtyp der Strahlpumpen darstellt. Die Seitendüsenejektorpumpe ist besonders geeignet, fluidisierten trockenen Feststoff oder Trüben, die grobkörnigen Feststoff enthalten, mit Hilfe von Wasser zu fördern. Die Studie enthält auch die Untersuchung der zweistufigen Wasserstrahltrübepumpe.

Im Aufsatz wird die energetische Kennzeichnung der Funktionsweise der Ejektorpumpe erörtert.

Eine Methode wird zur Dimensionierung der Strahltrübepumpen dargestellt, die anhand einer Verwendung im Bergbau veranschaulicht wird.

Bezeichnungen

d_i	Durchmesser des Querschnitts i	
c_i, c_i^*	Geschwindigkeit im Querschnitt i	
Q_i	Volumenstrom im Querschnitt i	
$\dot{m}_i$	Massestrom in Querschnitt i	
$\psi_m = \frac{\dot{m}_2}{\dot{m}_1}$	dimensionsloses Massestromverhältnis	
$\psi = \frac{\dot{Q}_2}{\dot{Q}_1}$	dimensionsloses Volumenstromverhältnis	
ε	dimensionslose Förderhöhe	
ε_p	dimensionslose Druckenergie	
ε_c	dimensionslose Geschwindigkeitsenergie	
ϱ_i	Dichte des Gemisches im Querschnitt i	
ϱ_{ij}	Durchschnittswert der Dichten in den Querschnitten i und j	
ϱ_s	Dichte des Feststoffes	
ϱ_f	Dichter der Flüssigkeit	
c_t	volumetrische Transportkonzentration	
c_{ti}	volumetrische Transportkonzentration im Querschnitt i	
ζ, ζ_0	Widerstandsbeiwert des Mischrohres	bei Strömung reiner Flüssigkeit, bzw. bei Gemischförderung
η_D, η_{D0}	Wirkungsgrad des Diffusors	
φ_1, φ_{10}	Verlustfaktor der Düse	
φ_2, φ_{20}	Verlustfaktor der Rohrverengung	
A_i	Fläche des Querschnitts i	
A	Querschnittsfläche des Mischraumes	
p_i	Druck im Querschnitt i	
p_0	atmosphärischer Druck	
p_t	Sättigungsdruck	

I, II	Indizes deuten an die Stufen I bzw. II hin
η, η_1, η_2	Wirkungsgrad der Ejektorpumpe bei den nützlichen Fördermengen $\dot{Q}$, $\dot{Q}_2$, $\dot{Q}_1+\dot{Q}_2$
η_{sz}	Wirkungsgrad der Treibpumpeneinheit
N_h	nützliche Leistung
N_b	zugeführte Leistung
H_1	Förderhöhe des Treibwassers
H_2	saugseitige Förderhöhe
H	hergestellte Förderhöhe
$\dot{Q}$	Volumenstrom der geförderten nützlichen Flüssigkeit
R^*	Strömungswiderstand in der Treibwasserleitung
R_2	Strömungswiderstand des Saugrohres
H_g^*, H^*	geodätische Förderhöhe, Förderhöhe in der Treibwasserleitung
N^*	durch die Treibpumpe aufgenommene Leistung
ϑ_1 ϑ_2 ϑ_3	Leistungskoeffizienten bei den nützlichen Fördermengen $\dot{Q}$, $\dot{Q}_2$, $\dot{Q}_1+\dot{Q}_2$
e, e_1, e_2	spezifischer Energieverbrauch bei den nützlichen Fördermengen $\dot{Q}$, $\dot{Q}_2$, $\dot{Q}_1+\dot{Q}_2$
k	Kavitationssicherheitsfaktor
ε_k	dimensionslose Förderhöhe zur Vermeidung der Kavitation

1. Erläuterung der Ejektorpumpentypen

Die Konstruktion, die Funktionsweise und die Betriebsverhältnisse der Ejektorpumpen, die auf gewöhnliche Weise ausgeführt sind und Wasser oder andere Flüssigkeit mit Hilfe von Wasser fördern, sind aus der Fachliteratur bekannt [1, 2 3]. Bei den Ejektorpumpen übergibt das Treibmedium, d. h. die Förderflüssigkeit, einen Teil ihres Energiegehaltes der zu fördernden Flüssigkeit. Nach der Vermischung beider Flüssigkeiten und dem Ausgleich der Flüssigkeitsenergien strömen beide Flüssigkeiten mit gleichem spezifischem Energiegehalt zum gewünschten Ort.

Die Förderflüssigkeit ist meistens Wasser oder ein anderes nicht absetzbares Gemisch, dessen Förderhöhe mittels Flüssigkeitspumpe unter günstigen Betriebsverhältnissen und bei gutem Wirkungsgrad auf den gewünschten Wert erhöht werden kann.

Auch die geförderte Flüssigkeit ist in der Regel Wasser, sie kann aber auch schlammiges, verschmutztes Wasser sein, da die Ejektorpumpe, die keine beweglichen Bestandteile enthält und von einfacher Ausführung ist, dagegen unempfindlich ist.

Die Verwendung der Ejektorpumpen gewinnt eine steigende Bedeutung auch für den Transport von Gemisch aus grobkörnigen Feststoffen mit einer Flüssigkeit oder Luft. Ihre Verwendung als Gemischförderpumpe wird vor allem durch die einfache, billige Konstruktion, die lange Lebensdauer und den geringen Raumbedarf begründet. Trotz ihres relativ kleineren Wirkungsgrades läßt sie sich im kontinuierlichen Betrieb wirtschaftlich verwenden, wenn das zu fördernde Gemisch stark abrasiv ist, da bei Verwendung von Kreiselpumpen teuere Bestandteile häufig zu ersetzen wären.

Bei den bekannten Ejektorpumpen geschieht die Energievermittlung mit Hilfe eines Wasserstrahles, der durch eine konzentrische Düse vom Kreis-

querschnitt, durch einen ringförmigen Seitenkanal oder durch Bohrungen bzw. Düsen in der Seitenwand des Gemischraumes eingeleitet wird (Abb. 1–4).

Die Ejektorpumpe in Abb. 1. hat die am meisten bekannte Ausführung und ist zur Förderung von reinen Flüssigkeiten oder von Gemischen aus Flüssigkeiten und sehr feinkörnigen Feststoffen geeignet. Die eine verstopfungsfreie Förderung gewährleistende zugelassene maximale Korngröße wird durch den ringförmigen freien Strömungsquerschnitt begrenzt, der durch die zentrale Düse verengt wird.

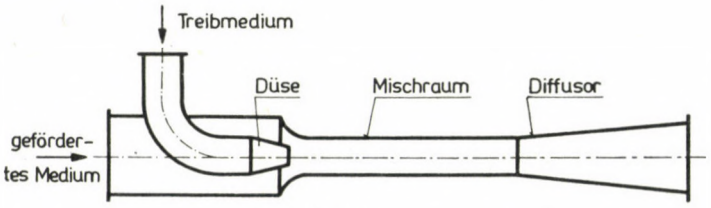

Abb. 1

Die Ejektorpumpe in Abb. 2 ist mit einem ringförmigen Seitenkanal ausgerüstet und kann als Trübepumpe mit geringer Förderhöhe gebraucht werden. Die maximale Korngröße bei dieser Pumpenart ist bereits durch den Durchmesser des Mischrohres bestimmt. Die Erzielung einer höheren Förderhöhe und eines besseren Wirkungsgrades ist durch den hohen Qualitätsanspruch an die Herstellung der ringförmigen Spalte erschwert.

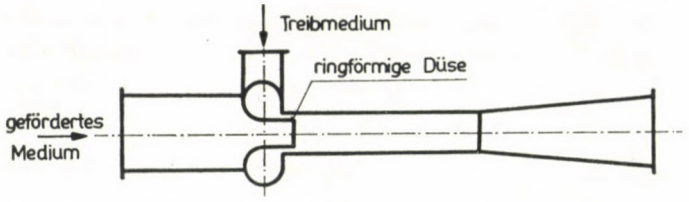

Abb. 2

Bei der Ejektorpumpe nach Abb. 3 wird die Treibflüssigkeit durch die Bohrungen in der Seitenwand des Mischraumes eingeleitet. Diese Ejektorpumpe verfügt über den Vorteil der vorigen Pumpenart: Der engste Strömungsquerschnitt ist nämlich auch in diesem Falle das Mischrohr, deshalb kann auch ein Gemisch mit relativ gröberen Körnern befördert werden. Die Seitenbohrungen entsprechen dagegen dem Begriff der Düse nicht, ihr Ausströmungsquerschnitt liegt nicht senkrecht zur Achse der Bohrungen. Der Wirkungsgrad dieses Pumpentyps ist deshalb ähnlich dem der Ejektorpumpe mit Seitenkanal.

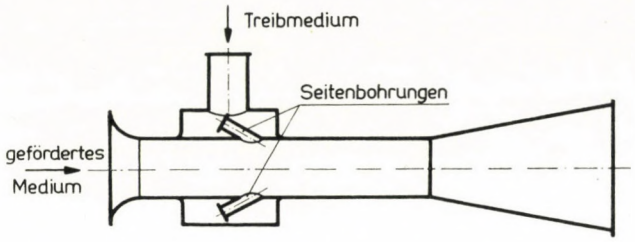

Abb. 3

Für Gemischförderung faßt die Konstruktion laut Abb. 4 die Vorteile aller erwähnten Ejektorpumpen zusammen. Hier werden strömungstechnisch richtig ausgeführte Düsen in die Bohrungen der Seitenwand des Mischrohres eingebaut.

Besonders bei größeren Förderhöhen ($H \geqq 40$ m) sollten die Ejektorpumpen zur Vermeidung der Kavitation mit einem so hohen Saugraumdruck betätigt werden, der in der Praxis nicht oder nur mit hohen Kosten erzeugt werden könnte. Bei Wasserförderung aus einem Brunnen sollte z. B. der Brunnen eigens deshalb tiefer abgeteuft werden, bei hydraulischer Förderung sollte der Flüssigkeitsspiegel über den Saugstutzen der Pumpe erhöht werden, oder bei Förderung von trockenem Material (Flugasche) sollte deshalb ein Überdruck im Behälter aufrechterhalten werden. Aufgrund der Obigen können die bekannten Ejektorpumpen höchstens eine Förderhöhe von $H = 60 - 80$ m herstellen. Um eine hohe Förderhöhe zu gewährleisten, kann eine zweistufige (Abb. 5), eventuell mehrstufige Ejektorpumpe konstruiert werden, die über Mischräume mit allmählich oder stufenartig abnehmenden Querschnitten verfügt. Die Energie, die in den Mischraum mit größtem Querschnitt eingeleitet wird, sichert die gute Saugfähigkeit, die weitere Energiezufuhr gewährleistet die notwendige hohe Förderhöhe. Auf diese Weise kann auch eine Förderhöhe von $H = 300$ m erreicht werden.

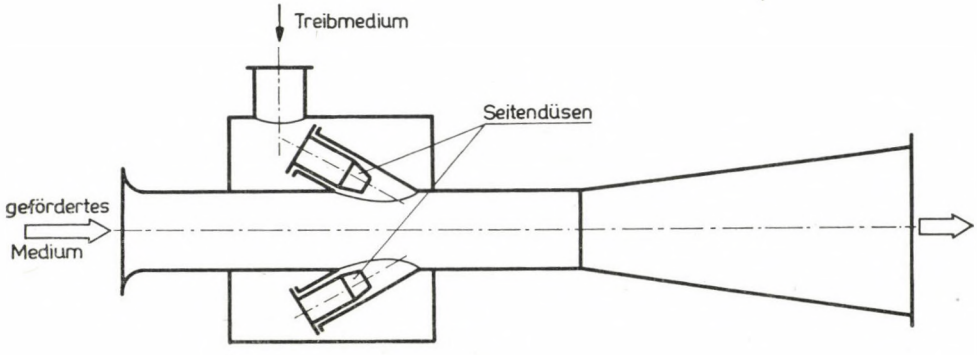

Abb. 4

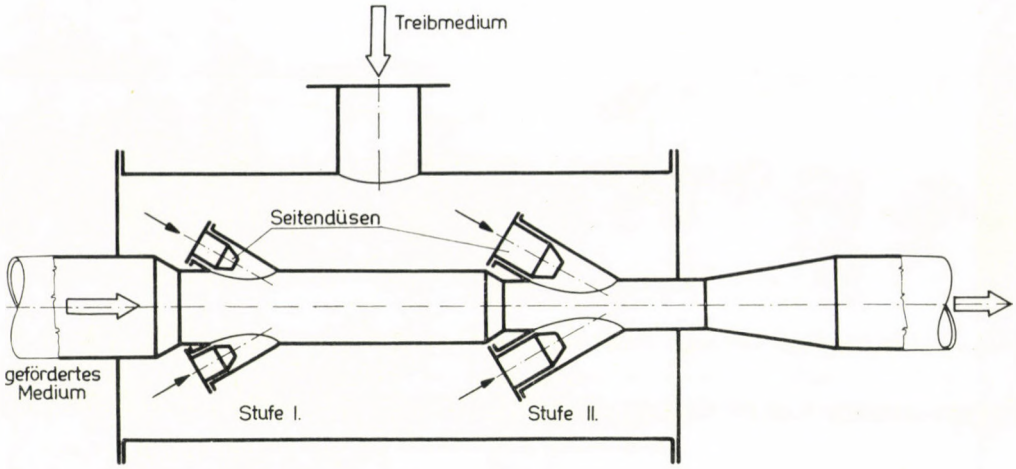

Abb. 5

In der Funktion der erläuterten Ejektorpumpen, in der Energieübergabe spielt der Zusammenstoß der zwei Flüssigkeiten von verschiedenen Strömungsenergiegehalten eine bedeutende Rolle. Das Verhältnis der Geschwindigkeiten beider Flüssigkeitsströme beträgt vor dem Zusammenstoß 10 – 20, das zur Befeuchtung von staubförmigen Materialien, zur Herstellung von Trüben aus körnigen Feststoffen und Wasser sehr vorteilhaft verwendet werden kann. Die Ausführung der Trübepumpe ist in Abb. 4 dargestellt. Das Saugrohr reicht in den Behälter oder ist zum Boden des Behälters geleitet, der das trockene staubförmige Material (eventuell im fluidisierten Zustand) enthält. Der durch die Ejektorpumpe entstehende Sog ruft eine Materialförderung im Luftstrom hervor, die die Umwelt nicht im geringsten Maße verschmutzt. Nach der Befeuchtung und Energievermittlung in der Ejektorpumpe strömt ein Dreiphasengemisch Luft-Wasser-Feststoff im Druckrohr. Dieses Gemisch soll zunächst in einem offenen Behälter entlüftet, danach die Konzentration auf einen geeigneten Wert eingestellt werden, worauf die Trübe (Wasser-Feststoff-Gemisch) an einen beliebigen Ort gefördert werden kann.

2. Allgemeine Untersuchung der Funktionsweise der Seitendüsenejektorpumpe

Zur Untersuchung soll die in Abb. 6 dargestellte einstufige Ejektorpumpe in Betracht gezogen werden, die aus dem sog. Mischrohr, aus dem Konfusor vor ihm, dem Diffusor hinter ihm, und aus den Düsen in der Seitenwand des Mischrohres besteht.

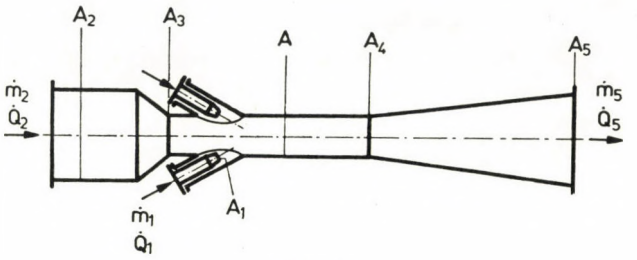

Abb. 6

Die Serienschaltung ähnlicher Einheiten kann in Kenntnis einer Stufe einfach berechnet werden, wodurch sich die technischen Kennwerte einer mehrstufigen Einheit bestimmen lassen.

Im Interesse einer allgemeinen Untersuchung soll angenommen werden, daß die durch die Querschnitte 1 und 2 in Abb. 6 durchströmenden Flüssigkeiten von verschiedener Qualität sind. Die durch den Querschnitt 2 einströmende Flüssigkeit kann sogar ein Flüssigkeit-Feststoff- oder Luft-Feststoff-Gemisch sein. Zur Kennzeichung der Menge des geförderten Flüssigkeitsstromes wird die dimensionslose Größe

$$\psi_m = \frac{\dot{m}_2}{\dot{m}_1} \tag{1}$$

unter Verwendung der Bezeichnungen in Abb. 6 gebraucht.

In Kenntnis der Dichte (ϱ), des Strömungsquerschnitts (A) und der Geschwindigkeit ist der Massestrom der Treib- oder Förderflüssigkeit:

$$\dot{m}_1 = \varrho_1 A_1 c_1 ; \tag{2}$$

der Massestrom der geförderten Flüssigkeit oder des Gemisches ($A_3 = A$):

$$\dot{m}_2 = \dot{m}_3 = \varrho_3 A c_3 ; \tag{3}$$

der Massestrom der ausströmenden Flüssigkeit ($A_4 = A$):

$$\dot{m}_4 = \dot{m}_1 + \dot{m}_2 = \varrho_4 A c_4 . \tag{4}$$

2.1. *Förderhöhe der Ejektorpumpen*

Zur Kennzeichnung der Förderhöhe der Ejektorpumpe wird wie gewöhnlich der dimensionslose Ausdruck

$$\varepsilon = \frac{p_5 - p_2}{p_1 - p_2} + \frac{c_5^2 - c_2^2}{p_1 - p_2} \frac{\varrho_{25}}{2} = \varepsilon_p + \varepsilon_c \tag{5}$$

eingeführt [1, 2, 4], in dem die absoluten statischen Drücke und Geschwindigkeiten im Querschnitt der Abb. 6 enthalten sind. Die Dichte ϱ_{25} ist die kennzeichnende oder reduzierte Dichte der strömenden Flüssigkeit oder des Gemisches in der Ejektorpumpe. Bei horizontalem Einbau der Ejektorpumpe stellt der Zähler von ε_p die Zunahme der Druckenergie und der Zähler von ε_c die der kinetischen Energie der geförderten Flüssigkeit dar.

Der Ausdruck im Nenner – die Druckabnahme der Förderflüssigkeit – kann mit Hilfe des Druckes p_3 der in das Mischrohr eintretenden Flüssigkeit bestimmt werden:

$$p_1 - p_2 = (p_1 - p_3) - (p_2 - p_3)\,, \tag{6}$$

wobei das erste Glied sich mittels des Bernoullischen Gesetzes bestimmen läßt:

$$p_1 - p_3 = \frac{c_1^2 - c_0^2}{2\varphi_1^2}\,\varrho_1 \cong \frac{c_1^2}{2\varphi_1^2}\,\varrho_1\,. \tag{7}$$

In diesem Zusammenhang wurde die Geschwindigkeit vor der Düse vernachlässigt. Unter den Größen der Gleichung ist c_1 die Geschwindigkeit im engsten Querschnitt der Düse, c_0 dagegen die im Querschnitt vor der Düse; p_1 ist der statische Druck vor der Düse und φ_1 der Verlustfaktor der Düse.

Die Druckabnahme der geförderten Flüssigkeit, die mit einer Geschwindigkeit c_3 in das Mischrohr eintritt, kann gleichfalls nach dem Bernoullischen Gesetz berechnet werden:

$$p_2 - p_3 = \frac{c_3^2 - c_2^2}{2\,\varphi_2^2}\,\varrho_{23}\,, \tag{8}$$

wobei φ_2 der Verlustfaktor der Rohrverengung, die Dichte ϱ_{23} die kennzeichnende Dichte der strömenden Flüssigkeit oder des Gemisches im Konfusor 2–3 ist.

Folgt man der obigen Methode, ergibt sich die durch die Ejektorpumpe hergestellte Druckzunahme für

$$p_5 - p_2 = (p_5 - p_4) + (p_4 - p_3) - (p_2 - p_3)\,. \tag{9}$$

Die ersten und zweiten Glieder in dieser Gleichung sind noch nicht bekannt.

Die im Diffusor entstehende Druckzunahme aus dem mit dem Druckverlust aufgeschriebenen Bernoullischen Gesetz:

$$p_5 - p_4 = \eta_D\,\frac{c_4^2 - c_5^2}{2}\,\varrho_{45}\,, \tag{10}$$

wobei η_D der Wirkungsgrad des Diffusors und ϱ_{45} die kennzeichnende Dichte des im Diffusor 4 – 5 strömenden Mediums ist.

Die Druckveränderung im Mischrohr kann mittels des Impulssatzes berechnet werden. Der Impulssatz wird für den Mischraum aufgeschrieben, der in Abb. 6 mit der gestrichelten Linie begrenzt ist:

$$\dot{m}_1 c_1 \cos\alpha + \dot{m}_2 c_3 - (\dot{m}_1 + \dot{m}_2) c_4 = A\left[(p_4 - p_3) + \zeta \frac{c_4^2}{2} \varrho_{34}\right]. \tag{11}$$

In diesem Zusammenhang soll bemerkt werden, daß die Schubspannungen am Zylindermantel bzw. die davon resultierenden Kräfte in Form des Druckverlustes in Betracht gezogen wurden, der mit dem Widerstandsbeiwert ζ des Mischrohers ausgedrückt werden kann. ϱ_{34} stellt dabei die charakteristische Dichte der strömenden Flüssigkeit dar.

Mit Hilfe des Kontinuitätsgesetzes können folgende Gleichungen in Kenntnis der Massenströme ($\dot{m}$) bzw. Volumenströme ($\dot{Q}$) und der geometrischen Abmessungen (A) aufgestellt werden:

$$\begin{aligned}
&\dot{m}_1 = \varrho_1 A_1 c_1 = \varrho_1 \dot{Q}_1 ;\\
&\dot{m}_2 = \varrho_2 A_2 c_2 = \varrho_2 \dot{Q}_2 = \varrho_3 \dot{Q}_3 = \varrho_3 A_3 c_3 = \dot{m}_3 ;\\
&\dot{m}_4 = \varrho_4 A c_4 = \varrho_4 \dot{Q}_4 = \varrho_5 \dot{Q}_5 = \varrho_5 A_5 c_5 = \dot{m}_5 ;\\
&\frac{c_2}{c_1} = \frac{A_1}{A_2}\frac{\dot{Q}_2}{\dot{Q}_1} = \frac{A_1}{A_2}\frac{\varrho_3}{\varrho_2}\frac{\dot{Q}_3}{\dot{Q}_1} = \frac{A_1}{A_2}\frac{\varrho_1}{\varrho_2}\frac{\dot{m}_2}{\dot{m}_1} ;\\
&\frac{c_3}{c_1} = \frac{A_1}{A}\frac{\dot{Q}_3}{\dot{Q}_1} = \frac{A_1}{A}\frac{\varrho_2}{\varrho_3}\frac{\dot{Q}_2}{\dot{Q}_1} = \frac{A_1}{A}\frac{\varrho_1}{\varrho_3}\frac{\dot{m}_2}{\dot{m}_1} ;\\
&\frac{c_4}{c_1} = \frac{A_1}{A}\frac{\dot{Q}_4}{\dot{Q}_1} = \frac{A_1}{A}\frac{\varrho_1}{\varrho_4}\frac{\dot{m}_4}{\dot{m}_1} = \frac{A_1}{A}\frac{\varrho_1}{\varrho_4}\left(1 + \frac{\dot{m}_2}{\dot{m}_1}\right) = \frac{A_1}{A}\frac{\varrho_1}{\varrho_4}\left(1 + \frac{\varrho_2}{\varrho_1}\frac{\dot{Q}_2}{\dot{Q}_1}\right);\\
&\frac{c_5}{c_1} = \frac{A_1}{A_5}\frac{\dot{Q}_5}{\dot{Q}_1} = \frac{A_1}{A_5}\frac{\varrho_1}{\varrho_4}\frac{\dot{m}_5}{\dot{m}_1} = \frac{A_1}{A_5}\frac{\varrho_1}{\varrho_5}\left(1 + \frac{\dot{m}_2}{\dot{m}_1}\right) = \frac{A_1}{A_5}\frac{\varrho_1}{\varrho_5}\left(1 + \frac{\varrho_2}{\varrho_1}\frac{\dot{Q}_2}{\dot{Q}_1}\right).
\end{aligned} \tag{12}$$

Nimmt man eine Gemischförderung und eine inkompressible Strömung an, ist die Dichte der einströmenden Flüssigkeit oder des Gemisches:

$$\varrho_2 = \varrho_s c_{t2} + \varrho_f(1 - c_{t2}) = \varrho_3 ; \tag{13}$$

wobei ϱ_2, ϱ_s und ϱ_f die Dichten des einströmenden Gemisches bzw. des in ihm enthaltenen Feststoffes und der Flüssigkeit sind, und c_{t2} die volumetrische Konzentration bezeichnet. Die Dichte der ausströmenden Flüssigkeit oder des Gemisches ist

$$\varrho_4 = \varrho_5 = \frac{\varrho_1 + \varrho_2 \psi}{1 + \psi}, \tag{14}$$

wobei ϱ_1 die Dichte der homogenen Förderflüssigkeit bedeutet.

Sind die Dichten bekannt, können die Geschwindigkeitsverhältnisse in folgender einfacher Form ausgedrückt werden:

$$\begin{aligned}
\frac{c_2}{c_1} &= \frac{A_1}{A_2}\,\frac{\dot{Q}_2}{\dot{Q}_1}\,; \\
\frac{c_3}{c_1} &= \frac{A_1}{A}\,\frac{\dot{Q}_2}{\dot{Q}_1}\,; \\
\frac{c_4}{c_1} &= \frac{A_1}{A}\left(1+\frac{\dot{Q}_2}{\dot{Q}_1}\right); \\
\frac{c_5}{c_1} &= \frac{A_1}{A_5}\left(1+\frac{\dot{Q}_2}{\dot{Q}_1}\right).
\end{aligned} \tag{15}$$

Die endgültigen Forme der dimensionslosen Größen ε_p und ε_c lassen sich nach der Einführung der Berechnung $\psi = \dfrac{\dot{Q}_2}{\dot{Q}_1}$ mit den aufgeschriebenen Geschwindigkeitsverhältnissen ausdrücken:

$$\varepsilon_p = \frac{B_1 - B_1\psi - B_3\psi^2}{B_4 - B_5\psi^2}\,;\quad \varepsilon_c = \frac{B_1^* - B_2^*\psi - B_3^*\psi^2}{B_4 - B_5\psi}, \tag{16}$$

wobei

$$\begin{aligned}
B_1 &= 2\frac{A}{A_1}\cos\alpha - \left(2+\zeta\frac{\varrho_{34}}{\varrho_1}\right) + \eta_D\frac{\varrho_{45}}{\varrho_1}\left[1-\left(\frac{A}{A_5}\right)^2\right]; \\
B_2 &= 2\left(1+\frac{\varrho_2}{\varrho_1}+\zeta\frac{\varrho_{34}}{\varrho_1}\right) - 2\eta_D\frac{\varrho_{45}}{\varrho_1}\left[1-\left(\frac{A}{A_5}\right)^2\right]; \\
B_3 &= \zeta\frac{\varrho_{34}}{\varrho_1} + \frac{1}{\varphi_2^2}\,\frac{\varrho_{23}}{\varrho_1}\left[1-\left(\frac{A}{A_2}\right)^2\right] - \eta_D\frac{\varrho_{45}}{\varrho_1}\left[1-\left(\frac{A}{A_2}\right)^2\right];
\end{aligned} \tag{17}$$

$$B_4 = \frac{1}{\varphi_1^2}\left(\frac{A}{A_1}\right)^2;\qquad B_5 = \frac{1}{\varphi_2^2}\,\frac{\varrho_{23}}{\varrho_1}\left[1-\left(\frac{A}{A_2}\right)^2\right];$$

$$\begin{aligned}
B_1^* &= \left(\frac{A}{A_5}\right)^2 \\
B_2^* &= -2\left(\frac{A}{A_5}\right)^2; \\
B_3^* &= \left(\frac{A}{A_2}\right)^2 - \left(\frac{A}{A_5}\right)^2.
\end{aligned} \tag{17}$$

Hier bedeuten ϱ_{23}, ϱ_{34} und ϱ_{45} die Dichten der im Konfusor, im Mischrohr bzw. im Diffusor strömenden Flüssigkeit oder des Gemisches, welche Dichten hinsichtlich der Berechnung der Druckverluste kennzeichnend sind. Alle

drei sind gleich der Dichte ϱ_1, wenn die geförderte und die Förderflüssigkeit identisch sind.

Im Falle von Förderung eines Gemisches von Wasser und Feststoff mit Hilfe von Wasser ist die Lage mit guter Annäherung ähnlich, besonders unter Berücksichtigung der hohen Strömungsgeschwindigkeiten. Mann könnte mit höherer Genauigkeit verfahren, wenn die Bezeichnungen

$$\begin{aligned} \zeta_0 &= \zeta \frac{\varrho_{31}}{\varrho_1}; & \eta_{D0} &= \eta_D \frac{\varrho_{45}}{\varrho_1}; \\ \varphi_{10} &= \varphi_1; & \varphi_{20} &= \varphi_2 \sqrt{\frac{\varrho_1}{\varrho_{23}}}; \end{aligned} \tag{18}$$

eingeführt und durch Experimente und Messungen an den zu fördernden Materialien bestimmt werden. Die vier strömungstechnischen Parameter werden als von den dimensionslosen Größen ε und ψ unabhängig betrachtet. Alle hängen von der *Reynolds-Zahl* der Strömung ab, der Koeffizient ζ ist außerdem vom Verhältnis der Länge und des Durchmessers des Mischrohres, die Werte φ_1 und φ_2 sind von Konizität des Diffusors oder Konfusors, ζ_0, η_{D0} und φ_{20} von den Dichteverhältnissen abhängig.

Ist es Aufgabe, ein Gemisch von Luft und Feststoff mittels Wassers zu fördern, dann strömt ein Zweiphasen- oder Dreiphasengemisch durch die erwähnten Querschnitte. In diesen Fällen (wie z. B. bei Verwendung der Ejektorpumpe als Befeuchtungsanlage) ist die Druckzunahme, d. h. der durch die Ejektorpumpe herstellbare Druck gering, die angenommene Inkompressibilität wird mit guter Annäherung erfüllt, der dadurch verursachbare Fehler kann vernachlässigt werden. Die experimentelle Bestimmung der strömungstechnischen Parameter ζ_0, η_{D0}, φ_{10}, φ_{20} ist in diesem Falle eine unvermeidbare, wichtige Aufgabe. Man muß besonders bei dem Diffusor sorgfältig verfahren, wo ein relativ größerer Druckunterschied auftritt.

2.2 *Saugfähigkeit von Ejektorpumpen*

Zur ungestörten Funktion der Ejektorpumpen ist auch ein kavitationsfreier Betrieb zu sichern. Der betreffenden Literatur nach [1, 2, 4] ist es dazu bei Flüssigkeitsförderung praktisch genügend, wenn die Ungleichheit

$$\frac{p_2 - p_t}{p_1 - p_2} \geqq \frac{\psi^2}{\left(\frac{\varphi_2}{\varphi_1}\right)^2 \frac{\varrho_1}{\varrho_{23}} \frac{\left(\frac{A}{A_1}\right)^2}{1 - \left(\frac{A}{A_2}\right)^2} - \psi^2} \tag{19}$$

erfüllt wird. In diesem Zusammenhang ist p_l der Sättigungsdruck bei der Temperatur der geförderten Flüssigkeit. Bei einer Befeuchtungsejektorpumpe, wo der eingesogene Materialstrom ein Gemisch von Luft und körnigem Feststoff ist, kann man nicht von Kavitation sprechen.

2.3 *Förderhöhe einer zweistufigen Seitendüsenejektorpumpe*

Eine größere Fördehöhe kann mittels zweistufiger Ejektorpumpe verwirklicht werden. Die gesamte statische Förderhöhe wird durch die zweite Stufe hergestellt, die erste Stufe dient als Zubringerpumpe.

Die schematische Anordnung der zweistufigen Ejektorpumpe und die gebrauchten Bezeichnungen sind in Abb. 7 dargestellt.

Der Grundzusammenhang der ersten Stufe ist mit der Modifizierung gültig, daß ein Konfusor statt eines Diffusors am Ende der ersten Stufe steht, deshalb ist $\eta_{D0} = \dfrac{1}{\varphi_{20}^2}$, wobei φ_{20} der Verlustfaktor des Konfusors ist.

Die Koeffizienten im Zusammenhang der dimensionslosen Fördehöhe mint den angewendeten Bezeichnungen sind wie folgt:

$$
\begin{aligned}
B_{1\mathrm{I}} &= 2\frac{A_{\mathrm{I}}}{A_{1\mathrm{I}}}\cos\alpha - (2+\zeta_0) - \frac{1}{\varphi_{20}^2}\left[\left(\frac{A_{\mathrm{I}}}{A_{\mathrm{II}}}\right)^2 - 1\right];\\
B_{2\mathrm{I}} &= 2\left(1 + \frac{\varrho_{2\mathrm{I}}}{\varrho_1} + \zeta_0\right) + \frac{2}{\varphi_{20}^2}\left[\left(\frac{A_{\mathrm{I}}}{A_{\mathrm{II}}}\right)^2 - 1\right];\\
B_{3\mathrm{I}} &= \zeta_0 + \frac{1}{\varphi_{20}^2}\left[1 - \left(\frac{A_{\mathrm{I}}}{A_{2\mathrm{I}}}\right)^2\right] + \frac{1}{\varphi_{20}^2}\left[\left(\frac{A_{\mathrm{I}}}{A_{\mathrm{II}}}\right)^2 - 1\right];\\
B_{4\mathrm{I}} &= \frac{1}{\varphi_{10}^2}\left(\frac{A_{\mathrm{I}}}{A_{1\mathrm{I}}}\right)^2;\\
B_{5\mathrm{I}} &= \frac{1}{\varphi_{20}^2}\left[1 - \left(\frac{A_{\mathrm{I}}}{A_{2\mathrm{I}}}\right)^2\right]\\
B^*_{1\mathrm{I}} &= \left(\frac{A_{\mathrm{I}}}{A_{\mathrm{II}}}\right)^2; \qquad B^*_{2\mathrm{I}} = -2\left(\frac{A_{\mathrm{I}}}{A_{\mathrm{II}}}\right)^2;\\
B^*_{3\mathrm{I}} &= \left(\frac{A_{\mathrm{I}}}{A_{2\mathrm{I}}}\right)^2 - \left(\frac{A_{\mathrm{I}}}{A_{\mathrm{II}}}\right)^2.
\end{aligned}
\tag{20}
$$

Die Zusammenhänge für die dimensionslosen Förderhöhen sind folgende:

$$
\begin{aligned}
\varepsilon_{1p} &= \frac{p_{5\mathrm{I}} - p_{2\mathrm{I}}}{p_{1\mathrm{I}} - p_{2\mathrm{I}}} = \frac{B_{1\mathrm{I}} - B_{2\mathrm{I}}\psi_{\mathrm{I}} - B_{3\mathrm{I}}\psi_{\mathrm{I}}^2}{B_{4\mathrm{I}} - B_{5\mathrm{I}}\psi_{\mathrm{I}}^2}\\
\varepsilon_{1c} &= \frac{c_{5\mathrm{I}}^2 - c_{2\mathrm{I}}^2}{p_{1\mathrm{I}} - p_{2\mathrm{I}}}\,\frac{\varrho_{25\mathrm{I}}}{2} = \frac{B^*_{1\mathrm{I}} - B^*_{2\mathrm{I}}\psi_{\mathrm{I}} - B^*_{3\mathrm{I}}\psi_{\mathrm{I}}^2}{B_{4\mathrm{I}} - B_{5\mathrm{I}}\psi_{\mathrm{I}}^2}.
\end{aligned}
\tag{21}
$$

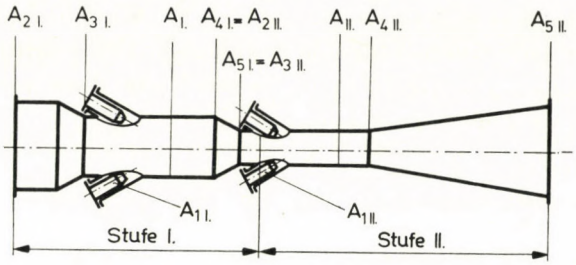

Abb. 7

In der zweiten Stufe sind die Koeffizienten der Grundgleichung einfacher, da hier $A/A_2 = 1$ ist, d. h.

$$
\begin{aligned}
B_{1\mathrm{II}} &= 2\frac{A_{\mathrm{II}}}{A_{1\mathrm{II}}}\cos\alpha - (2 + \zeta_0) + \eta_{D0}\left[1 - \left(\frac{A_{\mathrm{II}}}{A_{5\mathrm{II}}}\right)^2\right];\\
B_{2\mathrm{II}} &= 2\left(1 + \frac{\varrho_{2\mathrm{II}}}{\varrho_1} + \zeta_0\right) - 2\eta_{D0}\left[1 - \frac{A_{\mathrm{II}}}{A_{5\mathrm{II}}}\right)^2\right];\\
B_{3\mathrm{II}} &= \zeta_0 - \eta_{D0}\left[1 - \left(\frac{A_{\mathrm{II}}}{A_{1\mathrm{II}}}\right)^2\right];\\
B_{4\mathrm{II}} &= \frac{1}{\varphi_{10}^2}\left(\frac{A_{\mathrm{II}}}{A_{1\mathrm{II}}}\right)^2; \qquad B_{5\mathrm{II}} = 0\,;\\
B^*_{1\,\mathrm{II}} &= \left(\frac{A_{\mathrm{II}}}{A_{5\mathrm{II}}}\right)^2\\
B^*_{2\,\mathrm{II}} &= -2\left(\frac{A_{\mathrm{II}}}{A_{5\mathrm{II}}}\right)^2\\
B^*_{3\,\mathrm{II}} &= 1 - \left(\frac{A_{\mathrm{II}}}{A_{5\mathrm{II}}}\right)^2.
\end{aligned}
\tag{22}
$$

Die dimensionslose Förderhöhe ergibt sich für

$$
\begin{aligned}
\varepsilon_{\mathrm{II}p} &= \frac{p_{5\mathrm{II}} - p_{3\mathrm{II}}}{p_{1\mathrm{II}} - p_{3\mathrm{II}}}\,\frac{B_{1\mathrm{II}} - B_{2\mathrm{II}}\,\psi_{\mathrm{II}} - B_{3\mathrm{II}}\,\psi_{\mathrm{II}}^2}{B_{4\mathrm{II}}};\\
\varepsilon_{\mathrm{II}c} &= \frac{c_{5\mathrm{II}}^2 - c_{3\mathrm{II}}^2}{p_{1\mathrm{II}} - p_{3\mathrm{II}}}\,\frac{\varrho_{35\mathrm{II}}}{2} = \frac{B^*_{1\mathrm{II}} - B^*_{2\mathrm{II}}\,\psi_{\mathrm{II}} - B^*_{3\mathrm{II}}\,\psi_{\mathrm{II}}^2}{B_{4\mathrm{II}}}\,.
\end{aligned}
\tag{23}
$$

Es gilt als offensichtlich, daß $p_{5\mathrm{I}} = p_{3\mathrm{II}}$ ist, weiterhin ist auch die Gleichheit $p_{\mathrm{II}} = p_{1\mathrm{II}}$ meistens gültig, d. h. beide Stufen werden mit Flüssigkeiten gleichen Druckes gespeist. Die Verhältnisse werden vereinfacht, wenn die geförderte

und die Treibflüssigkeit identisch sind ($\varrho_1 = \varrho_2$). Die Lage wird dagegen kompliziert und die erhaltenen Zusammenhänge werden von annähernder Gültigkeit, wenn ein Gemisch von Flüssigkeit und körnigem Feststoff oder von Luft und Feststoff gefördert wird. Bei Transport eines Gemisches aus Flüssigkeit und Feststoff wird angenommen, daß die volumetrische Transportkonzentration des Feststoffes am Anfang des Mischrohres

$c_{t2} = \dfrac{\dot{Q}_s}{\dot{Q}_2}$ ist ($\dot{Q}_s$ ist der Volumenstrom des Feststoffes, $\dot{Q}_2$ der des Gemisches), die Dichte des Feststoffes ist ϱ_s und die der Flüssigkeit ϱ_f. Mit diesen Größen ist der Wert des Dichteverhältnisses im Koeffizienten B_2:

$$\frac{\varrho_2}{\varrho_1} = \frac{\varrho_f}{\varrho_1} + c_{t2}\left(\frac{\varrho_s}{\varrho_1} - \frac{\varrho_f}{\varrho_1}\right). \tag{24}$$

Je homogener das Gemisch ist, desto genauer ist der Zusammenhang. Ist $\varrho_f = \varrho_1$, wird das Dichteverhältnis vereinfacht:

$$\frac{\varrho_2}{\varrho_1} = 1 + c_{t2}\left(\frac{\varrho_s}{\varrho_1} - 1\right). \tag{25}$$

Die Befeuchtungsejektorpumpe saugt ein Gemisch von Luft und Feststoff ein. Es kann mit Recht angenommen werden, daß die Luft im praktischen Druckbereich (0,8 – 1.2 bar) für inkompressibel gehalten werden kann. Jetzt ist $\varrho_f = \varrho_1$ die Dichte der Luft in obiger Dichtefunktion. Nur der Vergleich der aufgeschriebenen Gleichungen mit den Resultaten von Laborversuchen der Ejektorpumpe liefert einen Beweis dafür, ob die Gleichungen zur Projektierung angewendet werden können.

2.4 *Energetische Charakterisierung der Funktionsweise der Seitendüsenejektorpumpe*

Die Qualität der Funktion der Ejektorpumpen kann mit dem bei Pumpen üblichen Wirkungsgrad-Ausdruck gekennzeichnet werden. Die Ejektorpumpe wird in diesem Falle unabhängig von der Benutzung untersucht. Bei den meisten Anwendungen aber – im Falle von Feststofftransport im Flüssigkeitsstrom ausschließlich – erhält man dann eine richtige Meßzahl zur Beurteilung der Funktion der Ejektorpumpe, wenn die Pumpenergiekosten für die Einheitsmenge des nützlichen Fördergutes (Flüssigkeit, Gemisch oder Feststoff) berechnet werden.

2.41 Wird die Ejektorpumpe als separate Einheit untersucht, führt die Treibflüssigkeit vom Massestrom $m_1 = \dot{Q}_1 \varrho_1$, von der Geschwindigkeit c_1^* und vom Druck p_1 die Leistung ein. Die Treibflüssigkeit wird aus einem Behälter

mit Druck p_0 gewonnen, der Wasserspiegel im Behälter ist gleichbleibend, deshalb ist $c_0 = 0$. Mit der

$$H_1 = \frac{p_1 - p_0}{\varrho_1 g} + \frac{c_1^{*2}}{2g} \tag{26}$$

spezifischen Energie bzw. Förderhöhe ergibt sich für den Wert Treibleistung:

$$N_b = \dot{Q}_1 \varrho_1 g H_1 . \tag{27}$$

Die nützliche Leistung der Ejektorpumpe kann von den meßbaren Drücken und Geschwindigkeiten in den Saug- und Druckstutzen bestimmt werden. Die hergestellte Förderhöhe ist:

$$H = \frac{p_5 - p_2}{\varrho_{25} g} + \frac{c_5^2 - c_2^2}{2g} \tag{28}$$

und die nützliche Leistung:

$$N_h = \dot{Q} \varrho g H . \tag{29}$$

In diesem Zusammenhang ist $\dot{Q}$ der Volumenstrom der zu fördernden nützlichen Flüssigkeit, der $\dot{Q}_2$ oder im glücklichen Fall $\dot{Q}_1 + \dot{Q}_2$ sein kann. Bei hydraulischer Förderung stellt nur der Transport des Feststoffes nützliche Arbeit dar, in diesem Fall kann der für Flüssigkeitstransport gültige Wirkungsgrad-Ausdruck nur gezwungen interpretiert werden.

Der Wirkungsgrad läßt sich bei Anwendung des Obigen in folgender Form ausdrücken:

$$\eta = \frac{N_h}{N_b} = \frac{\varrho}{\varrho_1} \frac{\dot{Q}}{\dot{Q}_1} \frac{H}{H_1} = \frac{\varrho}{\varrho_1} \frac{\dot{Q}}{\dot{Q}_1} \frac{\dfrac{p_5 - p_2}{\varrho_{25} g} + \dfrac{c_5^2 - c_2^2}{2g}}{\dfrac{p_1 - p_0}{\varrho_1 g} + \dfrac{c_1^{*2}}{2g}} = \tag{30}$$

$$= \frac{\varrho}{\varrho_{25}} \frac{\dot{Q}}{\dot{Q}_1} \frac{\dfrac{p_5 - p_2}{p_1 - p_2} - \dfrac{c_5^2 - c_2^2}{p_1 - p_2} \dfrac{\varrho_{25}}{2}}{1 - \dfrac{(p_0 - p_2) - c_1^{*2} \varrho_1/2}{p_1 - p_2}} .$$

Der Druckunterschied $p_0 - p_2$ im letzten Zusammenhang kann mittels der saugseitigen spezifischen Energie der durch den Saugstutzen einströmenden Flüssigkeit bzw. mittels der saugseitigen Förderhöhe angegeben werden:

$$H_2 = \frac{p_0 - p_2}{\varrho_2 g} - \frac{c_2^2}{2g} . \tag{31}$$

Der Wirkungsgrad der Ejektorpumpe ist also:

$$\eta = \frac{\varrho}{\varrho_{25}} \frac{\dot{Q}}{\dot{Q}_1} \frac{\varepsilon}{1 - \frac{H_2 \frac{\varrho_2}{\varrho_1} - \frac{c_2^2}{2g} \frac{\varrho_2}{\varrho_1} - \frac{c_1^{*2}}{2g}}{p_1 - p_2} \varrho_1 g} \tag{32}$$

Man kann mit Recht annehmen, daß die Dichten und Geschwindigkeiten in der Saugleiting und in der Leitung der Treibflüssigkeit unter den Größen im Nenner annähernd gleich sind, d. h.

$$\frac{c_2^2}{2g} \frac{\varrho_2}{\varrho_1} \cong \frac{c_1^{*2}}{2g} . \tag{33}$$

Auch das zweite Glied des Nenners darf vernachlässigt werden, da

$$p_1 - p_2 \gg H_2 \varrho_2 g . \tag{34}$$

Die Wirkungsgrad-Beziehung für die Ejektorpumpe hat also folgende endgültige Form:

$$\eta = \frac{\varrho}{\varrho_{25}} \frac{\dot{Q}}{\dot{Q}_1} \varepsilon . \tag{35}$$

In den meisten Fällen ist die nützliche Flüssigkeit lediglich die geförderte Flüssigkeit mit dem Volumenstrom $\dot{Q}_2 (\varrho = \varrho_2;\ \dot{Q} = \dot{Q}_2)$, die Gleichheit $\varrho_2 \cong \varrho_{25}$ ist sogar nur mit Annäherung gültig; der Wirkungsgrad ist in diesem Fall

$$\eta_1 = \frac{\varrho_2}{\varrho_{25}} \frac{\dot{Q}_2}{\dot{Q}_1} \varepsilon = \psi \varepsilon . \tag{36}$$

In einigen Fällen kann die Förderung auch der Flüssigkeit $\dot{Q}_1$ für nützlich gehalten werden, dann ist $\dot{Q} = \dot{Q}_1 + \dot{Q}_2$ und $\varrho \cong \varrho_{12} \cong \varrho_{25}$, demgemäß ist der Wirkungsgrad

$$\eta_2 = \frac{\varrho_{12}}{\varrho_{25}} \frac{\dot{Q}_1 + \dot{Q}_2}{\dot{Q}_1} \varepsilon \cong (1 + \psi) \varepsilon . \tag{37}$$

2.42 Die Funktion der Pumpe kann im Falle irgendwelcher Verwendung aufgrund der spezifischen Pumpenergiekosten gekennzeichnet werden.

Die zur Betätigung der Ejektorpumpe benötigte Flüssigkeit wird durch eine Pumpe gefördert. Die Energiespesen werden von der aufgenommenen Leistung der Pumpeneinheit berechnet.

