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THE APPENDIX
CONTAINS A PAPER, PUBLISHED IN 1922, NOW
COMPLETELY OUT OF PRINT, BY ROLAND EÖTVÖS
AND HIS CO-WORKERS

APPRECIATION OF ROLAND EÖTVÖS

by

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The feeling of a deep reverence is arising in my mind as I fulfil my authority to show the portrait of Roland Eötvös by force of personal commemoration in course of our session. On and on lessens the number of those who saw still Roland Eötvös with their own eyes and heard him with their own ears. They participated in the fortunate experience, to obtain an idea about strict morals and scientific notability on the basis of a living example. Still it is ringing in my ears the utterance of Géza Bartoniek, – the director of Eötvös College –, who returned thanks to his good fortune for passing away his life in the shadow of a great individuality.

Roland Eötvös was with distinguished respect in regard to the reputation of his father as author and statesman but he never had a claim of inheritance on this reputation. He was full of ambitions, and was determined on dedicating his life to great products worthy of him, but for the appreciation followed in theirs footsteps he wanted to have him to thank entirely. It is characteristic of his impatience, he was ready of the interruption of his studies at the university reporting to a pole-expedition. It was only a momentary dream.

Even the fatherly influence could not divert from his intention giving over the juristic studies insured a straight course of his life and to turn to the study of physics. It is not probable at all that he would have been droven by his natural bent towards this profession. It is still less possible when in all fields the political discussions were the questions of the day, problems of natural science would have been displayed the directing influence on the young Eötvös decisively. He refers to the real reason of his decision himself. When he matriculated to the Heidelberg' University, he considered almost incredibile that he could enjoy the same air as Kirchhof and Helmholtz. Their names encircled with honour got to all corners of the cultural world. The young student could dream about similar works with them he might rise to the resembling degree of the scientific greatness. It is remarkable, he is not thinking about the two first names of the list of German physicists, neither of Mayer, nor of Clausius. These two giants are definitely with theoretical attitude, they cannot be ranged among the experimental physicists. The admiration of Eötvös was fascinated rather by the great results of the experimental researches, this is a clear sign that this form of the examination exercises a pecular attraction on him. Indeed, when he gets later authority to give a course on the theoretical physics at the Budapest' University, he keeps the management of this proffessorship only for a short time and as soon as possible he passes on the guiding of the experimental institute. His experimental vocation was obviously already at his student time. For measuring of capillary constant he designed during the laboratory training such an instrument, that gave possibility to contain undecided liquids into a closed wessel and with them their surfaces were protected from impurity. As far as the theory was concerned he was in contradiction to such difficulties in his student time, which almost finished catastrophically. He was very desperate in consequence of the theoretical lectures hard to understand of professor Neumann, that is, he was firmly determined to change his profession. Only his father's words gave him persistency for the future too, when his father declared before him that fulfilling his commitments is a question of honour.

Eötvös attained the analytical method of the research at the German universities. His most significant theoretical thesis – as it is called "Eötvös's law" – was still derived by meditative methods. He paid the tribute of admiration to those properties of bodies being in van der Waalsian proper state, that they retain their similarity in mechanical respect too. From that he drew the conclusion, if he proportions the vapour pressure effected on plate of a liquid cube to the surface tension operated edgewise he gets such a quotient which is the same for all liquids. The circumstance that we can today include the "Eötvös' law" into the frame of a more common law, it does not conclude anything from its value.

After taking over the experimental institute Eötvös got into far closer relationship with the secondary school. Surely the physicist probationer teachers could get lectures of physics in their first year directly from Eötvös. The circle of knowledge what they were brought by them characterized their preliminary training. The public average in that time could get only a negative criticism. The lowly scientific level of the teaching body was complained reightfully with the exception of a certain. On that condition Roland Eötvös made en effort to change in his Minister of Education's time as he wanted to create an institute included all the branches of study of secondary school. To the memory of his father he founded the József Eötvös College, that in one respect in the richness of the library but on the other hand in the active force of its attitude stimulating to labour indicated till now the most eminence in the history of our colleges.

No the exacting selection at the registration was the guarantee of full value of the prospective advance of the student but the competition sprung from traditions. It would be worthy to count up whether howmany former members of the College work till now at our universities. Eötvös suggested by his own example to his College the slogan of today: "Work has became a matter of honour and glory " Every student who passed through the Garden of University could see day after day the well-lighted figure of Eötvös in the library of the building "D" of today, as he was wrapped up in his calculations sitting at his desk. Whoever cogitating objectively wishes without doubt, if only it were successful to raise from the dead this unsurpassed institute. But with a necessary supplement ! The old College does not know the ideological and political education. Such a standpoint is today unimmaginable of course.

By all means we have to propound the question, what is Eötvös himself like from the point of view of ideology and of social policy It is not possible to apply for him the same requirement and units of meassurement as for one or another of the excellent persons of our age. His age was full of passion but in the point of view of socialist it was the age of unfruitful policy. The governing party preserved rigidly its position at the royal court because he did not touch to the compromise of 1867 according to the exhortation of Franz Josef I. Opposite of him there stands the party of the opposition of 1848 which received for example with cry of victory the result of his fighting that it succeded in transmuting the name of the imperial-royal army to the name of imperial and royal army but he beheld with celestial tranquillity that many hundred thousands Hungarian citizens escaped to America away from the unbearable misery. Eötvös abandoned to this policy already with his choice of profession. He saw clearly with his searching glance that the science, the technical work and the high-class civilization set to the service of the national economy could garantee for our fatherland the prosperity in the country and the prestige in foreign lands. He declared also his opinion plainly as President of Academy He denounced the worthlessness of the patriotical catchphrese with the courage of great souls, the audience never heard idle phrases from him.

Doubtless he was a great scientist with an advanced mentality and this is the only requirement that may be claimed to the judgement of the notabilities of earlier ages. Roland Eötvös was chosen as the new denominator of our University by the excellent fine feeling of the Hungarian Republic.

It was unexpected for the national physicists the decision of Eötvös, that after the deduction of the law denominated from him he turned away from the problems connected with the microphysics and transfer his experimental activity to the field of gravity. We could believe that he detected any want in the Newtons theory of gravity and he endeavoured reform it. It is remarkable that this want exists indeed, but its authenticity came only to light long later, and then Eötvös – at least indirectly – contributed to its elimination. The main cause of his decision was his callings for the experiments, as a purpose of him was the powerfull refining of the gravitational measurements.

It is obvious, the result of this work was productiv of an exceptional practical advantage. The very place of the application of the gravitational instruments is not the laboratory but the open nature where the balance spicul looks under the earth and gives precise information about the unvisible trends. The instrument would be entlisted in the service of geology as a reliable and ready attendant.

Eötvös himself spent his days on the ice of Lake Balaton with pleasure to investigate with his equipment the trends under the basin. Expeditions were organised according to his initiations, wich investigate the terrain under the surface on prescribed direction. The Eötvös torsion balance gained a pecular fame then it was obvious that it might be produced in a well carrying form for heavy fields and it was the correct indicator of the occurrences of mineral oil by its precise measurements. This circumstance made the name of Eötvös world-famed and his instrument a required investigating article. It is hardly believable that Eötvös could extort the exceptional sensitivity of his torsion pendulum from the form of torsional balance.

During the first stage of investigations it looked like we could not expect any fundamental interference in question of the force of gravity. However this problem existed already in the very moment of its conceptual birth. The gravity is the most striking case of the longdistance operating force. We do not know any materially medium, mechanical alteration of which we could explain as a force. When the Earth rushes with her full speed round the Sun she always occupies other and other places compared with them. That is the question, what a mechanism could make possible that the attractive force of the Sun could follow the Earth and it is present ready for action in all position of the Earth. Newton himself refuses very definitely that assumption as if he had believed in the longdistance operating force. However he does not endeavour to a positive standpoint and he is content with that declaration he does not pry into the question of "causa gravitatis".

All the more this question was investigated by another notability of the world appearing far later: Albert Einstein. After ten year's strenuous mental work he gave also answer to the question but it was only possible that he was supported by such a thesis, faultless experimental verification of which was due to Eötvös. We have to think of the identity between the heavy and the inert mass. It was well know on experimental basis, if we determined on balance the weight of two masses, from them the havier has a greater resistance towards the same acceleration. In other words, to greater weight it corresponds a greater inertia. But the fact, that the weight of a certain body is srictly proportionate to its inertia was not demonstrated by the previous measurements precisely. The exact verification was so much wanted that the Göttinga's University announced a competition in 1906 for the final solving. Eötvös and his collaborators undertook the work and they won the offered prize on the basis of precize execution. The fundamental idea came from Eötvös and the measurement connected again with the sometimes well proved torsion balance. The previously measurements made with other methods which was similarly accomplished by excellent experimental physicists in point of view of accuracy could not compared with the Eötvös's result at all. The limits of inherent error in measurements fall into 1/200 000 000 part of the mass of the employed body.

Do not mind, that the scientific importance of this measurement is perhaps a thing of the past. Recently it was raised a premise on the part of Bondi – an English physicist –, that between particle and antiparticle a negative gravity comes into being. He wanted to solve the question supported on the accuracy attained by Eötvös. As it turned out eventually this accuracy is not enough for the definition. But it became known in this time, one of the Eötvös' collaborators – D. János Renner – perfecting the carefullness of construction corrected the accuracy of result with a complete order of magnitude. The methode of Eötvös turned to be excellent. Einstein could ground undoubtedly his famous equivalence equation on the measurements of Eötvös. In all the manuals and school-books of the world which introduce the theory of relativity we can find there the name of Roland Eötvös. We give also evidence today that the precize account of identify of two kinds of mass is till now the greatest accomplishment of the Hungarian physics.

The strict social judgement of today could also find those features of the spiritual likeness of Eötvös absence of which do not overlook even to the man of old ages. You see well the man full of constructive ambitions in him who separated himself from his cast, threw down the aristocratic Hungarian gala-dress and put on the white laboratory smock. You see in his person the active minister who was then "His Excellency" according to the official approaches, but in reality he is the creator of the working method of the youth in his college by his own example. And they recognize at last the eminent scientist, who discovers new veracities but his knowledges are not locked in the safe of abstract science but they are set to the service of technical advancement.

Already four decades passed away since the death of Eötvös but this time was enough to set us against such an extremely new physics about which the age of Eötvös could not take only any notice of the idea of daybreak. The pursuance of the work on this field was maintained for the following generation. But we do not make a mistake in time when our festal commemoration was attached to the personage of Roland Eötvös. His genius crossed also over with tragical grandiosity the threshold of our age. All who visited him in his last days, passed on to others with wonder, the dying scientist wanted to talk only about atom and electron. His name, who is accompanied to coffin by the science does real honour to our University.

THE EÖTVÖS EXPERIMENT

With 2 figures

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SUMMARY

Among the significant scientific works of Roland Eötvös his investigations concerning the proportionality between the attraction of masses and the inertia are of a special importance from the view point of modern scientific development. By means of these investigations he has proved, that even if there were a difference in the attraction of materials of various quality, it ought to be smaller than 1/200 000 000. Subsequent investigators have exaggerated this accuracy. The accuracy has been raised by one order of magnitude in the 1930 by the author of this paper. In the last years it was found by R. H. Dicke that the attraction of gravity is independent from the quality of materials to an accuracy as high as $1/10^{11}$. In connection with his experiments Eötvös has also investigated the absorption of massattraction with his very sensible compensator and at his experiments of high accuracy he has not found any absorption.

The various significant scientific works of Roland Eötvös are usually denoted by brief characteristic denominations. Thus we speak about the Eötvös law of surface tension, the Eötvös effect expresses the gravitational changes of moving bodies and mainly recently his significant investigations concerning the proportionality between gravitational attraction and inertia are referred to as Eötvös experiment. It is well known for the professional that Eötvös and his collaborators Dezső Pekár and Jenő Fekete won the Benecke Prize of the University of Göttingen in 1909 with their paper describing these investigations. The prize-winning paper was not published by Eötvös himself, it was published by his collaborators as a posthumous work several years after his death in the 1922 volume of Annalen der Physik. This work contains the prize-winning paper in a slightly abridged form. The original paper was published to somewhat fuller extent in the collection "Eötvös's Gesammelte Arbeiten", published in 1953 edited by Pál Selényi.

The investigations, that served as a basis for the paper, were carried out by Eötvös and his collaborators around 1900, his investigations concerning this object go back, however, to earlier times, they are virtually equal in age with his gravity researches. A document of his earlier investigations on this object is his report in the Academy of Sciences of Hungary on 20. th of January 1889, the material of wich appeared in 1890 in Hungarian and German.

The problem is a very old one, since it contains the question whether the acceleration depends on composition of materials in the gravitational field. This question was answered by the classical falling experiments of Galilei and later by the observations of the swinging time on pendulums, loaded by various materials, conducted by Newton. These experiments lead to the conclusion, that the gravitational attraction is independent from the composition of materials.

The experiments conducted by excellent investigators during centuries showed a gradual development, since the accuracy of the measurements constantly improved. The accuracy of Newton's experiments achieved 1/1000, in the first half of the 19-th century the experiments of Bessel with pendulums of various materials achieved the accuracy of 1/50 000.

In the 80-es of the last century Eötvös used the torsion balance constructed by him for his experiments concerning the proportionality between gravitational attraction and inertia and the same method was used by him with an improved process and greater accuracy for his investigations carried out at the beginning of our century. According to his report on the Academy in 1889 the accuracy achieved at that time was 1/20 000 000, while the accuracy of the experiments described in the prize-winning paper of Göttingen reached 1/200 000 000, i.e. it was 10 times higher. In his former experiments Eötvös tested brass, glass, antimonite and cork. In the experiments for the prize work of Göttingen he tested the following materials: magnalium, snakeweed, copper, water, crystalline cupric sulfate, solution of cupric sulfate, asbestos, tallow, silver sulfate and iron sulfate. The above mentioned materials were compared with platinum. The experiments on the proportionality between gravitational attraction and inertia as conducted by Eötvös were founded on an original principle, completely differring from the earlier ones, therefore they deserve the name "Eötvös experiment". The principal basis of Eötvös investigations is, that since the centrifugal force from the rotation of the Earth — as an inertial force — is independent from the material composition, if the gravitational attraction caused by the Earth were different for various materials, then the resultant, i.e. the gravity would change according to the quality of materials and the direction of gravity would also be different for various materials. By other words it means that for every kind of material a different niveau surface would form. The torsion balance of Eötvös is excellently suitable for measuring very small deviations in the direction of gravitational force.

In the followings I shall briefly decribe Eötvös's method. On fig. 1. ε is the angle between the force of attraction and the direction of resultant gravitational force. η is the assumed deviation of direction caused by differences of material composition. The value of angle ε depends on geographical latitude and its maximum is about 6' on the latitude of 45°. From the triangle on the figure PBB_1 $\eta = \frac{G_1 - G}{g} \sin \varepsilon$, where, G and G_1 represent the varoius accelerations according to material composition.

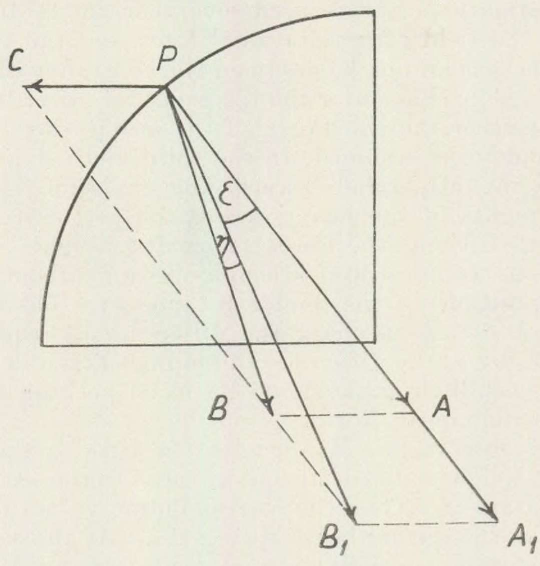

Fig. 1

Let us assume, that $G_1 - G = \varkappa G$, hence
$\varkappa$ is a factor depending on material composition.

Let us assume, that the ends of the beam of the torsion balance are loaded with weigths of various compositions, and the corresponding factors of material composition is $\varkappa_a$ and $\varkappa_b$. The horizontal torque arising from the difference of material composition in the position of the beam characterized by the angle α is:

$$(M_b l_b \varkappa_b - M_a l_a \varkappa_a) G \sin\varepsilon \sin\alpha$$

In this expression M_a and M_b are the weights loading the balance, l_a and l_b are the corresponding arms of rotation.

The condition of equilibrium as known from the theory of the torsion balance, adapted for the horizontal variometer and considering the assumed difference due to material composition is

$$\vartheta = \frac{1}{2}\frac{K}{\tau} U_\Delta \sin 2\alpha + \frac{K}{\tau} U_{xy} \cos 2\alpha - \frac{M_a l_a h}{\tau} U_{xz} \sin\alpha +$$

$$+ \frac{M_a l_a h}{\tau} U_{yz} \cos\alpha + \frac{M_a l_a}{\tau} (\varkappa_b - \varkappa_a) G \sin\varepsilon \sin\alpha$$

In this formula K is the moment of intertia of the suspended system, τ is the torsional moment of the wire, h is the distance from the centre of the lower suspended weight to the horizontal plain of the beam. U_{xz}, U_{yz}, U_Δ and U_{xy} are the second derivatives, characterising the gravitational field.

The angle ϑ is the deviation from the untorqued position of equilibrium.

In these investigations Eötvös used several methods. In his first method he assumed, that the field of gravitational force and the torque of the wire are constant. In the second one he assumed the constancy of the gravitational field, but allowed a slow change for the torque of the wire. In the third most accurate method neither the constancy of the gravitational field nor that of the torsion wire had to be assumed. In the third method Eötvös used double torsion balance with antiparallel beams. The experiment was arranged so, that the upper weights of the beam were made of the same material with $\varkappa_b$ characteristic coefficient, the lower suspended weights were made of the materials of various composition, selected in comparison with coefficients $\varkappa_a$ and $\varkappa'_a$. The positions of the double balance were observed in azimuthal angles shifted for 90° Let us mark the difference of readings, obtained in positions of $E-W$ by v, the difference of readings taken in positions $N-S$ by m and be $\Delta\alpha$ the very little angle closed in initial position by the axis of the beam and the astronomical North.

Two series of observations are needed: at first to the end of the first beam is attached a weight with coefficient $\varkappa_a$, next to the second beam another weight with coefficient $\varkappa'_a$. Then the corresponding values for the first beam are v_1, m_1, $\Delta\alpha_{1_I}$, for the second beam v'_2, m'_2, $\Delta\alpha_{2_I}$. At the second series of observations the suspended weights are mutually changed, then the values for the first beam are v'_1, m'_1, $\Delta\alpha_{1_{II}}$, and the values for the second beam are v_2, m_2, $\Delta\alpha_{2_{II}}$.

The ratio of the quantities v and m is not subjected neither to the casual changes of sensibility, nor to the slow change of the gravitational field. From the above mentioned position of equilibrium one can compute the difference of coefficients, characterising the material composition.

$$\varkappa_a - \varkappa'_a = \frac{m\tau}{8LM_a l_a G \sin\varepsilon}\left\{\left(\frac{v_1}{m_1} - \frac{v'_2}{m'_2}\right) + \left(\frac{v_2}{m_2} - \frac{v'_1}{m'_1}\right)\right\} +$$

$$+ \frac{m\tau}{8LM_a l_a G \sin\varepsilon}\left(1 + \frac{v^2}{m^2}\right)\{(\Delta\alpha_{1_I} - \Delta\alpha_{1_{II}}) - (\Delta\alpha_{2_I} - \Delta\alpha_{2_{II}})\}$$

In this formula L is the distance of the scale.

In connection with these experiments Eötvös examined very carefully the achieved accuracy. It can be seen from the above formula, that if there were any difference in the gravitational attraction of various materials, it ought to be appear in the value of the ratio v/m. Since the values v and m were directly observed, according to the rules of error propagation the mean error of the mean values of the quantity v/m from the mean errors of the observed mean values is

$$\Delta\left(\frac{v}{m}\right) = \sqrt{\left(\frac{v}{m^2}\Delta m\right)^2 + \left(\frac{1}{m}\Delta v\right)^2}$$

The mean errors Δv and Δm were calculated by Eötvös in the usual way with the formula $\sqrt{\frac{\Sigma\delta^2}{n(n-1)}}$.

The individual series of observation consisted of about 100 readings, and since within the individual series the values v and m had a minute of any change, the δ deviations as calculated from the mean values were very little and in consequence of this the mean errors Δv and Δm of the mean values proved to be very small.

This can be expected, of course, if any disturbing circumstance is eliminated, which may change the position of the beam of the torsion balance. Eötvös paid particular attention to the elimination of disturbances like these. He used carefully prepared and checked wires in the torsion balance. He cancelled the occasional electrostatic and magnetic influences and created a temperature protection for the instrument that no disturbances due to changes in temperature could take place. The mentioned disturbing effects may appear in the values for v and m, and if they were not eliminated in a corresponding manner, they might cover the effect, that may be caused by the differences of material compositions in the gravitational attraction. The order of magnitude of the accuracy achieved by Eötvös are characterised by the following data. In the series of about 100 observations the mean errors of the mean values of the quantities v and m are generally of $\pm 0{,}01$ scale division. According to the rules of error propagation the mean errors of the ratios v/m are of the order of about $\pm 0{,}002$; the error of the sum of the differences of ratios v/m is about $\pm 0{,}004$. If this is multiplied by the outer factor of appr. $0{,}4 \cdot 10^{-6}$, the mean error has the value of about $\pm 0{,}0016 \cdot 10^{-6}$ Let us take into consideration, that in the several series of Eötvös experiments the investigated material was placed is some container, metallic or glass tube and therefore the error must be related only to the part of mass made of the investigated material. In consequence of this the mean error increases to about $\pm 0{,}003 \cdot 10^{-6}$. The value of the differences $\varkappa_a - \varkappa_a'$ was determined from the observations to be $0{,}001 \cdot 10^{-6}$ $0{,}006 \cdot 10^{-6}$, thus they are partly bigger and partly smaller, than the computed average errors, their order of magnitude, however, corresponded to the order of magnitude of the mean errors. From these results Eötvös drew the conclusion, if there were any deviation between the gravitational attractions of various materials, it ought to be smaller than the value of $0{,}005 \cdot 10^{-6}$

In such a way the accuracy, achieved by Eötvös was 1/200 000 000.

Apart from the investigation of the above mentioned materials Eötvös conducted special experiments with radioactive materials. The age, in which Eötvös carried out his very accurate experiments on the proportionality between gravitational attraction and inertia, witnessed the discovery of radioactive phenomena. At that time R. Geiger maintained the idea, that the radioactive radiation absorbs the energy of attraction. Eötvös placed a radium-compound in a little glass tube in the proximity of the platinum weight on the beam of the torsion balance and experienced a little effect of repulsion or attraction depending on the position of the glass tube. The experiment was repeated in such a way, that the glass tube containing the radium-compound had been replaced by another glass tube, in which a thin platinum wire was welded and connected in an electric current. When the quantity of heat, produced in the platinum wire by electric current, was equal to that, performed by the radiation of the radium-compound, the effect of repulsion and attraction was observed to be the same, thus the phenomenon was completely explained by the temperature effect. There was no trace of the absorption of gravitational attraction.

Following the investigations of Eötvös, in the years 1930—1935 I have also dealt with the question of proportionality between gravitational attraction and inertia. My endeavour was to increase further the accuracy and to extend the investigations to various materials, not included in experiments of Eötvös. For the purpose of the experiments I have used the double torsion balance No. III of the Geophysical Institut, which at that time proved to be the most reliable laboratory instrument. It was especially remarkable for being almost unsensible to temperature influences. The torsion wires were long ago prepared platinum — iridium wires, the temperature coefficient of which was practically zero and had no elastical drift at all. A special care was taken of the constant temperature of the surroundings. I have compensated the geomagnetic field similarly to Eötvös and apart from this I have used the very observations for determining the occasional magnetic effect. This was achieved by carrying out the observations besides the chief directions of E—W and N—S also in intermediary positions making an angle of 45° with these directions. From the latter the occasional magnetic effect could be computed. This method was used by me at the comparison of brass and diamagnetic bismuth. Otherwise the observations in the intermediary positions of 45° represent a series in themselves, from which the differences $\varkappa_a - \varkappa_a'$ can be determined independently from the result of the observations in the main positions, thus they serve as a control. This investigation was carried out at the comparison of copper and an alloy of manganous copper and from the two series of observations I have recieved the following independent results: in the main positions:

$$\varkappa_a - \varkappa_a' = +0{,}08 \cdot 10^{-9} \pm 0{,}20 \cdot 10^{-9}$$

in the intermediary positions.

$$\varkappa_a - \varkappa_a' = +0{,}12 \cdot 10^{-9} \pm 0{,}22 \cdot 10^{-9}$$

The differences of the coefficients characterising the material composition were within the limits of error in both series and the error of the mean values was about $\pm 0{,}2 \cdot 10^{-9}$, i. e. 1/5 000 000 000, that exceeds 25 times the accuracy of the experiments conducted by Eötvös

At the subsequent experiments the order of mean errors was about $\pm 0{,}5 \cdot 10^{-9}$, i. e. 1/2 000 000 000, hence the accuracy exceeds in general the accuracy of the experiments of Eötvös 10 times.

Besides of the already mentioned materials the following materials were compared:

platinum — brass
batavian glass beads — brass
batavian scrap glass beads — brass
paraffin — brass
aluminium fluoride — copper.

The differences $\varkappa_a - \varkappa_a'$ computed from the series of observations had partly positive, partly negative signs, the mean errors were of the same order, their mean value being $\pm 0{,}52 \cdot 10^{-9}$.

Eötvös used another method for examining the proportionality between gravitational attraction and inertia, too. This method is founded on the comparison of the tidal force raised by the Sun and of the centrifugal force appearing on the orbit of the Earth. The torsion balance is very suitable for investigating this phenomena. Let us assume, that there are masses of various quality attached to the ends of the beam, orientated along the meridian, and that the Sun exerts a greater attraction to the mass on the northern end of the beam, than to the one on its southern end. In this case the mass on the northern end would move towards the Sun, when it rises and cause a corresponding turn of the beam; at Sunset the mass at the northern end would move towards west and the beam would turn in a direction opposite to the previous one. Thus owing to the different specific gravitational attraction the beam of the torsion balance would show up an oscillation with a 24-hour period, that could be observed with a balance of appropriate sensibility.

In comparison with the above discussed method the latter one has the advantage that the torsion balance remains in the same azimuthal position during the whole series of observations and the object of the observation must be the occasional change in the state of equilibrium. But the sensibility of this method is only a third part of the sensibility of the previous one.

Eötvös and his collaborators compared the coefficient of attraction of magnalium and platinum by the help of this method and the difference $\varkappa_a - \varkappa_a'$ was determined to be of the same order of magnitude, as in the experiments conducted with the other method. For the purpose of such experiments the highly sensitive instrument of Eötvös, the gravitational compensator is particulary suitable.

In the last years the same principle was used essentially for the investigation of the proportionality between gravitational attraction and inertia by the American investigator R. H. Dicke, who however had designed an instrument with modern technics for his experiments.

Dicke suspended a frame in the form of a triangle on three wires, and the corners of the triangle were loaded by weights of the same mass. Two of the three weights were made of copper, and the third was lead chloride sealed in a cylindrical flask. From one silvered side of the triangle the reflected light is brought through an adequate optical device to a wire oscillating at 3,000 cycles per second, from there it strikes a photocell having a certain current intensity at constant illumination, hence a constant current intensity corresponds to the unchanged position of the triangle. If the triangle turns but slightly, the signal from the photocell changes and gives rise to a direct-current voltage which by the help of a servo-mechanism exerts a restoring force on one of the copper weights, thus it makes the whole system to return into its initial position. The restoring force is registered and this measures the rotation angle of the suspended system. Dicke placed the whole system in a metal can and the air was evacuated from it to a pressure of 10^{-6} millimeter of mercury. By such a way he cancelled the disturbances from air convection. The whole apparatus was mounted in a chamber at a depth of 4 m According to an information from him after having overcome many technical difficulties he succeded in improving on the accuracy of the experiments Eötvös by a factor of 50 and on the accuracy of my experiments conducted in the 1930-es by 5. According to latest informations the accuracy in Dicke's experiments raised further to the order of 10^{-11}.

The proposal of Pál Selényi deserves attention, according to which it would be worth-while to extend the investigations on living material.

In the following I should like point briefly at the significance of the Eötvös experiment from the viewpoint of the general theory of relativity, on the one hand, and of the modern atomic physics, on the other

It is known, that the general theory of relativity of Einstein is based on the principle of equivalence, according to which the attractive and inertial masses are equal. It is probable, that Einstein did not know the results of Eötvös experiments as he formulated the theory and he became acquainted with them only afterwards. In any case, undependently from the result of the experiments Einstein was convinced of the correctness of the principle of equivalence. He endeavoured to determine the laws of motion for accelerating systems. Therefore he had to identify the inertial forces with the gravitational ones, that means, that in an accelerating system an observer closed from the outer world can not distinguish whether the motion of a certain mass is called forth by the gravitational field or by the inertia.

Recently the Eötvös experiment has obtained significance also in atomic physics. Namely among the many elementary particles known at present there are ones differring from one another only in the sign of their electric charge. Particles like this are the electron and positron, proton and antiproton etc. There are so called anti-particles that appear mostly in the big accelerators and have a very short period of life. The English physicist Bondi created a hypothesis according to which the gravitational effect from antiparticles would be repulsing. According to this hypothesis the proportionality between gravitational attraction and inertia would not be valid between particles and anti-particles. Since the anti-particles appear most rarely in our world it seems impossible to examine experimentally the principle of equivalence. According to the opinion of the Californian investigator L. J Schiff in the atoms of our world there can be also found anti-particles, since the electric fields in the inner part of the atom create virtual pairs of electron and positron, and if the positrons had a gravity with sign contrary to that of electrons, it ought to appear in the experiments of high accuracy concerning the proportionality of gravitational attraction and inertia. Nevertheless at the high accuracy achieved in these experiments there are no traces of a phenomenon like this, consequently there exists no antigravitation. In any case this problem contributed to bringing the experiments on the proportionality of gravitational attraction and inertia into prominence and to-day's investigators strive to improve further on the accuracy.

The investigations of Eötvös concerning the absorption of gravitational attraction are of great interest, too, the results of which were also described in the prize-winning paper of Göttingen. Eötvös used for these investigations the gravitational compensator, constructed by him. This was a very sensitive instrument and its sensitivity can be virtually raised beyond all limits by appropriately regulating the lead masses in the form of a quadrant, mounted in the proximity of the ends of the beam. If the central line of the pairs of lead quadrants are adjusted in 45° and 225° to the vertical, then one of the quadrants falls under the horizontal plane, the other above it; if the centre lines are adjusted in 135° and 315°, those quadrants raise above the horizontal plane, that were previously below it and conversely. In each case the attraction from one half of the earth exerts its influence through the lead mass of the quadrant, and the attraction from the other half of the earth is not shaded by the lead mass. If the gravitational attraction had an absorption, then the gravitational attraction ought to be greater on that side, where is no shading mass. (Fig. 2.)

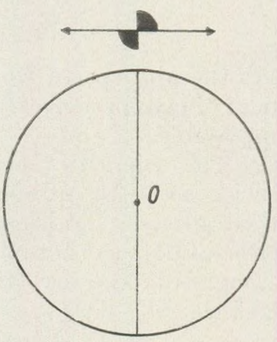

Fig. 2.

If the readings on the torsion balance in the subsequent positions of the quadrants are marked by n_1, n_2, n_3, n_4 then according to Eötvös computations the coefficient of absorption can be expressed as follows:

$$\mu = \frac{n_2 + n_4 - n_1 - n_3}{47890 \cdot 10^6} + \frac{608\zeta}{47890 \cdot 10^6}$$

where ζ means the vertical coordinate of the gravitational centre of the mass at the end of the beam in a coordinate system, the origin of which takes place on the horizontal line passing through the centre of the opposite quadrants. If the beam is adjusted centrically in relation to the quadrants, which must be achieved upon the orientation, then the term containing ζ in the above expression can be neglected.

According to the results of the performed experiments the numerator of the first term is of the order of unit, thus it can be stated if the gravitational attraction had an absorption, then the absorption of the lead quadrants of the compensator must be smaller than one in fifty milliards. This shading corresponds to a lead layer with a thickness of about 5 cm. Recalculated this for a lead layer of 1 m thickness the absorption would be smaller than one in two thousend five hundred millions. Calculated on this base the absorption of the whole earth along its diameter would be at the most one part in eight hundred.

If the absorption is applied to the tidal phenomena, then for the effect from the Sun the effect at the comparison of the zenith positions of 0° and 180° can be expressed as follows:

$$-Z = 2f\frac{M}{D^2} \cdot \frac{a}{D} (1 + 11800\mu)$$

In this expression a is the radius of the Earth, D – distance to the Sun; M – mass of the Sun. If, using the result of the experiments with the compensator, 1/1600 is substituted for μ, the tidal force created by the Sun is:

$$-Z = 2f\frac{M}{D^2} \cdot \frac{a}{D}(1 + 7{,}4)$$

i. e. the absorption would increase the tidal force 8 times, which fully contradicts the experiences. According to this the above mentioned absorption is impossible.

The scientific activity of Roland Eötvös, our great scientist, embraces a wide scope of problems in the field of gravity, the results achieved by him have a basic significance. The problems, he dealt with so extensively and successfully are actual even at present after a half of a century, they inspire thoughts in the investigators of today and stimulate them to further investigations.

GRAVITY, GEOPHYSICS AND ASTRONOMY

With 2 tables

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SUMMARY

Author discusses the astronomical and geophysical aspects of the Dirac's cosmology. It is shown that the existence of the reversible high-pressure phases in a Dirac cosmology gives a very simple explanation for the origin and structure of Solar system, as well as a quantitative explanation for the expansion of the Earth. The expansion derived on the basis of this model is in a very surprising agreement with the different observational date derived from continental surface, decrease of water-covered continental areas (palaeogeographic maps) slowing down of the Earth's rotation (astronomical and paleontological observations), and energies. The most probable radius increase derived amount to 0,5 – 1,5 mm/year

At the time when Roland Eötvös began his research work on gravity, mass attraction was considered as a problem entirely solved. Nevertheless in such a situation Eötvös attacked the most delicate problems of gravity and his outstanding result was the high-precision prove of the proportionality of inertial and gravitative mass. Besides this prominent result, however, which became the basis of the theory of general relativity, he has carried out another measurement with a more modest result. He determined the value of the gravity constant. These two results of different character, however, met in a very interesting manner in one of Dirac's suggestions. Namely, in 1938, Dirac came to the conclusion that the gravity constant is a value decreasing in time proportionally with a time parameter of an order of magnitude comparable with the age of the Earth of the Solar System. Before analyzing the geophysical and cosmological consequences of this suggestion, let us emphasize its philosophical importance.

Our modern science is based on observations of only a few centuries old. In spite of this the physical laws are applied to geological, geophysical and astronomical problems for periods of several billion years. Now the question arises whether the supposition of the constancy of physical laws for such a long period may be considered as correct ? As Newton's physics is not suitable for the description of phenomena of high velocity, moreover, the classical electromagnetic theory is not able to describe the phenomena of microsystems, it is probable that classical physics is not suitable to describe phenomena having a duration equal to the geological periods. Dirac's result proves that physical laws, if they refer to a time interval comparable with the age of the universe, may depend on a time parameter, too.

In the following it will be shown that the supposition of the validity of Dirac's cosmology, in addition to the existence of high pressure and degenerated phases, is suitable to give a general explanation for geological-geophysical phenomena of a long duration, as well as a consistent explanation for the origin of the Solar System.

The existence of Ramsey's high-pressure degeneration phases inside the Earth and the stars is equivalent, in the case of an Earth of homogeneous composition, to the establishment that the density inside the Earth is pressure dependent only. This conclusion together with Dirac's expression results in the expansion of the Earth and planets. The rate of expansion can be calculated from the known physical data of the Earth and its value depends only on the time parameters of Dirac's equation. Table I. contains the rate of radius increase in terms of the value the time parameter.

Table I.

Recent value of time parameter 10^9 years	Average rate of radius increase in mm/year		The average refers to a time-interval of (in million years)
	Bullen–Bullard model	Bullen–Jeffreys mode	
4,1	0,5	1,0	600
4,5	0,46	0,92	650
5,0	0,41	0,83	720
5,5	0,37	0,75	800
6,0	0,34	0,69	870
7,0	0,30	0,60	1000
8,0	0,26	0,52	1160
9,0	0,23	0,46	1300
10,0	0,21	0,42	1440

Only two of the geological-geophysical observations supporting these data may be mentioned. The first is the decrease of water-covered continental areas, a fact supporting the expansion of the Earth. The rate of decrease of water-covered continental areas can be determined from paleogeographic maps. On the basis of this the rate of expansion may be derived. It amounts to 0,5 to 1,6 mm/year, i. e. of the same order of magnitude as the theoretical value.

Another observation is connected with Middle-Devonian corals. On the basis of these fossils it has been shown that in the Middle-Devonian period the Earth made 400 rotations during a whole revolution around the Sun. Conform to the constancy of the momentum of inertiae this may be explained only as an increase of the Earth's radius, with a yearly rate of 0,7 mm/year. Taking into account that the year itself was shorter, the rate of radius increase might have amounted to 1,5 mm/year.

The agreement between theoretical and observed values is shown in Table II.

Table II.

Rate of the annual radius increase

Means of Determination	Minimum mm/year	Maximum mm/year
Theoretical	0,5	1,0
Palaeogeographic maps	0,5	1,6
Continental surface	0,8	–
Slowing down of the Earth's rotation	0,6	1,5
Earthquakes and magmatic activity	~ 0,5	~ 1,0

According to a personal communication by Prof. Petrova, Kramov has determined the rate of yearly radius increase of the Earth from paleomagnetic measurements by a method suggested formerly by me, (1960) and this determination resulted in a rate equal to the above data.

Concerning the problem of the origin of the Solar system the most delicate question arises in connection with the distribution of the angular momentum. If the constancy of the angular momentum is applied to any of the planets of the solar system, the following equation may be derived (Jordan, 1952):

$$fR_n = \text{const.}$$

where f is the gravity coefficient, R_n means the radius of the planetary orbit. If we accept the Dirac-equation $f = x/t$ for the gravity coefficient where x is a constant and t a time parameter equivalent to the age of the solar system then the following equation may be written:

$$R_n = \lambda \cdot t \qquad (\lambda = \text{const.})$$

This equation has the consequences that, in the beginning, the Sun contained the mass of the planet. Namely, in the case of a corresponding small value of t, R_n may have become arbitrarily small. On the other hand, the planets could be originated from the Sun in the case of the validity of Dirac's equation only.

Moreover, if we accept the existence of highpressure degenerated phases – and the existence of white dwarfs are supporting this supposition – then Dirac's expression is suitable to give a very simple explanation for the origin of the solar system. It may be shown on the basis of the constancy of angular momentum of the system that – neglecting a common factor – the value of mass attraction exerted on the equator of the Sun has a form of B/t, while that of the centrifugal force the form of the function $A/r_0 + \alpha \cdot t$. Therefore, it is clear that there was an instant when the centrifugal force and the mass attraction on the equator of the Sun became equal and a weightless mass ring has formed around it, and coagulated into a planet as a result of some smaller perturbations. On the residual mass of the Sun this phenomenon iterated but in longer and longer periods, while it ceased altogether in the interval where the decrease of mass attraction was small. According to this mechanism it is clear that the planets are orbiting around the Sun in a plane coinciding with the equator of the Sun and having the same direction of revolution. Moreover, this mechanism may be applied also to the planets in a time interval of strong decrease of mass attraction. Therefore, the outer planets must have several satellites, while the inner planets (except Mars which is the most external among the inner planets) possesses no satellites. Finally, the mass of the separated parts of the Sun (i. e. the mass of the planets) is relatively small in comparison with the mass of the Sun.

These theoretical consequences, however, are in good agreement with the regularities observed in the Solar system. Moreover, the formation of meteorites from small particles is a clear consequence of the above mechanism, as regards the possibility that, in case of the coagulation of rings into a planet, a dustlike residue could have remained is the surrounding space. Gravity showing in the beginning the characteristics of a strong interaction was suittable to collect this small particles by collision in meteorites or comets in a manner suggested by Urey, but carried out in a strong field. These accretion phenomenon was, however, a secondary one and not the primary process of planet formation.

The equation $fR_n = \text{const.}$ has the consequence that the duration of orbiting around the Sun, i. e. the lenght of a ,,year" is in connection with time parameter in the following way: $T = \mu \cdot t^2$. In this case the lenght of a ,,year" 400 million years ago, i. e. around the Middle-Devonian consisted of 303 contemporary days, consequently, the length of the day in the case of 400 rotations derived from corals had a duration of 65,500 sec instead of 86,400 sec, which corresponds to a very high angular velocity.

According to the theory sketched above the distribution of angular momentum in the planetary system may also be explained. At the formation of a planet the escaped mass had always the greatest specific angular momentum and, as a consequence of the high primaeval angular velocity of the Sun, the relativistic mass of the escaped material was very large.

This relativistic mass, however, became slowly equal to the statical mass as a consequence of the decrease of velocity following the increase of orbital radius, while the angular momentum did not change.

The above mechanism of the origin of the Solar system facilitates the comprehension of the high number of binary stars in our galaxy, too.

It may be concluded that phenomena having a duration comparable with the age of the universe may be explained only on the basis of laws similar to Dirac's hypothesis.

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DIE EÖTVÖSSCHE DREHWAAGE IM UNTERTAGEEINSATZ

Mit 7 Abbildungen

von

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Eingegangen 28. 9. 1963.

ZUSAMMENFASSUNG:

Ausgehend von den klassischen Arbeiten R. v. Eötvös' werden historisch die technischen Entwicklungen der Drehwaage und ihr weltweiter Einfluß auf die geophysikalische Erdölsuche dargestellt. Um auch für heute noch weitere Anwendungsbeispiele zu geben, wird der Einsatz der Drehwaage im Bergwerk aufgezeigt. Die konstruktiven Möglichkeiten, die Ausschöpfung der theoretischen Grundlagen des Schwerefeldes und der in der Praxis erprobte Einsatz lassen auch in Zukunft von Fall zu Fall in der Bergwerksgeophysik eine Verwendung der Drehwaage erwarten.

Die Drehwaage, wie sie in ihrer allgemeinen und klassischen Form von R. Eötvös [1]* bereits 1896 zur Bestimmung des horizontalen Schweregradienten und der Krümmungsglieder veröffentlicht wurde, ist bis auf geringfügige technische Verbesserungen nach Theorie, Handhabung und Interpretationsgrundlagen kaum noch grundsätzlich ergänzt worden. Die Arbeit von Eötvös und die diesbezüglichen Beiträge seiner unmittelbaren Schüler stellen auch heute noch eine in sich abgeschlossene Grundlagenarbeit dar, die rückblickend stets als eine außergewöhnliche Leistung gewürdigt werden muß.

Technisch sind die Einführung der automatischen photographischen Registrierung bei der Doppelgehängewaage durch O. Hecker [2] und die systematische Verkleinerung der Gehänge-Dimensionen für das ungarische Modell durch I. Rybár [4] als Weiterentwicklung zu nennen. Theoretische Überprüfungen durch K. Mader [3, S. 83] und K. Kilchling [5] ergaben, daß der lineare Ansatz für das Schwerefeld innerhalb der Drehwaage praktisch ausreichend ist.