Die Treibpumpe erhält die Flüssigkeit im allgemeinen aus einem Behälter, der um eine geodätische Höhe H_g^* oberhalb der Ejektorpumpe placiert

ist (Abb. 8). Das Flüssigkeitsniveau im Behälter ist gleichbleibend, der Behälter ist mit der Ejektorpumpe durch eine Rohrleitung von einem Strömungswiderstand R* verbunden, im Rohr strömt die Treibflüssigkeit mit dem Volumenstrom $\dot{Q}_1$. Aufgabe der Treibpumpe ist, die aus dem Behälter zur Ejektorpumpe strömende Flüssigkeit vom Druck p_1, vom Volumenstrom $\dot{Q}_1$ und von der Geschwindigkeit c_1^*zu fördern. Die Förderhöhe der Treibpumpe ist gleich der Belastung:

$$H^* = \frac{p_1 - p_0}{\varrho_1 g} + \frac{c_1^{*2}}{2g} + R^* \dot{Q}_1^2 - H_g^* \,. \tag{38}$$

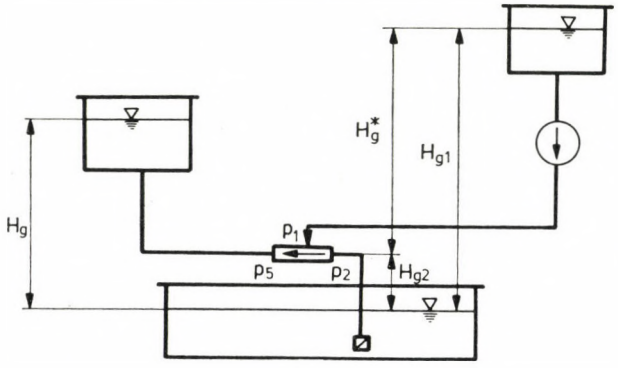

Abb. 8

Beträgt der Wirkungsgrad der Pumpeneinheit η_{sz}, ist die aufgenommene Leistung der Treibpumpe:

$$N^* = \frac{\dot{Q}_1 \varrho_1 g H^*}{\eta_{sz}} \,. \tag{39}$$

Der Wert der Förderhöhe H^* hängt von der Verwendung im großen Maße ab. Im glücklichen Fall (wenn H_g^* groß ist) kann man sich auch mit einer kleinen Treibpumpe begnügen. Im ungünstigen Fall dagegen (wenn R^* groß ist) braucht man eine Pumpe von sehr großer Leistung.

Das durch die Ejektorpumpe geförderte Nutzgut kann $\dot{Q} = \dot{Q}_2$ oder im glücklichen Fall $\dot{Q} = \dot{Q}_1 + \dot{Q}_2$ sein. Bei hydraulischer Materialförderung wird vorausgesetzt, daß das Gemisch vom Volumenstrom $\dot{Q}_2$ eine Feststoffmenge von volumetrischer Konzentration c_{t2} enthält. So ist das Nutzgut $\dot{Q} = c_{t2}\dot{Q}_2$. Der spezifische Energieverbrauch ist also:

$$\begin{aligned} e &= \frac{N^*}{\dot{Q}} = \frac{\dot{Q}_1}{\dot{Q}} \frac{\varrho_1 g}{\eta_{sz}} H^* = \\ &= \frac{\dot{Q}_1}{\dot{Q}} \frac{\varrho_1 g}{\eta_{sz}} \left[\frac{p_1 - p_0}{\varrho_1 g} + \frac{c_1^{*2}}{2g} + R^* \dot{Q}_1^2 - H_g^* \right] . \end{aligned} \tag{40}$$

Man erhält eine dem Wirkungsgrad ähnliche dimensionslose Größe, den sog. Leistungskoeffizienten zur energetischen Beurteilung der Verwendung der Ejektorpumpe, wenn die jetzt bestimmte zugeführte Leistung N^* zur nützlichen Leistung ins Verhältnis gestellt wird:

$$\vartheta = \frac{N_h}{N_x} = \eta_{sz} \frac{\varrho}{\varrho_1} \frac{\dot{Q}}{\dot{Q}_1} \frac{H}{H^*} = \\ = \eta_{sz} \frac{\varrho}{\varrho_{25}} \frac{\dot{Q}}{\dot{Q}_1} \frac{\varepsilon}{1 - \dfrac{H_g^* - R^* \dot{Q}_1^2 - \dfrac{c_1^{*2}}{2g} - \dfrac{p_0 - p_2}{\varrho_1 g}}{\dfrac{p_1 - p_2}{\varrho_1 g}}} . \tag{41}$$

In dieser Beziehung ist der saugseitige Überdruck der Ejektorpumpe

$$p_0 - p_2 = \varrho_2 g \left(H_{g2} + \frac{c_2^2}{2g} + R_2 \dot{Q}_2^2 \right), \tag{42}$$

wobei R_2 der Widerstandsbeiwert des Saugrohres ist. Wird das in den Ausdruck des Leistungskoeffizienten eingesetzt, ergibt sich die folgende Beziehung:

$$\vartheta = \eta_{sz} \frac{\varrho}{\varrho_{25}} \frac{\dot{Q}}{\dot{Q}_1} \frac{\varepsilon}{1 - \dfrac{H_g^* + H_{g2} \dfrac{\varrho_2}{\varrho_1} - \dfrac{c_2^2}{2g} \dfrac{\varrho_2}{\varrho_1} - \dfrac{c_1^{*2}}{2g} - R^* \dot{Q}_1^2 + R_2 \dot{Q}_2^2}{\dfrac{p_1 - p_2}{\varrho_1 g}}} . \tag{43}$$

Es ist eine begründete Voraussetzung, daß die Geschwindigkeiten in der Saugleitung und in der Leitung der Treibflüssigkeit unter den Größen des Nenners annähernd gleich sind, d. h.

$$\frac{c_2^2}{2g} \frac{\varrho_2}{\varrho_1} = \frac{c_1^{*2}}{2g} . \tag{44}$$

Weiterhin kann leicht eingesehen werden, daß mit den Angaben der Abb. 8 ($\varrho_2/\varrho_1 \simeq 1$):

$$H_g^* + H_{g2} \frac{\varrho_2}{\varrho_1} \simeq H_{g1} , \tag{45}$$

sowie der Widerstand im kurzen Saugrohr vernachlässigt werden kann:

$$R_2 \dot{Q}_2^2 \frac{\varrho_2}{\varrho_1} \simeq 0 . \tag{46}$$

Diese in Betracht gezogen ist der Leistungskoeffizient

$$\vartheta = \eta_{sz} \frac{\varrho}{\varrho_{25}} \frac{\dot{Q}}{\dot{Q}_1} \frac{\varepsilon}{1 - \dfrac{H_{g1} - R^* \dot{Q}_1^2}{p_1 - p_2} \varrho_1 g} . \tag{47}$$

2.43 Zwei typische Verwendungen der Ejektorpumpe werden in Abb. 9 und 10 erläutert.

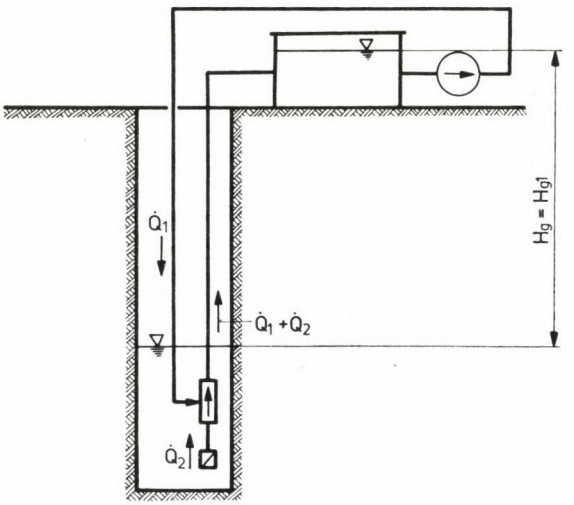

Abb. 9

Im ersten Fall (Abb. 9) fördert die Ejektorpumpe Wasser, Abwasser oder Gemisch aus einem Brunnen mit einer Förderhöhe H_g. Die Treibpumpe ist am Tage, direkt neben dem Brunnen angebracht; deshalb kann die zum Hochpumpen des Treibwassers gebrauchte potentielle Energie mit Ausnahme der Strömungsverluste zurückgewonnen werden. Die potentielle Energie H_{g1} ist in diesem Fall gleich der geodätischen Förderhöhe H_g. Wird die Rohrleitung, die das Treibwasser zurückführt, richtig ausgelegt (die Strömungsgeschwindigkeit $c_1^* \leqq 2{,}5$ m/s), können die Strömungsverluste und die kinetische Energie im Verhältnis zu H_{g1} vernachlässigt werden. Der spezifische Energieverbrauch ist also:

$$e_1 = \frac{\dot{Q}_1}{\dot{Q}_2} \frac{\varrho_1 g}{\eta_{sz}} \left(\frac{p_1 - p_2}{\varrho_1 g} - H_g \right) . \tag{48}$$

Auch der Leistungskoeffizient läßt sich bestimmen:

$$\vartheta_1 = \eta_{sz} \frac{\varrho_2}{\varrho_{25}} \frac{\dot{Q}_2}{\dot{Q}_1} \frac{\varepsilon}{1 - \dfrac{H_g \varrho_1 g}{p_1 - p_2}} . \tag{49}$$

Mit der leicht einsehbaren Substituierung

$$H_g = \frac{p_5 - p_2}{\varrho_{25} g} - R(\dot{Q}_1 + \dot{Q}_2)^2 + \frac{c_5^2 - c_2^2}{2g} \cong \cong \frac{p_5 - p_2}{\varrho_{25} g} + \frac{c_5^2 - c_2^2}{2g} \tag{50}$$

und mit der Voraussetzung $\varrho \cong \varrho_2 \cong \varrho_1 \cong \varrho_{25}$ erhält der Ausdruck des spezifischen Energieverbrauches folgende einfache Form:

$$e_1 = \frac{p_1 - p_2}{\eta_{sz} \psi} (1 - \varepsilon) . \tag{51}$$

Muß man die Flüssigkeit nach der vertikalen Förderung auf eine erhebliche horizontale Entfernung transportieren, dürfen $R^* \dot{Q}_1^2$ und $R(\dot{Q}_1 + \dot{Q}_2)^2$ nicht vernachlässigt werden. Die Zusammenhänge e_1 und ϑ_1 verändern sich in ungüstiger Richtung, beim Hochpumpen der Treibflüssigkeit können die Strömungsverluste nämlich nicht zurückgewonnen werden, im Gegensatz zur Zunahme der potentiellen Energie.

Bei hydraulischer Materialförderung, wenn $\dot{Q}_2$ das zu fördernde Gemisch darstellt und die Treibflüssigkeit die Ejektorpumpe im geschlossenen Kreislauf betätigt, sind die gleichen energetischen Parameter gültig. Man soll aber mit besonderer Sorgfalt bei der Berücksichtigung der Dichteverhältnisse vorgehen. Ist die Förderung nur des Feststoffes nützlich, so soll die Einsetzung $\dot{Q} = \dot{Q}_2 c_{t2}$ in den erläuterten energetischen Parametern durchgeführt werden, die erhaltenen Ergebnisse sind also mit der volumetrischen Konzentration c_{t2} zu multiplizieren.

Bei der anderen Verwendung (Abb. 10) fördert die Ejektorpumpe Trübe aus einer Grube zu Tage. Die Treibpumpe, die die Ejektorpumpe betätigt, arbeitet am Tage, auch das Hochpumpen des die Ejektorpumpe treibenden reinen Wassers und des im Gemisch enthaltenen Wassers ist eine nützliche Arbeit. Im allgemeinen Zusammenhang der spezifischen Energiekosten können $\dot{Q} = \dot{Q}_1 + \dot{Q}_2$ sowie $R^* \dot{Q}_1^2$ und $\dfrac{c_1^{*2}}{2g}$ in der kurzen Leitung der Treibflüssigkeit im Verhältnis zu den anderen Gliedern vernachlässigt werden; weiterhin ist $H_g^* = -H_{g2}$. Läßt man die in der kurzen Saugleitung der Ejektorpumpe

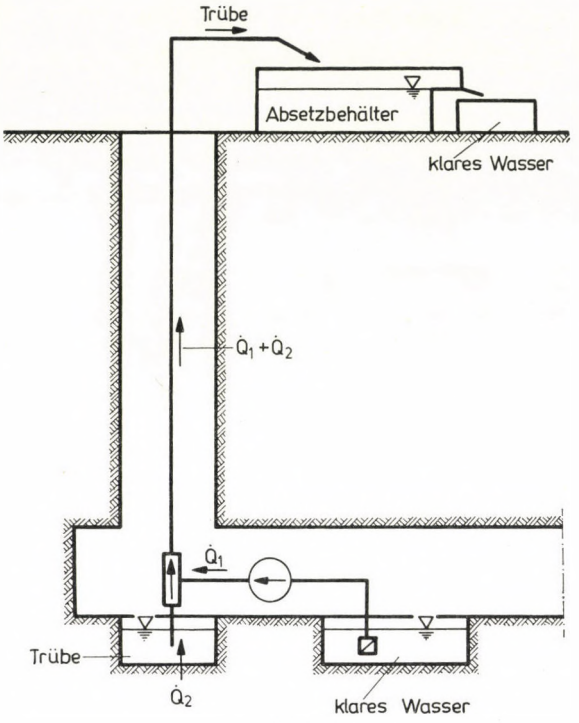

Abb. 10

auftretende Druckabnahme $R_2\dot{Q}_2^2$ und $\frac{c_2^2}{2g}$ außer acht, so erhält man $p_0 - p_2 = \varrho_2 g H_{g2}$

Der spezifische Energiebedarf mit den erläuterten begründeten Vereinfachungen beträgt:

$$e_2 = \frac{p_1 - p_2}{\eta_{sz}(1 + \psi)} \,. \tag{52}$$

Der Leistungskoeffizient kann mit Hilfe ähnlicher Vernachlässigungen ($\varrho_{125}/\varrho_{25} \cong 1$) in folgender Form geschrieben werden:

$$\vartheta_2 = \eta_{sz}(1 + \psi)\, \varepsilon \,. \tag{53}$$

3. Die Bestimmung der strömungstechnischen Parameter durch Messungen

Die zur Auslegung der Ejektorpumpe abgeleiteten allgemeinen Zusammenhänge können dann angewendet werden, wenn die strömungstechnischen Parameter φ_{10}, φ_{20}, ζ_0, η_{D0} in diesen Gleichungen bekannt sind. Diese Werte

hängen von der Ausführung der Ejektorpumpe und von der geförderten Flüssigkeit oder dem Gemisch ab, deshalb können sie für einen Ejektorpumpentyp im gegebenen Fall mittels Messung bestimmt werden. Dazu sind halbtechnische oder Laboranlagen am zweckmäßigsten zu gebrauchen.

Die Bestimmung der Werte φ_{10}, φ_{20} und η_{D0} unter den strömungstechnischen Parametern kann auf Messung des Druckunterschieds und des Volumenstroms zurückgeführt werden. Folgende Zusammenhänge werden hier angewendet:

$$\varphi_{10} = \frac{c_1}{\sqrt{2\dfrac{p_1 - p_3}{\varrho_1}}} = \frac{\dot{Q}_1}{A_1\sqrt{2\dfrac{p_1 - p_3}{\varrho_1}}}; \tag{54}$$

$$\varphi_{20} = \frac{\sqrt{c_3^2 - c_2^2}}{\sqrt{2\dfrac{p_2 - p_3}{\varrho_{23}}}} = \frac{\dot{Q}_1\sqrt{\dfrac{1}{A^2} - \dfrac{1}{A_2^2}}}{\sqrt{2\dfrac{p_2 - p_3}{\varrho_{23}}}}; \tag{55}$$

$$\eta_{D0} = \frac{2\dfrac{p_5 - p_4}{\varrho_{45}}}{c_4^2 - c_5^2} = \frac{2\dfrac{p_5 - p_4}{\varrho_{45}}}{\left(\dfrac{1}{A^2} - \dfrac{1}{A_5^2}\right)(\dot{Q}_1 + \dot{Q}_2)^2}. \tag{56}$$

Die in diesen Gleichungen enthaltenen Größen können direkt gemessen werden. Der Widerstandsbeiwert des Mischraumes ζ_0 läßt sich nur indirekt, aus einem vom Impulssatz abgeleiteten Zusammenhang, d. h. von den zusammengehörigen Punkten der Kennlinie der Ejektorpumpe berechnen.

Aus der Gleichung der dimensionslosen Kennlinie der Ejektorpumpe ergibt sich der folgende Zusammenhang für den Widerstandsbeiwert:

$$\zeta_0 = \zeta\frac{\varrho_{31}}{\varrho_1} =$$

$$= \frac{2\dfrac{A}{A_1}\cos\alpha - 2 - 2\left(1 + \dfrac{\varrho_{23}}{\varrho_1}\right)\varphi - (1 - \varepsilon_p)\dfrac{1}{\varphi_{20}^2}\left[1 - \left(\dfrac{A}{A_2}\right)^2\right]\psi^2 - \varepsilon_p\dfrac{1}{\varphi_1^2}\left(\dfrac{A}{A_1}\right)^2}{(1 + \psi^2)^2} + \tag{57}$$

$$+ \eta_{D0}\left[1 - \left(\frac{A}{A_5}\right)^2\right].$$

Alle Größen in diesem Zusammenhang (A, A_1, A_2, A_5, α, ψ, ε_p, ϱ_1, ϱ_{23}, φ_1, φ_{20}, η_{D0}) können gemessen werden. Die Gültigkeit des dimensionslosen Zusammenhanges wird beruhigend bewiesen, wenn die Widerstandsbeiwerte bei verschiedenen Werten von φ und ε_p innerhalb enger Grenzen verbleiben.

Berechnet man den Durchschnitt der aus verschiedenen Messungen berechneten Widerstandsbeiwerte, deuten die Abweichungen der einzelnen Messungen die Zuverlässigkeit des Durchschnittes an.

4. Auslegung der zweistufigen Ejektorpumpe

Als eine Anwendung der erläuterten theoretischen Zusammenhänge für Auslegung von Ejektortrübepumpen werden die Hauptabmessungen einer zweistufigen Seitendüsenejektortrübepumpe im folgenden bestimmt. Der herzustellende Förderdruck p_{5II}, der Druck an der Saugseite p_{2I}, der Druck des Treibwassers vor den Düsen $p_{1I} = p_{1II}$ und der Sättigungsdruck der an der Saugseite einströmenden Flüssigkeit p_t werden als Ausgangsdaten betrachtet. Weiterhin ist auch der Volumenstrom des geförderten Gemisches $\dot{Q}_{21}$ bekannt.

Von vorherigen Messungen sind die strömungstechnischen Parameter (φ_1, φ_2, ζ, η_D, α) und die Dichten der geförderten Flüssigkeit und der Förderflüssigkeit(ϱ_{2I}, ϱ_1) bekannt. Die Querschnittverhältnisse A_I/A_{2I} und A_{II}/A_{5II} können geschätzt werden. Auch ein größerer Fehler in der Schätzung beeinflußt das Ergebnis der Auslegung nicht beträchtlich.

Bei der Auslegung ist eine Ausgangsvoraussetzung, daß die erste Stufe als eine Vorsatzpumpe betrachtet wird, die nur die Kavitation zu verhindern hat. Man erhält die kleinste Vorsatzpumpe, die den kavitationsfreien Betrieb gewährleisten kann, wenn keine Druckzunahme in der ersten Stufe auftritt, sondern lediglich die kinetische Energie wächst. Die dimensionslose Förderhöhe mit den früher erläuterten Gleichungen:

$$\begin{aligned} \varepsilon_{Ip} &= 0 \\ \varepsilon_I &= \varepsilon_{Ic} \,. \end{aligned} \tag{58}$$

Selbstverständlich muß man auch in der ersten Stufe die Kavitation vermeiden, deshalb muß die frühere Bedingung erfüllt werden:

$$\frac{p_{2I} - p_t}{p_1 - p_{2I}} \geq \frac{\psi_I^2}{\left(\frac{\varphi_2}{\varphi_1}\right)^2 \frac{\varrho_1}{\varrho_{23}} \frac{\left(\frac{A_I}{A_{1I}}\right)^2}{1 - \left(\frac{A_I}{A_{2I}}\right)^2} - \psi^2} = \varepsilon_k \,. \tag{59}$$

Der Koeffizient

$$k = \frac{\frac{p_{2I} - p_t}{p_1 - p_{2I}}}{\varepsilon_k} \geq 1 \tag{60}$$

wird zur Kennzeichnung der Sicherheit gegen Kavitation eingeführt. Je größer der Wert von k ist, desto weniger tritt die Kavitation auf. Bei der Auslegung gehen wir mit Sicherheit von dem Wert $k = 2$ aus.

Bei der zweistufigen Ejektrorpumpe haben wir eine extra Bedingung: Der aus der ersten Stufe ausströmende Gemischstrom $\dot{Q}_{5\mathrm{I}}$ ist dem in die zweite Stufe einströmenden Strom $\dot{Q}_{3\mathrm{II}}$ gleich. Man erhält in Kenntnis der geometrischen Abmessungen und mit Hilfe der Kontinuitätsgesetze:

$$\frac{\dot{Q}_{5\mathrm{I}}}{\dot{Q}_{\mathrm{II}}} = \frac{\varrho_1}{\varrho_{5\mathrm{I}}}\left(1 + \frac{\varrho_{2\mathrm{I}}}{\varrho_1}\psi_1\right) \tag{61}$$

$$\frac{\dot{Q}_{3\mathrm{II}}}{\dot{Q}_{1\mathrm{II}}} = \psi_{\mathrm{II}}\,. \tag{62}$$

Aus dem Quotient beider Gleichungen ($\varrho_{5\mathrm{I}} = \varrho_{3\mathrm{II}}$) ergibt sich die Gleichung

$$\frac{\dot{Q}_{1\mathrm{II}}}{\dot{Q}_{1\mathrm{I}}} = \frac{\varrho_{2\mathrm{I}}}{\varrho_{5\mathrm{I}}}\left(\frac{\varrho_1}{\varrho_{2\mathrm{I}}} + \psi_{\mathrm{I}}\right)\frac{1}{\psi_{\mathrm{II}}} = \frac{c_{1\mathrm{II}}\,A_{1\mathrm{II}}}{c_{1\mathrm{I}}\,A_{1\mathrm{I}}} = \frac{c_{1\mathrm{II}}\,a_{\mathrm{II}}\,A_{\mathrm{II}}}{c_{1\mathrm{I}}\,a_{\mathrm{I}}\,A_{\mathrm{I}}}\,, \tag{63}$$

wobei

$$c_{1\mathrm{II}} = \varphi_{10}\sqrt{\frac{2\,(p_1 - p_{2\mathrm{I}})}{\varrho_1 - \varrho_{23\mathrm{I}}\left(\frac{\varphi_{10}}{\varphi_{20}}\right)a_{\mathrm{I}}^2\,\psi_{\mathrm{I}}^2\left[1 - \left(\frac{A_{\mathrm{I}}}{A_{2\mathrm{I}}}\right)^2\right]}} \tag{64}$$

$$c_{1\mathrm{II}} = \varphi_{10}\sqrt{\frac{2(p_1 - p_{3\mathrm{II}})}{\varrho_1}}\,. \tag{65}$$

Durch die Einsetzung obiger Werte ergibt sich folgende wichtige Bedingungsgleichung für die Kopplung beider Stufen:

$$\frac{A_{\mathrm{II}}}{A_{\mathrm{I}}} = \frac{\frac{\varrho_{2\mathrm{I}}}{\varrho_{5\mathrm{I}}}\left(\frac{\varrho_1}{\varrho_{2\mathrm{I}}} + \psi_{\mathrm{I}}\right)\frac{a_{\mathrm{I}}}{a_{\mathrm{II}}}\,\frac{1}{\psi_{\mathrm{II}}}}{\sqrt{1 - \frac{\varrho_{23\mathrm{I}}}{\varrho_1}\left(\frac{\varphi_{10}}{\varphi_{20}}\right)^2\left[1 - \left(\frac{A_{\mathrm{I}}}{A_{2\mathrm{I}}}\right)^2\right]a_{\mathrm{I}}^2\,\psi_{\mathrm{I}}^2}}\,. \tag{66}$$

Die Aufgabe ist damit gegeben: Im Falle der dimensionslosen Kennzahlen

$$\varepsilon_{1p} = 0\,; \quad \varepsilon_{\mathrm{II}p} = \frac{p_{5\mathrm{II}} - p_{3\mathrm{II}}}{p_1 - p_{3\mathrm{II}}} = \frac{p_{5\mathrm{II}} - p_{2\mathrm{I}}}{p_1 - p_{2\mathrm{I}}} \tag{68}$$

hat man solche Abmessungsverhältnisse $a_{\mathrm{I}} = \frac{A_{1\mathrm{I}}}{A_{\mathrm{I}}}$, $a_{\mathrm{II}} = \frac{A_{1\mathrm{II}}}{A_{\mathrm{II}}}$ und $\frac{A_{\mathrm{I}}}{A_{\mathrm{II}}}$

zu wählen, daß die mit diesen berechneten ψ_{I} und ψ_{II} minimalen Treibwasseranspruch $\dot{Q}_{1\mathrm{I}} + \dot{Q}_{1\mathrm{II}}$ zur Folge haben, d. h. die dimensionslose Flüssigkeitsbeförderung

$$\psi = \frac{\psi_{\mathrm{I}}\psi_{\mathrm{II}}}{1 + \psi_{\mathrm{I}} + \psi_{\mathrm{II}}} = \frac{\dot{Q}_{2\mathrm{I}}}{\dot{Q}_{2\mathrm{I}} + \dot{Q}_{1\mathrm{II}}} \tag{69}$$

maximal ist.

Mit Hilfe der erläuterten Grundzusammenhänge kann unser Ziel mit folgenden Schritten erreicht werden:

– Aus der für die zweite Stufe gültigen dimensionslosen Kennlinie in Kenntnis von

$$\varepsilon_{\mathrm{II}p} = \varepsilon_{\mathrm{II}p}\,(a_{\mathrm{II}}, \psi_{\mathrm{II}}) \tag{70}$$

kann das Abmessungsverhältnis

$$a_{\mathrm{II}} = a_{\mathrm{II}}\,(\psi_{\mathrm{II}}) \tag{71}$$

berechnet werden.

– Aus der zur Vermeidung der Kavitation benötigten Bedingung

$$\varepsilon_k = \frac{1}{k}\,\frac{p_{2\mathrm{I}} - p_t}{p_1 - p_{2\mathrm{I}}} = \frac{\psi_{\mathrm{I}}^2}{\left(\frac{\varphi_{20}}{\varphi_{10}}\right)^2 \frac{\varrho_1}{\varrho_{23}}\,\frac{\frac{1}{a_{\mathrm{I}}^2}}{1 - \left(\frac{A_{\mathrm{I}}}{A_{2\mathrm{I}}}\right)^2} - \psi_{\mathrm{I}}^2} \tag{72}$$

ergibt sich mit $k = 2$:

$$a_{\mathrm{I}}\,\psi_{\mathrm{I}} = \sqrt{\frac{\varepsilon_k}{1+\varepsilon_k}}\,\sqrt{\frac{\left(\frac{\varphi_{20}}{\varphi_{10}}\right)^2 \frac{\varrho_1}{\varrho_{23}}}{1 - \left(\frac{A_{\mathrm{I}}}{A_{2\mathrm{I}}}\right)^2}} \tag{73}$$

– Aus der Gleichung der dimensionslosen Kennlinie der ersten Stufe

$$\varepsilon_{\mathrm{I}p} = \varepsilon_{\mathrm{I}p}\left(a_{\mathrm{I}},\ \psi_{\mathrm{I}},\ \frac{A_{\mathrm{I}}}{A_{\mathrm{II}}}\right) = 0 \tag{74}$$

kann

$$\frac{A_{\mathrm{I}}}{A_{\mathrm{II}}} = f_{\mathrm{I}}(\psi_{\mathrm{I}}) \tag{75}$$

in Kenntnis von Gl. (73) bestimmt werden.

– In Bedingungsgleichung für die Kopplung beider Stufen können a_{I} und a_{II} durch die Einsetzung von Gl. (71) und (73) eliminiert werden. Danach

wird $\frac{A_I}{A_{II}} = f_2(\psi_I, \psi_{II})$ in Gl. (75) eingesetzt, und man erhält die Gleichung

$$f_3(\psi_I, \psi_{II}) = 0 . \tag{76}$$

– In Kenntnis von Gl. (76) kann die Gleichung

$$\psi = \frac{\psi_I \psi_{II}}{1 + \psi_I + \psi_{II}} = \psi(\psi_{II}) \tag{77}$$

als Funktion einer einzigen Variable durch die Eliminierung von ψ_I bestimmt werden. Der günstigste Wert von ψ_{II}, d. h. (ψ_{max}), kann mit Hilfe von Extremwertrechnung aus der Gleichung

$$\frac{\partial \psi}{\partial \psi_{II}} = 0 \tag{78}$$

bestimmt werden.

– In Kenntnis von ψ_{II} kann ψ_I aus Gl (76), A_I/A_{II} aus Gl. (75), a_I aus Gl. (73) und a_{II} aus Gl. (71) berechnet werden.
– Der günstigste Wert ψ_{max} läßt sich aus Gl. (77) bestimmen. Danach ist $\dot{Q}_{2I}$ bekannt, in Kenntnis von ψ_I ist auch $\dot{Q}_{1I}$ bekannt. Man kann jetzt auch die Ausströmungsgeschwindigkeit der Düse c_{1I} berechnen, und so sind die geometrischen Abmessungen der ersten und zweiten Stufe:

$$A_{1I} = \frac{\dot{Q}_{1I}}{c_{1I}} ; \qquad A_I = \frac{A_{1I}}{a_I} . \tag{79} \tag{80}$$

Mit dem bereits bekannten Verhältnis A_I/A_{II} ist jetzt A_{II} zu bestimmen. Zuletzt:

$$A_{1II} = a_{II} A_{II} . \tag{81}$$

Die Auslegung ist bei gegebener konkreter Aufgabe am zweckmäßigsten mit Komputer durchzuführen.

Als eine Anwendung der Dimensionierungsmethode der zweistufigen Ejektorpumpe soll hier ein Beispiel aus dem Bergbau veranschaulicht werden. In den Sumpfstrecken der Grubenentwässerungsstationen, die auch als Absetzbecken funktionieren, häuft sich der feinkörnige Feststoff an. Dieser Feststoff muß regelmäßig herausgefördert werden. Eine der vorteilhaften Methoden der Sumpfreinigung ist die Verwendung von Ejektorpumpen, wobei die in der Sumpfstrecke eingebaute Wasserstrahlpumpe das Gemisch unmittelbar zu Tage fördert. Der Förderdruck bei der Ejektorpumpe, der sich aus dem geodätischen Förderdruck und aus den in der Druckleitung auftretenden Verlusten

addieren läßt, beträgt 23 bar. Der zu transportierende Feststoff ist Sand, seine Dichte beträgt $2{,}65 \cdot 10^3$ kg/m³, die durchschnittliche Korngröße ist 0,1 mm. Die Treibpumpe ist gegeben, ihre Förderhöhe ist 80 bar und die geförderte Wassermenge 3,5 m³/min. Die Ejektorpumpe saugt eine Trübe von volumetrischer Konzentration $c_{t2} = 0{,}1$ ein, deren Dichte $\varrho_2 = c_{t2}\varrho_s + (1 - c_{t2})\,\varrho_1 = 1{,}165 \cdot 10^3$ kg/m³ ist. Die Aufgabe kann mit Hilfe von ein- und zweistufigen Ejektorpumpen gelöst werden. Die Dimensionierungsmethode fördert die Lösung in folgenden Schritten:

— der dimensionslose Förderhöhe der Ejektorpumpe in der I. und II. Stufe:

$$\varepsilon_{Ip} = \varepsilon_{Ip}\left(a_I, \psi_I, \frac{A_I}{A_{II}}\right) = 0$$

$$\varepsilon_{IIp} = \varepsilon_{IIp}\,(a_{II}, \psi_{II}) = \frac{p_{5II} - p_{3II}}{p_{1II} - p_{3II}} = \frac{23}{80} = 0{,}287;$$

— zur Vermeidung der Kavitation ($k = 2 \cdot \varphi_{10} = 0{,}92$; $\varphi_{20} = 0{,}96$; $\eta_D = 0{,}9$, $A_I/A_{2I} = 0{,}25$)

$$\varepsilon_k = \varepsilon_k\,(a_I, \psi_I) = \frac{1}{k}\,\frac{p_{2I} - p_t}{p_1 - p_{2I}} = 0{,}00625,$$

$$\frac{c_{3I}}{c_{1I}} = a_I \cdot \psi_I = 0{,}0787.$$

— aus den Gleichungen $\varepsilon_{Ip} = \varepsilon_{Ip}(a_I, \psi_I\, A_I/A_{II}) = 0$ und $a_I\psi_I = 0{,}787$ kann a_I eliminiert werden und man erhält die Kurve $A_I/A_{II} = f_1(\psi_I)$ aus Abb. 11;

— in Kenntnis der Gleichung $\varepsilon_{IIp} = \varepsilon_{IIp}(a_{II}, \psi_{II}) = 0{,}287$ wurde die Kurve $c_{3II}/c_{1II} = a_{II}\psi_{II} = f_4(\psi_{II})$ gezeichnet (Abb. 12).

— aus der Gleichung für die Kopplung der zwei Stufen ergibt sich für das Geschwindigkeitsverhältnis der zweiten Stufe:

$$\frac{c_{3II}}{c_{1II}} = a_{II}\,\psi_{II} = f_5\left(\frac{A_I}{A_{II}}, a_I, \psi_I\right) =$$

$$= f_5\left[f_1(\psi_I); \frac{0{,}0787}{\psi_I}; \psi_I\right] = f_5(\psi_I)\,.$$

Auch diese Funktion läßt sich darstellen (Abb. 12.);

— aus den beiden letzten Gleichungen:

$$\psi = \psi(\psi_{II})\,.$$

Als Resultat der Rechnung und Konstruktion (Abb. 13.) ergibt sich die opti-

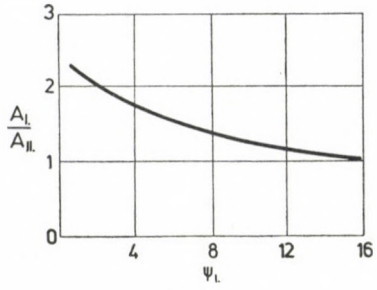

Abb. 11

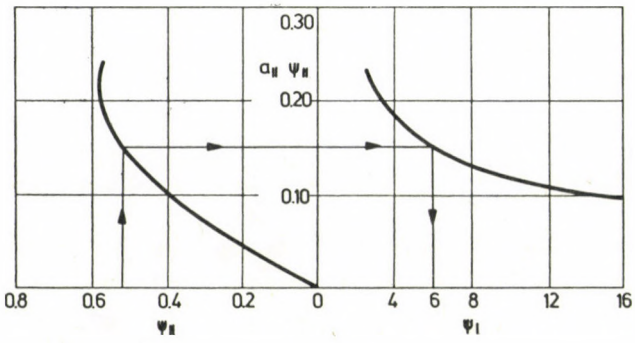

Abb. 12

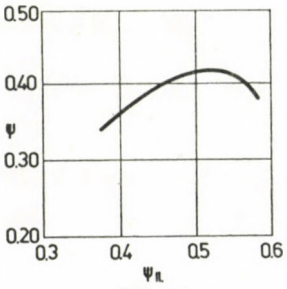

Abb. 13

male Lösung bei $\psi = 0{,}42$ ($\psi_{II} = 0{,}52$ und $\psi_I = 6{,}0$). Die geometrischen Abmessungsverhältnisse sind $a_{II} = 0{,}29$; $A_I/A_{II} = 1{,}52$ und $a_I = 0{,}012$. Bei der bekannten Treibwassermenge ist die eingezogene Gemischmenge $\dot{Q}_{2I} = (\dot{Q}_{1I} + \dot{Q}_{1II})\psi = 3{,}5 \cdot 0{,}42 = 1{,}47$ m³/min. Die Treibwassermenge der ersten Stufe ist $\dot{Q}_{1I} = \frac{\dot{Q}_{2I}}{\varphi_I} = \frac{1{,}47}{6} = 0{,}24$ m³/min, und die der zweiten Stufe beträgt $\dot{Q}_{1II} =$

$= 3{,}26$ m³/min. Die Ausströmungsgeschwindigkeit in der ersten Stufe ist $c_{1I} = 116$ m/s, der Durchschnitt der Düsen $A_{1I} = \dot{Q}_{1I} = \frac{0{,}245}{60.116} = 35{,}2$ mm², der Durchmesser der drei Düsen in der ersten Stufe: $d_{1I} = 3{,}86$ mm. Der Mischraum der ersten Stufe ist $A_I = \frac{A_{1I}}{a_I} = \frac{35{,}2}{0{,}012} = 2933$ mm², sein Durchmesser: $d_I = 61{,}11$ mm. Der Mischraum der zweiten Stufe: $A_I = \frac{2933}{1{,}52} = 1930$ mm², sein Durchmesser: $d_{II} = 49{,}57$ mm. Der Gesamtdurchschnitt der drei Düsen in der zweiten Stufe ist $A_{1II} = A_{1I} a_{II} = 1930 \cdot 0{,}29 = 559{,}7$ mm². Durchmesser einer Düse: $d_{III} = 15{,}41$ mm.

Die Aufgabe läßt sich mit Hilfe einer einstufigen Ejektorpumpe bei $a = 0{,}245$ lediglich mit einer dimensionslosen Wassermenge $\psi = 0{,}31$ im kavitationsfreien Betrieb lösen. Mit anderen Worten: Wenn eine Treibpumpe von einer Fördermenge 3,5 m³/min zur Verfügung steht, kann man mit Hilfe einer zweistufigen Ejektorpumpe höchstens eine Gemischmenge 1,47 m³/min, mit einstufiger Ejektorpumpe 1,08 m³/min fördern. Im gegebenen Fall ist also die Anwendung einer zweistufigen Ejektorpumpe begründet.

Weitere Untersuchungen wurden mit verschiedenen Ausgangsdaten durchgeführt. Aufgrund der Rechnungen wurde festgestellt, daß die Förderhöhe 80 – 100 m eine Grenzlinie für die Entscheidung zwischen ein- und zweistufigen Ejektorpumpen darstellt. Bei kleineren Lasten als diesem Wert ist die Anwendung von einstufigen, bei größeren Förderhöhen die von zweistufigen Ejektorpumpen begründet.

SCHRIFTTUM

1. Schulz, F.—Fasol, K. H.: Wasserstrahlpumpen. Wien, 1968. Springer Verlag.
2. Fuchslocher—Schulz: Die Pumpe (11. Aufl.). Berlin—Göttingen—Heidelberg, 1963. Springer-Verlag.
3. Vogel, R.: Theoretische und experimentelle Untersuchungen an Strahlapparaten. *Maschinenbautechnik*, 5 (1956), H. 12.
4. Tarján, I.—Debreczeni, E.: Das Bemessen von Wasserstrahlpumpen. Acta Geod. Geoph. Mont. Hung., 8 (1973), 115—141.

ИССЛЕДОВАНИЕ И РАСЧЕТ РАБОТЫ СТРУЙНОГО НАСОСА С БОКОВЫМ СОПЛОМ

Э. ДЕБРЕЦЕНИ – И. ТАРЯН

РЕЗЮМЕ

В статье рассматривается работа и теоретическое определение характерной кровой одного типа струйных насосов, насосов с боковым соплом. Этот насос особенно выгоден для транспортировки водой зернистого твердого или флудизированного сухого зернистого материала. В статье исследуется и двустеппенной струйно-смывочный насос.

Рассматривается энергетическая характеристика струйного насоса.

Дается метод для расчета струйно-смывочных насосов. Разработанный метод покаеан на примере применения в горном деле.

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CONTRIBUTION TO THE DETERMINATION OF THE GRAVITATIONAL EFFECT OF RECTANGULAR PRISMS

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[Manuscript received September 25, 1976]

The paper proves that the gravitational effect of rectangular prisms can be well approximated by concentrating 2/3 of the prismatic mass into the gravity center and distributing the remaining 1/3 of the mass into the eight vertices. This simple approximation is of very high accuracy, if the side lengths are small in respect of the distance of the gravity center from the reference point.

The approximation is from a computational point of view very advantageous, e.g. if for gravity interpretation a greater part of the space is divided into rectangular prisms of identical basis which can be taken already homogeneous, and the gravity effect is computed for them. The near lying prisms can be computed with exact formulas, whereas the effect of the much more numerous distant prisms can be determined by the method suggested in this article, needing much shorter (by about one order of magnitude) computer time.

1.

For the determination of the gravity effect of rectangular prisms several formulas can be used which are equally correct in principle (see e. g. [3, 4, 7]). The cause of this great choice is that this problem is one of the most inv estigated ones; the first articles appeared at the beginning of the last century. (The interesting fact may be mentioned that even the first use of the potential function was made in connection with these investigations.)

A nearly complete list of references on this theme is given by H. HAÁZ in his articles [1, 2, 3]. As in [2, 3] the previous works are critically studied, it would be superfluous to deal with them further here. It must be, however, said what is the aim of further investigations.

It can be stated shortly that *the determination of the gravity effect of rectangular prisms has changed from a theoretical problem to a computational one.* Namely, if in the course of quantitative gravitational studies a space is divided, in the depth interval studied, into elements which can be already supposed homogenous and which have an identical structure as seen from each observation point on the surface, then, according to the structure of the surface data system (mostly given in a rectangular grid) the space division into rectangular prisms is the most adequate one. Namely if in such a case the effect of all elementary prisms with unit density to a point of reference is known (in form of a three dimensional data matrix), then in possession of the actual density

distribution (being similarly a three dimensional data matrix) the determination of the effect of the structure is reduced to the computation of product sums whose number is equal to the number of prisms taken into account in the determination of a single effect.

From a computational point of view, the first step is to determine the gravitational effect for each prism of the space division (in case of density 1). That means a rather great number of prisms in case of a sufficiently fine division, and if we use e. g. the formulas given in (2), 32 series expansions of high accuracy are needed for a single prism, as the effect is the difference of 8 nearly equal values determined for the vertices of the prisms. Another important circumstance is: the effect of the elementary prisms must be known with much higher accuracy than the accuracy of the measurements, as the error increases in case of n prisms to $\sqrt{n}$-times (in case of density 1), therefore an accuracy of one or two orders of magnitude higher than the accuracy of the measurements must be achieved in the computations.

The computational burden of the determination of *accurate* values for a *very high number* of prisms can be greatly decreased, if for the prisms lying far from the reference point, i. e. for the greatest part of the space, a good approximation is used. In part 2, after referring to an exact formula, ideas about this problem shall be described. In part 3 examples illustrate the accuracy of the approximation, its advantages from a computational point of view, and the more general application possibilities of the method are discussed in part 4.

2.

Let us use the notations of I. HAÁZ. The co-ordinates of the reference point are x, y, z, the co-ordinates of the anomalous body ξ, η, ζ, further $\xi - x = a$, $\eta - y = b$ and $\zeta - z = c$, and $a^2 + b^2 + c^2 = r$. The gravitational effect of an anomalous body of any shape $g(x, y, z)$ is:

$$g(x, y, z) = f\sigma \iiint c \cdot r^{-3}\, da\, db\, dc = f\sigma \iiint G(a, b, c)\, da\, db\, dc\,, \tag{1}$$

where f is the gravity constant, σ the density of the homogeneous anomalous body (and $G(a, b, c,) = c \cdot r^{-3}$).

In case of a prism whose sides are parallel with the co-ordinate axis, the result of integration is in the possibly simplest form (see [2], p. 109):

$$g(x, y, z) = f\sigma \Big[a \cdot \ln(r + b) + b \cdot \ln(r + a) + 2c \arctan \frac{r + a + b}{c} \Big]_{a_2\, b_2\, c_2}^{a_1\, b_1\, c_1} \tag{2}$$

where a, b, c with indices mean the co-ordinates of the vertices of the prism according to Fig. 1. The co-ordinate triplets of all the eight vertices must be substituted into the expression denoted by $F(a, b, c)$ in the square brackets, and additions must be made according to the following sign-rule:

$$[F(a, b, c)]_{a_0 b_0 c_0}^{a_1 b_1 c_1} = F(a_1, b_1, c_1) + Fa_1, b_2, c_2) + \\ + F(a_2, b_1, c_2) + F(a_2, b_2, c_1) - F(a_2, b_2, c_2) - \\ - F(a_2, b_1, c_1) - F(a_1, b_2, c_1) - F(a_1, b_1, c_2) . \tag{2a}$$

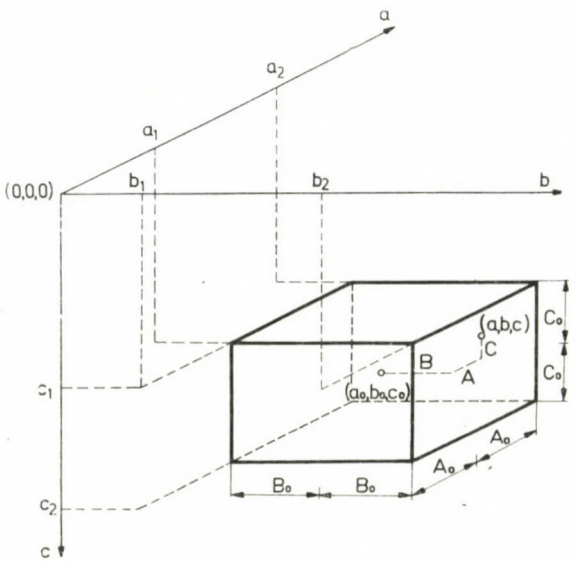

Fig. 1. Notations for the considerations in the second part

Let the half-lengths of the prism sides be A_0, B_0 and C_0 (see Fig. 1), further for the sake of simplicity, let the reference point be in the origin, therefore in the argument of g the co-ordinates of this point can be dropped. Let appear on the other hand all parameters of the prism here: the co-ordinates of its gravity center $a_0 = a_1 + A_0$, $b_0 = b_1 + B_0$ and $c_0 = c_1 + C_0$, and the half-lengths of its sides A_0, B_0 and C_0.

Let us expand the function $G(a, b, c)$ around the center of the prism (a_0, b_0, c_0):

$$G(a, b, c) = G(a_0, b_0, c_0) + G_a \cdot A + G_b \cdot B + G_c \cdot C + \\ + \frac{1}{2}(G_{ab} \cdot AB + G_{ac} \cdot AC + G_{bc} \cdot BC + G_{aa} \cdot A^2 + \\ + G_{bb} \cdot B^2 + G_{cc} \cdot C^2) + \ldots . \tag{3}$$

Here G-s with indices mean the first and second derivates of the function G after the corresponding variables in the point (a_0, b_0, c_0), the definition of the quantities in capitals is: $A = a - a_0$, $B = b - b_0$, $C = c - c_0$. If the half-lengths of the sides of the prisms are small in respect to the distance of the gravity center of the prism (a_0, b_0, c_0) from the origin, then inside of the prism the terms given in Eq. 3 can be taken as $G(a, b, c)$.

Let us integrate $G(a, b, c)$ for the prism with the gravity center (a_0, b_0, c_0), with sides of the lengths $2A_0$, $2B_0$ $2C_0$ parallel to the co-ordinate axis:

$$
\begin{aligned}
g(a_0, b_0, c_0, A_0, B_0, C_0) &= \int_{a_0-A_0}^{a_0+A_0} \int_{b_0-B_0}^{b_0+B_0} \int_{c_0-C_0}^{c_0+C_0} G(a, b, c)\, da\, db\, dc = \\
&= \int_{-A_0}^{A_0} \int_{-B_0}^{B_0} \int_{-C_0}^{C_0} \Big[G(a_0, b_0, c_0) + \\
&+ G_a \cdot A + G_b \cdot B + G_c \cdot C + \frac{1}{2} (G_{ab} \cdot AB + G_{ac} \cdot AC + G_{bc} \cdot BC + \\
&+ G_{aa} \cdot A^2 + G_{bb} \cdot B^2 + G_{pc} \cdot C^2) \Big] dA\, dB\, dC = \qquad (4) \\
&= \Big[G(a_0, b_0, c_0) \cdot ABC + \frac{1}{2} (G_a \cdot A^2 BC + G_b \cdot AB^2 C + G_c \cdot ABC^2) + \\
&+ \frac{1}{8} (G_{ab} \cdot A^2 B^2 C + G_{ac} \cdot A^2 BC^2) + G_{bc} \cdot AB^2 C^2 + \\
&+ \frac{1}{6} (G_{aa} \cdot A^3 BC + G_{bb} \cdot AB^3 C + G_{cc} \cdot ABC^3) \Big]_{-A_0 - B_0 - C_0}^{A_0 B_0 C_0} .
\end{aligned}
$$

After re-arranging this expression:

$$
\begin{aligned}
g(a_0, b_0, c_0, A_0, B_0, C_0) &= \Big[\frac{2}{3} ABC \cdot G(a_0, b_0, c_0) + \\
&+ \frac{1}{3} ABC \Big\{ G(a_0, b_0, c_0) + G_a \cdot A + G_b \cdot B + G_c \cdot C + \\
&+ \frac{1}{2} (G_{ab} \cdot AB + G_{ac} \cdot AC + G_{bc} \cdot BC + \\
&+ G_{aa} \cdot A^2 + G_{bb} \cdot B^2 + G_{cc} \cdot C^2) \Big\} \Big]_{-A_0 - B_0 - C_0}^{A_0 B_0 C_0} + \qquad (5) \\
&+ \frac{1}{6} \Big[ABC \cdot \Big\{ G_a \cdot A + G_b \cdot B + G_c \cdot C - \\
&- \frac{1}{4} (G_{ab} \cdot AB + G_{ac} \cdot AC + G_{bc} \cdot BC) \Big\} \Big]_{-A_0 - B_0 - C_0}^{A_0 B_0 C_0} .
\end{aligned}
$$

In the expression of the second square bracket, the coefficients of all first and second partial derivates of G are zero after substitution of the upper and lower limits and some easy algebra, therefore the whole expression is equal to zero. (This can be easily proved by simple substitution, perhaps using also Eq. 2a for checking.) The figure bracket of the first square bracket is according to Eq. 3 equal to $G(a, b, c)$; the substitution of the upper and lower limits is identical with the substitution of all co-ordinate-triplets of the eight prism vertices. The signs are determined by the followings: 1) the rule of Eq. 2a; 2) the actual upper and lower limits, which are in pairs equal with each other with opposite sign; 3) the appearance of the product ABC as well in $G(a, b, c)$ as in $G(a_0, b_0, c_0)$. According to these the final sign is in all cases positive (and the product $A_0B_0C_0$).

If the mass of the prism is m, it is evident that

$$m = \sigma \cdot 8A_0 B_0 C_0 .$$

Taking this into account:

$$g(a_0, b_0, c_0, A_0, B_0, C_0) = fm \left\{ \frac{2}{3} G(a_0, b_0, c_0) + \frac{1}{3} \left[\frac{1}{8} \sum G(a, b, c) \right] \right\}. \qquad (7)$$

The summation is to be carried out for all the eight vertices.

The essence of formula (7) can be expressed in the following form: if we concentrate two third of the mass of the prism into the gravity center, and distribute the remaining one third to the eight vertices, then the effect of this system consisting of 9 points will approximate the effect of the homogenous prism the better the more the condition $A_0, B_0, C_0 \ll \sqrt{a_0^2 + b_0^2 + c_0^2}$ is fulfilled.

3.

In possession of formula 7, the computation of the gravity effect of the prisms can be organized in case of the division of the space into prisms of identical base and of a height being on all depth levels also identical as follows.

The effect of each prism is computed according to Eq. 7, then, departing from the prism below the reference point and proceeding along any of the co-ordinate axes, the effect of the prism is also determined with an exact formula (e. g. with Eq. 2) for all depth levels. If the difference of the two values is greater than a prescribed minimum value, then the element of the data matrix corresponding to this prism is changed into that computed with the exact formula. If the difference against the exact formula is for three consecutive prisms less than a prescribed maximum difference, then the exact formula must be no more used in the corresponding direction, but the values computed with the exact formula are substituted into the data matrix for all prisms

of the depth level, which lie nearer to the reference point than the distance of the last prism of the treble coincidence.

This algorithm makes an analysis of the deviations (e. g. the use of some remainder formula) superfluous.

The accuracy of the approximative values from formula 7 can be estimated from the values of the prism series of Fig. 2, referred to the origin, illustrating the percentual differences between the values determined with the exact formula and those with Eq. 6 (the dotted curve 2 in Fig. 2; the corresponding scale can be seen on the right side of the figure).

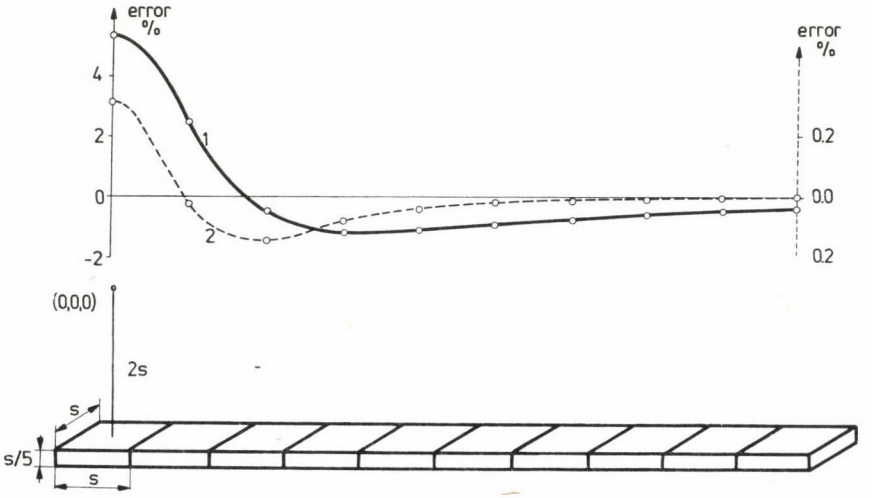

Fig. 2. In case of the division of the space into rectangular prisms, only two root extractions are needed in addition to elementary operations for the determination of the gravitational effect of the elementary prisms using the method described in the paper. The dotted curve (2) in the figure shows the values of the corresponding prisms in respect to the origin in order to show the negligible small errors in comparison to the exact values (the scale is shown on the right-hand side of the figure). — If the full mass of the prism is concentrated into the gravity center, the approximation contains only one extraction of the root, the errors, however, can be greater by one or two orders of magnitude (see the continuous curve (1); the scale is at the left-hand side)

It is natural if the whole mass of the prisms is concentrated into the gravity center, the one-point approximation is considerably worse than with the ninepoint approximation of Eq. 7. In order to show the differences, Fig. 2 contains also the error curve for that case (continuous line, curve 1, left side scale).

It is important to note that the prism vertices of Eq. 7 appear in 7 further prisms, i. e. each vertex plays a role in 8 prisms with the exception of prisms on the border of the space studied. This situation can be advantageously used in a program for the computation of the effect of the prisms in a complete

space division as the G-determinations for a vertex must be made only once (taking the concrete form of G in the gravitational case, see Eq. 1, into account), and therefore in addition to elementary operations only two square roots are to be determined for each inner prism of the space division. A comparison of the curves 1 and 2 of Fig. 2 shows that with a doubling the demand for computer time of the one-point approximation the error in respect of the values determined with the exact formula diminishes by one order of magnitude, or, in case of greater distances, by two orders of magnitude.

A computer program for the computer ODRA 1304 has been written by eng. D. Kiss.

4.

The actual form of the G-function (see Eq. 1) has not been used in the ideas developed in part 2 (which are in fact connected to the general scheme developed in (5)). Therefore Eq. 7 can be, in its domain of validity, more generally used than the exact formulas, as the followings show.

a) Formula 7 can be used for a rectangular prism in any position

If the origin of the co-ordinate system is shifted to the reference point and the axes lie parallel to the direction of the prism sides, the form of the function G will be different from that given in Eq. 1; but in part 2 this special form has not been used.

b) The integral of $w(x, y, z)$ for a rectangular prism can be also computed according to Eq. 7

Accordingly it is sufficient to determine the function w (see (6)) in the points of a special grid determined by the division into prisms, if for an interpretation of the filtered maps the w-integral of the prismatic space division is needed.

As a proof, it is sufficient to hint at the arbitrariness of the G-function in part 2, similarly to case *a)*. From the same cause the following statement is also valid:

c) Equation 7 can be also used for the determination of the magnetic effect of rectangular prisms.

Finally it should be reminded that the approximation presented here gives the exact value of the effect of prisms if the function G is a third order polynomial. In case of third order terms, namely, at least of one of the integration variables is at an odd power, therefore the integrals, being symmetrical to the origin, have the value zero. In the series expansion Eq. 3, the third order terms could also be included without any change in the results, a further discussion would be, with these terms, too lengthy.

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REFERENCES

1. HAÁZ, I. B.: Kapcsolat a derékszögű hasáb tömegvonzásának potenciálja és a potenciál deriváltjai között (Connection between the potential of the mass attraction of rectangular prisms and the derivatives of the potential). *Geofizikai Közlemények*, 2, No. 7. (1953), 1—10.
2. HAÁZ, I. B.: Történeti, elvi és gyakorlati adalékok a derékszögű hasáb tömegvonzásának számításaihoz (Historical, theoretical and practical contributions to the computation of the attraction of rectangular prisms). *Geofizikai Közlemények*, 18, No. 4 (1969), 103—110.
3. HAÁZ, I. B.: Gravitációs és földmágneses anomáliák háromdimenziós értelmezése derékszögű hasábalakú vagy ilyen hasábokból összetehető test gravitációs, illetve mágneses hatásával. (Three-dimensional interpretation of gravity and geomagnetic anomalies with the gravity or magnetic effect of rectangular prism-shaped bodies or of bodies constituted of such prisms). Dissertation, Manuscript, Budapest, 1971.
4. NAGY, D.: The gravitational attraction of a right rectangular prism. *Geophysics*, 31 (1966), 362—371.
5. STEINER, F.: Die Rolle der Polynomdarstellungen in der Lösung geophysikalischer Probleme. *Acta Geod. Geoph. Mont. Hung.*, 6 (1971), 63—81.
6. STEINER, F.: Characterization and derivation of map transformations on the basis of the anomalous body-map relationship. *Acta Geod. Geoph. Mont. Hung.*, 8 (1973), 71—84.
7. ZILAHI-SEBESS, L.: Háromdimenziós tömeg gravitációs hatásának kiszámítása (Computation of the gravitational effect of three-dimensional bodies). *Geofizikai Közlemények*, 15 (1966), 149—157.

ПРИЛОЖЕНИЕ К ВЫЧИСЛЕНИЮ ГРАВИТАЦИОННОГО ВЛИЯНИЯ ПРЯМОУГОЛЬНОЙ ПРИЗМЫ

Ф. ФТЕЙНЕР

РЕЗЮМЕ

Относительно гравитационного влияния прямоугольной призмы с однородной плотностью, в статье доказывается, что, сконцентрировав две третьих массы призмы в ее центре и равномерно распределив остаточную одну треть массы по восьми вершинам призмы, влияние этой, состоящей из 9 точек системы хорошо аппроксимирует влияние однородной призмы. Эта простая аппроксимация будет высокой точности, если длины ребер малы по отношению расстояния от точки относимости до центра призмы.

Аппроксимация с точки зрения вычислительной техники может быть очень выгодно применена, например, тогда, когда для целей гравитационной интерпретации разделяется большая область поля на прямоугольные призмы с одинаковым основанием, считаемые уже однородными, и, вычисляются их гравитационные влияния. В этом случае для близких призм можно применять точную формулу, а в случае удаленных в большинстве друг от друга призм целесообразно применять аппроксимацию, заданную в этой статье и требуемую машинного времени на целый порядок ниже.

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DAS AUSSTECKEN DES SCHACHTANSATZPUNKTES AUSSERHALB DES PRODUKTIVEN GEBIETES

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Bei reichen und in großer Tiefe gelagerten, oder steil einfallenden Schichten kann der im Schachtsicherheitspfeiler gebundene Nutzmineralvorrat und die damit verbundenen Wertverluste sehr beträchtlich sein. Erwünscht ist demnach, den optimalen Ort für außerhalb des produktiven Schachtfeldes angelegte Schächte mit der Bedingung zu bestimmen, daß dabei die Kosten für Förderung und Fahrung als Funktionen des Schachtansatzpunktes am kleinsten seien. Vom Verfasser werden die bezüglichen theoretischen Zusammenhänge abgeleitet und einige Beispiele aus der Praxis dargelegt, aus denen hervorgeht, daß die kompliziert scheinenden Aufgaben verhältnismäßig einfach gelöst werden können. Es ist zweckmäßig, die Lösungen zeichnerisch durchzuführen.

Der Kohlenvorrat, der durch den Sicherheitspfeiler eines im produktiven Gebiet niedergebrachten Schachtes blockiert wird, kann sehr beträchtlich sein, und zwar umso größer, je mächtiger die produktive Formation, und je tiefer sie gelagert ist. Der im Sicherheitspfeiler gebundene Mineralvorrat kann selbst bei steilgelagerten Vorkommen relativ groß sein. Naturgemäß ist die Menge des im Schachtsicherheitspfeiler enthaltenen, nicht greifbaren Kohlenvorrates bei einem gegebenen Vorkommen vor der Lage des Schachtansatzpunktes, dem für das Gebiet kennzeichnenden Grenzwinkel und auch davon abhängig, ein wie großes Gelände in der Umgebung des Schachtes gegen schädliche Bodenbewegungen geschützt werden muß. Außerdem spielt hier auch der spezifische Wert des gebundenen Mineralvermögens eine Rolle. Je höher dieser Wert ist, umso größer ist auch der Wertverlust, der durch den im Schachtsicherheitspfeiler gebundenen, nutzbaren Mineralvorrat erwächst.

Die neuen Mineralvorkommen werden immer häufiger nur in größeren Teufen entdeckt; deswegen ist es von Interesse, sich mit der Wahl von Schachtansatzpunkten außerhalb des produktiven Gebietes zu befassen. Die vorliegenden Untersuchungen gehen deshalb von zwei grundlegenden Bedingungen aus: Die eine lautet, daß man den Schacht, beziehungsweise die Schächte so anlegen muß, daß der Schutzpfeiler keine nutzbaren Mineralvorräte enthalte. Die zweite Bedingung aber ist, daß bei der obigen Zwangsbedingung die Kosten für Förderung und Fahrung möglichst einen Mindestwert betragen sollen.

Das allgemeine Verfahren kann der theoretischen Grundlage nicht entraten, deswegen soll diese Frage zunächst theoretisch betrachtet werden.

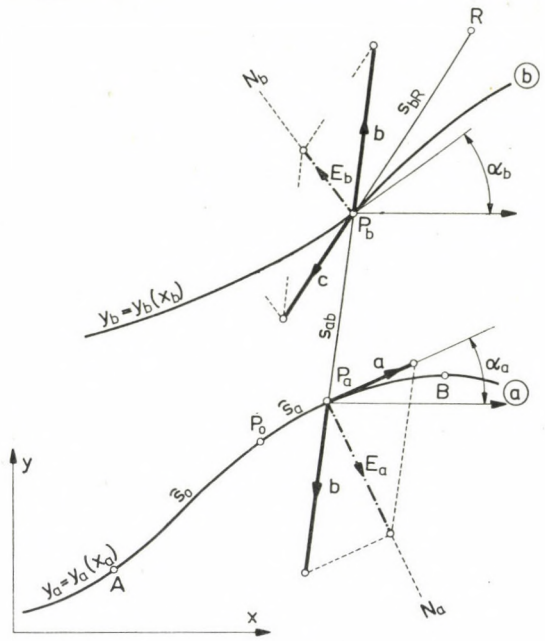

Abb. 1

Nach Abb. 1 seien in der Ebene zwei gekrümmte Linien gegeben: $y_a = y_a(x_a)$ und $y_b = y_b(x_b)$. Die Gleichungen dieser Kurven sind natürlich unbekannt, doch ist dies – wie wir sehen werden – praktisch auch gar nicht nötig; es genügt, wenn die Kurven kartierungsmäßig bekannt sind. Im Bogenabschnitt $\widehat{AB}$ der mit *a* bezeichneten Kurve entfalle auf die Längeneinheit das Gewicht Q_0; somit beträgt das ganze Gewicht $Q = Q_0 \cdot \widehat{AB}$.

Die Aufgabe lautet nun: Das entlang des Bogens $\widehat{AB}$ gleichmäßig verteilte Gewicht soll in einem Punkt $P_a[x_a, y_a(x_a)]$ gesammelt werden, wobei die Förderung entlang des Bogens erfolgt; danach aber soll das gesammelte Gewicht Q aus dem Punkt P_a auf der geraden Strecke s_{ab} bis zum $P_b[x_b, x_b(x_a)]$ der mit *b* bezeichneten Kurve, und schließlich aus dem Punkt P_b auf der Geraden s_{bR} zu einem gegebenen Punkt $R(x_R, y_R)$ gefördert werden. Die spezifischen Kosten der Förderung auf dem Bogenabschnitt $\widehat{AB}$ seien k_a, die auf der geraden Strecke s_{ab} seien k_{ab}, und die Kosten für den geraden Abschnitt s_{bR} seien k_{bR}.

Es frägt sich nun: Wo müssen die beiden Punkte P_a und P_b auf den Kurven *a* bzw. b gewählt werden, um für die ganze Förderkosten ein Minimum zu ergeben?

Es sei der Punkt $P_0[x_0, y_0]$ der Halbierungspunkt des Bogens $\widehat{AB}$ und es sei $\widehat{AP_0} = \widehat{P_0B} = \widehat{s}_0$. Dann ergeben sich die ganzen Kosten aus

$$K = Q_0 (\widehat{s}_0^2 + \widehat{s}_a^2)\, k_a + Q(s_{ab}\, k_{ab} + s_{bR}\, k_{bR}) ,$$

worin

$$\hat{s}_a = \int_{x_0}^{x_a} \sqrt{1 + [y'_a(x_a)]^2}\, dx_a$$

$$s_{ab} = \sqrt{(x_a - x_b)^2 [y_a(x_a) - y_b(x_b)]^2},$$

$$s_{bR} = \sqrt{(x_b - x_R)^2 + [y_b(x_b) - y_R]^2},$$

ferner

$$y'_a(x_a) = \operatorname{tg} \alpha_a$$

$$y'_a(x_b) = \operatorname{tg} \alpha_b .$$

Die ganzen Kosten für Förderung haben einen Extremwert, wenn

$$\frac{\partial K}{\partial x_a} = \frac{2Q_a k_a}{\cos \alpha_a} + Q (\cos \varphi_{ab} + \sin \varphi_{ab} \operatorname{tg} \alpha_a) k_{ab} = 0 ,$$

$$\frac{\partial K}{\partial x_b} = (\cos \varphi_{ba} + \sin \varphi_{ba} \operatorname{tg} \alpha_b) k_{ba} + (\cos \varphi_{bR} + \sin \varphi_{bR} \operatorname{tg} \alpha_b) k_{bR} = 0,$$

worin

$$Q_a = Q_0 \hat{s}_a \text{ ist.}$$

Der Einfachheit halber sei

$$\frac{Q_a}{Q} = \lambda .$$

Hiernach lautet unser Gleichungssystem:

$$2\lambda + \frac{k_{ab}}{k_a} \cos (\varphi_{ba} - a_a) = 0$$

$$\frac{k_{ab}}{k_a} \cos (\varphi_{ba} - \alpha_b) + \frac{k_{bR}}{k_a} \cos (\varphi_{bR} - \alpha_b) = 0 .$$

Hierin bedeuten:

φ den Richtungswinkel

α_a und α_b den Neigungswinkel der zum Punkt P_a, bzw. Punkt P_b gehörigen Tangente.

Das Gleichungssystem kann auch kürzer, wie folgt geschrieben werden:

$$a + b \cos (\varphi_{ab} - \alpha_a) = 0$$

$$b \cos (\varphi_{ba} - \alpha_b) + c \cos (\varphi_{bR} - \alpha_R) = 0 .$$

Es kann somit festgestellt werden: Die Punkte P_a und P_b bezeichnen dann den Extremwert der ganzen Förderkosten, wenn die Resultierende (E_a) der im Punkt P_a angreifenden Vektoren **a** und **b** auf die zum Punkt P_a gehörige

Normale (N_a), die Resultierende (E_b) der im Punkte P_b angreifenden Vektoren **b** und **c** aber auf die zum Punkt P_b gehörigen Normale (N_b) entfällt. Prinzipiell sind auch mehrere solcher Punkte möglich; unter diesen wird das Minimum von jenen bezeichnet, zu welchen die kleinsten Resultierenden (E_a, E_b) gehören.

Der Vektor **a** ist eigentlich ein Resultierungsvektor. Es werde der Wert $Q_0 \cdot \widehat{P_a A}$ mit Q_b und $Q_0 \cdot \widehat{P_a B}$ mit Q_r bezeichnet, dann ist

$$2\,Q_a = Q_b - Q_r\,.$$

Der Vektor **a** ist also wirklich ein Resultierungsvektor; der eine Vektor ist $Q_l k_{al}$, der andere $Q_r k_{ar}$, wobei die Richtung der beiden Vektoren identisch, d.h. die zu Punkt P_a gehörige Tangente, ihr Sinn jedoch entgegengesetzt ist. Demzufolge kann auch gesagt werden, daß Punkt P_a drei Vektoren angreifen, deren Resultierende auf die zu Punkt Pr gehörige Normale entfällt.

Ist $k_{ab} = k_{bR}$, dann entfallen nach der zweiten Gleichung s_{ab} und s_{bR} auf eine Gerade und damit wird die erste Gleichung zu

$$a + b\cos(\varphi_{aR} - \alpha_a) = 0\,.$$

Dies besagt, daß die Aufgabe in diesem Falle so zu betrachten ist, als ob die Kurve b gar nicht vorhanden wäre.

Auch eine solche Variante der Aufgabe wäre möglich, bei welcher der Punkt R fehlt, d.h. das Gewicht Q nur bis zu irgend einem Punkt der Kurve b gefördert werden muß. In diesem Fall lautet die zweite Gleichung:

$$b\cos(\varphi_{ba} - \alpha_b) = 0$$

Dies ist nur dann möglich, wenn

$$\varphi_{ba} - \alpha_b = 90^\circ \qquad \text{ist,}$$

d.h. nur dann, wenn der gerade Abschnitt s_{ab} normal auf die Kurve b verläuft. Auch hier kann es möglich sein, daß auch mehrere Punkte der Kurve b diese Bedingung erfüllen. In der Praxis ist dies aber sehr selten der Fall; sollte er trotzdem eintreten, so kann jener Fall leicht ausgesucht werden, für den das Kostenminimum gilt.

Derartige Aufgaben können nur mit dem Iterationsverfahren oder zweckmäßig graphisch gelöst werden. Das Iterationsverfahren ist – selbst wenn es zeichnerisch erfolgt – ziemlich schwerfällig. Wesentlich ist dabei die Wahl des ersten Schrittes. Soll beispielsweise die in Abb. 1 gegebene Aufgabe gelöst werden, so kann der erste Schritt zweckmäßig jener sein, daß auf der Kurve a ein solcher Punkt P'_a gesucht wird, von dem aus man mit einer geraden Strecke

zum Punkt R gelangen kann (Abb. 2), d.h. wo die Resultierende (E'_a) der im Punkt P'_a angreifenden Vektoren **a'** und **b'** auf die zum Punkt P'_a gehörige Normale (N'_a) entfällt, wobei sich die im Vektor **b'** erscheinenden spezifischen Kosten aus k'_{ab} und k'_{bR} im Verhältnis der dazugehörigen Entfernungen s'_{ab} und s'_{bR} zusammensetzen. Der nächste Schritt muß schon sinngemäß festgelegt werden. Ist $k'_{bR} < k'_{ab}$, dann muß der Punkt P'_b so verschoben werden, daß die Strecke s'_{bk} zunimmt, s'_{ab} aber abnimmt. Im gleichen Sinne und ähnlich verfährt man auch beim Verschieben des Punktes P'_a.

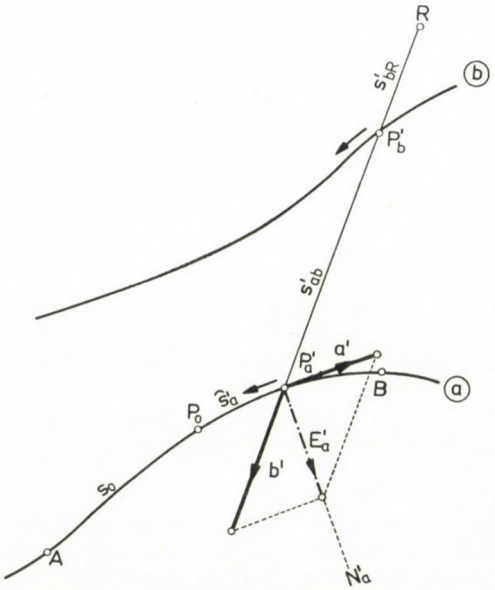

Abb. 2

Die in Abb. 3 dargestellte Aufgabe ist nach dem Vorhergesagten leicht zu verstehen. Ein bestimmtes Gewicht soll vom Punkt S zum Punkt R bewegt werden, wobei zwischen den beiden Punkten zwei Kurven liegen, und zu jedem Abschnitt jeweils andere spezifische Förderkosten gehören. Die beiden Bedingungsgleichungen für die Extremwerte können leicht, wie folgt aufgestellt werden:

$$\cos(\varphi_{Sa} - \alpha_a) + \frac{k_{ab}}{k_{Sa}} \cos(\varphi_{ab} - \alpha_a) = 0$$

$$\frac{k_{ab}}{k_{Sa}} \cos(\varphi_{ba} - \alpha_b) + \frac{k_{bR}}{k_{Sa}} \cos(\varphi_{bR} - \alpha_b) = 0 .$$

Oder kurz:

$$a \cos(\varphi_{Sa} - \alpha_a) + b \cos(\varphi_{ab} - \alpha_b) = 0$$

$$b \cos(\varphi_{ba} - \alpha_b) + c \cos(\varphi_{bR} - \alpha_b) = 0 .$$

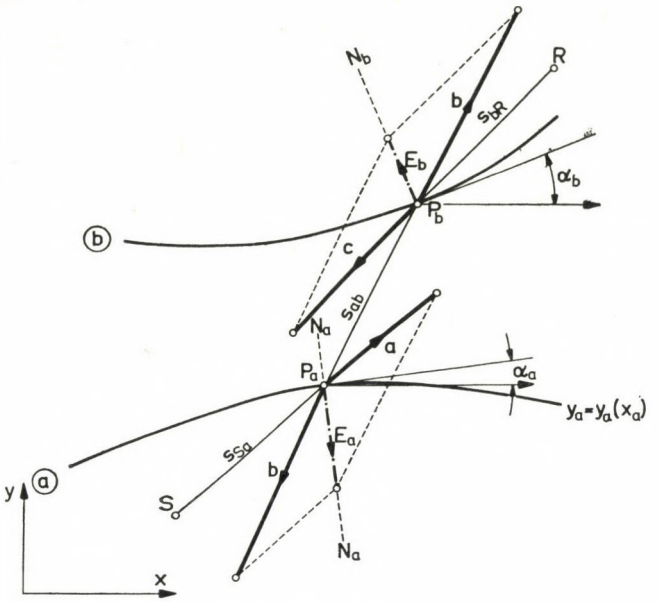

Abb. 3

Den Bedingungen können auch hier mehrere Punktpaare genügen: Zum absoluten Minimum aber gehört der kleinste resultierende Wert. Bei gewissen Kurven kann die Bedingung auch am Ort eines relativen Maximum erfüllt werden; deswegen ist hier Vorsicht geboten. Bei den in der Praxis möglichen Kurven aber kann das Minimum von dem eventuell möglichen Maximum leicht unterschieden werden.

Es ist leicht einzusehen, daß bei $k_{Sa} = k_{ab} = k_{bR}$ die Punkte S und R durch eine gerade Strecke verbunden werden.

In gewissen Fällen ist das Iterationsverfahren theoretisch umständlich, praktisch jedoch verhältnismäßig einfacher, u. zw. nicht nur deshalb, weil es graphisch mit genügender Genauigkeit durchgeführt werden kann, sondern weil es auch genügt, den oder die das Minimum bezeichnenden Punkte nur angenähert zu suchen, da sich die Kosten in der Umgebung des Minimums nicht wesentlich ändern.

Die spezifischen Förderkosten können Funktionen der Weglänge sein. Dies ist beim Iterationsverfahren kein Problem, da bei jedem Schritt auch die zu diesem gehörigen Längen gegeben sind, und die spezifischen Kosten dementsprechend ausgewählt werden können.

In Abb. 4 ist eine verhältnismäßig einfache Anwendung dargestellt. Bei einem steilgelagerten Vorkommen sind folgende fünf Fördersohlen geplant: +120, +20, −80, −180, −280 m. Die streichenden Sohlenstrecken werden im Liegenden aufgefahren. Der abbaubare Mineralvorrat des Vorkommens ist

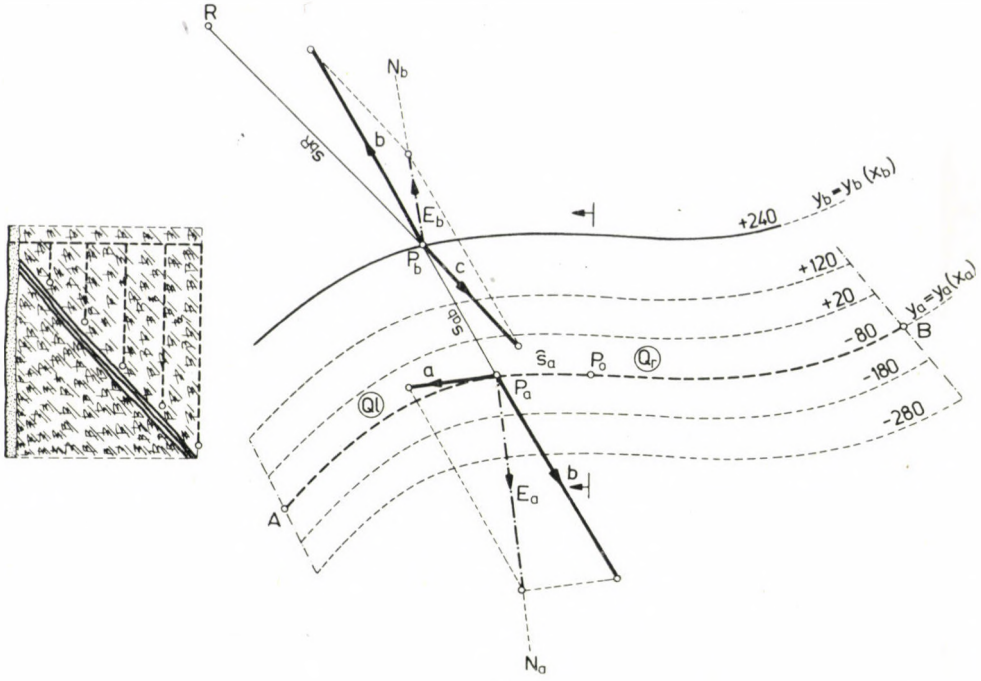

Abb. 4

Q. Das Fördern im Streichen wird so betrachtet, als ob sich Q auf der Sohle -80 m verteilt. Der Förderschacht P_b muß über Tage auf der Linie $y_b = y_b(x)$ so angesetzt werden, daß im Schachtsicherheitspfeiler kein nutzbarer Mineralvorrat verbleibe. Demzufolge ist s_{ab} der Hauptquerschlag der Sohle -80. Vergleicht man Abb. 4 mit Abb. 1, so ist damit schon die Lösung der Aufgabe gegeben.

Da im vorliegenden Falle die Kurve $y_a = y_a(x_a)$ nahezu parallel zur obertägigen Kurve $y_b = y_b(x_b)$ verläuft, so ist das Aufsuchen des Punktes P_b verhältnismäßig einfach.

Vektor **a** ist die Resultierende von zwei Vektoren: der eine ist $Q_l k_{al}$, der andere $Q_r k_{ar}$, wobei der Punkt P_a das Gewicht Q in zwei Teile teilt: $Q_l + Q_r = Q$. Natürlich können auch aus k_{al} und k_{ar} im Verhältnis von Q_l und Q_r die spezifischen Kosten k_a gebildet werden. Somit kann der Vektor **a** auch $2\,Q_a k_a$ sein, wobei $Q_a = Q_r - Q$ ist, d.h. der Mineralvorrat zwischen den Punkten P_a und $P_0 \cdot P_0$ halbiert Q. Wird dieses praktische Verfahren mit dem theoretischen Verfahren verglichen, so kann festgestellt werden, daß das praktische nur ein Näherungsverfahren ist. Dies ist jedoch durchaus zuläßig, da es in der Praxis kein exaktes Verfahren gibt.

In Abb. 5 ist ein Kohlenvorkommen zu sehen. Das gewinnbare Kohlenvermögen beläuft sich auf $Q = 168 \cdot 10^6$ t. Die untertägige, fast streichende

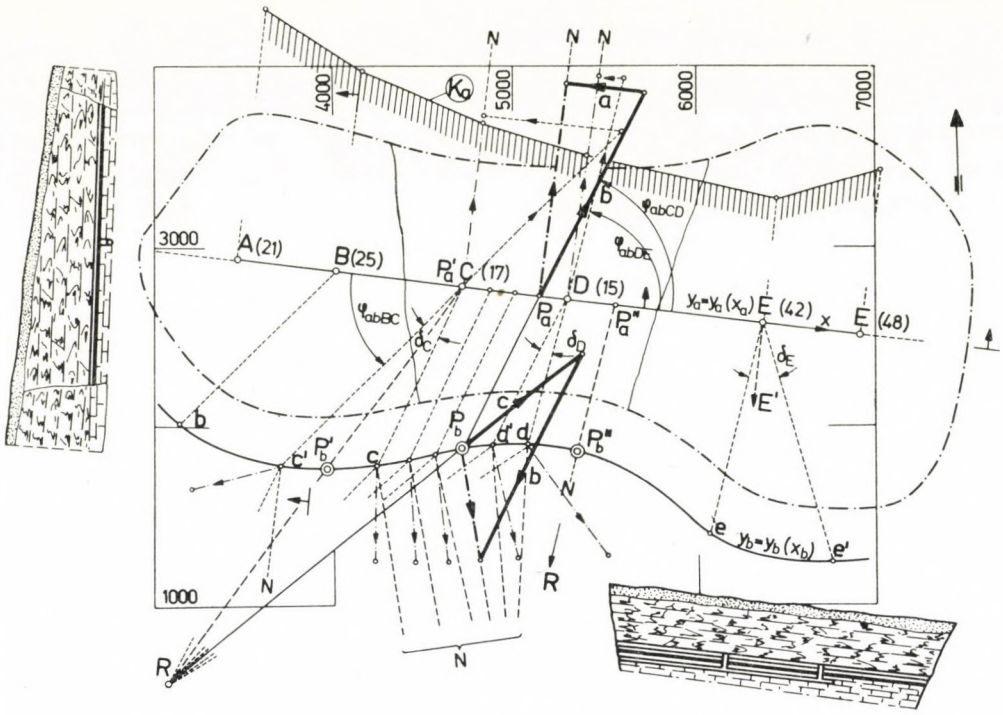

Abb. 5

Hauptförderstrecke ist im festen Liegenden geplant. In diese Hauptförderstrecke wird die Kohle aus den Flözen mit den Spiralrutschen in den Punkten A, B, C, D, E, F abgelassen. Zu den einzelnen Spiralrutschen gehören der Reihe nach folgende Kohlenmengen: 21, 25, 17, 15, 42 und 48 · 10^6 t. Geplant wird nur ein einziger vertikaler Förderschacht in einem Punkt (P_b) der obertägigen Linie $y_b = y_b(x_b)$, um im Schachtsicherheitspfeiler keinen Kohlenvorrat zu blockieren. Obertags muß die Kohle zum Wärmekraftwerk in Punkt R befördert werden. Selbstredend muß der gesuchte Punkt P_b mit den auf dem Geraden $y_a = y_a(x_a)$ gesuchten Punkt P_a unter Tage verbunden werden.

Wird in der untertägigen Hauptförderstrecke A . . . . F der Ort des Sammelpunktes P_a geändert, dann ändern sich nur die Kosten K_a für die Förderung in der Hauptstrecke gemäß der abschnittsweise gebrochenen Geraden, wenn die spezifischen Kosten bis dahin unverändert waren, wobei sich Punkt P_a auf dem gleichen Abschnitt plaziert. Praktisch ist dies auch zulässig.

Es ist leicht zu erkennen, daß die Punkte P_a und P_b in den Quadranten $A-E-E'$ entfallen, da in diesem der Punkt R zu finden ist, ferner Punkt E das Minimum der Kosten K_a bezeichnet.

In irgend einem Abschnitt der gebrochenen Geraden K_a ist der Neigungswinkel nur sehr selten gleich 0°, d.h. nur dann, wenn $Q_l k_{al} = Q_r k_{ar}$ ist. Prak-

tisch also gehört fast immer das Kostenminimum K_a zu irgendeinem Punkt. Im vorliegenden Fall erfolgt dies im Punkt E, da die Resultierende der von links und rechts angreifenden Vektoren hier am kleinsten ist.

Ein Abschnitt $C-D$ werde gewählt, d.h. es sei angenommen, daß der gesuchte Punkt P_a auf diesen Abschnitt entfällt. Die relativen spezifischen Kosten seien: $k_{aCD} = 1$; $k_{ab} = 0{,}75$ und $k_{bR} = 0{,}50$.

Für den Abschnitt $C-D$ kann demnach (mit Ausnahme der Punkte C und D) die Bedingungsgleichung aufgeschrieben werden, wobei $\alpha_a = \text{const} = 0^0$ ist. Somit ist

$$-42 + 168 \cdot 0{,}75 \cos \varphi_{abCD} = 0$$

worin $Q_l - Q_r = 21 + 25 + 17 - 15 - 42 - 48 = -42$ ist. Da $\alpha_a =$ konstant ist, gehört zu diesem Abschnitt ein einziger Richtungswinkel φ_{abCD} laut Beziehung

$$\cos \varphi_{abCD} = \frac{42}{126}$$

das heißt

$$\varphi_{abCD} = 70^\circ\, 32' .$$

Natürlich können auch die zu den anderen Abschnitten gehörigen Richtungswinkel berechnet werden. Sind beispielsweise die relativen spezifischen Kosten im Abschnitt $D-E$ 1 : 0,77 : 0,48, dann ist $\varphi_{abDE} = 84^0 41'$, im Abschnitt $B-C$ bei relativen spezifischen Kosten von 1 : 0,73 : 0,54 aber wird $\varphi_{abBC} = 51^0\, 42'$.

Aus den Abschnitten wurden die Endpunkte ausgelassen, da in diesen der für den Abschnitt kennzeichnende Richtungswinkel einen Grenzwert darstellt. Selbstredend gehören zu jedem Punkt zwei Grenzwerte. Zwischen den beiden Grenzwerten kann zu jedem Punkt *(A, B, C, D, E, F)* jede beliebige Richtung gehören, d.h. diese Richtungen können sich innerhalb des Winkels δ bewegen. Z. B.

$$\delta_c = \varphi_{abCD} - \varphi_{abBC} = 18^\circ 50', \text{ oder}$$

$$\delta_D = \varphi_{abDE} - \varphi_{abCD} = 14^\circ 9' .$$

Man kann es auch so sagen, daß zu jedem Abschnitt der Hauptstrecke $A \ldots . F$ bzw. zu jedem ihrer Punkte ein bestimmter Bogenabschnitt der Kurve $y_b = y_b(x_b)$ gehört. Auf diese Weise gehört zum Abschnitt $C-D$ der Bogenabschnitt $\overset{\frown}{c\,d}$, zum Punkt C aber $\overset{\frown}{c'c}$ und zum Punkt D der Bogenabschnitt $\overset{\frown}{dd'}$, usw. Es kann demnach festgestellt werden, daß bei dem Suchen nach Punkt P_a die einzelnen Punkte *(A, B, C, D, E, F)* mit wesentlich besseren Chancen erscheinen, als jeder beliebige andere Punkt. Im vorliegenden Fall aber ist der gesuchte Punkt P_a ein Zwischenpunkt des Abschnittes $C-D$,

und dementsprechend P_b ein Zwischenpunkt des Bogenstückes $\widehat{cd}$ ist, wobei Punkt P_b den Ansatzpunkt des Vertikalschachtes bezeichnet.

Die Teufe des Vertikalschachtes kann sich je nach der Lage des Punktes P_b ändern, doch davon sei hier abgesehen.

Mit Hilfe von Abb. 5 können auch in drei Spezialfällen die auf der Hand liegenden Lösungen erkannt werden. Der erste Spezialfall sei, wenn $k_{ab} = k_{bR}$ ist. Im vorliegenden Fall sei $k_{ab} = k_{bR} = 0{,}75$. Man sieht nun sofort, daß Punkt P'_a der C-Punkt ist, also der Schachtort, während sich der Punkt P'_b als Schnittpunkt des geraden Abschnittes $R-C$ mit der Kurve $y_b = y_b(x_b)$ ergibt.

Der zweite Spezialfall aber sei, wenn Punkt R fehlt. Dann ist $P''_a - P''_b$ normal auf die Kurve $y_b = y_b(x_b)$. Im vorliegenden Fall kann P''_a nur ein Punkt des Abschnittes $D-E$ sein.

Der dritte Spezialfall sei, wenn Punkt R nicht vorher bestimmt wurde, sondern nur die Bedingung besteht, daß der Punkt R von den durch die Kurve $y_b = y_b(x_b)$ bestimmten zwei Raumhälften in jener liege, in welcher sich das improduktive Gebiet befindet.

In diesem Fall ist Punkt R der Punkt jener Halbgeraden, deren Anfangspunkt P''_b, ihre Richtung $P''_a - P''_b$ ist.

Es können ferner Fälle vorkommen, bei denen auch der frei bezeichenbare optimale Ort des vertikalen Förderschachtes der Bedingung entspricht, daß in dem Schachtschutzpfeiler kein abbaubarer Mineralvorrat vorhanden sein soll. Ein solcher Fall ist in Abb. 6 dargestellt. Die abbaubaren Vorräte dreier, von einander unabhängiger Vorkommen seien Q_A, Q_B, Q_C, und es sei $Q_A + Q_B + Q_c = Q$. Die untertätigen Sammelpunkte der Vorkommen aber seien: A, B, C.

Die Mengen Q_A, Q_B und Q_C sollen unter Tage in einem Punkt P_0, dem Ansatzpunkt des Vertikalschachtes so gesammelt werden, daß dabei die Förderkosten ein Minimum betragen. Die spezifischen Kosten seien k_A, k_B und k_C, wobei natürlich die spezifischen Kosten auch die spezifischen Rücklaufskosten der Investitionen und die spezifischen Betriebskosten enthalten. Der Punkt P_0 bezeichnet dann das Kostenminimum, wenn das folgende Gleichungssystem erfüllt wird:

$$\lambda_A \frac{k_A}{k_R} \cos \varphi_A + \lambda_B \frac{k_B}{k_R} \cos \varphi_B + \lambda_C \frac{k_C}{k_R} \cos \varphi_C + \cos \varphi_R = 0$$

$$\lambda_A \frac{k_A}{k_R} \sin \varphi_A + \lambda_B \frac{k_B}{k_R} \sin \varphi_B + \lambda_C \frac{k_C}{k_R} \sin \varphi_C + \sin \varphi_R = 0$$

oder kurz

$$a \cos \varphi_A + b \cos \varphi_B + c \cos \varphi_C + d \cos \varphi_R = 0$$

$$a \sin \varphi_A + b \sin \varphi_B + c \sin \varphi_C + d \sin \varphi_R = 0 .$$

Hierin ist

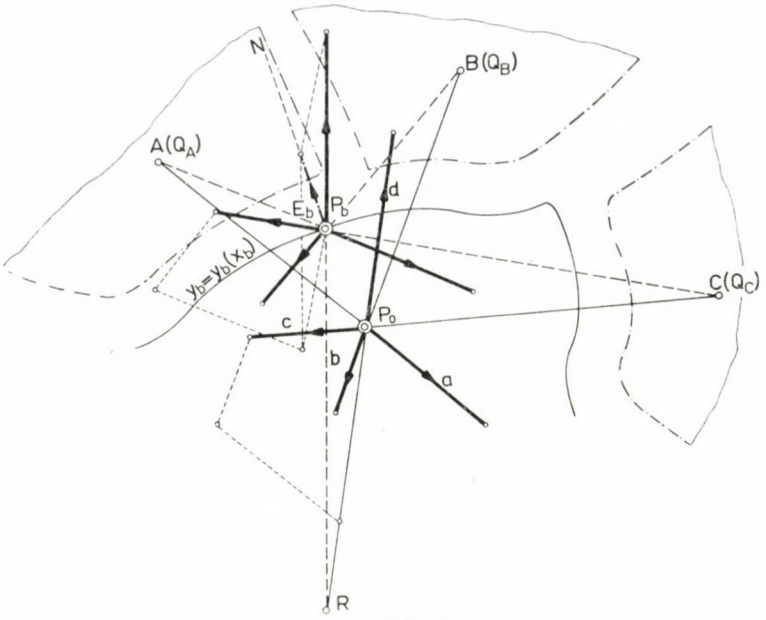

Abb. 6

$$\lambda_A = \frac{Q_A}{Q}$$

$$\lambda_B = \frac{Q_B}{Q}$$

$$\lambda_C = \frac{Q_C}{Q}$$

$$d = 1 .$$

Der Punkt P_0 kann selbstverständlich nur mit dem Iterationsverfahren gesucht werden und erfüllt die Bedingung dann, wenn die Resultierende der im Punkt P_0 angreifenden Vektoren gleich Null wird.

Würde zur Bedingung gestellt, daß der Vertikalschacht ein Punkt der Kurve $y_b = y_b(x_b)$ sei, dann gelangt man zum Punkt P_b, wobei die Resultierende des Vektorsystems auf die zu Punkt P_b gehörige Normale entfällt. Ansonst ist der Punkt P_b leicht zu finden, da er nahe bei jenem Punkt der Kurve $y_b = y_b(x_b)$ liegt, der P_b am nächsten ist.

Man sieht also, daß der das Kostenminimum bezeichnende Punkt auch außerhalb der Kurve $y_b = y_b\,(x_b)$ liegen kann.

ШАХТЫ, УСТАНОВЛЕННЫЕ ВНЕ ПРОДУКТИВНОЙ ТЕРРИТОРИИ

Й. ЗАМБО

РЕЗЮМЕ

При богатых и глубоко залегающих или крутых месторождениях могут быть значительными привязанные в защитной опоре шахт полезные ископаемые и связанные с ними потери стоймости. Поэтому, желательно определить оптимальное место установки шахт вне продуктивной территории с тем условием, чтобы расходы на транспорт, выражаемые в зависимости от места установки, были наименьшими. Автор выводит теоретические зависимости и показывает некоторые практические примеры, из которых ясно, что кажущиеся сложными задачи могут решаться сравнительно просто. Решения целесообразно проводить в чертежах.

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NUMERISCHE UNTERSUCHUNG DES ZUSTANDES VON GESTEINSKOMPLEXEN

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Die Abhandlung befaßt sich mit der Ausarbeitung solcher Rechenmethoden, welche bei der Untersuchung des mechanischen Zustandes einige charakteristische Eigenschaften der Gesteine in Betracht nehmen. Der bröckelige, lockere, wenig zugfestige Charakter des Gesteins wird durch ein aus starren Blöcken und festen Verbindungen bestehendes endliches System modelliert, dessen Berechnung zu einer Aufgabe führt, die im Falle des nur dem Druck widerstehenden Materials, sowie im Falle des den Mohr'schen Druckbedingungen folgenden Materials, weiters im Falle des sich verdichtenden Materials gleicherweise mit einer der simplexen Methoden lösbar ist. Die Untersuchung des begrenzten Sickerns kann genauso behandelt und mit den vorherigen Problemen in Zusammenhang gebracht werden.

Das elastoviskose Zweiphasenmaterial wird mit einem mit kontinuierlichen Festigkeitseigenschaften ausgestatteten abgegrenzten Elementensystem modelliert. Seine Berechnung führt zu einem Differenzialgleichungssystem.

1. Einleitung

Am Lehrstuhl für Mechanik der Fakultät für Bauwesen der Technischen Universität, Budapest hat man die Methode der endlichen Elemente bei vielerlei Aufgaben mit Erfolg angewendet. Für die Forschungen im Bergbau sind die Untersuchungen der in der Halbebene oder im Halbraum sich entwickelnden statischen Zustandsveränderungen, sowie des in den porösen Gesteinen auftretenden Sickerns von größtem Interesse. In der vorliegenden Arbeit sind wir bestrebt, die Untersuchung des Zustandes der Gesteinskomplexe auf jene Gesteinseigenschaften auszudehnen, die bei den in der Praxis des Bergbauingenieurs vorkommenden Materialien speziell wichtig sind.

Wir schenken den folgenden drei Haupteigenschaften des Materials besondere Beachtung:

a) dem bröckeligen, lockeren, wenig zugfestigen Charakter der Gesteine,

b) dem sog. standarden, elastoviskosen Material,

c) der Wechselwirkung von statischer Zustandsveränderung und transientem Sickern.

In jedem der Fälle wenden wir die Methode der endlichen Elemente an, wobei das Komplex durch Dreieckelemente begrenzter Anzahl ersetzt wird. Das Ziel unserer Arbeit ist derzeit die Entwicklung verschiedener Rechenmodelle. Diese Modelle führen zu numerischen Verfahren, die auf den ein-

heimischen elektronischen Rechenmaschinen vorteilhaft bearbeitet werden können. Die Programmierung der einzelnen Algorithmenteile wird – auf die bereits bestehenden Programme basierend – als weitere Aufgabe aufgefaßt. Die praktische Verwendbarkeit der Modelle entscheidet letzten Endes der Vergleich der vom Computer gewonnenen Ergebnisse mit den Daten der Versuche und der Erfahrung.

2. Berechnungsmethode der Zustandsuntersuchung von lockeren Gesteinskomplexen

2.1 *Allgemeine Bemerkungen*

Die Grenzfestigkeit der Gesteine ist für Druck viel höher, als für Ziehen, so ist es bei der Berechnungen im Tunnelbau und im Bergbau eine häufige Voraussetzung, daß das Material nicht zugfestig ist. Das nach dieser Voraussetzung untersuchte Material hat die Eigenschaft, daß wenn je eine Spannungskomponente die Gleichgewichts- und Deformationsbedingungen nur als Ziehen befriedigen könnte, das Material in Richtung der fraglichen Spannungskomponente (bzw. entsprechend dem Charakter der Komponente) birst und an der fraglichen Stelle eine Lücke von bestimmbarem Ausmaß entsteht.

Die lockeren Gesteine werden unter den Gesteinen verschiedener Qualität auch dadurch gekennzeichnet, daß die Gesteinsprobe oder die Gesteinsmasse auf Einfluß von Druck zunächst – obwohl die Belastung klein ist – eine bedeutende spezifische Deformation aufweist, danach wird sie jedoch auch durch erhebliche Belastung nicht bedeutend deformiert. Der Grund hierfür ist, daß am Beginn der Belastung die Kompaktheit des Materials (Steingeröll, bestehend aus kleineren und größeren Körnchen und Stücken) auf Kosten einer makroskopischen Umstrukturierung zunimmt, und die Stücke sich ordnen. Eine weitere Deformation des Systems ereignet sich danach nurmehr auf Kosten einer tatsächlichen Deformation der festen Teile. Es ist offenkundig, daß der erste Effekt mit kleinen Kräften und großen Verschiebungen, der zweite mit großen Kräften und kleinen Verschiebungen verbunden ist. Es ist zweckmäßig diese Erscheinung durch die Zuspitzung entgegengesetzter Tendenzen zu modellieren. Es sei vorausgesetzt, daß im Material Lücken von gegebenen Dimensionen vorhanden sind. Falls sich die Lücke im Laufe der Deformation des Komplexes nicht schließt, so kann sich die entsprechende Spannung nicht ausbilden, falls sie sich aber schließt, entsteht Druckspannung, die durch das bekannte physikalische Gesetz des Materials gekennzeichnet wird [3].