Wegen der langen Beobachtungszeiten wurden Viergehängewaagen gebaut [Hecker, Poddubnyj [6]]. Eine wesentliche Verkürzung der Beobachtungs-

* [] Literaturhinweis, () Formelhinweis.

zeit läßt sich jedoch nur über eine ausgewogene Dimensionierung des Gehänges erreichen. Betragen die Konstanten eines Drehwaagegehänges [3, S. 83]:

m	punktförmige gleiche beiderseitige Massen des Gehänges
21	horizontaler Abstand der beiden Gehängemassen m
h	vertikaler Abstand der beiden Gehängemassen m
τ	Torsionskonstante des Aufhängedrahtes
K	Trägheitsmoment des Gehänges ($K \sim 21^2\ m$)
T	Schwingungsdauer des Gehänges $\left(T \sim 2\pi \sqrt{\frac{K}{\tau}}\right)$,

so sind die durch die Abmessung bedingten Empfindlichkeitsfaktoren für die

$$\text{Gradientenkomponenten:} \quad \frac{hlm}{\tau} \sim \frac{T^2}{8\pi^2} \cdot \left(\frac{h}{1}\right) \tag{1}$$

$$\text{Krümmungsglieder.} \quad \frac{K}{\tau} \sim \frac{T^2}{4\pi^2} \tag{2}$$

Die neue Tendenz ist, T bis in die Größenordnung einer Minute zu verkleinern [6], die Genauigkeit der Krümmungsglieder entsprechend zu reduzieren und die Drehwaage unter Ausschöpfung des Faktors $h/1$ betont als Gradientenmesser (etwa $\pm$ einige E) zu benutzen. Einer optischen oder elektrischen Vergrößerung des Winkelausschlages wird nur durch den mikroseismischen Störpegel, durch sonstige Erschütterungen im Gelände und durch eine geeignete Dämpfung von schädlichen Gehängeschwingungen eine praktische Grenze gesetzt.

W i r t s c h a f t l i c h hat die Eötvössche Drehwaage bei der Erdölerschließung in den zwanziger Jahren einen umfassenden weltweiten Einsatz gefunden, der Anfang der dreißiger Jahre durch das Aufkommen von feldbrauchbaren Gravimetern praktisch zum Erliegen kam. Trotzdem kann man auch noch heute in geeignetem Gelände kleinere strukturelle Teilprobleme unter Umständen mit der Drehwaage geeignet bearbeiten. Wegen der Geländekorrektion und einer vernünftigen Beobachtungszeit ist die Wirtschaftlichkeit jedoch stark beschränkt.

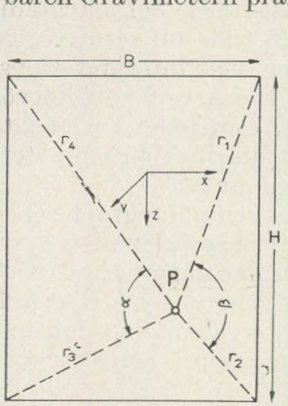
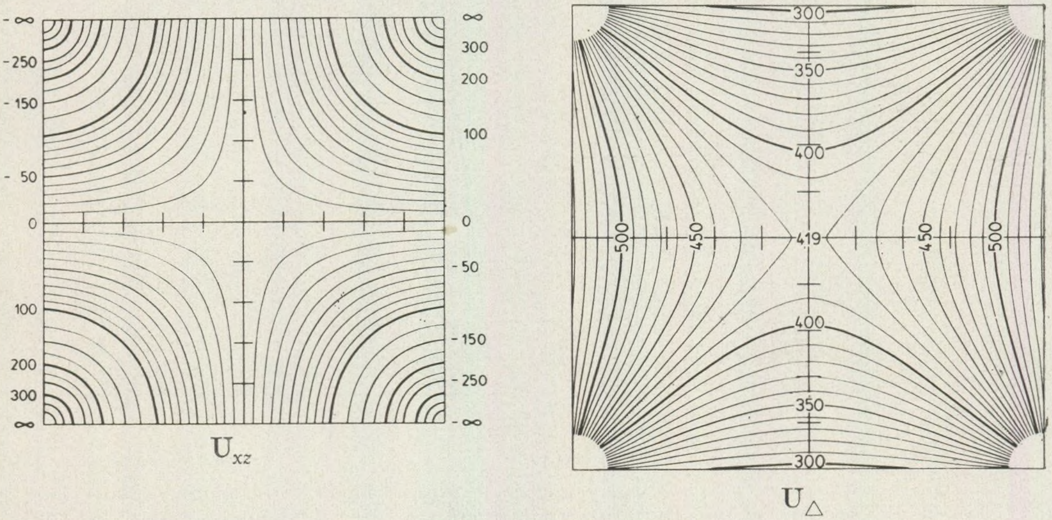


Abb. 1
Koordinaten des Punktes P im rechteckigen Streckenquerschnitt.

Einen nicht voll ausgeschöpften Einsatz kann die Drehwaage in der Grube finden. Erste Versuche fanden in Ungarn 1928 von Oszlaczky und Bakos in der Doroger Kohlengrube in 250 m Tiefe statt [7]. Eigene Messungen [3, S. 100] habe ich Ende der zwanziger Jahre im Werrakaligebiet (Thüringen) ausgeführt. Die Anlage der Untertagemessungen wird durch die Korrektionen für den Streckenquerschnitt sowohl instrumentell wie aufstellungsmäßig bedingt.

Unter Verwendung der Ableitungen für das Potential U eines Prismas mit rechteckigem Querschnitt [8] und [3 S. 74] erhält man mit den Bezeichnungen nach Abbildung 1 für den Punkt P die a



b

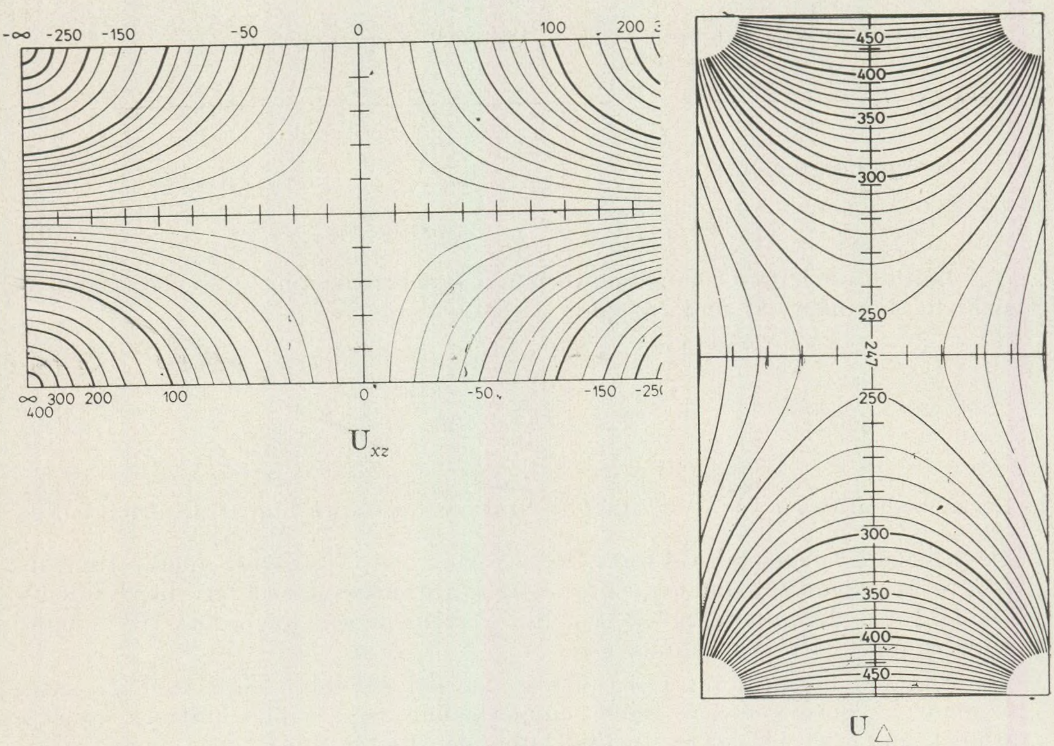


Abb. 2a und b

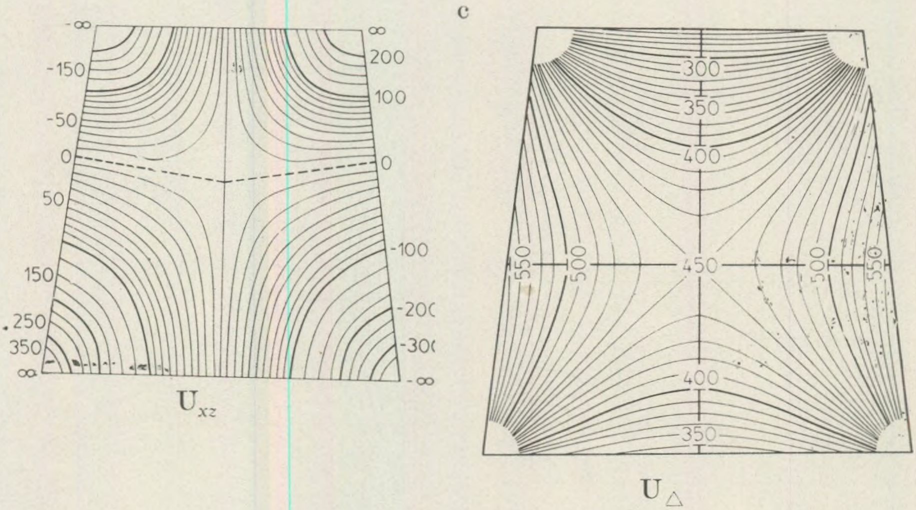


Abb. 2

a) Quadratischer, *b)* rechteckiger und *c)* trapezförmiger Streckenquerschnitt mit Linien gleicher Werte des Horizontalgradienten und des Krümmungsgliedes für die Dichte Eins

Gradientenkomponenten: $$U_{xz} = 2k \cdot \sigma \ln \frac{r_2 \cdot r_4}{r_1 \cdot r_3} \tag{3}$$

$$U_{yz} = 0 \tag{4}$$

Krümungsglieder: $$U_{xy} = 0 \tag{5}$$

$$U_\Delta = U_{yy} - U_{xx} = 2k\sigma(\alpha + \beta) \tag{6}$$

Befindet sich der Gehängeschwerpunkt P in Streckenmitte, so vereinfachen sich die Formeln (3) und (6) zu

$$U_{xz} = 0 \tag{7}$$

$$U_\Delta = 8k\sigma \operatorname{arctg} \frac{H}{B} \tag{8}$$

Dabei bedeuten $k = \frac{200}{3} \cdot 10^{-9}$ die Gravitationskonstante und σ die Dichte des die Strecke umgebenden Gesteins. In der Abb. 2 sind für einen quadratischen *(a)*, rechteckigen *(b)* und trapezförmigen *(c)* Streckenquerschnitt die Gradienten und das Krümmungsglied für die verschiedenen Lagen des Gehängeschwerpunktes im Querschnitt gegeben.

Aus den Formeln (3) und (6), wie aus den Abbildungen 2, ergeben sich folgende Folgerungen bei einem einigermaßen regelmäßigen Streckenquerschnitt, der auch bei Unregelmäßigkeiten als „Reduktionsfläche" dienen soll:

Der Gehängeschwerpunkt ist in den Streckenmittelpunkt zu bringen. Hier ist der durch den Streckenquerschnitt bedingte Gradient Null, und das Krümmungsglied, das zur Dichte (σ) bestimmung benutzt werden kann, besitzt ein Maximum. Daraus ergeben sich für die Aufstellung insbesondere die Forderung nach einem in der Höhe verstellbaren Stativ und für die Größe der Drehwaage eine Beschränkung in der Dimensionierung. Eine solche Anordnung zeigt Abb. 3. [9]

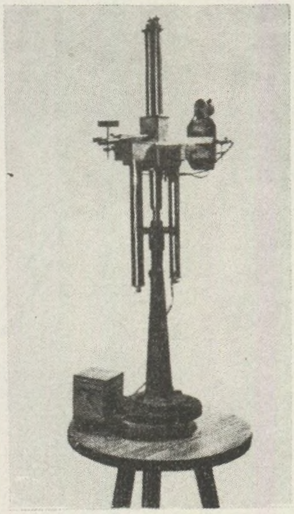


Abb. 3
Kleindrehwaage auf vertikal verstellbarem Stativ

Eine wesentliche Bedeutung bei der Kleindimensionierung kommt der ,,Behandlung" des Torsionsdrahtes zu. Eötvös benutzte Drähte aus einer Legierung von Pt und 10 – 20% Ir. Seine Wärmebehandlung und elastische Alterung war zeitraubend [1, S. 97/98 und S. 236 ff.], daher hat man später versucht, die Technologie zu verbessern. Bereits in den zwanziger Jahren ging man zu Wolframdrähten [3, S. 87] über, da dieses Material eine erhöhte Tragfähigkeit zeigt. Man kann die Wolframdrähte durch eine entsprechende Wärmebehandlung im Schutzgas in der Zugfestigkeit verbessern und gleichzeitig auch den Torsionsmodul in gewissen Grenzen ,,einstellen." Darüber hinaus muß der Temperaturgang des Nullpunktes überprüft werden.

In Abb. 4a ist die Belastbarkeit eines Wolframdrahtes vom Durchmesser $\varnothing = 0{,}03$ mm in Abhängigkeit von der Temperung dargestellt, während in Abb. 4b die Abhängigkeit der Belastbarkeit von der Temperatur wiedergegeben ist. Die Einstellung der Torsionskonstante τ z. B. für Wolframdrähte $\varnothing = 0{,}03$

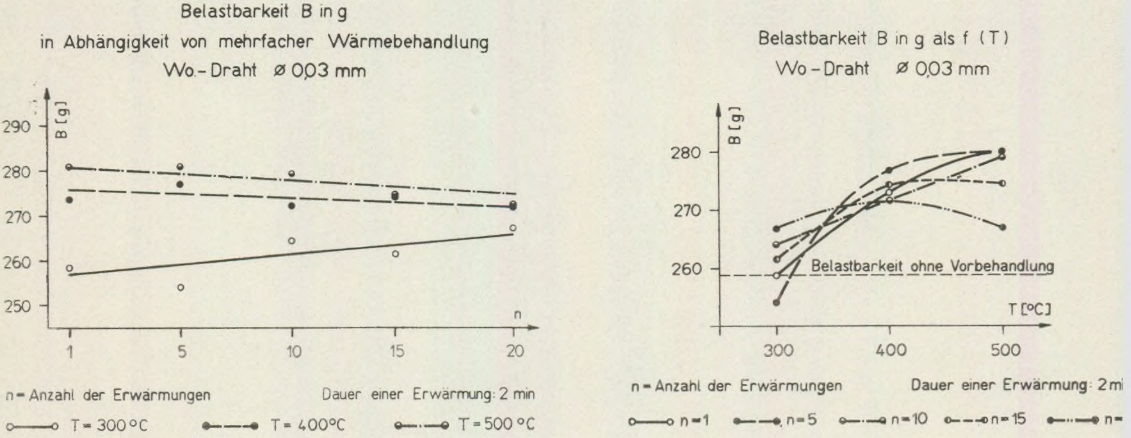


Abb. 4
Belastbarkeit von Wolframdrähten (ø = 0,03 mm) in Abhängigkeit von der Temperung (*a*) und der Glühtemperatur (*b*)

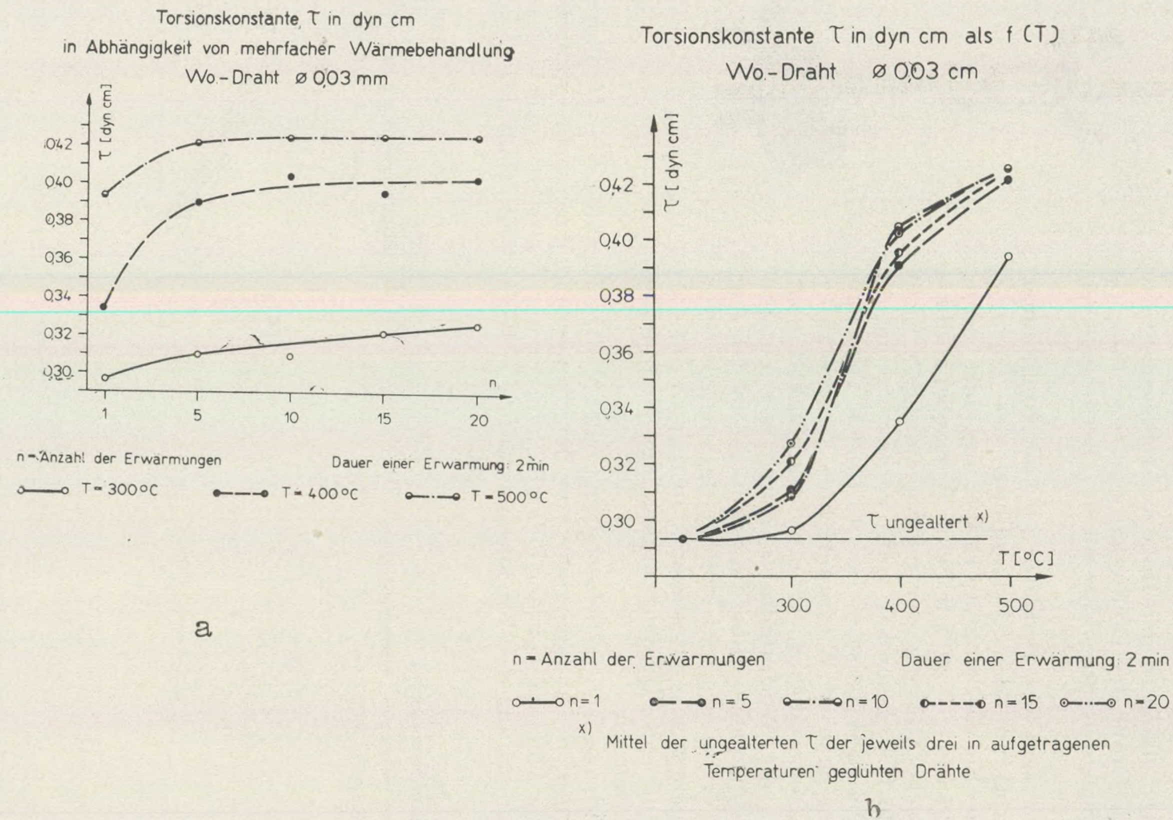


Abb. 5

Torsionskonstante τ von Wolframdrähten ($\varnothing = 0{,}03$ mm) in Abhängigkeit von der Temperung (*a*) und der Glühtemperatur (*b*) mm in Abhängigkeit von der Temperung zeigt Abb. 5a, während in Abb. 5b die entsprechende Angabe in Abhängigkeit von der angewandten Temperatur dargestellt ist. Man kann nach vorliegenden Diagrammen die Drahtbehandlung empirisch gut ausführen. Schließlich sind in Abb. 6 die technischen Daten eines fertigen Drahtes einschließlich des „Temperaturkoeffizienten" für seinen Nullpunkt angegeben.

Draht-Nr.: 15 Werkstoff: Wo Durchmesser: 0,03 mm Länge: 260 mm
Belastbarkeit: 275 g Torsionskonstante τ (ungealtert): 0,29 dyn cm

Behandlung in N_2-Atmosphäre

Datum		4.3.55	
Belastung	g	5,3	
Raumtemperatur	°C	20	
Anzahl d. Erwärm.		10	
Temperat. d. Drahtes	°C	450	
Dauer einer Erwärm.	min	2	

Erwärmen im Ofen

Datum		16.3.55	
Belastung	g	81,85	
Raumtemperatur	°C	20	
Temperatur im Ofen	°C	93	
max. Auslenkung a	mm	1,3	
Torsionskonstante τ	dyn cm	0,42	
Temperaturkoeff. TK	mm/°C	0,057	

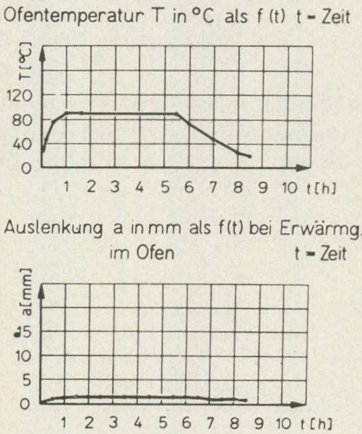


Abb. 6

Die Daten und Nullpunktskonstanz eines präparierten Wolframdrahtes (Ø = 0,03 mm) in Abhängigkeit von der Temperatur

Die vorstehenden technologischen Angaben zeigen, daß es keine Schwierigkeiten macht, auch für kleinere Drehwaagen mit relativ kurzer Schwingungszeit, insbesondere für Untertagemessungen, einen brauchbaren Gradientenmesser zu bekommen, der selbst die großen Krümmungsglieder (Größenordnung fast 1000 E) noch hinreichend genau mit erfassen läßt.

Es wäre sehr erwünscht, wenn diese rein experimentell gewonnenen Ergebnisse auch von metallurgischer Seite noch theoretisch untermauert werden könnten, um gegebenenfalls noch günstigere Verhältnisse auch bei anderen Drahtmaterialien für dünne Drähte zu gewinnen.

In Abb. 7 ist die Wirkung einer Basalteinlagerung im Salz wiedergegeben [9]. Größe und Verlauf der Gradienten und Krümmungsglieder zeigen die Brauchbarkeit dieses Verfahrens.

Zusammenfassend möchte ich bemerken, daß die geniale Entwicklung der gravimetrischen Drehwaage von R. Eötvös auch heute noch Anwendungsperspektiven hat, insbesondere im Einsatz unter Tage als Schnellmeßinstrument, um besondere bergbauliche Probleme zu lösen. Daneben besteht eine beschränkte Anwendungsmöglichkeit in der Übertageprospektion und für geodätische Pro-

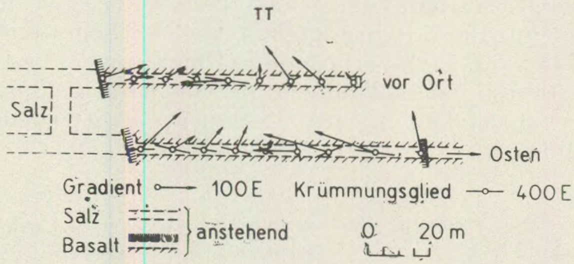


Abb. 7
Gradienten und Krümmungsglieder in einem Salzbergwerk mit Basalteinlagerung

bleme. Darüber hinaus empfiehlt es sich, die jarzehntelang angefallenen wertvollen instrumentellen Erfahrungen in der Behandlung von dünnen Torsionsdrähten für eine genauere Bestimmung der Gravitationskonstante mit auszuwerten.

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SOME REMARKS ON EQUIVALENCE PRINCIPLES

by

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Received 28. 9. 1963.

The experiment of Baron Roland v Eötvös on the equivalence of intertial and gravitational mass was more than just an inprovement by one or two orders of magnitude in a result already known to be true. His result constitutes one of the primary pillars upon which general relativity rests. His result, that with an accuracy of better than 1 part in 10^8 the gravitational acceleration of a body is independent of the structure of the body, is a necessary condition to be satisfied if the formalism of general relativity is valid.

The experiment of Eötvös is sometimes called an experiment on the equivalence principle. It is necessary to remark, however, that the direct test of the equivalence principle represented by the Eötvös experiment is a test of a limited form of equivalence principle, a form which I have called the "weak principle of equivalence". Einstein's theory is based on the "strong equivalence principle". This principle states that, neglecting effects due to gravitational gradients, the laws of physics seen in a freely falling non-rotating laboratory are the same, including all their numerical content, independent of the position and velocity of the laboratory. It is interesting that Eötvös's experiment is capable of supplying valuable information about the strong equivalence principle in addition to the weak one. Thus, using the results of the modern version of his experiment carried out recently at Princeton, an experiment which gave an equivalence of inertial and gravitational mass with an accuracy of 1 part in 10^{11}, it has been possible to conclude that certain parts of the numerical content of physical laws are indeed constant. Thus most of the dimensionless numbers encountered in physics, such as ratios of intertial masses, and the coupling constants for the electromagnetic and nuclear forces, are constant, that is independent of the position of the laboratory. It has not been possible from the experiment to conclude that the gravitational coupling "constant" is constant or that the β decay coupling "constant" is fixed. However, it is interesting and remarkable that this one experiment is capable of supporting such a large part of the strong equivalences principle.

I wish to express my regrets at being unable to be present at the conference and wish you and the whole conference a very informative and productive session in honour of Baron Roland v. Eötvös.

A POSSIBILITY FOR THE EXPERIMENTAL PROOF OF GENERAL RELATIVITY*

by

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The direct experimental consequences of the general relativity are rather few and restricted to static fields. An other experimental proof of the general relativity going beyond the equivalence principle and only static field problems was proposed by Schiff; it consists in studying the exact motion of a giroscope in an atrifical satellite. Author has given a series of arguments about the experimental problems and difficulties in carrying over this program, and expressed his personal hope, that these difficulties could be overcome in USA during the next five years.

* Abstract of this talk has been given by the committee of Redaction.

PRINCIPLES OF RELATIVITY AND OF EQUIVALENCE IN THE EINSTEINIAN GRAVITATION THEORY

by

V. FOCK

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The role of the concepts of principles of general relativity and of equivalence in Einsteinian Gravitation theory was analysed. It was argued that this latter principle considered from the point of view of the theory itself, plays only a restricted part and is only approximate. Not this principle, but the idea of the unification of space and time and the assumption that physical processes can influence this metric constitute the very foundation of Einstein's gravitation theory.

* This absract has been given by the Redaction based on the lecture given by the Author. The subject material has been published in more detail in Вопросы философии No. *12*, 46/1961) and "Det Kongelige Norske Videnskabers Selskabs Vorhandliger, Bind 36, (1963) p. 16.

ON THE PROBLEM OF THE ORIGIN OF INERTIA*

by

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SUMMARY

The problem of the origin of inertia is investigated in connention with Mach's principle. After a short review of the problem it is argued that Mach's principle in a sense near to the original ideas of Mach, is not incorporated in the general theory of relativity. On the other hand one can formulate a selection principle which enables us to divide the solutions of the gravitational equations into two classes. The members of the first class do, the members of the second class do not contradict to the appropriate inheritance of Mach's-original requirements. Thus these latter solutions may perhaps remain permissible even in the future when at a stage of deeper knowledge the others turn out to be a not permissible extrapolation beyond the domain of validity of Einstein's equations. It has been given a precise formulation of the selection principle, which seems to be more general and flexible than the previous ones.

* To be published in the Acta Phys. Hung.

OBSERVATIONAL BASIS OF MACH'S PRINCIPLE*

by

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The status of the observational control of the main idea of Mach's principle was reviewed. It was stated, that if Mach's principle is to have any meaning, it gives a method of selection of the inertial coordinate system. This system will be that one on which some average of the motions of the distant masses is uniform. Some opinions concerning the observational errors in specifying this coordinate system with respect to rotation were analysed and it was claimed, that the recently discovered distant galaxies, in conjunction with strong radio sources may provide the most precise connection between the local inertial system and the distant masses of the universe and hence the surest observational basis for Mach's principle.

* The abstraçt was given by the Redaction, based on the paper submitted by the Author to the Session. The paper has been published in preprint form by the Institute of Theoretical Physics, Stanford University, Publication No. 97, and will be published in Phys. Rew.

PROJEKTIVE FELDTHEORIE UND VARIABILITÄT DER GRAVITATIONSZAHL

von

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Der vorangegangene sehr interessante Vortrag von Prof. Egyed sowie weiteres umfangreiches empirisches Material aus den Gebieten der Geo- und Astrophysik legen die Vermutung einer Expansion sowohl unserer Erde als auch übriger Himmelskörper im Laufe der Zeit nahe. Um eine physikalische Deutung dieses Effektes zu geben, wird meist auf der Diracschen Hypothese von 1937 aufgebaut, wo nach die Gravitationszahl umgekehrt proportional mit dem Weltalter abnimmt. Da diese Hypothese keinen relativistisch kovarianten Charakter besitzt, so muss sie, falls ihr Wahrheitsgehalt zukommt, aus einer allgemeinen kovarianten Feldtheorie zu deduzieren sein. Als die am besten ausgebaute und auf Grund ihrer logischen Einfachheit am glaubwürdigsten erscheinende Theorie bietet sich dafür die 5-dimensionale projektive Feldtheorie an, deren mathematischer Apparat in bezug auf eine variable Gravitationszahl von Jordan weitgehend entwickelt wurde. Der gehaltene Vortrag skizzierte die Grundlagen dieses Apparates, durch Benutzung des vom Referenten deduzierten Vektorformalismus, wodurch sich der Projektions-mechanismus in die 4-dimensionale Raum-Zeit besonders übersichtlich gestaltet. Das Ergebnis der Projektion der 5-dimensionalen Feldgleichungen wurde dargelegt. Die Interpretation wurde so durchgeführt, dass diese Feldtheorie sowohl zu einer Variabilität der Gravitationszahl im Sinne von Thiry als auch zu einer geometrischen Polarisation des elektromagnetischen Feldes Anlass gibt.

* Zusammenfassung des Verfassers.

EQUATIONS OF MOTION AND RADIATION REACTION

by

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SUMMARY

The Einsteinian field equations and the equations of motion are expanded into powers of the gravitational coupling constant. Starting from the equations for continuously distributed matter we arrive at the equations of motion for a system of point-like particles. We deal with the first and second approximations. Finally we get the equations of motion for point-like particles free from infinities and containing the radiation reaction terms.

The discussion, whether gravitational radiation exists or does not exist, is not yet finished. Various people have various opinions Einsteinian equations have wave-like solutions as well as the exact equations as well as approximate ones. Whether these waves have a real physical meaning or not, it is connected with the question, whether they transfer energy or not. To decide this question is very difficult, because the definition of energy in general relativity is not yet finally solved. A usable method to decide the question about the existence of gravitational radiations, it seems to us, is the investigation of the equations of motion of a system of interacting particles. From the kind of the motion one should conclude, whether the particles radiate energy to each other or not.

This program was suggested by Havas [1]. To deal with the full Einsteinian theory is mathematically too difficult. Therefore one needs approximat methods. The EIH-method is not suitable in this case, because radiation terms appear only in higher orders. Havas therefore used the fast motion approximation. He gets, besides of the expression for the self-energy, an equation of motion with radiation damping terms:

$$m_{A,ren}\ddot{a}^{\alpha} + \frac{11}{3}\frac{\gamma m_{A}}{c^2}(\dddot{a}^{\alpha} + \dot{a}^{\alpha}\dot{a}^2) +$$

+ force originated by the other mass points = 0 (the dot means $\frac{d}{d\tau}$, τ is the Minkowskian proper time, γ is the Newtonian gravitational constant).

With this equations of motion Havas and Smith calculated the motion of two masses around each other. They found a spiral outward. It would mean, that a moving mass in gaining energy by the gravitational radiation. This result is not yet fully convincing. Up to the first order the equations of motion contain the velocities in zeroth-order (it means $\dot{v}^\alpha = 0$).

The radiation damping terms are proportional to ε ($\varepsilon = 4\gamma c^{-2}$) the influence of the other masses on the velocity, too. Therefore one wants to use the second order equation of motion, which contain the velocities in first order. With this equation one could calculate the motion of two mass points (two-body-problem) and then could conclude reliably on the existence or non-existence of the gravitational radiation.

For this purpose we need the equations of motion for a system of point-like particles up to second order in ε without infinities. To remove these infinities after the application of δ-functions (s. e. g. the paper of Bertotti and Plebanski [2]) has not been reached up with the methods suitable for the linear approximation; either the definition of energy is needed or the mathematical difficulties are not overcome.

We will deduce the equations of motion for a system of point-like particles by the transition from extended fluid-drops to point-like particles. We start from the field equations for a perfect compressible fluid and use de Donder's coordinate condition. Then we expand the $\mathfrak{g}^{\alpha\beta}$($\mathfrak{g}^{\alpha\beta} = \sqrt{-g}g^{\alpha\beta}$) instead of the metric quantities $g_{\alpha\beta}$.

We do not accept a definite equation of state, but the pressure shall be a unique function of the mass density. For we deal with a finite-sized fluid in its gravitational field, there exists a condition of stability for this "drop". From this condition we conclude, that the expansion of the pressure starts with a term of first order in ε.

The field equations in the linear approximation are

$$\Box \underset{1}{\mathfrak{g}}^{\alpha\beta} = -4\pi\varepsilon\mu v^\alpha v^\beta$$

($\Box$ is the d'Alembertian, μ is the mass density). The solution is:

$$\underset{1}{\mathfrak{g}}^{\alpha\beta}(x) = \varepsilon \int_\Omega \mu(x')v^\alpha(x')v^\beta(x')D(x-x')d_4x'$$

Ω is the fourdimensional volume of the fluid, D is the retarded Green's function with $\Box D(x) = -4\pi\delta_4(x)$.

The equation of motion up to the first order in ε is

$$\mu\ddot{a}^\alpha + \frac{1}{2}\mu(\eta^{\alpha\varrho} - \dot{a}^\alpha\dot{a}^\varrho)\left\{\varepsilon\int_\Omega\left[\left((\dot{a}\dot{b})^2 - \frac{1}{2}\right)D_{1\varrho}(a-b) - 2\dot{b}_\varrho(\dot{a}\ddot{b})\frac{d}{d\tau_a}D(a-b)\right]\cdot\right.$$

$$\left.\cdot\,\mu(b)d_4b - \frac{1}{c^2}p_{1\varrho}\right\} = 0 \qquad (1)$$

$$\text{with} \quad \eta_{\alpha\beta} \dot{a}^\alpha \dot{a}^\beta = \dot{a}^2 = 1$$

We imagine the fluid consisting of drops separated from each other. We investigate the equation of motion for one of these drops and single out the terms from which infinities may arise after the transition to point-like particles.

We make the following assumptions: A drop can be described by one unique velocity; we do not consider internal motions (rotations, turbulence, deformations). This velocity does not depend on the spatial coordinates. We imagine the drops to be small spheres, the mass distribution shall be spherically symmetric.

For a system of drops separated from each other the integral in equ. (1) becomes a sum of integrals over the regions of the single drops. First we integrate this equation over the region of the A-th drop, to introduce the mass m_A. Only the one term, the ranges of integrations is which coincide, will yield infinite contributions. From the other terms of the sum one gets the same contribution as Bertotti and Plebanski found with help of δ-functions.

We transform the interesting part of equ. (1) into a more suitable form and make then an expansion analogous to those in Dirac's paper [3]. Finally we get an equation of stability, a mass renormalization terms

$$\delta m_A = -\frac{7}{8}\,\varepsilon \int_A \mu(a) d_3 a \int_A \mu(b) d_3 b \cdot \lambda^{-1}, \quad \lambda = |a-b|,$$

a Havas' radiation reaction terms.

The linear approximation was only a test for our method.

The equation of motion up to second order in ε is a rather long equation. It contains already the non-linear features of the Einsteinian theory. There appear products of two and three D-functions. We can apply our method in this case and get a further mass renormalisation of the coupling constant $(\varepsilon\, m_A^2)$ before the radiation reaction terms of the first order. The equation of stability is much more involved than that in first order. We need not only a scalar pressure but a stress tensor, therefore we must leave the perfect fluid model in some respects. But the choice of this special model does not influence the results we are interested in which up to the second order in ε.

The remaining and after renormalised finite part of the equations of motion coincides with the equation of Bertotti and Plebanski for point-like particles and derived with help of δ-functions. In addition there appears a terms like a radiation reaction terms: $-\frac{253}{192}\,(\ddot{a}^\alpha + 3\dot{a}^\alpha \dot{a}\dddot{a})$. This terms is, however, not due tot he radiation field, because it is invariant with respect to a time reversal.

In this way we have got the equation of motion up to the second order fast motion approximation free from infinities, and we have seen that no new radiation reaction force is appearing in the second order equation.

To decide the question about the existence of gravitational radiation, from our point of view it is necessary to calculate a special motion, say an elliptic orbit, and to find the derivations from this orbit produced by the gravitational radiation. But it is still a future program.

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VARIATIONSPRINZIPIEN FÜR KONSERVATIVE SYSTEME IN DER RELATIVISTISCHEN KONTINUUMSMECHANIK

von

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ZUSAMMENFASSUNG*

Es wird gezeigt, dass im Falle eines konservativen Systems, die Mechanik des Kontinuums in der allgemeinen Relativitätstheorie mit Hilfe einer gewöhnlichen Variationsmethode behandelt werden kann.

* Zusammenfassung der Redaktion.

In einer relativistischen Feldtheorie hat man ein Variationsprinzip

$$\delta \int \left\{ L(\psi^A, g_{\mu\nu}) + \frac{1}{2\varkappa} R \right\} \sqrt{-g}\, d^4x = 0, \tag{1}$$

wobei L ein Funktional der Feldrößen ψ^A und des metrischen Tensors $g_{\mu\nu}$ ist. Aus den Feldgleichungen

$$\frac{\delta L}{\delta \psi^A} = 0 \tag{2}$$

folgt dann zusammen mit der Definition des Energie-Impulstensors

$$\sqrt{g}\, T^{\mu\nu} \equiv -2 \frac{\delta \sqrt{-g} L}{\delta g_{\mu\nu}} \tag{3}$$

die Divergenzfreiheit desselben:

$$T^{\mu\nu};^{\nu} = 0.$$

Die Variation bezűglich $g_{\mu\nu}$ in 1. liefert die Einsteinschen Gleichungen:

$$-\varkappa T^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R. \tag{5}$$

Es ist wünschenswert, die allgemeinrelativistische Kontinuumsmechanik in dieses Schema einzuordnen. Zu diesem Zweck muß man überlegen, welche Größen jetzt den Feldvariablen ψ^A entsprechen. Ferner wird man eine geeignete Definition für ein konservatives mechanisches Kontinuum zu treffen haben, da in Anwesenheit von dissipativen Effekten die Existenz eines Variationsprinzips nicht zu erwarten ist. Nun suchen wir in der Kontinuumsmechanik letztlich eine Kongruenz zeitartiger Weltlinien mit dem normierten Tangentenvektor u^μ

$$u^\mu u_\mu = -1. \tag{6}$$

Anstatt das kontravariante Vektrofeld u^μ zu suchen, kann auch eine Koordinatentransformation.

$$a^\varkappa = A^\varkappa(x^1, x^2, x^3, x^4) \tag{7}$$

gesucht werden, derart daß das neue Koordinatensystem ein mitbewegtes ist, d. h. daß in ihm die Weltlinienkongruenz durch

$$a^k = \text{const.}, \quad a^4 = \text{variabel} \tag{8}$$

beschrieben wird. (Griechische Indizes laufen von 1 bis 4, lateinische von 1 bis3).

Das erwähnte Programm läßt sich nun durchführen, indem man die ψ^A mit den $a^\varkappa$ (7) identifiziert. Von diesem Standpunkt aus wird die Kontinuumsmechanik konservativer Systeme als eine Feldtheorie erscheinen, die insofern entartet ist, als die Gleichungen (2) und (4) identisch werden.

Im folgenden sollen alle Größen im $a^\varkappa$-System mit einem Querstrich versehen werden. Es ist dann:

$$\bar{u}^\alpha = \frac{\delta_4^\alpha}{\sqrt{-\bar{g}_{44}}}; \quad \bar{u}_\beta = \frac{\bar{g}_{4\beta}}{\sqrt{-\bar{g}_{44}}}. \tag{9}$$

Bezeichnen wir die zu (7) inversen Transformationen durch

$$x^\mu = \varphi^\mu(a^1, a^2, a^3, a^4), \tag{10}$$

und benutzen wir die Abkürzung

$$\varphi_\lambda^\mu \equiv \frac{\partial \varphi^\mu}{\partial a^\lambda}, \tag{11}$$

so ergibt sich

$$u^\mu = \frac{\varphi^\mu}{\sqrt{-g_{\alpha\beta}\varphi_4^\alpha \varphi_4^\beta}}. \tag{12}$$

Auf diese Weise bereits u^μ als Funktional der $g_{\alpha\beta}$ und der ,,Feldgrößen'' $a^\varkappa$ ausgedrückt.

Um den konservativen Charakter des Systems auszudrücken, führen wir zunächst den Projektionstensor $S_{\alpha\beta}$ vermöge

$$S_{\alpha\beta} \equiv g_{\alpha\beta} + u_\alpha u_\beta \tag{13}$$

ein Es ist

$$\overline{S}_{ik} = \gamma_{ik}, \tag{14}$$

wobei γ_{ik} gemäß

$$dl^2 = \gamma_{ik} da^i da^k \tag{15}$$

für den räumlichen Abstand von benachbarten weltlinien maßgebend ist. Weiter benutzen wir die Identität von C. Eckart [1]

$$T^{\mu\nu} \equiv w u^\mu u^\nu + u^\mu w^\nu + u^\nu w^\mu + w^{\mu\nu}, \tag{16}$$

wobei

$$w \equiv T^{\varrho\sigma} u_\varrho u_\sigma$$

die Energiedichte im lokalen Minkowskischen Ruhsystem,

$$w^\alpha \equiv -S^\alpha_\varrho T^{\varrho\sigma} u_\sigma$$

der Wärmestrom und

$$w^{\alpha\beta} \equiv S^\alpha_\varrho S^\beta_\sigma T^{\varrho\sigma}$$

der Drucktensor ist. Der konservative Charakter des Systems erfordert zunächst

$$w^\alpha = 0 \tag{17}$$

anzunehmen. Weiter fordern wir, daß die Spannungen $-\overline{w}^{ik}$ im mitbewegten Koordinatensystem aus einem Potential Φ gemäß

$$2\varrho \frac{\partial \Phi}{\partial \overline{e}_{ik}} = -\overline{w}^{ik} \tag{18}$$

zu bestimmen sind. Hierbei erklären wir den Deformationstensor im mitbewegten System durch die Differenz zweier metrischer Tensoren

$$\overline{e}_{ik} = \gamma_{ik} - \gamma^*_{ik}, \tag{19}$$

wobei für unsere Zwecke nur

$$\frac{\partial \gamma^*_{ik}}{\partial a^4} = 0 \tag{20}$$

verlangt werden muß. Hängt $g^*_{\mu\nu}$ mit γ^*_{ik} in derselben Weise zusammen wie $g_{\mu\nu}$ mit γ_{ik} und bedeutet „$||$" die mittels $g^*_{\mu\nu}$ gebildete kovariante Ableitung, so ist (20) gleichbedeutend mit

$$u_{\nu||\mu} + u_\nu u_{\mu||\varkappa} u^\varkappa + u_{\mu||\nu} + u_\mu u_{\nu||\varkappa} u^\varkappa = 0, \tag{21}$$

dh. bezüglich der Hilfsmetrik $g^*_{\mu\nu}$ beschreibt die wirkliche Weltlinienkongruenz eine starre Bewegung im Sinne von Born und Rosen. Wir sind hier einer Idee von Rayner [2] gefolgt um den Begriff der Deformation in der allgemeinen Relativitätstheorie zu definieren.

Die Dichte ϱ kam durch

$$\frac{1}{\varrho} \equiv v \equiv \frac{\sqrt{\gamma}}{\sqrt{\gamma^*}} \tag{22}$$

definiert werden. v ist ein Analogon zum spezifischen Volumen. Es ist das verhältnis der Volumina ein und derselben kleinen Materiemenge im mitbewegten System einmal gemessen mittels der wirklichen Metrik, zum andern mittels der Hilfsmetrik, bezüglich derer die Bewegung eine starre ist.