Durch die geeignete Wahl des statischen Modells des Komplexes kann eine derartige Rechenmethode entwickelt werden, die beide oben genannten Eigenschaften in Betracht zieht; eine gute Übersicht bietet und die Aufgabe der Bestimmung der Zustandscharakteristiken (innere Kräfte, Verschiebung

usw.) auf ein in die Rechenmaschine programmiertes mathematisches Verfahren reduziert. Im weiteren wird das Modell, der aufgrund des Modells aufgebaute rechnerische Algorithmus, der zur *linearen komplementären Aufgabe* führt [5], sowie der Grundgedanke der Lösung dieser Aufgabe angegeben.

2.2 *Der Aufbau des Grundmodells*

Anordnung

Jenen Teil des Komplexes, der den Abbau umgibt und dessen Zustand durch die Abbaubedingungen beeinflußt wird, teilen wir der Anordnung und den Schichtenverhältnissen entsprechend zweckmäßig in Dreiecke auf (Abb. 1).

Über die Dreieckelemente wird annäherungshalber angenommen, daß sie unendlich starr sind [9]. Es sei bemerkt, daß diese Annäherung durch eine Serienberechnung genauer sein kann, falls man auch die in Punkt 3 angegebenen elastischen Dreieckelemente verwendet.

Das Komplex wird durch das aus Dreieckelementen und den sich zwischen ihnen befindenden Knotenpunkten ohne Ausdehnung bestehende System ersetzt (Abb. 2a). Die statischen Eigenschaften werden an die Anpassungsstellen der Dreiecke und der Knotenpunkte konzentriert (Abb. 2b).

Zustandskennwerte

Unter den die Zustandsveränderung des Modells charakterisierenden Größensystemen heißen die auf den Schwerpunkt der Elemente reduzierte Verschiebung charakterisierenden Vektoren — im »globalen« Koordinatensystem des Ganzen der Struktur gedeutet :

$$\mathbf{u}^*_{E;i} = [u_{E,x} \; u_{E,y} \; \Phi_z]_i \, .$$

Die die Verschiebung der Knotenpunkte charakterisierenden Vektoren sind:

$$u^*_{C;j} = [u_{C,x} \; u_{c,y}]_j \, .$$

Die Verschiebung der Spitzen der Elemente ist:

$$\mathbf{u}^*_{Si;i,l} = [u_{S,x} \quad u_{S,y}]_{i,l} \, .$$

Die auf der gleichen Stelle, zwischen den Spitzen und den Knotenpunkten auftretenden relativen Verschiebungen sind:

$$\Delta\mathbf{u}^*_{i,l} = [\Delta u_x \quad \Delta u_y]_{i,l} \, .$$

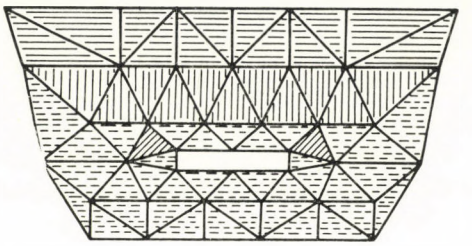

Abb. 1

In Einklang damit ist der Vektor der auf den Schwerpunkt der Elemente reduzierten Belastungsdynamen:

$$^{P}\mathbf{q}^{*}_{E;i} = [q_{E,x} \quad q_{E,y} \quad w_z]_i$$

(letzterer ein Kraftpaar), der der Belastung an den Knotenpunkten:

$$\mathbf{q}^{*}_{C;j} = [q_{C,x} \quad q_{C,y}]_j$$

(Abb. 3). Hier bedeuten $i = 1, 2, \ldots, N_i$ die Elemente, $j = 1, 2, \ldots, N_j$ aber die Numerierung der Knotenpunkte. l zählt die zu je einem Element sich anschließenden Knotenpunkte.

Die weiteren Zustandskennwerte werden in jenen »lokalen« Koordinatensystemen gedeutet, die für je eine Elementenspitze an der Anpassungsstelle zum Knotenpunkt ausgewählt wurden. Ihre Zeichen sind $\xi_{i,l}$ und $\eta_{i,l}$, ihre Stelle wird von Punkt zu Punkt durch zwei, die statischen Verhältnisse charakterisierenden, aufeinander senkrechten Richtung bestimmt, z.B. durch die Hauptrichtungen der orthogonalen Anisotropie, oder durch die Schichtenlage und deren Normale, oder durch die Rißebene und deren Normale, usw.

Die Vektoren der an den Anpassungspunkten auftretenden, vorgeschriebenen Belastungsglieder sind:

$$\mathbf{t}^{*}_{l,l} = [t_\xi \quad t_\eta]_{i,l} .$$

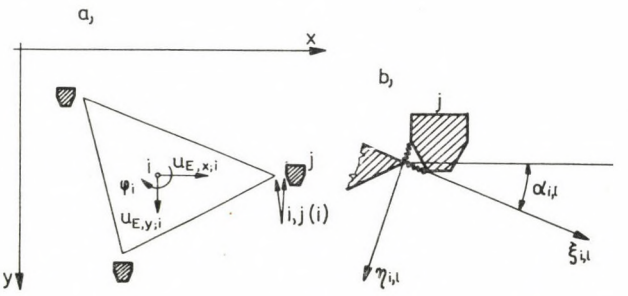

Abb. 2

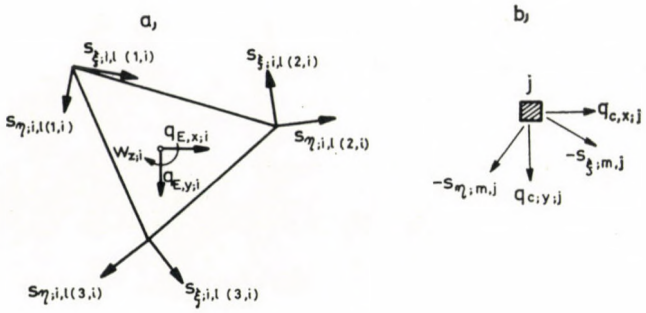

Abb. 3

Die an denselben Stellen möglichen, unbekannten Rißbreiten sind:

$$\mathbf{v}^*_{i,l} = [v_\xi \quad v_\eta]_{i,l} \,.$$

Der Vektor der Verbindungskräfte ist:

$$\mathbf{s}^*_{i,l} = [s_\xi \quad s_\eta]_{i,l} \,.$$

Zur Aufschreibung der Gleichungen benötigt man noch weitere orthogonale Matrizen, die an je einem Verbindungspunkt die im globalen Koordinatensystem gedeuteten Komponenten der zweidimensionalen Vektoren in die in den lokalen Systemen gedeuteten transformieren. Eine dieser Matrizen ist:

$$\mathbf{T}_{i,l} = \begin{bmatrix} \cos \alpha_{i,l} & \sin \alpha_{i,l} \\ -\sin \alpha_{i,l} & \cos \alpha_{i,l} \end{bmatrix} .$$

Stoffgesetz

Das Stoffgesetz des Modells besagt, daß bei jedem der Verbindungen die inneren Kräfte mit der relativen elastischen Verschiebung proportional sind:

$$\mathbf{s}_{i,l} = \mathbf{M}_{i,l} \, \Delta \mathbf{u}_{R;i,l} \tag{1}$$

Dabei ist:

$$\mathbf{M}_{i,l} = \begin{bmatrix} M_{\xi,\xi} & M_{\xi,\eta} \\ M_{\eta,\xi} & M_{\eta,\eta} \end{bmatrix}_{i,l} ; \quad M_{\xi,\eta} = M_{\eta,\xi}$$

die die Starrheitsverhältnisse des Materials bei der Verbindung charakterisierende Matrix, deren Elemente aus petrophysikalischen Labormessungen stammen, und

$$\Delta \mathbf{u}^*_{R;i,l} = [\Delta u_{R,\xi} \;\; \Delta u_{R,\eta}]_{i.l}$$

ist der Vektor der relativen elastischen Verschiebung im lokalen Koordinatensystem.

Bedingung ist noch, daß die vorgeschriebenen Elemente des Vektors der inneren Kräfte nicht positiv sein können.

Es sei der Vektor der Kräftekomponenten mit beschränkten Vorzeichen durch

$$\mathbf{s}_o^* = [s_{o,1}\ s_{o,2}\ \ldots\ s_{o,N_o}]$$

bezeichnet, wo N_o die Anzahl aller Kräftekomponenten mit beschränktem Vorzeichen ist.

Laut Bedingung ist:

$$\mathbf{s}_o \leqq \mathbf{0}\,. \tag{2}$$

Zugleich kann beim Bersten kein Riß mit negativer Breite auftreten. Gruppiert man die Rißbreiten den inneren Kräften ähnlich, so ist der Vektor der Breiten jener Risse, die tatsächlich auftreten können

$$\mathbf{v}_o^* = [v_{o,1}\ \ldots\ v_{o,N_o}]\,.$$

Das heißt:

$$\mathbf{v}_o^* \geqq \mathbf{0}\,. \tag{3}$$

Ist eine innere Kräftekomponente nicht positiv, so tritt entsprechend der Kräftekomponenten kein Bersten auf; ist jedoch ein Entfernungselement des Berstens nicht negativ, so tritt entsprechend der Rißbreite keine innere Kraft auf. Kurz: an derselben Stelle können innere Kraft und Riß gleichzeitig nicht vorhanden sein.

Da die Vorzeichen beider Vektoren gleichsam beschränkt sind, kann diese Alternative in folgender Form aufgeschrieben werden:

$$\mathbf{v}_o^*\,\mathbf{s}_o = 0\,. \tag{4}$$

2.3 *Die lineare komplementäre Aufgabe der Zustandsuntersuchung*

Infolge der bei der Aufstellung des Modells vorgeschriebenen Bedingungen stehen die den Spannungszustand des Komplexes beschreibenden inneren Verbindungskräfte in linearer Abhängigkeit von den Kräftebelastungen, von den vorgeschriebenen Verschiebungen belastenden Charakters, und von den eventuellen Rißausmaßen des Systems

$$\mathbf{s} = \mathbf{S}_A\,\mathbf{q}_A + \mathbf{S}_B\,\mathbf{q}_B + (\mathbf{S}_T - \mathbf{M}_T)(\mathbf{t} + \mathbf{v})\,,$$

wo $\mathbf{q}_A$ die auf die Elemente einwirkenden Belastungen, $\mathbf{q}_B$ die auf die Knotenpunkte einwirkenden Belastungen, $\mathbf{t}$ die kinematischen Belastungen, und $\mathbf{v}$ die eventuellen Riß-Intervalle aufzählt, $\mathbf{S}_A$ die Einflußmatrix des Elementen-

schwerpunktes der inneren Kräfte, S_B die allgemeine auf die Knotenpunktbelastungen bezogene Einflußmatrix darstellt. $\mathbf{M}_T$ ist die die Steifigkeit der Verbindungen zusammenfassende Hypermatrix, S_T drückt das Zusammenwirken der durch das System hervorgerufenen Verbindungen aus.

Eine kürzere Form des obigen Zusammenhanges lautet:

$$\mathbf{s} = \mathbf{s}_q + \mathbf{s}_t + \mathbf{S}_v \mathbf{v} . \tag{5}$$

Hier bedeuten $\mathbf{s}_q$ bzw. $\mathbf{s}_t$ die von den Kräftebelastungen, bzw. belastenden Bewegungen entstehenden elastischen inneren Kräfte. Die einzelnen Spalten der Matrix $\mathbf{s}_v$ werden durch die inneren Kräfte, entstehend von der kinematischen Einheitsbelastung, geliefert.

Den Vektor der Kräftekomponenten mit tatsächlich beschränkten Vorzeichen wählen wir vom Vektor sämtlicher inneren Kräftekomponenten formell so, daß eine Matrix $\mathbf{I}_o$ als Koeffizient angewendet wird. Diese Matrix hat die Ordnung $N_o \times 6N_i$, und nur jene ihrer Elemente weichen von 0 ab, die sich an den Kreuzungen derselben Kräftekomponenten in Zeilen als beschränkte Komponenten und in Spalten als allgemeine Kräftekomponenten zu finden sind. Der Wert solcher Elemente beträgt 1.

Verwendet man die Bezeichnung der Auswahl auch bei den Rißbreiten, so wird

$$\mathbf{s}_o = \mathbf{I}_o \mathbf{s} \qquad \mathbf{v}_o = \mathbf{I}_o \mathbf{v} .$$

Die Nichtpositivitätsforderung (2) wird mit Hilfe einer Schlupfvariable in eine Gleichung umgestaltet:

$$\mathbf{s}_o - \Phi = \mathbf{0} \qquad \Phi \leqq \mathbf{0} .$$

Der unbekannte nicht positive Vektor Φ hat die Ordnungszahl N_o. Bei Anwendung von (5) wird

$$\mathbf{I}_o(\mathbf{s}_q + \mathbf{s}_t) + \mathbf{I}_o \mathbf{S}_v \mathbf{v} - \Phi = \mathbf{0} .$$

Nun sei das zweite Glied in zwei Blöcke zerspaltet und die Spalten so umgeordnet, daß die tatsächlich möglichen Rißbreitenausmaße in den ersten fallen und die sicher rißfreien Stellen, bzw. die nullwertigen Risse der Richtungen in den zweiten:

$$\mathbf{I}_o \mathbf{S}_v \mathbf{v} = \mathbf{I}_o \mathbf{S}_v|_1 \mathbf{v}_o + \mathbf{I}_o \mathbf{S}_v|_2 \mathbf{0} .$$

Dieses Umordnen kann formell durch die Anwendung der Matrix $\mathbf{I}_o^*$ als Multiplikator durchgeführt werden:

$$\mathbf{I}_o \mathbf{S}_v|_1 = \mathbf{I}_o \mathbf{S}_v \mathbf{I}_o^* .$$

Nach Einführung der Bezeichnungen:

$$\mathbf{A} = \mathbf{I}_o \mathbf{S}_v \mathbf{I}_o^* \qquad \mathbf{b} = -\mathbf{I}_o(\mathbf{s}_q + \mathbf{s}_t) \tag{6}$$

liefert die vorige Gleichung mit den Alternativen der Druckkräfte und der Risse gemeinsam folgende Aufgabe:

$$\mathbf{A}\,\mathbf{v}_o - \Phi = \mathbf{b}$$
$$\mathbf{v}_o \geqq \mathbf{0} \quad \Phi \leqq \mathbf{0} \quad \mathbf{v}_o^* \Phi = 0\,. \tag{7}$$

Die Aufgabe wird derart belöst, daß ein Vektor **w** als neue Unbekannte eingeführt wird; diese ist nicht negativ und ihre Elemente bedeuten fiktive Beanspruchungen. Außerdem wird zum Problem eine Zielfunktion angeordnet. Die Zielsetzung wird nur dann erfüllt, wenn die fiktiven Kräfte eliminiert sind.

$$\mathbf{A}\mathbf{v}_o - \Phi + \mathbf{D}\,\mathbf{w} = \mathbf{b} \quad \sum_{i=1}^{N_o} w_i = \min!$$
$$\mathbf{v}_o,\ \mathbf{w} \geqq \mathbf{0} \quad \Phi \leqq \mathbf{0} \quad \mathbf{v}_o^* \Phi = 0\,. \tag{8}$$

Hier stammt **D** aus der Einheitsmatrix durch den Wechsel der Vorzeichen so, daß man nach der Wahl $\mathbf{v}_o = \mathbf{0}$, $\Phi = \mathbf{0}$ zum Simplexverfahren eine triviale Basis erhalte.

Die Berechnung kann mit dem Simplexverfahren durchgeführt werden. Die Bedingung $\mathbf{v}_o^* \Phi = 0$ muß im Falle von Basiswechseln in Betracht gezogen werden.

Der technische Inhalt der Methode besteht darin, daß die statische Grenzbedingung als Funktion der Rißausmaße und der inneren Kräftevorräte aufgeschrieben wird. Danach werden zuerst mit Hilfe der entsprechenden fiktiven Kräfte sämtliche Lücken geschlossen und auch die Kräftevorräte aber noch nicht ausgeschöpft, danach die fiktiven Kräfte in geeigneter Reihe aus dem System eliminiert und die Kräftevorräte nach Bedarf erschöpft.

Der Kräftevorrat ist deshalb negativ, weil die innere Kraft keine Ziehung sein kann.

Für das Verfahren ist im Computer CDC 3300 des Forschungsinstitutes für Angewandte Rechentechnik der UAdW ein Programm in FORTRAN vorhanden.

2.4 *Die Ausdehnung der Rechenmethode auf den Fall der Mohrschen Anstrengungshypothese*

Stoffbeziehungen

Die Mohrsche Bruchbedingung kann durch ein lineares Ungleichheitssystem angenähert werden, das durch die Ersetzung der Hüllkurve der Spannungskreise als Limitationsbedingung geliefert wird. Die Ungleichheiten kön-

nen zweckmäßig im Koordinatensystem der Hauptspannungen charakterisiert werden [7].

Am Beispiel dieser Ungleichheitssysteme können für die Elemente der Kraftvektoren der Verbindung von Punkt zu Punkt auch solche Begrenzungen vorgeschrieben werden, die nicht nur ausdrücken, daß das Material allein druckwiderständig ist, sondern auch den Zustand des kombinierten Druckes umreißen. Eine derartige Begrenzung zeigt Abb. 4a. Demnach geht das Material bei jeder Verbindung i, l dann zugrunde, wenn der durch die Elemente des Vektors $\mathbf{s}_{i,l}$ gekennzeichnete Zustandspunkt P vom Innenbereich des durch die – ein konvexes Vieleck umgrenzenden – Geraden der Anzahl N_z umgrenztes Gebiet, an dessen Rand gerät.

An Abb. 4b sieht man die z-este Abgrenzungsgerade. Der Punkt P liegt dann auf der Gerade, wenn seine Koordinaten, d.h. die Elemente $s_{\xi;\,i,l}$ und $s_{\eta;\,i,l}$ des inneren Kräftevektors die folgende Gleichung der Gerade befriedigen:

$$\frac{s_{,\xi;\,i,l}}{s_{z,\xi;\,i,l}} + \frac{s_{\eta;\,i,l}}{s_{z,\eta;\,i,l}} = 1\,. \tag{9}$$

Hier sind $s_{z,\xi;\,i,l}$ und $s_{z,\eta;\,i,l}$ vorgeschriebene Druckkräfte. Auf Grund dessen, sowie in Betracht genommen, daß der Bereich durch zwei Koordinatenachsen begrenzt ist, kann man mit i, l bezeichneten Verbindungspunkt die Limitationsbedingung der inneren Kräfte durch folgende Matrizenungleichheit ausgedrückt werden:

$$\begin{bmatrix} 1 & 0 \\ \dfrac{1}{s_{2,\xi;\,i,l}} & \dfrac{1}{s_{2,\eta;\,i,l}} \\ \vdots & \vdots \\ \dfrac{1}{s_{z,\xi;\,i,l}} & \dfrac{1}{s_{z,\eta;\,i,l}} \\ \vdots & \vdots \\ \dfrac{1}{s_{N_z-1,\xi;\,i,l}} & \dfrac{1}{s_{N_z-1,\eta;\,i,l}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s_{\xi;\,i,l} \\ s_{\eta;\,i,l} \end{bmatrix} \leqq \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \\ 0 \end{bmatrix}. \tag{10}$$

Kurz:

$$\mathbf{B}_{i,l}\,\mathbf{s}_{i,l} \leqq \mathbf{e}_{i,l}\,.$$

Es sei angenommen, daß an allen Verbindungen eine derartige Bedingung gültig ist; es ist innerhalb deren möglich, daß die Anzahl der in den einzelnen Bedingungen vorkommenden Relationen, bzw. die in den Relationen

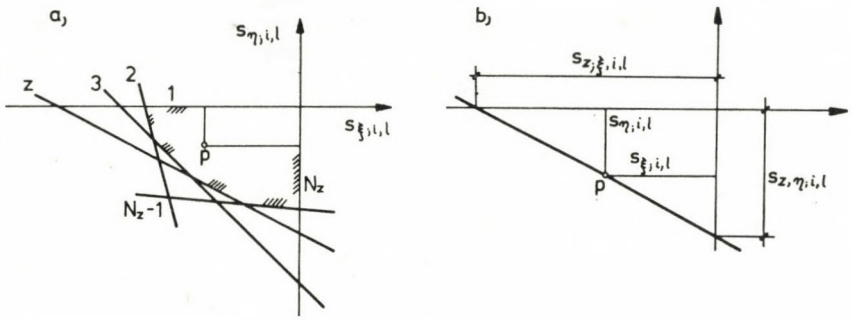

Abb. 4

auffindbaren Konstanten sich verändern. In diesem Fall können die Bedingungen in einer Hypermatrix-Ungleichheit zusammengefaßt werden:

$$\mathbf{Bs} \leqq \mathbf{e} .$$

Hier ist

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{1,l(1,1)} & & & & \\ & \mathbf{B}_{1,l(2,1)} & & & \\ & & \mathbf{B}_{1,l(3,1)} & & \\ & & & \ddots & \\ & & & & \mathbf{B}_{N_i,l(3,N_i)} \end{bmatrix} \quad \mathbf{e} = \begin{bmatrix} \mathbf{e}_{1,l(1,1)} \\ \mathbf{e}_{1,l(2,1)} \\ \mathbf{e}_{1,l(3,1)} \\ \vdots \\ \mathbf{e}_{N_i,l(3,N_i)} \end{bmatrix} . \tag{11}$$

Im Falle identischer Bruchbedingungen ist **B** eine Matrix $3N_iN_z \times 6N_i$-er Ordnung, **s** ein Vektor $6N_i$-er Ordnung und **e** aber ein Vektor $3N_iN_z$-er Ordnung.

Kinematische Bedingungen

Befindet sich der Zustandspunkt innerhalb des Bereiches, so gibt es weder einen Riß, noch einen Bruch. Falls die mit i, l bezeichnete Dyname der Verbindung einen der Begrenzungsbedingungen befriedigt, entsteht an der Verbindung die relative Bruchverschiebung $\mathbf{v}_{i,l}$. Die Stellung des Verschiebungsvektors ist vertikal zur Geraden der Limitationsbedingung, seine Richtung zeigt vom Bereich hinaus.

Dementsprechend sind die zu den Geraden der Anzahl 1 und N_z vertikal stehenden möglichen Vektoren **v** positiv, alle übrigen aber negativ.

Falls die Verbindungsdyname gleichzeitig zwei Limitationskonditionen ausgeschöpft hat, d.h. der Zustandspunkt auf einen der Spitzen des Bereiches geraten ist, so wird der relative Verschiebungsvektor durch die lineare Kombination jener **v**-Vektoren gekennzeichnet, die zu den sich im entsprechenden Spitzpunkt schneidenden Geranden gehören würden.

Fällt nun der Zustandspunkt genau auf die z-ste Bedingungsgerade, dann besteht infolge der Orthogonalitätsforderung zwischen den beiden Komponenten des Vektors $\mathbf{v}_{i,l}$ der Zusammenhang:

$$\frac{v_{\xi;i,l}}{v_{\eta;i,l}} = \frac{s_{z,\eta;i,l}}{s_{z,\xi;i,l}} .$$

Diese können auch als zwei gesonderte Gleichungen behandelt werden:

$$v_{\xi;i,l} = \frac{1}{s_{z,\xi;i,l}} \lambda_{z;i,l} \qquad v_{\eta;i,l} = \frac{1}{s_{z,\eta;i,l}} \lambda_{z;i,l} \tag{12}$$

wo $\lambda_{z;i,l}$ ein unbekannter Parameter ist.

Es besteht in Verbindung mit der Geraden Nr. 1 speziell:

$$v_{\xi;i,l} = \lambda_{1,i,l} \qquad v_{\eta;i,l} = 0 \tag{13}$$

und mit der N_z-sten

$$v_{\xi;i,l} = 0 \qquad v_{\eta;i,l}\, \lambda_{N_z,i,l} .$$

Von den Zusammenhängen (12, 13) ist entweder auf einmal nur einer gültig, oder, falls der Zustandspunkt auf einer der Spitzen des Bereiches liegt, gleichzeitig beide. All diese Bedingungen können zur gleichen Zeit in den Verbindungsstellen folgenderweise befriedigt werden:
Für die Gleichung

$$\begin{bmatrix} 1 & \frac{1}{s_{2,\xi;i,l}} & \cdots & \frac{1}{s_{z,\xi;i,l}} & \cdots & \frac{1}{s_{N_z-1,\xi;i,l}} & 0 \\ 0 & \frac{1}{s_{2,\eta;i,l}} & \cdots & \frac{1}{s_{z,\eta;i,l}} & \cdots & \frac{1}{s_{N_z-1,\eta;i,l}} & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_z \\ \vdots \\ \lambda_{N_z-1} \\ \lambda_{N_z} \end{bmatrix}_{i,l} = \begin{bmatrix} v_\xi \\ v_\eta \end{bmatrix}_{i,l} \tag{14}$$

wird eine Lösung gesucht, in der entweder nur einer der λ-Werte von 0 abweicht, oder zwei, wobei aber in den Indizen dieser beiden Werte die Differenz nur 1 sein kann (diese zusätzliche Bedingung soll als λ-Beschränkung genannt werden).

Infolge der über die Vorzeichnen der Elemente des Vektors $\mathbf{v}$ behaupteten, ist $\lambda \geqq \mathbf{0}$, da die in der Koeffizientenmatrix vorkommenden Beschrän-

kungskräfte Druck bedeuten. Gleichung (14) kann in Kurzform folgend geschrieben werden:

$$\mathbf{B}^*_{i,l}\,\lambda_{i,l} = \mathbf{v}_{i,l} \qquad \lambda_{i,l} \geqq \mathbf{0}$$

(die λ-Beschränkung mit inbegriffen).

Die Zusammenhänge obigen Typs in eine Hypermatrix-Gleichung zusammengefaßt, erhält man:

$$\mathbf{B}^*\lambda = \mathbf{v} \qquad \lambda \geqq \mathbf{0} \tag{15}$$

wobei

$$\lambda^* = [\lambda_{1,l(1,1)}\ \lambda_{1,l(2,1)}\ \lambda_{1,l(3,1)}\ \ldots\ \lambda_{N_i,l(3,N_i)}]$$

ein Vektor $3N_iN_z$-ster Ordnung (vorausgesetzt, daß die Bruchbedingung an jeder Stelle durch die gleiche Anzahl von Geraden charakterisiert ist), und $\mathbf{v}$ hat die Ordnung $6N_i$.

Die Aufstellung der algebraischen Aufgabe

Die lineare komplementäre Aufgabe des modifizierten Problems wird so zusammengestellt, daß allererst der Schlupfvariabel-Vektor

$$\Phi \leqq \mathbf{0}$$

der Dimension $3N_iN_z$ eingeführt wird, mit dessen Hilfe (11) sich zu einer Gleichung

$$\mathbf{Bs} - \Phi = \mathbf{e} \tag{16}$$

formt. Durch die Anwendung von (5) und (15) erhält man:

$$\mathbf{B}(\mathbf{s}_q + \mathbf{s}_t) + \mathbf{B}\,\mathbf{S}_v\,\mathbf{B}^*\,\lambda - \Phi = \mathbf{e}$$

Die entsprechenden Elemente von λ und Φ können auch jetzt nur abwechselnd von 0 abweichen, weil nur dann eine relative Bruchverschiebung auftritt, wenn die entsprechende Bedingung lückenlos erfüllt ist. Daher

$$\lambda^*\Phi = 0\,.$$

Werden nun die Bezeichnungen:

$$\begin{aligned} \mathbf{A} &= \mathbf{BS}_v\,\mathbf{B}^* \\ \mathbf{b} &= \mathbf{e} - \mathbf{B}(\mathbf{s}_q + \mathbf{s}_t) \end{aligned} \tag{17}$$

eingeführt, kann die Aufgabe folgenderweise aufgestellt werden:

Man sucht bei Einhaltung der λ-Beschränkung die Vektoren λ und Φ, die das System

$$\mathbf{A}\lambda - \Phi = \mathbf{b}$$

$$\lambda \geqq \mathbf{0} \quad \Phi \leqq \mathbf{0} \quad \lambda^* \Phi = \mathbf{0} \tag{18}$$

befriedigen.

Die Aufgabe kann wiederum mit dem Simplexverfahren behandelt werden, für die Einhaltung der λ-Beschränkung muß in der Serie der Basiswechsel gesorgt werden, was aber keine besondere Schwierigkeiten bedeutet.

2.5 *Komplexe Bewegungsbeschränkung*

Die lineare komplementäre Aufgabe der Zustandsuntersuchung kann auch für Fälle solcher Materialien angewendet werden, wo zwar die inneren Kräfte nicht beschränkt sind, anstatt dessen aber im Material Verbindungsdislokationen komplexerweise limitierten Ausmaßes möglich sind [6, 10]. Das bedeutet, daß an den Verbindungen schließbare Lücken vorhanden sind, die Aufbaustruktur infolge der Belastung sich geringfügig umordenen kann, das Komplex kompakter wird. Die Limitationsgerade der Dislokationen lautet:

$$\frac{v_{\xi;i,l}}{v_{z,\xi;i,l}} + \frac{v_{\eta;i,l}}{v_{z,\eta;i,l}} = 1 \,. \tag{19}$$

Werden auf Grund dessen die Matrizen

$$\mathbf{C}_{i,l} = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{v_{2,\xi;i,l}} & \dfrac{1}{v_{2,\eta;i,l}} \\ \vdots & \vdots \\ \dfrac{1}{v_{z,\xi;i,l}} & \dfrac{1}{v_{z,\eta;i,l}} \\ \vdots & \vdots \\ \dfrac{1}{v_{N_z-1,\xi,i,l}} & \dfrac{1}{v_{N_z-1,\eta;i,l}} \\ 0 & 1 \end{bmatrix} \tag{20}$$

bzw. die aus ihnen zusammengestellte Hyper-Diagonalmatrix

$$\mathbf{C} = \langle \mathbf{C}_{i,l} \rangle$$

eingeführt, so kann das Beschränkungsgesetz der relativen Verschiebung wie folgt vorgeschrieben werden:

$$\mathbf{C}\,\mathbf{v} \leqq \mathbf{e} \,. \tag{21}$$

Für die inneren Kräfte der sich schließenden Verbindungen wird die Annahme gestellt, daß im Falle des Zustandekommens des Kontaktes die Kontaktkräfte zu den durch die vorgeschriebenen Verschiebungen v_z charakterisierten Geraden, die die Lücken umgrenzen, senkrecht stehen. Die Reibung wird also vornachlässigt.

Daher können nach dem Beispiel des im vorangehenden Punkt erörterten, erneut Orthogonalitätsgleichungen aufgeschrieben werden, die sich jetzt aber auf die inneren Kräfte beziehen:

$$\mathbf{C}^*\lambda = \mathbf{s} \qquad \lambda \leqq \mathbf{0} \tag{22}$$

Bei Anwendung von (5), (21) und (22) gelangt man wieder zu einem Problem, ähnlich wie bei (18):

$$\mathbf{A}\lambda - \Phi = \mathbf{b}$$
$$\lambda \leqq \mathbf{0} \qquad \Phi \geqq \mathbf{0} \qquad \lambda^*\Phi = \mathbf{0} \tag{23}$$

Nun ist aber

$$\mathbf{A} = \mathbf{C}^* \mathbf{S}_v^{-1} \mathbf{C}^* \quad \mathbf{b} = \mathbf{e} - \mathbf{C}\mathbf{S}_v^{-1}(\mathbf{s}_q + \mathbf{s}_l) .$$

Verwendet man anstatt der Matrizen **C** die im Punkt 2.3 erscheinenden Auswahlmatrizen $\mathbf{I}_o$, bedeutet weiters **e** beliebig vorgeschriebene, kleine positive Rißdimensionen, dann gelangt man, als zu einem dualen Begriff des Komplexes aus *nur druckfestem Material*, zu einem Komplex, der *nur in den gegebenen Richtungen kompakter* werden kann, bzw. zu den diesen Zustand charakterisierenden Zusammenhängen.

2.6 *Die Zustandsgleichungen des Systems, die Bestimmung der Verschiebungen*

Befaßt man sich mit einem Komplex, der nur dem Druck wiedersteht, der vom Mohrschem Typ ist oder der verdichtbar ist, so erhält man aus der Lösung der komplementären Aufgabe die Rißverschiebungen, bzw. Dislokationen, sowie die inneren Kräfte. Bei der Bestimmung des Verschiebungsfeldes des Komplexes, genauer gesagt der diskreten Verschiebungswerte, die diese charakterisieren, müssen die vervollkommneten Zustandsgleichungen des Systems zusammengestellt werden. Die besagen, daß die Belastungen und die inneren Kräfte sowohl an den dreieckförmigen Elementen, als auch in den Knotenpunkten in Gleichgewicht sind. Bei deren Aufstellung müssen die sich auf die kleinen Verschiebungen der Elemente in den Gleichungen als starren Körper bezogenen geometrischen Gleichungen, die Zusammenhänge zwischen den Verschiebungen der Knotenpunkte und der der Elementenspitzen, sowie das Stoffgesetz gleichsam in Betracht gezogen werden. Hinweisend darauf, daß einzelne Zustandskennwerte in dem mit dem Ganzen

des Komplexes verbundenen Koordinatensystem x, y, andere in den zu den einzelnen Verbindungen zugeordneten Koordinatensystemen $\xi_{i,l}$; $\eta_{i,l}$ zu behandeln zweckmäßig ist, weiters noch die Reduktionsmatrizen

$$\mathbf{G}_{i,l} = \begin{bmatrix} 1 & -(y_l - y_i) \\ & 1 \quad (x_l - x_i) \end{bmatrix}$$

einführend, erhält man die Zustandsgleichungen in der Form:

$$\begin{aligned} &\mathbf{K}_{E;i|E;i}\,\mathbf{u}_{E;i} + \sum_{l}{}' \mathbf{K}_{E;i|C;l}\,\mathbf{u}_{C;l} = \mathbf{q}_{E;i} + \mathbf{q}_{t,E;i} + \mathbf{q}_{v,E;i} \\ &\sum_{m}{}' \mathbf{K}_{C,j|E;m}\,m\,\mathbf{u}_{E;m} + \mathbf{K}_{C;j|C;j}\,\mathbf{u}_{C;j} = \mathbf{q}_{C;j} + \mathbf{q}_{t,C;j} + \mathbf{q}_{v,E;j}. \end{aligned} \tag{24}$$

Die erste Blockmatrixgleichung ist im Falle von $i = 1,2, \ldots, N_i$, die zweite aber im Falle von $j = 1,2, \ldots, N_j$ gültig.

Die Summierung nach l erstreckt sich bei jedem der Elemente auf alle anschließenden Knotenpunkte, die nach m bei jedem der Knotenpunkte auf alle anschließenden Elemente.

$$\begin{aligned} \mathbf{K}_{E;i|E;i} &= -\sum_{l}{}' \mathbf{G}^*_{i,l}\,\mathbf{T}^*_{i,l}\,\mathbf{M}_{i,l}\,\mathbf{T}_{i,l}\,\mathbf{G}_{i,l} \\ \mathbf{K}_{E;i|C;l} &= \mathbf{G}^*_{i;l}\,\mathbf{T}^*_{i;l}\,\mathbf{M}_{i;l}\,\mathbf{T}_{i;l} \\ \mathbf{K}_{C;j|C;j} &= -\sum_{m}{}' \mathbf{T}^*_{m;j}\,\mathbf{M}_{m;j}\,\mathbf{T}_{m;j}. \end{aligned} \tag{25}$$

Die Koeffizientenhypermatrix ist symmetrisch. Die Gleichungen konventieren die spannungserregenden Effekte der Belastungsglieder und Dislokationen in kinematischen Belastungen. Es können vier solche unterschieden werden. An der rechten Seite der auf die Elemente bezogenen Gleichgewichtsgleichungen:

$$\begin{aligned} &-\sum_{l}{}' \mathbf{G}^*_{i;l}\,\mathbf{T}^*_{i;l}\,\mathbf{M}_{i;l}\,\mathbf{t}_{i;l} = \mathbf{q}_{t;E;i} \qquad -\sum_{l}{}' \mathbf{G}^*_{i;l}\,\mathbf{T}^*_{i;l}\,\mathbf{M}_{i;l}\,\mathbf{v}_{i;l} = \mathbf{q}_{v;E;i} \\ &\sum_{m}{}' \mathbf{T}_{m;j}\,\mathbf{M}_{m;j}\,\mathbf{t}_{m;j} = \mathbf{q}_{t,C;j} \qquad \sum_{m}{}' \mathbf{T}_{m;j}\,\mathbf{M}_{m;j}\,\mathbf{v}_{m;j} = \mathbf{q}_{v,C;j}. \end{aligned} \tag{26}$$

Da im Laufe der Untersuchung die auf die Elementengewichtspunkte, bzw. auf die Knotenpunkte reduzierten Belastungen des Komplexes, sowie auch alle Belastungsglieder angegeben sind, und die Dislokationen aber festgestellt wurden, sind laut den voran gesagten *alle* diese Vektoren bekannt.

Die Zustandsgleichungen werden durch die Einsetzung der Verschiebungs- bzw. Belastungs-Randbedingungen bestimmt. Da die bewegungslosen Randpunkte des Bereiches, sowie die Belastungen der Oberfläche und die eventuellen Stützkräfte des Abbaus ebenfalls als bekannt betrachtet werden, steht einer endgültigen Lösung der Zustandsgleichung nichts im Wege.

2.7 *Sickerungsuntersuchungen*

Die lineare komplementäre Aufgabe kann auch zur Untersuchung der im Gesteinskomplex auftretenden Sickerung von Formationswasser benutzt werden, vorausgesetzt, daß die Sickerung Darcys Gesetz folgt, die Strömung jedoch nur bei der Präsenz einer bestimmten und fürs Material charakteristischen Schwellengradienten beginnt.

Die bekannten Grundgleichungen der permanenten Sickerung legen fest, daß die Komponenten des hydraulischen Falles die Elemente des Druckpotenzialgradienten sind, und weiters, daß wegen der Kontinuität der Strömung die Divergenz des Geschwindigkeitsvektors sich mit den verschiedenen spezifischen Wassererträgen ausgleicht. Diese Zusammenhänge werden im Falle eines finit Modells durch die Gleichungen:

$$\mathbf{i} = \mathbf{D}\Phi \quad \mathbf{D}^*\mathbf{v} = \mathbf{q} \tag{27}$$

ausgedrückt, in denen $\mathbf{i}$ der Spaltenvektor der Elemente der hydraulischen Fallkomponente ist, Φ punktweise die Werte des Potentials enthält, $\mathbf{v}$ die diskreten Werte der Geschwindigkeitskomponenten, $\mathbf{q}$ die erwähnten Erträge bezeichnen. $\mathbf{D}$ ist eine die Rolle der Bildung von Differenzen-Quotienten erster Ordnung erfüllende Matrix.

Es ist zweckmäßig Darcys Gesetz im Falle eines Schwellengradienten in folgender Form zu schreiben:

$$\begin{gathered} \mathbf{v} = -\mathbf{K}(\mathbf{i} - \mathbf{i}_0 + \mathbf{h}) \\ \mathbf{h} \geqq \mathbf{0} \qquad \mathbf{v} \geqq \mathbf{0} \qquad \mathbf{h}^*\mathbf{v} = 0\,. \end{gathered} \tag{28}$$

Hier ist $\mathbf{K}$ eine aus den Matrizen der Koeffiziententensoren der Sickerung zusammengestellte Hypermatrix, $\mathbf{i}_0$ der Spaltenvektor der Elemente des Schwellengradientes, $\mathbf{h}$ ein Vektor, der das eventuelle Fehlen der hydraulischen Fälle charakterisiert.

Als Erklärung wird erwähnt, daß im Falle von $\mathbf{h} = \mathbf{0}$

$$\mathbf{v} = -\mathbf{K}(\mathbf{i} - \mathbf{i}_0)$$

ist, was Darcys Gesetz entspricht, aber nur den *effektiven* hydraulischen Fall in Betracht zieht. Ist jedoch $\mathbf{v} = \mathbf{0}$, dann ist

$$\mathbf{i}_0 \geqq \mathbf{i} \qquad \mathbf{h} = \mathbf{i}_0 - \mathbf{i}\,,$$

da $\mathbf{K}$ nicht singulär ist, und die Strömung, da es keinen Fall $\mathbf{h}$ gibt, nicht zustande kommen kann. Die Formel $\mathbf{h}^*\mathbf{v} = 0$ bedeutet wieder eine Alternative:

wo die Strömung mit endlicher Geschwindigkeit einsetzt, dort gibt es keinen Fall, wo aber der Fall nicht vorhanden ist, dort setzt die Strömung nicht ein.

Man erhält aus den Gleichungen (27) und (28) nach dem Beispiel der bei den Aufgaben der Gesteinsstatik erläuterten Berechnungen eine lineare komplementäre Aufgabe der Form:

$$\mathbf{Ah} - \mathbf{v} = \mathbf{b}$$

$$\mathbf{h} \geqq \mathbf{0} \qquad \mathbf{v} \geqq \mathbf{0} \qquad \mathbf{hv} = 0\,. \tag{29}$$

Der schematische Algorithmus der Berechnung von $\mathbf{A}$ und $\mathbf{b}$ lautet:

$$\begin{aligned} \mathbf{D} &= [\mathbf{D}_b \;\; \mathbf{D}_p] \\ \mathbf{K}_{gl} &= \mathbf{D}_b^* \, \mathbf{K} \, \mathbf{D}_b \\ \mathbf{A} &= \mathbf{K}\mathbf{D}_b \, \mathbf{K}_{gl}^{-1} \, \mathbf{D}_b^* \, \mathbf{K} - \mathbf{K} \\ \mathbf{q}_{b,\,\mathrm{red}} &= -\,\mathbf{q}_b + \mathbf{D}_b^* \, \mathbf{K}(\mathbf{i}_0 - \mathbf{D}_p \, \Phi_p) \\ \mathbf{b} &= \mathbf{K}(\mathbf{D}_b \, \mathbf{K}_{gl}^{-1} \, \mathbf{q}_{b,\,\mathrm{red}} + \mathbf{D}_p \, \Phi_p - \mathbf{i}_0)\,. \end{aligned} \tag{30}$$

Hier bedeuten $\mathbf{D}_b$ und $\mathbf{D}_p$ die durch die *inneren* Netzknotenpunkte unbekannten Zustandes und *äußeren* Netzknotenpunkte bekannten Zustandes generierten Blöcke, $\mathbf{K}_{gl}$ kann als globale Durchlaßmatrix bezeichnet werden. Φ_p ist der Vektor der an den Rändern vorgeschriebenen Druckpotential-Werte, $\mathbf{y}_b$ aber der im Inneren des Netzes angegebenen spezifischen Erträge. Die einfache Erklärung der Aufschreibung der Formel (29) leutet:

Es sei einstweilen angenommen, daß $\mathbf{h}$ und $\mathbf{i}_0$ je einen wirklichen aber bekannten hydraulischen Gradienten charakterisieren, die von den Druckpotentialen unabhängig sind. In einem solchen Fall würde die Geschwindigkeit aus vier Komponenten resultieren:

a) aus den an den inneren Knotenpunkten des Netzes auftretenden und den Potentialen Φ_b entsprechenden Geschwindigkeiten $\mathbf{v}_{\Phi_b}$,

b) aus den den Potentialen Φ_p der Randpunkte entsprechenden Geschwindigkeiten $\mathbf{v}_{\Phi_p}$,

c) aus den Folgen der $\mathbf{h}$-Gradienten, die mit $\mathbf{h}$ in linearem Zusammenhang stehen:

$$\mathbf{v}_h = \mathbf{Ah} \tag{31}$$

d) aus den mit $\mathbf{v}_{i_0}$ bezeichneten Folgen der Gradienten $\mathbf{i}_0$. (Welche der Möglichkeiten realisiert wird, entscheidet die Lösung der Aufgabe.)

Infolge dieser vier Teileffekte ist:

$$\mathbf{v} = \mathbf{v}_{\Phi b} + \mathbf{v}_{\Phi_p} + \mathbf{Ah} + \mathbf{v}_{i_0} \tag{32}$$

bzw.

$$\mathbf{Ah} - \mathbf{v} = \mathbf{b}$$

hier ist

$$\mathbf{b} = -(\mathbf{v}_{\Phi_b} + \mathbf{v}_{\Phi_p} + \mathbf{v}_{i_0}) .$$

Die Gleichung (29) läßt auch durchleuchten, daß das lineare komplementäre Problem auch für die komplexe Untersuchung vom Zustand zweiphasiger Komplexe, d. h. ihrer Verschiebungen, Spannungen und Sickerungs-Druckverhältnissen geeignet sein wird. Diese Frage wird hier nicht weiter analysiert, da wir uns damit in einer anderen Beziehung befassen werden.

3. Zustandsuntersuchung elastoviskoser Komplexe

3.1. *Die Gleichungen der Zustandsuntersuchung*

a) Annahmen; Ableitungen der Gleichungen

Es ist allgemein zweckmäßig bei der Zustandsuntersuchung elastoviskoser Komplexe das Poynting-Thomsonsche Materialmodell [2] als Grundlage zu nehmen. In solchen Fällen werden die dreieckförmigen Elementen des endlichen Mosaiksystems nicht als starre Blöcke, sondern als Kontinua standarden elastoviskosen Materials betrachtet. Das ist also ein festes Modell, besteht aber aus nicht starren Elementen und Knotenpunkten unendlich kleiner Ausdehnung, es enthält keine inneren Verbindungen.

Die Zustandsuntersuchung des Komplexes kann mit der Bewegungsuntersuchung des darin sickernden Wassers verbunden werden [1, 4].

Unsere Annahmen sind die folgenden:

a) Das Komplex ist zweiphasig. Ein Teil der darin auftretenden Spannung bildet die tatsächliche, effektive Spannung des Gesteins, der andere Teil stammt aus dem Wasserdruck, dies ist die neutrale Spannung [8].

b) Die Bewegung ist so langsam, daß die Trägheitskräfte vernachlässigt werden können.

c) Das Material fester Phase kann mit den Stoffbeziehungen des standarden elastoviskosen Körpers charakterisiert werden.

d) Der physikalische Zusammenhang der Sickerung ist durch Darcys Gesetz charakterisiert. Das Gestein ist vom Gesichtspunkt der Durchlassung aus heterogen. Der Koeffiziententensor der Sickerung kann an jedem seiner endlichen Elemente anders sein. Zwecks Vereinfachung der Berechnung wird dennoch angenommen, daß das gemeinsame Koordinatensystem x, y an jedem endlichen Element Sickerungshauptrichtungen bedeutet.

e) Die zeitliche Änderung des Lückenvolumens kann vernachlässigt werden.

f) Die Quelle der Sickerung ist in die Knotenpunkte des als endliches Elementensystem aufgefaßten Komplexes konzentriert.

g) Die Theorie erster Ordnung ist gültig.

h) Der Komplex befindet sich in einem ebenen Formänderungszustand.

Zuerst werden die Differentialgleichungen des Gleichgewichtes des elementaren Mediumteils aufgeschrieben, diese beziehen sich jetzt auf die *totalen* Spannungen. Im Cartesianischen Koordinatensystem:

$$\begin{aligned} \frac{\partial g_x}{\partial x} dxdy + \frac{\partial g_{yx}}{\partial y} dydx + q_x dxdy = 0 \\ \frac{\partial g_{xy}}{\partial x} dxdy + \frac{\partial q_y}{\partial y} dydx + q_y dxdy = 0 . \end{aligned} \tag{1}^*$$

Die effektive Spannung greift auf dem festen, die neutrale am flüssigphasigen Volumelement an. Angenommen, daß die differentialen Längen des Elementes aus festem Material die ω-tel Teile der totalen Differentiallängen sind, kann statt (1) folgendes geschrieben werden:

$$\begin{aligned} \frac{\partial \sigma_x}{\partial x} dxdy\omega + \frac{\partial \tau_{xy}}{\partial y} dydx\omega - \frac{\partial p_x}{\partial x} dxdy(1 - \omega) + q_x dxdy = 0 \\ \frac{\partial \tau_{xy}}{\partial x} dxdy\omega + \frac{\partial \sigma_y}{\partial y} dydx\omega - \frac{\partial p_y}{\partial y} dydx(1 - \omega) + q_y dxdy = 0 . \end{aligned}$$

Hier sind σ_x, σ_y, τ_{xy}, τ_{yx} die Komponenten der effektiven Spannung, p_x und p_y die des Druckes.*

Die Komponenten des Sickerungsdruckes werden mit dem hydraulischen Potential ausgedrückt

$$\frac{\partial p_x}{\partial x} = \gamma_{\text{wasser}} \frac{\partial \Phi}{\partial x} \qquad \frac{\partial p_y}{\partial y} = \gamma_{\text{wasser}} \frac{\partial \Phi}{\partial y} . \tag{2}$$

Die Gleichungen des Gleichgewichtes mit Hilfe von (3) transformiert und die Bezeichnungen

$$\gamma_{\text{wasser}} \frac{1 - \omega}{\omega} = b_p \qquad \frac{1}{\omega} = b_q$$

einführend, erhält man:

$$\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} - b_p \frac{\partial \Phi}{\partial x} + b_q q_x = 0 \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} - b_p \frac{\partial \Phi}{\partial y} + b_q q_y = 0 . \end{aligned} \tag{3}$$

* Die Numerierung in diesem Kapitel ist von den vorangehenden unabhängig.