Es gilt identisch die Kontinuitätsgleichung

$$(\varrho u^\mu)_{;\mu} = 0. \tag{23}$$

Schließlich setzen wir noch

$$w = \varrho \Phi. \tag{24}$$

Sodann kam die Theorie aus einem Variationsprinzip (1) hergeleitet werden, wenn

$$L = \varrho \Phi(\bar{e}_{ik}) \tag{25}$$

augenommen wird. Hierbei ist γ^*_{ik} als Funktion von a^1, a^2, a^3 auzusehen, während für γ_{ik} gemäß (14)

$$\gamma_{ik} = \varphi^\varrho_i \varphi^\sigma_\varkappa S_{\varrho\sigma} \tag{26}$$

geschrieben werden muß.

Auf den Beweis gehen wir hier nicht ein. Er läst sich am besten durch eine leichte Weiterentwicklung der Methode von Fock [3] erbringen.

Der Spezialfall einer idealen Flüssigkeit, in welchem

$$T^{\mu\nu} = (w+p)u^\mu u^\nu + g^{\mu\nu} \cdot p \tag{27}$$

ist, erhält man, indem man

$$\Phi = \Phi(v) \tag{28}$$

annimmt, und

$$\frac{\partial \Phi}{\partial v} = -p \tag{29}$$

setzt. Dieser Fall gestattet noch eine andere Variationsmethode. Um sie darzulegen, ersetzen wir (28) durch

$$\Phi = \Phi(v, \eta) \tag{30}$$

wobei η die spezifische Entropie ist. Außerdem sei in Ergänzung zu (29) die Temperatur T durch

$$\frac{\partial \Phi}{\partial \eta} = T, \tag{31}$$

gegeben. Schließlich führe man die spezifische Enthalpie h vermöge

$$h \equiv \Phi + pv \tag{32}$$

ein. Für sie gilt gemäß (29) und (31)

$$\frac{\partial h}{\partial p} = \frac{1}{\varrho}, \qquad \frac{\partial h}{\partial \eta} = T. \tag{33}$$

Sodann kan durch eine Betrachtungsweise, die eine geeignete relativistische Modifikation der Clebsch-Transformationen darstellt, gezeigt werden, daß die mit (27) formulierten Ausgangsgleichungen äquivalent ist:

$$\begin{cases} hu_\mu = \psi,_\mu + \alpha\eta,_\mu. \\ \eta,_\mu u^\mu = 0, \qquad \alpha,_\mu u^\mu = T \end{cases} \tag{34}$$

sowie in Ergänzung dazu (6) und (23).
Man setzte nun in (1).

$$L = \varrho \left\{ -\frac{h}{2} g_{\mu\nu} u^\mu u^\nu + u^\mu(\psi,_\mu + \alpha\eta,_\mu) + \frac{h}{2} \right\} - p \tag{35}$$

und fasse h als Funktion von η und p auf, welche (33) genügt, sowie ϱ als Funktion dieser Variablen. Variiert man dann unabhängig voneinander die vier u^μ sowie die vier Skalare ψ, α, η, p und schließlich die $g_{\mu\nu}$, so liefert die Variation nach den ersten acht jener Variablen die acht Gleichungen (34), (6), und (23), während die Variation nach den $g_{\mu\nu}$ wieder die Einsteinschen Feldgleichungen (5) ergibt.

Ausführliche Rechnungen zu den hier behandelten Problemen sowie inhaltliche Ergänzungen nebst weiteren Literaturangaben finden sich in zwei Arbeiten des Verf. [4], [5].

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РАЗВИТИЕ ИДЕЙ Р. ЭТВЕША В СССР В ОБЛАСТИ ГРАВИМЕТРИИ

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SUMMARY

The achievements of the distinguished Hungarian physicist in the field of gravimetry and gravitational attraction did not diminish in value up to the present. His method, ideas and instruments have been continued to develop and accomplish and are extensively employed in many countries, so in the USSR too.

In the present we are going to give an account about the development of his ideas in the USSR, in the field of gravimetry.

Работы выдающегося венгерского физика Роланда Этвеша в области гравиметрии и тяготения не потеряли своего значения до настоящего времени. Его методы, идеи и приборы продолжают развиваться, совершенствоваться и получают все большее применение во многих странах, в том числе и в СССР.

В настоящем докладе остановимся кратко на развитии в СССР идей Р. Этвеша в области гравиметрии.

Известно, что гравитационное поле Земли может быть охарактеризовано различными величинами: силой тяжести, потенциалом силы тяжести, уклонениями отвеса, первыми, вторыми и более высшими производными силы тяжести.

Одной из замечательных заслуг Р. Этвеша является то, что для геодезических и геологических целей он предложил изучать не только силу тяжести и первые производные потенциала силы тяжести (сила тяжести, условия отвеса), но и показал на практике возможность использования вторых производных силы тяжести (горизонтальные градиенты силы тяжести и кривизна уровенной поверхности).

Использование высших производных потенциала силы тяжести в ряде случаев, особенно в гравиразведочной практике, может быть более выгодным, чем использование самой силы тяжести. Если сравнить отношение величины аномалий горизонтальных градиентов силы тяжести к нормальной величине этих градиентов, обусловленной фигурой Земли и ее суточным вращением, с соответствующим отношением для силы тяжести, то первое будет во много раз больше второго. Это обстоятельство позволяет при помощи вторых производных потенциала силы тяжести более детально изучать уровенную поверхность и аномальные массы, особенно поверхностные, что важно для гравиметрической разведки.

Р. Этвеш не только предложил использовать для изучения гравитационного поля Земли вторые производные, но и, что особенно ценно, разработал теорию и конструкцию высокоточного гравиметрического прибора — гравитационного вариометра и применил его на практике. Стоит подчеркнуть, что теория и конструкция гравитационных вариометров Р. Этвешом была разработана в такой степени, что они остались практически теми же самыми и по сей день.

Гравитац ионные вариометры Этвеша получили всемирное признание и повсеместно, широко вошли в практику гравиметрических работ, в том числе и в СССР.

Вначале в СССР гравитационные вариометры Этвеша применялись лишь для геодезических целей, для детального исследования фигуры уровенной поверхности силы тяжести. Так, в России были выполнены еще в 1915–16 гг. наблюдения с гравитационным вариометром вдоль ряда первоклассной триангуляции Пулково — Николаев. Эти работы с гравитационным вариометром были вторыми в мире после наблюдений Этвеша в Венгрии.

Развитие промышленности и ее потребности в сырье привели к бурному развитию в 20-х годах гравиметрической разведки. С этого времени в практике гравиметрических работ в СССР широкое применение получают гравитационные вариометры, которые являлись долгое время единственными приборами, позволяющие выполнять детальную гравиметрическую съемку с разведочными задачами.

В 1926 г. начинаются массовые измерения гравитационными вариометрами в районе Курской Магнитной Аномалии (КМА) с целью определения глубины залегания и формы железорудных месторождений. Работа на КМА явилась важной школой развития советской геофизики. Здесь была разработана и затем эффективно применена в СССР методика проведения количественной интерпретации результатов детальных вариометрических гравиразведочных работ на пластовых железорудных месторождениях (Шмидт, Никифоров, Сорокин).

Успехи вариометрических работ на КМА показали, какие огромные возможности имеет вариометрический метод гравиразведки.

Вслед за КМА гравитационные вариометры были использованы при разведке промышленно-важного Криворожского железорудного месторождения. Здесь, как и на КМА, выяснялись структуры железорудных месторождений и в соответствии с результатами наблюдений выбирались места для последующего бурения.

С конца 20-х годов вариометрический метод гравиразведки используется при поисках каменного угля. Такие работы велись в 1929 г. в Донецком каменноугольном бассейне, а последующем на Урале, Кузбассе, Караганде, Грузии и в других районах страны. При этом в основном решается задача расширения известных угленосных площадей, поскольку залежи каменного угля бывают скрыты мощными поверхностными отложениями.

В 30-х годах гравитационные вариометры получают все большее и большее применение в гравиразведке. Они успешно используются при изучении пластовых месторождений калийной соли (в районе Соликамска), выполняются работы по поискам хромитов и сульфидов на Урале (Юньков, Андреев). Здесь следует отметить, что результаты применения гравитационных вариометров для разведки хромитовых и сульфидных месторождений представляли особый методический интерес, так как они практически показали возможность на основе исследования аномального гравитационного поля изучать не только крупные геологические структуры, но и небольшие геологические объекты, какими являются хромитовые, сульфидные и другие изолированные рудные месторождения.

Особенно большой вклад внес вариометрический метод гравиразведки в деле поисков нефтяных месторождений.

Начало гравиметрических работ в нефтяной промышленности в СССР было положено производством вариометрической съемке на реке Эмбе в 1925 г.

На первом этапе (с 1925 г. до 1930 г.) работы велись в небольшом объеме в Урало – Эмбенском районе, Азербайджане, на Северном Кавказе и Средней Азии с целью выяснения гравиметрического (вариометрического) метода при разведке нефти. Напомним, что к этому времени были достигнуты первые успехи вариометрической разведки в изучении железнорудных месторождений.

Результаты вариометрических работ в районах, где предполагалось наличие нефти, показала действенность этого метода в поисках нефти, что способствовало резкому увеличению объема гравиметрических работ. Только в 1931 – 32 гг. число вариометрических работ было в 2 раза больше, чем за все предыдущие годы вместе взятые.

Начиная с середины 30 годов ликвидируется прежняя разрозненность гравиметрических работ в СССР, разорванность детальных и региональных исследований, осуществляется широкая связь с другими геофизическими методами разведки.

В 40-х годах на техническое вооружение гравиметрической разведки стали входить гравиметры, которые позволили значительно расширить фронт гравиразведочных работ и производить их на огромных площадях.

Расширяются возможности гравиразведки. Наряду с открытием и изучением отдельных нефтяных структур, данные гравиметрической (гравиметровой и вариометрической) съемки используются для выделения целых областей, перспективных для поисков нефти для осуществления тектонического районирования. Так были, например, оконтурены Урало – Змбенский р-н, Апшероно – Нижне – Куринский нефтеносный район и т. п.

И в настоящее время, несмотря на широкое внедрение в производство гравиметрических работ гравиметров, гравитационные вариометры занимают почетное место в гравиразведке.

Успешному развитию гравиразведки в СССР способствовали работы советских ученых по разработке отечественных гравитационных вариометров, первые из которых были изготовлены в 1924 г. под руководством Никифорова. В последующие годы были сконструированы и изготовлены вариометры различных систем: двухрычажные, трехрычажные, вариометры с короткой крутильной нитью и т. д. Большое внимание при этом уделялось повышению чувствительности вариометров и повышению производительности с ними. В настоящее время в СССР разрабатываются и широко используются в гравиразведке короткопериодические горизонтальные градиентометры с кварцевой крутильной системой (Поддубный). Эти градиентометры миниатюрны, просты и надежны в обращении. Основным их преимуществом перед обычными гравитационными вариометрами является значительно большая производительность съемки при сохранении почти той же точности наблюдений.

Известно, что новые перспективы в гравиразведке могла бы открыть возможность использования второй вертикальной производной потенциала силы тяжести.

В 1922 году известный русский математик Стеклов, проведя в более общей постановке анализ уравнения движения коромысла гравитационного вариометра, составленного Этвешем, пришел к выводу о том, что Этвеш рассматривал лишь одно из возможных движений крутильной системы. Оказалось, что два других уравнений момента позволяют не только определять горизонтальные градиенты, но и вертикальный градиент силы тяжести, чего не предполагал Этвеш. Стеклов показал также, что для получения величин градиентов силы тяжести можно значительно упростить принятую методику наблюдений. Он предполагал также внести некоторые изменения в конструкцию прибора. К сожалению, подробности работы Стеклова не опубликованы, а небольшая статья по этому вопросу была незаслуженно забыта.

Позднее, в начале 30-х годов вопрос об измерении вертикального градиента силы тяжести рассматривался Садовским, показавший технические трудности в изготовлении в то времена вертикального гравитационного градиентометра.

Возросшие технические возможности позволили в настоящее время поставить вопрос о создании подобного прибора и в этом направлении в Советском Союзе ведутся работы (Федынский).

Вертикальный градиентометр интересен еще тем, что его возможно конструктивно осуществить таким образом, что на его показания не будут сказываться вертикальные и горизонтальные поступательные перемещения основания прибора, т. е. при помощи его можно будет производить измерение вертикального градиента силы тяжести с самолета или корабля.

Практика использования вторых производных потенциала силы тяжести привела к развитию в 20—30-х годах методов количественной интерпретации, ориентированных на использовании этих производных. В эти годы были разработаны методы расчета поля вторых производных для некоторых тел простой геометрической формы (ступени, параллелепипеда, шарового сегмента и т. д.), а также для двумерных тел произвольного сечения с помощью палеток. Были даны формулы для определения координат центра тяжести и избыточной массы по аномалиям градиента силы тяжести (Нумеров, Гамбурцев, Садовский, Заморев и др.). В те годы были разработаны способы тщательного учета рельефа местности при вариометрической съемке, получившие всеобщее признание (Нумеров и др.). Был изготовлен, прибор, позволяющий сразу получать поправки за рельеф (Леонтовский), предлагались особые рейки для нивелировки местности, упрощающие дальнейшее вычисления и т. д.

На современном этапе в разведочной гравиметрии широко используются способы пересчета карт аномалий силы тяжести в карты аномалий вторых и более высоких производных силы тяжести (Веселов, Сагитов, Шванк и др.), что также в известной мере является развитием идей Этвеша, который, как отмечалось, предлагал изучать гравитационное поле Земли с помощью вторых производных потенциала силы тяжести.

В свое время Этвеш рассмотрел вопрос о введении в показания гравиметрических приборов, находящихся на движущемся основании, поправки за влияние силы Кориолиса. В настоящее время в СССР и других странах проводятся гравиметрические работы на кораблях и самолетах. Большие скорости движения самолетов, а также сложная траектория полета заставили заново проанализировать вопрос об учете эффекта Этвеша с тем, чтобы учитывать влияние квадратичных членов и кривизну траекторий полета. С этой целью были проведены специальные исследования.

Не потеряли своей актуальности и работы Этвеша по определению постоянной тяготения. В этих его работах, на наш взгляд, ценным являются не числовые значения определенной им постоянной тяготения, а метод определений. Заслуживают внимания два способа регистрации малых эффектов притяжения, с которыми обычно приходится иметь дело при опытах по тяготению, а именно применение для этой цели компенсатора тяготения и мультипликатора тяготения.

Этвеш, наряду с опытами по выявлению степени эквивалентности инертной и тяготеющей массы, ставил опыты по определению эффекта экранизации тяготения. С помощью вариометра II рода он показал, что поверхностный пласт Земли в 1 км толщиною не изменяет притяжения Солнца даже на 1 : 100 000 000 часть. В 1961 г. опыты по проверке экранизации тяготения в СССР были проведены В. Б. Брагинским и Рукманом в виде установки, которая периодически экранизировала некоторую массу. Экранизирующего эффекта не было обнаружено.

Одним из авторов доклада предпринимается попытка разработки нового определения постоянной тяготения. Излишне говорить, что задача более точного численного определения постоянной тяготения представляет большое теоретическое и практическое значение.

Мы не останавливаемся на опытах Этвеша по определению тождественности инерциональной и тяготеющей массы, так как этот вопрос относится больше к компетенции физиков. Стоит лишь отметить, что и в этой области в Советском Союзе проводятся соответствующие исследования.

На грани 19—20 вв, когда вышли в свет работы Этвеша по тяготению, начинали решаться лишь некоторые геодезические и геологические задачи. В настоящее время в изучении гравитационного поля Земли нуждаются ряд отраслей науки; геодезия, геология, физика Земли, астрономия, теория и практика полетов искусственных спутников Земли. Отмечая прогресс и успехи современной гравиметрии, мы всегда с благодарностью вспоминаем Роланда Этвеша, который заслуженно занимает одно из первых мест среди видных гравиметристов мира.

ROLAND EÖTVÖS AND PALEOMAGNETISM

With 3 figures

by

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SUMMARY

At the end of the previous century Roland *Eötvös* investigated by the aid of his translatometer the remanent magnetism of earthenwares and stamped bricks. From the measured components of the remanent moment he determined the angle of inclination contemporary to the burning. It is interesting, that he found negative inclinations for the centuries B. C. – similarly to the data found before him by the Italian *Folgheraiter.*

Eötvös investigated with his translatometer in each case the magnetization of rocks which caused magnetic anomalies. He constructed also a device, which is in steady use, for taking oriented samples.

He did not publish his translatometer-method for the determination of the induced and remanent magnetic moment. The author makes known the practice followed by the Eötvös Geophysical Institute, and shows the method how to calculate declination and inclination of past times from measurements made by the translatometer.

It has been well known a good while back that *fired clay products* get magnetized in the earth's magnetic field in the direction of the field during their cooling after being burnt out and they retain their magnetism as *remanent magnetism* with a great stability Thus these fired clay products may keep at least the *direction* of the geomagnetic field prevailing during their firing (more correctly speaking: during their cooling), hence providing a possibility for the determination of the direction of the ancient geomagnetic field, taking into account the remanent magnetisation of the clay product and its site during the firing process.

The Italian Folgheraiter investigated the remanent magnetization of *Etruscan ceramic relics* from this point of view. Obviously, such earthen vessels had been standing upright when being burnt, therefore the vertical direction may be marked on them easily, though their horizontal directional site might be varied. Thus the direction of their remanent magnetization can only indicate the angle between the direction of the ancient magnetic field and the vertical, respectively its complementary, i. e. the *inclination*, while the declination, which characterises the direction of the horizontal projection of the field-vector remains unknown. Folgheraiter based his conclusions concerning the direction of the remanent magnetization of the ceramic vessels — and concerning so the inclination prevailing during their burning — on the distribution of the *free magnetism* manifesting itself on the surface of these vessels. He found that in Italy in the 8th century B. C. the inclination was small and its *direction was opposite* to the actual, going through zero a few centuries later and turning after all to *positive values* now prevailing.[1]

In Hungary Roland Eötvös conducted investigations of this kind, using a more developped method — not only on ceramic vessels, but on *baked bricks*, too. It is namely well known in the case of marked bricks, on wich side they come to lie during their firing, while their directional position in the horizontal plane could be varied. Therefore this kind of investigation is able also to define only the *inclination of ancient times*. Eötvös did not consider the distribution of the free magnetism, but he determined the components of the remanent magnetic moment by his sensitive *magnetic translatometer*. The resultant of these gave him the direction of the resultant magnetization from which the magnetic inclination of those times could be computed. His main results were as follows:

Era	Inclination
B. C. IV. century	−35°
,, III. ,,	−20°
A. D. about 1400	58°
,, 1669	72°
,, 1748	68°
,, 1870	62°

As we see the inclination had been also *negative* in Hungary in the centuries B. C., it turned to be zero in one of the first centuries A. D. and it has been remaining positive since then reaching its maximum value at the end of the 17th century.

The principle of the procedure and the results for the inclination were not published by Eötvös in print but they had been dealt with in the session of the Hungarian *Mathematical and Physical Society* on the 1st February, 1900. The lecture was reviewed by Alexander Mikola in the *Gazette of Natural Sciences*.[2] According to this Eötvös had the intention to continue the investigation and he wanted to concentrate especially on clay products of the first millenium A. D. We have no further information on this matter.

At a later date — in his report presented to the General Conference of the International Geodetic Association in 1909 — Eötvös published the susceptibility-data of some rock samples from the Fruska-Gora mountains.[3] He promised a more intensive investigation of the magnetization of these rocks and the explanation of this method but he could not keep his promise in his life time. His method of determining susceptibility and remanent magnetic moment was briefly dealt with by Eugene Fekete in the

Eötvös *Memorial Number of the Mathematical and Physical Journal* (of Hungary),[4] nevertheless no details were given.

Eugen Fekete also told that Eötvös had constructed a simple *magnetic compass,* too, to sample some rocks for the purpose of such investigations. By means of this instrument the position of the sample in its site, its orientation respectively the north, the east and vertical direction can be marked and so the three rectangular components of the magnetic moment can be determined. We know that the remanent magnetization of eruptive rocks is also the result of the magnetization obtained during the cooling process following the state of outflowing hot lava and stabilized afterwards. Therefore the determination of the remanent magnetization of such *oriented rock samples* may lead us to the information about the direction of the magnetic field intensity being active in the era of rock-genesis. On the other hand if we know the geological era in which the magnetic field assumed a given direction, we are in the position to compute the era of the origin of the rock (showing the remanent magnetization dealt with) and so the time of eruption of the volcano as well.

This method of investigation of rock magnetism is widely applied nowadays under the name of *paleomagnetic research.* We may state therefore that Eötvös with his investigations of the remanent magnetization of oriented rock samples quite closely approached to this modern branch of research, nevertheless we have no information about whether he attempted to apply his method to the determination of the magnetization of geological eras or to the determination of the age of rock genesis.

* * *

The above mentioned instrument of Eötvös had been called magnetic translatometer by him because it is suitable to measure the *translational force* exerted upon magnetic bodies. It is well known that in a homogeneous magnetic field magnetic bodies are affected only by rotational or directing force. A translational – attractive or repelling – action can only be present in case of an *inhomogeneous magnetic field.* If we denote the rectangular components of the magnetic field intensity by X, Y, Z, the derivatives with respect to the coordinates x, y and z by the respective indices, then the inhomogeneity of the magnetic field is characterized by the components of the following gradient-vectors:

$$\begin{aligned} \text{grad } X &= (X_x, \ X_y, \ X_z) \\ \text{grad } Y &= (Y_x, \ Y_y, \ Y_z) \\ \text{grad } Z &= (Z_x, \ Z_y, \ Z_z) \end{aligned}$$

Further we know that the *components of the translational force* acting in the inhomogeneous magnetic field upon the magnetic body with the moment $\mathfrak{M}(M_x, \ M_y, \ M_z)$ are the scalar products of the $\mathfrak{M}$ moment of the magnet and of the gradient-vector of the individual components of the field intensity:

$$\begin{aligned} P_x &= (\mathfrak{M}, \text{grad } X) = M_x X_x + M_y X_y + M_z X_z \\ P_y &= (\mathfrak{M}, \text{grad } Y) = M_x Y_x + M_y Y_y + M_z Y_z \\ P_z &= (\mathfrak{M}, \text{grad } Z) = M_x Z_x + M_y Z_y + M_z Z_z \end{aligned}$$

The magnetic translatometer of Eötvös is based on the principle of the Coulomb-balance. In its form it is like the horizontal gravitational variometer of Eötvös – the Eötvös-balance of today – with the usual weighing mass on the one end of the balance beam and with the small magnet suspended on the other end. (Fig. 1.) Obviously, upon the balance beam hanging on a vertical wire only horizontal forces, therefore only the *horizontal components* P_x and P_y of the translational force exerted in the inhomogeneous field will be acting, thus producing a rotational couple. According to this the instrument is suitable to measure the horizontal components P_x and P_y of the translational magnetic force. In the original form of the instrument the inclination of the measuring magnet to the horizontal plane could be varied. Eötvös has shown that if we make the measurement both with a magnet inclined under the angle i downwards and then upwards, we obtain components P_x, P_y and P'_x, P'_y from which the quantities

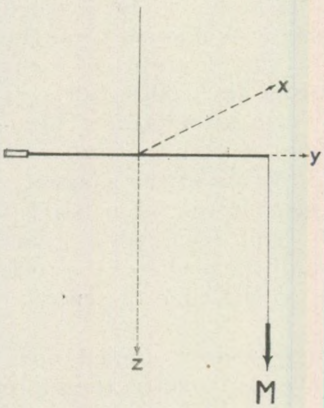

Fig. 1.

$$X_x,\ X_y = Y_x, \quad X_z = Z_x \quad \text{and} \quad Y_z = Z_y$$

i. e. *4 of the 6 data* needed for the characterization of the inhomogeneity of the magnetic field can be determined. (As a matter of course we have to measure and to take into account the rotational couple originated from the inhomogeneity of the gravitational field, too.)[5]

Later on Eötvös made the remark that he had not reached with his instrument such a degree of sensitivity which could enable him not only to demonstrate the existence of the minute normal variations of the magnetic field but to measure them, too. But for the determination of the far greater *anomalous variations* the device proved itself to be more then sufficiently sensitive.[6]

* * *

As I have mentioned before, Eötvös did not publish – and even Fekete has given a rather brief explanation of it – how the translatometer could be applied to the determination of the induced and remanent moment of the rocks. Therefore it will be perhaps of some interest if on this occasion I should deal with the matter in a few details basing upon the routine of measurement followed for many decades after the death of Eötvös, too, in the Hungarian Geophysical Institute named after him.

In the course of this application the inclination of the *measuring magnet* is not changed, but it is hanged so that its *axis* be directed permanently *vertically* downwards. Assuming this direction as the direction of the positive z-axis, we have

$$M_x = M_y = 0, \quad M_z = M$$

and the horizontal components of the translational force (exerted upon the magnet of moment M) producing a couple upon the translatometer are:

$$P_x = MX_z$$

$$P_y = MY_z$$

Let a horizontal direction be assumed arbitrarily as the direction of the positive x-axis and let us set the horizontal beam of the translatometer perpendicular to it, so that the direction of the beam showing towards the suspension point of the measuring magnet M, be the direction of the positive y-axis (Fig. 1.) In this case from the components of the translational force exerted upon the measuring magnet it will be only the component

$$P_x = MX_z$$

which produces a couple on the horizontal balance beam. (The horizontal component of the magnetic field, H could also produce a couple, but the value of this would be zero because the moment of the measuring magnet has no horizontal component.)

Taking as origin of the coordinate system the intersection of the suspension wire and the horizontal balance beam and denoting by l the length of the positive half part of the horizontal balance beam, we have for the coordinates of the measuring magnet M (in its centre)

$$x = 0, \quad y = l, \quad z$$

Therefore the torque of the forces acting upon the measuring magnet M relating to the z axis coinciding with the suspension wire is

$$F = xP_y - yP_x = -lP_x = -lMX_z$$

The suspension wire of a torsion-coefficient τ will be twisted owing to the action of the torque by an angle ϑ, this twisting producing an opposite torque of value $\tau\vartheta$, balancing the torque F in case of equilibrium, i. e.

$$\tau\vartheta = -lMX_z$$

Applying the common scale-reading by means of a mirror and denoting the distance of the scale and the mirror by D, the scale-reading corresponding to the equilibrium position, as producing itself in case of a homogeneous magnetic field, by s_0; the reading in case of the position with the twisting by the angle ϑ let be: s, the difference $s - s_0$ let be: n; taking into account that with increasing of the positive value of ϑ the scale reading s diminishes, we get

$$\vartheta = \frac{s_0 - s}{2D} = -\frac{n}{2D}$$

Therefore we have

$$X_z = \frac{\tau n}{2DlM}$$

The factor $\tau : 2\,DlM$ may be called the sensitivity (more correctly: scale value) of the instrument with respect to X_z; it may be denoted by ε.

With this we have:

$$X_z = \varepsilon n.$$

Now if we have to determine the magnetic moment $\mathfrak{m}(m_x, m_y, m_z)$ of some rock sample or other magnetic body by means of our translatometer measuring the translational action of the inhomogeneous magnetic field of the body, then from the data characterizing the inhomogeneity of the magnetic field of the body we have only to deal with the derivative X_z and we have to express this by means of the magnetic moment of the body, $\mathfrak{m}$, and the data characterizing its position with respect to the measuring magnet.

The *potential of the magnetic field of a magnet* being at the point (ξ, η, ζ) and having the moment $\mathfrak{m}$ can be expressed at the point (x, y, z) as follows:

$$V = -\left(\mathfrak{m}, \operatorname{grad}_{\xi\eta\zeta}\frac{1}{r}\right) = \left(\mathfrak{m}, \operatorname{grad}_{xyz}\frac{1}{r}\right) =$$

$$= m_x\frac{\partial\frac{1}{r}}{\partial x} + m_y\frac{\partial\frac{1}{r}}{\partial y} + m_z\frac{\partial\frac{1}{r}}{\partial z}$$

$$V = m_x\left(\frac{1}{r}\right)_x + m_y\left(\frac{1}{r}\right)_y + m_z\left(\frac{1}{r}\right)_z$$

Therefore

$$X_z = V_{xz} = m_x\left(\frac{1}{r}\right)_{xxz} + m_y\left(\frac{1}{r}\right)_{xyz} + m_z\left(\frac{1}{r}\right)_{xzz}$$

where

$$r = [(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2]^{\frac{1}{2}}$$

and

$$\left(\frac{1}{r}\right)_{xxz} = 15\frac{(\xi - x)^2(\zeta - z)}{r^7} - 3\frac{\zeta - z}{r^5}$$

$$\left(\frac{1}{r}\right)_{xyz} = 15\frac{(\xi - x)(\eta - y)(\zeta - z)}{r^7}$$

$$\left(\frac{1}{r}\right)_{xzz} = 15\frac{(\xi - x)(\zeta - z)^2}{r^7} - 3\frac{\xi - x}{r^5}$$

Let us consider two characteristic positions of the body to be investigated; in these cases the expressions for the derivatives of $\frac{1}{r}$ and the derivative X_z itself will be considerably simpler. In one of the cases the body to be investigated will be placed on the *level of the measuring* magnet in the direction of the $+x$ axis; in the other case it is assumed to be *under the measuring magnet* at a given distance from it.

I. In this case the body of a magnetic moment $\mathfrak{m}(m_x, m_y, m_z)$ let be placed on the *level of the measuring magnet* at a *distance ϱ from M in the direction of the $+x$-axis,* the angle of wich with respect of the balance beam is $-90°$, i. e. the coordinates of the body will be

$$\xi = \varrho, \quad \eta = y = l, \quad \zeta = z$$

At this point we have obviously

$$\xi - x = \varrho$$

$$\eta - y = 0$$

$$\zeta - z = 0$$

$$r = \varrho$$

$$\left(\frac{1}{r}\right)_{xxz} = 0$$

$$\left(\frac{1}{r}\right)_{xyz} = 0$$

$$\left(\frac{1}{r}\right)_{xzz} = -\frac{3}{\varrho^4}$$

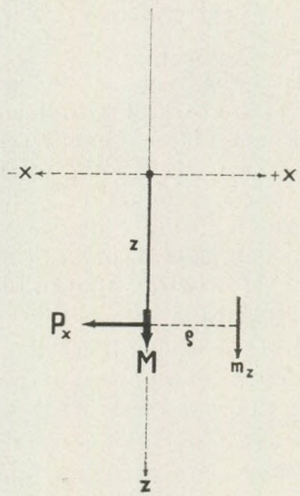

Fig. 2.

Therefore

$$X_z = -\frac{3m_z}{\varrho^4}$$

$$P_x = -\frac{3m_z}{\varrho^4} M$$

and

$$F = \tau\vartheta = -lMX_z = \frac{3Mm_z l}{\varrho^4}$$

We see that in this case the torque F produced as a consequence of a negative gradient X_z and a likewise negative translational force P_x, is positive, further we see that this rotating effect is brought about solely by the *vertical component* of the magnetic moment of the body under investigation.

Let us solve our equation for m_z:

$$m_z = -\frac{\varrho^4 X_z}{3} = -\frac{\varrho^4 \varepsilon}{3} n,$$

and the factor of the elongation n, i. e. the sensitivity (or scale value) of the instrument with respect to m_z denoted by R, then we have

$$R = \frac{\varrho^4 \varepsilon}{3} = \frac{\varrho^4}{3} \frac{\tau}{2DMl}$$

and

$$m_z = -Rn.$$

Thus the magnetic moment of the investigated body pointing vertically downwards may be very simply computed from the observed value of the elongation n, supposing that the characteristic data ε and ϱ are given.

As a rule, the sensitivity-constant ε is not determined by computation — using the data τ, D, M, l —; preferably we get it experimentally, using a magnet of known moment m_z at a given distance ϱ and observing the elongation n. The quantity R will be computed in each case by means of ε and ϱ.

The component m_z of the magnetic moment of the body under discussion is composed of two parts: one of them is the vertical component $\varkappa v Z$ of the magnetic moment *induced* in the body by the actual magnetic field vector $\mathfrak{H}(X, Y, Z)$, the other is the component μ_z of the *remanent* magnetic moment of the body, where we have denoted the magnetic susceptibility of the body by $\varkappa$ and its volume by v. In case of a homogeneous isotropic body the z-component of the induced moment amounts always to $\varkappa v Z$ irrespective of whether that or other direction (axis) of the body should be in vertical position or whether it pointed upwards or downwards. On the other hand the remanent moment $\vec{\mu}$ is a vector bounded to the body, i. e. its vertical component depends on that which axis of the body (and in which direction, upwards or downwards) would occupy the vertical position.

Therefore if we set one of the axes of the body in vertical position and we revert it afterwards by turning the body upside down, the vertical component of the magnetic moment induced in the body remains unchanged, i. e. $\varkappa v Z$, while the vertical component of the remanent magnetic moment of the body reverts its sign however keeping its absolute value. Thus the action of the body exerted upon the measuring magnet of our device is in one of the cases proportional to the *sum* of the vertical components of the induced and remanent moments, in the other to the *difference* of the same two moments.

Let us mark the end of the axis of our body in the positive direction and the elongation brought about by the action of the body if the positive end of the axis dealt with points *downwards*, let be benoted by n_a and in the *reversed* case by n_f. According to the above discussion, we have

$$\varkappa v Z + \mu_z = -Rn_a$$

$$\varkappa v Z - \mu_z = -Rn_f$$

Adding and substracting these two equations and dividing by $2vZ$ resp. by 2, we get the susceptibility $\varkappa$, respectively the vertical component μ_z of the remanent magnetic moment of the body. Denoting the sum $n_f + n_a$ by c, the difference $n_f - n_a$ by d we have

$$\varkappa = -\frac{R}{2vZ}c$$

$$\mu_z = \frac{R}{2}d.$$

In the course of paleomagnetic researches we have to determine the remanent magnetic moment of *oriented rock samples*. We may represent the orientation of the sample, i. e. its position at the original place of occurrence most simply by suitable denoting in the body the ends of the three axes taken parallelly to the geographical coordinate-axes. Then we take the body – keeping a position as mentioned above – nearby the instrument and we set the *marked axes one after another* in *vertical position*, executing two observations in each position (one whith the axis pointing upwards and one by reversing it). We may easily derive the following formulae:

$$\mu_N = \frac{R}{2}d_N$$

$$\mu_E = \frac{R}{2}d_E$$

$$\mu_Z = \frac{R}{2}d_Z$$

where N denotes the northern (positive) end of the x-axis, E the eastern end of the y and Z the end of the z-axis poiting downwards, all three being bounded to the rock body in its position as occupied at the original place of occurrence.

If it may be supposed that the remanent magnetic moment of the rock should be due to the inducing effect of the *magnetic field of the Earth prevailing at the genesis of* the rock, $\mathfrak{H}_0(X_0, Y_0, Z_0)$ and if we may suppose – at least on the average of many samples – that the orientation of the sample-rock has not changed – or only in a way to be taken into account – than we are entitled to assume that the values of μ_N, μ_E and μ_Z are proportional to the one-time components of the magnetic vector, i. e. X_0, Y_0 and Z_0 respectively. The factor of proportionality, the one-time magnetic susceptibility of the rock being unknown we are able to determine only the direction of $\mathfrak{H}_0$, its magnitude remaining indeterminable.

The magnetic *declination* D_0 and *inclination* I_0 characterizing the one-time field vector will be determined by the folloving formulae:

$$\cos D_0 = \frac{X_0}{\sqrt{X_0^2+Y_0^2}} = \frac{\mu_N}{\sqrt{\mu_N^2+\mu_E^2}} = \frac{d_N}{\sqrt{d_N^2+d_E^2}}$$

$$\sin D_0 = \frac{Y_0}{\sqrt{X_0^2+Y_0^2}} = \frac{\mu_E}{\sqrt{\mu_N^2+\mu_E^2}} = \frac{d_E}{\sqrt{d_N^2+d_E^2}}$$

$$\operatorname{tg} I_0 = \frac{Z_0}{\sqrt{X_0^2+Y_0^2}} = \frac{\mu_Z}{\sqrt{\mu_N^2+\mu_E^2}} = \frac{d_Z}{\sqrt{d_N^2+d_E^2}}$$

It is obvious that the declinational angle D_0 will be determined unequivocally by $\cos D_0$ and $\sin D_0$ and the same is true for I_0 being defined by $\operatorname{tg} I_0$ because the inclination may be only an angle lying always between $-\frac{\pi}{2}$ and $+\frac{\pi}{2}$.

II. Let us set now our body with the magnetic moment $\mathfrak{m}(m_x, m_y, m_z)$ vertically *under the measuring magnet* at a distance ϱ, i. e. to the point with the coordinates

$$\xi = x = 0, \quad \eta = y = l, \quad \zeta = z + \varrho$$

At this point we have obviously

$$\xi - x = 0$$

$$\eta - y = 0$$

$$\xi - z = \varrho$$

$$r = \varrho$$

$$\left(\frac{1}{r}\right)_{xxz} = -\frac{3}{\varrho^4}$$

$$\left(\frac{1}{r}\right)_{xyz} = 0$$

$$\left(\frac{1}{r}\right)_{xzz} = 0$$

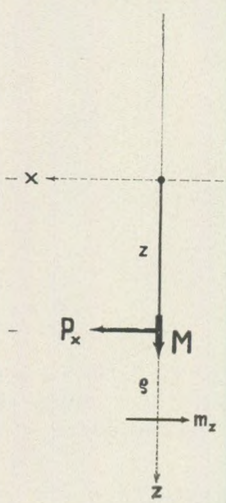

Fig. 3.

Therefore

$$X_z = -\frac{3m_x}{\varrho^4}$$

$$P_x = \frac{3m_x}{\varrho^4} M$$

and

$$F = \tau\vartheta = -lMX_Z = \frac{3Mlm_x}{\varrho^4}$$

We may see that the torque F originating as a result of the negative gradient X_z and the equally negative translational action P_z, is positive in this case, too. This rotating effect is due also in this case only to one of the components of the magnetic moment of the body under investigation, but now it is not the vertical component which exerts its effect but the *horizontal one in the direction of the x-axis*, m_x.

Solving our equation for m_x we obtain:

$$m_x = -\frac{\varrho^4 X_z}{3} = -\frac{\varrho^4 \varepsilon}{3} n = -Rn$$

$$m_x = -Rn$$

The angle of the positive direction of the x-axis with respect to the positive direction of the y-axis is $-90°$; therefore we are now able to determine – knowing ε and the distance ϱ and observing the elongation n, – the component m_x of the magnetic moment of the body placed under the measuring magnet, this component being perpendicular (according to the interpretation given above) to the balance beam.

This component consists of two parts again, of the $\varkappa vX$ *induced and* μ_x *remanent* components of the moment. As to their separation and to the determination of the components of moment μ_N, μ_E and μ_Z as well as of the D_0 declination and I_0 inclination characterizing the *one-time magnetic field of the Earth* the same procedure can be applied as with case *I*.

It is worth while mentioning however, that if the balance beam be set in the magnetic meridian, then the magnetic field vector has no component in the direction of the x-axis, therefore the body does not possess an induced magnetic moment in the *x-direction*. Thus, in this case the magnetic moments m_N, m_E and m_Z – observed on the oriented rock samples – will furnish themselves the μ_N, μ_E and μ_Z components of the remanent magnetization.

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* * *

Galina Nikolayevna P e t r o v a the renowned Soviet researcher of paleomagnetism added a remark to my lecture. Previously she lectured namely at the 9th International Symposium of the Association of Hungarian Geophysicists on the paleomagnetic researches, performed in the Academic Geophysical Institute of Moskow, and presented the diagrams of the variations of the paleomagnetic data in the USSR. Joining to her lecture, I directed her attention to the results of E ö t v ö s' similar investigations. She received Eötvös' data with great interest and plotted these data relating to about the last 500 years on her diagram, then discussing on my lecture, she showed on the screen the completed diagram. A good accordance became visible, if one considered the lag due to the „westward drift" in the secular variation of the geomagnetic elements. The negative inclinations found by E ö t v ö s (and F o l g h e r a i t e r) for the age B. C. awoke especially her attention, as for lack of soviet data for that age, she interpolated on the basis of earlier data found by Japanese researchers, on *positive level.* This is the reason, why should it be important, if additional investigations were corroborate the reality of Eötvös' negative data. It should mean that the 5 to 600 years period of the variation in the inclination should be considered as an oscillation of a shorter period superimposed on the larger variation of about 5000 years.

THE SECULAR VARIATION IN THE GEOMAGNETIC FIELD AND OTHER GEOPHYSICAL PHENOMENA

With 7 figures

by

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SUMMARY

The author makes a comparison between the pulsation recognizable in the secular variation of the geomagnetic field of about a 50 years period and similar periods observable in the speed of the Eart's rotation, in the amplitude of the variation of the polar altitude, and in the variation of sea level. From the conformity of the periods he draws the conclusion, that the secular variation of the geomagnetic field is connected with a large-scale movement of masses in the interior of the Earth.

Supposed, that the eccentricity of about 350 – 400 km of the Earth's magnetic centre is running together with the Earth's inner core, then the eccentricity of the masses in that dimension produces with respect to direction and size the triaxiality of the Earth known from geodesy, i.e. the equatorial ellipticity. It is possible, that the secular variation of the geomagnetic field can be attributed to tidal forces acting on the inner core, and in consequence, to displacement of the core.

It is well known that the magnetic field of the Earth is slowly changing. Long series of observations have proved that the total period of a variation amounts to about 500 years. This period is long in the history of magnetic measurements, but very short as compared with the periods of geological ages. It is, therefore, a great problem whether a slow variation bound to the interior of the Earth, i. e. presumably one of geological character of the Earth, i. e. presumably one of geological character, could cause a significant process of such a short period.

The investigations are rendered difficult by the fact that each magnetic component is separately measured and maps are plotted on the basis of measuring points spread all over the surface of the Earth. This mode of representation is very artificial. In the case of secular variations it is not the individual components which change but the whole space vector; another extraordinary difficulty for our observations is the plane representation of the globe. It is not surprising at all that with so many arbitrary elements nothing but a decomposed irregular image of the secular magnetic variation could be obtained.

Inspite of the difficulties mentioned above, it could be shown as a result of the mathematical analysis of these maps and of their direct comparison that a certain western tendency is observable in the process of variation.

From time to time the coefficients of the spherical functions of the geomagnetic field were computed from the magnetic world charts valid for different epochs. Even by making comparisons of the coefficients dating from different times, conclusions might be drawn to certain general characteristics of the secular variations, such as, for instance, that the seat of the secular variations is in the interior of the Earth, further, that the magnetic centre of the Earth is by 300 to 400 km eccentric in the direction of the western part of the Pacific Ocean. The point of eccentricity is wandering slowly towards West, in accordance with the secular variation.

For studying the details of secular variations, it is more advisable to make use of the magnetic data of certain observatories, these meet much better the requirements for uniformity and homogenity than the geometrical charts, or rather the coefficients of spherical functions computed from the same.

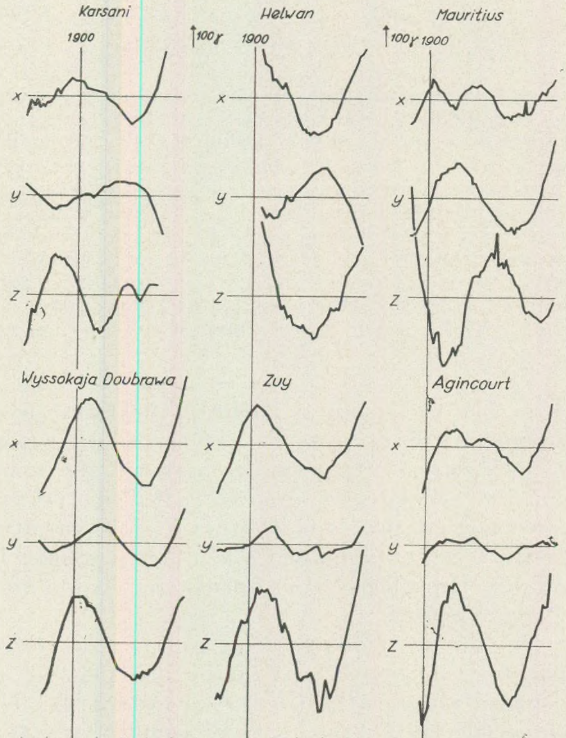

Fig. 1. **The 50 years period of the secular variation of the earthmagnetic field.**

By approximating the data of the observatories in the temperate zone with the power function of time in applying the method of least squares, one sees that over the mathematical curve, which may be considered as the average cours, is superposed a very considerable wave of 200 to 300 gamma amplitude and of about a 50 years period (Fig. 1).

In order to avoid the arbitrariness implied in the separation of the individual coefficients, it is advisable to study this phenomenon in the system of co-ordinates given by the tangent, the normal and the binormal of the adjusted secular variation.

The projection of the superposed vector on the direction of the space curve tangent is – within certain periods – positive, then negative, i.e. the measured point as compared with the adjusted one is from time to time fast, respectively late. This phenomenon is a result of the acceleration, respectively retardation of the secular variation, i.e. it corresponds with a longitudinal wave. In the observatories of the Northern hemisphere this accelaretion and retardation appears at the same time and the longitudinal wave of secular variation has its minimum around 1910 and its maximum around 1935 [4] (Fig. 2.).

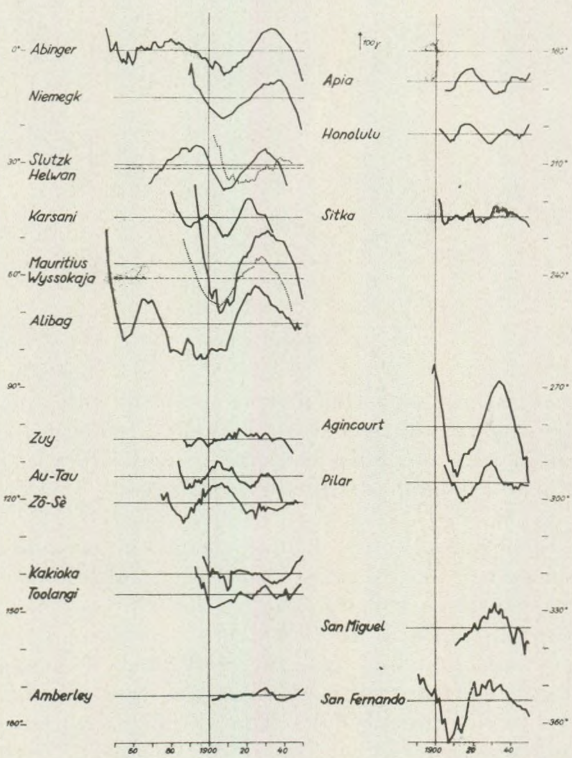

Fig. 2. **The longitudinal wave of the secular variation of the earthmagnetic field.**

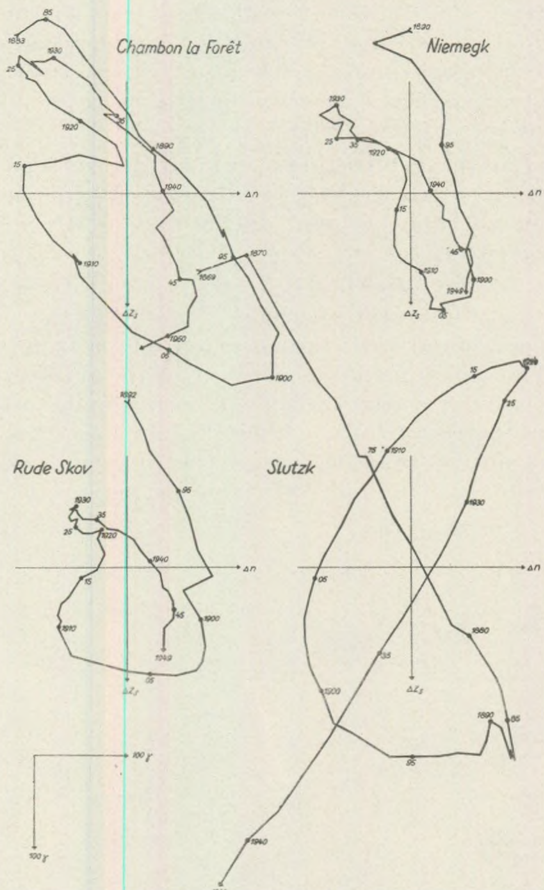

Fig. 3. The transversal effect of the secular variation of the earthmagnetic field.

The coefficients falling into the direction of the binormal and the main normal are representing the spiral-like movement of the end-point of the vector. This phenomenon may be called the transversal effect of the secular variation. The direction of turning of the wave vector is clockwise in the northern temperate zone [4] (Fig. 3.).

If one part of the process of secular variation is uniformly appearing nearly all over the Earth, then the main variation itself will be even more generally. The coordinate system X, Y, Z used to determi ne the magnetic vector is varying from point to point at the surface of the globe, it is, therefore, not suitable for recognizing the more general character of secular variation. In our further investigations the adjusted variation vectors may, as a consequence, be projected orthogonically onto the three coordinate planes perpendicular to each other and passing the centre of the Earth. If the centre of projection is lying around Pakistan, the picture of variation shows remarkable symmetry, since the variation vectors are turning against one another around the centre of the projected picture, whilst the vectors at the observatories at the border are showing radially outwards (Fig. 4.). From this fact and from suitable side views it may be concluded that a circular current of a radius of about 3000 km is flowing around Pakistan as a centre in a depth of about 3000 km. The situation and the intensity of this circular current is varying in time and this could possibly be the cause of the general part of secular variation [3].

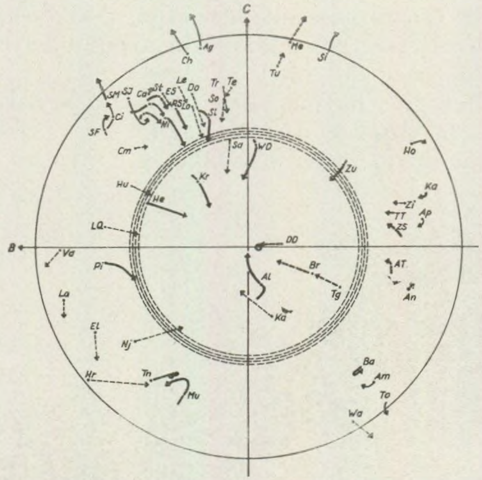

Fig. 4. The orthogonal projection of the adjusted vector diagrams of the secular variations as seen from Pakistan.

means the assumed circuit; *o* is the magnetic centre of the Earth. (The courses at the observatories lying at the side of the viewpoint are represented by thick solid lines, those on the opposite side by thin solid lines, and the courses extrapolated from the shorter courses at the observatories for a 50 years long period are represented by broken lines.)

It is noteworthy that the magnetic centre of the Earth is perpendicularly eccentric to the direction showing from the geometric centre towards Pakistan, and is moving directly towards Pakistan. Supposed the internal core of the Earth is the holder of the terrestrial magnetism, this means that the Earth's core is moving towards this direction. Consequently, the magnetic field of the Earth has three essential directions: the magnetic axis, the direction connecting the geometrical centre with the magnetic centre and the direction of movement of the magnetic centre. The three directions are about perpendicular to each other.

As a consequence of the aforesaid fact, it is quite natural that – if a fifty years period is observable in the secular variation of the geomagnetic field – this variation should appear in the movement of the internal core of the Earth as well. And this phenomenon should manifest itself at the surface of the Earth in other phenomena too.

The definite 50 years period of certain phenomena is indicating, as a matter of fact, a remarkable shift of masses in the interior of the Earth. A periodical fluctuation of about 50 years is observable in the speed of rotation of the Earth during the last fifty years. Around 1910, the Earth was late, and around 1935 she was fast as compared with the uniformly rotating Earth [11]. The amplitude of retardation and acceleration is $\pm 0,9$ sec. During a period like this, such a variation would be caused by the sinking or the rise of the sea level. Mass shifts of this size are not observed at the surface of the Earth and cannot be supposed neither in the crust nor in the outer mantle. The effective cause, accordingly, ought to be in the Earth's core (Fig. 5.).

A large-sized movement of masses is indicated, moreover, by the fact that a period of about fifty years is easily observed in the amplitude of the polar distance variation, which had its maximum around 1910 and 1958, and its minimum around 1935 (Fig. 5.) [10].

If the displacement of the magnetic centre is related to the large-sized mass movements in the interior of the Earth, this variation has to appear in the secular variation of the gravity field as well. We do not dispose of gravi-

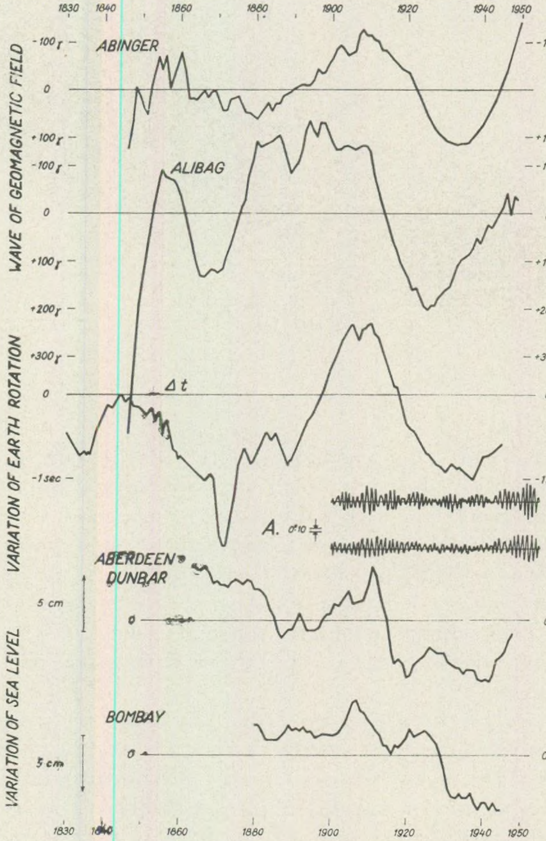

Fig. 5. The secular variations of various geophysical phenomena.
A. The variation of the amplitude of the oscillation of the polar heigth, and the same after the elimination of the shorter periods [10].

tational observational series of sufficient length and accuracy, but our long since observed sea-level height data correspond, after all, with the measurements of the height of the equipotential surface of the gravity field. It may be observed a certain connection between the long series sea-level observational data and the periodicity of previous phenomena (Fig. 5.) [5].

When comparing long series sea-level data systems submitted to overlapping averaging even the sea-level fluctuations show a certain general character. Thus the level surface variation in Sydney and Aberdeen, for instance, – which may be considered as opposite points of the globe – shows a definitely parallel course. The level surface height of Bombay and Tunis situated between them, as well as of Honolulu, lying in a far distance too, is showing, on the contrary, a definite counter-course. These features are justifying the allover character of the level surface variation. There is recognizable a periodicity of about fifty years in these data systems too (Fig. 6.).

The eccentric position and movement of the magnetic centre of the Earth are playing an important part in the above observations. Hence a possibly accurate knowledge of this very significant group of phenomena is required. Eccentricity is manifested, among others, by the fact that the horizontal intensity in the surroundings of the Western Pacific is by 10 000 gamma greater than in the Atlantic area situated at the opposite side of the Earth. Both the

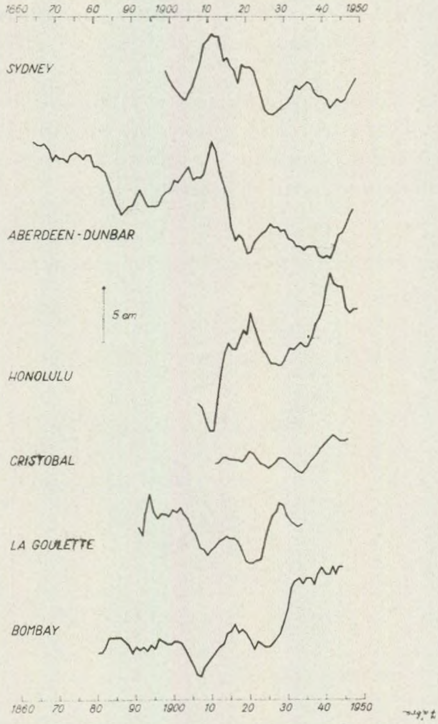

Fig. 6. The secular variation of the sea level at differents points of the Earth.

other magnetic components are similarly clear proofs of eccentricity. It is clear, accordingly, that the distortion of the geomagnetic field related to the eccentricity of the centre is by some orders of magnitude greater than the measuring accuracy of today.

By comparing the data of different epochs we can see that the magnetic centre is performing certain regular movements. It moved since Gauss in the W-NW direction with a speed of about 0,2° a year, whereas its eccentric position has moved between 300 and 400 km [6].

Magnetic eccentricity varied in the course of times, it is true, but it could always be followed continuously. Its demonstration is therefore based not on one single data system, but on numerous ones. We may consequently declare that the eccentricity of the geomagnetic field is an undeniable physical fact based on a great number of measuring data. Should this be true, a similar eccentricity must also be found in the internal material structure of the Earth.

It follows from the temporal displacement of the magnetic centre that it cannot be directly connected with the a continental structure of the Earth, not even with the structure of the crust and mantle. The change in time of these latters is not so essential [that an immediate periodic variation night be logically connected to them] similar to the magnetic secular variation. The origin of this effect can only be found under the crust and mantle, i.e., in the core. And from the long periodic character of the variation a process of great inertia may be concluded. We could presume therefore, that it concerns the eccentricity and slow shift of huge masses lying in great depths, as compared to the surface.

Now quite obviously the problem is raised that such an enormous eccentricity of huge gravitating masses may cause the static deformation of the globe. The direction of eccentricity of the magnetic field corresponds, as a matter of fact, with the direction of the equatorial great axis of the triaxial Earth

THE SIZE AND ORIENTATION OF THE AEQUATOR ELLIPSE ACCORDING TO DIFFERENT AUTHORS.

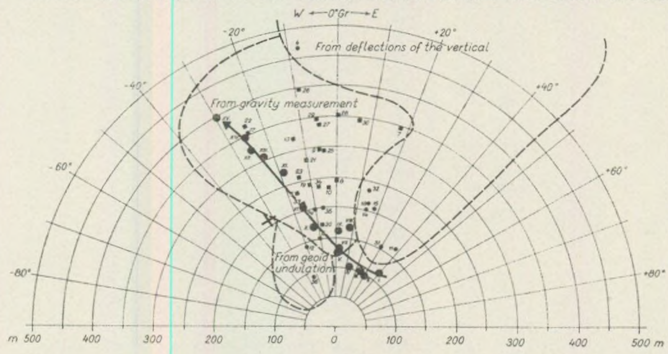

Fig. 7. The difference of the major and minor axis of the equatorial ellipse and the direction of its major axis calculated on the basis of gravity data (small circles and square points), orbits of artificial satellites (cross), and magnetic eccentricity (wide cirles). The solid arrow indicates the variation of the equatorial ellipse, calculated from magnetic data between 1550 and 1955 (2, 8, 12).

within the limits of measuring accuracy Supposed that the solid state internal core of a radius of 1250 km and of a density of 17 g/cm^3 is lying excentrically in the substance of the external core of liquid state and of a density of 11 g/cm^3, the equipotential surface deformation at the surface of the Earth indicates the equatorial oblateness obtained from different geodetic measurements and observations of artificial satellites are not only according to its direction, but also according to its magnitude (Fig. 7.) [2].

Material eccentricity has, of course, considerable consequences and, by emphasizing it, many phenomena can easily be explained. The eccentrical terrestrial core is, as a matter of fact, unbalanced within the solar system, Sun and Moon having a gravitational influence similar to the tide-producing force. As a consequence of the attractive force, the internal core eccentrically floating in the fluid external core is moving from east to west just as the tide-wave does. The western tendency of secular magnetic variation has thus become quite natural. The problem of the origin of the energy necessary to maintain the secular variation has been cleared as well. As long as the Earth's core is floating eccentrically in the fluid external core and around us enormous celestial bodies are moving, this eccentrical core is necessarily moving in the western direction. The energy required to maintain this movement has its origin from the energy of movement of the Earth, the Sun and the Moon.

The assumption of the eccentricity of the Earth's core requires, of course, the explanation of the eccentricity as well. The Earth's core is obviously developing round the point of maximum pressure. The point of maximum pressure, on the other hand, in case of an inhomogeneous Earth does not coincide with the geometrical centre of the Earth. If the Earth's substance on the oceanic hemisphere -- i.e. around the area of the Pacific Ocean – is more dense than on the continental one, the point of maximum pressure is shifting from the centre towards the Pacific, and thus the eccentricity is established.

There are even some geological comsequences of this idea. At the boundary surface of the two media of different densities has developed the volcanic zone of high seismicity around the Pacific. The rotating energy contents of the denser hemisphere are greater than those of the thinner one and when the speed of rotation of the Earth is reducing, the material falls forwards, piles up the substance ahead and is breaking off from that behind. This phenomenon can be seen in the foldings of the Cordilleras and Andes as well as in the formation of Eastern Asian-Australian deep-sea trenches. There are deep-sea trenches along the western shores of America too, it is true, but they are insignificant in their depth and width as compared with those of the Western Pacific, and their formation is in all probability rather due to piling than to rupturing phenomena.

When assuming the eccentricity and movement of the Earth's core it should be studied whether the consequences hereof would not render impossible such an hypothesis.

The movement of the Earth's core is accompanied by the alteration of the form and the gravity field of the Earth. According to our computations (carried out by Miss Anna Pintér in 1962), these variations reach in the height of the equipotential surface the order of one meter a year and in the intensity of the gravity field the order of a tenth milligal per annum.

It is well-known that after having repeated the Kühne-Furtwängler measurement carried out at the beginning of this centruy, a difference of 10 to 15 milligal has been observed. Certain objections were raised against this measurement, it is true, the first measurement, consequently, cannot be considered as a solid infallible starting basis, nevertheless it ought to be mentioned that in the last fifty years the intensity of the gravity in Potsdam has – as a consequence of the movement of the Earth's core – decreased by 11 mgal, this value gives the observed difference with the correct sign.

It is remarkable that the results of the absolute gravity measurements repeated after decades are differring from each other beyond the probable error of measurements. When comparing Potsdam with several other absolute base points, various differences are obtained as well. On the other hand there have been relative gravity profiles measured along rather long lines and repeated after ten years where no measurable variation has been indicated. In order to solve this problem we have obviously to carry out for a long period a great number of highly accurate relative and absolute gravity measurements. The possibility of the secular variation of the gravity field should, however, theoretically never be precluded.

Long ago it has been observed that the geographical coordinates of the astronomical or geodetic points are varying in time. As an explanation of this phenomenon it was used to presume that the position of the points at the Earth's surface is varying in time. When assuming the movement of the Earth's core, this problem is seen under quite another light. In consequence of the core movement the direction of the gravity field is varying too. This change of direction plays a part as well in the locations performed in different periods.

When computing the changes of directions occurring in consequence of the shift of the Earth's core we found that in extreme cases they can reach 0,08'' a year (E. Aczél, 1962.). The places of greatest variations are always the seas, whilst on continental cultured areas the variation amounts only to 0,02'' a year. About these $\Delta\lambda$ and $\Delta\varphi$ speeds of variation are shown by the results of our highly accurate locating measurements performed in the last decades. This computation did not thus lead to an insolvable contradiction.

From the coincidence of the periods of certain phenomena and from other natural phenomena we may conclude that our Earth is not a homogeneously balanced formation and its departure from it is manifested by the phenomena of considerable secular variations and movements. The study of these very phenomena permits a more detailed knowledge of the internal structure and features of the Earth. Therefore it is one of the most important works for the geophysicist dealing with the Earth as a whole to determine, how far the conditions of the homogeneity and equilibrium could be valid and when ought to be already used the inhomogeneous Earth's model satisfying the demands of the accuracy in higher degree.

As to our future aims, we intend to disclose the energy conditions of the group of phenomena and to investigate the natural phenomena which may give further informations on the degree of inhomogeneity of our Earth. A certain inhomogeneity is existing without any doubt, since a complete homogeneity in substance and energy would preclude any movement and accumulation of tension, whereas our experiences with the Earth are strictly contradictory.

The investigation of the relationships may bring results in many scientific branches and, following this way, we could perhaps better approach to an explanation of the physical background of phenomena. Unfortunately, the consequent pursuance of the theorem of inhomogeneity requires to abandon the concentric structure of our Earth, and the consequence of which is a sudden increase of the mathematical difficulties.

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SALIENT LATITUDINAL GEOTECTONIC ZONES IN CHINA WITH NOTES ON THE RELATED MAGNETO-GRAVITY ANOMALIES

With 9 figures

by

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SUMMARY

In recent years geological and geophysical explorations revealed in China at least three major tectonic zones of E-W trend, as follows:

1. The Inshan zone between the latitudes 40° and 43° N, as a boundary between North China Plain and the Mongolian Plateau.

2. The Tsinling zone between the latitudes 34° and 35° N as a natural divide between North and South China, which deflecting underneath the North China Plain, reappears on the eastern side of Yellow Sea in Japan.

3. The Nanling zone between 23° and 26° N latitudes is separating the Yangtze drainage system from that of the East, North, West, and Red Rivers.

All these zones have undergone mighty North-South compressions. Some parts of them are covered by younger beds, and traversed by faults.

The zones were already existing in Sinian times, but in places they were thrown apart in Mesozoic times, or later.

The existence of these zones, where they are sunken, was discovered by magnetic and gravimetric surveys. The isogams run mainly parallel to these structures.

Such geotectonic zones appearing in certain definite latitudes occur not only in China, but in many parts of the world. The author concludes, that they are of planetary origin, and connected with the present axis of rotation of the Earth. He refers to the tidal analyses of G. H. Darwin, then the researches of J. S. Lee, B. L. Lethchkov, and M. V. Stowas. The latter's theory gives account of the presence of some latitudinal zones, e. g., the Tsinling-zone.

J. S. Lee conceives that changes in the Earth's rotational speed may be caused by the uplifting and sinking of the various portions of the continents and oceans. As the Earth's internal portions are more sturdy to comply with the changed rotational speed than the crust, the superficial layers would tend to shift towards lower latitudes. Moreover, his theory offers an explanation for the occurrence of longitudinal tectonic zones, like the Cordilleran geosyncline and the longitudinal mountain ranges of western China, continuing southward to form the Indonesian arc. The same way, along the eastern margins of the continents were brought about the tensional zones, like the ocean deeps along the eastern Asiatic continent.

The existence of east-west geotectonic zones in China and elsewhere in Eastern Asia [1, 2] at definite intervals of latitude was first recognized by Professor J S. Lee in the early twenties of the present century. Since that time powerful structural disturbances of different geological ages along the several latitudes have been steadily brought to light from place to place [3] not only in other continents but also in parts of the great ocean basins.

In recent years extensive geological and geophysical explorations in numerous mountainous areas in China and in some of the adjoining plains have further revealed the subterranean existence of these tectonic zones. They are here and there interfered with by tectonic zones of other trend and are often covered by subsequent sediments.

It is now well known that there are in China at least three major tectonic zones [4, 5] of east-west trend (Fig. 1). Enumerating from north to south, they are as follows:

(1) The Inshan zone spreads between the latitudes 40° and 43° N, more intensely developed in latitudes 41°–42°. It marks the boundary between the North China Plain and the Mongolian Plateau. To the west it is more or less in

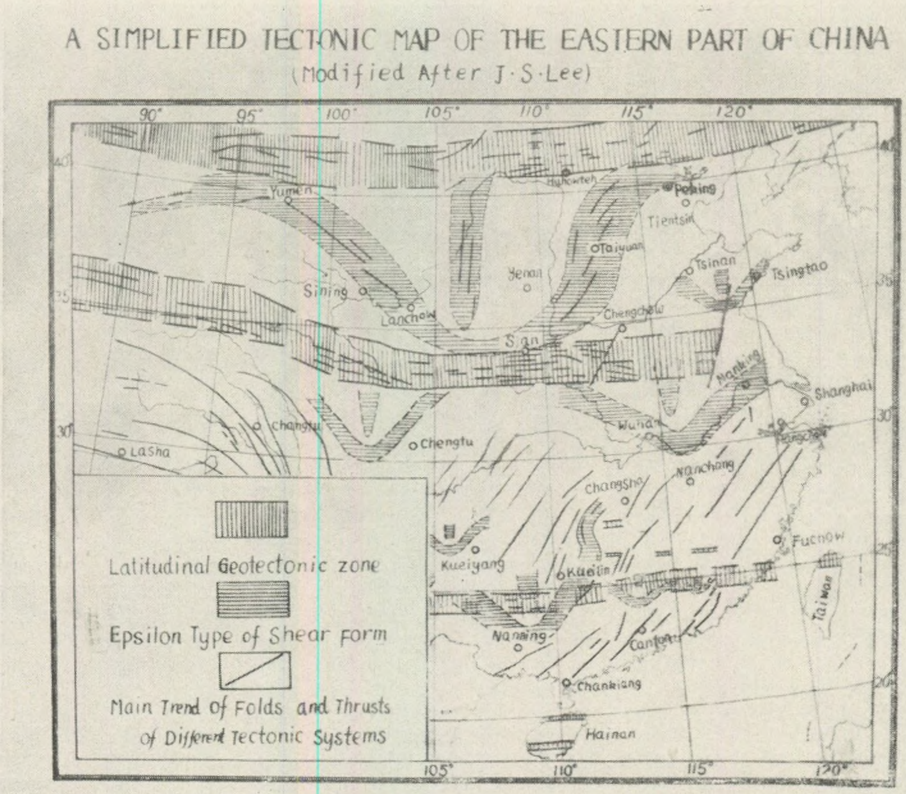

Fig. 1. Sketch map showing the distribution of the east-west geotectonic zones in China

line with the Tienshan Range, though the latter is somewhat deflected to the north. To the east it transects the Sungliao Plain and the Changpeishan Range.

(2) The Tsinling zone constitutes the natural divide between North and South China. This zone is spread between the latitudes 33° and 36° N more powerfully compressed in latitudes 34° – 35° and somewhat deflected to the north in its western continuation, the Kuenlun Range. Towards the east it dives underneath the North China Plain but reappears on the eastern side of Yellow Sea in Japan.

(3) The Nanling zone stretches along latitudes 23° – 26° N, roughly forming the divide of the drainage system of the Yangtze on the one hand and that of the East, North, West and Red Rivers on the other.

Of these three zones the Inshan and the Tsinling are far more prominent than the Nanling both in morphological and tectonic senses, the latter, though does not persistently trend east-west in its superficial structure, but the emplacement of a linear series of granite along this zone in a general east-west direction proving the deep-seated nature of this zone.

There is one common tectonic feature about all these zones, that is, they all have undergone mighty north-south compression more than once resulting in the formation of highly compressed folds, sometimes recumbent, and overthrusts with a general east-west strike.

The Inshan zone consist of folds of grand scale (Fig. 2.).

Its subsided eastern part runs across the Sungliao Plain forming the divide between the Sunghua-Kiang in the north and the Liaoho in the south. It is known as the Tiehling anticline, situated in latitude about 32° N, north of Mukden. It stretches east-west for hundreds of kilometres. Archaean gneiss and schist occupy a wide area in its axial part. These ancient rocks are unconformably overlain by a great thickness of Palaeozoic strata ranging from the Sinian to Permo-Carboniferous. Silurian and Devonian are absent on the southern side of the said anticline. On its southern side there occurs the Yenshan depression [6], likewise running east-west along the border between the Autonomous Inner Mongolian Region and the Hopei Province. Circumstantial evidence indicates that the Tiehling anticline has been uplifted in post-Sinian times probably by the Luliang movement.

In the area of northern Hopei [7, 8] of the Yenshan area and further west in the Tachingshan Range (Fig. 3), the Palaeozoic sediments and the Jurassic coal-bearing series are all involved in asymmetrical or overturned folds of subordinate magnitude [9, 10]. Their axes all trend east-west. They are best developed in the axial part of a synclinorium. Schuppen structures and rumping faults are sometimes observable. Such a state of intense diastrophism is always confined to a number of narrow belts, e. g., one (Fig. 4) of these belts lies about latitude 40° 30′ N in the northermost part of Hopei Province. The other belt runs along latitude 41° 40′ N on the southern side of Tachingshan anticline. Within this belt Jurassic coal-bearing basins of east-west trend are scattered here and there. They vary in size. The largest attains the length of 60 kilometres. On the border of these basins, such as the Shihguaitze coalfield (Fig. 5) northwest of Huhohaoteh city, a sequence of Palaeozoic strata and pre-Sinian metamorphics is overturned to the north. The intensity of deformation is how-

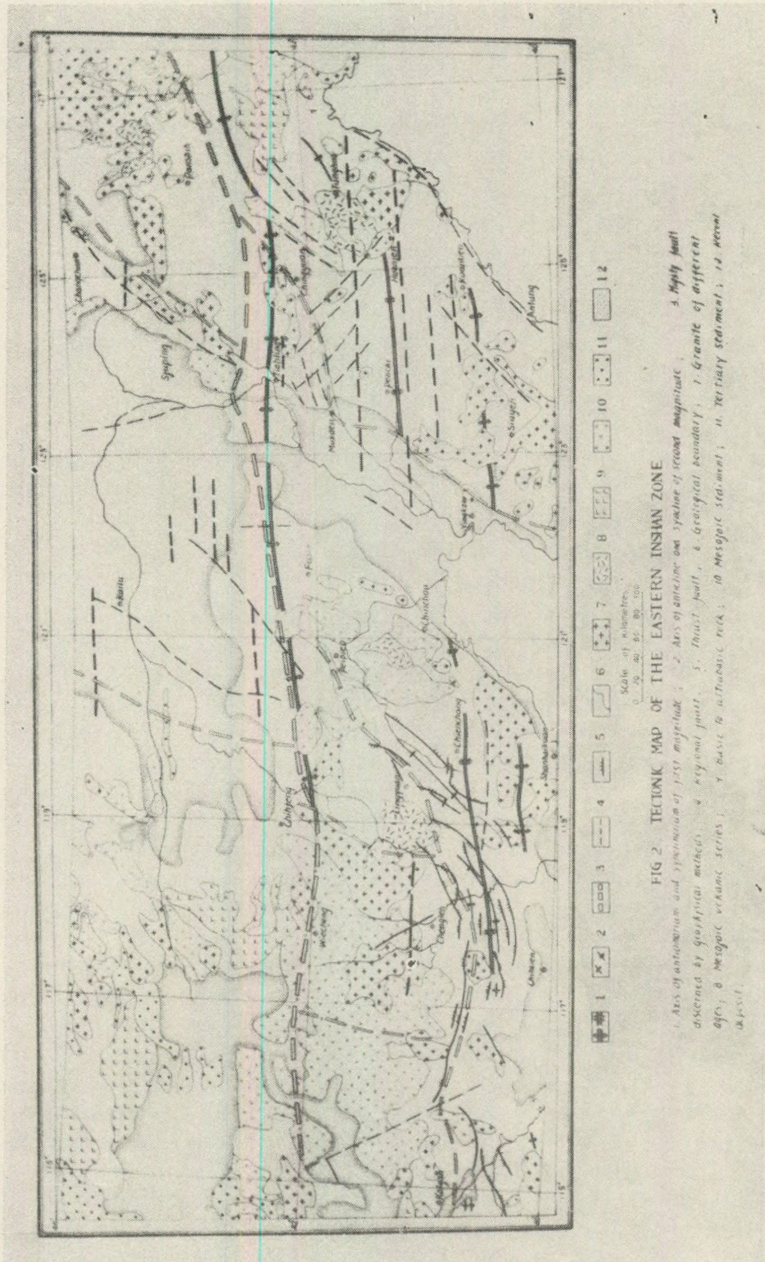

Fig. 2. Tectonic map of the eastern part of the Inshan Zone

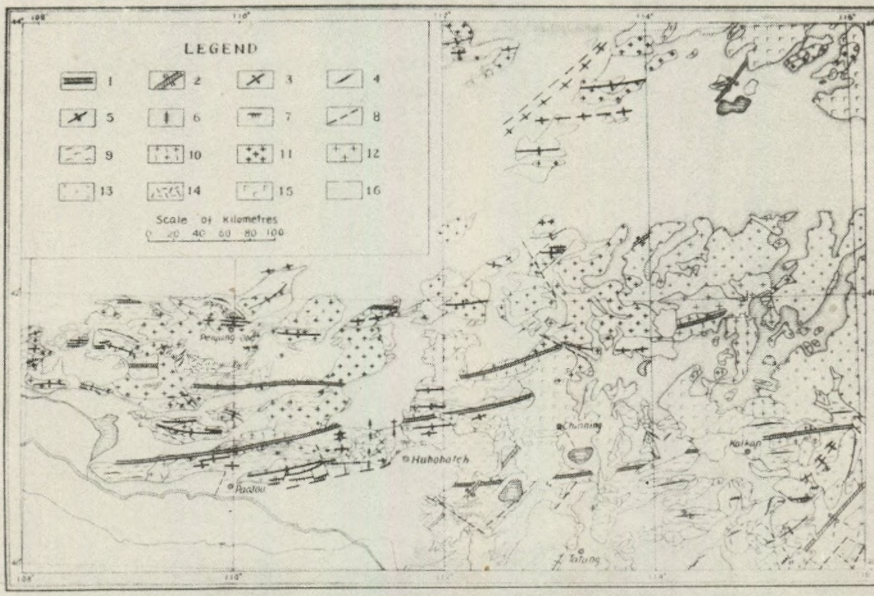

FIG. 3 TECTONIC MAP OF THE TACHINGSHAN AND PEIYUNG-OBE REGIONS

1. East-west fold-jone; 2. Northeast trending fold of first magnitude; 3. Anticline of second magnitude; 4. Doubly-pitching anticline; 5. Syncline of second magnitude; 6. Compressed jone; 7. Thrust fault; 8. Fault suspected; 9. Archaean gneiss; 10. Pre-Sinian gneissic granite; 11. Palaeojoic granite; 12. Mesojoic granite; 13. Jurassic coal-bering series; 14. Cretaceous tuff conglomerate; 15. Tertiary to Quarternary Basalt; 16. Cenojoic sediment.

Fig. 3. Tectonic map of the Tachingshan and Peiyung-Obe region.

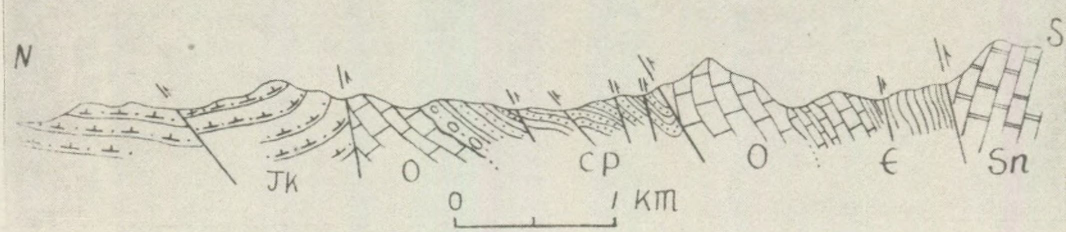

Fig. 4. Section across the Sinlung coal basin of northern Hopei, showing the east-west folds and thrusts.

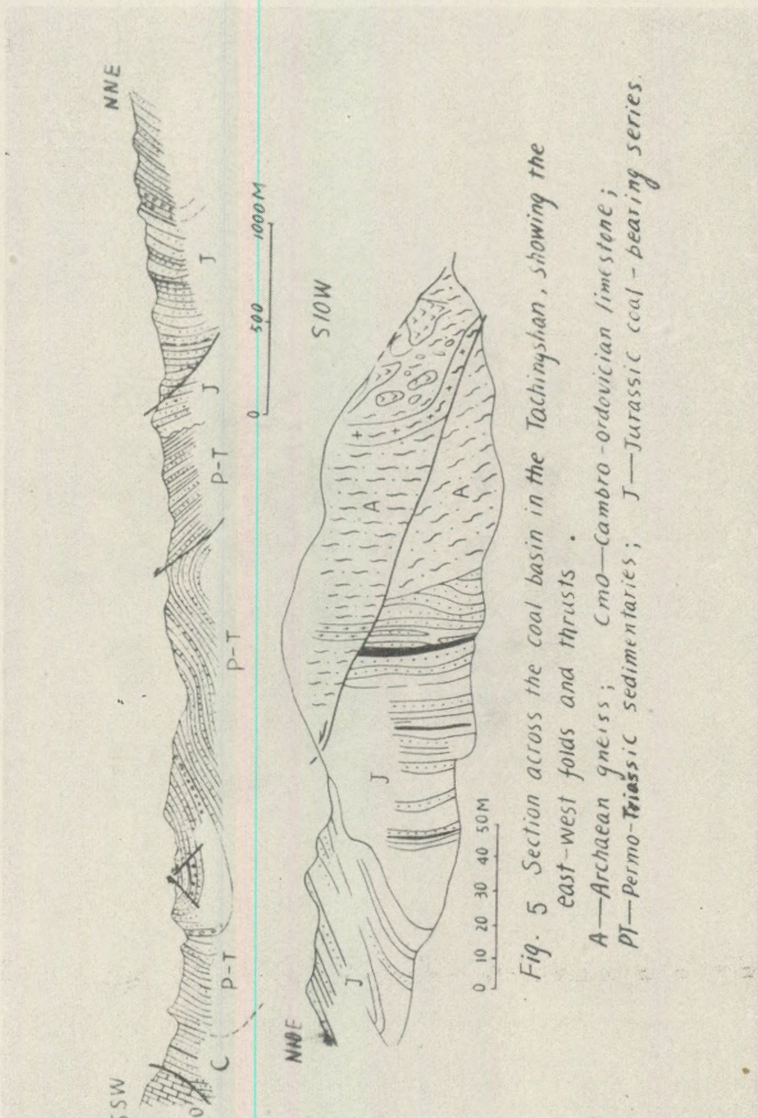

Fig. 5. Section across the coal basins showing east-west folds and thrusts in the Tachingshan region ever ostensibly decreasing towards the north. On the northern border of the basin only undulation of the exposed strata is observed.

Still further west similar intense folding [11, 12] is recorded in Archaean, Sinian and Palaeozoic rocks, being covered from place to place by Mesozoic and sometimes even by Tertiary beds. These latter are often also thrown into gentler folds.They may agree in trend with the underlying older formations, or may be entirely discordant with older rocks. Igneous activities predominate in the Inshan Zone. Elongate large masses of batholithic granite supposed to be of Hercynian age, occur in the axial part of some of the anticlines. Mesozoic intrusions appear, however, occurring in a scattered manner, but towards the east of Kalgan, these later intrusions appear to be more widespread. Bodies of ultrabasic rocks mainly of peridotite and dunite crop out sporadically all along this zone.

The Tsinling zone is also noted for a formidable pack of highly compressed folds [13, 14] and mighty thrusts (Fig. 6) with persistent east-west strike. They are well developed within the Tsinling Range proper which stands aloft in the southern part of Shensi Province. There, this zone comprises two great fold-groups, one is generally called the North Tsinling Anticlinorium, facing the

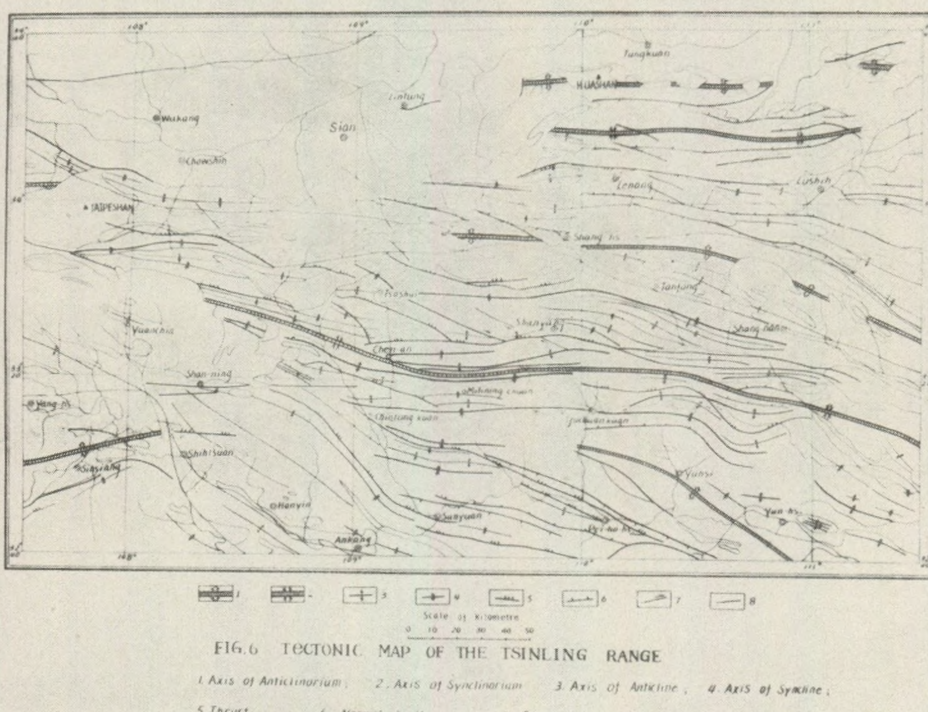

Fig. 6. Tectonic map of the Tsinling Range, southern Shensi

Shensi basin on the north with a wall-like front due to a clearcut prominent fault. The Archaean and proterozoic gneisses, schists with interbedded lenticles of marbles occupy a wide area of this remarkable anticlinorium. In this belt of ancient metamorphic rocks tight folds of subordinate magnitude are numerous, and their axial planes always dip steeply to the north. They are accompanied by some thrusts of similar trend. The other great fold-group is a wide synclinorium, well known as the South Tsinling Synclinorium. The boundary of the two great fold-groups is marked by a thrust which brings in, in the neighbourhood of Tsoshui district, the Archaean gneiss. Southward these ancient rocks thrust upon the Sinian marmorized limestones and slates.

In the South Tsinling Synclinorium a succession of Palaeozoic rocks ranging from Sinian to Permian are all disturbed by a north-south compression, forming a series of folds and faults which run almost east-west [15]. The limestones and slates from Carboniferous to Permian occupy frequently the axial part of synclines, whereas the core of anticlines is mostly composed of middle Devonian stratified rocks. They strike in general eastwest or north-west. In places the contact between different systems of Palaeozoics is often, if not all, marked by a thrust running along the same latitudes [16]. Generally a group of minor isoclinal folds (Fig. 7) occurs at certain intervals in such a manner that the strata are overturned one after another towards the axial part of a syncline from both of its limbs. These isoclinal or overturned folds are accompanied by numerous ramping faults. They usually dip at an angle somewhat larger than 60°.

Emplacements of igneous masses, largerly granites, supposedly of Hercynian age occupy a broad belt in the axial portion of some of the east-west trending folds. Mesozoic granites occur in most cases as small elongate bodies. Outcrops of a few basic to ultrabasic rocks have been observed in recent times.