Die geometrischen Gleichungen sind:

$$\varepsilon_x = \frac{\partial u_x}{\partial x} \qquad \varepsilon_y = \frac{\partial u_y}{\partial y} \qquad \gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} . \tag{4}$$

Hier sind ε_x, ε_y, γ_{xy} die Elemente des auf das ganze Gebietselement bezogenen Deformationstensors, u_x und u_y die Verschiebungskomponenten.

Den auf den festen Teil bezogenen Deformationstensor erhält man durch eine Reduktion. Die spezifische Längenänderung dieses Teiles in der Richtung x ist z.B.:

$$\varepsilon_{xs} = \frac{u_x(x + \omega dx) - u_x(x)}{\omega\, dx} = \frac{1}{\omega} \frac{\partial u_x}{\partial x} = b_q\, \varepsilon_x \tag{5}$$

ähnlich

$$\varepsilon_{y,s} = b_q\, \varepsilon_y \qquad \gamma_{x,y,s} = b_q\, \gamma_{xy} .$$

In den Stoffbeziehungen spielt die Zeit eine wichtige Rolle und es muß auch beachtet werden, daß diese Gleichungen die Verbindung zwischen den *effektiven Spannungen* und den *Deformationen der festen Teile* charakterisieren. Nach Einführung des Parameters b_q in die Materialkennwerte können auf der rechten Seite der Gleichungen doch noch die ursprünglichen Deformationen stehen:

$$\begin{aligned}
\sigma_x + \theta \frac{\partial \sigma_x}{\partial t} &= S_{11}^{(r)} \varepsilon_x + S_{12}^{(r)} \varepsilon_y + S_{11}^{(v)} \frac{\partial \varepsilon_x}{\partial t} + S_{12}^{(v)} \frac{\partial \varepsilon_y}{\partial t} \\
\sigma_y + \theta \frac{\partial \sigma_y}{\partial_t} &= S_{12}^{(r)} \varepsilon_x + S_{22}^{(r)} \varepsilon_y + S_{12}^{(v)} \frac{\partial \varepsilon_x}{\partial t} + S_{22}^{(v)} \frac{\partial \varepsilon_y}{\partial t} \\
\tau_{xy} + \theta \frac{\partial \tau_{xy}}{\partial t} &= S_{33}^{(r)} \gamma_{xy} + S_{33}^{(v)} \frac{\partial \gamma_{xy}}{\partial t} .
\end{aligned} \tag{6}$$

Hier bedeuten $S_{ij}^{(r)}$ die mit b_q multiplizierten Werte der Elastizitätskoeffizienten, $S_{ij}^{(v)}$ die der Viskositätskoeffizienten.

Die partiellen Differentialgleichungen der Gleichgewichts-Zustandsänderung des Kontinuums wird auf Grund der Deformationsmethode der Elastizitätslehre, d.h. nach der Methode der *Laméschen Gleichungen* aufgeschrieben. Deshalb müssen im Interesse der Aufstellung der in (6) vorkommenden Glieder auch die Zusammenhänge (3) deriviert werden. Als Ergebnis der Ableitung wird die Gleichgewichtsbedingung:

$$\begin{aligned}
&S_{11}^{(r)} \frac{\partial^2 u_x}{\partial x^2} + S_{33}^{(r)} \frac{\partial^2 u_x}{\partial y^2} + \left(S_{12}^{(r)} + S_{33}^{(r)}\right) \frac{\partial^2 u_y}{\partial x\, \partial y} - b \frac{\partial \Phi}{\partial x} + \\
&+ S_{11}^{(v)} \frac{\partial^3 u_x}{\partial x^2 \partial t} + S_{33}^{(v)} \frac{\partial^3 u_x}{\partial y^2\, \partial t} + \left(S_{12}^{(v)} + S_{33}^{(v)}\right) \frac{\partial^3 u_y}{\partial x\, \partial y\, \partial t} - b\theta \frac{\partial^2 \Phi}{\partial x\, \partial t} + \\
&+ q_x + \theta \frac{\partial q_x}{\partial t} = 0 ,
\end{aligned} \tag{7}$$

sowie eine analoge Gleichung in Richtung y.

Die Bilanzgleichung der Materialströmung der Sickerung lautet:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial Q}{\partial t} = m_2 \frac{\partial \Phi}{\partial t} . \tag{8}$$

Hier bezeichnen die ersten beiden Glieder die Differenz der in das Elementarviereck ein- und ausströmenden Flüssigkeit, die durch die Geschwindigkeitsdivergenz charakterisiert wird. Das zweite Glied ist im allgemeinen der durch die innere (verteilte) Quelle oder Senke verursachte spezifische Ertrag, in diesem Fall die spezifische Bilanzdifferenz. Auf der rechten Seite ist:

$$m_2 = \frac{\gamma_{\text{wasser}}}{1 - \omega} \tag{9}$$

eine vom Porenanteil abhängige Konstante, $\frac{\partial \Phi}{\partial t}$ mißt die Geschwindigkeit der Druckänderung.

Das Glied $\frac{\partial \Phi}{\partial t}$ wird aus der Erwägung ausgehend analysiert, daß nach der Verschiebung des Komplexes das Maß des differentialen Gebietselementes annähernd

$$dA' = (1 + \varepsilon_x + \varepsilon_y)\, dxdy$$

wird.

Dies besteht aus drei Teilen:
aus dem veränderten Bereich des festen Teils

$$dA'_1 = [1 + b_q (\varepsilon_x + \varepsilon_y)]\, dxdy$$

aus dem nicht zusammendrückbaren Flüssigkeitsvolumen

$$dA'_2 = (1 - \omega^2)\, dxdy$$

und schließlich aus der Differenz, die nach der Annahme *f)* letztlich den konzentrierten Quellen und Senken zu verdanken ist:

$$dA'_3 = Q dxdy .$$

Aus der Gleichung

$$dA' = dA'_1 + dA'_2 + dA'_3$$

erhält man

$$Q = (1 - b_q)(\varepsilon_x + \varepsilon_y) - 1 + \omega^2$$

und laut Annahme *e)*, bei Inbetrachtnahme von (4) ist

$$\frac{\partial Q}{\partial t} = (1 - b_q) \frac{\partial}{\partial t} \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) . \tag{10}$$

Laut Annahme *d)* ist

$$v_x = -k_x \frac{\partial \Phi}{\partial x} \qquad v_y = -k_y \frac{\partial \Phi}{\partial y} \,. \tag{11}$$

Die Bezeichnung

$$m_1 = 1 - b_q \tag{12}$$

einführend, erhält man schließlich aus (8), mit Hilfe von (10) und (11)

$$-k_x \frac{\partial^2 \Phi}{\partial x^2} - k_y \frac{\partial^2 \Phi}{\partial y^2} + m_1 \left(\frac{\partial^2 u_x}{\partial x \, \partial t} + \frac{\partial^2 u_y}{\partial y \partial t} \right) - m_2 \frac{\partial \Phi}{\partial t} = 0 \,. \tag{13}$$

Auf die Zusammenhänge (7) und (13) hinweisend wird das partielle Differentialgleichungssystem in der Matrix-Operator Form

$$\mathbf{D}^{(r)} \mathbf{u} + \mathbf{D}^{(v)} \dot{\mathbf{u}} + \mathbf{q} + \theta \dot{\mathbf{q}} = \mathbf{0} \tag{14}$$

zusammengestellt, hier bedeutet $\cdot$ die Ableitung nach der Zeit, weiters:

$$\mathbf{D}^{(r)} = \begin{bmatrix} S_{11}^{(r)} \frac{\partial^2}{\partial x^2} + S_{33}^{(r)} \frac{\partial^2}{\partial y^2} & \left(S_{12}^{(r)} + S_{33}^{(r)}\right) \frac{\partial^2}{\partial x \partial y} & -b \frac{\partial}{\partial x} \\ \left(S_{12}^{(r)} + S_{33}^{(r)}\right) \frac{\partial^2}{\partial y \partial x} & S_{33}^{(r)} \frac{\partial^2}{\partial x^2} + S_{22}^{(r)} \frac{\partial^2}{\partial y^2} & -b \frac{\partial}{\partial y} \\ & & k_x \frac{\partial^2}{\partial x^2} + k_y \frac{\partial^2}{\partial y^2} \end{bmatrix}$$

$$\mathbf{D}^{(v)} = \begin{bmatrix} S_{11}^{(v)} \frac{\partial^2}{\partial x^2} + S_{33}^{(v)} \frac{\partial^2}{\partial y^2} & \left(S_{12}^{(v)} + S_{33}^{(v)}\right) \frac{\partial^2}{\partial x \partial y} & -b\theta \frac{\partial}{\partial x} \\ \left(S_{12}^{(v)} + S_{33}^{(v)}\right) \frac{\partial^2}{\partial x \partial y} & S_{33}^{(v)} \frac{\partial^2}{\partial x^2} + S_{22}^{(v)} \frac{\partial^2}{\partial y^2} & -b\theta \frac{\partial}{\partial y} \\ m_1 \frac{\partial}{\partial x} & m_1 \frac{\partial}{\partial y} & -m_2 \end{bmatrix}$$

und

$$\mathbf{u} = \begin{bmatrix} u_x(x,y,t) \\ u_y(x,y,t) \\ \Phi(x,y,t) \end{bmatrix} \quad \mathbf{q} = \begin{bmatrix} q_x(x,y,t) \\ q_y(x,y,t) \\ 0 \end{bmatrix} \,.$$

Zur Lösung der Differentialgleichungen wird die Methode der endlichen Elemente mit einer für die Integration nach Zeit geeigneten Methode kombiniert verwendet.

b) Die Analyse der auf den Porenanteil bezogenen Annahme

Ein fundamentaler Punkt unserer Erwägungen ist die Annäherung *e)*. Ist sie nicht gültig, so wird die Theorie nichtlinear. Man kann sich einfach darüber überzeugen, daß die Annahme nach *g)* keine neuere strenge Einschränkung bedeutet.

Der elementare Bereich ist in einem Zustand $dA = dxdy$. Ist das Material zweiphasig und vom Gesichtspunkt der Porosität aus isotrop, dann wird das Maß des mit Boden erfüllten Gebietsteiles

$$dA_1 = \omega\, dx \cdot \omega dy = \omega^2 dxdy\,.$$

Der Porenanteil ist:

$$n = \frac{dA - dA_1}{dA} = 1 - \omega^2\,. \tag{15}$$

Im Laufe der Zustandsänderung verändert sich das Gebietselement dA in das Maß

$$dA' = (1 + \varepsilon_x)(1 + \varepsilon_y)\, dxdy \cong (1 + \varepsilon_x + \varepsilon_y)\, dxdy\,,$$

daraus fällt auf den festen Teil nach (5)

$$dA_1' = \left[1 + \frac{1}{\omega}(\varepsilon_x + \varepsilon_y)\right] \omega^2 dxdy$$

also fällt auf den neuen Porenanteil

$$n' - \frac{dA' - dA_1'}{dA'} = 1 - \frac{1 + \frac{1}{\omega}(\varepsilon_x + \varepsilon_y)}{1 + \varepsilon_x + \varepsilon_y}\,\omega^2\,.$$

Im Sinne der Annahme *g)* sind die Verschiebungen klein:

$$\varepsilon_x + \varepsilon_y \ll 1\,.$$

Wenn außerdem auch

$$\varepsilon_x + \varepsilon_y \ll \omega \tag{16}$$

gültig ist, dann wird

$$n' \cong n\,. \tag{17}$$

Ist z. B. $n = 0{,}36$, dann wird laut der Annahme $\varepsilon_x + \varepsilon_y \ll 0{,}8$, was eine ziemlich milde Forderung bedeutet.

c) Rand-, Anfangs- und Übergangsbedingungen

Bei der Aufstellung der Rand- und Anfangsbedingungen der Aufgabe müssen die herkömmlichen Bedingungen der Zustandsuntersuchung der Statik und der Sickerung vereint werden.

Im Falle der Treibung eines Stollens gibt es z.B. im Moment $t = 0$ keine Randkraft, es gibt weder am Rande noch im Inneren des untersuchten Bereiches eine Bewegung, dessenungeachtet ist der Spannungszustand des Eigengewichtes und des Ruheseitendruckes entsprechend. Die freie Oberfläche ist gegeben.

Im weiteren kann angenommen werden, daß der Raum unbelastet bleibt, daß entlang des den Halbraum des Bereiches abgrenzenden Konturs keine Verschiebung stattfindet, entlang des Konturs jedoch die primären Spannungen nach der vorgeschriebenen Zeitfunktion vermindert werden. Im Abbaufeld gibt es keine Quelle, vom Gesichtspunkt der Strömung aus herrscht jedoch dort Druckgleichgewicht.

Die konzentrierten Quellen und Senken der Knotenpunkte sind gegeben und verändern sich mit der Zeit in der vorgeschriebenen Weise. Dasselbe besteht auch für die eventuellen Knotenpunktskräfte. Mit den bekannten Knotenpunktseffekten werden wir uns im nächsten Punkt befassen.

3.2. *Die Annäherungsmethode der Lösung*

Die Behandlung der Anfangswertaufgabe

Jene Bereich A des Komplexes, dessen Zustand sich infolge des Bergbaus verändert, teilt man nach dem Grundprinzip der Methode der endlichen Elemente in A_i Teilbereiche auf. Diese sind dreieckförmig gewählt und es werden für jede einzelnen Annahmefunktionen bezüglich des in der Differentialgleichung (14) auftretenden Zustandsvektors $\mathbf{u}(x, y, t)$ vorgeschrieben. Sie haben die Form:

$$\mathbf{u}_i = \mathbf{U}_i(x,y)\ \ \mathbf{w}_i(t)\ . \tag{18}$$

Hier bedeutet $\mathbf{U}_i$ aus bekannten Funktionselementen bestehende Matrizen und $\mathbf{w}_i$ den Vektor der den Zustand des Teilbereiches charakterisierenden und allein von der Zeit abhängigen unbekannten Parameter.

Den Wert sämtlicher Annahmefunktionen deklarieren wir außerhalb ihrer eigenen Gültigkeitsbereiche für Null, wogegen wir jeden im eigenen Gültigkeitsbereich gemeinsam groß wählen. Danach kann statt $\mathbf{U}_i$, $\mathbf{U}$ geschrieben werden.

Eine Interpolation gleicher Form kann in Verbindung mit den Vektoren $\mathbf{q}(x, y, t)$ durchgeführt werden:

$$\mathbf{q}_i = \mathbf{Q}(x, y)\ \mathbf{z}_i(t)\ , \tag{19}$$

hier ist nicht nur $\mathbf{Q}$ sondern auch $\mathbf{z}$ gegeben.

Vorausgesetzt, daß der Anfangsvektor $\mathbf{w}_i(0)$ bekannt ist, wird der Zeitbereich $0 \leqq t \leqq T$ in die Zeitabschnitte Δt_j aufgeteilt und während dieser

Abschnitte die Änderung des Parameters $\mathbf{w}_i(t)$ als parabolisch angenommen:

$$\mathbf{w}_i(t) = \mathbf{a}t^2 + \mathbf{b}t + \mathbf{c} \quad t_j \leqq t \leqq t_{j+1}\,. \tag{20}$$

In Sinne dieser Voraussetzung und nach Einführung der Schreibweise $\mathbf{w}_i(t = t_j) = \mathbf{w}_{i,j}$ kann in Verbindung mit den Grenzmomenten der Zeitabschnitte folgendes aufgeschrieben werden:

$$\mathbf{w}_{i,j+1} - \mathbf{w}_{i,j} = \frac{\Delta t_j}{2}(\dot{\mathbf{w}}_{i,j+1} + \dot{\mathbf{w}}_{i,j}) \tag{21}$$

Nun sei (19) und (20) in den Zusammenhang (14) eingeführt und die Differentialgleichung sowohl für den Moment t_j, als auch für t_{j+1} aufgeschrieben:

$$\begin{aligned} \mathbf{D}^{(r)}\mathbf{U}\mathbf{w}_{i,j} + \mathbf{D}^{(v)}\mathbf{U}\mathbf{w}_{i,j} + \mathbf{Q}(\mathbf{z}_{i,j} + \theta\dot{\mathbf{z}}_{i,j}) = \mathbf{0} \\ \mathbf{D}^{(r)}\mathbf{U}\mathbf{w}_{i,j+1} + \mathbf{D}^{(v)}\mathbf{U}\mathbf{w}_{i,j+1} + \mathbf{Q}(\mathbf{z}_{i,j+1} + \theta\dot{\mathbf{z}}_{i,j+1}) = \mathbf{0} \end{aligned} \tag{22}$$

Nach Addierung der beiden Zeilen von (22) und die Bezeichnung

$$\mathbf{r}_{i,j} = \mathbf{z}_{i,j} + \mathbf{z}_{i,j+1} + \theta(\dot{\mathbf{z}}_{i,j} + \dot{\mathbf{z}}_{i,j+1}) \tag{23}$$

eingeführt, sowie auch (21) eingesetzt, erhält man

$$\left\{\mathbf{D}^{(r)} + \frac{2}{\Delta t_j}\mathbf{D}^{(v)}\right\}\mathbf{U}\mathbf{w}_{i,j+1} + \left\{\mathbf{D}^{(r)} - \frac{2}{\Delta t_j}\mathbf{D}^{(v)}\right\}\mathbf{U}\mathbf{w}_{i,j} + \mathbf{r}_{i,j} = \mathbf{0}\,.$$

In der Aufgabe des Anfangswertes der Integration nach der Zeit von Schritt zu Schritt schreitend kann $\mathbf{w}_{i,j}$ immer als bekannt betrachtet werden, und daher ist der Vektor

$$\mathbf{r}_{i,j,\mathrm{red}} = \mathbf{r}_{i,j} + \left\{\mathbf{D}^{(r)} - \frac{2}{\Delta t_j}\mathbf{D}^{(v)}\right\}\mathbf{U}\mathbf{w}_{i,j} \tag{24}$$

der Reihe nach als gegeben zu betrachten. Gleichwohl wird die Bezeichnung

$$\mathbf{D}^{(r)} + \frac{2}{\Delta t_j}\mathbf{D}^{(v)} = \mathbf{D}_{j,\mathrm{red}}$$

angewendet, wonach:

$$\mathbf{f}_{i,j} = \mathbf{D}_{j,\mathrm{red}}\mathbf{U}\mathbf{w}_{i,j+1} + \mathbf{r}_{i,j,\mathrm{red}} = \mathbf{0}\,. \tag{25}$$

In Anbetracht, daß $\mathbf{D}^{(r)}$ und $\mathbf{D}^{(v)}$ partielle Differentialoperatoren beinhalten, und $\mathbf{U}$ eine Funktionsmatrix ist, repräsentiert der Zusammenhang

(25) ein solches Gleichungssystem, dessen Koeffizienten nicht Zahlen, sondern Funktionen sind. Die im Teilbereich A_i gültige Annäherung der Lösung suchen wir auf Grund des Galerkinschen Prinzips. Die derart erhaltenen Ergebnisse approximieren in A_i sämtliche der Mechanik [4, 15].

Die ortsabhängige Änderung, die Steifigkeitsmatrix der Elemente

Die ortsabhängige Änderung der Zustandskennwerte erscheint im Index i der Vektoren w. Im Zusammenhang mit der Beschreibung dieser Änderung ist auch notwendig, daß die Gleichung (25) für Berechnungszwecke weiterentwickelt wird und schließlich müssen die durch die im Knotenpunkt konzentrierten Effekte charakterisierten Übergangsbedingungen auch erfüllt werden. Alldies wird mit Hilfe der zustandsbestimmenden Matrizen der Elemente, die die Verallgemeinerung der in der Elastizitätslehre auftretenden *Steifigkeitsmatrizen* sind, erledigt. Die bekannte Bezeichnung wird auch auf die zustandbestimmenden Matrizen ausgebreitet.

Im Sinne des *Galerkinschen Prinzips* muß die annähernde Lösung der Gleichung (25) die Bedingungen:

$$\int_{A_i} \mathbf{w}^*_{i,j+1}\, \mathbf{U}^*\, \mathbf{f}_{i,j}\, dA_i = 0\,, \tag{26}$$

in jedem der Teilbereiche befriedigen. Demnach wird:

$$\mathbf{w}^*_{i,j+1} \left\{ \int_{A_i} \mathbf{U}^* (\mathbf{D}_{j,\mathrm{red}}\, \mathbf{U})\, dA_i\, \mathbf{w}_{i,+1} + \int_{A_i} \mathbf{U}^*\, \mathbf{r}_{i,j,\mathrm{red}}\, dA_i \right\} = \mathbf{0}\,,$$

woraus folgt, daß falls $\mathbf{w}_{i,j+1}$ ungebunden ist, der Inhalt der Klammern ein Nullvektor wird. Führt man die Steifigkeitsmatrix

$$\int_{A_i} \mathbf{U}^* (\mathbf{D}_{j,\mathrm{red}}\, \mathbf{U})\, dA_i = \mathbf{K}_{i,j}\,,$$

sowie die Belastungsvektoren

$$\int_{A_i} \mathbf{U}^*\, \mathbf{r}_{i,j,\mathrm{red}}\, dA_i = \mathbf{g}_{i,j}$$

ein, so kann kurz geschrieben werden:

$$\mathbf{K}_{i,j}\, \mathbf{w}_{i,j+1} + \mathbf{g}_{i,j} = \mathbf{0}\,. \tag{27}$$

Die Zusammenstellung der Steifigkeitsmatrizen ist eine langwierige Aufgabe, worauf man u. a. in [11] ein Beispiel findet. Damit befassen wir uns hier

eingehend nicht, wir weisen nur auf den Inhalt der Vektoren $\mathbf{w}_i$ und auf die Auswahl der Interpolationsmatrizen $\mathbf{U}$ hin.

Im Falle dreieckförmiger endlicher Elemente ist es sinnvoll z. B. den Zustandskennwert an den Eckpunkten der Elemente, sowie an je 3 Punkten der Schwergerade zu suchen. Der Schwerpunktswert ist gemeinsam und so handelt es sich um insgesamt $3 \times 10 = 30$ Skalarkoordinaten auf jedem Element:

$$\mathbf{w}^*_{i,j+1} = [\mathbf{w}^*_{x;i,j+1} \quad \mathbf{w}^*_{y;i,j+1} \; \Phi^*_{i,j+1}] \,.$$

Hier sind:

$$\mathbf{w}^*_{x;i,j+1} = [u_{x;i,j+1,1} \; \cdots \; u_{x;i,j+1,10}]$$
$$\mathbf{w}^*_{y;i,j+1} = [u_{y;i,j+1,1} \; \cdots \; u_{y;i,j+1,10}]$$
$$\Phi^*_{i,j+1} \; = [\Phi_{i,j+1,1} \; \cdots \; \Phi_{i,j+1,10}]$$

die aktuellen Werte der Verschiebungsfunktionen, bzw. des hydraulischen Potentials in den aufgezählten Punkten.

Dementsprechend wird die Matrix $\mathbf{w}$ aus 3×30 Größen von der Voraussetzung

$$u_x = \xi^* \, \mathbf{e}_1 \qquad u_y = \xi^* \, \mathbf{e}_2 \qquad \Phi = \xi^* \, \mathbf{e}_3$$

ausgehend aufgebaut. Hier wird:

$$\xi^* = [1 \; \xi \; \eta \; \xi^2 \; \xi\eta \; \eta^2 \; \xi^3 \; \xi^2\eta \; \xi \; \eta^2 \; \eta^3]$$

in welchem ξ und η die lokalen Koordinaten des dreieckförmigen endlichen Elementes ist (Abb. 5). $\mathbf{e}_1$, $\mathbf{e}_2$ und $\mathbf{e}_3$ sind solche Vektoren, deren Elemente durch die Bedingung, daß die Elemente von $\mathbf{w}^*_{i,j+1}$ wirklich die gewünschten Zustandskennwerte sein sollen, festgelegt sind.

Es muß bemerkt werden, daß die Matrizen $\mathbf{K}_{i,j}$ von den zitierten abweichend jetzt nicht mehr symmetrisch sind.

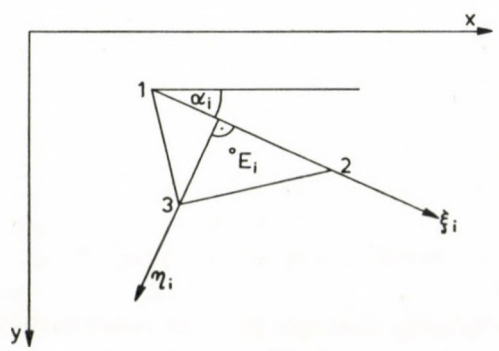

Abb. 5

Die Befriedigung der Übergangsbedingungen

Die Gleichung (27) kann – nach eventuellen Ordnen – in einer in zweimal zwei Blöcke zerlegten Form aufgeschrieben werden:

$$\mathbf{K}^{11}_{i,j}\,\mathbf{w}^1_{i,j+1} + \mathbf{K}^{12}_{i,j}\,\mathbf{w}^2_{i,j+1} + \mathbf{g}^1_{i,j} = \mathbf{0}\,, \tag{28}$$

$$\mathbf{K}^{21}_{i,j}\,\mathbf{w}^1_{i,j+1} + \mathbf{K}^{22}_{i,j}\,\mathbf{w}^2_{i,j+1} + \mathbf{g}^2_{i,j} = \mathbf{0}\,. \tag{29}$$

Die oberen Indizes sind keine Exponenten, sondern zeigen die Gruppierung an: $\mathbf{w}_1$ zählt die im Inneren des Elementes, $\mathbf{w}^2$ die an den Spitzen, bzw. an den Seiten gültigen Größen auf, dementsprechend drückt die erste Gleichung das im Inneren des Elementes gültige momentane Zustandsgleichgewicht aus, die zweite bezieht sich auf die Randpunkte.

Im Falle einer richtigen Annäherung ist $\det \mathbf{K}^{11}_{i,j} \neq 0$ und

$$\mathbf{w}^1_{i,j+1} = -(\mathbf{K}^{11}_{i,j})^{-1}\,(\mathbf{K}^{12}_{i,j}\,\mathbf{w}^2_{i,j+1} + \mathbf{g}^1_{i,j})\,, \tag{30}$$

d. h. die Zustandskennwerte des Inneren des Elements können aus den Daten des Randes ausgedrückt werden. (Dazu kommt es nach einer umfangreichen Gleichungslösung.) Deshalb genügt es, uns mit (29) zu befassen.

Darauf hinweisend, daß die Randpunkte der endlichen Elemente gleichzeitig auch die Knotenpunkte des Dreiecksnetzes sind, werden die geometrischen Übergangsbedingungen so befriedigt, daß bei der serienweisen Anwendung von (29) die Zustandskennwerte der sich zu den gemeinsamen Knotenpunkten anpassenden Elementspunkte als gemeinsam betrachtet werden.

Die dynamischen Bedingungen der Anpassung werden so befriedigt, daß der physikalische Inhalt der nach dem Galerkinschen Prinzip aufgeschriebenen Gleichungen in Betracht gezogen wird, in welchem Sinne die linke Seite von (29) die an den ausgewählten Randpunkten des Dreieckselements gültigen Gleichgewichtsbedingungen, bzw. die Sickerungsbilanz ausdrückt.

Die obige Form von (29) könnte nur an einem aus einem einzelnen Element bestehenden endlichen System gültig sein, dort würde sie jedoch eine falsche globale Bilanz zeigen. So muß statt (29) an jedem Anschlußpunkt je eine Summierungsgleichung aufgeschrieben werden:

$$\sum_i \left\{\mathbf{K}^{22}_{i,j} - \mathbf{K}^{21}_{i,j}(\mathbf{K}^{11}_{i,j})^{-1}\,\mathbf{K}^{12}_{i,j}\right\}\mathbf{w}^2_{i,j+1} = \mathbf{p}_j + \sum_i \left\{\mathbf{K}^{21}_{i,j}(\mathbf{K}^{11}_{i,j})^{-1}\,\mathbf{g}^1_{i,j} - \mathbf{g}^2_{i,j}\right\}. \tag{31}$$

Die Summierung nach i erstreckt sich auf sämtliche sich an Punkt anpassende Elemente. In der Formel ist $\mathbf{p}_j$ ein die im Knotenpunkt konzentrierten Kräfte und Quellen (Senken) charakterisierender Vektor, der auf die Art von (23) aufgeschrieben wird.

Die den momentanen Zustand des Systems beschreibende Matrixgleichung wird – entsprechend der serienweisen Aufschreibung von (31) – durch

das in der Methode der endlichen Elemente übliche Sammeln der Koeffizienten aufgestellt. Nach der serienweisen Lösung der Gleichung kann die Lösung der Anfangswert-Aufgabe von Schritt zu Schritt ausgearbeitet werden.

4. Schlußbemerkungen

Die in den Kapiteln 2 und 3 erörterten Modelle sind alle derartig, daß zu ihrer Verwirklichung elektronische Rechenmaschienen großer Leistungsfähigkeit benötigt werden. In diesem Zusammenhang erheben sich zwei berechtische Fragen:

a) Ist die ungarische Computerisierung bereits auf dem Stand, den die Lösung dieser Aufgaben erfordert, wobei der riesig große Freiheitsgrad der Aufgaben besonders berücksichtigt werden muß?

b) Wird die Realität der Computerlösungen befriedigend sein und besteht die Aussicht auf die Rückerstattung der in deren Aufstellung investierten Summe?

Unserer Ansicht nach können beide Fragen bejahend beantwortet werden. Bezüglich der ersten möchten wir darauf hinweisen, daß es bereits gelungen ist Aufgaben bedeutenden Umfanges zu lösen [12, 13] und dieselben bilden die Grundlage der in der Arbeit dargelegten Methoden. Außerdem versorgt uns das Programm der Automatisierung der Rechentechnik mit elektronischen Rechenanlagen vor Jahr zu Jahr größerer Leistung, wodurch unsere Ziele verwirklicht werden können.

Zur zweiten Frage muß betont werden, daß die Ergebnisse mit den Erfahrungen an Ort und Stelle laufend verglichen werden und die Ausgangsdaten dynamisch verändert werden müssen.

Bedenkt man, daß eine richtig durchgeführte Zustandsuntersuchung zur Abwehr von Bergbauschäden und zum Wasserschutz des Bergbaus Informationen liefert, die mit anderen Methoden nicht erhältlich sind, und weiters, daß die aufgeworfenen Aufgaben des Bergbaues mit ziemlich hohen Investitionen und manchmal auch mit regionalen Problemen verbunden sind, dann müssen wir der Ansicht sein, daß die Kosten der Automatisierung der Rechentechnik im Vergleich zu denen der Investitionen unbedingt verschwindend klein, die erreichbaren Ersparnisse aber nicht unbedeutend sind.

SCHRIFTTUM

1. Asszonyi, Cs.: A kőzetmechanika egy új felfogásáról (Über ein neues Konzept der Gesteinsmechanik). *Bányászati és Kohászati Lapok — Bányászat*, 107 (1974), 145—150; 250—253; 325—332.
2. Asszonyi, Cs.—Richter, R.: Bevezetés a kőzetmechanika reológiai elméletébe (Einführung in die Rheologietheorie der Gesteinsmechanik). NIM Továbbképző Központ, Budapest, 1974.

3. Corradi Dell'Aqua, L.—Maier, G.: A matrix theory of elastic locking structures. Ist. di Scienza e Tecnica delle Costr. del Politecnico di Milano Publ. N. 452. Anno Ac. 1968—1969.
4. Duguid, J. O.—Abel, J. F.: Finite element Galerkin method for analysis of flow in fractured porous media. Finite Element methods in flow problems. A collection of papers edited by J. T. Oden et al. UAH Press, Hunstsville Alabama, 1974, 599—617.
5. Eaves, B. C.: The linear complementarity problem Management Science, 17 (1971), 612—634.
6. Kaliszky, S.: Feltételes kapcsolatokat tartalmazó szerkezetek vizsgálata (Untersuchung von bedingte Verbindungen enthaltenden Tragwerken. *Építés-Építészettudomány*, 6 (1974), 325—339.
7. Kaliszky, S.: Képlékenységtan. Elmélet és mérnöki alkalmazások (Plastizitätslehre, Theorie und Anwendung in der Ingenieurpraxis). Akadémiai Kiadó, Budapest, 1975.
8. Kézdi, Á.: Handbuch der Bodenmechanik. Bd. I. Bodenphysik. Akadémiai Kiadó, Budapest, 1969.
9. Roller, B.: Rugalmasan kapcsolt merev elemekből álló héjszerkezet számítási elmélete (Die Berechnungstheorie von Schalentragwerken, bestehend aus elastisch verbundenen starren Elementen). NIM. *Számítástechnikai Közlemények*, No. 14, 1970, 24—48.
10. Roller, B.: Korlátozott kapcsolatú keretszerkezetek megoldása mátrix-elmozdulásmódszerrel (Die Lösung von Rahmentragwerken beschränkter Verbindung mit Hilfe der Matrizengleichgewichtsmethode). *Mélyépítéstudományi Szemle*, 25 (1975), 90—93.
11. Roller, B.: 7. Bányavízvédelmi Konferencia (7. Konferenz über Bergbauwasserschutz) Budapest, Oktober 27—29, 1976. Manuskript des Vortrages 1—4.
12. Roller, B.—Kesserü, Zs.: Anwendung der Methode der endlichen Elemente für die Untersuchung der Unterbauten Hangendschichten im Falle veränderlicher materieller Eigenschaften. III. Internationales Symposium für Markscheidewesen, Leoben (Österreich) 28. Juni—3. Juli 1976. Bd. 4. S. 189—199.
13. Roller, B.—Nagy, T.—Györgyi, J.: Szivárgási áramképek vizsgálata véges elemek módszerével (Untersuchung von Strömungsbilder der Sickerung mit Hilfe der Methode der endlichen Elemente). *Hidrológiai Közlöny*, Budapest, No. 2. 1976, 78—82.
14. Szabó, J.—Roller, B.: Rúdszerkezetek elmélete és számítása (Theorie und Berechnung von Stabwerken). Műszaki Könyvkiadó, Budapest, 1971.
15. Scharle, P.: On the relationship between different approximating methods. Acta Technica Ac. Sci. Hung., 82 (1976), 53—59.

ЧИСЛОВОЕ ИССЛЕДОВАНИЕ ПОРОДНЫХ КОМПЛЕКСОВ

Р. РОЛЛЕР

РЕЗЮМЕ

В статье рассматривается разработка числовых методов, учитывающих некоторые характерные особенности пород при исследовании механического состояния. Облимочный, рыхлый, мало противостоящий растяжению характер породы моделируется конечной системой, состоящей из жестких блоков и прочных связей и вычисление которой приводит к задаче, решаемой симплексным методом, одинаково для случаев вещества, противостоящего только сжатию, вещества, отвечающего условиям разлома *Мора*, а также уплотняющегося вещества. Также можно обращаться и исследованием ограниченной утечки, связываемой с упомянутыми уже проблемами.

Эластовискозное, двуфазное вещество моделируется конечной системой элементов, к которой приписываются непрерывные прочностные свойства. Ее вычисление приводит к системе дифференциальных уравнений.

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A STUDY OF GEOMAGNETIC PULSATION PERIODS

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The study of coherencies between pulsations recorded at distant observatories has special importance for the knowledge of the microstructure of pulsations. The paper presents results of a comparison of amplitude and coherence spectra of magnetic and telluric pulsations in 5 observatories lying far from each other. The results are also compared with earlier results

1. One of the best means to study the microstructure of geomagnetic pulsations is the latitude dependence of the pulsation periods and the coherency between pulsations in distant observatories. For the latter purpose magnetic and earth current records of the distant observatories listed in Table I have been compared. A detailed investigation has been carried out for pulsation events of September 17, 1968 between 02 and 07 UT. Each event has a duration of about 20 min. The investigations were carried out in several steps.

Table I

Station	Sign	Geographical co-ordinates		Geomagnetic co-ordinates	
		φ	λ	Φ	Λ
Tromsø	Tr	69°40′ N	18°57′ E	67.1° N	116.7° E
Nagycenk	Nc	47°38′	16°43′	47.2°	98.3°
Zavhanmandal	Zh	47°	97°		
M'Bour	MB	14°23′	16°57′ W	21.3°	55.0°
Addis Abeba	AA	9°02′	38°46′ E	5.4°	109.1°

In the first step earth current records of Nagycenk (Hungary) and Zavhanmandal (Mongolia) have been used. The records were run with paper speeds of 20 and 30 mm/min respectively. From the events studied, one had Pi 2 character, the other three Pc 3—4 character. The events were digitized with lags of 5 sec, then amplitude spectra and coherence spectra were computed for all possible combinations of the components. A statistics of the maxima in these spectra has shown that they appear more often at certain values. These periods are about 33, 48, 80 and 110 sec, i. e. they approximately correspond to the 2., 3., 5. and 7. harmonics of a period of 220—240 sec. The Pi 2

event showed this structure in a lesser degree, in the other spectra the structure was quite distinct, and the position of the maxima changed only a little during the time of the investigation [1].

2. We have supposed on the basis of the earlier investigations [2] that at least the Pc 4 pulsations are the harmonics of a basic period of 200 –220 (or 100 –110 sec) and hence the harmonics appear as maxima in the spectra. STUART and USHER [3] made similar suppositions, namely, that pulsations are the 2 –5. harmonics of a basic period of 100 –120 sec. STUART et al. [4] found that the greatest part of the maxima in the pulsation spectra can be identified as the 1 –8. harmonics of a basic period of 100 –300 sec. Further it is not excluded that this harmonic structure plays an important role in the latitude dependence of pulsation periods.

Earlier own investigations have shown that the most common latitude dependence of pulsation periods are 0, 4 and 10 per cent for a degree of latitude difference, being valid for middle latitudes and for Pc-type pulsations [5]. The first group without latitude dependence could mean cases when the same harmonics is present at both investigated stations; in the second (4 %/°) and third (10 %/°) group the order of the dominant harmonics differ at the stations by 1 or by 2, the higher harmonics being present at lower latitudes. According to the theory of oscillating shells this can be so explained that the stations receive signals from the same shell, from adjacent ones or from the next to the adjacent one. A difficulty of this explanation is that identical periods appear sometimes at stations which can be hardly on the same shell (the width of the shell is typically 100 or a few 100 km-s).

3. In the last years a new idea was publicized according to which the pulsations are generated outside of the magnetosphere, or more exactly before the magnetopause. In this case the generation mechanism for latitude dependent periods must be re-interpreted, as the period of the pulsations is determined by $160/B$, where B is the interplanetary magnetic field in nT [6]. Several possibilities emerge in this respect. The first is that the source determines a "primary" spectrum which changes by crossing the magnetosphere; even the theory of oscillating shells could be used. In the latter case it is problematic how can be the equation $160/B$ valid for different latitudes?

A second possibility is that only a part of the pulsations has extramagnetospheric origin, namely those having latitude-independent periods. The resulting pulsations can be mixtures of extra-magnetospheric pulsations and those generated in the magnetosphere. Even in the latter part pulsation from the plasmapause may have identical periods on a global scale.

In a certain degree the situation can be checked by comparing the parameters of the interplanetary magnetic field with the parameters of the pulsations. If the values (e. g. the periods) do not agree, an extramagnetospheric origin is unlikely. If they coincide, the extramagnetospheric origin is possible.

4. For a detailed study records of some additional observatories were also used.

In a second phase, the magnetic records of Tromsø (6 mm/min), have been added to those of Nagycenk and Zavhanmandal but only the three Pc-events were analyzed. The maxima of the spectra were used again and a basic period has been looked for whose harmonics could be the maxima. Maxima of the spectra could be approximated by the 2., 3., 5. and 7., sometimes also by the 9. harmonics of a basic period of 210 –270 sec. If only the maxima lying near to the 3., 5. and 7. harmonics were taken into account then the basic period was 220 –260 sec [7].

In possession of magnetic records from M'Bour (4.7 mm/min) and Addis Abeba (6 mm/min) the investigations could have been extended to 5 stations and 51 spectra determined for each station. The maxima of the complete spectra lie in this case at the harmonics of a basic period of 210 –300 sec, or if only the 3., 5. and 7. harmonics are taken into account, of 210 –290 sec (Table II). A comparison of the results shows that the most deviating basic periods in the coherence spectra were found, if one of the stations was Tromsø i. e. a station in the polar zone.

The average values of the basic periods and their scatter have been determined for each event and for each station (Table III). This comparison shows that the most stable results were yielded by the middle latitude stations Nagycenk and Zavhanmandal, where the time resolution of the records was also the highest.

Figure 1 shows amplitude spectra of the magnetic D or telluric N-components of the stations for the three Pc events. Figure 2 shows the geometric average of these spectra for each event. The spectrum gets somewhat smoother during the period of the investigation, and its maxima shift towards the longer periods.

Figures 3, 4 and 5 present examples for the coherence spectra. In case of the station pairs (e. g. for Tromsø – Nagycenk) the coherence spectra were united so that the highest coherency was used for each period value out of 4 possible ones (Tr D –Nc N, Tr D –Nc E, Tr H –Nc N, Tr H –Nc E). Out of the 30 coherence spectra shown only 3 do not reach the value 0.7, but even these reach 0.6.

5. A further comparison can be made with earlier investigations carried out in our Institute. ÁDÁM et al. [8] determined the maximum amplitudes of the pulsations for 6 non-consecutive hours in 19 observatories on June 9, 1958. Correlation factors were computed from the 2.5-minute maximum amplitudes for each station pair, and the medians of these correlation factors were used further on ([8] Table III). From this material we used 13 observatories (Apia, Nagycenk, Tamanrasset, Watheroo, Eskdalemuir, Onagawa, Rude Skov, Tucson, Honolulu, Nurmijärvi, Mirny, Reykjavik, Guam), thus we had 78

station pairs. The correlation factors are shown in Fig. 6 in function of the distance between the station pairs, and their averages yield curve *a*.

It has already been shown in an earlier investigation that the amplitudes of Pc 3 pulsations are correlated even in very distant observatories, if the averages are well chosen [9]. The correlation factors between the average

Table II

September 17,1968	Basic period	
	from the full spectra	from the 3., 5. and 7. harmonics
02 44 — 03 05 UT		
Amplitude spectra	230 sec	230 sec
Coherence spectra	220	220
Nc—Zh	230	230
Nc—Tr	220	240
Nc—AA	230	250
Nc—MB	230	230
Zh—Tr	220	220
Zh—AA	230	240
Zh—MB	230	230
Tr—AA	210	210
Tr—MB	230	220
AA—MB	230	240
Σ	230	240
05 16 — 05 37 UT		
Amplitude spectra	230 sec	260 sec
Coherence spectra	300	240
Nc—Zh	230	240
Nc—Tr	210	240
Nc—AA	210	210
Nc —MB	270	260
Zh—Tr	220	240
Zh—AA	230	260
Zh—MB	210	240
Tr—AA	290	290
Tr—MB	220	220
AA—MB	270	260
Σ	220	230
06 08 — 06 29 UT		
Amplitude spectra	260 sec	260 sec
Coherence spectra	250	260
Nc—Zh	260	250
Nc—Tr	240	250
Nc—AA	230	260
Nc—MB	240	260
Zh—Tr	260	260
Zh—AA	260	260
Zh—MB	280	260
Tr—AA	270	270
Tr—MB	280	260
AA—MB	300	290
Σ	260	270
The three cases together:	230	260

Table III

September 17, 1968

	02 44–03 05 U T		05 16–05 37 UT		06 08–06 29 UT	
	Average basic period	scatter	Average basic period	scatter	Average basic period	scatter
Nc	228	± 4 sec	230	± 25 sec	242	± 10 sec
Zh	228	± 4	222	± 8	265	± 8
Tr	220	± 7	235	± 34	265	± 15
AA	225	± 8	250	± 32	265	± 25
MB	230	± 0	242	± 28	275	± 25
Σ	226	± 6	236	± 28	262	± 20

hourly amplitudes of the pulsations in the observatories from this investigation are also shown in Fig. 6 (curve *b*). These values show a good agreement with the values from the other investigation mentioned.

For the present case the values of the 4 coherence functions at the 3., 5. and 7. harmonics were compared and the greatest picked out. Then the coherencies from the three events were averaged for this maximum value, and the averages can be seen in Fig. 7 for all 10 station pairs. Highest values are reached for the 3 harmonics (80 – 100 sec), the coherencies for the 5. (45 – 60 sec) and for the 7. (30 – 40 sec) are somewhat lower. The decrease is rather slow. Supposing the exponential decrease with the distance $\mathrm{Coh} = e^{-A \cdot t}$, where t is the distance in 10^3 km, and its coefficient A has the following values: for the 3. harmonics $A_3 = 0.942/10^3$ km, for the 5. harmonics $A_5 = 0.930/10^3$

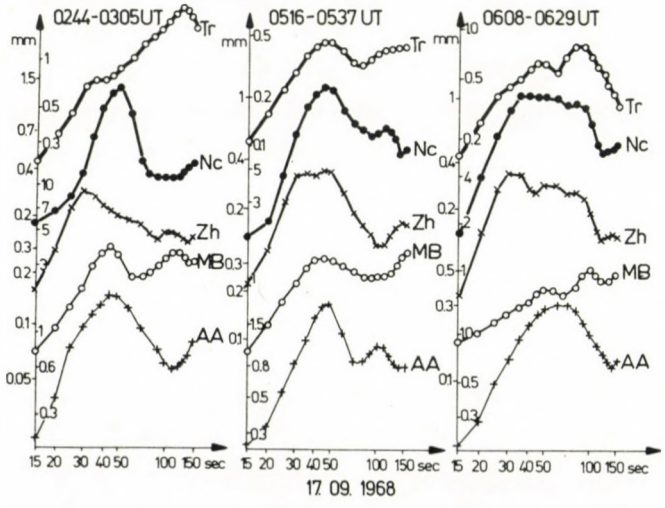

Fig. 1. Amplitude spectra of the events

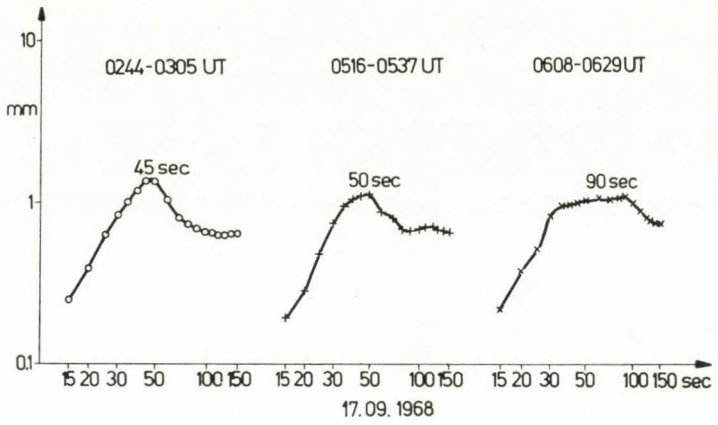

Fig. 2. Average amplitude spectra

km, for the 7. harmonics $A_7 = 0.897/10^3$ km. The averages for the three periods are shown again in Fig. 6 with dots (∘). Thus, Fig. 6 shows together the results of three independent investigations.

For a comparison of the data in Fig. 6 it should be noted that curve *a* is the result of a correlation analysis from 2.5 min maximum amplitudes, and the events include Pc and Pi 2 as well. The exponential curve $b(A_b = 0.7946/10^3$ km) results from hourly averages of pulsation amplitudes without selection. It has been shown [10] that it is just the 1-hour interval which yields lowest correlation coefficients between average amplitudes in the 15 – 30 sec band. In the present case filtered data were used for selected pulsation events. Coherence values can be $\sqrt{2}$-times greater than corresponding correla-

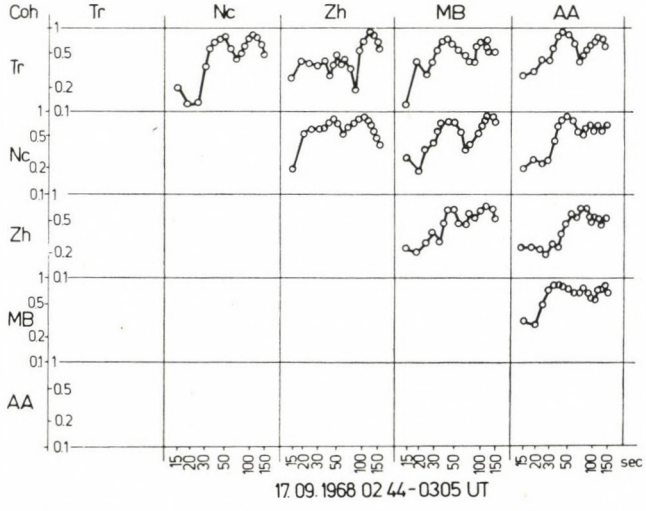

Fig. 3. United coherence spectra for the event 02 44 – 03 05 UT containing Pc pulsations

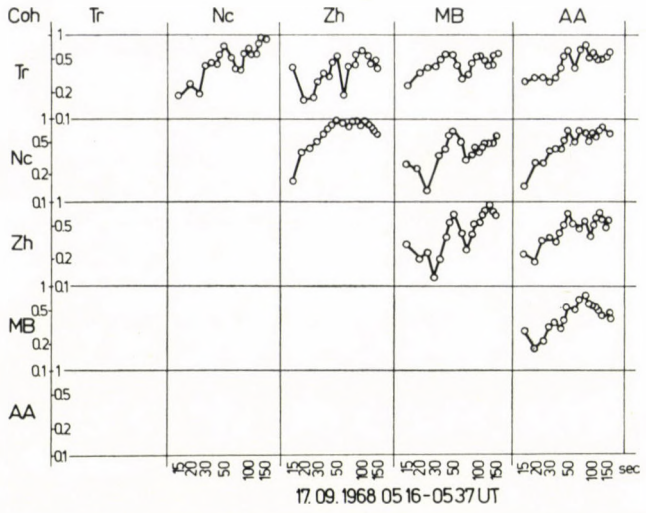

Fig. 4. United coherence spectra for the event 05 16 – 05 37 UT containing Pc pulsations

tion factors. If we take this into account, the values of the curve *c* would lie up to 30 per cent lower than given in the figure. The differences between different investigations are less in case of smaller distances between the observatories (up to 3 –4000 km).