Going westward from the main Tsinling Range, the east-west geotectonic zone stretches to the Kuenlun Mountains deflecting gradually somewhat to

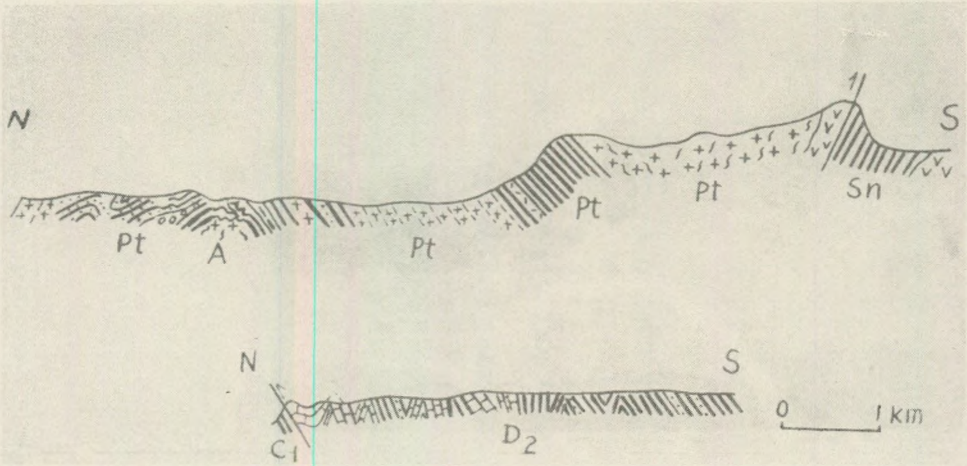

Fig. 7. Sections across the Tsinling Range from Tayukou southward to Chengan district, showing the isoclinal folds of east-west trend

the north, to the latitudes 34° – 36° N. There, folds of Palaeozoic and Mesozoic strata together with some elongated granitic intrusions all run nearly east-west. The upper Devonian limestones and a series of Permo-Carboniferous sediments are observed to have thrust northward upon the Cenozoic formations in the southern border [17] of Tsaidam Basin.

From the main Tsinling Range eastward, the east-west belt of strong folding [18] continues to extend to the districts of Chengchow, Nanyang and Hsiangyang. A line drawn to link the capital of these three districts is a line along which the said tectonic belt is suddenly intercepted as it approaches the great plain spreading out from the eastern extremity of the Sungshan anticline. Nevertheless, traces of similar structural elements are now and then brought to notice along the same latitudes in this extensive downwarped area and the Tapeishan region [19, 20]. This subsided eastern part of the Tsinling zone will be dealt with later in connection with the remarks on geophysical observations.

Unlike in the Inshan and Tsinling zones, the east-west tectonic elements of the Nanling zone are as a rule expressed by a number of thrusts [21, 22], some elongated domes and basins and several uplifted terrains of similar trend. They are often manifested by a series of acidic stocks and cupolas and especially by the batholithic intrusions of biotite granite. Northern Kwangtung (Fig. 8) and

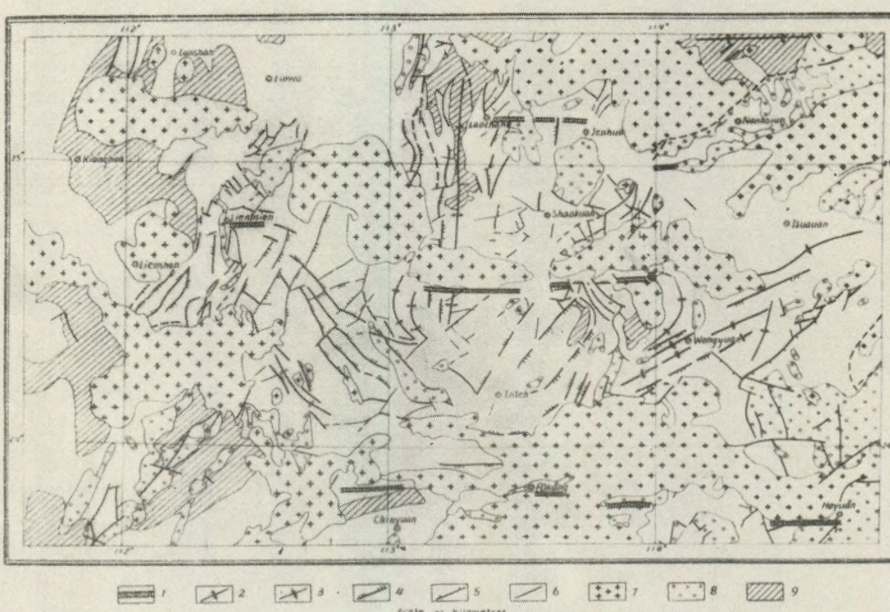

FIG 8 TECTONIC SKETCH OF THE NORTHERN KUANTUNG SHOWING EAST WEST ZONES AHD SHEWR FORMS

1 East west trending zone. 2 Axis of anticline. 3 Axis of syncline. 4 Thrust; 5 Normal fault;
6 Uncertain fault. 7 Granite chiefly of mesozoic age; 8 Cenozoic sediment; 9 Pre Devonian metamorphic rock.

Fig. 8. Outline of tectonic features of northern Kwangtung

southern Kiangsi afford typical examples [23, 24]. There several intrusive bodies of granite extend east-west for tens of kilometres. The country rocks range from Pre-Devonian to lower Jurassic in age. These latter are disturbed by the north-south compression. Pressure joints and thrust-faults of east-west trend are rampant. In southern Kiangsi province, wellknown for tungsten deposits, the east-west large-scaled anticlines and synclines are brought into prominence by virtue of elongated granitic batholiths [25] of different ages and also by a series of elongated structural basins. The latter lies in the axial part of a syncline.

Going westward along the same latitudes, numerous folds of short axis and thrusts are observable here and there in the provinces of Hunan [26, 27, 28], Kueichow and Kwangsi (Fig. 9) [29]. In northern Yunnan [30, 31] stretches a

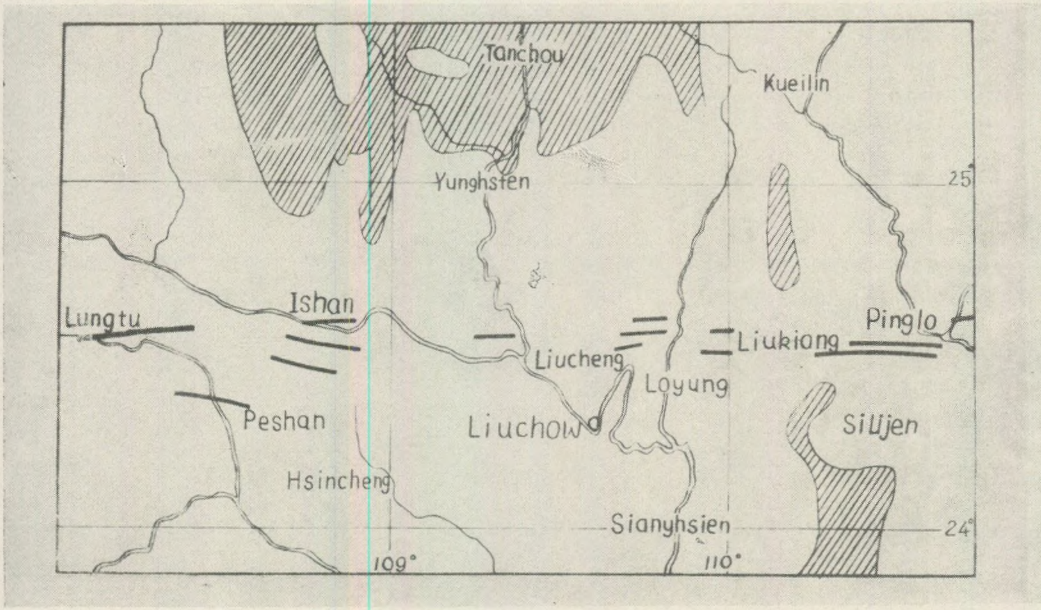

Fig. 9. Outline of the east-west folds and thrusts of northern Kwangsi.

broad and complicated anticline the core of which is chiefly constituted of pre-Sinian metamorphic rocks. The Kinshakiang suddenly bends to the west on this account. Thrust-faults are frequently present in this anticline formed by Palaeozoic and Mesozoic strata. A number of striking stratigraphical unconformities have been observed there throughout the sequence of pre-Cretaceous rocks. Structural features with east-west strike have been observed also in the vicinity of Kunming and the other districtes in a discontinuous manner. They are somewhat widely spread.

From the above-mentioned facts it appears evident that the latitudinal zones of China are highly complicated not only in specific characters but also in the history of development. Attempt has been made to look for the process of generation of these zones by palaeo-geographical methods as follows. Both the Inshan and Tsinling zones were already in existence at least in a rudimentary east-west trend early in Sinian times due to the Luliang movements. Hercynian movements undoubtedly played a part in the development of these two belts of disturbance. They began to take shape through the Yenshan movements of Mesozoic age. There is evidence to show that the Nanling tectonic zone came into being mainly in Mesozoic times, though its western part was possibly uplifted in an embryonic form by the Luliang movements. That these three tectonic zones have undergone regeneration in post-Cretaceous times is supported by certain observational facts.

As regards the inter-relation between these zones and the relation between these and the other structural systems which came to interfere with them since Mesozoic times, it may be said that the east-west zones have been in places thrown apart or even transected by a number of distorsional or vortex structures of different types and sizes and also by numerous north-north-east trending thrust and folds. In most cases, the latitudinal zones are apparently dislodged by the epsilon type of tectonic systems. They occur in mutual adaptation and sometimes penetrate each other.

In complicated tectonic regions and plains where the east-west zones run across, it is often of paramount importance to resort to geophysical investigation in order to detect the existence of the east-west tectonic zones. In China, until now, aeromagnetic works have been fairly extensively carried out, while gravity measurements are restricted only to definite areas. It hardly needs to be remarked that the isomagnetic lines and the magnetic anomalies are closely related to each other, both being determined by the distribution of different rock types possessing or underwent different degree of magnetization. Since different granitic rocks and basic and ultrabasic rocks of different ages usually occur in the east-west tectonic zones, and since the amount of magnetic minerals present in sedimentary rocks as a rule varies, aeromagnetic investigation often proves to be a method more rapid and more efficacious than any other in detecting the existence of the east-west tectonic zones.

For similar reasons gravity survey also turns out to be equally useful, since the more outstanding structures of the basement rocks under the plains are often revealed by various density of the rocks involved in the structures. Needless to say, in the case of gravity measurement, latitudinal corrections must be made.

Geophysical surveys [32] along the Tiehling anticline and the adjacent Yenshan syncline on the south has established the fact that the isanomals both in the magnetic and gravitational fields run essentially parallel to each other. It is further shown that in magnetic field three belts are discernible. The northern belt is more or less wide-spread, the anomalous value ranges from -250 to $+250\ \gamma$. The middle belt, situated in latitudes $40°\ 20' - 41°\ 20'$, runs almost strictly east-west, and can be subdivided into a number of sub-zones of positive and negative anomalies. The positive sub-zones vary in their magnetic anomalous values from 500 to 1000 γ, at points even rise to 4000 γ. In the negative sub-zones the values $-250\ \gamma$ to $+250\ \gamma$ are recorded. The southern belt, situated in latitude about 40° N., also shows anomalous values ranging from $-250\ \gamma$ to $+\ 250\ \gamma$. Gravitational anomalies in this area are also distributed in three sub-zones, all running east-west: in the northern subzone -85 to -135 mgl, in the middle zone -50 to -100 mgl, in the northern sub-zone $+10$ to $+20$ mgl. The most conspicuous anomalous zone lies in latitude 41° N

Gravity survey shows that in the vast plain of northern China, there exist east-west trending depressions and swells in the eastern part of Honan province, for example, the Kaifeng depression and the Tungshu swell. These are situated in the neighbourhood of latitude 34° 20′ which agrees with the latitudinal position of the main Tsinling Range. This anomalous gravity zone continues to run farther east, coinciding with the east-west structural zone between Hsuchow and Yihsien.

On the northern side of the Tapei Range and in the plain of the Huai River latitudes 32°–33°, both the isogams in the magnetic field and isanomals in the gravitational field run nearly parallel to each other, all trending approximately east-west, disclosing subterranean existence of an east-west zone of dense rocks with marked magnetic property.

In the Nanling Range, the superficial structure is highly complex as already alluded to. Geophysical investigation shows, however, that there exists underneath the sedimentary cover a persistent east-west zone of granite together with associated metamorphics. A prominent example occurs in northern Kwantung where an elongate porphyritic granite zone extends both westward and eastward for a considerable distance. Towards the east it reaches the Hoyuan basin, in the north-eastern part of Kwantung. In the basin itself, though flat on the surface, gravity anomalies have been encountered with a north-south gradient amounting to 1.5–2 mgl/km, when latitudinal correction is made. It means that under the cover of young sediments, denser rocks form a buried hill trending east-west. Similar examples are also found in northeastern Kwantung (about latitudes 23° 40′–24°) where magnetic disturbance is particularly pronounced, valuing 100–200 γ, and in the granitic region of the southern part of Fukien province, where the iso-magnetic lines and the transitional line along which the region with positive anomalies changes into negative anomalies run east-west. A significant example is found to the north of Lungyen, southern Fukien.

The fact of far-reaching significance is that such prominent geotectonic zones, as are mentioned here, do not only occur in China but in many parts of the world in the continents as well as in ocean basins. Although they apparently differ in their tectonic nature, and perhaps rose or have become active in different geological times, they generally appear in certain definite latitudes. Some of them agree in their latitudinal position with those which we have dealt with here. It is therefore admissible to conclude that prominent latitudinal geotectonic zones, whatever their real nature, are of planetary origin, and have had a long history in the geological past.

This conclusion leads at once to the inference which is equally far-reaching in an attempt to elucidate the history of the dynamic behaviour of our Earth. Since these large-scaled complex geotectonic zones persist to run east-west, it is further deducible that their origin might, in some way, be connected with the present axis of rotation of the Earth.

From harmonic analysis G. H. Darwin first pointed out the possible existence of deformational zones on the Earth's surface parallel to the equator.

He called them harmonic mountain ranges [33]. In Darwin's time nothing however is known of any such para-equatorial ranges or of structural zones. About three decades ago Prof. J. S. Lee [34] and Prof. B. L. Lithchkov [35] independently and almost simultaneously called attention to the presence of such zones from the geotectonic point of view. These authors traced the origin of these structural zones to the effect of Earth's rotation, and in some cases coupled with tidal attraction. More recently Dr. M. V. Stowas [36] elaborated mathematical analyses with a view to unfolding the reason why large-scaled geotectonic zones should be located in certain definite latitudes. It is of interest that Stowas' theory does account for the presence of some latitudinal zones, notably those corresponding to our Tsinling. There remain, however, others which still require explanation with the same precision as attained by Stowas.

In this connexion a few words may be said of the hypothesis offered by Prof. J. S. Lee [37]. He stresses the change of the rate of the Earth's rotation around its own axis throughout geological times. Lee conceives that such changes of the Earth's rotational speed may be caused by the uplifting and sinking of the various portions of the continents and the oceans or by the contraction of the entire Earth due to the change of the Earth's thermal conditions. Gravitational differentiation of the materials within the Earth and the tidal pull on the superficial layers of the Earth must be, according to him, also be taken into account.

When the rate of the Earth's rotation increases, the ellipticity also increases, but the rigidity of the internal portions of the Earth is conceivably greater. Hence the internal portions are more sturdy to comply with the changed rotational speed than the crust. Under those circumstances the superficial layer of the Earth would tend to shift towards the lower latitudes in order to accomodate the required ellipticity. The results are: along those belts where such thrust movements are strong and uniform, they give rise to the formation of the east-west tectonic zone; whereas along those belts where such movements are not uniform and unbalanced, they give rise to the formation of a variety of horizontal shear [38] and vortex structural types [39]. As a corollary to follow the above reasoning we can readily offer an explanation for the occurrence of longitudinal tectonic zones [40].

If certain portions of the Earth's crust, as in the case of some continents, cannot follow up the increasing rate of rotation, they will lag behind those neighbouring portions. Consequently, along their western margins and along certain longitudinal lines in the interior of the continents where the landmass on their eastern side fails to keep pace with the western as the Earth spins faster, there will arise longitudinal compressional zones. Spectacular examples of such zones are the Cordilleran geosyncline and the lofty longitudinal mountain ranges of western China continuing southward to form the Indonesian arc. Along the eastern margins of the continents where they fail to keep pace with the ocean floor in their eastward march, there may be brought about tensional zones. Those ocean deeps occurring along the eastern Asiatic continent are presumably attributable of this origin.

When the Earth's rate of rotation decreases the entire tendency of the crustal movement will proceed in the opposite manner

Numerous other hypotheses have been proposed by geologists and geophysicists from time to time to account for the origin of geotectonic movements. None can however claim to be founded on substantial tectonic grounds unless an adequate account is given as to why there exist on planetary scale such extensive and deep-seated tectonic zones as those which we have dealt with here.

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ON THE ASTRONOMICAL TESTS OF GENERAL RELATIVITY

With 5 figures and 2 tables

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SUMMARY

The present state of observational verification of the three classical effects + 1. Perielion advance of planets, 2. Gravitational shift of spectral lines, 3. Light-deflection in graviational field – is outlined.

For forty years after the foundation of the general theory of relativity the astronomical observations of those three effects constituted merely the emprirical tests of the theory which Einstein himself has already mentioned: 1. the precession of the perihelion of Mercury, 2. the deflection of light rays passing near the Sun, 3. the gravitational shift of spectral lines. During the last years new methods have been suggested, particularly with the use of artificial satellites. For the empirical verification of the theory a new possibility of laboratory experiment opened by the Mössbauer effect, which enables us to measure extremely small changes of the frequency of gamma rays and, consequently, laboratory measurement of the gravitational red-shift has become possible [1, 2]. This is the more fortunate since astronomical observations concerning this effect are contradictory. This effect follows from the principle of equivalence* but it can be considered also as the energy-change of photons performing work in the gravitational field. Thus it happens that exactly this phenomenon proved in the laboratory does not present a crucial test of General Relativity.

The great elegance in principle and construction of the general theory of relativity has without doubt contributed to the fact that text–books of

* Gy. Marx had the kindness to call my attention to the fact that L. J. Schiff considers also the light-deflection as a consequence of the principle of equivalence [3]. A. Schild, however, is opposing Schiff, the more so, as Nordström's theory of gravitation, for instance, which also satisfies the principle of equivalence, gives a different value for the light-deflection [4]. See also another paper by R. U. Sexl [5].

the theory, particularly those published in the twenties, judged far too optimistically the first astronomical observations made to test the theory. We can read in several text-books that the gravitational shift of spectral lines has been successfully proved in the solar spectrum and for the white dwarf companion of Sirius, whilst light-deflection had already been demonstrated on the occasion of the total solar eclipse in 1919. This is not true at all, and we shall see that the astronomical verification of these two effects has remained uncertain even nowadays. Recently alternative theories of gravitation are frequently discussed, which either do not accept the principle of equivalence or that of covariance, such as, particularly, the theories of Whitehead [6] and Birkhoff [7]. These theories give the same values for the three above mentioned effects, which therefore do not afford decisive empirical criterions between the different theories.

The three theories give different numerical values for the rotation effect, i.e. the influence of the rotation of a body on the motion of its planet or satellite. The astronomical observation of this effect is nowadays impossible, since the Sun is rotating very slowly, and even the distance between the Sun and its nearest planet is far too great in this respect. In the solar system the effect is significant only for the fifth satellite of the quickly rotating Jupiter, this could be measured, however, with too much difficulty

It should be mentioned that formulae given on this effect regard all weak fields, and the accordance with experiment would not yet guarantee the strict accuracy of Einstein's field equations. The basic assumptions of the theory of relativity, on the other hand, are not sufficient for the unambiguous determination of the field equations, and if some experimental or observational results would contradict the computations based on the field equations, this would not mean that the basic ideas of the general theory of relativity are not valid.

Relativistic effect in celestial mechanics.

Even according to classical mechanics the perihelion of a planet moves within its orbit plane in the direction of the motion of the planet, as a consequence of perturbations by other planets. In the theory of General Relativity such an effect occurs already in the one-centre problem, and the predicted advance in the longitude of the perihelion, D, expressed in seconds of arc per century amounts to

$$D = 3.34 \times 10^{33} \cdot a^{-5/2} \quad (1-e^2)^{-1} \tag{1}$$

wherin a is the semi major axis of the planetary orbit expressed in cm, and e is the excentricity of the orbit. D is greatest for Mercury; 43″, 03, whereas it is 8″, 63 for Venus, 3″, 84 for the Earth, 1″,35 for Mars, and 0″, 06 for Jupiter. In case of Venus the determination of D from observations is difficult, because of the small excentricity the location of the line of the apsides in the nearly circular orbit is rather uncertain.

On the other hand, the observation of Mercury belongs to the difficult tasks of positional astronomy, Mercury being observable but in daytime, and since Mercury always is near the Sun, the turbulence of the atmosphere heated by the Sun leads to serious errors in the determination of its position. The task becomes even more complicated by the variations of the shape of the planetary disk. Besides, the determination of the positions cannot be performed in a Newtonian coordinate-system, the observations refer to the moving equinox, and the exact determination of the precession of the equinox is no easy task at all. The perihelion advance due to the perturbations of the planets is considerably greater than the relativistic effect, hence a complete perturbation-theory of the planets should be worked out for the verification of the relativistic effect.

The US Naval Observatory in Washington has accomplished this immense work after World War II by means of electronic computing machines, thus bringing up to date Newcomb's computations performed around the tur ı of century. In Table 1 we show the results obtained by C. M. Clemence for Mercury [8] and by H. H. Morgan for the Earth [9]. We see which factors contribute to the perihelion movement according to Newtonian mechanics. The differences between observations and Newtonian theory is in good accordance with values predicted by formula (1), in case of the Earth, however, this is not of great weight, the error being 60 per cent of the result. It is the same with the results obtained for Mars and Venus. In all events, none of these results contradicts formula (1), and for Mercury the accordance is excellent.

Since the difference between Newtonian theory and observations has already been remarked by Leverrier, an explanation for this difference was endeavoured long ago before the theory of relativity became known. Leverrier himself tried to explain the difference by a planet within the orbit of Mercury, and he called this hypothetical planet Vulcan. Nowadays, we know that there is no such planet, but it may be that several planetoids are revolving near Mercury. Seeliger tried to give an explanation for the perihelion advance of Mercury by considering the effect of the dust-cloud causing the zodiacal light, but he assumed for it a density exceeding by several orders of magnitude the value derived from modern observations. Harzer tried to explain the difference by the oblateness of the Sun, but this effect is too small. It may be, of course, that axial rotation in the interior of the Sun is much quicker than at the surface, and the equipotential surfaces are there strongly flattened. As a matter of curiosity it should be mentioned that Grossman in the twenties computed the internal structure of the Sun on the basis of Jean's theory of radiative retardation. According to this theory the angular velocity is strongly increasing inwards. By taking into consideration the oblateness of the interior equipontential surfaces, Grossman obtained for the perihelion movement of Mercury exactly the value resulting from formula (1).

The orbits of several minor planets have a very great excentricity, and within a certain period it will be possible to study the movement of their perihelia resulting from (1). A particularly high value is expected for the minor planet Icarus 1566. In this case $a = 1{,}6 \times 10^{13}$ cm, $e = 0{,}8265$. From (1) we get for the perihelion advance during a century the value $10'',05$ [10]. For the planet Hermes the effect is $2'',62$, for Apollo $2'',10$, for Adonis $1'',80$. But all these planets can be seldom observed, and decades are required for the accu-

Table 1

Contributions to the perihelion movement of Mercury and the Earth according to G. M. Clemence and H. R. Morgan

	Mercury		Earth	
Mercury	0",03 ±	0,00	− 13",75 ±	2,3
Venus	277,85	0,68	345,49	0,8
Earth, Moon	90,04	0,08	7,68	0,0
Mars	2,54	0,00	97,69	0,1
Jupiter	153,58	0,00	696,85	0,0
Saturn	7,30	0,01	18,74	0,0
Uranus, Neptune	0,18	0,00	0,75	0,0
Solar oblateness	0,01	0,02	0,00	0,0
Precession	5025,65	0,50	5025,65	0,5
Sum	5557,19 ±	0,85	6179,1 ±	2,5
Observed motion	5599,74	0,41	6183,7	1,1
Difference	42,55 ±	0,94	4,6 ±	2,7
Relativistic effect	43,03 ±	0,03	3,8 ±	0,0

rate determination of the effect. By and by it will be possible to test relativistic movement of perihelion for close double stars with great excentricity. A very suitable case is DI Herculis having an excentricity 0,453 [11].

Nowadays the possibilities to determine the movement of perihelion and the relativistic effect of the Earth's rotation by artificial satellites are much talked over. The artificial satellites with their small masses, however, will hardly be suitable to demonstrate effects like that, since collisions with meteorites as well as air resistance may influence their orbit.

An interesting new suggestion came from L. I. Schiff [12] for the observation of the precession of the axis of a gyroscope in a gravitational field. The axis of a gyroscope on the Earth, if the centre of mass of the gyroscope rests with respect to the Earth, would precede 0",4 in a year. This precession could be increased, if the gyroscope would be placed on an artificial satellite, because then, beside the rotation-effect of the Earth there is a precession of the axis depending on the velocity of the satellite. This experiment is by no means easy, neither on the Earth where the gyroscope ought to be suspended against gravity, nor on an artificial satellite where suspension is not necessary, but other difficulties would arise.

Astronomical Measurement of the Gravitational Shift of Spectral Lines

According to the theory of relativity, the spectral lines in a gravitational field of potential Φ show – as compared with those in a gravitation-free field – a red-shift $\Delta\lambda$ according to the quation

$$\Delta\lambda/\lambda = \Phi/c^2 \tag{1}$$

The shift of the lines in the solar spectrum can be computed from this formula, since the gravitational field of the Earth can be neglected. A red-shift of 0,0063 A can be expected for the wave-length of 3000 A, and one of 0,0148 A for 7000 A. Accordingly, the red-shift is equivalent to an apparent

Doppler-effect of +0,636 km/sec. For a star of an arbitrary mass M and radius R, the relativistic shift corresponds to a Dopplershift of

$$v = 0{,}636 \ \mathrm{M/R} \ (\mathrm{km/sec}) \tag{3}$$

if M is given in solar mass and R in solar radii. Since in the case of stars of great mass also the radius is usually large, no greater values than 4 km/sec can be expected for v even for stars of the greatest mass. Most hopeful is the situation for the white dwarfs having radii smaller than that of the Sun by two orders of magnitude. As their mass is about $M=1$, some white dwarfs may be expected to show a red-shift equivalent to $v=+60$ km/sec. The radii of the recently discovered sub-white-dwarfs are still smaller, but their masses are small as well.

When studying the red-shift in the solar spectrum, we are compensated for the smallness of the effect by being able to use high dispersion spectrographs. Measurement of the effect is difficult even then, the effect being a small fraction of the widths of the spectral lines. In the first measurements a great number of lines were used, later on the measurements were rather concentrated on a few carefully chosen Fraunhofer-lines. The most careful measurements have been performed by Miss M. G. Adam [13]. According to the results:

1. the red-shift shows a systematic trend with the intensity of the lines.

2. the red-shift in the centre of the Sun's-disk is much smaller than required, near the limb of the Sun it suddenly becomes greater, and beyond the limb it reaches high values (limb-effect).

Schrőter very carefully studied all data of observations concerning the red-shift of solar spectral lines [14]. It turned out, that among the 1500 spectral lines measured by St. John, only four per cent were free of blends. The wave-length errors of the lines used for comparison contributed to the errors of observation. When comparing the solar Fraunhofer-lines with the emission lines of the comparison spectra, systematic errors depending on the width of the Fraunhofer-lines may occur as well. Much trouble is caused by the circulations in the solar atmosphere. According to Schrőter, the differences between theory and measurements arise, above all, from the up- and down movements of the solar granulae. In consequence of local Dopplershifts, each Fraunhofer-line is the superposition of lines originating in granulae *and* intergranulae. The small asymmetry of the resulting line-contours causes an apparent line-shift. Thus Schrőter succeeded in explaining the limb-effect as a function of line-strength.

Recently the French astronomers, J. E. Blamont and F. Roddier determined the profile of the solar strontium line at 4607 angstroms with great precision [15]. They brought the light to be analyzed into the vapour of strontium, and they measured the intensity of the narrowband resonance radiation induced in the metal vapour, at right angles to the exciting beam, by photomultipliers. The intensity of the resonance radiation re-emitted by the vapour is proportional to the incident intensity. The resonance frequency was shifted by applying a magnetic field to the vapour In this way the profile of the exciting light could be determined from the dependence of the intensity of the re-emitted light on the magnetic field. The profile of the strontium-line thus determined resulted in a red-shift which agreed well with the relativity prediction.

As to the red-shift of the companion of Sirius, the first measurements by A d a m s and M o o r e gave a value of 30 km/sec apparently in full accordance with the theory. But it soon turned out that in formula (3) for the radius of Sirius B a too great value was used. The radius was computed from the spectral type and from the absolute luminosity of Sirius B. The luminosity is distorted by Sirius A which is 20000 times brighter than Sirius B and the separation of the two stars never exceeds 10″ The diffuse light of Sirius A influences in the same way the lineshift, because the same spectral lines occur in the spectra of both stars. Sirius B is, consequently, not suitable at all for testing the red-shift.

More suitable is for this purpose another white dwarf, 40 Eridani B. It is a close companion of an M-type star, 40 Eridani C. As the two stars differ strongly in spectral type, the spectral lines of the white dwarf companion are not influenced by its close neighbour. P o p p e r obtained from 37 spectrograms taken at the Mt. Wilson-reflectors a red-shift of 21 ± 4 km/sec [16]. The theoretical value based on $M=0{,}43$ computed from the orbit of the BC-system, and on $R=0{,}016$ obtained from the spectral type and the photometric data proved to be 17 ± 3 km/sec in good agreement with the observations.

Many suggestions have already been made for using artificial satellites to test the gravitational line-shift. Such an experiment has so far not yet been carried out, so I only refer here to a paper by G i n z b u r g [17]. In the light-source placed into the artificial satellite a violet-shift is, as a matter of fact, expected relative to light sources on the Earth.

The Light-Deflection

According to the general theory of relativity a light ray passing at a distance d from the centre of the Sun is subjected to a deflection

$$L = 1'', 75/d \tag{4}$$

directed radially outwards from the centre. d should be given in this formula in solar radii.

Even in the distance of several solar radii from the Sun's centre the deflection could easily be measured under ordinary conditions with the usual methods of positional astronomy. But measurements have so far been possible only during total solar eclipses, because only then are stars visible sufficiently near to the Sun's limb. A total solar eclipse is a rare opportunity and the longest possible duration of the totality is at most 7,5 minutes. The shadow of the Moon passes only a narrow belt of the Earth and within this belt it is mostly difficult to find a favourable place for the observations. Hence it is easy to understand that since 1916 there were only ten expeditions which succeeded in making photographs for the study of the light-deflection, on the occasion of six different solar eclipses, sometimes under rather bad weather conditions. At some eclipses there were rather few stars near the Sun, or the stars were situated asymmetrically around the Sun, and all these factors influenced unfavourable the accuracy of the results. The stars nearest to the Sun's disk, where the effect is the greatest, often cannot be used at all, since the light of the solar corona suppresses them. The greatest number of measurable stars are in a distance of $d=4-10$ from the Sun's centre, where the hyperbola representing the deflection as a function of d has already a linear course.

Since the co-ordinates of the stars around the Sun are not known accurately enough, the light-deflection can be determined only by means of differential measurements. The photographs taken at the solar eclipse should be compared with night photographs of the same area of the sky. These must be taken some months after or before the eclipse, and even when they are made at the same place, with the same instrument in the same arrangement, it is still inevitable that there will be change in the focus of the telescope between the two exposures and thus the scale of the two plates will differ as well. The resulting error is increasing proportionally with d, and influences with full weight the results to be obtained for the light-deflection, since this latter is varying also linearly with d if $d>3$. In case of a focal distance of 3,5 m, for instance, a focal change of 0,1 mm would cause at $d=8$ a scale correction of the same order of magnitude as the Einstein-effect itself at the same distance. The scale difference, ΔS, between the two exposures causes an error

$$\Delta L=-\overline{d_i^2}\cdot\Delta S$$

in the value of the light-deflection at the Sun's limb, whereby $\overline{d_i^2}$ is the mean value of the squares of star distances from the Sun. $\overline{d_i^2}$ being rarely smaller than 20, every error in the scale causes an error in L which is twenty times greater.

The observations should be arranged therefore so that the scale of the plates could be determined independently from the stars around the Sun. This was done at the expedition of the Potsdam Observatory in 1929 when the scale-value was determined by an independent star-field during the eclipse itself, furthermore photographic reseaux were printed on all plates to determine scale changes between the exposures.

Table 2 shows the data characterizing the photographs at different expeditions and the values obtained for L. The results are judged, however, quite otherwise when the details are shown as well. For this purpose, I show you some figures taken from an elaborate paper by H. von K l ü b e r [18]. Figure 1

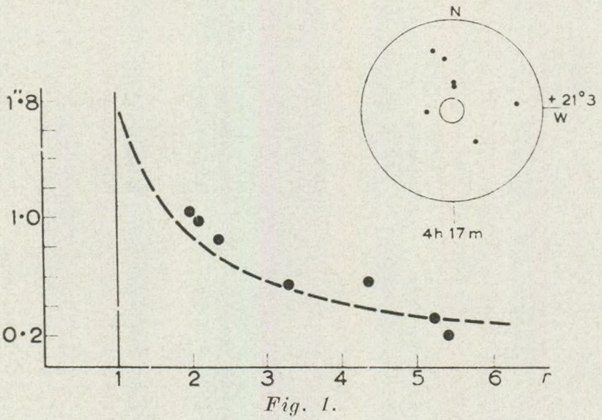

Fig. 1.

Table 2.

Observation of Light-Deflection at Different Solar Eclipses

Date	Station	f (cm)	No. of plates	Time of exposure (sec)	No. of stars	d	L	Observatory
1919 May 29	Sobral	570	7	26	7	2 – 6	1,78 ± 0,16	Greenwich
	,,	343	16	5 – 10	11	2 – 6	0,93	,,
	Principe	343	2	2 – 20	5	2 – 6	1,61 ± 0,40	,,
1922 Sep. 21	Cordillo Dawns	160	2	20 – 30	11 – 14	2 – 10	1,77 ± 0,40	Adelaide-Greenw.
	Wallal	330	2	45	18	2 – 10	1,74 ± 0,30	Victoria
	,,	450	4	120 – 125	62 – 85	2,1 – 14,5	1,72 ± 0,15	Lick
	,,	150	6	60 – 102	145	2,1 – 42	1,82 ± 0,20	Lick
1929 May 9	Takengon	850	4	40 – 90	17 – 18	1,5 – 7,5	2,24 ± 0,10	Potsdam
1936 June 19	Kuybyshevka	600	2	25 – 35	16 – 29	2 – 7,2	2,73 ± 0,31	Sternberg
	Kosimizu	500	2	80	8	4 – 7	2,13 ± 1,15 } *	Sendai
							1,28 ± 2,67 }	
1947 May 20	Bocajuva	609	1	185	51	3,3 – 10,2	2,01 ± 0,27	Yerkes
1952 Feb 25	Khartoum	609	2	60 – 90	9 – 11	2,1 – 8,6	1,70 ± 0,10	,,

f = focal length of telescope, d = distance of stars from centre in solar radii.

* Two comparison plates gave these alternative values when in different combinations with the eclipse plate.

shows the results of the Greenwich expedition in 1919. The figure shows the actually measured light-deflection for each star as function of d. Right above you see the corresponding starfield around the Sun. Coordinates of the Sun's centre for the equinox 1855 are indicated. The dotted curve represents the theoretical hyperbola. There are very few stars, as you see, and even these are rather asymmetrically distributed around the Sun. A representation of the observed values as accurate as through the theoretical hyperbola could be obtained also through a straight line, but this would lead to a value $L = 1,''05$. We must mention that the photographs have been taken with horizontal telescopes fed by coelostats. The mirror of the coelostat got deformed by the Sun's heat, and thus the photographs were deformed too.

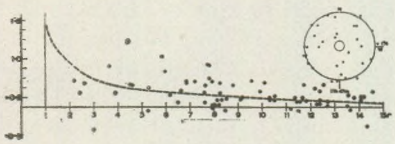

Fig 2

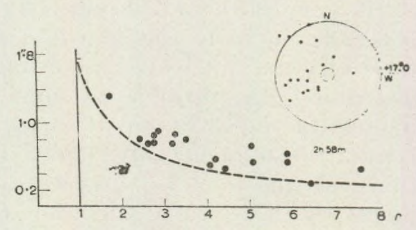

Fig. 3.

At the solar eclipse in 1922 (Fig. 2) the number of stars was sufficient, but I think I do not exaggerate when declaring that many different values can be derived for L depending on the method of reduction. The measurements of the Potsdam expedition in 1929 prepared with the greatest care led, using the theoretical hyperbola, to the value $L = 2,''24$. The accuracy looses much by the asymmetrical distribution of the stars (Fig. 3). It is difficult to understand how the observations in 1936 could result in $L = 2,''73$ (Fig. 4), or those in 1947 in $L = 2,''01$. (Fig. 5). Here all stars have been in a distance of $d > 3$ from the Sun's centre.

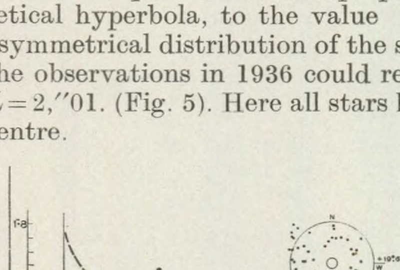

Fig. 4.

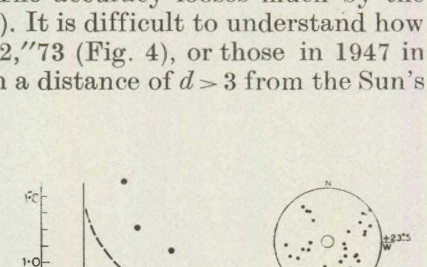

Fig. 5.

I think these figures could convince everybody of the fact that we cannot speak about the astronomical verification of the relativistic light-deflection. It is not impossible that the values obtained for L are influenced by systematic errors unknown so far. I am afraid there was some negligence when considering atmospheric refraction. The refraction-tables used were derived from night observations of stars and it is not impossible that in day time considerable differences may occur, even if the anomal refraction which might develop along the shadow-path is not considered as essential. A good improvement in the observation of the effect may be hoped for only by observations to be carried out from outside the atmosphere. Star light may be influenced also by the plasma clouds continuously flowing from the Sun, particularly at times of sun-spot maximum. It is, perhaps, worth-while to mention that the L-values in Table 2 are strongly correlating with the solar cycle: the highest L-values have been obtained at sun-spot maxima (in 1929 and 1947), and the lowest ones at sun-spot minima (in 1922 and 1952).

Summing up, it may be stated that there is no astronomical verification of the general theory of relativity, it is true, but on the other side there was no observation made which would contradict it. Apart from this, the theory of relativity did not solve the problem of gravitation, giving only the method for describing gravitational phenomena. From a satisfactory theory of gravitation it must be required that it should give an idea of the physical nature of gravitation. Consequently, the most important experiments made so far in connection with the nature of gravitation are even nowadays the fundamental experiments by Eötvös and their repetition with a more powerful apparatus by Dicke.

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CLOSING SPEECH OF THE PRESIDENT:

When the Board of our Faculty decided to arrange an international scientific session in honour of Roland Eötvös and the principal topic of the session should be connected with Eötvös' work in the field of gravimetry and geomagnetism, we were worried about the problem whether these were a satisfactory base for a whole scientific session. It is known that Roland Eötvös has dealt with but four or five topics all his life and these were elucidated by him to such a degree that there was nothing to add to or take off them.

The Session, however, relieved us of our doubts appreciating in a worthy manner Eötvös' life work. At the same time, the results became manifest which developed from a foundation harder than granite created and laid down by him. Physics developing in an absolutely new space and time, as well as the secrets of Earth and Solar system, none the less than the foundation of oil industry may take their source in Eötvös' work.

Taking into consideration the high standard and deep philosophical and physical contents of the lectures read in the course of the Session we may rest assured that Eötvös' memory was worthily honoured.

Now, thanking our honoured guests to have promoted by their lectures the success of our Session I hope that at the same time they have enjoyed their stay in Hungary and return home with the conviction that our small country contributed in an essential manner to build the palace of Science!

L. Egyed

APPENDIX

CONTRIBUTION TO THE LAW OF PROPORTIONALITY OF INERTIA AND GRAVITATION*

by

R. v. EÖTVÖS, D. PEKÁR and E. FEKETE

Bibliography:
Annalen der Physik, (4) 68, 1922, 11 – 66.

„ARS LONGA, VITA BREVIS"

The warning of this ancient maxim induces the authors of this paper to edit the results of their investigations and submit it to the judgement of a higher scientific Areopagus.

Methods of observations are naturally improving and refining in the course of observations and thus no mortal being could be through with his task, would he follow without reserve his meritorius urge to steadily replace the utilizable by something better.

The authors submit to man's fate of finiteness and cede the task of improving these observations to future times and future workers who believe to be able to refine them by mature experience.

This treatise is the paper presented for the competition and rewarded with the first prize of the Benecke-fund for 1909 by the Philosophical Faculty of the Göttingen University[1]. Its publication was postponed so far, for the reason,

* This is a translation of the original paper, first published in Annalen der Physik (in German) as given in the Bibliography, then for the second time in „Roland Eötvös Gesammelte Arbeiten", edited by P. Selényi, Budapest, 1953. – Some of his notes were completed by the translator, marked by [*]

1. S. the literal text of the theme of competition in VIII. 78, p. 336. [*This citation of the „Gesammelte Arbeiten" refers to the paper of R. Eötvös: Über Geodätische Arbeiten in Ungarn, besonders über Beobachtungen mit der Drehwage. Bericht an die XVI. Allgemeine Konferenz der Internationalen Erdmessung", Budapest, 1909. The citation is, as follows:

„A very sensitive method was given by Eötvös to make a comparison between the inertia and gravitation of matter. Considering this and the new developments of electrodynamics, as well as the discovery of radioactive stuffs, Newton's law concerning the proportionality of inertia and gravitation is to be proved as extensively as possible."

The first prize, Mark 4500, was awarded to the abovenamed authors for their collective work in March, 1909.]

that similar new investigations with accomplished Eötvös' torsion balances were promising even a greater accuracy[2]. In recent times, however, Eötvös' torsion balance was applied to practical mining prospections, which in ever expanding frames, hindered the above-mentioned investigations. But with regard to the great interest – expecially for the postulate of general relativity given by Einstein – the authors of this treatise think not to retain it anymore from publicity. By so doing they believe to comply with the intention of the Baron *Roland* v. *Eötvös* who himself had already prepared the publication, but the completion was hindered by his death on the 8th April, 1919. The original of this competition essay had an extent of about 10 sheets whereby a considerable abridging of the paper became necessary, but without getting lost the originality of the work. So the long tables containing the readings and those parts which did not touch the essence of the whole were principally omitted[3].

1. The task as it was conceived and treated Newton's law may be expressed as follows: Every smallest part of a body attracts every other one with a force whose direction coincides with the connecting line of both parts, its magnitude being directly proportional to the product of the masses and inversely proportional to the square of their distance from one to the other. So. if M, m are these masses and r their distance, the mutual attraction has the value

$$P = f\frac{Mm}{r^2}$$

according to the principles of *Galilei's* and *Newton's* mechanics, the acceleration of the part of mass m toward M is

$$\gamma = f\frac{M}{r^2}.$$

Consequently, the proportionality of inertia and gravitation is equivalent to the constance of f (the constant of gravitation).