6. In a next step, the parameter of the pulsations have been compared with the data of the solar wind and interplanetary magnetic field. Pioneer 6 measured on September 17, 1968, at 13 UT a bulk velocity of 576.4 km/sec, and a density of 2.7 cm^{-3}. The data of the interplanetary magnetic field

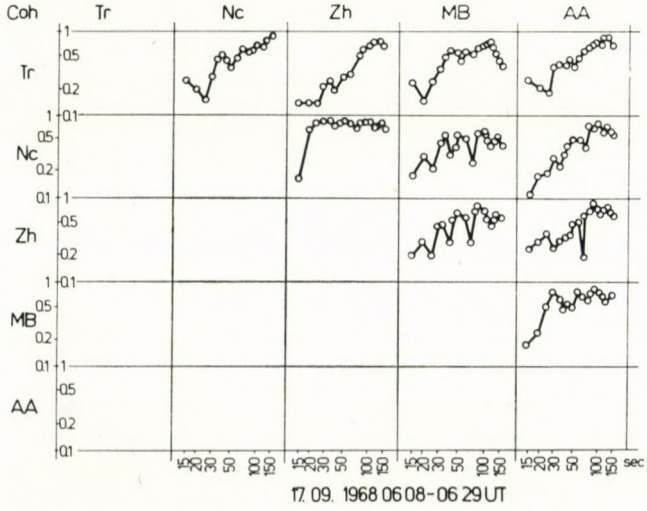

Fig. 5. United coherence spectra for the event 06 08 – 06 29 UT containing Pc pulsations

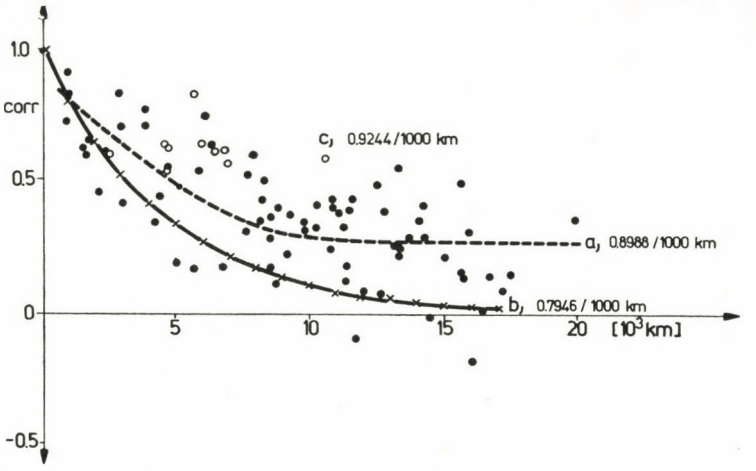

Fig. 6. Correlation factors between distant observatories from different investigations (detailed explanations see in text). *a)* • Ádám et. al. [8]; *b)* × Cz. Miletits and Verő [9]; *c)* ○ from the present investigation

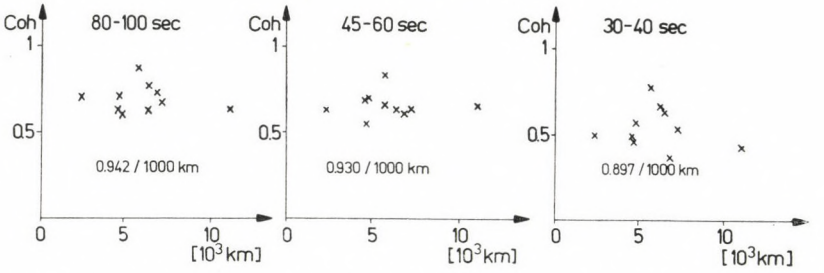

Fig. 7. Values of the coherencies for the 3., 5. and 7. harmonics in function of the distance between the observatories

delivered by the measurements of the satellite IMP 4 are shown in Table IV. Here B denotes the field in nT, Θ and Φ are the direction angles of the field in ecliptic co-ordinates. I, II and III denote the times hours of the events investigated.

Table IV

September 17, 1968

	Hour	B	$\Theta°$	$\Phi°$
I.	2	3.0	— 9	142
	3	3.0	1	178
	4	3.3	4	173
II.	5	3.4	40	153
III.	6	3.2	40	129
	7	2.8	—16	127

The indices Kp show for this interval rather low activity (2_+, 2_0, 3_-). The solar wind velocities measured by Pioneer 6 are somewhat too large for such a geomagnetic activity. The formula by SNYDER et al. [11]:

$$V \approx 330 + 8.5 \sum \text{Kp}$$

yields velocities of 488, 466 and 511 km/sec for the Kp-values valid for the events. They are all smaller than that measured by Pioneer 6, the time of the measurement is, however, somewhat later than the events.

Table V

Amplitude values in units of 0.15 *mV/km*

Period (sec)											
−5	−10	−15	−20	−25	−30	−40	−60	−90	−120	−300	−600
—	—	—	—	—	.09	1.09	.82	.45	.18	1.50	—
—	—	—	.20	.20	.70	1.90	1.60	—	—	—	—
—	—	.14	.68	1.14	.68	2.55	.91	1.55	.45	.14	1.50

Figure 8 shows with full line the amplitude spectra of Fig. 2, with dotted lines the spectra from Table V. Table V contains spectra for the appropriate 3-hours interval of the day in case of the parameters of the interplanetary magnetic field present in that time from the material of the records of the year 1972 in the Nagycenk observatory. The spectra are in the latter case less broad, but the general shape corresponds rather well to the amplitude spectra measured, and the peaks occur in all cases between periods 30 −40 sec. Further as Fig. 9 shows for one of the cases (the other two are of similar character), the periods of the maximum amplitudes do not show here any latitude dependence.

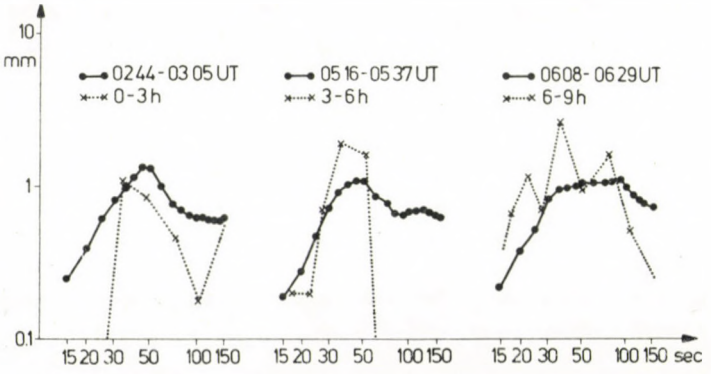

Fig. 8. Average measured amplitude spectra and average spectra for the momentanous parameter of the interplanetary magnetic field (1972 Nagycenk data)

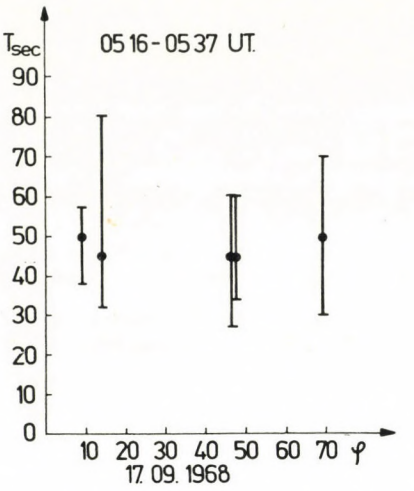

Fig. 9. Amplitude maxima (dots) for the case 05 16 – 05 37 UT in the different observatories with bars corresponding to periods where amplitudes are more than 70 % of the maxima, in function of the geographic latitude

Thus it cannot be excluded that the pulsations investigated in the present case are of extra-magnetospheric origin, a harmonic structure seems, nevertheless, to be present.

It is a rather diffucult problem to have pulsation periods determined by the interplanetary magnetic field and periods depending on the latitude at the same time. In order to solve this problem the fine structure of the pulsations must be investigated in full details including coherencies between distant observatories. The differences of the amplitude and coherence spectra can add valuable information to the study of the latitude dependence of pulsation periods.

REFERENCES

1. Cz. Miletits, J.—Verő, J.: Vergleich simultaner Pulsationsspektra bei grossen Längenunterschieden. *Acta Geod. Geoph. Mont. Hung.*, 10 (1975), 147.
2. Ádám, A.—Cz. Miletits, J.—Verő, J.: Das Mikropulsationsfeld in Osteuropa (Ergebnisse der KAPG-Synchronmessungen vom Jahre 1969). *Acta Geod. Geoph. Mont. Hung.*, 7 (1972), 289.
3. Stuart, W. F.—Usher, M. J.: An investigation of micropulsations at middle latitudes. Geoph. J. R. A. S.. 12 (1966), 71.
4. Stuart, W. F.—Sherwood, V. E.—Macintosh, Sh. M.: An analysis of micropulsation spectra (Pc) obtained from the U. K. observatories. *J. Atm. Terr. Phys.*, 33 (1971), 869.
5. Cz. Miletits, J.: On the latitude dependence of Pc-type pulsations. *Acta Geod. Geoph. Mont. Hung.*, 6 (1971), 141.
6. Gul'elmi, A. V.: Diagnostics of the magnetosphere and interplanetary medium by means of pulsations. *Space Sci. Rev.*, 16 (1974), 331.
7. Czuczorné, Miletits, Judit: Pulzációk spektrum-vizsgálata (Investigations of pulsation spectra). Physics of ionosphere and magnetosphere, 4 (1977). 139.
8. Ádám, A.—Verő, J.—Wallner, Á.: Die räumliche Verteilung der Pulsationen des elektromagnetischen Feldes der Erde. *Acta Geod. Geoph. Mont. Hung.*, 1 (1966), 379.

9. Cz. Miletits, J.—Verő, J.: Correlated Pc type micropulsations observed at separate places. *J. Atm. Terr. Phys.*, 33 (1971), 967.
10. Takács, L.: Experimental results with the characterization of geomagnetic micropulsations V.: Comparison of the geomagnetic pulsation activity at the observatories Nagycenk, Niemegk and Sodankylä. *Acta Geod. Geoph. Mont. Hung.*, 10 (1975), 307.
11. Snyder, C. W.—Neugebauer, M.—Rao, U. R.: The solar wind velocity and its correlation with cosmic ray variation and with solar and geomagnetic activity. *J. Geoph. Res.*, 68 (1973), 6361.

ИССЛЕДОВАНИЕ ПЕРИОДА ГЕОМАГНИТНЫХ ПУЛЬСАЦИЙ

Й. Ц. МИЛЕТИЧ

РЕЗЮМЕ

С точки зрения изучения микроструктуры пульсаций особое значение имеет исследование наблюденных на большом расстоянии когерентностей. В статье были сопоставлены спектры амплитуд и когерентностей, вычисленных на основе съемок магнитных и земных потоков, зарегистрированных одновременно в 5 удаленных друг от друга обсерваториях. Полученные результаты были сопоставлены и с нашими более ранними исследованиями.

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ON THE DISTRIBUTION OF THE ELECTRICAL CONDUCTIVITY IN THE AREA BETWEEN THE RHINEGRABEN, THE BOHEMIAN MASSIF AND THE CENTRAL ALPS*

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[Manuscript received January 10, 1977]

In the southern part of the Federal Republic of Germany and in Austria geomagnetic depth sounding has been done at about 80 stations by different geophysical institutes. For most of the stations "Wiese" induction arrows are plotted in Fig. 1 for variations with a period of about 20 min. The spatial variation of the length and direction of the induction arrows depends on at least 3 stronger conductivity anomalies: the sedimentary troughs of the Rhinegraben striking about NS and the molasse basin striking about EW and further on a lateral gradient of the electrical conductivity in greater depth which strikes about 20 km north of the river Danube (Donau) parallel to the average striking direction of the river. Below the Eastern Alps no distinct anomaly of the electrical conductivity could be found.

The area, treated in this paper (Fig. 1), is roughly bordered by the river Main in the north (about latitude 50°.N), the Rhinegraben in the west, the frontiers FRG/CSSR respectively Austria/Hungary in the east and the southern boundary of the Alps in the south. In this area geomagnetic depth sounding has been done at 80 stations. At 29 stations — in Fig. 1 marked by circles — the magnetic variations have been measured by the geophysical institutes of the Universities in Göttingen, Braunschweig and Münster. At 51 stations — in Fig. 1 marked by dots — the magnetic variations have been measured by the author. A lot of magnetotelluric measurements have been done also they are not treated in this paper.

The author is working on a homogeneous evaluation of all geomagnetic depth sounding measurements in the area stated above to determine the distribution of the electrical conductivity in the crust and upper mantle of this area. A first compilation of these evaluations and those of other authors is given in this paper.

For most of the stations "Wiese" induction arrows have been determined by the different authors as a function of the period T of the variations (in Fig. 1 for $T = 20$ min). The spatial variation of the length and direction of

* Paper presented at the IAGA Workshop on Electromagnetic Induction, Sopron, July 1976.

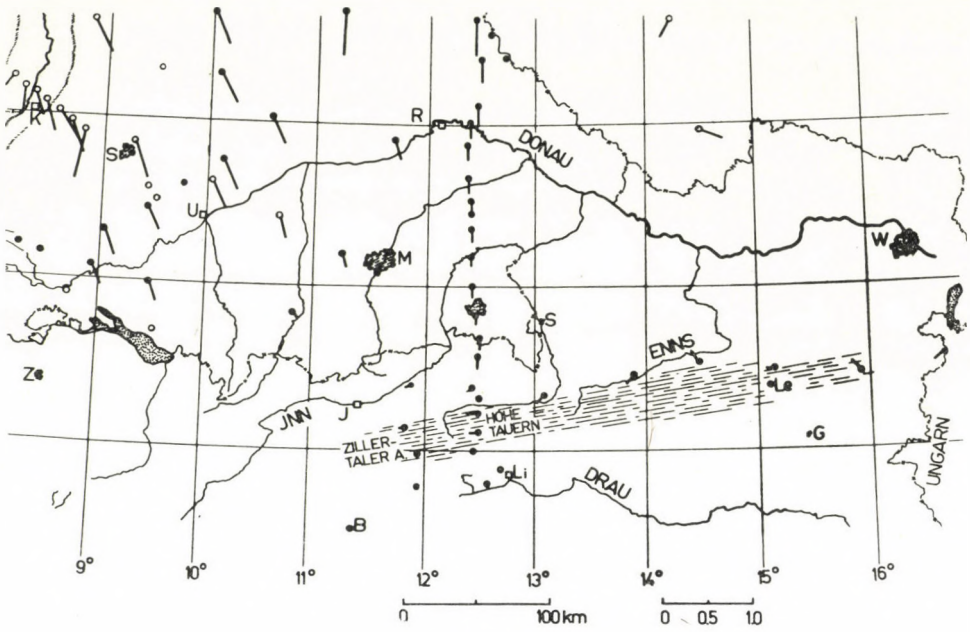

Fig. 1

the induction arrows gives an indication to the existence, location and striking direction of conductivity anomalies. (The length of the induction arrow represents the quotient $Z/H_{\text{corr.}}$, where $H_{\text{corr.}}$ is the horizontal component, which is in maximum correlation with Z. Near to a two-dimensional conductivity anomaly the induction arrows are directed perpendicularly to the striking direction of the anomaly. The "*Wiese*" arrows point from areas of higher conductivities to those of lower conductivity. If there are several conductivity anomalies in an area, the induction arrows result from the magnetic fields of the different anomalies in a complex and not simple additive superposition.)

In the whole area between the North German conductivity anomaly and the central Eastern Alps the induction arrows are directed more or less to the south. Stronger deviations from the south direction are observed mainly near the conductivity anomaly of the Rhinegraben striking about NS. In a large area north of the river Donau (Danube) the length of the induction arrows is rather constant. The average value for the length is about 0.3 within a wide period range. In an area, which is situated ca. 10 – 20 km north of the river Donau and which strikes about parallel to the average striking direction of the river (between the latitudes 9° E and 13° E), the length of the induction arrows starts to decrease in southern direction. From this area to the south resp. south-east the length of the induction arrows decreases to about 0.15 in the northern part of the Bavarian molasse basin and to a rather small and

constant value of about 0.10 in the Eastern Alps (in the area between Innsbruck and the frontier Austria/Hungary).

The author supposes that the part of the Z variations which is measured at all sites and which causes the south direction of the induction arrows is of internal origin. The decrease of the induction arrows to the south near the river Danube may be caused by a lateral gradient of the electrical conductivity in greater depth – from an increased electrical conductivity north of the river Danube to a decreased electrical conductivity south of the river.

The part of the Z variations measured at all sites, is locally superimposed by a second part which is correlated with the superficial conductivity anomalies of the Rhinegraben striking about NS and the molasse basin striking about EW. Both troughs are filled with well conducting sediments of several km thickness. The effect of the Rhinegraben anomaly may be seen in Fig. 1 by the deviation of the induction arrows from the south direction near the Rhinegraben.

Model calculations have been done by WINTER [1], REITMAYR [2] and the author [3] to determine the conductivity-depth-distribution in the area stated above. For the calculations the "thin sheet" algorithm [4] and the finite difference algorithm [5] have been used. For the northern part of the Rhinegraben WINTER supposes a well conducting layer in the depth range between 25 and 75 km. Further to the east – north of the river Danube – the author supposes a well conducting layer in the depth range of 20 – 40 km. Thus, in the present state of evaluation it seems that the well conducting zone below the northern part of the Rhinegraben is not typical for the Rhinegraben itself but it is extended over a large area from the northern Rhinegraben along the river Danube (and the area north of it) to the Bohemian Massif. For the southern part of the Rhinegraben REITMAYR could not find such a clear indication for a well conducting layer in the lower crust as WINTER did. He supposes a well conducting zone in a depth range of 80 – 100 km. This coincides more or less with the results of the present author who supposes a well conducting layer in a depth range of 60 – 80 km below the Bavarian molasse trough. REITMAYR, on the other hand, introduces a well conducting zone in the shallow depth of 25 – 45 km below the southern Rhinegraben, derived from the rather slow decrease of the amplitude of Z from the Rhinegraben to the southeast. But this slow decrease of Z can be interpreted also as the decrease of the Z amplitude to the south which is caused by the conductivity anomaly north of the river Danube.

In the Eastern Alps the amplitude of the geomagnetic variations is rather homogeneous. The vertical component Z of the variations is small in a wide period range ($\widetilde{Z}/\widetilde{H}_{\text{corr.}} \approx 0.15$). The amplitude of the horizontal components is increased somewhat in the area of the Hohe Tauern. Below the Eastern Alps no distinct anomaly of the electrical conductivity exists.

Acknowledgements

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REFERENCES

1. WINTER, R.: Der Oberrheingraben als Anomalie der elektrischen Leitfähigkeit, untersucht mit Methoden der Erdmagnetischen Tiefensondierung. Dissertation, Universität Göttingen, 1973.
2. REITMAYR, G.: Elektromagnetische Induktion im Erdinneren, studiert am Rheingraben. Dissertation, L—M-Universität München, 1974.
3. BERKTOLD, A.: Erdmagnetische Tiefensondierung in Süddeutschland und in den Alpen. Protokoll über das Kolloquium Erdmagnetische Tiefensondierung in Grafrath/Bayern, März 1974.
4. SCHMUCKER, U.: Anomalies of geomagnetic variations in the southwestern United States. Bull. Scripps Inst. Oceanography, 13, 1970.
5. HAAK, V.: Berechnung von induzierten elektrischen und magnetischen Feldern in Modellen mit zwei-dimensional verteilter Leitfähigkeit. Protokoll über das Kolloquium Erdmagnetische Tiefensondierung in Grafrath/Bayern, März 1974.

РАСПРЕДЕЛЕНИЕ ЭЛЕКТРИЧЕСКОЙ ПРОВОДИМОСТИ НА ТЕРРИТОРИИ МЕЖДУ ПРИРЕЙНСКОЙ ВПАДИНЫ, ЧЕШСКОГО МАССИВА И ЦЕНТРАЛЬНЫХ АЛЬП

А. БЕРКТОЛЬД

Резюме

В южной части ФРГ и в Австрии различными геофизическими учреждениями были проведены геомагнитные глубинные зондирования на примерно 80 станциях. На рист. 1 показаны индукционные стрелы Визедля Вариаций с периодом примерно в 20 минут. Пространственные изменения длин и направлений индукционных стрел связаны по крайней мере с темя более мощными аномалиями проводимости: направленная примерно от севера к югу Прирейнская впадина и моласский бассейн примерно востоко-западной направленности со своими осадочными мульдами, а также электрическая проводимость с более глубоким боковым гвадиентом, простирающаяся километров на 20 от Дуная к северу параллельно среднему направлению реки. В электрической проводимости под восточными Альпами не нашли определенной аномалии.

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ON THE NUMERICAL MODELLING OF TWO-DIMENSIONAL CONDUCTIVITY STRUCTURES IN CYLINDRICAL AND CARTESIAN CO-ORDINATES

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[Manuscript received April 30, 1977]

The paper discusses some problems concerning the use of the finite difference technique in the analysis of alternating electromagnetic fields in two-dimensional cylindrical co-ordinates. Symmetrical and asymmetrical conductivity structures are studied in both cylindrical and Cartesian co-ordinate systems when applying the Gauss–Seidel as well as the successive over-relaxation method. Also, electromagnetic sounding curves are presented for two models in Cartesian co-ordinates.

1. Introduction

The perturbations of alternating electromagnetic fields in two-dimensional structures were analyzed by JONES and PASCOE [1] in Cartesian co-ordinates and by TÁTRALLYAY and JONES [2, 3] in cylindrical co-ordinates. Their basic equation was

$$\nabla^2 F = i\eta^2 F \tag{1}$$

where η^2 was a function of the conductivity configuration and the frequency of the alternating field. In this, F represented the field component which was parallel to the strike of the structure: the total magnetic field in the H-polarization case and the total electric field in the E-polarization case. To solve Eq. 1 a numerical finite difference technique was used employing the *Gauss-Seidel* iterative method.

Many workers have applied similar approaches to study such problems and much attention has been paid to this method of solution. WILLIAMSON et al. [4] improved the finite difference equations used in the technique. Instead of the simple central differences used by JONES and PASCOE [1] for the derivatives, weighted averages of forward and backward differences were applied which gave more accurate results for variable grid spacings.

In a similar approach, HAAK [5] used a mesh with smaller grid dimensions. Also, in that work the field values at the external boundaries of the grid did not correspond to an unperturbed configuration, but were changed during the iteration procedure.

In order to increase the rate of convergence, the successive over-relaxation method was used instead of the *Gauss–Seidel* iterative method by HAAK [5] and by TÁTRALLYAY [6].

BREWITT-TAYLOR and WEAVER [7] showed that conductivity gradients must not be neglected in the *H*-polarization case. The following equation was used by them:

$$\nabla^2 \mathbf{B} - \text{grad } \varrho \times \text{curl } \mathbf{B} = i\,\mathbf{B} \tag{1a}$$

where $\varrho = 1/\eta^2$ and $\mathbf{B} = \mu\,\mathbf{H}$ is the magnetic induction vector which is parallel to the strike of the structure.

A discussion of some problems concerning the above papers is presented here.

2. The finite difference equations in cylindrical co-ordinates

To write Eq. 1 in finite difference form, *Taylor's* theorem is used:

$$F(x + \Delta x) = F(x) + \Delta x \frac{dF}{dx} + \frac{(\Delta x)^2}{2} \frac{d^2 F}{dx^2} + \ldots \tag{2}$$

If the change in the field is not rapid or the Δx distances are small, terms of higher order may be neglected. Figure 1 illustrates an arbitrary part of a general polar mesh. The field values at the gridpoints surrounding point "0" are:

$$F_1 = F(R, \Phi - \Delta\Phi_1) = F_0 - \Delta\Phi_1 \frac{\partial F}{\partial \Phi} + \frac{(\Delta\Phi_1)^2}{2} \frac{\partial^2 F}{\partial \Phi^2}, \tag{2a}$$

$$F_2 = F(R + \Delta r_2, \Phi) = F_0 + \Delta r_2 \frac{\partial F}{\partial r} + \frac{(\Delta r_2)^2}{2} \frac{\partial^2 F}{\partial r^2}, \tag{2b}$$

$$F_3 = F(R, \Phi + \Delta\Phi_3) = F_0 + \Delta\Phi_3 \frac{\partial F}{\partial \Phi} + \frac{(\Delta\Phi_3)^2}{2} \frac{\partial^2 F}{\partial \Phi^2}, \tag{2c}$$

$$F_4 = F(R - \Delta r_4, \Phi) = F_0 - \Delta r_4 \frac{\partial F}{\partial r} + \frac{(\Delta r_4)^2}{2} \frac{\partial^2 F}{\partial r^2}. \tag{2d}$$

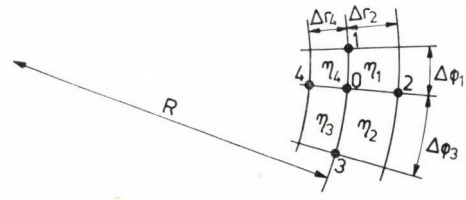

Fig. 1. An arbitrary part of the polar mesh

In the paper by TÁTRALLYAY and JONES [2] simple central differences were assumed for the first derivatives:

$$\frac{\partial F}{\partial \Phi} = \frac{F_3 - F_1}{\Delta\Phi_1 + \Delta\Phi_3}, \tag{3a}$$

$$\frac{\partial F}{\partial r} = \frac{F_2 - F_4}{\Delta r_2 + \Delta r_4}. \tag{3b}$$

The second derivatives were expressed from Eqs 2a and 2c and Eqs 2b and 2d, respectively:

$$\frac{\partial^2 F}{\partial \Phi^2} = \frac{F_3 - F_0}{(\Delta\Phi_3)^2} - \frac{F_0 - F_1}{(\Delta\Phi_1)^2} - \frac{F_3 - F_1}{\Delta\Phi_1 + \Delta\Phi_3}\left(\frac{1}{\Delta\Phi_3} - \frac{1}{\Delta\Phi_1}\right), \tag{4a}$$

$$\frac{\partial^2 F}{\partial r^2} = \frac{F_2 - F_0}{(\Delta r_2)^2} - \frac{F_0 - F_4}{(\Delta r_4)^2} - \frac{F_2 - F_4}{\Delta r_2 + \Delta r_4}\left(\frac{1}{\Delta r_2} - \frac{1}{\Delta r_4}\right). \tag{4b}$$

When using these formulae errors may occur if there are considerable differences between neighbouring grid spacings. This was discussed by WILLIAMSON et al. [4] and JONES and THOMSON [8] for rectangular meshes. The proper substitution for the first derivative is a weighted average of backward and forward differences, which in cylindrical co-ordinates gives:

$$\frac{\partial F}{\partial \Phi} = \left(\frac{F_3 - F_0}{\Delta\Phi_3/\Delta\Phi_1} + \frac{F_0 - F_1}{\Delta\Phi_1/\Delta\Phi_3}\right)\Big/(\Delta\Phi_1 + \Delta\Phi_3), \tag{5a}$$

$$\frac{\partial F}{\partial r} = \left(\frac{F_2 - F_0}{\Delta r_2/\Delta r_4} + \frac{F_0 - F_4}{\Delta r_4/\Delta r_2}\right)\Big/(\Delta r_2 + \Delta r_4), \tag{5b}$$

$$\frac{\partial^2 F}{\partial \Phi^2} = \left(\frac{F_3 - F_0}{\Delta\Phi_3} - \frac{F_0 - F_1}{\Delta\Phi_1}\right)\Big/\frac{\Delta\Phi_1 + \Delta\Phi_3}{2}, \tag{6a}$$

$$\frac{\partial^2 F}{\partial r^2} = \left(\frac{F_2 - F_0}{\Delta r_2} - \frac{F_0 - F_4}{\Delta r_4}\right)\Big/\frac{\Delta r_2 + \Delta r_4}{2}. \tag{6b}$$

In the works by TÁTRALLYAY and JONES [2, 3] the differences between any two neighbouring grid spacings were under 25 per cent in both directions. The only exception was in the vicinity of the singular centre point. The ratio of the radius of the smallest arc to that of the next arc was 3 to 8 for the different meshes. In order the check the effect of the two different substitution formulae, some of the models have been further analyzed. There is no change in the basic character of the anomalies produced by these conductivity structures. The maximum differences between the corresponding surface profiles occur near the centre point.

For model B of TÁTRALLYAY and JONES [2], which corresponds to model E_p in Fig. 4, the maximum deviation between the profiles given by the two calculations is about 1.5 per cent in the electric field values and about 3 –4 per cent in the derived apparent resistivity and magnetic field values in the E-polarization case for $T = 100$ sec. In the H-polarization case the differences are smaller.

For the sloping contact models [3] the deviations between the corresponding apparent resistivity profiles calculated using the two different substitutions for the derivatives decrease with increasing steepness of the slope. The maximum differences between the apparent resistivity profiles are under 4 per cent in the E-polarization case and about 10 per cent in the H-polarization case. In the latter case the field changes rapidly near the centre point.

3. An analysis of sloping contact models applying meshes of different maximum arc and over-relaxation

Some of the sloping contact models analyzed by TÁTRALLYAY and JONES [3] were recalculated for the E-polarization case where meshes of larger maximum arc which better satisfy the external boundary conditions were applied. The mesh must be large enough so that the field values along its external arc correspond to an unperturbed field configuration. In order to satisfy this condition, the intersection point of the sloping discontinuity and the external arc has to be at a depth which is over three skin depths.

For the vertical contact model V_p in Fig. 4, there was no significant difference between the corresponding surface profiles calculated over the different meshes. The maximum difference between the two apparent resistivity curves occurs on the higher resistivity side of the model and it is less than 4 per cent. The differences observed in the central region increase with decreasing slope. For the smallest slope the external boundary of the mesh used by TÁTRALLYAY and JONES [3] intersected the sloping discontinuity at a depth which slightly exceeded the skin depth.

Figure 2 illustrates the surface profiles of the electric field, apparent resistivity and the normalized vertical magnetic field for the 11.7° slope. The broken lines represent the surface values over the mesh of smaller maximum arc published by TÁTRALLYAY and JONES [3]. The profiles shown by the continuous lines are calculated over a mesh with maximum arc twice as large. The maximum deviations between the corresponding curves are about 10 per cent in the electric field values and 20 per cent for the apparent resistivity profiles. The local minimum at the centre point of the mesh of smaller maximum arc disappears in both profiles when applying the mesh

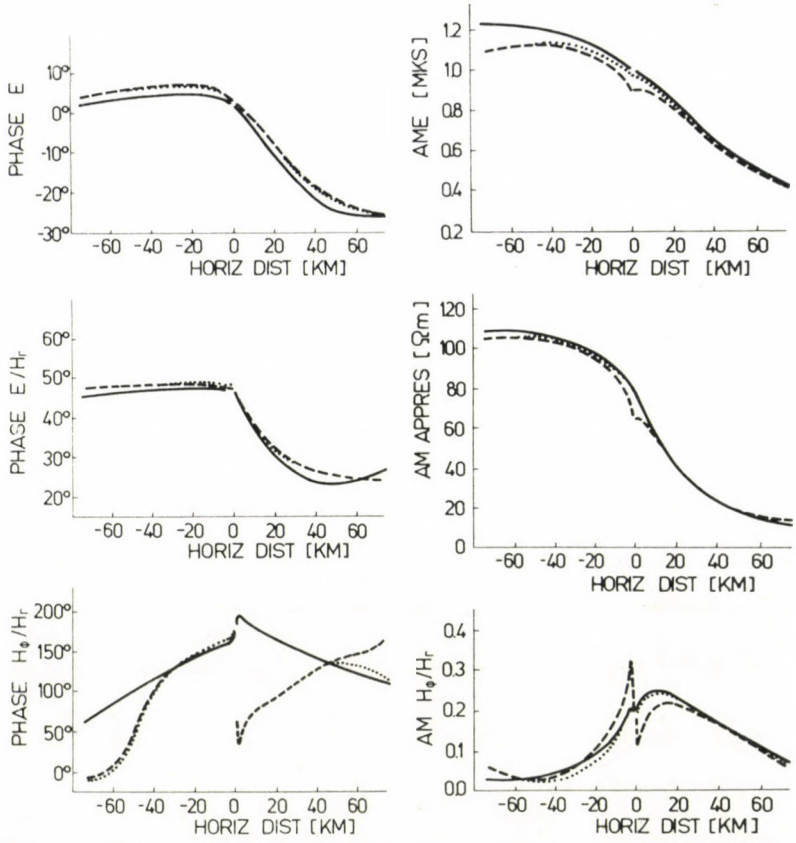

Fig. 2. Surface profiles of the electric field, apparent resistivity and the normalized vertical magnetic field for the 11.7° slope [3] in the *E*-polarization case. Dashed lines: smaller mesh, $\omega = 1.0$; dotted lines: smaller mesh, $\omega = 1.5$; continuous lines: greater mesh, $\omega = 1.5$

of larger dimension. This distortion of the electric field at the centre point produced the maximum on the high resistivity side and the minimum on the low resistivity side in the normalized vertical magnetic field curves in the previous results. This effect is not significant in the profiles calculated over the mesh of greater maximum arc. The differences between the phase profiles on the right-hand side of the contact will be discussed later.

Dotted lines in Fig. 2 represent the surface profiles calculated over the mesh of TÁTRALLYAY and JONES [3] where instead of the *Gauss–Seidel* iterative method ($\omega = 1.0$), the successive over-relaxation method with an accelerating factor $\omega = 1.5$ has been used. Near the external boundaries these profiles coincide with the previous results [3]. Near the centre point the curves approach the ones calculated over the mesh of greater maximum arc. Better satisfied external boundary conditions and the application of a relaxation factor $\omega > 1$ produce smooth electric field and apparent resistivity

profiles. Also, as a result of this the local maximum and minimum at the centre point in the vertical magnetic field profiles disappear.

The H_Φ/H_r profiles calculated over the mesh of larger maximum arc are presented for three slopes in Fig. 3. The maximum appearing above the smaller slopes shifts toward the high resistivity side when α increases. The small local maximum and minimum at the centre point are the effects of the singularity of the mesh. It is possible that they could be eliminated by continuing the iteration to greater accuracy, but this would require much computing time.

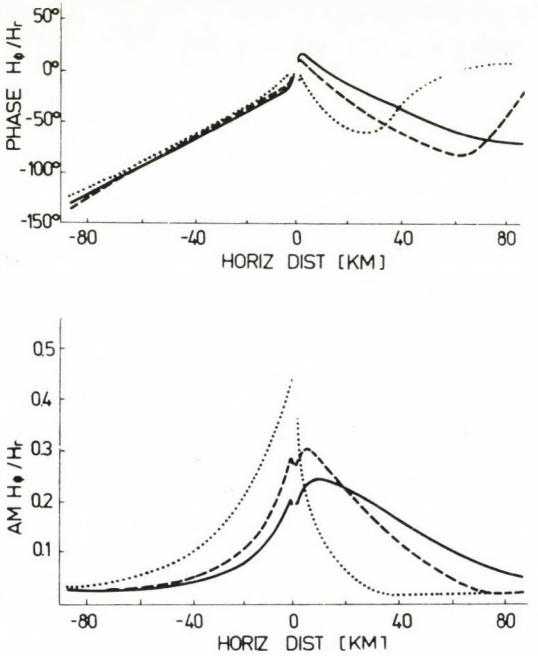

Fig. 3. The normalized vertical magnetic field profiles for three sloping contact models [3] in the *E*-polarization. Continuous lines: $\tan\alpha = 0.1$; dashed lines: $\tan\alpha = 0.2$; dotted lines: $\tan\alpha = \infty$

In the papers by TÁTRALLYAY and JONES [2, 3] the phase profiles of H_Φ and H_Φ/H_r are incorrect on the right-hand side of all models. The phase of the vertical magnetic field

$$\text{Arg}\ (H_\Phi) = -\frac{T}{2\pi}\,\text{Arg}\left(i\,\frac{\partial F}{\partial r}\right) \tag{7}$$

is calculated in e. m. u. using Eq. 5b where F represents the surface value of the electrical field. Clearly, the radial derivative changes sign when moving from the left-hand side of the model through the centre point to the right-

hand side, and at a distance R

$$\left(\frac{\partial F}{\partial r}\right)_{R\text{rhs}} = -\left(\frac{\partial F}{\partial r}\right)_{R\text{lhs}} \tag{8}$$

and so

$$\text{Arg}\,(H_\Phi)_{R\text{rhs}} = \text{Arg}\,(H_\Phi)_{R\text{lhs}} + 180^\circ\,. \tag{9}$$

In the paper by TÁTRALLYAY and JONES [2, 3]

$$\text{Arg}\,(H_\Phi)_{R\text{rhs}} = 360^\circ - \text{Arg}\,(H_\Phi)_{R\text{lhs}} \tag{10}$$

was presented for all models as illustrated by the broken line in the H_Φ/H_r phase profiles in Fig. 2.

4. Comparison of calculations in different co-ordinate systems

For the conductivity structures illustrated in Fig. 4 the associated electromagnetic fields were determined using both Cartesian and cylindrical co-ordinate systems. The differences between the apparent resistivity profiles calculated in cylindrical co-ordinates and those calculated in Cartesian co-ordinates are under 4 per cent for the vertical contact and the 45° slope models in the E-polarization. In the H-polarization case the field values change rapidly in the vicinity of the centre point and the construction of the mesh may influence the results. Though, the character of the changes in the surface profiles is the same in the different co-ordinates systems, the maximum difference is about 25 per cent in the apparent resistivity values.

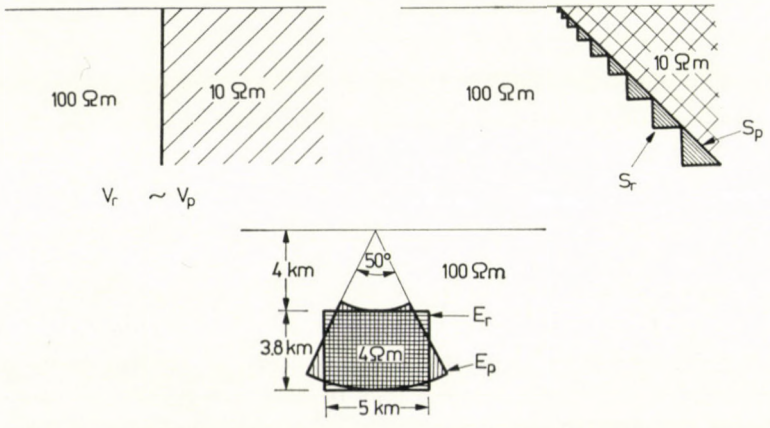

Fig. 4. Two-dimensional conductivity structures studied applying both rectangular and polar meshes

Figure 5 presents the apparent resistivity profiles over an embedded anomaly of higher conductivity for the *H*-polarization case. The period of the alternating field is 40 sec. For model E_p the irregularity of the polar mesh causes the asymmetry of the curves illustrated by continuous lines. The profiles marked by asterisks are the results of an iteration using the *Gauss—Seidel* method. The other pair of continuous curves was calculated

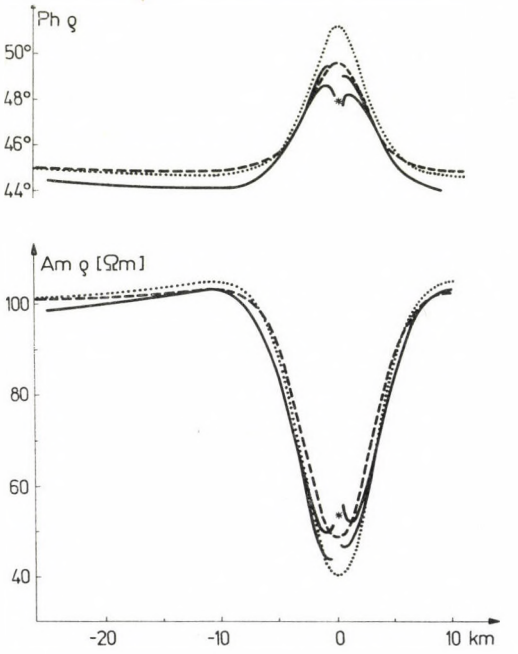

Fig. 5. Apparent resistivity profiles for the embedded anomaly models E_p (continuous lines — with asterisk: $\omega = 1.0$, without asterisk: $\omega = 1.3$) and E_r ($\omega = 1.4$ — dotted line: from Eq. 1, dashed line: from Eq. 1a) in the H-polarization case

applying the successive over-relaxation method. In this latter case there are greater corrections to the field values at the beginning of the iteration than for the *Gauss—Seidel* method. These weighted corrections smooth the irregularly distorted field configuration to a certain extent. The relative error of the apparent resistivity values produced by the numerical method is over 10 per cent in the vicinity of the centre point. The asymmetry of the profiles and the deviations between the curves determined using different relaxation factors are within this limit of error. The asymmetry is negligible in the surface profiles calculated for model E_r in Cartesian co-ordinates. The profiles illustrated by dashed lines in Fig. 5 were determined applying Eq. 1a and the dotted curves were calculated using Eq. 1. The maximum difference between the two amplitude profiles is about 20 per cent above

the middle of the embedding. Sudden changes in the field values are smoothed when the conductivity gradients are not neglected in the basic equations. (A similar comparison was effected for model V_r. In this case the resistivity curve determined by Eq. 1 deviates from the profile determined by Eq. 1a less than 2 per cent.)

5. Surface configuration of alternating electromagnetic fields for different frequencies

The vertical contact model and the embedded anomaly model (V_r and E_r in Fig. 4) were studied for different periods of the alternating field. Magnetotelluric sounding curves and geomagnetic sounding curves (the normalized vertical magnetic field in the E-polarization case) were determined.

The sounding curves can be calculated from the field configuration for a single period of the alternating field for the vertical contact model. In this case there is only one discontinuity which is parellel to the direction of the normal exponential decrease of the field in a homogeneous half-space. The field configuration is a function only of the skin depth:

$$d_s = \frac{1}{\eta} = K \sqrt{\frac{T}{\sigma}}, \tag{11}$$

where T is the period of the alternating field, σ is the conductivity, and K is a constant. For the surface field values $F(d)$ where d is the distance from the contact at the surface

$$F_1 = F_2 \quad \text{if} \quad d_1/\sqrt{T_1} = d_2/\sqrt{T_2}. \tag{12}$$

In the E-polarization case the longitudinal apparent resistivity curves are influenced by inductive effects caused by excessive currents flowing parallel to the strike of the structure. In the H-polarization case the currents flow from one layer to the other. Galvanic effects caused by surface charges on the conductivity boundaries are more significant. Their effect which is a distortion of the transverse apparent resistivity curves does not disappear with increasing period.

Figure 6 presents the magnetotelluric sounding curves calculated over model V_r at different distances from the point where the discontinuity intersects the surface. In the E-polarization ($\varrho^{\parallel}$) there is a gradual transition between the two extreme resistivity values 10 Ω m and 100 Ω m. In the H-polarization case ($\varrho^{\perp}$) the curves on the high resistivity side of the model are over 100 Ω m while the curves on the low resistivity side are under 10 Ω m. Geomagnetic sounding curves are presented in Fig. 7 only for the left-hand

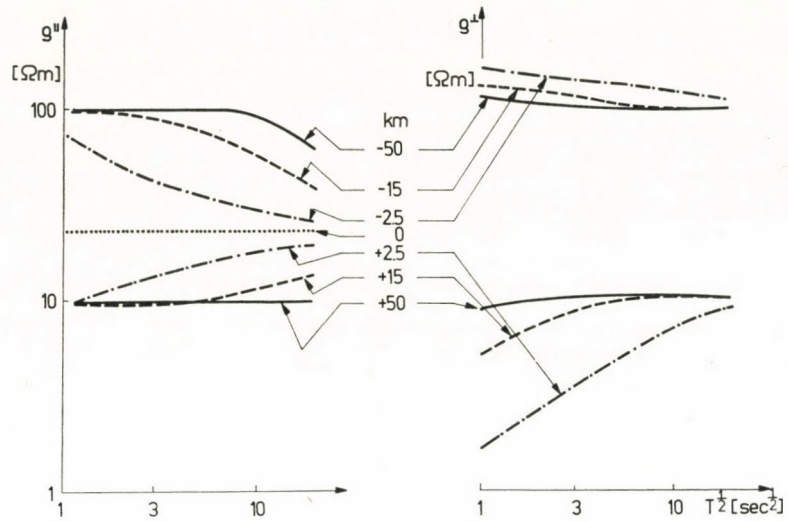

Fig. 6. Magnetotelluric sounding curves for the vertical contact model V_r at different distances from the surface contact of the discontinuity

side of the model. The normalized vertical magnetic field curve calculated on the high resistivity side at a distance $-d$ corresponds to the curve at a distance $d/\sqrt{10}$ on the low resistivity side. The maximum values occur above the intersection point.

Figure 8 presents the MTS curves calculated for model E_r at different distances from the symmetry plane of the structure. The difference between the longitudinal and transverse apparent resistivity curves is shown. The exist-

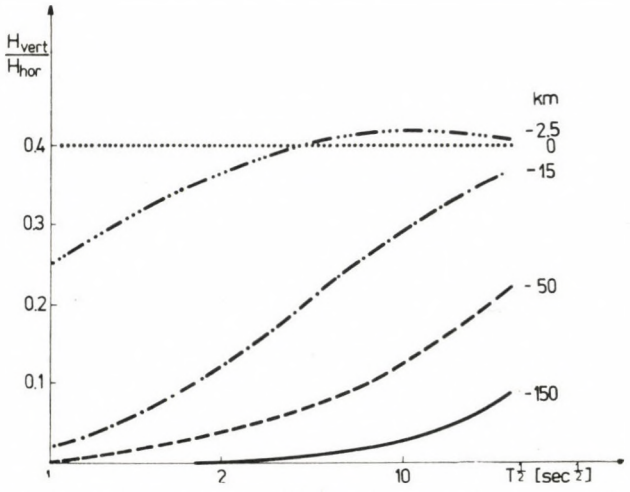

Fig. 7. Geomagnetic sounding curves at different distances from the surface contact of the discontinuity over the high resistivity side of model V_r

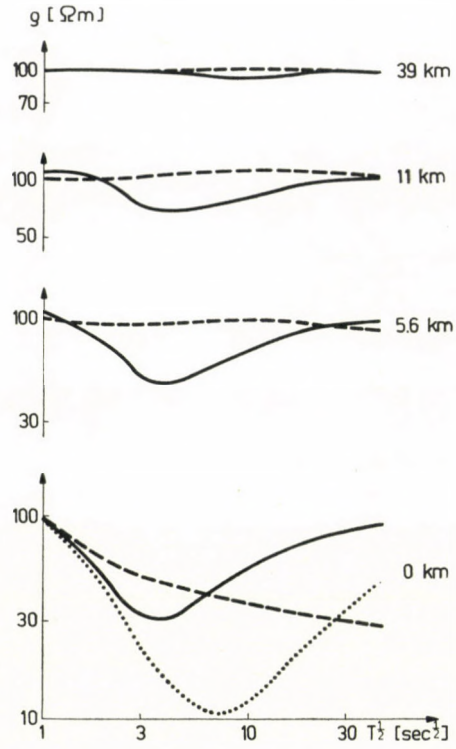

Fig. 8. Magnetotelluric sounding curves at different distances from the symmetry plane of model E_r. Continuous lines: $\varrho^{\parallel}$; dashed lines: $\varrho\perp$; dotted line: theoretical curve for a horizontally layered half-space

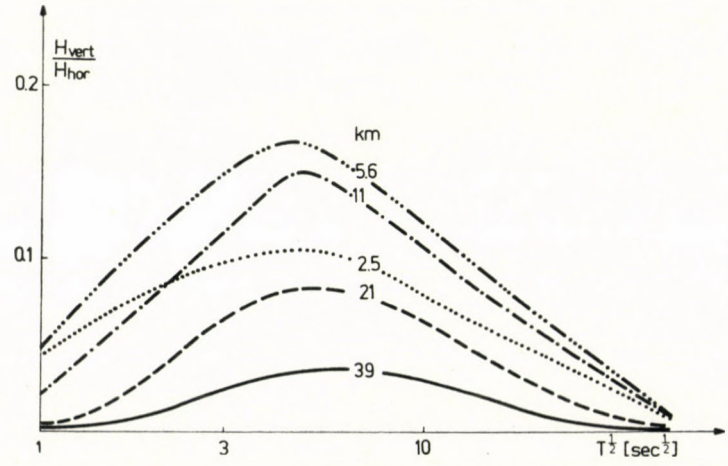

Fig. 9. Geomagnetic sounding curves at different distances from the symmetry plane of model E_r

ence of the embedded anomaly is not apparent in the curves of the H-polarization case. In the E-polarization case the effect of the anomaly is significant also at those surface points which are not directly over the inhomogeneity. Since the width of the anomaly is not large, the MTS curve at the surface point in the symmetry plane of the model does not coincide with the theoretical curve calculated for horizontally layered strata of three layers: $\varrho_1 = 100\ \Omega$ m, $h_1 = 4$ km; $\varrho_2 = 4\ \Omega$ m, $h_2 = 3.8$ km; $\varrho_3 = 100\ \Omega$ m, $h_3 = \infty$ (see dotted line). At a distance of about 50 km the effect of the anomaly disappears. The geomagnetic sounding curves are presented in Fig. 9. The maximum change in the normalized vertical field values occurs outside the inhomogeneity at a distance of 5.6 km.

6. Conclusions

From the calculations presented in this work it is found that for the analyzed cylindrical models only small differences occur in the results when weighted derivatives are used rather than central differences in the finite difference scheme. The greatest differences occur near the centre point of the co-ordinate system for the H-polarization case where the fields are rapidly changing.

The application of the successive over-relaxation method together with the choice of more suitable external boundaries has improved the solution for these models. The field profiles are smoother than the previous results [2, 3] and more rapid convergence is achieved.

Sounding curves have been presented which show the variation of the apparent resistivity and that of the normalized vertical magnetic field with distance from the surface contact for a vertical discontinuity model.

From the calculation of the magnetotelluric and geomagnetic sounding curves for an embedded two-dimensional anomaly it is seen that the detection of a localized anomaly by these methods depends upon the proximity of the measuring location to the anomaly as well as on the orientation of the sensors relative to the strike of the anomaly.

REFERENCES

1. JONES, F. W.—PASCOE, L. J.: A general computer program to determine the perturbation of alternating electric currents in a two-dimensional model. *Geophys. J. R. astr. Soc.*, 23 (1971), 3—30.
2. TÁTRALLYAY, M.—JONES, F. W.: The perturbation of geomagnetic fields by cylindrical structures I. *Geophys. J. R. astr. Soc.*, 38 (1974), 449—462.
3. TÁTRALLYAY, M.—JONES, F. W.: The perturbation of geomagnetic fields by cylindrical structures II. *Geophys. J. R. astr. Soc.*, 38 (1974), 463—477.
4. WILLIAMSON, K.—HEWLETT, G.—TAMMEMAGI, H. Y.: Computer modelling of electrical conductivity structures. *Geophys. J. R. astr. Soc.*, 37 (1974), 533—536.

5. Haak, V.: Berechnung von Induzierten Elektrischen und Magnetischen Feldern in Modellen mit Zwei-dimensional Verteilter Leitfähigkeit. Prot. Koll. Erdmagn. Tiefens. Grafrath, 1974, 400—428.
6. Tátrallyay, M.: Perturbations of an alternating EM field by conductivity inhomogeneities in the Earth. Proceedings of the Symposium "Electromagnetic Field of the Earth", Hurbanovo—Smolenice, 1975 (in press).
7. Brewitt-Taylor, C. R.—Weaver, J. T.: On the finite difference solution of two-dimensional induction problems. *Geophys. J. R. astr. Soc.*, 47 (1976), 375—396.
8. Jones, F. W.—Thomson, D. J.: A discussion of the finite difference method in computer modelling of electrical conductivity structures. *Geophys. J. R. astr. Soc.*, 37 (1974), 537—544.

ЧИСЛОВОЕ МОДЕЛИРОВАНИЕ ДВУМЕРНЫХ СТРУКТУР ПРОВОДИМОСТИ В ЦИЛИНДРИЧЕСКОЙ И ДЕКАРТОВОЙ СИСТЕМАХ КООРДИНАТ

М. ТАТРАЛЯИ

РЕЗЮМЕ

В статъе рассматриваются некоторые проблемы метода конечных разностей в двумерной цилиндрической системе координат. Метод применен для определения вариирующих электромагнитных полей. Исследуются симметричные и несимметричные структуры проводимости как в цилиндрической, так и в прямоугольной системах координат в случае применения методов Гаусса—Зейделя, а также сукцессивной сверхрелаксации. Показаны кривые электромагнитного зондирования для двух моделей, заданных в декартовых координатах.

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PROPOSALS FOR THE STANDARDIZATION OF MAGNETOTELLURIC DIGITAL TAPES*

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[Manuscript received May 2, 1977]

After providing a short review of the technical characteristics of the main types of magnetotelluric field and laboratory equipment, we propose a standardizable method for the acquisition of experimental and additional data. For this purpose a header block was made, as an example, in which the additional data, indispensable for the processing of the experimental data, appears in digital form.

We foresee the possibility of facilitating in this way the exchange of magnetotelluric information, even between research teams using different magnetotelluric equipment.

Introduction

The introduction of digitization methods into magnetotellurics for experimental data, operating both in the laboratory (play-back) and on the field, has meant that methods of analysis by electronic filtering have practically disappeared. This analysis involved electronic filtering and subsequent graphic calculations, whereas digitization in this field now exploits the enormous operational capacity of computers.

Magnetotellurics has reached a significant point in its progress by the use of automatic digital data processing: in fact, as well as the basic parameters, such as the electrical conductivity and depth which are already calculated by employing the magnetotelluric method in analog form, we can now instantly know the phase (which suggests new theoretical and applicative methods) and the ensemble of parameters and physical quantities such as the spectral density of the various components of the magnetotelluric field, coherence, degree of polarization, impedance tensor, strike, skew, etc.

Obtaining digitized data involves many new problems, which are sometimes typically technical or which depend on the requirements of the experimental trials. Similar problems have also been taken into consideration in the field of geophysics, mostly in working out seismic data [1, 2, 3, 4]. However, the peculiar physical features of the natural electromagnetic field required a critical revision of standard methodologies, so that they could be better adapted to the special requirements of magnetotellurics.

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As well as the experimental data of the magnetotelluric field, a series of identification and instrumental data exists, indispensable for processing the magnetotelluric phenomena on the computer. It seemed to us convenient that these additional data should be stored on the same tape as that already holding the magnetotelluric field data.

We therefore propose an ordered and standardizable list for the acquisition of additional data, with the aim of aiding exchange between teams using different equipment.

The two types of data are put into two separate block sequences, called respectively header blocks and data blocks.

Before showing how the two types of blocks are built, it is important to remember the general characteristics of computer-compatible magnetic tapes (e.g. the IBM, 0.5 inch, 2400 ft. tapes). In particular:

a) density of 800 bpi for binary or EBCDIC coded data, 1600 bpi for PE (phase-encoded) data, for 9-track tape write-up. Conversion to and from 7-track tapes can easily be carried out using the system routines common to all data processing centers;

b) all data are arranged on two or more bytes (4, 8 or 10, including the identification number of the data – see below) of 6 or 8 bits. Conversion to and from 6 or 8 bit bytes is also easy to carry out, using routine computer programs;

c) inter-block gap (IBG) of $^3/_4$ inches of erased tape or blank EBCDIC between one block and the next;

d) an end-of-file (EOF) consisting of a tape mark (TM) preceded by an IBG to the end of the recording of all data of a particular station. If registrations from different stations are arranged on the same tape, each one is separated from the next by an EOF; an end-of-information (EOI) of two or more EOF then ends the last registration.

Better standardization and exchange of tapes also require conventions on all measurement units used. Therefore, for all data we propose that distances be measured in meters, times in seconds, (except for the sampling times of the high-frequency bands in msec, frequencies in cycles/sec, and variations of frequency responses in dB/octaves).

Requirements

The data which accompany a magnetotelluric sounding, and for which it would be convenient to have a recording on the compatible magnetic tape in one or more header blocks, are as follows:

1. *place and date of the magnetotelluric sounding* (it is to be remembered that the intensity of the magnetotelluric field is closely linked to the variations

of the terrestrial magnetic field: for example, it is necessary to know whether the date of field recording coincides with a magnetic storm);

2. *geographic directions of measurement.* Crossed soundings are by now general when wishing to find present structural directions, in view of the determination of the principal directions of the impedance tensor;

3. *performance of the magnetic sensors*, pass-band and gain of the magnetic lines;

4. *length of the telluric lines* ;

5. *number and width of the frequency bands* into which the magnetotelluric sounding is subdivided. The use of different frequency bands becomes necessary because of the different responses of the magnetic sensors at different frequencies;

6. *transfer functions* of any type of equipment. To these characteristics, we must also add those relative to digitization; and, in particular;

1. *sampling time for each way* (in seconds or msec, according to whether frequencies are low or high);

2. *delay time* in the subsequent sampling of the ways (if this exists: see below);

3. *value of the least significant bit* (LSB);

4. *length of each physical data block*, i.e. number of data per way in each physical block;

5. *number of bits on which a single datum is placed.*

It must be remembered that similar problems in the acquisition of field data have even been encountered in the field of seismics.

In such a case, the data processing center, requests further information accompanying a survey which, for greater convenience, are to be placed on the same tape which contains the data in a separated block (normally on top). Of course, the quantity and kind of information to be stored on tape in seismics can be of any other type (e.g. number and position of geophones, etc.). A solution of the kind suggested here has already been proposed in the above-mentioned work.

Proposals

Presuming that bytes of 8 bits are used, and that the number of bits indicated is sufficient both for binary and EBCDIC coded data, the structure of the header block (see Fig. 1) containing all mentioned data will be as follows:

1. identification number of the tape (2 bytes);

2. place of the magnetotelluric sounding, and, if necessary, the geographic co-ordinates (10 bytes);

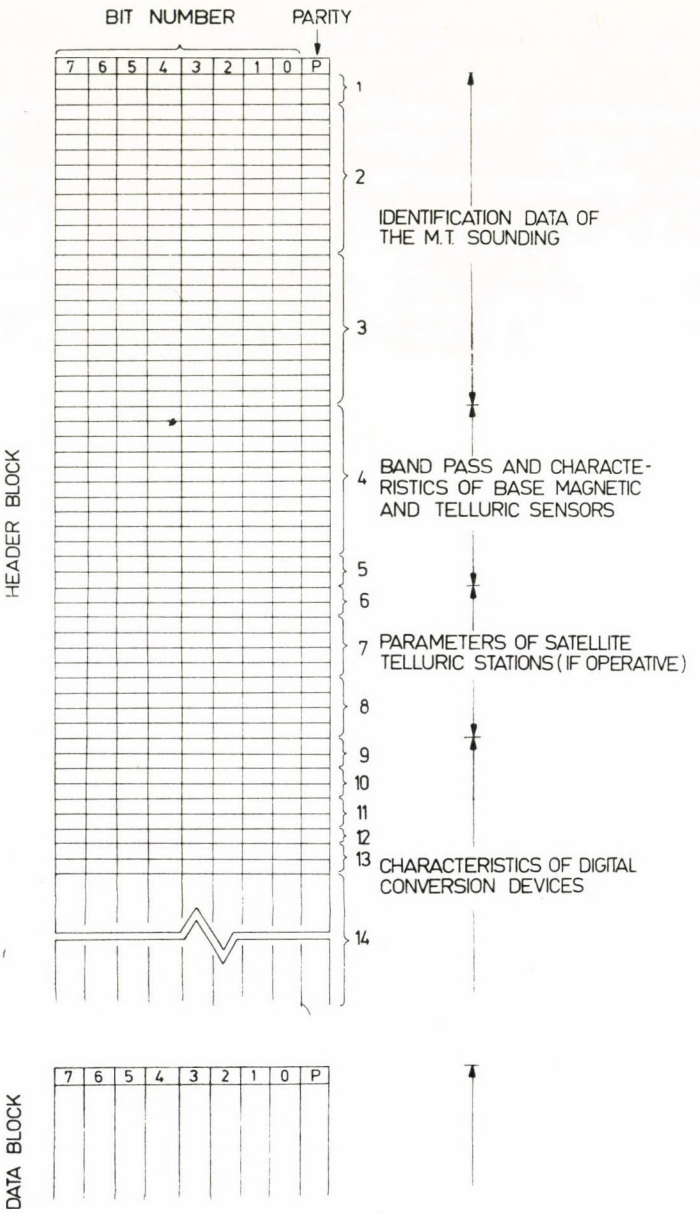

Fig. 1. Model of a header block, followed by a data block

3. sounding date: year, month, day, hour (and also minutes and seconds if conversion takes place directly in the field and if the converter has a clock) (10 bytes);

4. pass-band width and preamplification of the magnetic sensors (10 bytes);

5. angle of the x-axis of measurement with the meridian passing by the station (2 bytes);

6. length of the telluric line in meters (2 bytes per line);

7. number, geographic disposition and distance in kms of each satellite telluric line with respect to the basic magnetotelluric station (4 bytes per station);

8. angle and gain for each satellite telluric station (4 bytes per station);

9. number of data per way for each physical block (2 bytes);

10. sampling time (in seconds or msec according to bands of low or high frequency). It should be remembered that the least recorded period must be at least double the sampling time (2 bytes);

11. delay time in msec, if the reading of each way is sequential (see below) (2 bytes);

12. number of bits on which each single datum is set out (1 or 2 bytes);

13. value in millivolts and position of the LSB (usually 2 bytes);

14. a number of bytes sufficient for the transfer functions of the filtering devices in the magnetotelluric system.

Points 7, 8 and 11 are closely linked to the technique used, and are therefore optional [5].

Observations on data block recording

As far as the transposition and data block structure is concerned, it should be remembered that there exist two ways of reading the data which reaches the digital converter either from the multiple-track analog tape or directly from the measurement devices themselves.

The first method consists in the simultaneous reading, at the sampling frequency, of all ways in the input. The second consists in the subsequent reading of each way with a delay time between one reading and the next which is generally about 1 msec (see Fig. 2).

In both cases a certain number of sampling cycles is run through, and the data read are transferred to a system of two temporary memories. When one of the two is filled, the data ensemble is transferred onto the tape to make a physical block, while the other memory carries on with the transfer of the next block without a break.

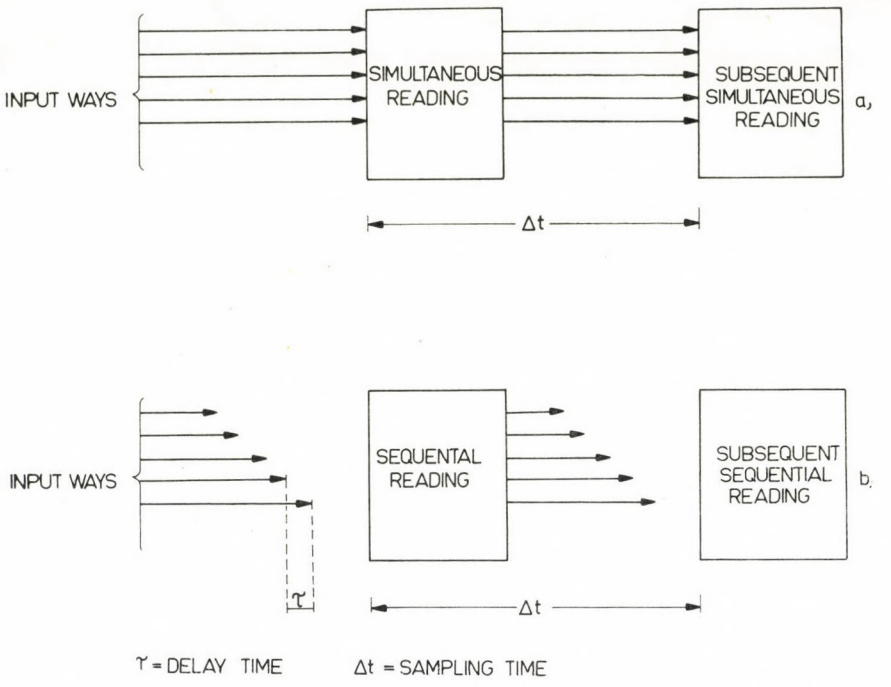

Fig. 2. Pattern of simultaneous (A) and sequential (B) digital reading of input magnetotelluric ways

The digital converters used in magnetotellurics are already able to record, at the beginning of each data block, a few bytes as a block label, generally containing the number of data per channel, the progressive numeration of each block in the interior of a data set of the same frequency band, and if there is a clock, the date, hour, minute and second of the block recording.

Our experience in digital data rccording is based on the instrumentation operating at the "Istituto di Fisica Terrestre, Geodesia e Geografia Fisica" of the University of Padua and at the "Centre des Recherches Géophysiques" at Garchy (Nièvre, France). There is no substantial difference between these two systems.

In both cases, the data is recorded on 9-track compatible tapes with odd vertical parity; each datum is binary integer and composed of 2 bytes of 8 bits, with the same value for the LSB (5 mV). The first 4 bits are for the identification of the magnetotelluric way.

The differences consist in the different length of the physical blocks and, due to the fact that while the instrumentation of the C. R. G. at Garchy keeps a specific bit for the sign, the two's complement is used in the instrumentation at Padua.

We describe below the structure of a datum as supplied by the Garchy instrumentation, as an example:

a) the first 4 bits on the left give the magnetotelluric method (e.g.: $1-E_x$, $2-H_y$, $3-E_y$, $4-H_x$, $5-H_z$);

b) the fifth bit (MSB) is the sign bit: 0 if the datum is positive, 1 if negative;

c) the remaining 11 bits contain the value of the datum; the 16th bit is the LSB (value: 5 mV). The maximum value of a magnetotelluric field datum is therefore about 10 volts.

Conclusions

The rapid evolution of the magnetotelluric method has meant that the additional data must be put in a proper field, and this is indispensable for the subsequent steps of processing the magnetotelluric data.

Moreover, the quantity of the additional data [5] has greatly increased with the introduction of experiments carried out together with the traditional survey at single station points, e.g. the recorded phenomena complete together in different stations, or just the telluric components in satellite stations.

We have therefore tried to propose a header block model allowing for the orderly grouping of data, in view of subsequent operations on the computer. Lastly, we foresee that, if magnetotelluric experiments carried out by various research and operative teams with different equipment are made so that the data acquired is recorded in a standard way, all interchange of information is much easier.

Acknowledgements

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REFERENCES

1. NORTHWOOD, E. J.—WEISINGER, R. C. — BRADLEY, J. J.: Recommended standards for digital tape formats. *Geophysics*, 32 (1967), 1073—1084.
2. BARRY, K. M.—CAVERS, D. A.—KNEALE, C. W.: Recommended standards for digital tape formats. *Geophysics*, 40 (1975), 344—352.
3. ALLSOPP, D. F.—BURKE, M. D.—RANKIN, D.—REDDY, I. K.: A wide-band digital magnetotelluric recording system. *Geoph. Prosp.*, 22 (1974), 272—278.
4. ILICETO, V.: Quelques considérations sur la digitalisation des signaux magnétotelluriques: description d'un équipement. *Geoph. Prosp.* 25 (1977), 291—301
5. HERMANCE, J. F.—THAYER, R. E.: The telluric-magnetotelluric method. *Geophysics*, 40 (1975), 664—668.

ПРЕДЛОЖЕНИЯ ДЛЯ СТАНДАРТИЗАЦИИ МАГНИТОТЕЛЛУРИЧЕСКИХ ЦИФРОВЫХ МАГНИТНЫХ ЛЕНТ

В. ИЛИЧЕТО – Г. САНТАРАТО

РЕЗЮМЕ

После краткого изложжения технических особенностей главнейших типов магнитотеллурических установок на местности и в лабораторных условиях, авторы предлагают метод для стандартизации бора экспериментальных и вспомогательных данных. С этой целью, в качестве примера, был составлен блок для подготовления заставки, в которой побочные данные, неизбежные для обработки экспериментальных данных, появляются в цифровом виде.

Авторы думают, что этим путем можно способствовать обмену магнитотеллурической информацией и среди групп, использующих отличные друг от друга типы магнитотеллурических установок.

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ADJUSTMENT OF LEVELLINGS FOR THE DETERMINATION OF THE VERTICAL MOVEMENTS OF THE EARTH'S SURFACE

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[Manuscript received May 11, 1977]

Up to now the vertical changes of the surface in unit time intervals (the velocities) had been determined by comparing the results of two levellings following each other with sufficient delay (at least 15 – 20 years). If the results of more levellings of acceptable accuracy will be at disposal, it will be in addition possible to determine the velocities of the changes, i. e. the accelerations, too. The paper gives a solution for the adjustment of levelling networks for the determination of surface movements, whereby the accelerations have also been taken into account. It shows the advantages of the proposed method in comparison with that neglecting the accelerations.

The study of the vertical movements of the Earth's surface is an up-to-date global problem. The Hungarian Geodetic Survey developed a levelling network for this purpose having in mind the plan to repeat the measurements e.g. in each 15 – 20 years, and to determine in this way the values of the movements for time units, i.e. the velocities with the necessary accuracy. Such a repetition of the measurements will also enable to take the changes of the velocities, i.e. the accelerations into account. However, at least three levellings of the highest possible precision carried out in adequate time intervals are necessary. At present neither generally nor in Hungary are the results of three such levellings at disposal, therefore the computations have been so far limited to the comparison of two levellings. In Hungary a computation of the accelerations will be possible in about 30 – 40 years. In spite of this it seems advantageous to develop a theory and method for their determination in order to show that the velocities computed now can be strongly biased.

In order to present the method developed for the adjustment of levelling networks aiming the study of crustal movements by taking into account the accelerations, and to show its advantages, it is necessary to review the methods used or proposed hitherto for the determination of the velocities, at least those used in Hungary or proposed by Hungarian authors.

The most simple method (let us call it the first stage) is an immediate comparison of the heights of the individual levelling stations received from levellings made with a certain time delay. Practical computations with this

method were made in Hungary at the first time around 1930 by J. GÁRDONYI. He compared heights determined by the earlier Military Geographical Institute in Vienna between 1873 and 1898, and by the Hungarian Hydrological Survey between 1889 and 1895, with those determined by the Geodetic Survey in 1921 a national levelling network. This comparison enabled, however, no reliable results, since the levellings made by the Geographical Institute were considerably biased by systematic errors due to different reasons as it has been shown by L. BENDEFY somewhat later.

The survey aspects have been treated intensively by BENDEFY. It should be emphasized that his work contributed considerably to the geodetic study of the vertical movements. He proposed in Hungary and propagated also abroad the following, second stage of the comparisons.

The levelling network consists of connected, closed polygonials. The stations of the connections are called junctions, the parts of the polygonials between two junctions are called levelling lines. Figure 1 shows a part of a levelling network: the arrows on the lines show the direction of the slope.

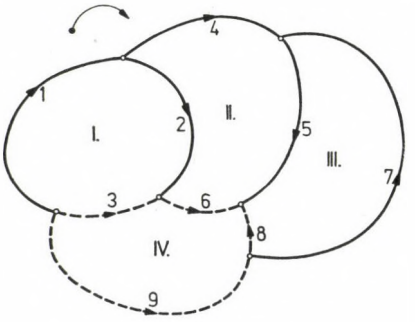

Fig. 1

At first let us neglect the difference between full and dotted lines in this figure. In an adjusted network the sum of the height differences must be zero in all closed polygonials — by taking the direction of ascent into account. Supposing a certain sense (clockwise or anti-clockwise), we compute the height differences positive, if the direction of the arrow is the same, and negative, if the direction of the arrow is opposite.

BENDEFY's proposal was to divide the difference of the unadjusted measurement results by the number of years between the two levellings (T), thus to get for each line a preliminary yearly velocity (Δ). Now these velocities should be taken as fictitious measurement results and adjusted with the condition that their sum should be in all closed polygonials zero — taking again into account the slope directions.

In my opinion this is a necessary but not a sufficient condition. The velocities determined so are, namely, separated from the final values of the levelling network, since in case of a separate adjustment of the results of the two levellings, the velocities received do not coincide with those determined by BENDEFY's method.

About one decade ago I proposed a third stage, remaining in the framework of the then actual problem, i.e. to determine the velocities from two. (and only two) levellings. I used the following theoretical conditions:

1. Let us suppose that the change in unit time, i.e. the velocity remains constant for one and the same levelling line within the time of investigation, i.e. the change is linearly proportional to the time.

2. It is requested that the velocities remain in accordance with the adjusted results of the two levellings. Therefore the two levellings should be adjusted together.

3. The adjustment should be unambiguous, i.e. the adjusted values of the measurements should be independent from the epoch for which the adjustment has been carried out.

4. The changes in unit time (the velocities) should be values, by which the adjusted measurement results could be so reduced to any epoch that the conditions for the adjustedness of the levelling networks should remain fulfilled.

Before the mathematical description of these conditions I made a thorough search in the literature. I found two Finnish methods – one with iteration and a direct one – which correspond essentially to the conditions mentioned.

For the common adjustment of the two levellings and for the determination of the velocities I proposed an easily surveyable and simply solvable form. That means that the results of the first levelling, and similarly those of the second should be reduced to certain epochs (being different from each other) by means of preliminary values determined from the measurements. The mathematical form following from the conditions and the reduction is the following. The change in unit time, i.e. the velocity is:

$$\Delta_i = \frac{L_i'' + v_i'' - (L_i' + v_i')}{T_i}, \tag{1}$$

where L is the height difference between the two endpoints of the line (taking into account the correction for the non-parallelity of the niveau surfaces, e.g. the orthometric correction, too), v is the correction from the adjustment, T the time interval between the two adjustments. (Values with $'$ refer to the first, with $''$ to the second levelling.)

If τ is the time which elapsed between the measurement and the reference epoch, the adjusted height differences referred to the epoch are:

$$H_i' = L_i' + v_i' + \frac{\tau_i'}{T_i}(L_i' + v_i'' - L_i' - v_i')$$
$$H_i'' = L_i'' + v_i'' + \frac{\tau_i''}{T_i}(L_i'' + v_i'' - L_i' - v_i') \tag{2}$$

as well as

$$H_i' = \left(1 - \frac{\tau_i'}{T_i}\right) v_i' + \frac{\tau_i'}{T_i} v_i'' + h_i'$$

and (3)

$$H_i'' = -\frac{\tau_i''}{T_i} v_i' + \left(1 + \frac{\tau_i''}{T_i}\right) v_i'' + h_i'' ,$$

where the free terms h (the measurement results reduced to the epoch) are:

$$h_i' = \left(1 - \frac{\tau_i'}{T_i}\right) L_i' + \frac{\tau_i'}{T_i} L_i''$$

and (4)

$$h_i'' = -\frac{\tau_i''}{T_i} L_i' + \left(1 + \frac{\tau_i''}{T_i}\right) L_i'' .$$

The condition for the adjustedness is now, that the sum of the height differences reduced to the epoch should be zero in each polygonial (taking into account the directions of the slopes):

$$[H'] = 0 \quad \text{and} \quad [H''] = 0 . \tag{5}$$

If the preliminary values of the velocities:

$$(\Delta_i) = \frac{L_i'' - L_i'}{T_i} \tag{6}$$

are computed, the terms h can be more easily obtained from the formulae:

$$h_i' = L_i' + (\Delta_i)\,\tau_i' \qquad h_i'' = L_i'' + (\Delta_i)\,\tau_i'' . \tag{7}$$

As the corrections are added to the original measurement results (and not to the values reduced to the epoch), any weights can be used in the adjustment similarly to the usual adjustment of levellings. It should be repeated that the corrections due to the deviations from the parallelity of the niveau surfaces are included into the measurement results.

It is also possible to connect the network to points whose heights are supposed to remain unchanged or whose absolute changes can be determined (e.g. sea level). If one has n such points, then $(n-1)$ so-called constraint equations must be added to the conditional equations of the closed polygonials. These do not differ anyway from the corresponding equations used in the simple adjustment of levelling networks. The common adjustment of the two networks becomes necessary as the two levellings are distributed in a longer space of time, and the measurement results can be reduced to the epoch only with the velocities determined from the results of both levellings.

I was glad to read that V. HRISTOW from Bulgaria dealt with the same problem independently of me and published recently a method in a somewhat different mathematical form which is based on the same principles and which yields the same numerical results. This strengthened my conviction that the way followed in the last decades was correct.

*

If several levellings of the same network are considered, the method outlined above cannot be considered as adequate. If namely the velocities Δ are supposed to be constant during the time intervals between two levellings, then at the boundary of consecutive, neighborous time intervals (e.g. at the fourth levelling from the point of view of the interval between the third and fourth, and the fourth and fifth levellings) there will be stepwise changes in the velocities. If there exists a change in the velocity this is continuous and not stepwise, except extraordinary events such as earthquakes or human interventions. In our newest investigations the time factor has also been taken into account by supposing not only constant velocity, but also acceleration. This would be the fourth step. It should be remarked that in the meantime HRISTOW has also dealt with the problem of the acceleration, but he treats the problem in a principally different manner. This will be discussed later.

The fourth stage presented in the following supposes the knowledge of the accelerations. Their determination will also be treated.

If the acceleration is α, and the index zero refers to an arbitrary time moment, and the time t is measured from this moment, the velocity in the moment t is:

$$\Delta_i = \Delta_{0i} + t\alpha_i \tag{8}$$

and the height difference along the line is:

$$m_{ti} - m_{0i} = t\Delta_{0i} + \frac{t^2}{2}\alpha_i \,. \tag{9}$$

At the horizontal (time-) axis of Fig. 2, M' and M'' mean the moments of the measurements, E' and E'' the epochs of the adjustment. K is the bisecting point of the time interval between the two measurements; F' and F'' are the bisecting points between the measurements and the corresponding epochs. As the acceleration is supposed to be constant (its change is not determinable either at the present accuracy of the near future), the preliminary value of the velocity corresponding to K is:

$$(\Delta_k) = \frac{L'' - L'}{T} \tag{10}$$

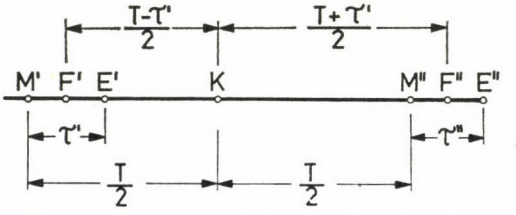

Fig. 2.

and the preliminary values for the moments of the measurements are:

$$(\Delta') = (\Delta_K) - \frac{T}{2}\alpha \quad \text{and} \quad (\Delta'') = (\Delta_K) + \frac{T}{2}\alpha\,. \tag{11}$$

The preliminary values for the moments F are:

$$(\Delta'_F) = (\Delta_K) - \frac{T - \tau'}{2}\alpha$$

and (12)

$$(\Delta''_F) = (\Delta_K) + \frac{T + \tau''}{2}\alpha\,.$$

It should be remarked that the moment of the measurements and the position of the epochs can be in another relation than that shown in the figure. In such cases the sign of the values τ is to be chosen according to the actual situation.

The measurement results reduced to the epoch are:

$$h'_i = L'_i + \tau'_i\,(\Delta')_i + \frac{\tau'^2_i}{2}\alpha_i$$

and (13)

$$h''_i = L''_i + \tau''_i\,(\Delta'')_i + \frac{\tau''^2_i}{2}\alpha_i$$

or further

$$h_i' = L_i' + (\Delta_F')_i \tau_i' \quad \text{and} \quad h_i'' = L_i'' + (\Delta_F'')_i \tau_i'' \,. \tag{14}$$

We get the conditional equations of the adjustment here again from Eqs 3 by substituting these values into them and similarly the conditions of the adjustment from Eqs 5. The adjusted velocity in the moment K is (using always the values corresponding to the levelling lines):

$$\Delta_K = \frac{L'' + v'' - (L' + v')}{T} \,. \tag{15}$$

The correction equations for the determination of the accelerations will have the form:

$$L_i + \lambda_i = L_{0i} + t_i \Delta_{0i} + \frac{t_i^2}{2} \alpha_i \tag{16}$$

where L is the result of measurements made at time t, measured from an initial (basic) epoch, λ is its correction, L_0 and Δ_0 the height differences and velocities computed from the adjustment for the initial epoch, and α the acceleration resulting from the adjustment.

Such correction equations must be set up for all levelling lines and for each levelling measured along them. If e.g. there are 9 lines (Fig. 1) and four levellings were carried out, the number of equations will be 36. Taking into account the precondition that the adjusted height differences of the network reduced to any epoch have to fulfil the condition of the adjustedness, three constraints are necessary for each closed polygonial. These are for the basic epoch as follows:

$$[L_0] = 0 \,, \; [\Delta_0] = 0 \quad \text{and} \quad [\alpha] = 0 \,. \tag{17}$$

The constraints decrease the number of the unknowns in the adjustment, and thus the number of normal equations, too. Figure 1 shows four polygonials with 9 lines. It is sufficient to use 3 unknowns each for the five levelling lines drawn with full lines, as the values for the four dotted lines can be determined from the constraints. (It is of course allowed to choose the lines with full and dotted lines differently, only their number must remain the same.) If V is the number of lines from which the unknowns must appear in the normal equations and N the number of these unknowns, then

$$V = 1 + (n - 2) \;\; 2 \quad \text{and} \quad N = 3V \,. \tag{18}$$

(If more than three closed polygonials meet in a junction, the values of V and N will correspondingly decrease.)

In the method of the determination of the acceleration we met HRISTOW's method that also takes the accelerations into account. He determines the heights of the points similarly at the initial epoch, as well as the velocity and acceleration of the movements of the Earth's surface. The difference between us is that he considers here the problem to be solved and the adjusted height values of the last levelling are according to him the values computed from the function determined in the adjustment. In our method, however, only the accelerations have been taken from the functions. Namely, it is evident that the accelerations are also burdened by errors. If the adjusted heights are requested for the 40th year from the initial epoch, the complete use of the functions transfers the error of the accelerations with their 800-fold value into the heights. Whereas, when according to our ideas the accelerations are used only for 10 – 12 years, at most about 50 – 70-fold error will burden the adjusted height differences. The accuracy of the adjusted heights calculated from the function will be decreased by the error of the velocities with a lower coefficient, too, e.g. in the last example about 40-fold. We do not use the velocities from this function, but the average of the velocity values (Δ_K) taken immediately from two consecutive levellings.

It could be argued whether the height values of the last levelling determined on the basis of the common adjustment of all previous levellings can be regarded as better, or those of the last two levellings carried out with more developed instruments and methods. I consider the complete functions too rigid, which follow less correctly the actual situation.

Hence, there are two variants of the fourth stage; our method yields the data of both methods following from the way of the determination of the accelerations.

*

In order to compare the results of adjustments with and without accelerations, we carried out the adjustment or a network consisting of three closed polygonials and 6 lines (Fig. 3) with both methods. We supposed that the first levelling (denoted by A) was carried out between the years 1 and 3, the second (B) between the years 20 and 25, the third (C) between the years 50 and 53, the fourth (D) between the years 75 and 77. The used data came from a simulation, as corresponding measurements are not available. Care was taken that the closures of the closed polygonials should be similar to those of high order levellings, and the random character should be ensured both in each simulated levelling and between the subsequent levellings. The weights p used in the computations are also shown in the figure. They were taken in the most simple way, namely as inversely proportional with the length of the lines. (The values of the weights are indifferent from the point of view of the present investigation.)

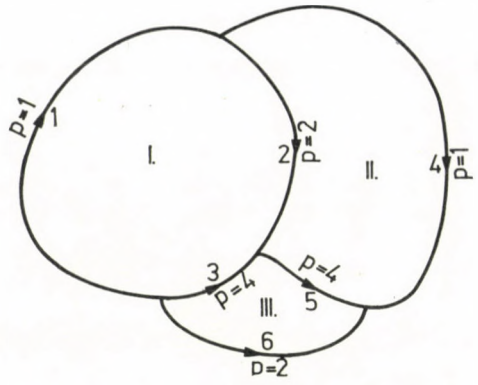

Fig. 3

The advantageous effect of the use of the acceleration is illustrated by some numerical values.

The upper part of Table I shows the adjusted values for line 2 from different pairs of levellings taking the accelerations into account, the lower part the same values computed without accelerations. Table II shows the velocities in the moments of the measurements in a similar arrangement. Column F shows the values from the adjustment on the basis of the 16 correction equations for the function of the line 2. The columns $S - D$ in both tables show the results of an experimental computation, in which a fictitious measurement result has been deduced for the mean time moment of the measurements in case of each levelling line at the levellings A, B and C, and

Table I

	Measurement	m	$A-B$	$B-C$	$C-D$	$A-C$	$S-D$	Average	Year
			mm						
With α	*A*	2.64	3.74			3.86		3.80	1
	B	2.62	2.30	2.28				2.29	21
	C	2.56		0.76	0.85	0.72		0.78	53
	D	2.49			8.27		8.33	8.30	77
Without α	*A*	2.64	3.85			4.15		4.00	1
	B	2.62	1.95	2.69				2.32	21
	C	2.56		0.91	0.81	0.85		0.86	53
	D	2.49			8.58		9.16	8.87	77

Table II

	Measurement	$A-B$	$B-C$	$C-D$	$A-C$	$S-D$	Average	F	Year
		mm							
With α	A	−0.80			−0.85		−0.83	−0.84	1
	B	−1.37	−1.45				−1.41	−1.41	21
	C		−2.37	−2.30	−2.34		−2.34	−2.33	53
	D			−2.96		−3.00	−2.98	−2.98	77
Without α	A	−1.10			−1.60		−1.35		1
	B	−1.10	−1.93				−1.52		21
	C		−1.93	−2.59	−1.60		−2.04		53
	D			−2.59		−2.12	−2.35		77

these were compared to the results of the levelling D. (In the practice such a computation is superfluous, for experimental purposes, however, it is of some interest.) It should be remarked that the choice of the results of line 2 has nothing to do with any outstanding property or more advantageous results; it is only superfluous to quote the results for all lines. A full picture about the deviations of the height differences is given in Table III, further about the deviations of the velocities in respect of the corresponding arithmetical means in Table IV. (In both tables, α denotes the accelerations, δ the deviations, and n is the number of values belonging together.) For the sake of completeness Table V shows the adjusted results for a measurement (the B measurement along line 4) where simulation seems to be less successful.

It should be further remarked that the closure errors of the closed polygonials and similarly the value of the mean square errors from the adjustment seems in both adjustments to be identical as expected, since the quality of the measurements depends on the accuracy of their realization. The differences of the adjusted values from the same measurement results are only due to the differences of the conditions used in the adjustment.

The greater is the number of repeated levellings, the more accurate are the values of the accelerations. Therefore I propose to our successors that they should re-determine the accelerations beginning from the third levelling prior to the adjustment of a new levelling, and in the computations the last values should be used. The re-determination of the accelerations means in each case some additional work, but using up-to-date computers this surplus does not mean any difficulties.

Table III

Line	α mm/t²	$\left\|\frac{\delta}{n}\right\|$	$\sqrt{\frac{\delta^2}{n}}$	max	$\left\|\frac{\delta}{n}\right\|$	$\sqrt{\frac{\delta^2}{n}}$	max
		10^{-2} mm					
		With α			Without α		
1	+0.0376	3.3	3.8	7	25.2	28.0	51
2	−0.0283	4.0	4.5	7	19.2	23.4	37
3	+0.0093	3.6	4.0	8	13.1	14.5	19
4	−0.0282	8.0	10.0	21	52.3	64.6	108
5	+0.0001	1.7	2.1	3	8.7	9.7	18
6	+0.0094	3.8	4.0	6	9.0	11.0	17
		4.1	5.3		21.2	31.4	

Table IV

Line	$\left\|\frac{\delta}{n}\right\|$	$\sqrt{\frac{\delta^2}{n}}$	max
	10^{-2} mm		
	With α		
1	4.3	4.5	6
2	2.7	3.0	4
3	2.3	2.6	4
4	3.6	3.9	6
5	0.6	0.8	1
6	1.7	1.8	3
	2.5	3.0	

Table V

	v mm	$L+v$ m	
$A-B$	−1.20	4.32032	With α
$B-C$	−1.61	4.31991	
diff.	0.41	0.00041	
$A-B$	−0.22	4.32130	Without α
$B-C$	−2.37	4.31915	
diff.	2.15	0.00215	

I think that realistic measurements follow better the reality as simulated ones, therefore their more exact knowledge together with that of the accelerations will bring the corresponding values nearer to each other, than shown by the present computations.

REFERENCES

1. GÁRDONYI, J.: A régi felsőrendű szintezési alappontok magasságainak változásai (Height changes of the old high-order basic points of levelling). *Geod. Közl.*, 8, 1932.
2. KÄÄRIÄINEN, E.: On the recent uplift of the Earth's crust in Finland. Veröff. Finn. Geod. Inst., No. 42, 1953.
3. KÄÄRIÄINEN, E.: On the use of different adjustment methods for the determination of land uplift from precise levellings. Veröff. Finn. Geod. Inst., No. 46, 1955.
4. BENDEFY, L.: Szintezési alappontok időközi magasságváltozásának meghatározása (Determination of the temporal height variations of basic points in levelling networks). *Geof. Közl.*, 4, No. 2 (1955), 3—30.
5. BENDEFY, L.: Módszer szintváltozások abszolút mértékének meghatározására (A method for the determination of the absolute value of height changes). *Geof. Közl.*, No. 1, 5, No. 4 (1956), 15—20.
6. BENDEFY, L.: Szintezési munkálatok Magyarországon 1820—1920. (Levellings in Hungary, 1820—1920). Akadémiai Kiadó, Budapest 1958.
7. KORHONEN, I.: Adjustment of levellings in region of slow vertical movement, Annales Acad. Sci. Fenn. Ser. A. III. Geol-Geogr., No. 61, 1961.
8. WYRZYKOWSKI, T.: The methods of geodetic determination of the recent vertical movements of the Earth's crust. Abh. Deutsch. Akad. Wiss. Berlin, Akad. Verlag, Berlin, 1962.
9. ZAKATOV, P. S.: On the application of the method of relevelling in a study of recent vertical movements of the Earth's crust. Abh. Deutsch. Akad. Wiss. Berlin, Akad. Verlag, Berlin, 1962.
10. HERMANOWSKI, A.: Scista metoda wyrowania i obliczania wielkosci przesumiec pionowych skornpy ziemskiej wyznaczanych podstawie okresowych pomiarow niwelacji precyzyjnoj. Politechn. Wars.z, Geod. Nr. 8, 1963 (in Plish).
11. HAZAY, I.: A vertikális kéregmozgási hálózatok kiegyenlítése (Adjustment of networks for the determination of vertical crustal movements). *Geod. Kart.*, 1967.
12. MISKOLCZI, L.: Kéregmozgások vizsgálata szabatos szintezésekkel (Study of crustal movements with precise levellings). Akadémiai Kiadó, Budapest 1973.
13. HRISTOV, W. K.: Gemeinsame Ausgleichung von Höhen- und Vertikalgeschwindigkeiten eines Nivellierungsnetzes. *Acta Geod. Geoph. Mont. Hung.*, 9 (1974), 147—151.

УРАВНИВАНИЕ НИВЕЛИРОВОК, СЛУЖАЩИХ ДЛЯ ИССЛЕДОВАНИЯ ВЕРТИКАЛЬНЫХ ДВИЖЕНИЙ ЗЕМНОЙ ПОВЕРХНОСТИ

И. ХАЗАИ

РЕЗЮМЕ

Изменения вертикальных движений земной поверхности в единицу времени (скорости) до сих пор определялись путем сопоставления результатов двух, последующих через определенное время (по крайней мере 15—20 лет) нивелировок. Если будут известны результаты большего числа нивелировок, удовлетворяющих требованиям точности, тогда, помимо скоростей движений, можно бедет учесть и их изменения, т. е. ускорения. В статье мается решение уравнивания нивелирных сетей с учетом ускорений. Показываются преидущества этого метода против такого, где ускорения еще пренебрегались.

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INTERPRETATION OF SIMULTANEOUS VARIATIONS OF GALACTIC COSMIC RAYS AND OF THE LEVEL OF ATMOSPHERIC RADIO NOISE

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The ionization due to the galactic cosmic radiation and the corresponding electron density profiles have been calculated in the lower ionosphere for Forbush-decreases. The attenuation of VLF electromagnetic waves in the Earth-ionosphere waveguide, computed for these events from the change of electron density, is in accordance with the experimental results.

In a previous paper it has been shown that during Forbush-decreases the level of atmospheric radio noise decreases, if the shock wave modulating the galactic cosmic ray particles presumably missed the Earth, or the increased plasma flow from the Sun did not produce an SSC, and increases when an SSC indicated the arrival of the interplanetary shock wave [1]. The interpretation of these effects may be divided into three steps.

1. Computation of ionization caused by galactic cosmic rays

The ionization caused by corpuscular radiation penetrating into the upper atmosphere may be computed by means of the following formula [2]:

$$q^{(h)} = \sum_j \int_E \int_\Omega \frac{1}{W} \left(- \frac{dE}{dh} \right)_j \varphi_j(E, \Omega)\, dE d\Omega \,, \tag{1}$$

where q in $\mathrm{cm}^{-3}\,\mathrm{s}^{-1}$ is the total electron production rate at a height h, due to particles of type j, W is the energy required for the formation of an electron-ion pair, which amounts to 35 eV for air, $(-dE/dh)$ and $\varphi_j(E, \Omega)$ are the ionization losses of particles, respectively their differential spectra belonging to the energy interval dE and the spatial angle $d\Omega$.

The ionization losses in case of relativistic heavy particles, neglecting the inner shell corrections are given by the Bethe—Bloch formula [3].

$$- \left(\frac{dE}{dh} \right) = \frac{4\pi Z_j^2 e^4}{m_e v^2} nZ \left\{ \ln \frac{2 m_e v^2}{I(1 - \beta^2)} - \beta^2 \right\} . \tag{2}$$

Here n and Z are the number density and the atomic number of the absorber atoms, respectively; $Z_j e$ and v are the charge, respectively the velocity of incident particles, m_c is the mass of electron; I is the average excitation energy, which is about 94 eV for air; $\beta = v/c$ is the relativistic factor.

If we use the rigidity R instead of energy, where $E = Z_j/M_j\, R$ (M_j is the atomic weight of the penetrating particle), the following expression is obtained from equation (2):

$$-\left(\frac{dR}{dh}\right) = \frac{0.3 Z_j^2}{\beta^2}\, \varrho(h) \left\{\ln\left(\beta \frac{Z_j}{M_j} R\right) + 4.5 - 0.5\, \beta^2\right\}, \tag{3}$$

where $\varrho\,(h$ is the density of the atmosphere.

Since the galactic cosmic radiation is of relativistic energy and isotropic, the condition $\beta = 1$ is valid and at the computation of ionization the integration according to the spatial angle can be performed.

For the computation of the differential spectrum of galactic cosmic radiation, the formula

$$\varphi_j\,(> R_c) = K_j\, R_c^{-\gamma}$$

has been used, from which the expression

$$\Phi_j\,(> R_c) = K_j \frac{R_c^{-(\gamma j - 1)}}{\gamma_j - 1} \tag{4}$$

can be obtained for the calculation of the integral spectrum.

R_c is the cut-off rigidity.

K_j and γ_j depend generally on rigidity and are different for different events.

Since in galactic cosmic rays among the separate groups of nuclei (p, α, L, M, H, VH) definite ratios exist, $q\,(h)$ may be calculated by the ratio of the integral spectrum of the protons $\Phi_p(R_c)$ to that of the other groups, expressed in terms of e.g. the integral spectrum of the H fraction [4].

In this way, using equations (2), (3) and (4), the following expression can be derived from (1) for the ionization:

$$q(h) = 1.8 \cdot 10^5\, \varrho(h) \Phi_p(> R_c)(\ln R_c + 4.15)\,. \tag{5}$$

According to DORMAN [4] the differential proton spectrum may be described by the following formula:

$$\varphi_p\,(> R_c) = 1.2\, R_c^{-2.5}\,,$$

assuming $\delta R_c = 0$. If the geomagnetic field is represented by a dipole, $R_c = 14.9 \cos^4 \lambda_m$, where λ_m is the geomagnetic latitude.

For the computation of the effect of Forbush-decreases according to equation (5) both the change of the cosmic ray flux and the variation of the cut-off rigidity must be known.

For the variation of the geomagnetic cut-off rigidity the following formula was used [5]:

$$\frac{\delta R_c}{R_c} = 1 - \text{const.} \left(1 - \frac{r_0^3 \, \delta H}{2M}\right)^2,$$

where r_0 is the Earth radius, $M = 8.1 \cdot 10^{25}$ G cm^3 is its magnetic moment and δH is the change of the geomagnetic horizontal component.

The variation of the primary cosmic rays can be given as

$$\frac{\delta \Phi (R_c)}{\Phi (R_c)} = - K R^{-n_0} .$$

where n_0 depends on the rigidity. In case of Forbush-decreases K and n_0 vary between 0.3 and 1, respectively between 0.5 and 1 [6].

The change of ionization due to the joint effect of the variation of the geomagnetic field and of the primary cosmic rays during Forbush-decreases can be expressed, as follows [6]:

$$\frac{\delta q}{q} = (\gamma - 1) \left[\frac{\delta R_c}{R_c} + \frac{1}{n_0} \frac{\delta \Phi (R_c)}{\Phi (R_c)}\right]. \tag{6}$$

$\varrho (h)$ has been taken from CIRA 1972 and ionization profiles were computed for different events and for the geomagnetic latitude of those observatories [Kühlungsborn ($\lambda_m = 54.36°$), Panska Ves ($\lambda_m = 50.41°$) and Toyokawa ($\lambda_m = 24.46°$)], where the effect of galactic cosmic rays in the level of atmospheric radio noise has been studied.