Now it should be examined by observations with Eötvös' torsion balance, how far do the gravitational phenomena agree with this postulate.

In this work the investigation will be conducted in two directions, at first about the question, whether the gravitational attraction is depending on the nature of the body, secondly as regards the question, whether an influence on the attraction of a body by the presence of other bodies would be perceptible, similarly to effects of different kind, like the phenomena of magnetic and electric inductions, and especially those of absorption of heat and light.

Experiments made with radioactive stuffs will be treated in a special chapter of this paper.

2. cf. the note at the end of this treatise.

[3] According to a discussion with Mr. D. P e k á r, who kindly placed the original manuscript at our disposal, we inserted here again those parts omitted at the first time. For their indication they were put in brackets.

2. On observations with the object to decide the question whether gravitation is dependent on the nature of bodies.

First of all we have to consider the principles of argumentation presented by *Newton* himself in his Principia for the proportionality of inertia and gravitation of different bodies. Those are of two kinds: *astronomical*, referring especially to the motion of the satellites of Jupiter, and *terrestrial*, resting on observations on the free fall, and the oscillating motion of materially different bodies. Both kinds of demonstration gave the result, that the gravitational attraction seemed to be independent of the material nature of the bodies, though through those observations merely a difference of 1/1000 in the gravitational attraction of different bodies with identical masses and positions was recognized.

After Newton's times the continuous advance in the art of observation of terrestrial and celestial motions rendered it possible to carry out more precise investigations resting upon his law So, above all, we want here to point to the classical pendulum observations of Bessel, by those the limit of a still possible difference in the attraction of different bodies was shifted from 1/1000 to 1/60 000. And this limit was recently still more significantly reduced to 1/20 000 000 by *Eötvös*' investigations, who had taken our most sensitive instrument, the torsion balance, for that purpose in service. As the methods and results of that research were published only in a short note in vol. 8. of Naturwissenschaftlichen Berichte aus Ungarn, 1890[1], we thought it for necessary to explain here somewhat more thoroughly the theme what was there only outlined.

We are regarding the force of gravity as the resultant of two forces, generally of different directions, one of which originates in the attraction of masses, the other one in the inertia of the bodies. For this reason observations directed to the direction of the gravity of different bodies may be used for the investigation of the relation between inertia and gravity.

The first force, the component of the gravity force of a unit mass at point P is expressed, according to the attraction of masses, by the integral

$$G = f \int \frac{dm}{\varrho^2},$$

and the *second one* is the component originated in the inertia, i. e. the centrifugal force expressed by

$$C = l\omega^2.$$

The notations used are, as follows: dm is an attracting unit mass, ϱ its distance from P, f the gravitational constant, b is the distance of P from the axis of rotation of the Earth, and a the angular velocity of the rotation. Fig. 1. shows a picture of these two components PG, together with their resultant Pg, i. e. the total force of gravity, shown by length and direction. One sees on the

[1] S. II. (43), pp. 17 – 20. [*Über die Anziehung der Erde auf verschiedene Substanzen, Akadémiai Értesítő 1, 1890, pp. 108 – 110, in Hungarian; Math. u. Naturw. Ber. aus Ungarn, 8, 1890, pp. 65 – 68.]

picture that the direction of the attractive force G deflects northwards from the direction of the force of gravity g by the angle ε on the northern hemisphere. Its value depending on the geographical latitude φ is calculated, as follows:

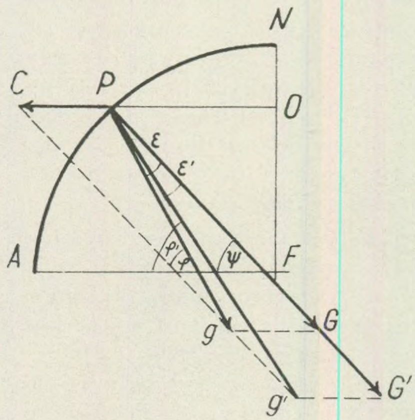

Fig. 1.

The total force at point P is in the vertical:

$$g = G\cos\varepsilon - C\cos\varphi, \qquad (1)$$

but in the tangent plane the components of C and G are in equilibrium at point P, so, that

$$C\sin\varphi = G\sin\varepsilon, \qquad (2)$$

and hence

$$\operatorname{tg}\varepsilon = \frac{C\sin\varphi}{g + C\cos\varphi} \qquad (3)$$

[For the sake of better orientation we calculated and assembled in the annexed table the values g, G, C and ε, according to the *Bessel* ellipsoid and *Helmert's* formula, for every 5° of geographical latitude in a quadrant of the Earth. The values used here are, as follows: for the major semi-axis of the ellipsoid of the Earth

$$a = 637\ 739\ 700 \text{ cm},$$

for the minor semi-axis

$$b = 635\ 607\ 900 \text{ cm}.$$

Further is

$$g = 978{,}00(1 + 0{,}005\ 31\ \sin^2\varphi).$$

The centrifugal force was calculated from the formula:

$$C = l\omega^2 = \frac{a\cos\varphi}{\sqrt{1 - \frac{a^2 - b^2}{a^2}\sin^2\varphi}}\,\omega^2,$$

where

$$\omega^2 = 5{,}31751 \cdot 10^{-9}$$

If we admit in the course of this research that the attraction of bodies with equal mass but of different nature could be different, so are the quantities G and f, consequently also g and ε to be considered as depending on that nature. Then we cannot speak shortly about gravity, or a level plane through a point, but distinction must be made between different gravities and different level planes according to the sorts of heavy bodies.

φ	G	$C = l\omega^2$	ε	G	$G \sin \varepsilon = C \sin \varphi$
0°	978,0000	3,3912	0' 0″	981,3912	0,0000
5	978,0394	3,3784	1' 2″	981,4049	0,2944
10	978,1566	3,3400	2' 2″	981,4461	0,5800
15	978,3479	3,2764	2'58″	981,5130	0,8480
20	978,6075	3,1871	3'49″	981,6038	1,0903
25	978,9275	3,0753	4'33″	981,7156	1,2997
30	979,2983	2,9393	5' 9″	981,8449	1,4697
35	979,7085	2,7848	5'35″	981,9878	1,5951
40	980,1457	2,6014	5'51″	982,1399	1,6721
45	980,5966	2,4019	5'57″	982,2969	1,6984
50	981,0475	2,1841	5'51″	982,4528	1,6731
55	981,4847	1,9495	5'35″	982,6042	1,5969
60	981,8949	1,6999	5' 9″	982,7459	1,4721
65	982,2657	1,4371	4'33″	982,8740	1,3025
70	982,5857	1,1633	3'49″	982,9842	1,0931
75	982,8453	0,8805	2'58″	983,0736	0,8505
80	983,0366	0,5908	2' 2″	983,1394	0,5818
85	983,1537	0,2966	1' 2″	983,1795	0,2954
90	983,1932	0,0000	0' 0″	983,1932	0,0000

Accordingly, even in an approximative representation of the gravity conditions, in place of an only *Bessel* ellipsoid and an only *Helmert's* formula, it would be necessary to put a lot of such ellipsoids and formulas adequate to the different bodies.]

It seems to be most practicable to fix the gravity conditions of a normal substance and to characterize those of other ones by the departures from those. As a normal substance could serve, e. g. the water

For our contemplations the difference in the directions of gravity of different bodies according to this idea is of first importance. Putting for a body

$$C \sin \varphi = G \sin \varepsilon,$$

and for an other one

$$C \sin \varphi' = G' \sin \varepsilon',$$

we can calculate the angle between the directions of their gravities:

$$\varepsilon' - \varepsilon = \varphi' - \varphi,$$

because the directions of the forces of attraction G and G' are the same, i. e., conforming to our figure

$$\varphi = \psi + \varepsilon \quad \text{and} \quad \varphi' = \psi + \varepsilon'.$$

Considering the smallness of these angles, we obtain

$$\varepsilon' - \varepsilon = \varphi' - \varphi = -\frac{G' - G}{G \cos \varepsilon - C \cos \varphi} \sin \varepsilon. \tag{4}$$

In view of equation (1), we have

$$\varepsilon' - \varepsilon = \varphi' - \varphi = -\frac{G' - G}{g} \sin \varepsilon.$$

If G is related now to the normal substance (water), and we write

$$G' = G(1+\varkappa),$$

it follows:

$$\varepsilon' - \varepsilon = \varphi' - \varphi = -\frac{G}{g} \varkappa \sin \varepsilon. \tag{5}$$

Thus, the quantity $\varkappa$ takes on the signification of a specific constant of attraction, for

$$\frac{G'}{G} = \frac{f'}{f},$$

consequently

$$f' = f(1-\varkappa).$$

Newton's pendulum experiments indicated just, that $\varkappa$ is less than 1/1000, those of *Bessel*, that $\varkappa$ is less than 1/60 0000, those of *Eötvös*, that $\varkappa < 1/20\,000\,000$.

To enlighten our next considerations we introduce in addition the angle of deflection η, what the direction of gravity of any substance makes with that of the normal substance (water) towards the poles, i. e., to the North on the Northern hemisphere. Being

$$\eta = \varepsilon - \varepsilon',$$

we can write

$$\eta = \frac{G}{g} \varkappa \sin \varepsilon. \tag{6}$$

Let us consider, in which way would be manifested a difference like this in the direction of the gravity of different bodies. First of all the demand arises that plumb lines determined by different stuffs and fluids would give different directions of the vertical, when they were in standstill. In general, the plumb line would also not be normal to the resting fluid level.

The differences of directions are at the 45° latitude:

for $\varkappa = 1/1\,000$	0,357 sec of arc
$\varkappa = 1/60\,000$	0,003 95 sec of arc
$\varkappa = 1/20\,000\,000$	0,000 018 sec of arc.

[No direct observations of such differences in the directions were carried out so far with the intention to solve the problem we are interested in, but we have to remember the experiments of *Guyot*, which aroused great interest in his days. *Guyot* observed in 1836 in the Paris Pantheon the mirror images of marks reflected by a resting mercury surface, the marks coming along a 57 m long pendulum and he found that its end deviates from the normal of the fluid surface by 4,5 mm to the South.[1]. The legality, to conclude from that to a deviation of the direction of the gravity was strongly disputed. The author had the opportunity to be convinced by suspending pendulums of different materials in a tower of 22 m height, using diverse wires for the suspension, that the ends showed really some deviations, which originated but in the pressure of the irregularly heated and moving air.

A further consequence of the direction of gravity depending on the material nature would be an inconformity of the gravitational level planes of different substances.

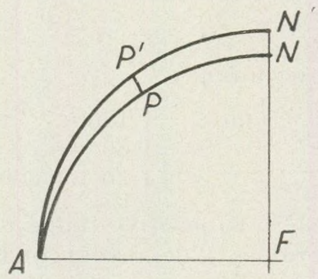

Fig. 2.

APN is a meridional quadrant of the equipotential surface belonging to the normal substance (water, s. fig. 2.), *A'P'N'* the same for an other substance having the coefficient of attraction $\varkappa$. The distance between both equipotential surfaces passing through the point of the equator is easily calculable. Do we move a unit mass of the second substance from A along the equipotential surface of the normal substance to P — then from P to P', and again along the second equipotential surface back to A, so is the whole performed work along this way equal to zero. Hence it is

$$\int_0^{\varphi} g'\eta ds + zg' = 0,$$

where ds is an element of arc of the meridional quadrant, z the positive distance to the equipotential surface $AP'N'$ downward.

Using the relations (6) and (2), we have

$$\int_0^{\varphi} \frac{g'}{g} \varkappa C \sin\varphi \, ds = -g'z.$$

To avoid tiresome calculations, with sufficient approximations we put here g for g', and

$$ds = r d\varphi$$

$$C = r\cos\varphi\omega^2,$$

denoting the mean of the Earth's semi-diameter by r. We have thus:

$$z = -\frac{1}{2}\frac{\varkappa}{g} r^2\omega^2 \sin^2\varphi$$

[1] Guyot: La pendule n'est pas perpendiculaire à la surface des liquides tranquilles. C. R. XXXII, Fortschritte der Physik. VI.

and for $\varphi=90°$, i. e. at a pole of the Earth:

$$z = -\frac{1}{2}\frac{\varkappa}{g}r^2\omega^2.$$

Taking the values

$$r=636\ 740\ 000\ \text{cm},\ g=983{,}19\ \text{cm sec}^{-2},\ \omega^2=5{,}31751\cdot10^{-9}$$

we obtain for the greatest distance between the equipotential surface of any substance and that of the water at the poles

$$z=-1\ 380\ 250\ \varkappa\ \text{cm},$$

accordingly

$$\begin{aligned}
\text{for } \varkappa &= 1/1000 & z &= -1\ 380\ \text{cm},\\
\varkappa &= 1/60\ 000 & z &= -\ 23\ \text{cm},\\
\varkappa &= 1/20\ 000\ 000 & z &= -\ 0{,}069\ \text{cm}.
\end{aligned}$$

To positive values of $\varkappa$ corresponds at the poles an elevation, to negative values a depression of the equipotential surface.

We could think on a separation of terrestrial substances so that those with positive $\varkappa$ should be piled up at the poles, on the other hand those with negative $\varkappa$ in the equatorial regions, but the eventual forces acting this way are certainly too small, and the resistances acting against them are too large to permit of separations of this kind].[1]

[1] We recognize by these never published reflections of E ö t v ö s the ideas disclosed in the footnote of VIII (78), p. 272. But the phenomenon in question is considered also here rather from a practical experimental standpoint, than as a matter of principle. [*The quotation refers to the treatise of E ö t v ö s: Bericht über Geodätische Arbeiten in Ungarn, besonders über Beobachtungen mit der Drehwaage, Verhandl. d. XVI. allg. Konferenze der Internat. Erdmessung in London – Cambridge, 1909. I. 319 – 350.

The treatise ends with the phrase: „Would ever the physicist detect by further refinement of his experimental methods even minor spoors of selective attraction of the Earth, the activity of the geodesist, should be confined just as before to measure out the dimensions of only one geoid, valid for every sort of substances."

The text of the quoted footnote is:

From these reflections of E ö t v ö s, especially from the above last phrase immediately follows that a body floating on the resting surface of a fluid, e. g., on the surface of water, should move by itself to the North, or to the South, respectively, if its gravitational constant were greater or less than that of the water, or generally: the rotation of a fluid or gaseous celestial body would operate the segregation of its constituting substances having different gravitational constants. The editor was apparently the first to point to these simple inferences, many years ago. It was shown in a short preliminary notice (M. R ó z s a and P. S e l é n y i, Über eine experimentelle Methode zur Prüfung der Proportionalität der trägen und gravitierenden Masse, ZS. f. Phys, 71, 814, 1931), that observations aiming at that and executed with the most primitive technique give for $\varkappa$ a relatively small value ($\varkappa < 1/100\ 000$). In two further notices (P. S., Inert and Heavy Mass, Term. tud. Közlöny, Supplement, Ápr. – Jún. 1940 (in Hungarian). and *P. S.* Inertia and Gravity of Matter, Hungarica Acta Physica, vol. 1. no. 5, 1949) this subject was considered from the side of principle and dared the assertion, that possibly the non-occurrence of the mentioned phenomena might be regarded as the most immediate and clearest evidence for the proportionality of inertia and gravity, what makes at the same time plainly represent the importance and far-reaching of this law of N e w t o n in the constitution of the universe.]

It is quite surprising that so tiny differences in the directions are sufficient to provoke mechanical impulses, which can be perceived and even measured with the torsion balance.

Is the swinging body of the torsion balance consisting of masses of different materials m_1, m_2, m_3 etc., so according to our considerations the axis of rotation represented by the measuring fibre should be deflected from the direction of the gravity of water to the North by an angle, that is easily calculable. Taken, namely, the conditions of equilibrium for such a body swinging about a horizontal axis 0, and oriented in West-East (fig. 3.) we have for the torque of gravity of a homogeneous part of mass m_1 the quantity:

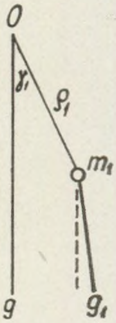

Fig. 3.

$$-m_1 \varrho_1 g_1 \sin(\gamma_1 - \eta_1),$$

and for the condition of equilibrium:

$$\Sigma\, m_\varkappa \varrho_\varkappa g_\varkappa \sin(\gamma_\varkappa - \eta_\varkappa) = 0,$$

where $\varrho_\varkappa$ means the radius of rotation of the centre of gravity for the mass $m_\varkappa$, $\gamma_\varkappa$ the angle between $\varrho_\varkappa$ and the direction of gravity of the water; $g_\varkappa$ is the gravity of the unit mass $m_\varkappa$; and $\eta_\varkappa$ the deflection of its direction from the gravity of water.

Placing approximately:

$$\cos\eta_\varkappa = 1,$$

$$\sin\eta_\varkappa = \eta_\varkappa,$$

$$g_\varkappa = g(1+\varkappa),$$

we have then

$$\Sigma\, m_\varkappa \varrho_\varkappa g \sin\gamma_\varkappa + \Sigma\, m_\varkappa \varrho_\varkappa g(\varkappa \sin\gamma_\varkappa - \eta_\varkappa \cos\gamma_\varkappa) = 0,$$

and denoting the mass of the whole swinging body by M, and the radius of rotation of its centre of gravity by R, we obtain

$$MRg \sin E + \Sigma\, m_\varkappa \varrho_\varkappa g(\varkappa \sin\gamma_\varkappa - \eta_\varkappa \cos\gamma_\varkappa) = 0,$$

whence it is easy to see, that E remains always a small angle, not surpassing the order of $\varkappa$ and $\eta_\varkappa$.

In the plane of rotation is acting hence on every part of mass $m_\varkappa$ a component of gravity directed to the pole, which can be expressed by

$$m_\varkappa g_\varkappa(\eta_\varkappa - E).$$

We want now to refer our further calculations to a rectangular coordinate-system, where the z-axis coincides with the axis of rotation (i. e., with the measuring fibre), directed downwards, while the x-axis should be directed to the North, and the y-axis to the East.

The torque of gravity originating in the before considered differences in directions is then

$$-\Sigma\, m_\varkappa g_\varkappa y_\varkappa (\eta\,\varkappa - E) = -\Sigma\, m_\varkappa g_\varkappa \eta_\varkappa + E\,\Sigma\, m_\varkappa g_\varkappa \eta_\varkappa\,;$$

but as on account of equilibrium about the z-axis we have

$$\Sigma\, m_\varkappa g_\varkappa y_\varkappa = 0,$$

the torque will be limited to the first member:

$$-\Sigma\, m_\varkappa g_\varkappa y_\varkappa \eta_\varkappa .$$

On the torsion balances used here were placed the different masses at the end of a straight beam. One end of the beam should be denoted by a, the other one by b, and we write then m_a, l_a, g_a, and η_a for the masses which lie along the beam between the axis of rotation up to its end a. For the other side of the beam similar notations are due. Introducing the notation α for the azimuth of the beam, denoting by it the angle what the beam oriented from b to a makes clockwise with the x-axis pointing to the North, we obtain the former torque in the form:

$$(\Sigma\, m_b l_b g_b \eta_b - \Sigma\, m_a l_a g_a \eta_a) \sin \alpha,$$

and using equation (6), but neglecting the terms which are multiplied by $\varkappa^2$ we become for the torque

$$D = (\Sigma\, m_b l_b \varkappa_b - \Sigma\, m_a l_a \varkappa_a)\, G \sin \varepsilon \sin \alpha. \tag{7}$$

The value of this eventual torque be illustrated by an example.

On both ends of a 40 cm long homogeneous beam should be suspended two masses of different stuff, 25 g each. At 45° latitude, where is

$$G \sin \varepsilon = 1{,}7,$$

in the case that the end a of the beam points to the North, we have for the torque

$$D = 25 \cdot 20 \cdot 1{,}7 (\varkappa_b - \varkappa_a) = 850 (\varkappa_b - \varkappa_a),$$

but when the end a points to the West:

$$D' = -850 (\varkappa_b - \varkappa_a),$$

hence

$$D - D' = 1700 (\varkappa_b - \varkappa_a).$$

Would be $\varkappa_b - \varkappa_a = 10^{-6}$, we had

$$D - D' = 0{,}0017,$$

and this torque would cause a torsion to a fibre, having a constant of torsion of 0,5 and the required weight-carrying capacity. Then the torsion, read at a distance of 1500 scale divisions, expressed in scale divisions, would be

$$n - n' = \frac{0{,}0017}{0{,}5} \cdot 3000 = 10{,}2 \text{ scale divisions.}$$

Yet, the matter is not so plain. The torsion of the measuring wire will not be effected by the just calculated torque D alone, but by the torque originated in the spatial changes of the force of gravity. In closed rooms of observation, namely in cellarlike spaces, it can be even quite considerable.[1]

A hanging system as shown in fig. 4 is consisting in a horizontal tube which is loaded at its end b by a weight M_b inserted to it, and by a suspended weight M_a at its end a, so, the centre of gravity of M_a being by h deeper than that of the weight at b. For this system *Eötvös* writes the torque effected by the increment of gravity, as follows:

$$F = \left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) K \frac{\sin 2\alpha}{2} + \frac{\partial^2 U}{\partial x\,\partial y} K \cos 2\alpha - \\ - \frac{\partial^2 U}{\partial x\,\partial z} Mhl \sin\alpha + \frac{\partial^2 U}{\partial y\,\partial z} Mhl \cos\alpha.$$

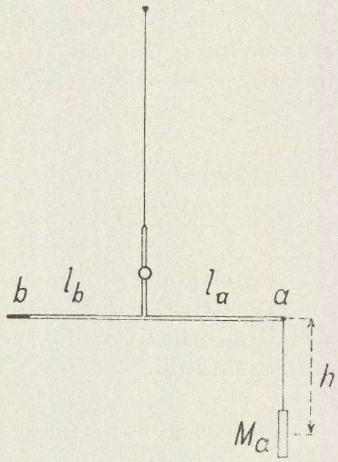

Fig. 4.

U means here the potential of gravity, K the moment of inertia of the suspended system. As the total torque of the forces of gravity acting at it is

$$D + F,$$

the angle of torsion ϑ according to the torsion of the fibre is in the position of equilibrium:

$$\vartheta = \frac{1}{2}\frac{K}{\tau}\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right)\sin 2\alpha + \frac{K}{\tau}\frac{\partial^2 U}{\partial x\,\partial y}\cos 2\alpha - \frac{M_a hl}{\tau}\frac{\partial^2 U}{\partial x\,\partial z}\sin\alpha + \\ + \frac{M_a hl}{\tau}\frac{\partial^2 U}{\partial y\,\partial z}\cos\alpha + \frac{1}{\tau}\left(\Sigma\, m_b l_b \varkappa_b - \Sigma\, m_a l_a \varkappa_a\right) G \sin\varepsilon \sin\alpha, \qquad (8)$$

where τ is the constant of torsion.

This equation was set up by taking into consideration that those small differences caused by the different attractions of different stuffs, which would only slightly alter the second derivatives of the potential, were negligible as compared with the last term of the equation. Attention must be paid to the fact, that the quantity

$$\Sigma\, m_b l_b - \Sigma\, m_a l_a$$

is here no more to be taken for zero, nevertheless its order of magnitude remains equal to that of the quantity

$$\Sigma\, m_b l_b \varkappa_b - \Sigma\, m_a l_a \varkappa_a,$$

[1] S. the treatise VI (76) in this volume. [*R. Eötvös Bestimmung der Gradienten der Schwerkraft und ihrer Niveauflächen mit Hilfe der Drehwaage, Verhandl. d. XV. Allg. Konferenz der internat. Erdmessung in Budapest, 1906, vol. I, pp 337 – 395]

because for the equilibrium about a horizontal axis the relation

$$\Sigma\, m_b l_b g_b - \Sigma\, m_a l_a g_a = 0$$

must be valid and the ratio g_b/g_a is differing from the unit only by a fraction λ which is of the same order as $\varkappa$.

For the swinging-system of the torsion balance of the described type we may write in the case that its both ends are of equal length and homogeneous, having everywhere the same thickness, we have

$$\Sigma\, m_b l_b \varkappa_b - \Sigma\, m_a l_a \varkappa_a = M_b l_b \varkappa_b - M_a l_a \varkappa_a,$$

thus, neglecting the terms multiplied by $\lambda\varkappa$, we obtain

$$\Sigma\, m_b l_b \varkappa_b - \Sigma\, m_a l_a \varkappa_a = M_a l_a (\varkappa_b - \varkappa_a). \tag{8'}$$

Equations (8) and (8') will indicate later, how to determine by the aid of observations the quantities $\varkappa_b - \varkappa_a$, after eliminating all other unknowns, and thus, the problem can be solved, whether their value regains the limit of measurability.

However, experiments like these do us furnish only with an information on the attraction of one single body, i.e. the Earth. It is, certainly, of interest to investigate, whether the attraction of the sun and moon, which is really manifested in the tidal phenomena and in the variation of the plumb line, could contribute to the elucidation of our question? We want here to give answer in a short approximate discussion to this complicated phenomenon.

The so called tidal force can be composed of two components.

One of these components is the attraction exerted by the sun or moon on a particle of mass of the Earth; its value referred to the unit mass will be by assumption of a barycentric attracting body

$$f\frac{M}{\varrho^2},$$

where M means the mass of the sun or moon, ϱ the distance from its centre of attraction. We want to regard here this force, which has different length and different direction toward the different parts of mass of the Earth, as depending on the material nature, consequently, on $\varkappa$.

The second component of force acting here according to the inertia is the centrifugal force of the revolving motion described by the Earth round the center of inertia of Sun and Earth, respectively of Moon and Earth. For every part of the Earth free from rotation, this force is equal by size and direction; we want to denote it by C, referred to the unit mass.

Since the attraction exerted on the whole Earth and the centrifugal force of the Earth's whole mass has the same size, we write

$$C = f_0 \frac{M}{D^2},$$

where D means the distance of the inertia-center of the Earth from the common inertia-center of Sun and Earth, or Moon and Earth, respectively. By f_0 is denoted here a mean value of the eventually different f values for the different substances of the Earth.

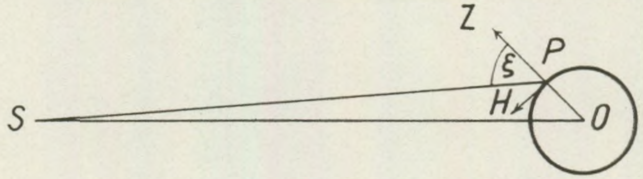

Fig. 5.

According to these considerations, and taking on a spheric form for the Earth, we obtain the components of the force as referred to a terrestrial co-ordinate-system: a vertical force directed upwards (fig. 5)

$$-Z = f\frac{M}{D^2}\cos\zeta - C\cos\zeta + fM\frac{a}{D^3}(2\cos^2\zeta - \sin^2\zeta),$$

and a horizontal force

$$H = f\frac{M}{D^2}\sin\zeta - C\sin\zeta + \frac{3}{2}\,M\,\frac{a}{D^3}\sin 2\zeta.$$

In these equations ζ is the zenith-distance of the Sun, or Moon, a the mean radius of the Earth, H is directed to that point of the horizon where the vertical plane of Sun, or Moon intersects the horizon, for which $\zeta = +\frac{\pi}{2}$.

A replacement of the approximate calculation brought up here by a more complete one would exceed the limits of this treatise.

Putting

$$f = f_0(1+\varkappa),$$

we obtain

$$-Z = \varkappa f_0\frac{M}{D^2}\cos\zeta + fM\frac{a}{D^3}(2\cos^2\zeta - \sin^2\zeta), \tag{9}$$

$$H = \varkappa f_0\frac{M}{D^2}\sin\,\zeta + \frac{3}{2}f\,M\frac{a}{D^3}\sin 2\,\zeta. \tag{10}$$

If $\varkappa=0$, these expressions give us the usual components of the tidal forces (cf. e.g., *Thomson* and *Tait*, Natural Philosophy vol. 1, § 812).

But if $\varkappa$ be different of zero, an other term with a diurnal period according to the first terms would come in beside of the semi-diurnal tidal phenomena expressed by the seconds terms of these equations.

For $\zeta=0$

$$-Z = \varkappa f_0 \frac{M}{D^2} + 2f \frac{M}{D^2} \frac{a}{D},$$

and for $\zeta=\pi$

$$-Z = -\varkappa f_0 \frac{M}{D^2} + 2f \frac{M}{D^2} \frac{a}{D}.$$

[The ratio of the first term to the second one is now $\varkappa$: 2a/D, so, if we take for the Earth and Sun a/D=1/23600, and for the Earth and Moon a/D=1/60.27 it would follow, that $\varkappa$ should have a value of 1/11800 to double the solar tide at the first time, then to annihilate it after half a day, and it ought not to be less then 1/30 to effect the same action against the semi-diurnal lunar tide.

Taking on that the force $-$Z would be determinable from the tidal phenomena up to 1/100 of its size, so the observation of the solar tides would still lead to the recognition of such values of $\varkappa$, which are not greater then 1.10^{-6}, i.e., one million of the unit. But such an accurate observation of the 24 hourly tidal wave originating in the attraction of the Sun is hardly conceivable, for it would be difficult to isolate it from the radiation effects of the Sun, which repeat themselves in same periods.]

It is easier to make use of the equations (9) and (10) for observations with the torsion balance. Orienting namely, a torsion balance of the above described type so that the azimuth α of the beam be zero, i.e., the axis of the beam be in the meridian and its end a point to the North, then two external torques are acting on it. One is due to the gravity of the Earth and results in a torsion ϑ_0 of the fibre, independently of the time, the second torsion is according to the force H given by equation (10) and depending on the time.

If A means the azimuth of Sun or Moon, the component of H normally to the beam-axis is $-$ $H sin A$, and we obtain for the torsion of the measuring fibre

$$\begin{aligned} \vartheta = \vartheta_0 - \frac{1}{\tau} f_0 \frac{M}{D^2} \Sigma\, m_a l_a \varkappa_a - \Sigma\, m_b l_b \varkappa_b) \sin\zeta \sin A \\ - \frac{3}{2}\frac{1}{\tau} f_0 \frac{M}{D^2}\frac{a}{D} (\Sigma\, m_a l_a \quad \Sigma\, m_b l_b) \sin 2\zeta \sin A \qquad (11) \\ - \frac{3}{2}\frac{1}{\tau} f_0 \frac{M}{D^2}\frac{a}{D} (\Sigma\, m_a l_a \varkappa_a - \Sigma\, m_b l_b \varkappa_b) \sin 2\zeta \sin A. \end{aligned}$$

The last term on the right side of this equation can be neglected because of the smallness of the factor a/D, likewise the term preceding it, because

$$\Sigma\, m_a l_a - \Sigma\, m_b l_b$$

is of the same order as

$$\Sigma\, m_a l_a \varkappa_a - \Sigma\, m_b l_b \varkappa_b,$$

so that it is admissible to use the approximate formula:

$$\vartheta = \vartheta_0 - \frac{1}{\tau} f_0 \frac{M}{D^2} \{\Sigma\, m_a l_a \varkappa_a - \Sigma\, m_b l_b \varkappa_b\} \sin\zeta \sin A. \tag{12}$$

We can get some information on the size and measurability of that torsion by means of an example. We are using the above described instrument for which we take

$$\Sigma\, m_a l_a \varkappa_a - \Sigma\, m_b l_b \varkappa_b = M_a l_a (\varkappa_a - \varkappa_b)$$

with the dates

$$M_a = 25g, \quad l_a = 20 \text{ cm}, \quad \tau = 0{,}5.$$

We take further

for the Sun: $\quad f_0 \dfrac{M}{D^2} = 0{,}586$

for the Moon: $\quad f_0 \dfrac{M}{D^2} = 0{,}00332.$

We obtain then a torsion according to the attraction of the Sun

$$\vartheta = \vartheta_0 \quad 586(\varkappa_a - \varkappa_b) \sin\zeta \sin A,$$

and to the Moon

$$\vartheta = \vartheta - 3{,}32(\varkappa_a - \varkappa_b) \sin\zeta \sin A.$$

We want to deal mainly with the first one, since the second one has little significance because of its many times less value.

If $(\varkappa_a - \varkappa_b)$ be different from zero and positive, i.e., the mass unit of the mass M_a suspended on the North-end of the beam be stronger attracted by the Sun than the mass unit of M_b, the torsion-beam should show a daily oscillation so that its end a should be deflected from the middle position to the East at sunrise, and to the West at sunset.

Since at sunrise and sunset $\sin\zeta = 1$, the value of this deflection is

$$\vartheta - \vartheta' = 586(\varkappa_a - \varkappa_b)(\sin A' - \sin A),$$

and in the case, when

$$\sin A' - \sin A = 2$$

as it turns out to be approximately so at equinox, the elongation is

$$\vartheta - \vartheta' = 1172(\varkappa_a - \varkappa_b)$$

or in scale divisions at a distance of 1500 scale units

$$n - n' = 3\,516\,000(\varkappa_a - \varkappa_b).$$

For $\varkappa_a - \varkappa_b = 1.10^{-6}$ we should have therefore an elongation

$$n - n' = 3,5.$$

For the observational method based on these reasonings the required sensitivity is therefore only the third part of that given by *Eötvös*, as long as the same instrument will be used. Notwithstanding, this new method is promising some advantages as leaning on observations made by a stable instrument and in this way a greater sensitivity is utilizable. Eötvös' gravity compensator[1] permits of increasing up to an arbitrary limit the sensitivity of such stable torsion balances, when perturbing influences are eliminated.

Both methods are complementing each other so that the first one furnishes the required information on the attraction of the Earth, the second on that of the Sun.

3. Particulars on the execution of the observations according to the method given by Eötvös.

There were applied two instruments of the same kind as those used by *Eötvös* for his investigations about the local variations of gravity, and described by him in the first volume of ,,Abhandlungen der XV. Allgemeinen Konferenz der Erdmessung, 1906." These are torsion balances of great sensitivity, rotatory about a vertical axis, very suitable thus to the investigations treated here.

[Fig. 6. depicts one of the instruments, the ,,*single gravity variometer*", so called by Eötvös. Its photo is shown on page 100[2].]

The housing is made of about 3 mm thick brass plates and pipes, which enclose the suspended system twofold, and even threefold at the hanging low part. This housing can be rotated about an adjustable vertical axis, and is resting on a solid base where a graduated circle is serving for indication of the angle of rotation. The graduation is by third degrees so that by the aid of a vernier one minute can be read.

The suspended system consists of a thin-shelled brass tube of about 40 cm length and 0,5 cm diameter; to its end b is inserted a platinum cylinder of about 30 g weight, while on the other end a were suspended by a thin cupper-bronze fibre the various bodies for examination. The weight of these bodies must be always so adjusted as to bring the other end loaded by a constant weight steadily in the same horizontal position. The suspension was done so that the inertia-center of the body came about 21 cm beneath the beam-axis. This length h had to be known more accurately especially for some parts of the experiments, to this end we used beside of the cathetometer a suitably shaped balance. By the aid of this we could determine the position of the gravity-center in the examined body, not always consisting of one single stuff; this position was determined by observing the change in the sensitivity of

[1] S. IV. (58), p. 63. [*The quoted treatise is: Untersuchungen über Gravitation und Erdmagnetismus, Math. és Term.-tud. Ért, 14, 1896, 221 – 266 (in Hung.); Math. u. Naturw. Ber. aus Ungarn, 13, 1896, 193 – 243; Ann d. Phys. u. Chem. Neue Folge, 59, 1896, 854 – 400.]

[2] [*The quotation refers to the above mentioned treatise.]

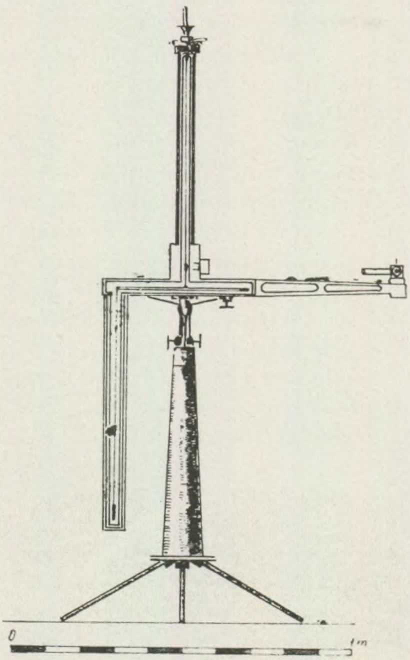

Fig. 6.

the balance occuring when the body was fixed to the beam of the balance. The attained accuracy for h was of about 0,1 mm, what was higher than needed.[1]

In order to read the position of the beam of the torsion balance, a mirror was fixed to it and a scale with half mm division at a distance of cca 62 cm from the axis of rotation. The readings were taken with a refracting telescope, with a view to set up and observe the instrument in a possibly small room.

We are using platinum-iridium fibres of 0,04 mm diameter and cca 60 cm length serving for the suspension of the swinging-system weighing cca 80 g and for measuring at the same time. The fibres loaded by 80 g weight were at first slowly heated over 100 C°, and then cooled down, after several repetitions of this procedure they attained in this way an almost perfect constancy of their equilibrium-positions after some months. Even the most violent shakings, accompanying the rotation of the instrument when the suspension-wires were in excentric positions, do not cause in general noticeable changes in the positions of equilibrium, but only seldom some small deviations.

[1] [*Determination of h was necessary, when the gravity-center of the body in question was not simply calculable from the form of the body. In this case a calibrating cylinder, then the body could be fixed to the beam of an analytical balance, near to the axis of rotation, specially designed for that purpose by E ö t v ö s. See the detailed description: J. R e n n e r, Experimental examination of the proportionality of gravitational attraction and inertia, Math. és Term. tud. Értesítő (in Hungarian), vol. 53, 1935, pp. 553 – 555.]

Experiments with quartz fibres did not give at all the same favourable results.

However, the position of equilibrium for loaded metal fibres is depending on temperature. It is the consequence of the torsion with vhich the fibres leave the eyelet, and so it is different for every piece of the fibre. This dependence is quite complicated namely the drift of the position of equilibrium is not depending on the variation of the temperature, but also on its course in time. Nevertheless, with so small and slow variations of temperature which occurred during the observations here treated, not exceeding some tenth of a grade in a day, this drift is satisfactorily represented by the individual temperature coefficient for each fibre. For the fibre in the simple variometer used by us this coefficient is $dn/dt = 0{,}4$, where n is the scale reading for the positions of equilibrium, and t is taken in centigrades. For the fibres in the second instrument used here it is still less.

That second instrument is *a double gravity variometer* (s. p. 101)[1], so called by *Eötvös*, for it consists in two parallelly suspended torsion balances which lying on a common base can be turned about the same axis. Both these single balances are of the type of the single gravity variometer; their beams are almost parallel, but so oriented as their suspended weights M_a lie on the opposite ends. Thus, when the suspended weight of one beam points to North, that of the other balance points to South.

In the beginning we followed *Eötvös*' instructions, but in the course of the observations we succeeded in a simple way to make the instruments more efficient. As according to our method of observation the position of equilibrium of the beam itself is read in the moment when it came just to rest, in this way the determination of a new position of equilibrium acted by a rotation requires a certain time depending on the resistance acting against the motion of the beam. Thus, the time-interval between two consecutive readings could not be fixed primarily for less then two, sometimes three hours. But simple calculations, the presentation of which would be here out of place, showed us, that this time-interval can be reliably reduced to one hour, when the resistance acting against the motion of the beam can be increased to the lowest limit required to make the motion aperiodic. The wanted increase was reached by inserting of brass plates of suitable size to the base and lid of the innermost housing. The innermost clearance was reduced so to 9 mm.

By applying those plates our performance could be increased by two-three-times higher than before.

Observations with so delicate instruments had to be done in shake-proof rooms protected at once against changes of temperature, and in consequence, possibly againts one-sided temperature radiations. Cellars without window would best fit to this condition. Unfortunately we did not dispose of such. Time was pressing, and so we had to be satisfield with a room for the observations, what lies at the first floor of the laboratory being at our disposal and has two windows opening to the South. Yet, higher buildings shadowed these windows for most part of the day, shutters did blanket them too, so the room was always held in dark. To complete this protection for each of the

[1] [*The quotation refers to Eötvös' treatise of 1906.]

instruments a celt was built, with strong double-lined walls between the frames, filled with sawdust, the linen stitched like counterpanes.

As the room used for observation lies out of the way of street traffic, we were not anxious about heavier shocks. Unfortunately, the conditions grew worse by a new building in progress, taken on in the immediate proximity, during the observations. Though the results of observations show no significant influence of these perturbances, we are aware of them, knowing that the observations disclosed here were not performed under the most favourable conditions and have not the perfection, as we thought to be able to reach. Well, "Ars longa, vita brevis", we must content ourselves with having proceeded a step forward.]

The considerations of the previous chapter serving for a theoretical basis of the experiments to be done suppose that the suspended parts of the torsion balance are not subject to other influences, but to those of the inertia and gravitational attraction of masses lying outside of them and the elastic force of the fibre acting againts the torsion.

Such a complete exclusion of all the effects which had to be preceeded by the knowledge of all natural forces, is beyond man's grasp, but at least those perturbing influences must be possibly avoided which are known to us to a certain extent.

We want to enumerate in order the most important influences and indicate also the way, how we invalidated them.

Magnetic forces, especially the geomagnetic force, must manifest themselves, if the swinging system contains some remanent magnetic parts. A fragment with a magnetic moment with only 1/1000 cgs magnetic moment, as about a fragment of a good steel magnet with 1/50 mg weight, could cause perturbing elongations of two scale divisions, after a rotation of the torsion balance. By careful selection of the parts composing the suspended system, it is attainable that it can be taken for non-magnetic, inspite of its great sensitivity; all the same, with our experiments attention had to be taken to this defect and prevent it in another way, while the suspended parts were repeatedly substituted by other ones. For this reason we compensated the horizontal component of the geomagnetic force so that in the space of the instruments H was reduced to zero by using permanent magnets and electromagnets.

[The compensating magnets had to be placed at a greater distance (about 1,4 m) so that they could not exert translatory forces on the temporary induced magnetism of the swinging system, effected mainly by the vertical component of the geomagnetic force. With extensive knowledge of the magnetic force it was easy to avoid its perturbance.]

The same can be said about the *electrostatic actions of outer bodies*, the influence of those on our torsion balance can be regarded as fully annulled by the threefold metal casing.

On the other hand, we have to regard for the *electrostatic forces between the suspended system and the enclosing housing* for they are not consisting of the same material. If the surfaces of the swinging system and the enclosing walls of the casing have different electric charges, electrostatic forces are produced, which might well be equal to null in a symmetrical mean position, but in the case of a deviation from it they can be sensible. Consequently, these forces must manifest themselves by that they influence the sensitivity of the instrument, i. e., the torque $\tau\vartheta$ acting againts the gravitational forces will be changed by them into $\tau'\vartheta$. In order to prevent the diverse electrical charges of the different parts of the surfaces we covered them with a uniform layer of soot. We also devised a method of observation, the results of which were not affected by small discrepancies in the quantity τ'.

The direct effects of irradiation caused by external bodies are not sufficiently known to us. But the multiple metal cover of the housing serves for the reduction to minimum of this unknown influence. [Also the dimensions of the instruments used were accordingly chosen and application of smaller anf lighter swinging systems was avoided, as we had to consider that the force to be measured, which is proportional to the mass, should be great with respect to eventual forces, which are proportional to the surface. These forces are certainly very small with our instruments and concealed in the hazards adhering in form of errors to every series of observation.]

Effects originating in the differences of temperature between the diverse parts of the housing and the swinging system. The external variations in the temperature, in consequence of which heat will be transported to the instruments or taken away from them through radiation and conduction, produce discrepancies in the temperature of the parts of the balance and the enclosed air. The multiple metal cover of the housing serves for reducing this discrepancy to as small as possible; the soot cover of all inner parts mentioned above has the same purpose. [Supposed that a distribution can be reached by it, which is on both sides symmetrical to the vertical plane passing through the mean position of the beam, only the sensitivity of the instrument, i. e. the quantity τ' is to be substituted for the constant of torsion, in the same way, as it was mentioned concerning the internal electrostatic forces. Traces of asymmetrical warming up existing inspite of all protection, maintain still to-day their accidental character.]

Changes in the temperature of fibres, if small and of slow course, can be calculated by their individual coefficients, or even discarded by a suitably chosen method of observation.

Shocks are not absolutely ineffective, too. Namely the position of equilibrium of the end of a loaded fibre changes with the load as a result of the remanent torsion of the fibre, consequently, vertical shocks shall cause jumps of the beam. But these jumps are negligeably small if caused by usual street traffic. [Only in case of earthquakes do they reach perceptible values, and at this time they amount to several scale units. In the course of observations of several years we noticed so many earthquakes the occurence of which was stated later by the seismological reports. Exceptional cases, like those, are easily recognizable and have no significance for the totality of the observations.]

In the order of possible perturbances we have to think on changes, which take place owing to *variations in mass distribution in the environment* as acting to the second derivatives of the gravitational potential and specially to $\partial^2 U/\partial x\partial z$ and $\partial^2 U/\partial y\partial z$ and which may have measurable though not great values. Displacements of objects in the building are scarcely to be considered, but much more the accumulation of masses of water as it used to happen after cloud-bursts. [1 cm thick layer of water surrounding the building causes an effect on the position of equilibrium of our instrument, what amounts to about one hundredth of a scale division. Observations concerning changes of this kind had to be carried out systematically, but for that we found no time yet. Nevertheless, parts of our observational results were freed also from that possible influence.]

The execution and evaluation of our observations considering all those circumstances was developing and improving in the course of work. [Shortness of time did not allow us to carry out all, what was regarded as the best for our scheme, but it would involve a waste of time.] The results disclosed here were obtained by three different procedures, discriminated by us as the firts, second, and third procedure.

The first procedure supposes, that the quantities, $\partial^2 U/\partial x\partial z$ and $\partial^2 U/\partial y\partial z$ are constant and also the sensitivity of the instrument, i. e., τ remains steadily the same.

The second procedure rests like the first one on the constancy of $\partial^2 U/\partial x\partial z$ and $\partial^2 y\partial z$ but it admits the possibility, that τ be different during experiments with different suspended bodies and also it changes steadily in time.

The third procedure renders us at last independent from the supposition of the constancy of the quantities $\partial^2 U/\partial x\partial z$ and $\partial^2 U/\partial y\partial z$, as well as of τ.

All the three procedures repose on equations (8) and (8′), what we want to unite, and transform, by putting for ϑ the value

$$\vartheta = \frac{n_0 - n}{2L}$$

where n means the scale reading due to the position of equilibrium, n_0 a constant, and L the scale distance expressed in scale divisions. So we have

$$n_0 - n = \frac{L}{\tau}\varkappa\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right)\sin 2\alpha + \frac{2L}{\tau}\varkappa\frac{\partial^2 U}{\partial x\,\partial y}\cos 2\alpha - \frac{2L}{\tau}M_a l_a h\frac{\partial^2 U}{\partial x\,\partial z}\sin\alpha +$$

$$+\frac{2L}{\tau}M_a l_a h\frac{\partial^2 U}{\partial y\,\partial z}\cos\alpha + \frac{2L}{\tau}M_a l_a \sin\varepsilon(\varkappa_b - \varkappa_a)\sin\alpha. \tag{13}$$

All observations were taken in four positions of the torsion balance, what we want to denote with respect to the end a of the beam as the northern, eastern, southern, and western positions by N, E, S, W and also the corresponding scale reading by n_N, n_E, n_S, n_W. Setting to N is easily done by the aid of a compass with knowledge of the magnetic declination; starting from that position the other ones are reached by successive rotations of the housing of the balance by 90°. However, in those positions the axis of the beam is not pointing precisely to the four quarters of the heaven. Be $\Delta\alpha$ the azimuth of the balance-axis in the initial N-position counted from North to the East, we obtain the following values for the four azimuths according to the four positions:

$$\begin{array}{lll}
\text{position} & N & \alpha_N = \varDelta\alpha, \\[1ex]
\quad ,, & E & \alpha_E = \varDelta\alpha + \dfrac{n_N - n_E}{2L} + \dfrac{\pi}{2}, \\[2ex]
\quad ,, & S & \alpha_S = \varDelta\alpha + \dfrac{n_N - n_S}{2L} + \pi, \\[2ex]
\quad ,, & \mathrm{W} & \alpha_W = \varDelta\alpha + \dfrac{n_N - n_W}{2L} + \dfrac{3\pi}{2}.
\end{array}$$

Considering, that $\varDelta\alpha$ and also the quantities

$$\frac{n_N - n_E}{2L} \text{ etc.}$$

are small, we compute for the four positions from equation (13) the approximate values:

$$\begin{aligned}
n_0 - n_N = {} & \frac{L}{\tau}\varkappa\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) 2\varDelta\alpha + \frac{2L}{\tau}\varkappa\frac{\partial^2 U}{\partial x\,\partial y} \\
& - \frac{2L}{\tau} M_a l_a h \frac{\partial^2 U}{\partial x\,\partial z}\varDelta\alpha + \frac{2L}{\tau} M_a l_a h \frac{\partial^2 U}{\partial y\,\partial z}, \\
n_0 - n_E = {} & -\frac{L}{\tau}\varkappa\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) 2\left(\varDelta\alpha + \frac{n_N - n_E}{2L}\right) - \frac{2L}{\tau}\varkappa\frac{\partial^2 U}{\partial x\,\partial y} \\
& - \frac{2L}{\tau} M_a l_a h \frac{\partial^2 U}{\partial x\,\partial z} \quad \frac{2L}{\tau} M_a l_a h \frac{\partial^2 U}{\partial y\,\partial z}\left(\varDelta\alpha + \frac{n_N - n_E}{2L}\right) \\
& + \frac{2L}{\tau} M_a l_a h G \sin\varepsilon\,(\varkappa_b - \varkappa_a), \\
n_0 - n_S = {} & \frac{L}{\tau}\varkappa\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) 2\left(\varDelta\alpha + \frac{n_N - n_S}{2L}\right) + \frac{2L}{\tau}\varkappa\frac{\partial^2 U}{\partial x\,\partial y} \\
& + \frac{2L}{\tau} M_a l_a h \frac{\partial^2 U}{\partial x\,\partial z}\left(\varDelta\alpha + \frac{n_N - n_S}{2L}\right) - \frac{2L}{\tau} M_a l_a R \frac{\partial^2 U}{\partial x\,\partial y}, \\
n_0 - n_W = {} & -\frac{L}{\tau}\varkappa\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) 2\left(\varDelta\alpha + \frac{n_N - n_w}{2L}\right) - \frac{2L}{\tau}\varkappa\frac{\partial^2 U}{\partial x\,\partial y} \\
& + \frac{2L}{\tau} M_a l_a h \frac{\partial^2 U}{\partial x\,\partial z} + \frac{2L}{\tau} M_a l_a h \frac{\partial^2 U}{\partial y\,\partial z}\left(\varDelta\alpha + \frac{n_N - n_W}{2L}\right) \\
& - \frac{2L}{\tau} M_a l_a G \sin\varepsilon\,(\varkappa_b - \varkappa_a).
\end{aligned}$$

We shall put in the following notations:

$$n_N - n_S = m \quad \text{and} \quad n_E - n_W = v,$$

and use the following equations as *basic equations:*

$$\left.\begin{aligned} m = & -\frac{4L}{\tau} M_a l_a h \frac{\partial^2 U}{\partial y\, \partial z} + \frac{L}{\tau} K\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) \frac{n_N - n_S}{L} \\ & + \frac{2L}{\tau} M_a l_a h \frac{\partial^2 U}{\partial x\, \partial z} \left(2\Delta\alpha + \frac{n_N - n_S}{2L}\right), \end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned} v = & +\frac{4L}{\tau} M_a l_a h \frac{\partial^2 U}{\partial y\, \partial z} - \frac{L}{\tau} K\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) \frac{n_E - n_W}{L} \\ & + \frac{2L}{\tau} M_a l_a h \frac{\partial^2 U}{\partial y\, \partial z} \left(2\Delta\alpha + \frac{2n_N - n_E - n_W}{2L}\right) + \frac{4L}{\tau} M_a l_a G \sin\varepsilon(\varkappa_a - \varkappa_b). \end{aligned}\right\} \tag{15}$$

For the gravity variometers applied here the values occurring in these equations are collected in the following table, where M^* and h^* denote mean values, which were substituted by more accurate ones for the single observations.

	τ	$\frac{K}{\tau}$	L	l_a	M_a^*	h^*	$G \sin\varepsilon$
Single Gravity variometer	0,5035	41 896	1232	20	25,4	21,2	1,6858
Double Gravity variometer							
Balance no. 1	0,5073	43 081	1258	20	25,4	21,2	1,6858
Balance no. 2.	0,5116	43 849	1258	20	25,8	21,2	1,6858

4. *Observations and their evaluation according to the first procedure*

Only one swinging system of the torsion balance was used, i. e., that of the single variometer, or only one of the double variometer. End b remained steadily loaded by the same piece of platinum inserted to the tube.

End a was loaded so as before by the investigated body (e. g., by platinum), and the North-position of the instrument (with the end a to the North) as approximately defined on the graduated circle by the aid of compass. The admissible departure, i. e., the value of $\Delta\alpha$ may here reach some degrees.

Now, the instrument was in regular time intervals repeatedly set in two positions being distant from the approximately defined North-position by 90°, and 270°, respectively, i. e., in East- and West-position.

By reading the positions of equilibrium, we obtained then

$$v = n_E - n_W$$

so that this distance in each position was determined from the mean of the immediately preceding and subsequent opposite positions. Moreover the n values of the single variometer were reduced by a temperature correction of 0,4.

The quantity m will then be likewise determined for the same body (e. g., platinum), for which purpose even a few observations are sufficient, because at this procedure only the knowledge of an approximate value of this quantity is needed.

Having carried out these observations with a body at the place of it on the same end a, we suspended an other one (e. g., magnalium), of about the same weight and determined for it v' and m'. As the exchange of the body can only be done with arrested instrument, a small displacement of the first North-position is inevitable, we put therefore $\Delta\alpha'$ for $\Delta\alpha$. However, the quantity $\Delta\alpha' - \Delta\alpha$, which is measured in scale divisions, hardly reaches the value 1/1000.

Taken on that during this whole series of observation lasting for some weeks, the value of τ and the second derivatives of the gravitational potential remained constant, we obtain for the calculation of $\varkappa_a - \varkappa_a'$ i. e., the difference of the coefficients of attraction of both bodies (e. g., platinum and magnalium) according to equation (15)

$$v - v' = \frac{4L}{\tau} M_a l_a \frac{\partial^2 U}{\partial x \, \partial z} (h - h') + \frac{4L}{\tau} M_a l_a \frac{\partial^2 U}{\partial y \, \partial z} (h\Delta\alpha - h'\Delta\alpha')$$

$$+ \frac{4L}{\tau} M_a l_a G \sin \varepsilon (\varkappa_a - \varkappa_a'),$$

where we neglected those terms which were multipled by vanishingly small quantities like

$$\left(\frac{n_E - n_W}{L}\right) - \left(\frac{n_E - n_W}{L}\right).$$

This expression is still capable of a further simplification in that we disregard the small quantities of second order; so we have

$$v - v' = v \frac{h - h'}{h} - m(\Delta\alpha - \Delta\alpha') + \frac{4L}{\tau} M_a l_a G \sin \varepsilon (\varkappa_a - \varkappa_a'), \tag{16}$$

consequently

$$\varkappa_a - \varkappa_a' = \frac{\tau}{4 L M_a l_a G \sin \varepsilon} (v - v') + \frac{m(\Delta\alpha - \Delta\alpha') - v \dfrac{h - h'}{h}}{4 L M_a l_a G \sin \varepsilon} \tau. \tag{17}$$

a) *Observations concerning the difference* $\varkappa_{magnalium} - \varkappa_{platinum}$ performed with the first procedure using the single gravity variometer.

First series of observation

On the end a of the beam there was suspended by a 0,9 mm cupper-bronze wire a magnalium cylinder of 11,92 cm length and 1,01 cm diameter. We had

$$M_a = 25{,}402\ g,\ h = 21{,}20\ \text{cm}.$$

From the observed 114 v-values we received the mean value

$$v = +1{,}983 \pm 0{,}008,$$

and from the observed 64 m-values the mean value

$$m = +8{,}138 \pm 0{,}009.$$

Second series of observation

On the end a of the balance a platinum cylinder of 6,01 cm length and 0,50 cm diameter was suspended. We had

$$M'_a = 25{,}430\ \text{g and } h' = 21{,}24\ \text{cm}.$$

From the observed 48 m' and 56 v' values we obtained the mean values:

$$m' = +7{,}534 \pm 0{,}004 \text{ and } v' = +1{,}799 \pm 0{,}006.$$

For the calculation of $\varkappa_{magn} - \varkappa_{plat}$ *after the formula* (17) we took for M_a its mean value from the two series of observation, namely 25,416 g. With the values given previously for the instrument we obtained

$$\frac{\tau}{4LM_a l_a G \sin\varepsilon} = 0{,}1192 \cdot 10^{-6}.$$

As the readings in the North-positions were

$$n = 209{,}5 \text{ and } n' = 206{,}5,$$

we have

$$\Delta\alpha - \Delta\alpha' = \frac{n' - n}{2L} = -\frac{3}{2464} = -0{,}0012,$$

and with the mean value of $m = 7{,}84$,

$$m(\Delta\alpha - \Delta\alpha') = -0{,}009.$$

Further, as we have

$$\frac{h - h'}{h} = -0{,}002,$$

the term multiplied by this is to be neglected.

We obtain thus with the values found for m, m' and v, v'

$$\varkappa_{magn} - \varkappa_{pt} = +0{,}022 \cdot 10^{-6} - 0{,}001 \cdot 10^{-6} = +0{,}021 \cdot 10^{-6},$$

or with the mean error of this result:

$$\varkappa_{magn}-\varkappa_{pt}=+0{,}021\cdot10^{-6}\ \pm0{,}001\cdot10^{-6}.$$

But this result indicating a value for the wanted difference surpassing the mean value should not mislead us. We mentioned already that with this first experimental disposition the constancy of τ was supposed; if we show together not only the values v, v' but also m, m', it is clearly seen that with the experiments with magnalium the value of τ was greater than with the platinum, namely

$$v=+1{,}983,\ v'=+1{,}799,\ m=+8{,}138,\ m'=+7{,}534.$$

Pursuing the second procedure of observation, that will be described shortly, we shall be free from such effect of the dissimilarity and variability of τ, and we may apply the equations set up for the second procedure to evaluate the first experiments, inasmuch as we suppose that τ was constant during the experiments with magnalium, as well as with platinum, but τ and τ' had different values. [Reasons capable to cause such dissimilarities were already treated above.] Calculating the results of the preceding experiments by equation (20), wich will follow later, we obtain

$$\varkappa_{magn}-\varkappa_{pt}=+0{,}004\cdot10^{-6}\ \pm0{,}001\cdot10^{-6}.$$

b) *Observations for the difference.* $\varkappa_{wood}-\varkappa_{platinum}$ performed after the first procedure, using beam 1 of the double gravity variometer

First series of observation

On the end a of the beam was suspended a cylindric piece of snakewood, with 24,00 cm length and 1,01 cm diameter There was

$$M_a=24{,}925\,\mathrm{g}\ \text{ and }\ h=21{,}03\,\mathrm{cm}.$$

From the observed 45 m values and 53 v values we obtained the means

$$m=+6{,}698\pm0{,}019\ \text{ and }\ v=-1{,}797\pm0{,}008$$

Second series of observation

On the end a of the beam was suspended a platinum cylinder of 6,00 cm length and 0,50 cm diameter. We had

$$M'_a=25{,}396\,\mathrm{g}\ \text{ and }\ h'=21{,}18\,\mathrm{cm}.$$

We obtained from the observed 14 m' and 34 v' values the means

$$m'=+6{,}595\pm0{,}016\ \text{ and }\ v'=-1{,}754\pm0{,}011.$$

For the evulation of $\varkappa_{wood}-\varkappa_{platinum}$ *after equation* (17) we take for M_a its mean value 25,160 g; we have then with the values given already for beam 1:

$$\frac{\tau}{4LM_a l_a G\sin\varepsilon}=0{,}1189\cdot10^{-6}.$$

In the North-position we had

$$n+187{,}5 \text{ and } n'=191{,}3,$$

consequently

$$\Delta\alpha-\Delta\alpha' = +\frac{3{,}8}{2516} = +0{,}0015,$$

and with the mean value $m=+6{,}65$

$$m=(\Delta\alpha-\Delta\alpha')=+0{,}010,$$

we had further

$$\frac{h-h'}{h}=-0{,}007 \quad \text{and} \quad v\frac{h-h'}{h}=+0{,}013,$$

then we got

$$\varkappa_{wood}-\varkappa_{Pt}=-0{,}005\cdot10^{-6}-0{,}000\cdot10^{-6}=-0{,}005\cdot10^{-6},$$

or with the computed mean error

$$\varkappa_{wood}-\varkappa_{Pt}= \quad 0{,}005\cdot10^{-6}\pm0{,}002\cdot10^{-6}.$$

But using equation (20) for the evaluation in the same way, as with the magnalium and platinum, we obtain

$$\varkappa_{wood}-\varkappa_{Pt}=-0{,}001\cdot10^{-6}\pm0{,}002\cdot10^{-6}$$

5. Observation and evaluation after the second procedure

Only one swinging system was used, like with the first procedure. The comparative body was suspended on end a. The instrument was set always in equal time intervals, in order in the N-, E-, S-, W-position; this operation was sufficiently repeated.

We assume now, that τ and together with it m and v are varying with the time, but at least during the length of time necessary to six settings those variations can be taken as proportional to time. We obtain thus the values m according to the moment of readings, taken in the meridional position as the differences of these readings and the mean values of the reading taken in the preceding and the next following opposite meridional positions. The momentary values were similarly taken from the values read in the positions in the prime vertical. Whereas we compute the momentary values of v for the moment of a meridional reading as the mean of the preceding and next following reading of this quantity. And so viceverse.

We calculate now the ratio v/m, for which we obtain from (14) and (15) by neglecting small quantities of second order:

$$\frac{v}{m} = -\frac{\dfrac{\partial^2 U}{\partial x\,\partial z}}{\dfrac{\partial^2 U}{\partial y\,\partial z}} + \frac{2K}{\tau}\,\frac{\dfrac{\partial^2 U}{\partial x\,\partial z}}{\dfrac{\partial^2 U}{\partial y\,\partial z}}\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) - \left(1 + \frac{v^2}{m^2}\right)\Delta\alpha$$
$$- \frac{v^2}{m^2}\,\frac{m}{4L} - \frac{2n_N - n_E - n_W}{4L} + \frac{4L}{m\tau}\, M_a l_a G \sin\varepsilon(\varkappa_a - \varkappa_b), \tag{18}$$

After that the body at a will be replaced by an other one, and a new series of observations renders us the value for v'/m'.

For the computation of the $\varkappa_a - \varkappa'_a$ serves the approximate formula

$$\frac{v}{m} - \frac{v'}{m'} = +\frac{4LM_a l_a G \sin\varepsilon}{m\tau}(\varkappa_a - \varkappa'_a) - \left(1 + \frac{v^2}{m^2}\right)(\Delta\alpha - \Delta\alpha'), \tag{19}$$

whence

$$\varkappa_a - \varkappa'_a = \frac{m\tau}{4LM_a l_a G \sin\varepsilon}\left(\frac{v}{m} - \frac{v'}{m'}\right) + \frac{m\tau}{4LM_a l_a G \sin\varepsilon}\left(1 + \frac{v^2}{m^2}\right)(\Delta\alpha - \Delta\alpha'), \tag{20}$$

where all quantities are neglected which contribute to the strict value of v/m by less then 1/1000.

a) *Observations concerning the difference* $\varkappa_{copper} - \varkappa_{platinum}$ carried out with beam 1 of the double gravity variometer, following the second procedure.

First series of observations

On end a of the beam was suspended a copper cylinder of 6,40 cm length and 0,77 cm diameter. We had

$$M_a = 25{,}441 \text{ g}, \quad h = 21{,}26 \text{ cm}.$$

From the 92 observed values we received the means:

$$m = +6{,}516 \ \pm 0{,}015 \text{ and } v = -1{,}923 \ \pm 0{,}005.$$

Second series of observations

On end a of the beam was suspended a platinum cylinder of 6,00 cm length and 0,50 cm diameter.
We had

$$M'_a = 25{,}437 \text{ g}, \quad h = 21{,}23 \text{ cm}.$$

From the observed 64 values we obtained the means:

$$m' = +6{,}536 \ \pm 0{,}013 \text{ and } v' = -1{,}982 \ \pm 0{,}011.$$

When computing $\varkappa_{Cu} - \varkappa_{Pt}$ *after equation* (20), we had to take exactly the mean values of the individually computed v/m and v'/m' values for the moments of readings. The laborious circumstantiality of computation of this nature moved us, however, instead of those to compute the mean values of v and m, as well as v' and m', and to form the ratios v/m and v'/m' from these. It is easy to prove that this mode of computation is permissible here within the limits of the accuracy found. By so calculating we obtained from the results of the two series of observations

$$\frac{v}{m} = -0{,}295 \pm 0{,}001, \quad \frac{v'}{m'} = -0{,}303 \pm 0{,}002,$$

we had further in the N-position the means

$$n = 214{,}5 \text{ and } n' = 208{,}0,$$

accordingly

$$\varDelta\alpha - \varDelta\alpha' = -0{,}002,$$

whence

$$\left(\frac{v^2}{m^2} + 1\right)(\varDelta\alpha - \varDelta\alpha') = -0{,}002.$$

Using the mean value $M_a = 25{,}439$ we obtain

$$\frac{m\tau}{4LM_a l_a G \sin\varepsilon} = 0{,}7687 \cdot 10^{-6}$$

and by this

$$\varkappa_{Cu} - \varkappa_{Pt} = +0{,}004 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}.$$

b) *Observations concerning an eventual change of $\varkappa$ with the reaction of silver-sulfate and ferrous sulfate.*

The great interest connected since the researches of *H. Landolt*[1] to the reaction

$$Ag_2SO_4 + 2\ F_0SO_4 = 2\ Ag + Fe_2(SO_4)_3,$$

who using an analytical balance proved recognizable changes in the weight, what made us investigate, whether that reaction had a result in the change of the coefficient $\varkappa$.

According to *Landolt's* dates we weighed first
1,56 g silver sulfate + 4,25 g water = 5,81 g,

then 4,05 g crystalline ferrous sulfate + 1,62 g water + 0,14 g dilute sulfuric acid = 5,81 g
and closed these two mixtures separately in two thin walled glass tubes. Then we have put these two mixtures commonly in a glass tube and then laid it aside, while after a week the perfect termination of the reaction was to be expected. In a series of observations the reacting mixtures being separate till now were introduced in a cylindric brass tube and suspended on the beam of the torsion balance. With a preceding series of observations the tube containing the products of reactions were examined in the same way.

[1] Zeitschr. f. physik. Chem, 12, 1893, p. 1.

First series of observations

Both glass tubes containing the two reacting mixtures placed one over the other were fixed in a cylindric brass tube of 12,91 cm length and 1,16 cm diameter, and suspended on the beam of the single gravity variometer. There was

$$M_a = 25{,}357 \text{ g}, \; h = 21{,}50 \text{ cm},$$

where 11,62 g falls to the share of the reacting mixture.

From observations of 132 values were derived the mean values

$$m = +7{,}590 \; \pm 0{,}011 \text{ and } v = -2{,}027 \; \pm 0{,}005.$$

Second series of observations

The glass tube containing the product of reaction was put into the brass tube used before and suspended together with the brass tube, we had

$$M'_a = 25{,}362 \text{ g}, \; h' = 21{,}34 \text{ cm},$$

where 11,62 g falls to the share of the products of reactions.

From the observed 132 values we obtained the mean values

$$m' = +7{,}622 \; \pm 0{,}08, \; v' = -2{,}032 \; \pm 0{,}005.$$

Using formula (20) *for the computation of* $\varkappa - \varkappa'$, we had

$$\frac{v}{m} = -0{,}267 \pm 0{,}001, \qquad \frac{v'}{m'} = -0{,}267 \pm 0{,}001,$$

further

$$n = 211{,}1 \text{ and } n' = 212{,}0,$$

consequently the term multiplied by $\Delta\alpha - \Delta\alpha'$ is to be neglected. With the mean value $M_a = 25{,}36$ we obtain

$$\frac{m\tau}{4LM_a l_a G \sin\varepsilon} = 0{,}9089 \cdot 10^{-6},$$

and so

$$\varkappa - \varkappa' = 0{,}000 \cdot 10^{-6} \; \pm 0{,}001 \cdot 10^{-6},$$

$\varkappa$ and $\varkappa'$ have the meaning of mean values for the inhomogeneous masses M_a and M'_a, which contained the reacting mixtures and products of reaction, respectively.

If we wanted to attribute an eventual, from zero different change of $\varkappa$ to the proceeded reaction, it should be

$$\varkappa_v - \varkappa_n = 0{,}000 \cdot 10^{-6} \; \pm 0{,}001 \cdot 10^{-6},$$

where $\varkappa_v$ relates to the same mass before the reaction and $\varkappa_n$ after it.

6. *Observations and their evaluation after the third procedure*

[This procedure yields values, which are independent not only from continuous changes of the sensitivity, but from changes of the local variations of the gravity at once.]

A double gravity variometer will be used to that purpose, but its beams should be only approximately parallel. The azimuth of the first beam in the N-position be $\varDelta\alpha_{1I}$ with the first series of experiments, with the second series $\varDelta\alpha_{1II}$, the azimuth of the second beam with the first series $\varDelta\alpha_{2I}$, and with the second series $\varDelta\alpha_{2II}$, this time the differences $\varDelta\alpha_2-\varDelta\alpha_1$, should not surpass about two degrees, what is easy to achieve.

While the b ends of both beams are loaded by the inserted platinum pieces, one of the comparative bodies with $\varkappa_a$ was suspended on the a end of the swingling system no. 1., and the other one with $\varkappa'_a$ suspended on the a end of the swinging system no. 2.

The observations will be then in the consecutive N-, E-, S-, W-positions so arranged, as with the second procedure.

We obtain, thus, after equation (18)

$$\frac{v_1}{m_1}=\frac{\dfrac{\partial^2U}{\partial x\,\partial z}}{\dfrac{\partial^2U}{\partial y\,\partial z}}+\frac{2K}{\tau}\,\frac{\dfrac{\partial^2U}{\partial x\,\partial z}}{\dfrac{\partial^2U}{\partial y\,\partial z}}\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)-\left(1+\frac{v_1^2}{m_1^2}\right)\varDelta\alpha_{1I}$$

$$\frac{v_1^2}{m_1^2}\,\frac{m_1}{4L_1}-\left(\frac{2n_N-n_E-n_W}{4L}\right)_1+\left(\frac{4LM_al_a}{m\tau}\right)_1G\sin\varepsilon(\varkappa_a-\varkappa_b),$$

and

$$\frac{v'_2}{m'_2}=\frac{\dfrac{\partial^2U}{\partial x\,\partial z}}{\dfrac{\partial^2U}{\partial y\,\partial z}}+\frac{2K}{\tau}\,\frac{\dfrac{\partial^2U}{\partial x\,\partial z}}{\dfrac{\partial^2U}{\partial y\,\partial z}}\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)-\left(1+\frac{v_2'^2}{m_2'^2}\right)\varDelta\alpha_{2I}$$

$$-\frac{v_2'^2}{m_2'^2}\,\frac{m_2}{4L_2}\quad\left(\frac{2n_N-n_E-n_W}{4L}\right)_2+\left(\frac{4LM_al_a}{m\tau}\right)_2G\sin\varepsilon(\varkappa_a-\varkappa_b),$$

whence by subtraction, then neglecting quantities under 1/1000 we have

$$\frac{v_1}{m_1}\quad\frac{v'_2}{m'_2}=\left(1+\frac{v^2}{m^2}\right)(\varDelta\alpha_{2I}\quad\varDelta\alpha_{1I})+\frac{4LM_al_a}{m\tau}G\sin\varepsilon(\varkappa_a-\varkappa'_a),\tag{21}$$

where for v, m, L, M_a, l_a their mean values are to be taken.

We exchange now the comparative bodies hung on the two half-instruments so that the body with the coefficient $\varkappa'_a$ be hung on beam 1 and that with $\varkappa_a$ on beam 2. We have now for the second series of observations.

$$\frac{v_2}{m_2}-\frac{v'_1}{m'_1}=\left(1+\frac{v^2}{m^2}\right)(\varDelta_{1II}-\varDelta\alpha_{2II})+\frac{4LM_al_a}{m\tau}G\sin\varepsilon(\varkappa_a-\varkappa'_a),$$

and we obtain by addition

$$\left.\begin{aligned}\left(\frac{v_1}{m_1}-\frac{v_2'}{m_2'}\right)+\left(\frac{v_2}{m_2}-\frac{v_1'}{m_1'}\right) &= \frac{8LM_a l_a G\sin\varepsilon}{m\tau}(\varkappa_a-\varkappa_a') \\ &+\left(1+\frac{v^2}{m^2}\right)[(\Delta\alpha_{2I}-\Delta\alpha_{2II})-(\Delta\alpha_{1I}-\Delta\alpha_{1II})].\end{aligned}\right\} \qquad (22)$$

and

$$\left.\begin{aligned}\varkappa_a-\varkappa_a' &= \frac{m\tau}{8LM_a l_a G\sin\varepsilon}\left\{\left(\frac{v_1}{m_1}-\frac{v_2'}{m_2'}\right)+\left(\frac{v_2}{m_2}-\frac{v_1'}{m_1'}\right)\right\} \\ &+\frac{m\tau}{8LM_a l_a G\sin\varepsilon}\left(1+\frac{v^2}{m^2}\right)[(\Delta\alpha_{1I}-\Delta\alpha_{1II})-(\Delta\alpha_{2I}-\Delta\alpha_{2II})].\end{aligned}\right| \qquad (23)$$

a) *Observations concerning the difference* $\varkappa_{water}+\varkappa_{Cu}$

First series of experiments

On beam 1. of the double gravity variometer was suspended a cylindric brass case filled with water, having 14,14 cm length and 1,16 cm diameter; there was

$$h=21{,}34 \text{ cm}, \quad M_a=25{,}447 \text{ g},$$

where the share of water alone was 12,82 g so that $M_{water}=0{,}504\ M_a$,

On beam 2. of the double variometer was suspended a copper cylinder with 6,50 cm length and 0,77 cm diameter

From 108 observed values were derived the mean values

$$m_1=+6{,}767 \pm 0{,}016, \quad v_1=-2{,}029 \pm 0{,}012,$$

$$m_2'=+6{,}611 \pm 0{,}012, \quad v_2'=-1{,}927 \pm 0{,}005,$$

accordingly

$$\frac{v_1}{m_1}=-0{,}300\pm 0{,}012, \qquad \frac{v_2'}{m_2'}= \quad 0{,}291\pm 0{,}001.$$

Second series of experiments

On beam 2 of the double gravity variometer was suspended a copper cylinder of 6,40 cm length and 0,77 cm diameter. There was

$$h=21{,}16 \text{ cm}, \quad M_a=25{,}441 \text{ g}.$$

On beam 2 was hung a brass case with 14,14 cm and 1,16 cm diameter, filled with water. There was

$$h=21{,}21 \text{ cm}, \quad M_a=25{,}809 \text{ g},$$

where the share falling alone on the water was 13,18 g so that

$$M_{water} = 0{,}511 \ M_a.$$

From the observed 92 values were derived the mean values

$$m_1' = +6{,}516 \ \pm 0{,}015, \ v_1' = -1{,}923 \ \pm 0{,}005,$$

$$m_2 = +6{,}786 \ \pm 0{,}010, \ v_2 = -2{,}016 \ \pm 0{,}009,$$

and accordingly

$$\frac{v_1'}{m_1} = -0{,}295 \pm 0{,}001, \quad \frac{v_2}{m_2} = -0{,}297 \pm 0{,}001$$

Computing $\varkappa_a - \varkappa_a'$ *after formula* (23) we get from the results of the first and second experimental series

$$\left(\frac{v_1}{m_1} - \frac{v_2'}{m_2'}\right) + \left(\frac{v_2}{m_2} - \frac{v_1'}{m_1'}\right) = -0{,}011 \pm 0{,}003,$$

we had further the means

$$n_{1I} = 216{,}4, \ n_{2I} = 594{,}7, \ n_{1II} = 214{,}5, \ n_{2II} = 595{,}3,$$

and accordingly

$$\left(\frac{v^2}{m^2} + 1\right) \{(\Delta\alpha_{1I} - \Delta\alpha_{1II}) - (\Delta\alpha_{2I} - \Delta\alpha_{2II})\} = -0{,}001$$

We found for the mean value

$$\frac{m\tau}{8LM_a l_a G \sin\varepsilon} = 0{,}3940 \cdot 10^{-6},$$

and so

$$\varkappa - \varkappa' = -0{,}005 \cdot 10^{-6} \ \pm 0{,}001 \cdot 10^{-6}.$$

Supposed that this difference is resulted from the difference $\varkappa_{water} - \varkappa_{cu}$ alone, so will be, as $M_{water} = 0{,}508 \ M_a$,

$$\varkappa_{water} - \varkappa_{Cu} = -0{,}010 \cdot 10^{-6} \ \pm 0{,}002 \cdot 10^{-6}.$$

Observations concerning the difference $\varkappa_{crystalline} - \varkappa_{cupric\ sulfate}$

First series of experiments

On beam 1 of the double variometer was suspended a cylindric brass case of 12,99 cm length and 1,16 cm diameter filled with crystalline cupric sulfate; here was

$$h = 21{,}22 \text{ cm and } M_a = 25{,}447 \text{ g},$$

from what the share of the cupric sulfate alone was 16,15 g so that $M_{cupric\ sulfate} = 0{,}635 \ M_a$.

On beam 2 was hung a second brass cylinder of 8,01 cm length and 1,16 cm diameter, filled with pieces of electrolytic copper wire; here was

$$h = 21{,}23 \text{ cm and } M_a = 25{,}810 \text{ g},$$

from what the share falling on the electrolytic copper alone was 18,83 g so that $M_{cu} = 0{,}730\ M_a$.

From 111 observed values were derived the mean values

$$m_1 = +6{,}676 \pm 0{,}011,\quad v_1 = -1{,}965 \pm 0{,}008,$$

$$m_2' = +6{,}684 \pm 0{,}010,\quad v_2' = -1{,}937 \pm 0{,}006,$$

and accordingly

$$\frac{v_1}{m_1} = -0{,}294 \pm 0{,}001, \qquad \frac{v_2'}{m_2'} = -0{,}290 \pm 0{,}001.$$

Second experimental series

On beam 1 was suspended a brass cylinder with 8,01 length and 1,16 cm diameter, containing pieces of electrolytic copper wire, and there was

$$h_1 = 21{,}16 \text{ cm and } M_a = 25{,}468 \text{ g},$$

from which the share of the electrolytic copper was 18,49 g so that $M_{Cu} = 0{,}726\ M_a$.

On beam 2 was suspended a 12,99 cm long brass case, with 1,16 cm diameter, filled with crystalline cupric sulfate and there was

$$h = 21{,}18 \text{ cm and } M_a = 25{,}842 \text{ g},$$

where the share of the cupric sulfate alone was 16,54 g so that $M_{cryst.\ cupric\ sulfate} = 0{,}640\ M_a$.

From the observed 132 values were derived the mean values

$$m_1' = +6{,}635 \pm 0{,}010,\quad v_1' = -1{,}984 \pm 0{,}005,$$

$$m_2 = +6{,}613 \pm 0{,}008,\quad v_2 = -1{,}923 \pm 0{,}007,$$

and accordingly

$$\frac{v_1'}{m_1'} = 0{,}298 \pm 0{,}001, \qquad \frac{v_2}{m_2} = -0{,}291 \pm 0{,}001.$$

By computing $\varkappa - \varkappa'$ after the formula (23) we obtain

$$\left(\frac{v_1}{m_1} - \frac{v_2'}{m_2'}\right) + \left(\frac{v_2}{m_2} - \frac{v_1'}{m_1'}\right) = +0{,}003 \pm 0{,}002,$$

and we had further

$$n_{1I} = 216{,}0, \;\; n_{2I} = 593{,}6, \;\; n_{1II} = 192{,}8, \;\; n_{2II} = 592{,}3,$$

accordingly

$$\left(1 + \frac{v^2}{m^2}\right) \{(\Delta\alpha_{1I} - \Delta\alpha_{1II}) - (\Delta\alpha_{2I} - \Delta\alpha_{2II})\} = -0{,}010.$$

For the mean value we obtain

$$\frac{m\tau}{8\tau M_a l_a G \sin\varepsilon} = 0{,}3898 \cdot 10^{-6},$$

and so

$$\varkappa - \varkappa' = -0{,}003 \cdot 10^{-6} \pm 0{,}001 \cdot 10^{-6}.$$

Supposed that this difference derives from the difference $\varkappa_{cryst.\ cupric\ sulfate} - \varkappa_{Cu}$ alone, being $M_{cryst.\ cupric\ sulfate} = 0{,}638\ M_a$,
we have

$$\varkappa_{cryst.\ cupric\ sulfate} - \varkappa_{Cu} = -0{,}005 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}$$

Observations concerning the difference $\varkappa_{solution\ of\ cupric\ sulfate} - \varkappa_{Cu}$

First series of experiments

On the beam 1 of the double gravity variometer was suspended a cylindric brass case of 13,50 cm length and 1,16 cm diameter, the inside of which was plated with silver and filled with solution of cupric sulfate.

The solution contained 20,61 g crystalline cupric sulfate in 49,07 g water, which ratio was according to the solution used by *Heydweiller* with his experiments.[1] We had

$$h = 21{,}22 \text{ cm}, \;\; M_a = 25{,}459 \text{ g},$$

from which the share falling to the solution of cupric sulfate alone was 15,38 g so that $M_{sol.\ cupric\ sulfate} = 0{,}730\ M_a$.

On beam 2 was hung a second brass cylinder of 8,01 cm length and 1,16 cm diameter, containing pieces of electrolytic copper wire. Here was

$$h = 21{,}13 \text{ cm}, \;\; M_a = 25{,}834 \text{ g},$$

from which the share falling on the electrolytic copper alone was 18,85 g, so that $M_{Cu} = 0{,}730\ M_a$.

From the 132 observed values we obtained the mean values

$$m_1 = +6{,}693 \pm 0{,}011, \;\; v_1 = -2{,}027 \pm 0{,}006,$$

$$m = +6{,}669 \pm 0{,}010, \;\; v_2' = -1{,}928 \pm 0{,}005,$$

[1] Über Gewichtsänderungen bei chem. und phys. Umsetzungen. Annd. Phys. 5, 1901, p. 394.

and accordingly

$$\frac{v_1}{m_1} = -0,003 \pm 0,001, \qquad \frac{v_2'}{m_2'} = -0,289 \pm 0,001.$$

Second series of observations

On beam 1 was suspended a brass cylinder of 8,01 cm length and 1,16 cm diameter, containing pieces of electrolytic copper wire; here was

$$h = 21,11 \text{ cm}, \; M_a = 25,468 \text{ g},$$

where the share of the electrolytic copper alone was 18,49 g so that $M_{Cu} = 0,726\ M_a$.

On beam 2 was suspended a cylindric brass case of 13,50 cm length and 1,16 cm diameter the inside of which was plated with silver and filled with solution of cupric sulfate, here was

$$h = 21,22 \text{ cm}, \; M_a = 25,833 \text{ g},$$

from which the share of the solution of copper sulfate alone was 15,40 g so that

$$M_{sol.\ cupric\ sulfate} = 0,596\ M_a.$$

From the 132 observed values we obtained the mean values

$$m_1' = +6,641 \pm 0,011, \; v_1' = -1,972 \pm 0,005,$$

$$m_2 = +6,766 \pm 0,010, \; v_2 = -1,982 \pm 0,007,$$

and accordingly

$$\frac{v_1'}{m_1'} = 0,297 \pm 0,001 \quad \text{and} \quad \frac{v_2}{m_2} = -0,293 \pm 0,001.$$

Computing $\varkappa - \varkappa'$ after formula (23) we obtained

$$\left(\frac{v_1}{m_1} - \frac{v_2'}{m_2'}\right) + \left(\frac{v_2}{m_2} - \frac{v_1'}{m_1'}\right) = -0010 \pm 0,002,$$

we had further the means

$$n_{1I} = 192,4, \; n_{2I} = 593,6, \; n_{1II} = 192,8, \; n_{2II} = 593,4,$$

accordingly to that the term, which is multiplied by

$$(\Delta\alpha_{1I} - \Delta\alpha_{1II}) - (\Delta\alpha_{2I} - \Delta\alpha_{2II}),$$

is to be disregarded.

We obtained the mean value

$$\frac{m\tau}{8LM_a l_a G \sin\varepsilon} = 0,3917 \cdot 10^{-6},$$

and hereby

$$\varkappa - \varkappa' = -0{,}004 \cdot 10^{-6} \pm 0{,}001 \cdot 10^{-6}$$

By taking on that this difference is arising only from the difference $K_{sol.\ cupr\ sulfate} - K_{C_u}$ we have, as $M_{sol.\ cupr.\ sulf.} = 0{,}600\ M_a$,

$$\varkappa_{sol.\ cupr.\ sulf.} - \varkappa_{Cu} = -0{,}007 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}$$

Observations concerning the difference: $\varkappa_{asbestos} - \varkappa_{Cu}$

First experimental series

On beam 1 of the double gravity variometer was suspended a cylindric brass case of 12,99 cm length and 1,16 cm diameter, loaded with asbestos. Here was

$$h = 21{,}31 \text{ cm}, \quad M_a = 25{,}462 \text{ g},$$

from which the share of asbestos alone was 15,25 so that $M_{asbest} = 0{,}599\ M_a$.

On beam 2 was suspended a second brass cylinder of 8,01 cm lenght and 1,16 cm diameter, containing pieces of electrolytic copper wire. Here was

$$h = 21{,}13 \text{ cm}, \quad M_a = 25{,}834 \text{ g},$$

from what the share of the electrolytic copper alone was 18,85 g so that $M_{Cu} = 0{,}730\ M_a$.

From the 110 observed values were derived the mean values

$$m_1 = +6{,}685 \pm 0{,}012, \quad v_1 = -2{,}024 \pm 0{,}006,$$

$$m = +6{,}705 \pm 0{,}009, \quad v_2' = -1{,}935 \pm 0{,}004,$$

accordingly

$$\frac{v_1}{m_1} = \quad 0{,}303 \pm 0{,}001 \quad \text{and} \quad \frac{v_2'}{m_2'} = -0{,}289 \pm 0{,}0001$$

Second series of experiments

On beam 1 was suspended a brass cylinder of 8,01 cm length and 1,16 cm diameter, containing pieces of electrolytic wire. Here we had

$$h = 21{,}10 \text{ cm}, \quad M_a = 25{,}469 \text{ g},$$

from which the share falling on the electrolytic copper alone was 18,48 g, accordingly $M_{Cu} = 0{,}726\ M_a$.