Figures 1, 2, 3 show ionization profiles for different geomagnetic latitudes and seasons for undisturbed conditions, in case of Forbush-decreases and of $\delta R_c \neq 0$. Profiles A refer to undisturbed conditions. In case of strong Forbush-effects and $\delta R_c = 0$, that is, if the shock wave modulating the galactic cosmic ray particles presumably missed the Earth (no geomagnetic storm was observed) (profiles B), the ionization decreases compared with undisturbed conditions (profiles A) and to a greater extent with increasing geomagnetic latitude. In case of weak Forbush-decreases and $\delta R_c \neq 0$, that is, when an SSC indicated the arrival of the interplanetary shock wave (a geomagnetic storm occurred), at the latitudes of Kühlungsborn and Panska Ves the ionization profiles hardly differ from profiles A. The difference between the profiles cannot be shown in the illustrations (Figs 1, 2). For

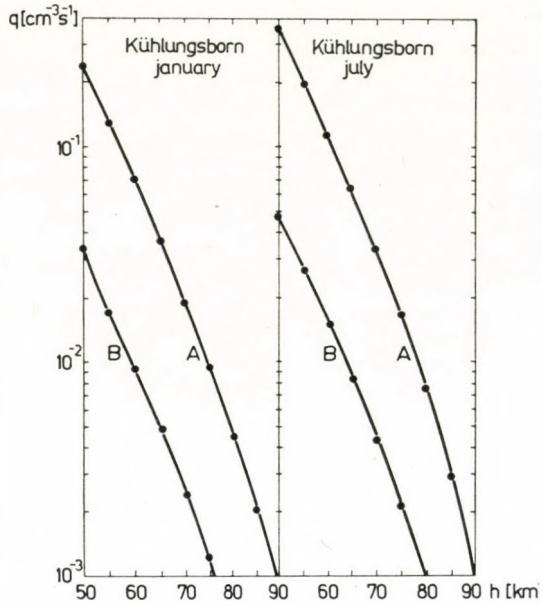

Fig. 1. Ionization profiles in different seasons (January, July), computed for the geomagnetic latitude of Kühlungsborn, referring to undisturbed conditions and to Forbush-decreases if geomagnetic storms occurred (profiles A), as well as to Forbush-decreases when no geomagnetic storms were observed (profiles B)

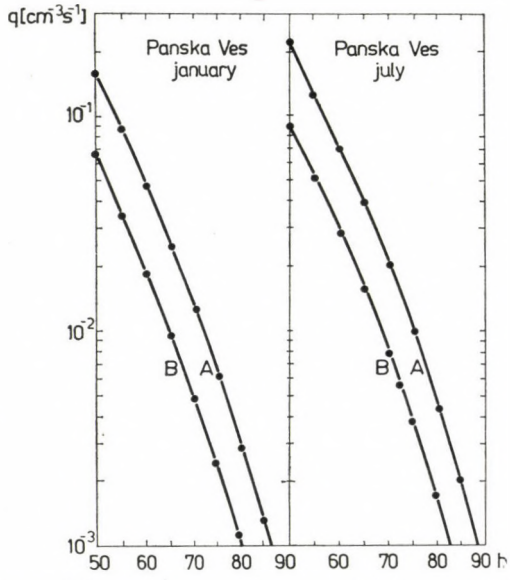

Fig. 2. Ionization profiles in different seasons (January, July), computed for the geomagnetic latitude of Panska Ves, referring to undisturbed conditions and to Forbush decreases, if geomagnetic storms occurred (profiles A), as well as to Forbush-decreases, when no geomagnetic storms were observed (profiles B)

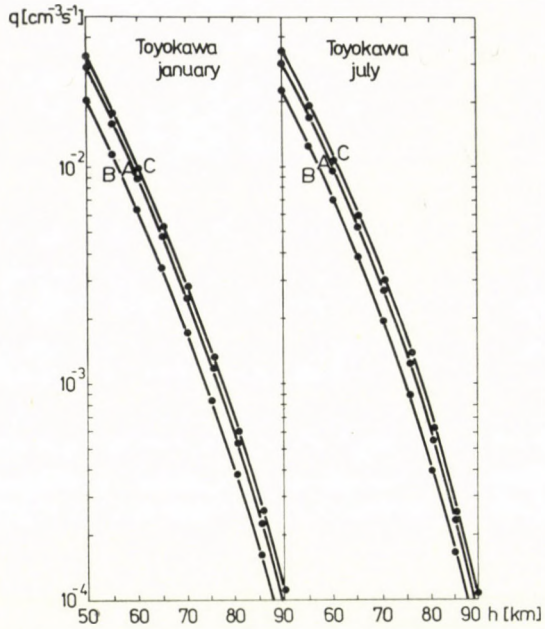

Fig. 3. Ionization profiles in different seasons (January, July), computed for the geomagnetic latitude of Toyokawa, referring to undisturbed conditions (profiles A), as well as to Forbush-decreases when no geomagnetic storms were observed (profiles B), and to Forbush-decreases, if geomagnetic storms occurred (profiles C)

these latitudes $\delta H \leq 200\ \gamma$ has been chosen, since in our earlier investigations concerning the years 1971–1973 geomagnetic storms of such intensity occurred, too. In case of Toyokawa $\delta H \leq 300\ \gamma$ has been used and the ionization (profiles C) may increase compared with undisturbed conditions (profiles A) depending on the relative magnitude of the terms in expression (6) (Fig. 3). In case of a very strong magnetic storm it can happen that in spite of the ionization decrease due to the Forbush-effect the ionization increases also at higher latitudes.

2. Calculation of electron density from ionization profiles

The electron density may be computed from

$$N(h) = \left[\frac{q(h)}{\alpha'(h)\,[1 + \lambda(h)]}\right]^{1/2},$$

where $\lambda\,(h)$ is the negative ion to electron ratio, and α' is the effective recombination coefficient [7]:

$$\alpha' = \alpha_d + \alpha_i + [N(1 + \lambda)]^{-1}\, d\lambda/dt\,.$$

In this expression α_d and α_i are the dissociative recombination coefficient and the ionic recombination coefficient respectively, the third term represents the time variation of the negative ion to electron ratio, which has been neglected. Further

$$\lambda = k\ n^2(O_2)\,[\gamma_0 + \gamma n\,(O_2)]^{-1}\,,$$

where $n\,(O_2)$ is the number density of molecular oxygen, $k = (2 \pm 1)\ 10^{-30}$ $cm^6\ s^{-1}$ and $\gamma_0 = 10^{-18}\ cm^3\ s^{-1}$ are the three body attachment, respectively the collisional detachment coefficients, $\gamma = 0.35$ is the photo detachment rate.

The electron density profiles have been computed for the same conditions as in case of the ionization profiles. Figures 4, 5, 6 show electron density profiles for the latitude of the observatories, mentioned above. At the higher latitudes the electron density is at all heights larger, than at lower latitudes. In case of Forbush-decreases the electron density decreases, when no geomagnetic storm was observed (profiles B) or it differs only slightly from the electron density corresponding to undisturbed conditions (profiles A), and so cannot be shown in the illustrations, if also a fairly strong geomagnetic storm occurs. In Toyokawa during strong geomagnetic storms the electron density may increase (profiles C) as compared with undisturbed conditions (profiles A).

The maximum of electron density is near 75 km, that is at the maximum of the CR layer. The computed values of electron density agree very well below 75 km with measured values, obtained at middle latitudes, at solar zenith angles $\chi = 60° - 80°$ [8].

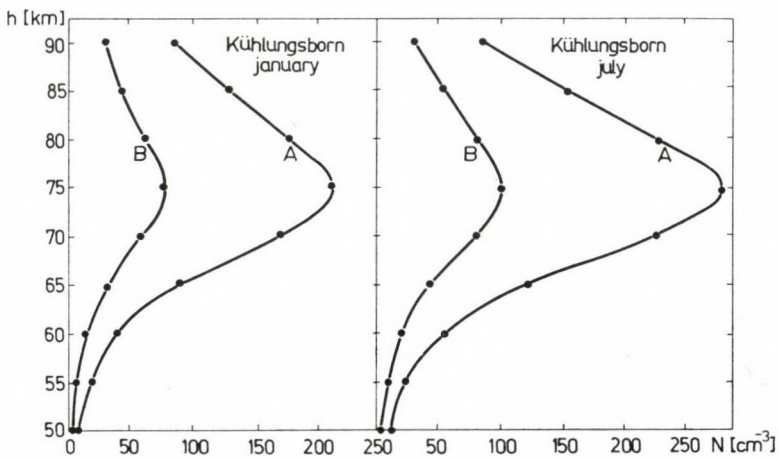

Fig. 4. Electron density profiles in different seasons (January, July), calculated for the geomagnetic latitude of Kühlungsborn, referring to undisturbed conditions and to Forbush decreases, if geomagnetic storms occurred (profiles A), as well as to Forbush-decreases, when no geomagnetic storms were observed (profiles B)

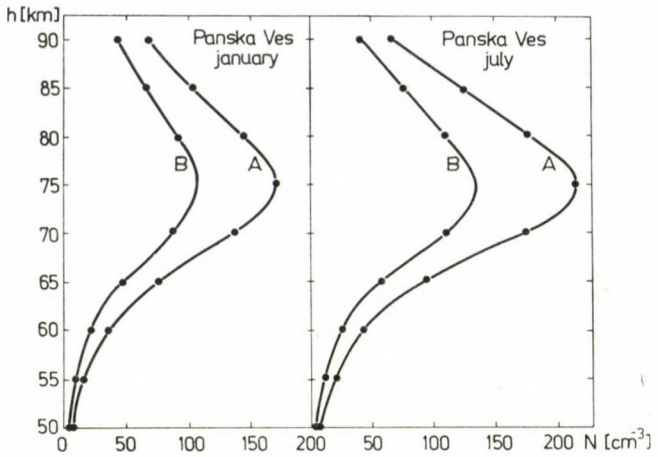

Fig. 5. Electron density profiles in different seasons (January, July), calculated for the geomagnetic latitudes of Panska Ves referring to undisturbed conditions and to Forbush decreases, if geomagnetic storms occurred (profiles A), as well as to Forbush-decreases, when geomagnetic storms were observed (profiles B)

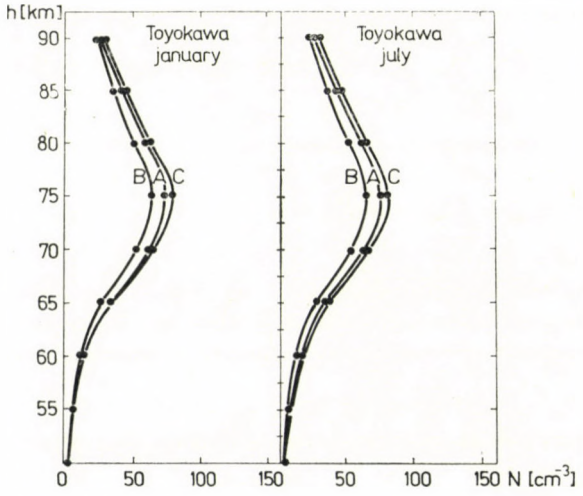

Fig. 6. Electron density profiles in different seasons (January, July), calculated for the geomagnetic latitude of Toyokawa, referring to undisturbed conditions (profiles A), as well as to Forbush-decreases, when no geomagnetic storms were observed (profiles B) and to Forbush-decreases, if geomagnetic storms occurred (profiles C)

3. Attenuation of VLF electromagnetic waves in the Earth-ionosphere waveguide

As we have seen, the galactic cosmic radiation is an important source of ionization in the lower D-region. The upper wall of the Earth-ionosphere waveguide controlling the propagation of terrestrial radio waves is formed

by the D-region. Thus, the variation of its conductivity influences the propagation properties of the medium and thereby, the attenuation of VLF electromagnetic waves.

The attenuation may be obtained from the mode equation [9]:

$$R_i(C)\, R_g(C) \exp(-i\, 2\, khC) = \exp(-i\, 2\, \pi n)\,.$$

Here R_g (C) is the reflection coefficient for a ray incident on the ground, and $R_i(C)$ is the reflection coefficient for the upper boundary of the waveguide, which coincides with the lower edge of the ionosphere. Further k is the value wave-number, h is the height of the Earth-ionosphere waveguide and n is an integer. C_n represents the complex roots of this equation, which are actually the cosine of the angle of incidence. The attenuation is given by the imaginary part of

$$S_n = (1 - C_n^2)^{1/2}.$$

In the VLF range the flat-earth mode theory may be considered as an acceptable approximation, especially below 10 kcps [10]. In this case the attenuation may be obtained from the following expression, regarding the ionosphere as an inhomogeneous isotropic medium [9]:

$$\operatorname{Im} S_n = \frac{-\lambda}{2\sqrt{2}\,\pi h\, \bar{S}_n}\left[\bar{C}_n^2\left(\sqrt{L} + \frac{1}{\sqrt{L}} + \sqrt{G}\right)\right]$$

where

$$\bar{C}_n = \frac{n - 1/2}{2h/\lambda}$$

and

$$\bar{S}_n = (1 - \bar{C}_n^2)^{1/2}\,.$$

Further, $L = \omega/\omega_r$, in which $\omega_r = \omega_0^2/\nu$, ω_0 and ν are the (angular) plasma frequency and the collision frequency, respectively, ω and λ are the angular frequency, respectively the wave length of the propagating electromagnetic wave. $G \cong \varepsilon\omega/\sigma_g$, where ε is the dielectric constant of vacuum and σ_g is the conductivity of the ground. The relation between plasma frequency and electron density at the reference level h is described by

$$\omega_0^2 = 3.18 \cdot 10^9 \cdot N(h)\,,$$

where $N(h)$ is expressed in electrons per cm^3.

On the basis of the computed electron density profiles the attenuation of the first mode has been computed for the same conditions as before (Figs 7, 8, 9). As reference level 60 km was taken, because in the VLF range electromagnetic waves are reflected day-time from these heights. The computed

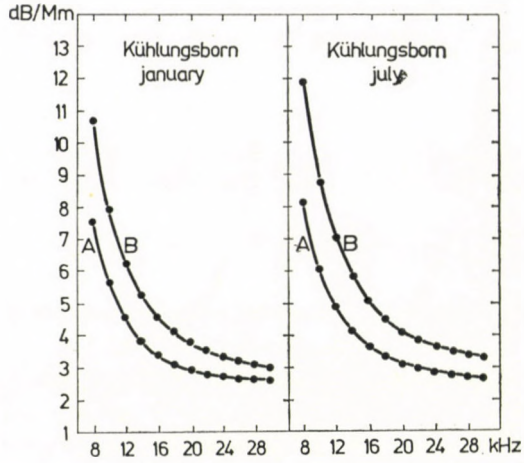

Fig. 7. Attenuation as a function of frequency in different seasons (January, July), determined for the geomagnetic latitude of Kühlungsborn in case of both undisturbed conditions and Forbush-decreases, if geomagnetic storms occurred (curves A), as well as in case of Forbush-decreases, when no geomagnetic storms were observed (curves B)

attenuation refers to 1000 kilometers. On this distance one obtains 0.6 – 0.7 dB for the change of attenuation at 27 kcps during Forbush-decreases in case of Kühlungsborn (Fig. 7) and 0.3 – 0.4 dB in case of Panska Ves (Fig. 8). For Toyokawa, during Forbush-decreases a change of attenuation of 0.5 – 0.8 dB at 10 kcps is found. The attenuation depends on frequency. This is shown

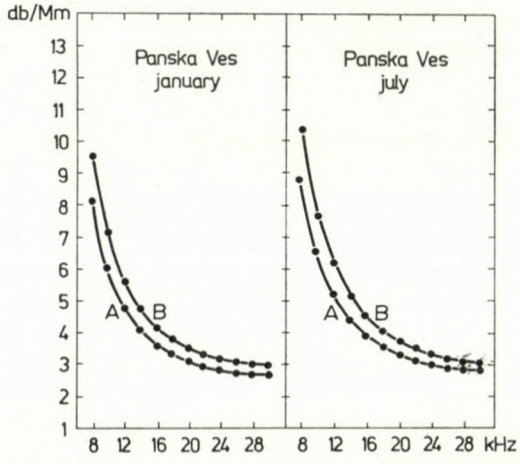

Fig. 8. Attenuation as a function of frequency in different seasons (January, July), determined for the geomagnetic latitude of Panska Ves in case of both undisturbed conditions and Forbush-decreases, if geomagnetic storms occurred (curves A), Forbush-decreases, as well as in case of Forbush-decreases, when no geomagnetic storms were observed (curves B)

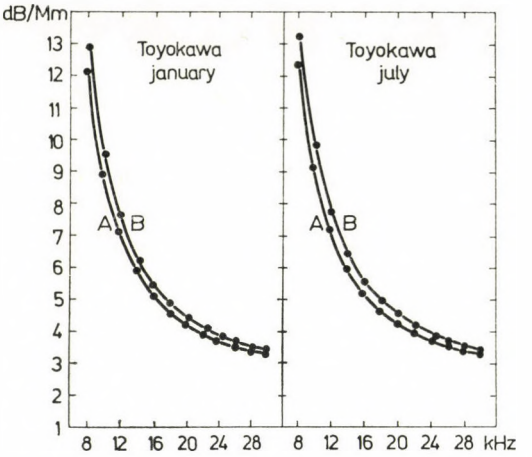

Fig. 9. Attenuation as a function of frequency in different seasons (January, July), determined for the geomagnetic latitude of Toyokawa in case of both undisturbed conditions and Forbush-decreases, if geomagnetic storms (curves A), as well as in case of Forbush-decreases, when no geomagnetic storms were observed (curves B)

especially well in case of Toyokawa by comparing the computed attenuation (Fig. 9) with the observed decrease of the level of atmospheric radio noise (Fig. 10). The influence of Forbush-decreases decreases gradually with increasing frequency (Fig. 10).

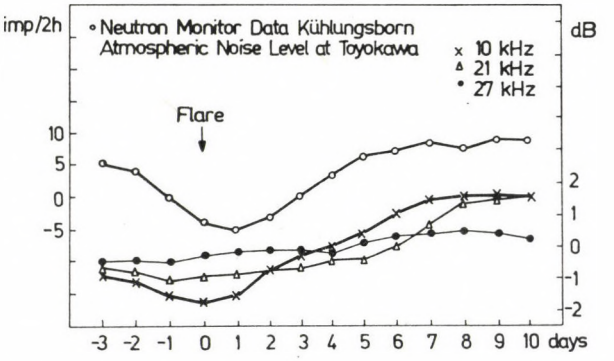

Fig. 10. Variation of the daily average of neutron monitor data (deviations from the normal value 26800 imp/2h) and that of the daily average deviation from the monthly median of the noise level measured in Toyokawa before and after the occurrence of solar flares (superposed epochs, number of cases: 7)

Discussion

In case of the European receiving sites, the data of which are applied in these investigations, the day-time level of atmospheric radio noise is produced by discharges occurring within a distance of about 1000 – 2000 km

[11]. Taking this circumstance into consideration, the change of attenuation computed for a Forbush-decrease and no geomagnetic storm is in accordance with the decrease of the level of atmospheric radio noise, given in a previous paper (1). In case of both a Forbush-effect and a geomagnetic storm, the computed attenuation differs only slightly from the undisturbed value (Figs 7, 8, 9; curves A), but at the same time the level of atmospheric radio noise increases. This shows that at a given geomagnetic latitude not only the decrease of the cut-off rigidity, but also other effects may appear and result in an increase of the radio noise level as it can be seen from an earlier investigation [12]. Thus the increase may be the consequence of the after-effect, which was not taken into consideration in the present calculations.

The geomagnetic field influences the attenuation, especially in case of a propagation towards the east and west; but this effect becomes less, if the magnetic inclination approaches the vertical, that is at higher latitudes [9]. In the present investigations in the computation of attenuation the geomagnetic field was not considered, since electromagnetic waves producing the level of radio noise arrive to the receiving site from different directions, and in our case only changes of attenuation are of interest.

Summing up, by means of the atmospheric radio noise the influence of the intensity variations of galactic cosmic rays in the lower ionosphere may be studied, especially at higher geomagnetic latitudes ($\geq 50°$ N) and at lower frequencies (≤ 10 kcps).

REFERENCES

1. Sátori, G.: Effect of the cosmic rays in the lower ionosphere as shown by the level of atmospheric radio noise. *Acta Geod. Geoph. Mont. Hung.*, 11 (1976), 229.
2. Whitten, R. C.—Poppoff, I. G.: Physics of the Lower Ionosphere. Prentice Hall, Englewood Cliffs, W. J. 1965, p. 40.
3. Bates, D. R.: Atomic and Molecular Processes. Academic Press, New York 1962.
4. Dorman, L. I.: Cosmic Ray Variation and Space Investigation. Acad. Sci. USSR. 1963.
5. Obayashi, T.—Hakura, Y.: Enhanced ionization in the polar ionosphere caused by solar corpuscular emissions. *Rep. Ionos. Space Res. Japan*, 14 (1960) 1.
6. Velinov, P.: On variations of the cosmic ray layer in the lower ionosphere. *J. Atm. Terr. Phys.*, 33 (1971), 429.
7. Velinov, P.: On ionization in the ionospheric D-region by galactic and solar cosmic rays. *J. Atm. Terr. Phys.*, 30 (1968), 1892.
8. Bremer, J.—Singer, W.: Diurnal, seasonal and solar-cycle variations of electron densities in the ionospheric D- and E-regions. *J. Atm. Terr. Phys.*, 39 (1977), 25.
9. Wait, J. R.: Electromagnetic Waves in Stratified Media. Pergamon Press Inc., New York 1962.
10. Wait, J. R.—Spies, K. P.: Characteristics of the Earth-ionosphere waveguide for VLF radio waves. Central Radio Propagation Laboratory, National Bureau of Standards Boulder, Colorado 1964.
11. Lauter, E. A.—Schäning, B.: Ergebnisse und Aspekte von Messungen des atomsphärischen Funkstörpegels im Längstwellenbereich in verschiedenen Breiten. NKGG Veröffentlichungen Reihe II. 3 (1966), 95.
12. Sátori, G.: An investigation of the invasion of particles into the lower ionosphere during the recovery phase of geomagnetic storms. *Acta Geod. Geoph. Mont. Hung.*, 7 (1972), 119.

ИНТЕРПРЕТАЦИЯ ОДНОВРЕМЕННЫХ ВАРИАЦИИ ГАЛАКТЕРИЧЕСКОГО КОСМИЧЕСКОГО ИЗЛУЧЕНИЯ И АТМОСФЕРНОГО РАДИОШУМА

Г. ШАТОРИ

РЕЗЮМЕ

Были вычислены присваиваемая галактическому космическому излучению ионизация и соответствующие профили електронных плотностей для нижней ионосферы во время Форбушева убывания. Затухание электромагнитных волн очень низкой частоты в волноводе Земля — ионосфера, вычисляемое по изменению плотности электронов во время этих событий, совпадает с экспериментальными результатами.

RECENSIONES

S. Flügge ed.

HANDBUCH DER PHYSIK — ENCYCLOPEDIA OF PHYSICS
Vol. XLIX/5, *Geophysics* III/5 (ed. K. Rawer)
Springer-Verlag, Berlin—Heidelberg—New York, 1976, 407 pp. 236 figs. DM 198,— $ 81,20

The fifth volume of the geophysical part of this outstanding enterprise is again completely devoted to the physics of the Earth's upper atmosphere. The fact that out of the seven volumes dealing with the various branches of geophysics five are written about the physics of the upper atmosphere, reflects again the tremendous development of this area in the space age.

The volume contains five contributions by the best experts of the topics all the five treating current problems of upper atmosphere physics. On the first pages the reader finds the useful introductory remarks by Prof. K. RAWER, the editor of the volume, on the systems of units and on mathematical signs. This facilitates the use of the book and the common application of it with other volumes, published earlier. The first contribution is an excellent summary of the most important results concerning *the nightglow*, i.e. the emission of the upper atmosphere in the optical band of the electromagnetic radiation. This part of the book is written by Dr. A. T. VASSY and Prof. E. VASSY in French. The importance of the study of the airglow is stated by the ever growing interest of aeronomers, who consider it as a promising method of investigation of energy transformation in the upper atmosphere. The review of the problems is divided into several sections, in which among others the observational methods, the colour of the nightglow, then one of the most important questions, the spectral analysis of the nightglow are discussed. Shorter sections dealing with the polarization of the emission and with the temporal variation of the nightglow are included. Further sections are devoted to the spatial variations and to the correlation of the nightglow with other phenomena. In the following two sections important questions are again discussed, i.e. the origin of the night-glow and its application to the study of the upper atmosphere. The last section deals with the twilight airglow and with the dayglow, the emission of the upper atmosphere before, during and after it is irradiated by the Sun's radiation.

The second contribution gives a review of the *dynamic structure of the stratosphere and mesosphere* summarized by Dr. W. L. WEBB. Nowadays one of the most exciting problem concerning the whole mankind is the influence of solar activity on weather. The investigations, described in this chapter, help us to learn more about the processes in these regions, which may "relay" solar effects. In the brief review of this constantly developing field of atmospheric sciences the structure, the motions and other features of the above-mentioned atmospheric regions are discussed. The section on structure deals with the origin of the ozonosphere and with problems such as the small scale features of the stratospheric circulation, thermal transport and eddy viscosity, as well as the thermal structure and its variations. Discussing the stratospheric circulation, thermal tides and upper atmospheric clouds, the results related to the stratospheric circulation index, the hemispheric similarities and the stratopause diurnal heat wave are reviewed. Finally other features, such as the atmospheric acoustical structure and the electrical structure of the atmospheric regions in question are briefly described. The most important conclusion of the summary is the need of the recognition of a unified atmosphere.

The third contribution deals with one of the most interesting problems of atmospheric dynamics, with the *linear internal gravity waves* and is written by Prof. W. L. JONES. In the last fifteen years the atmospheric waves are recognized as an important factor redistributing the solar energy absorbed by the different layers of the atmosphere. This part of the book is a summary of the most important theoretical results concerning the generation, propagation and dissipation of internal atmospheric gravity waves. The wave equations taking into account of the mean flow and approximate techniques for solving the wave equations are given.

The fourth contribution written by Prof. Ja. L. ALPERT deals with *wave like phenomena in the near-earth plasma* and *interactions with man-made bodies.* The satellite era enabled the scientists to study the near-earth plasma in situ and to get acquainted with the wealth of wave-like phenomena occuring in the huge laboratory of nature. The first part of this review is devoted to the flow around solid bodies moving in a plasma, which is an important question from the point of view of the arrangment of probes and traps on satellites. In the second part the waves and oscillations of the ionospheric and near-Earth plasma are discussed, which have been detected in the different radio frequency (ELF, VLF, LF and HF) bands.

The fifth and last contribution points already ahead. The scientists exploring the near-Earth environment nowadays do not limit their investigations to our planet, but started to study the ionospheres of near-Earth planets, hoping to learn more about the development of the Earth's atmosphere too. *Some characteristic features of the ionospheres of the near-Earth planets* are summarized by Prof. K. I. GRINGAUZ and Dr. T. K. BREUS. The reader gets acquainted with the methods used for the investigation of planetary ionospheres by means of spacecraft. Methods of plasma diagnostics are described here, which have been successfully applied on deep space probes. The experimental results of the exploration of the Martian and Venusian inonospheres are those, which enabled us first to obtain an insight into the life of foreign planets. The section dealing with the models of the Martian and Venusian ionospheres show, how many information has been collected in a rather short interval. Perhaps this is one of the most convincing evidence of the power of space science.

P. Bencze

CRUSTAL AND UPPER MANTLE STRUCTURE IN EUROPE
Monograph No. 1. European Seismological Commission
Publication of the Deutsche Geophysikalische Gesellschaft
Explosion Seismology in Central Europe. Data and Results.
Editors: *P. Giese, C. Prodöhl, A. Stein*
248 figs, 3 maps inside front cover, 22 tables
XIV. 429 pages. 1976, Berlin—Heidelberg—New York,
Springer Verlag
Cloth DM 146,— US $ 59,90

The structure of the Earth's crust is important for both geophysicists and geologists from the point of view of exploration of mineral resources and of basic research. The determination of the inner (deep) structure of the Earth can be carried out mainly by seismology, and this way is only possible if a correct picture of the structure of the crust is at disposal. Geometrical and physical parameters from explosions are more accurate than data from natural earthquakes, therefore seismic deep sounding data have special importance. It has been chosen for a main task of the European Seismological Commission to promote investigations on the structure of the Earth's crust. This book contains a part of results from such investigations.

The chapters summarize shortly and clearly, with many figures the geophysical and geological results mostly measurements from the territory of the Federal German Republic. In addition several scientific papers are also included which are in close connection with the geophysical problem.

The book describes the main geologic features of the area given in the title, and discusses investigations on the seismic noise, magnetism, geothermics, gravity and geoelectrics, further problems in the interpretation of different seismic methods, results of seismic deep soundings etc.

The estetically beautiful book offers more on the peculiarities of the Earth's crust than promised in the title.

E. Bisztricsány

W. Schröder
ENTWICKLUNGSPHASEN DER ERFORSCHUNG DER LEUCHTENDEN NACHTWOLKEN.

Akademie der Wissenschaften der DDR.
Veröffentlichungen des Forschungsbereiches Geo- und Kosmoswissenschaften, Heft 5.
Akademie-Verlag, Berlin, 1975. 64 S.

Es ist sehr erfreulich, daß der Verfasser die Geschichte der Erforschung der Leuchtenden Nachtwolken und ihren heutigen Stand, sowie dessen Entwicklungstendenzen mit viel Umsicht und Kenntnis bearbeitet hat.

Die Geschichte der Erforschung der Hochatmosphäre geht bis 1883, zum Krakatau-Ausbruch zurück, mit dem sich in 1885 und 1888 KIESSLING eingehend beschäftigt hat. Es folgen weiters die Forschungsarbeiten der Berliner Sternwarte 1885—1901, die die Vorläufer der heutigen internationalen Forschungsvorhaben waren.

Es werden dann die internationalen Forschungsvorhaben zwischen 1900 und 1956, dann im Internationalen Geophysikalischen Jahr 1957—1958 und weiters bis 1974 angeführt.

Sehr interessant sind die Erörterungen über die Entwicklungslinien der Theorie der Leuchtenden Nachtwolken, und über die Beziehungen zwischen kosmischen Staubeinbrüchen in die Hochatmosphäre und den Leuchtenden Nachtwolken. Nicht minder interessant sind die kritische Behandlung der neueren Vorstellungen über die Leuchtenden Nachtwolken und der, das Internationale Jahr der ruhigen Sonne (1964—1965) berücksichtigende Ausblick mit dem, von Ertel stammenden Ausspruch: »Eine atmosphärische Erscheinung gilt demzufolge als erklärt, wenn es gelungen ist, sie aus den bewährten Sätzen der Physik deduktiv abzuleiten.«

Das Schröder'sche Büchlein stellt eine wertvolle Bereicherung der geophysikalischen Fachliteratur dar.

A. Tárczy-Hornoch

H. Piller
MICROSCOPE PHOTOMETRY
Springer Verlag, Berlin, Heidelberg, New York, 1977, 253 pp. 79. Fig.

PILLER's book on Microscope Photometry describes the photometric measurements on microscopic range materials so that the work may be particularly useful for scientists who deal with Mineralogy, Petrology and Biology. But in a wider sense of the word microphotometry and complex image analyzing technology which use the common building stones of microscope photometry may enrich themselves with this volume.

The author's concern is restricted basically on the classical light microscope which various mechanical, optical and electrical modules are added to in order to measure the optical properties of light transmitted or reflected by original microscopic specimens. The scope of the book is extended to the interference and fluorescence techniques too.

The author himself who is with the Opton Oberkochen sticks to short and essential explanations of physical laws and phenomena with bright figures and legends instead of complicated deductions. Consequently he makes the heart of the matter understandable for users being less familiar with this problem. Because of the restricted extent of the volume a lot of references is made to other sources.

The first chapters of the book deal with the geometry and energy transfer of light microscopic image formation which carries the photometric information from the objet to the image.

Further chapters are devoted for the description of the function of equipment parts, starting from stabilized light sources, monochromatizing devices, filters, condensers, reflectors and lenses to the photosensor types.

After studying the different equipment parts, the geometric and photometric adjustments are treated and the procedures in photometry with microscopes are introduced. Those procedures involve measurements with a fixed wavelength, spectral measurements on a fixed measuring area and special attention is paid to the reference standards used in transmission and reflection measurements mode.

Special techniques are also described involving the microscope fluorometry, the determination of optical path differences, the measurements of specular and diffuse surface reflections, techniques for measuring microradiographs, the microdensitometry of developed photographic layers and determination of basic optical constants with interference layer techniques, too.

Practical chapters deal with the special adjustments of microscopic photometers, with the focusing of illuminating and imaging systems, levelling of stage object in specularly reflected light, control of angle of coincidence, spectral bandwidth and systematic errors having an instrumental origin. The stage object factors are also interpreted which affect the light supply to the photosensor and measuring results.

After studying the source of statistical errors which influence the measurements, the application of microscope photometry is introduced for biological and non-biological subjects. Special attention is paid to the utilisation of photographs as input, too.

A further short chapter summarizes the parameters measured in reflected light and in microscope fluorometry.

In the Appendix the symbols and equations are collected, which are referred to in this book. About 150 references and a well constructed Subject Index completes the volume.

M. Gerencsér

R. Meissner—L. Stegena:
PRAXIS DER SEISMISCHEN FELDMESSUNG UND AUSWERTUNG.
Akadémiai Kiadó Budapest und Gebrüder Borntraeger, Berlin—Stuttgart, 1977. p. 276

Die Verfasser haben in 13 Kapiteln jene Kenntnisse über die Seismik zusammengefaßt, die die Studenten der Geophysik und der Nachbargebiete über die alltägige Praxis haben müssen. Neben dieser Aussage im Vorwort wurde noch festgelegt, daß die Arbeit in erster Linie für die in der Erdöl- und Erdgaserkundung Tätigen zusammengestellt worden ist.

Diese einleitenden — und vom Gesichtspunkt der Empfehlung her wichtigen — Bemerkungen müssen damit ergänzt werden, daß dieses Buch der erste Band der Reihe »Studienhefte zur Angewandten Geophysik« (Herausgeber: Gebrüder Borntraeger, Berlin—Stuttgart) ist und dessen Fortsetzungen die physikalischen, instrumentellen und filtertheoretischen Grundlagen der Seismik besprechen. Daher befaßt sich der vorliegende erste Band mit Fragen der theoretischen Grundlage nur in dem zum Verständnis unbedingt notwendigen Maße, d. h. es werden diesbezügliche Kenntnisse vorausgesetzt.

Der kurze Inhalt der einzelnen Kapiteln sowie einige bescheidene Bemerkungen sollen im weiteren angeführt werden.

Kapitel 1 »Petrologische und geologische Grundlagen der Seismik« enthält wichtige jedoch nur in kurzen Umrissen ausgeführte Feststellungen. Auch diese Feststellungen legen in erster Linie die petrophysikalischen Zusammenhänge dar, von Hinsicht der geologischen Strukturen her werden nur schematische Skizzen für sehr allgemeine Erdölfallen angegeben (S. 4—17).

Über »Die Planung seismischer Feldarbeiten« (Kapitel 2) — gesondert über Refraktions- und Reflexionsmessungen — erfährt man nebst einigen sehr wichtigen Daten der Praxis, sowie Grundrissen der Beobachtungsanordnungen, auch die Schranken der beiden Meßverfahren. So wird auch der Begriff der durchgeschossenen Schicht, die Ausbildung der Reflexionssignalform ebenfalls in diesem Kapitel eingehend diskutiert (S. 17—31).

»Die Erregung seismischer Wellen« (Kapitel 3) wird für Sprengungen und sprengstofflosen Verfahren getrennt behandelt. Bei Sprengungen werden die Zusammenhänge Ladung-Amplitude sowie der Gruppenordnungen besonders hervorgehoben. In einem kurzen Abschnitt werden die Methoden der Wellenerregung bei Seemessungen dargelegt (S. 31—44).

In Kapitel 4 »Das Schwingungssystem Geophon/Erdboden« werden — von der vorangehenden Praxis abweichend — die theoretisch-experimentellen Resultate detailliert angegeben. Hierbei wäre es vielleicht kein Fehler gewesen, wenn die Verfasser ihre Ansichten, bzw. Erfahrungen in Beziehung Bodenqualität, Überdeckung und Festigung dargelegt hätten (S. 44—50).

Unter »Störwellen bei seismischen Messungen« (Kapitel 5) werden von der Bodenunruhe bis zu den in der Tiefe sich ausbildenden multiplen Reflexionen bzw. refraktierten Wellen alle Erscheinungen aufgezählt und eingehend diskutiert, in einigen Fällen — wie z. B. bei den Geist-Reflexionen — werden auch die Methoden der Filterung analysiert. In diesem Kapitel ist mir nicht einleuchtend, weshalb die Tauchwellen unter die Störwellen gezählt werden (S. 50—79).

Die Kapiteln 6 und 7 (Reflexions- und Refraktionsverfahren im Feld) beginnen mit den Regeln der Überprüfung der Apparaturen. Es ist mir jedoch nicht ganz klar, warum die Richtungscharakteristiken der Geophonbündelungen, das System der Mehrfachüberdeckung, das Beobachtungssystem mit gemeinsamen Reflexionselement oder gar die Summierung mit Magnetköpfen und der Vibrator in diesem Kapitel und nicht bei der Planung behandelt wurde. Ähnliche Bemerkung gilt auch für die Beschreibung des Refraktionsverfahrens, da es die Beobachtung der Wechselwellen und des weitwinkligen Reflexionssignals enthält, die ebenfalls viel eher zur Planung der Lösung eines gegebenen geologischen Problems, als zur Beschreibung der Feldmethoden behilflich sind (S. 79—104).

Ein kurzes Kapitel (8) befaßt sich mit dem Problemenkreis der Korrelation der Reflexions- und der Refraktionswellen. Die Beschreibung ist gut, gibt aber nur eine Übersicht in Umrissen; es wäre vielleicht noch günstiger gewesen, diese Fragen als Teile späterer Kapiteln (11 und 12) zu behandeln, da ja jede Auswertung mit der Lösung dieser Aufgabengruppe beginnt (S. 104—112).

Die Korrektion (Kapitel 9) bedeutet heutzutage für jede seismische Messung, aber besonders für die Reflexionsseismik eine sehr zeitraubende und arbeitsaufwändige Operation. Das kann dem Kapitel entnommen werden, bzw. bereits ein Leser mit geringer Praxis kann darauf folgern. Meiner Meinung nach ist die Anzahl der erwähnten Methoden zur derzeitigen seismischen Praxis zu wenig, weil diese nur im Idealfall eine Lösung geben und auch so nur für die statische Annäherungskorrektur ersten Grades. Die Frage der Korrekturen zweiten und dritten Grades (letztere ist die automatische Korrektur) wird nur kurz aufgeworfen (S. 112—125).

Das umfassendste, wenn auch nicht umfangreichste Kapitel (10) des Bandes enthält die Kenntnisse über die Geschwindigkeitsbestimmungen. Es beginnt mit eindeutigen Definitionen und schließt mit der Bearbeitung der Laufzeitkurven der Tauchwellen. Es fehlt jedoch der Begriff der Effektivgeschwindigkeit oder rms (root mean square) Geschwindigkeit und deren Zusammenhang mit den Intervallgeschwindigkeiten, die ja eines der prekärsten Fragen der dynamischen Korrektur in der Praxis ist (S. 126—155).

Kapitel 11 »Auswertung von Reflexionsmessungen« befaßt sich mit allen im vorangehenden nicht behandelten Fragen und behandelt sogar, wenn auch auf geringen Umfang eingeschränkt, die geologische Interpretation. Das Kapitel enthält alles, was zum Verstehen der Auswertungstechnologie notwendig ist. Bemerkungen habe ich dennoch, worauf ich schon bei Kapitel 8 hingewiesen habe. Zu diesem Fragenkomplex gehört auch die Frage der Reihenfolge der Besprechung, was die Signalerkennung, den Begriff des Verhältnisses Signal/Rausch und dessen Verbesserungsmöglichkeiten betrifft. Meine zweite Bemerkung knüpft sich zu den Zweifeln, die bei den Verfassern von Lehrbüchern allgemein auftreten: Ist es notwendig und zweckmäßig zur Zeit der zeitgemäßen Auswertungstechnologie mit EDV-Anlagen auf die Auswertung mit Papier und Bleistift zurückzugreifen? Meiner Meinung nach ja, weil die Reflexionsseismik derzeit — im großbetrieblichen Ausmaß — nur in der Kohlenwasserstofferkundung eingeführt ist, bei der Erkundung anderer Mineralrohstoffe, wegen des großen Kostenaufwandes jedoch noch nicht. So ist es bei der Einführung der Methode zweckmäßig, aus Kostengründen zur »klassischen« Auswertung mit Papier und Bleistift — wenn auch nicht im großen Außmaß — zurückzugreifen. Die Kenntnis der klassischen Auswertung ist aber auch deshalb notwendig, weil sie den Anfänger in der Seismik oder den die Ergebnisse anwendenden Geologen zur einer Anschauung hilft, die ihm aus der Auswertung mit Rechenmaschinen nicht immer ins Auge fällt (S. 155—223).

Für Kapitel 12 »Auswertung von Refraktionsmessungen« ist alles was ich zum vorangehenden geschrieben habe mit der Ausnahme gültig, daß bei dieser Methode die Papier-Bleistift Auswertung heutzutage noch allgemein ist. Dies folgt aber nur aus der derzeitig relativ zurückgedrängten Lage der Meßmethode, wird aber auch in der Erdölerkundung nicht zu lange andauern (S. 223—252).

Das letzte Kapitel (13) befaßt sich mit der räumlich Darstellung seismischer Messungen und zeigt auch einige Beispiele (S. 252—260).

Die über das Buch gemachten Bemerkungen zusammenfassend, kann ich mit Freude feststellen, daß dieser Band für jene, die die Praxis erlernen wollen, aber auch für die in der Praxis arbeitenden eine nützliche Lektüre ist. Die Nutzung sowie die darauf basierende Erweiterung der Kenntnisse wird im großen Maße durch die nach jedem Kapitel stehende Literaturzusammenfassung gefördert. Diese enthalten die Titeln von annähernd 200 Büchern, bzw. Artikeln. Meiner Ansicht nach haben die Verfasser eine sehr wertvolle und nützliche Arbeit geleistet.

Seitens der Seismiker verdienen beide Verlage einen besonderen Dank für die sehr sorgfältige und fehlerfreie Ausstattung und Ausgabe.

O. Ádám

GEOELECTRIC AND GEOTHERMAL STUDIES
KAPG GEOPHYSICAL MONOGRAPH

Editor-in-chief: *A. Ádám*
Akadémiai Kiadó, Budapest, 1976. 752 p. $ 48.

The Committee of Academies of Sciences of Socialist Countries for Planetary Geophysical Investigation (so called KAPG program) has the task to organize the participation in international (UGGI) projects. The investigation of the distribution of thermal energy, playing dominant role in the earth's structure and development, was from the beginning on an important part of the KAPG activity. The strong dependence of electrical conductivity on temperature makes the electromagnetic induction studies a very effective tool to give ideas about geothermal activity in the depth.

The main task of this monograph is to analyze the connections between the results of geothermic and magnetotelluric investigations. It contains papers of 93 experts presenting the main achievements of 6 countries in these disciplines during the period 1960—1973.

Chapter I. (ed. by CH. OELSNER, p. 19—99.) — Collecting and Processing of Geothermal Data — deals in 9 papers with geothermal measuring methods, measurements of thermal properties of rocks, processing and correction of geothermal data and theoretical considerations on inhomogeneous geothermal fields.

Chapter II. (ed. by G. FANSELAU and G. PORSTENDORFER, p. 100—318) — Collecting of Geoelectric Data; Theory, Data Processing, Limitations — treats in 13 papers the most important problems of interpretation and data processing of magnetotelluric methods. At first we can read about the theory, the data analysis of magnetic variation profiling and geomagnetic induction in several models. Then the problems in the interpretation and data processing of magneto-telluric sounding curves above inhomogeneous and anizotropic geological structures are considered in several papers. Two papers discuss the homogeneity of the inducing external geomagnetic field in respect of substroms and pulsations. In the end homological relations of the electromagnetic and other geophysical methods are given from the point of view of the complex interpretation.

Chapter III. (by M. P. VOLAROVICH and E. I. PARKHOMENKO, p. 319—369) — Electrical Properties of Rocks at High Temperatures and Pressures — deals separately with the main rock types of the crust and upper mantle, and gives on the basis of laboratory data electrical resistivity distributions with depth.

Chapter IV. (ed. by V. ČERMÁK, p. 371—480) — Geothermal Data and their Complex Interpretation on Test Areas — gives in 13 papers a summary of heat flow measurements in Central and Eastern Europe and Soviet Asia. Here are introduced the appendices of the monograph, i.e. the maps of the KAPG Working Group for Geothermics "Heat Flow Map of Central and Eastern Europe" and "Geoisotherms at the Depth of 1 Km in Central and Eastern Europe". Separate papers are dealing with the interpretation of heat flow as well as deep temperature measurements in the GDR, Poland, Czechoslovakia, Hungary, Bulgaria and the USSR (European Part, Caucasus, Baikal Rift, Siberia, Kamchatka and Okhotsk Sea, Yakutia).

Chapter V. (ed. by A. ÁDÁM, and M. N. BERDICHEVSKY, p. 481—717) — Deep Electromagnetic Investigations and their Geothermic Interpretation On Test Areas — is a rich summary of the results obtained by deep geoelectromagnetic measurements. Inside the Chapter the 31 papers are divided in five parts. The first four deal with the geoelectric structure of the German–Polish Depression, the Czech Massive, the Pannonian Basin and the Carpathians. The fifth part contains electromagnetic induction data from 16 areas of the Soviet Union and their interpretation. They are as follows: Baltic-Scandinavian Shield, Belorussian Crystalline Massif, North-Western and Central Part of the Russian Platform, Ukraine, Ural, Eastern Pre-Caucasus, Turkmenia and Azerbaijan, Tungus Syneclise and West Siberian Plate, South Siberian Plate and Baikal Rift Zone, Yakutia, Soviet Far East, Sakhalin, Kamchatka, Arctic Ocean. Each part contains besides the data details about the processing and interpretation from geoelectric, geothermal as well as tectonical point of view.

Chapter VI. (by I. S. FELDMAN, p. 719—730) — On the Nature of Conductive Layers in the Earth's Crust and Upper Mantle — discloses models for the development of conductive layers.

The monograph is from both theoretical and practical aspects as well as considering his informative data a unique synthesis of deep geothermal and magnetotelluric investigations. The editors did not try in any way to bring the contributions in any uniforme scheme so the description of the various ideas and methods used in the measurements, analysis and interpretation gives a wide survey of the subject and transmits actual international experiences to the readers.

E. Takács

INDEX

Karl Rinner – 65 Jahre – *J. Somogyi* 303
Halmos, F.: Methodische Fragen der geodätischen Anwendung von Dopplerschen Satelliten-Beobachtungen 305
Lakatos, I. J.: The role of wettability in polymer flooding 325
Hristow, W. K.: Allgemeiner Fall der gemeinsamen Ausgleichung der Höhenwerte und Geschwindigkeiten eines Nivellements 337
Debreczeni, E. – Tarján, I.: Dimensionierung und Untersuchung der Funktionsweise von Seitendüsenejektorpumpen 347
Steiner, F.: Contribution to the determination of the gravitational effect of rectangular prisms 375
Zambó, J.: Das Ausstecken des Schachtansatzpunktes außerhalb des produktiven Gebietes 383
Roller, B.: Numerische Untersuchung des Zustandes von Gesteinskomplexen 395
Cz. Miletits, J.: A study of geomagnetic pulsation periods 425
Berktold, A.: On the distribution of the electrical conductivity in the area between the Rhinegraben, the Bohemian Massif and the Central Alps 437
Tátrallyay, M.: On the numerical modelling to two-dimensional conductivity structures in cylindrical and Cartesian co-ordinates 441
Iliceto, V. – Santarato, G.: Proposals for the standardization of magnetotelluric digital tapes 455
Hazay, I.: Adjustment of levellings for the determination of the vertical movements of the Earth's surface 463
Sátori, G.: Interpretation of simultaneous variations of galactic cosmic rays and of the level of atmospheric radio noise 475

Recensiones

Flügge, S. ed.: Handbuch der Physik – Encyclopedia of Physics – *P. Bencze* 487
Crustal and upper mantle structure in Europe – Monograph No. 1 – *E. Bisztricsány* .. 488
Schröder, W.: Entwicklungsphasen der Erforschung der leuchtenden Nachtwolken – *A. Tárczy-Hornoch* 488
Piller, H.: Microscope Photometry – *M. Gerencsér* 489
Meissner, R. – Stegena, L.: Praxis der seismischen Feldmessungen und Auswertung – *O. Ádám* 490
Geoelectric ond Geothermal Studies – KAPG Geophysical Monograph. Ed. in-chief: A. Ádám – *E. Takács* 491

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