On beam 2 was suspended a cylindric brass case of 12,99 cm length and 1,16 diameter, loaded by asbestos. Here was

$$h = 21{,}24 \text{ cm}, \quad M_a = 25{,}833 \text{ g},$$

falling on the asbestos alone a weight of 15,25 g so that $M_{asbest} = 0{,}596\ M_a$.

From the observed 106 values were derived the mean values

$$m = +6{,}591 \pm 0{,}012, \quad v_1' = -1{,}946 \pm 0{,}005,$$

$$m_2 = +6{,}736 \pm 0{,}013, \quad v_2 = -1{,}933 \pm 0{,}008,$$

accordingly

$$\frac{v_1'}{m_1'} = -0{,}295 \pm 0{,}001, \quad \text{and} \quad \frac{v_2}{m_2} = -0{,}287 \pm 0{,}001.$$

Computing $\varkappa - \varkappa'$ *after formula* (23) we obtain

$$\left(\frac{v_1}{m_1} - \frac{v_2'}{m_2'}\right) + \left(\frac{v_2}{m_2} - \frac{v_1'}{m_1'}\right) = -0{,}006 \pm 0{,}002,$$

and we had the means

$$n_{1I} = 193{,}8, \; n_{2I} = 592{,}8, \; n_{1II} = 193{,}8, \; n_{2II} = 593{,}7,$$

whereby the term multiplied by $(\Delta\alpha_{1I} - \Delta\alpha_{1II}) - (\Delta\alpha_{2I} - \Delta\alpha_{2II})$ is to be disregarded.

We obtain the mean value

$$\frac{m\tau}{8LM_a l_a G \sin\varepsilon} = 0{,}3909 \cdot 10^{-6},$$

and so

$$\varkappa - \varkappa' = -0{,}002 \cdot 10^{-6} \pm 0{,}001 \cdot 10^{-6}.$$

Supposed that this difference derives from the difference $\varkappa_{asbest} - \varkappa_{Cu}$ alone, being $M_{asbest} = 0{,}598\ M_a$, we have

$$\varkappa_{asbest} - \varkappa_{Cu} = -0{,}003 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}.$$

Observations concerning the difference $\varkappa_{tallow} - \varkappa_{Cu}$

First experimental series

On beam 1 of the double gravity variometer was suspended a cylindric brass case of 15,60 cm length and 1,16 cm diameter, filled with pure tallow. The specific density of the tallow used was 0,918 (23,9 C°), and accordingly its mean molecular volume was about 53 times that of the water. It was further

$$h = 21{,}21 \text{ cm}, \; M_a = 25{,}470 \text{ g},$$

from what the share falling on the tallow alone was 13,78 g, so that $M_{tallow} = 0{,}541\, M_a$.

On beam 2 was suspended a second brass cylinder of 8,01 cm length and 1,16 cm diameter, containing pieces of electrolytic copper wire. Here we had

$$h = 21{,}13 \text{ cm}, \; M_a = 25{,}834 \text{ g},$$

from what the share of the tallow alone was 18,85 g, so that $M_{cu} = 0{,}730\ M_a$.

From the 118 observed values were derived the mean values

$$m_1 = +6{,}575 \pm 0{,}013, \; v_1 = -1{,}917 \pm 0{,}012,$$

$$m_2' = +6{,}637 \pm 0{,}013, \; v_2' = -1{,}877 \pm 0{,}007,$$

and accordingly

$$\frac{v_1}{m_1} = -0{,}292 \pm 0{,}002, \quad \frac{v_2'}{m_2'} = -0{,}283 + 0{,}001.$$

Second experimental series

On beam 1 was suspended a brass cylinder of 8,01 cm length and 1,16 cm diameter, containing pieces of electrolytic copper wire. Here was

$$h = 21{,}10 \text{ cm}, \; M_a = 25{,}469 \text{ g},$$

where the share of the electrolytic copper alone was 18,48 g so that $M_{C_u} = 0{,}726 \; M_a$.

On beam 2 was suspended a cylindric brass case of 15,60 cm length and 1,16 cm diameter, filled with tallow. Here was

$$h = 21{,}16 \text{ cm}, \; M_a = 25{,}847 \text{ g},$$

where the share of the tallow alone was 13,78 g so that $M_{tallow} = 0{,}533 \; M_a$.

We obtained from the 115 observed values the mean values

$$m_1' = +6{,}655 \pm 0{,}09, \; v_1' = -1{,}881 \pm 0{,}007,$$

$$m_2 = +6{,}831 \pm 0{,}005, \; v_2 = -1{,}930 \pm 0{,}006,$$

and accordingly

$$\frac{v_1'}{m_1'} = -0{,}283 + 0{,}001 \quad \text{and} \quad \frac{v_2}{m_2} = \quad 0{,}283 \pm 0{,}001.$$

Computing $\varkappa' - \varkappa$ after the formula (23) we obtain

$$\left(\frac{v_1}{m_1} - \frac{v_2'}{m_2'}\right) + \left(\frac{v_2}{m_2} - \frac{v_1'}{m_1'}\right) = -0{,}008 \pm 0{,}003.$$

We had further the means

$$n_{1I} = 195{,}2, \; n_{2I} = 593{,}7. \qquad n_{1II} = 196{,}2, \; n_{2II} = 593{,}9,$$

whereby the term which is multiplied by $(\Delta\alpha_{1I} - \Delta\alpha_{1I}) - (\Delta\alpha_{2I} - \Delta\alpha_{2II})$ is to be neglected.

We obtain the mean value

$$\frac{m\tau}{8LM_a l_a G \sin\varepsilon} = 0{,}3907 \cdot 10^{-6},$$

and hereby

$$\varkappa - \varkappa' = 0{,}003 \cdot 10^{-6} \pm 0{,}001 \cdot 10^{-6}.$$

Supposed that this difference is resulted from the difference $\varkappa_{tallow} - \varkappa_{Cu}$ alone, we have, as $M_{tallow} = 0{,}537 \; M_a$,

$$\varkappa_{tallow} - \varkappa_{Cu} = -0{,}006 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}.$$

7. Observations made with the purpose to determine the difference $\varkappa-\varkappa'$ concerning the attraction of the Sun

For basis there are serving equation (12) and the considerations preceding it. We shall use the simple gravity variometer, for what we put

$$\Sigma m_a l_a \varkappa_a - \Sigma m_b l_b \varkappa_b = M_a l_a(\varkappa_a \;\; \varkappa_b),$$

and

$$\vartheta = \vartheta_0 - \frac{1}{\tau}\left(f_0 \frac{M}{D^2}\right) M_a l_a(\varkappa_a \;\; \varkappa_b) \sin\zeta \sin A,$$

for the attraction of Sun

$$f_0 \frac{M}{D^2} = 0{,}586,$$

and for the single gravity variometer

$$M_a = 25{,}4 \text{ g}, \quad l_a = 20 \text{ cm}, \quad \tau = 0{,}5035, \quad \vartheta - \vartheta_0 = \frac{n_0 - n}{2464},$$

accordingly

$$n - n_0 = 1\,457\,000(\varkappa_a - \varkappa_b) \sin\zeta \;\sin\; A,$$

and

$$n' - n = 1\,457\,000(\varkappa_a - \varkappa_b)\;(\sin\zeta' \;\sin A' - \sin\zeta \;\sin A),$$

consequently

$$\text{(a)} \qquad \varkappa_a - \varkappa_b = 0{,}6863 \cdot 10^{-6} \frac{n' - n}{\sin\zeta' \sin A' - \sin\zeta \sin A}.$$

Applying this formula it seems to us that we can determine the difference $\varkappa_a - \varkappa_b$ by a sole series of experiments, where the two heterogeneous bodies are separately suspended on the two ends of the torsion balance set into the meridian. But we can expect that the daily oscillation of the balance loaded in this way, deriving eventually from the different gravitation of these different bodies, will be accompanied by such other osciallations with the same period, which are originating in perturbing influences which were not perfectly eliminated.

In order to eliminate, if possible, the influence of these latter ones on the result, we chose the following way for the observations.

In a *first series of observations* there was a platinum cylinder suspended on end a of the beam and the scale positions of the beam, as well as the temperatures were read hourly, during two weeks. The values of these readings were then collected and after reduction to the same temperature, the hourly mean values n were computed.

After this, in *a second series of observations* there was suspended on the end a a magnalium cylinder and we followed the same way of observation and computation as with the first series of observations. Denoting now the difference $n' - n$ observed during two sections of the day in the first series by

$$(n'-n)_I$$

and by

$$(n'-n)_{II}$$

observed for the same length of two sections of the day in the second series, we compute on the basis of formula (a)

$$\text{(b)} \qquad \varkappa_{\text{magnalium}} - \varkappa_{Pt} = 0{,}6863 \cdot 10^{-6} \frac{(n'-n)_I - (n'-n)_I}{\sin\zeta' \sin A' - \sin\zeta \sin A}$$

Though $\sin\zeta' \sin A' - \sin\zeta \sin A$ is changing in the time between the first and second experimental series, nevertheless it is certainly satisfactory to introduce here in the calculation the mean value of this quantity.

First series of observations

On the end a of the single gravity variometer was suspended a platinum cylinder of 6,01 cm length and 0,5 cm diameter and the beam was set in the meridian, with its end a pointing to the North. Here was

$$h = 21{,}24 \text{ cm}, \quad M_a = 25{,}421 \text{ g}.$$

The observations took place between June 18 and July 2, 1908, and they were taken hourly, from which the hourly mean values, as well as their departure from the total mean were formed.

Second series of observations

On the end a of the single gravity variometer a magnalium cylinder of 11,91 cm length and 1,01 diameter was suspended and the beam was set in the meridian with its end a pointing to the North. Here was

$$h = 21{,}24 \text{ cm and } M_a = 25{,}362 \text{ g}$$

The observations were taken between July 21 and August 4, 1908, and arranged so as in the first experimental series.

For the computation for $\varkappa_{magnalium} - \varkappa_{Pt}$ those n and n' values were used, for which $\zeta = 90°$, i. e., the reading was taken at sunrise and sunset.

Approximately we take for the value n at sunrise the mean of two hourly values taken at sunrise at $4^h\,0^m$ and $5^h\,0^m$ a. m. and for n' the mean of two values read at $7^h\,0^m$ and $8^h\,0^m$ p. m., at sunset, i. e., when $A = -120°$ and $A' = +120°$.

We had from the readings in the first series

$$n' - n = -0{,}062,$$

and in the second series

$$n' - n = -0{,}046.$$

Would we base our calculations on a sole series of observations, that is, to apply formula (a), we had

$$\varkappa_{magnalium} \quad \varkappa_{Pt} = -0{,}018 \cdot 10^{-6}$$

With exclusion of perturbing influences causing fluctuations at daily periods i. e. using the results of both series of observations and formula (b) we obtain more correctly

$$\varkappa_{magnalium} - \varkappa_{Pt} = +0{,}006 \cdot 10^{-6}.$$

In a more complete evaluation of the observed material, and in a desirable extension of it, unfortunately, we were hindered by shortness of time.

8. On observations to decide whether an absorption of the gravitation by an intermediate body is taking place.

With our former considerations is very closely connected the question whether the attraction exerted by a body A to an other body B be depending on a third body C lying between them, more particularly, whether for the attraction an absorbing capacity be attributable to bodies. For, if this would be the case, so had bodies of different shape and size to be differently attracted by an other one. Surely, this attraction should depend on the orientations of the single parts of the attracted body with respect to the attracting one, the front parts of the attracted body would then modify the attraction of the parts lying in the background. In this comprehension the above described observations concerning the attraction of different bodies can be considered as serving for settling the question, though the possibility of a direct experiment is not precluded.

I don't think, however, on experiments, like those of *Messrs. L. V. Austin and C. B. Thwing*[1], who endeavoured by interposing some cm thick layers of water, lead, and mercury to measure the effect exerted on the position of a torsion balance, what was deflected by the attraction of masses weighing several kilograms.

Experiments of this sort, carried out with utmost care, hardly can lead to more accurate results than those presented by the aforeasaid Gentlemen in their paper of 1897, proving that the influence of the attraction by those interposed layers is less than 1/500 of it. A result like this is much easier obtainable by considering that a balance, though subject to the much greater attraction of the Earth, undergoes no perceptible change in its equilibrium position, when layers of the above mentioned sort would be put beneath one of its scale-pans, with due protection, the accuracy attainable by this latter instrument could be increased even to the millionth of the weight. And still much more accurate results can we attain with the torsion balance.

We have carried out experiments of this sort, namely with the gravitational compensator already in 1902[2].

[Those experiments have of course, the character of trials, and if we still make known them here, we should like to consider them as preliminary tests. The time required to perform them satisfactorily, expecially to build more complete instruments, was not at our disposal.

The instrument was very like that described by Eötvös, therefore we need not discuss here its particularities.]

[1] An experimental research on gravitational permeability, Phys. Rev., 5, 1897, p. 294.

[2] S. treatise IV (58). [*The title is given in the foot-note on p. 126.]

The brass spheres, 50 g each, fixed to both ends of a 50 cm long torsion balance beam enclosed besides a metal tube for protection also by arrangements for compensation (S. Fig. 7. and 8.)

Each of these two arrangements applied on both ends consists of a cylindric metal case of 5 cm diameter encircling the protecting tube. The metal cases bear two oppositely lying cylinder quadrants (compensating masses), made of cast-lead, which are resting on horizontal shafts, by the aid of which the angle of inclination φ of the middle line K K of the quadrants can be altered against the horizon. The dimensions of the quadrants are: inner-radius 2,5 cm, outer radius 12 cm, thickness, i. e. the distance between the two boundary planes

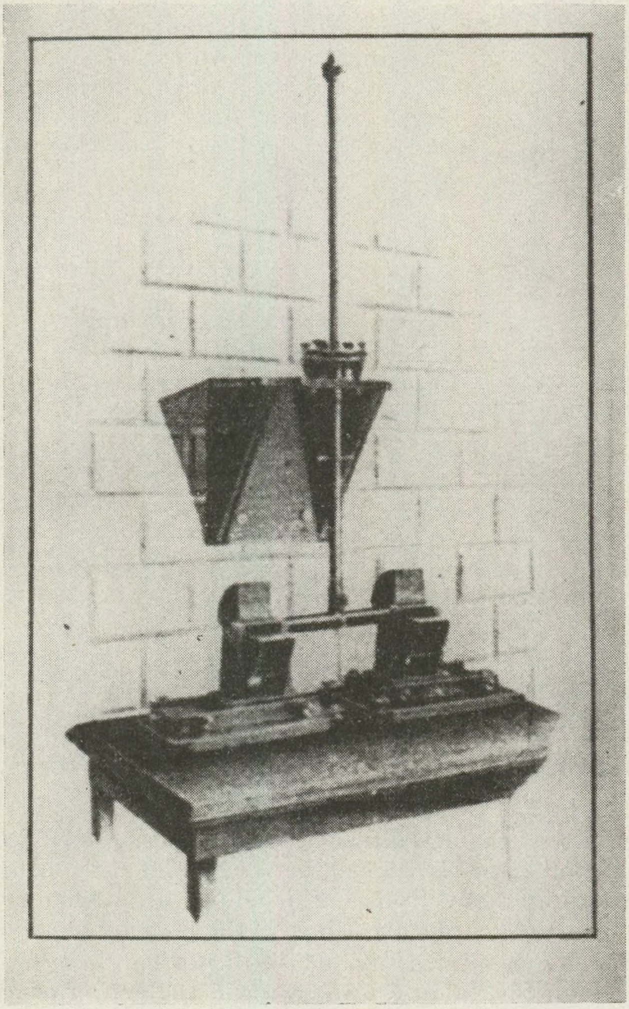

Fig. 7.

9,5 cm. The ends of the balances, more precisely the brass spheres fixed to them are oscillating in the centre of each compensating quadrant pair.

The centre P of the spheres on the balance ends should be in the axis of rotation C of the compensators, when the adjustment of the instrument is perfect. But, as perfection cannot be achieved, we have shown in the figur 8. the points P and C as detached from each other, and we shall accordingly denote the coordinates of P by ξ, ζ, relating to the coordinate-system X, Z passing through C.

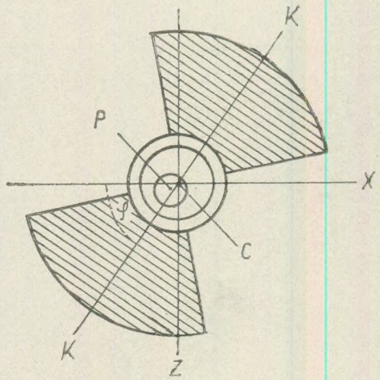

Fig. 8.

In the present case the instrument was steadily used so that both compensators had the same position against the beam ends enclosed by them. The figure shows the cross-section of the compensator and the spheres oscillating in it so as they appear to an observer who standing at one end looks toward the axis of rotation, and the figure shows just so for the observer who standing at the other end is looking also toward the axis of rotation.

In this case we can express the torque exerted on the beam by the attraction of the compensators in the following form:

$$F = A\,\xi + B\cos\varphi + \xi\,C\cos 2\varphi + \xi D\sin 2\varphi.$$

In the course of the investigations in question the compensators were set nto four different positions, distant from each other by right angles, namely

position I

$$\varphi = 45°$$

$$F_1 = A\xi_1 + B\cos\frac{\pi}{4} + \zeta D$$

position II

$$\varphi_2 = 135°$$

$$F_2 = A\xi_2 - B\cos\frac{\pi}{4} - \zeta D$$

position III

$$\varphi_3 = 225°$$

$$F_3 = A\xi_2 - B\cos\frac{\pi}{4} + \zeta D$$

position IV

$$\varphi_4 = 315°$$

$$F_4 = A\xi_4 + B\cos\frac{\pi}{4} - \zeta D.$$

If we suppose that the attraction of the Earth's masses acting upon the masses of the torsion balance is affected by the masses of the compensators like absorbing bodies, to the above torque F is adding an other torque Φ, wich is oriented forwards or backwards, according to the position of the compensators.

Thinking namely the Earth divided into two halves by a vertical plane, which passes through the balance beam, so will the action of one half of the Earth pass through the compensator, while that of the other half will not (s.

fig. 9.). Each of these halves produces a horizontal component of the attraction, the value of which as referred to the unit mass is G/π, disregarding a possible absorption, and it is directed to the side where the attracting half-Earth is lying. For the case of absorption the attraction of the half-Earth affected by it is to take for

$$\frac{G}{\pi}(1-\mu)$$

where μ is depending on the absorbing capacity of the intermediate body, further on its shape, size and position.

In this manner, the action of both half-Earth, results in an horizontal component directed to that side where the absorption is smaller. Denoting by m the mass of one sphere on the balance-beam, by l its radius of rotation, we have the torques in the four positions of the balance resulting from the one-side absorption, as follows:

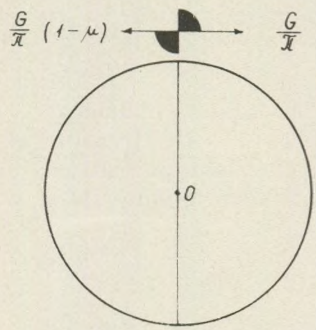

Fig. 9.

position I

$$\Phi_1 = -2ml\frac{G}{\pi}\mu$$

position II

$$\Phi_2 = +2ml\frac{G}{\pi}\mu$$

position III

$$\Phi_3 = -2ml\frac{G}{\gamma}\mu$$

position IV

$$\Phi_4 = +2ml\frac{G}{\pi}\mu.$$

We admit the position of equilibrium as being accomplished partly by the sum of the torques F and Φ, partly by the torques acting against the torsion. We express this latter in the form $\tau\vartheta_0+\tau\vartheta$ where ϑ_0 means the position of the beam, when $\xi=0$ and $\vartheta_0+\vartheta$ represents the total angle of torsion. Writing further

$$\xi = l\vartheta$$

we obtain for the conditions of equilibrium in the four positions I – IV:

$$\tau\vartheta_0+\tau\vartheta_1 = Al\vartheta_1 + B\cos\frac{\pi}{4} + \zeta D - 2ml\frac{G}{\pi}\mu,$$

$$\tau\vartheta_0+\tau\vartheta_2 = Al\vartheta_2 - B\cos\frac{\pi}{4} - \zeta D + 2ml\frac{G}{\pi}\mu,$$

$$\tau\vartheta_0+\tau\vartheta_3 = Al\vartheta_3 - B\cos\frac{\pi}{4} + \zeta D - 2ml\frac{G}{\pi}\mu,$$

$$\tau\vartheta_0+\tau\vartheta_4 = Al\vartheta_4 + B\cos\frac{\pi}{4} - \zeta D + 2ml\frac{G}{\pi}\mu.$$

Subtracting the sum of the second and fourth equations from the sum of the first and third one, we obtain

$$(\tau - Al)(\vartheta_1 + \vartheta_3 - \vartheta_2 - \vartheta_4) = 4\zeta D - 8ml \frac{G}{\pi} \mu.$$

Observing by the aid of mirror and scale, denoting the scale reading by n and the distance between scale and mirror by L, expressed in scale divisions we have

$$n_1 + n_3 - n_2 - n_4 = \frac{8LD\zeta}{\tau - Al} - \frac{16Lml}{\tau - Al} \frac{G}{\pi} \mu.$$

For the evaluation of the observations, according to the apparatus used, we took the sufficiently approximating values:
$L = 1315$ scale division, $m = 30$ g, $l = 25$ cm, $G = 982$ cgs, $\tau - Al = 0{,}103$ cgs.

The last quantity was determined by deflection experiments of the compensator-beam. Using those values we have

$$n_1 + n_3 - n_2 - n_4 = \frac{8LD}{\tau - Al}\zeta - 47890 \cdot 10^6 \mu$$

Although we could the factor of ζ quite easily determine from the dimensions of the apparatus, we determined it rather from observations so that we observed two values of the quantity $n_1 + n_3 - n_2 - n_4$ due to different values of ζ. We have so:

$$(n_1' + n_3' - n_2' - n_4') - (n_1 + n_3 - n_2 - n_4) = \frac{8LD}{\tau - Al}(\zeta' - \zeta).$$

Such change in the value of ζ can be easily operated and measured by sinking or lifting the compensator, what is resting on plate screws. From these kinds of experiments we obtained, if ζ is expressed in cm,

$$\frac{8LD}{\tau - Al} = 608,$$

and so

$$n_1 + n_3 - n_2 - n_4 = 608\zeta - 47\,890 \cdot 10^6 \mu,$$

from which

$$\mu = \frac{n_2 + n_4 - n_1 - n_3}{47890 \cdot 10^6} + \frac{608\zeta}{47890 \cdot 10^6}.$$

The numerical values of this formula are proving the high accuracy obtainable with the determination of μ, but on the other hand they point to difficulties to be surmounted. They are consisting not only in the protection from perturbing effects, which are doubly effective in the case of a high accuracy like this, but expecially also in that the influence of the term multiplied by ζ should be possibly avoided or confidently determined.

With our experiments the mounting of the apparatus in a cellar of uniform temperature afforded the necessary protection and by the aid of cathetometers we succeeded in adjusting the compensators so that ζ differed from zero by less then 1/500 cm. We carried out three series of observations under these circumstances, already some years ago; the readings are collected in the following short table.

Readings

Position		17 April	20 April	23 April
I.	n_1	246,2	264,0	266,2
II.	n_2	247,4	264,6	268,0
III.	n_3	246,3	263,8	267,1
IV.	n_4	246,0	262,5	266,6
I.	n_1	246,0	263,9	265,9

Omitting the term proportional to ζ we computed μ from the observations

17th April $$\mu = -\frac{1}{47890 \cdot 10^6} \cdot 1{,}0,$$

20th ,, $$\mu = -\frac{1}{47890 \cdot 10^6} \cdot 0{,}6,$$

23th ,, $$\mu = +\frac{1}{47890 \cdot 10^6} \cdot 1{,}4.$$

Considering that an inaccuracy of 1/50 mm = 1/500 in the adjustment of ζ should cause an error slightly over one unit, values of μ differing from zero by about one unit can be ascribed to this imperfection. As far as it is permissible, on the basis of a few experiments we may assert, that μ, *i. e., the diminution of the Eart's attraction effected by the interposed compensating quadrants was less, than a fifty thousand millionth part of it.*

Experiments, like these, should be many times repeated to become conclu sive, moreover their accuracy should be increased, what is attainable by according dimensions of the apparatus in order to get rid of the influence of the factor of ζ. [The compensating masses should be placed at a greater distance from the beam. Unfortunately we found no time for that as yet.]

Examining the meaning of the results concerning μ, we think ourselves absolved from the trouble of a more accurate calculation of this quantity having the supposition that the absorption is proportional to the radiated section, and so the question is to fix a limiting value. But we have the right to state that those sections of the straight lines, which coming from points of the half-Earth pass through the compensator mass before reaching the attracted sphere, have an average length over 5 cm. We may say, therefore, that the attraction of the Earth passing through a 5 cm thick layer of lead is not affected by an absorption which is over the fifty thousand millionth of its value.

For a 1 m thick layer of lead their limit would be 1/2500 million and for the absorption through the whole length of the Earth's diameter about 1/400. But supposing that the absorption be proportional to the mass passed through, according to our experiments *the absorption of the whole Earth along its diameter should be less than about 1/800.*

Observations on ebb and flow and its generating forces show however, that this limit for an eventual absorption of the gravitation caused by the whole Earth along one of its diameters is even smaller.

We can convince ourselves of this in the simplest way by considering the vertical forces due to the attraction of Sun and Moon at two points of the Earth, where $\zeta=0$ and $\zeta=\pi$.

In place of the value

$$-Z = 2f\frac{M}{D^2}\frac{a}{D}$$

of the force without absorption, we have now

$$-Z = 2f\frac{M}{D^2}\frac{a}{D}+\mu f\frac{M}{D^2},$$

or

$$-Z = 2f\frac{M}{D^2}\frac{a}{D}\left(1+\mu\frac{D}{2a}\right),$$

where μ means the part of the attraction absorbed by the Earth's mass along the length of its diameter.

In this way we have to write for the tide caused by the sun:

$$-Z = 2f\frac{M}{D^2}\frac{a}{D}(1+11800\,\mu)$$

and for the lunar tide:

$$-Z = 2f\frac{M'}{D'^2}\frac{a}{D'}(1+30{,}14\mu).$$

Would μ touch the limit 1/1600 found by our torsion balance observations, we had for the Sun

$$-Z = 2f\frac{M}{D^2}\frac{a}{D}(1+7{,}4),$$

and for the Moon

$$-Z = 2f\frac{M'}{D'^2}\frac{a}{D}(1+0{,}002).$$

In this case the solar tide should be magnified about eight-times, while the lunar tide should be hardly noticeable.

Even the roughest observations of the tidal phenomena contradict a supposition like that. But one might think on the observation of the tidal forces in order to determine the value μ or at least to fix more exactly the limit, which this value cannot surpass.

Namely the proportion of the tidal force of the Sun to that of the Moon according to our previous considerations is

$$\frac{Z}{Z'} = \frac{Z_0}{Z_0'}(1 + 11770\mu)$$

where Z_0/Z_0' represent the theoretical value of this proportion when $\mu = 0$.

Many years of observations of tidal phenomena do us entitle to say that at least the magnitude of the solar tides does not surpass that of the lunar tides, by that is given us the prove that the attraction of sun along an Earth's diameter has no more loss than a tenthousandth part of it. We obtain this result by the aid of our last formula if we put there

$$\frac{Z}{Z'} = 1 \quad \text{and} \quad \frac{Z_0}{Z_0'} = \frac{1}{2{,}2}.$$

More accurate results of this sort are well to be expected from observations of the tidal force.

[We have already such observations at our disposal. We have in our hands the fair work of *O. Hecker* ,,Beobachtungen an Horizontal-Pendeln usw." (Veröffentlichung des k. preuss. Geodetischen Institutes. Neue Folge No. 32, 1907), which is reach in observations and in very interesting conclusions derived from those.

The observations were taken on two horizontal-pendulums, and the last results are collected on pp. 31 and 32 of the treatise in the following formulas

Pendulum I	computed attraction of the Moon	$0''00922 \cos(2t - 305°{,}5)$
	observed lunar wave	$0''00622 \cos(2t - 285°{,}4)$
Pendulum II	computed attraction of the Moon	$0''00900 \cos(2t - 48°{,}7)$
	observed lunar wave	$0''00543 \cos(2t - 63°{,}2)$
Pendulum I	computed attraction of the Sun	$0''00399 \cos(2t - 305°{,}5)$
	observed solar wave	$0''00244 \cos(2t - 273°{,}6)$
Pendulum II	computed attraction of the Sun	$0''00389 \cos(2t - 48°{,}7)$
	observed solar wave	$0''00585 \cos(2t - 48°{,}3)$

We don't want to deal here more closely with the satisfactory agreement of the computed and observed phases, we are rather interested in the ratio of the amplitudes for Sun and Moon. Denoting the amplitudes by A_s and A_m we obtain

$$\text{Pendulum I} \begin{cases} \dfrac{A_s}{A_m} \text{ computed} = 0{,}432 \\[2ex] \dfrac{A_s}{A_m} \text{ observed} = 0{,}392 \end{cases}$$

$$\text{Pendulum II} \begin{cases} \dfrac{A_s}{A_m} \text{ computed} = 0{,}432 \\[2ex] \dfrac{A_s}{A_m} \text{ observed} = 1{,}077 \end{cases}$$

The good accord between the computed and observed values for pendulum I makes us to take the limit of μ lower than before. The pity is that pendulum II hinders us to do so with full conscience, though Prof. *Hecker* emphasizes several times in his paper that pendulum II suffered by many disturbances and it was even less accurate in its dates.

But using the dates of this pendulum II alone, we arrive to the result stated before that the loss of the Sun's attraction suffered along an Earth's diameter is less than one tenthousandth part. This result is ten times more accurate than that received by the gravitational compensator, but we should point again to the fact, that the observations made by that instrument were only of preliminary character and they promise us a much higher degree of accuracy, if carried out more carefully.]

9 Experiments with radioactive substances

Investigations on radioactive substances were carried out in two directions, at first, concerning the proportion of their masses to the attraction of Earth exerted on them, secondly about the question if these substances do exert an absorption on the attraction or even a specific attraction or repulsion.

a) *Observations concerning the proportion of mass and attraction*

We carried out experiments with a preparative of radium which derived from the *Curie* Laboratory and was put at our disposal. The total weight of this specimen closed in a small tube was 0,200 g, containing 0,100 g pure $RaBr_2$, with 1 500 000 – fold activity of that of metallic uranium. This specimen was available to us only for a short while at the beginning of this work, that is why we could perform our observations but by the first procedure.

The small tube containing the radium was carefully fixed in the midle of a closed brass tube, and suspended on the end of beam, the observations were then carried out in the same manner as with the magnalium and wood.

We had to consider at this time, that the suspended mass M_a was not homogeneous, as only a 1/250 part of it was consisting of $RaBr_2$.

The direct determination concerned an average coefficient $\varkappa_a$ of the attraction of the total mass M_a, where the coefficient $\varkappa_{Ra}$ of the attraction of the radium bromide added only a contribution with 1/250 of M_a. In this way, if we attribute the value $\varkappa_a - \varkappa_{Pt}$ to the effect of the radium specimen alone, we have to put

$$\varkappa_{Ra} - \varkappa_{Pt} = 250(\varkappa_a - \varkappa_{Pt})$$

First series of experiments

On end a of beam 1 of the double gravity variometer was suspended the closed brass tube of 9,62 cm length and 0,90 cm diameter, containing the radium specimen. There was

$$M_a = 25{,}396 \text{ g}, \quad h = 21{,}55 \text{ cm}.$$

From 15 observed values we obtained the mean value

$$m = +6{,}566 \pm 0{,}028,$$

and from 43 values

$$v = -1{,}736 \pm 0{,}008.$$

Second series of experiments

For this purpose are serving the experiments made with the same apparatus for the determination of $\varkappa_{wood} - \varkappa_{Pt}$ (s. p. 136.), where the end a was loaded with a platinum cylinder.

The results of that series of experiments were

$$m' = +6{,}595 \pm 0{,}016, \qquad v' = -1{,}754 + 0{,}011.$$

For the computation of $\varkappa_a - \varkappa_{Pt}$ *after formula (17)* we take the mean values

$$M_a = 25{,}396\ g \quad \text{and} \quad \frac{\tau}{4LM_a l_a G \sin\varepsilon} = 0{,}1178 \cdot 10^{-6}.$$

In the N-position we had

$$n = 191{,}5 \text{ and } n' = 191{,}3,$$

whence $n - n' = +0{,}2$, consequently $m(\Delta\alpha - \Delta\alpha')$ is to be neglected. Here was

$$\frac{h-h'}{h} = +0{,}017 \quad \text{and} \quad v\frac{h-h'}{h} = -0{,}029$$

so that we obtain

$$\varkappa_a - \varkappa_{Pt} = 0{,}005 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}.$$

But computing after formula (20) so as in the case of magnalium and wood, we obtain

$$\varkappa_a - \varkappa_{Pt} = -0{,}001 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}$$

and at last after our preceding determination:

$$\varkappa_{Ra} - \varkappa_{Pt} = -0{,}25 \cdot 10^{-6} \pm 0{,}50 \cdot 10^{-6}.$$

b) *Observations concerning a specific mechanical effect of specimens of radium*

In addition we want to report here on experiments carried out by us years ago (1904) with the purpose to discover possible mechanical effects of radium specimens on the torsion balance beam.

The researches have lead us in the domain travelled by the treatise of *Mr. Robert Geigel:* ,,On absorption of gravitational energy by radioactive substances[1] After the remarks made shortly by *Mr. W. Kaufmann* on the work of *Mr. R. Geigel*[2], the publication of any further remarks seemed us superfluous at that time; we think, however, that our experiments are worth mentioning in the frame of the present treatise.

[1] R. Geigel, Ann. d. Phys. 10, 1903, p. 429.
[2] R. Geigel, Ann. d. Phys. 10, 1903, p. 894.

With the experiments we used 10 mg of a radium specimen; its activity was about million-fold that of uranium metal.

The radium specimen was closed in a small tube, of 4,5 cm length, 0,5 cm outer diameter, and 0,66 g weight.

Experiment no. 1

After reading the equilibrium position of the gravity variometer, the small tube containing the radium specimen was placed into the housing of the torsion balance and set there up on a light frame of wire so that the tube was lying at the end *b* of the beam, parallel to the platinum cylinder inserted to the beam, and in the same height.

The radium tube was put once to one side, then to the other side of the oscillating platinum cylinder, and the position of equilibrium was read in each case. The distance H between platinum cylinder and tube could be figured out from the scale reading where the oscillating beam was repulsed by the tube.

At a distance of $H=50$ sc. division = 4,05 mm the tube pushed away the platinum cylinder by 1,8 scale divisions, according to a force P, the value of which is easily calculated from

$$\frac{lP}{\tau}=\frac{n'-n}{2L},$$

i. e.,

$$\frac{20P}{0,5}=\frac{1,8}{2464}.$$

Repulsing force $P=0{,}000\,018_2$.

With repetition of the experiment we obtained:

repulsing force $P=0{,}000\,018_8$.

The pulverized radium specimen was lying during the experiments dispersed on the base of the tube, about 2 mm deeper than the axis of the platinum cylinder.

Experiment no. 2

All was arranged so as with experiment no. 1, with the difference, that the radium tube was lifted above the platinum cylinder by about 3 mm. At $H=41$ sc. div. = 3,2 mm the tube attracts the platinum cylinder by 2,5 scale divisions, with the

force of attraction $P=0{,}000\,025_3$.

Experiment no. 3

Instead of the tube containing the radium specimen we placed into the housing of the instrument an empty glass tube having the same form and size as that one with experiment no. 1. Traces of a repulsion were showing themselves not exceeding the value $P=0{,}000\,001$, and it seemed to be decreasing in time.

We repeated these experiments several times and proved by them the result first found. The simplest, but rather light-minded interpretation would be the supposition of a specific attractive force of the radioactive substance, according to the attraction found by experiment no. 2, moreover an absorption of the Earth's attraction by the substance, what could cause the repulsion found by experiment no. 1. Experiment no. 3., where such substance was not present, looks rather to corroborate interpretations like this, but we have to bear in mind that the glass tube here used replaced the radium tube with respect only to its mass-attraction, and not to other effects, especially its heat effect.

In order to suppress every doubt about this very important question we examined the effect of a glass tube, which was like the radium tube not only with respect to form and mass, but also to the heat it was steadily radiating.

Into the glass tube was sealed a short piece of platinum wire, of 0,04 mm diameter and 1,41 ohm electrical resistance, and it was heated by a current of according intensity.

At first, careful comparison was made of the quantities of heat radiated in the same time by the tube heated by current and the radium tube. A comparison like this performed by thermoelectric method rendered us the result, that the radium specimen radiated 0,169 g calorie in an hour according to 0,0118 ampere intensity. The tube with the platinum wire was then placed into the inside of the housing, while the current was conducted to it through carefully packed holes.

Experiment no. 4

At $H = 32$ sc. division = 2,4 mm appeared a repulsion of 1,8 sc. div. so that there was a

repulsing force $P = 0{,}000\,018$,

equal to the repulsing force found by experiment no. 1.

Experiment no. 5

The torsion balance beam was lowered by about 3 mm. Here appeared an attraction, as with experiment no. 2,

attracting force $P = 0{,}000\,024_8$.

With the repetition of experiments no. 4. and no. 5. applying a higher intensity of current of $i = 0{,}0250$ amp, when platinum cylinder and heating wire were in the same height (experiment no. 4.), we found a repulsion by 9 sc. division, and an attraction up to touch when the wire was in a higher position. (experiment no. 5.)

We believe to have satisfactorily proved, that attraction and repulsion appearing in experiments no. 1. and no. 2. were not effected by a specific mechanical action of the radium specimen, not even by absorption of the Earth's attraction, but they were solely caused by thermic effects, exerting mainly a mechanical action as a result of warming up the air.

The radium specimen of about 10 mg weight exerted, thus, on the torsion balance beam at a distance of about 4 mm certainly no specific attractive or repulsive force, the value of which could touch one unit of 10^{-6} order. There was also no recognizable trace of an absorbing effect on the Earth's attraction

10. *Grouping of the results*

1. Observations after Eötvös' method

Considering in *Newton's* formula

$$P = f\frac{mm'}{r^2}$$

the quantity f as depending on the nature of the attracted body and putting

$$f = f_0(1 + \varkappa),$$

the results of our observations can be represented by values of the quantity $\varkappa$ computed from those. We show there the values $\varkappa - \varkappa_{Pt}$ found, together with the mean errors of the determinations, where

$$\varkappa_{Pt} = 0, \text{ if } f_{Pt} = f_0.$$

Magnalium	$+0{,}004 \cdot 10^{-6} \pm 0{,}001 \cdot 10^{-6}$
Snake-wood	$-0{,}001 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}$
Copper	$+0{,}004 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}$
Water	$-0{,}006 \cdot 10^{-6} \pm 0{,}003 \cdot 10^{-6}$
Crystalline cupric sulfate	$-0{,}001 \cdot 10^{-6} \pm 0{,}003 \cdot 10^{-6}$
Solution of cupric sulfate	$-0{,}003 \cdot 10^{-6} \pm 0{,}003 \cdot 10^{-6}$
Asbestos	$+0{,}001 \cdot 10^{-6} \pm 0{,}003 \cdot 10^{-6}$
Tallow	$-0{,}002 \cdot 10^{-6} \pm 0{,}003 \cdot 10^{-6}$

The mean values found for $\varkappa - \varkappa_{Pt}$ are smaller in four cases, slightly greater in three cases than their average errors, and equal in one case.

The probability of a value different from zero for the quantity $\varkappa$ even in these cases is vanishingly little, as a review of the according observational data shows quite long sequences with uniform departure from the average, the influence of which on the average could only be annulled by much longer series of observations.

Among the bodies subjected to the observations are to be found those with very different specific gravities, molecular gravities, molecular volumes, and also with different states and structures.

We believe to have the right to state that $\varkappa$ relating to the Earth's attraction does not reach the value of $0{,}005 \cdot 10^{-6}$ for any of those bodies.

In connection with the question if the attraction would change following a chemical reaction or dissolution taken place in the attracted bodies, we obtained even smaller limits.

For *Landolt's* silver sulfate — ferrous sulfate reaction we found namely

$$\varkappa_{before} - \varkappa_{after} = 0{,}000 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6},$$

and for the solution of cupric sulfate in water after the proportion given by *Heydweiller*

$$\varkappa_{before} - \varkappa_{after} = 0{,}002 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}$$

2. Observations in the meridian

For the Sun's attraction we obtained

$$\varkappa_{magnalium} - \varkappa_{platinum} = +0{,}006 \cdot 10^{-6}$$

3. *Observations concerning an absorption-like influencing or attraction through intermediate bodies.*

By experiments with the gravitational compensator it was shown, that a 5 cm thick lead layer causes no absorption touching a value of $0{,}00002 \cdot 10^{-6}$. Accordingly:

the absorption through a 1 m thick lead plate is less than $0{,}0004 \cdot 10^{-6}$ times the Earth's attraction, absorption through the Earth along one of its diameters is $< 1/800$ part of the Earth's attraction.

4. *Observations on radioactive substances*

From experiments with a radium specimen of 0,20 g weight we obtained

$$\varkappa_{RaBr_2} - \varkappa_{Pt} = 0{,}25 \cdot 10^{-6} \pm 0{,}50 \cdot 10^{-6}.$$

From experiments with other specimens we noticed,

a) that the specimen exerts on a platinum cylinder of 30 g weight laying at 4 mm distance from it no specific attraction or repulsion what would reach one unit of the order 10^{-6};

b) that this specimen causes no noticeable absorption of the Earth's attraction.

We can express in a few words the final results of our work.

We have carried out long series of observations, the accuracy of which is surpassing all previous ones, but we were not able in any case to discover noticeable departures from the law of the proportionality of inertia and gravity.[1]

(Manuscript received on February 27, 1922)

[1] The measurements carried out by Dr. J. Renner (Math. u. Naturwiss. Anz. d. Ung. Akademie d. Wiss, vol. LIII, 1935, pp. 542 – 570, in Hungarian, with German résumé) in 1935, in the Eötvös-Institute after the third procedure fully affirmed the conviction of the authors expressed in the introduction of this treatise, that the accuracy of measurements of this sort can be still considerably increased. He succeeded in that by careful choice of the torsion balance and excellent torsion wires, as well as by perfect elimination of the disturbing effects of variations in temperature. He was able to show that in the case of platinum, brass, glass drop, smashed glass drops, paraffine, ammonium fluoride, manganese-copper alloy, and bismuth the difference of the gravitational constants does not surpass in any case the average error $0{,}52 \cdot 10^{-9}$ of the measurements, i. e., the value 1 : 2 000 000 000, in one case (brass – bismuth) it remained even under 1 : 5 000 000 000.

As we can hardly expect any further increase of the measuring accuracy and the discovery of a specific gravitation, this fundamental problem of physics appears to be definitely solved. – And still remains here the question to be answered: how do things stand with live matter? Newton had already investigated corn and wood; tallow as an organic stuff was drawn here in the measurements, but we are thinking on real living organisms, actively developing by cell-division, which were never chosen for objects of similar investigations. The difficulties would somewhat increase, probably the attainable accuracy somewhat decrease, and a negative result may be foreseen. Anyhow, inertia and gravitation are both inseparable universal properties of matter; besides, they let themselves compare with $10^{-8} - 10^{-9}$ accuracy. Therefore it would be doubtless of certain natural philosophical significance to make sure, that even in this regard there is no difference between living and lifeless matter.

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