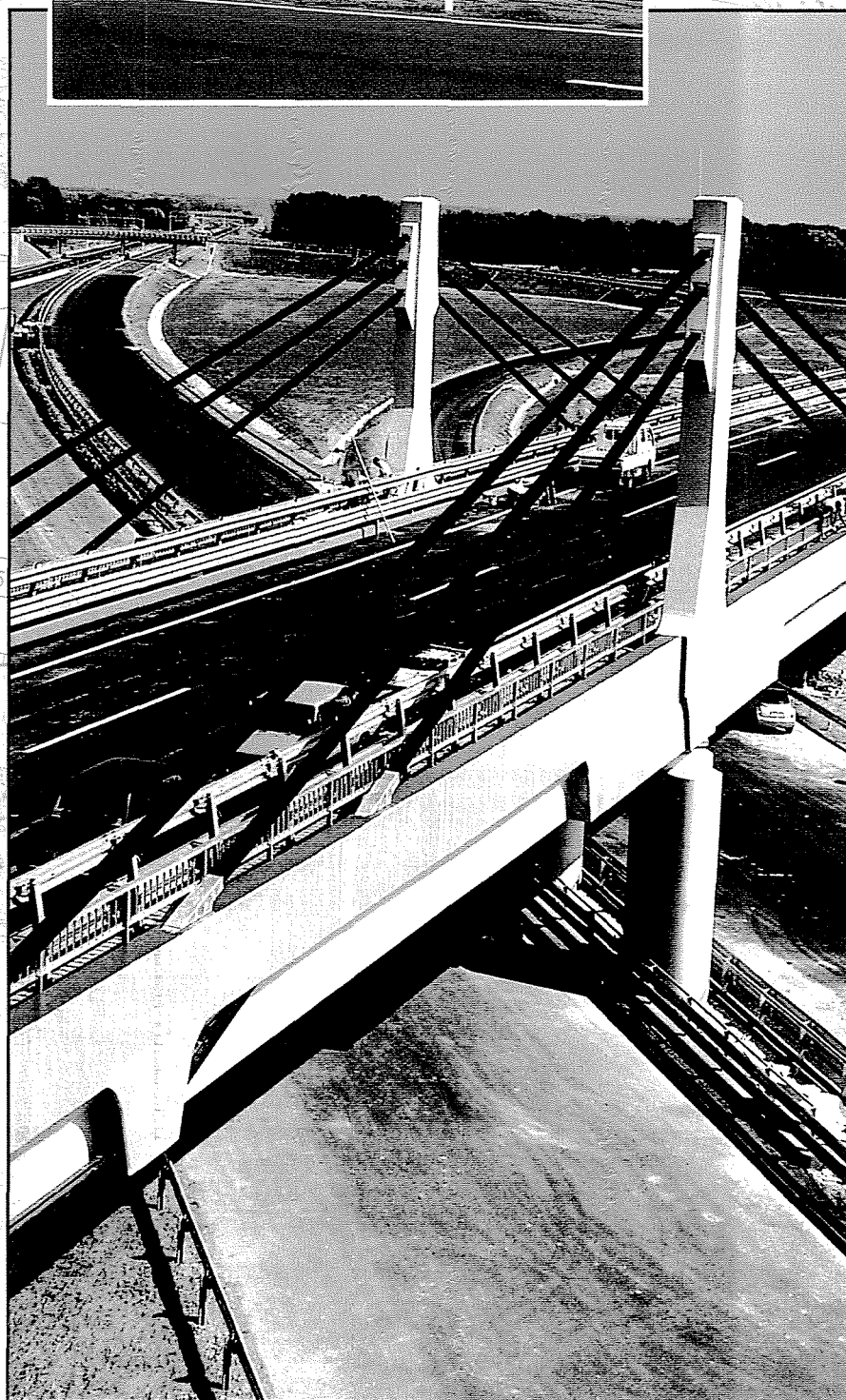


# CONCRETE STRUCTURES

ANNUAL TECHNICAL JOURNAL



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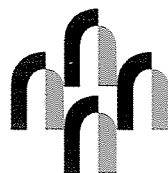
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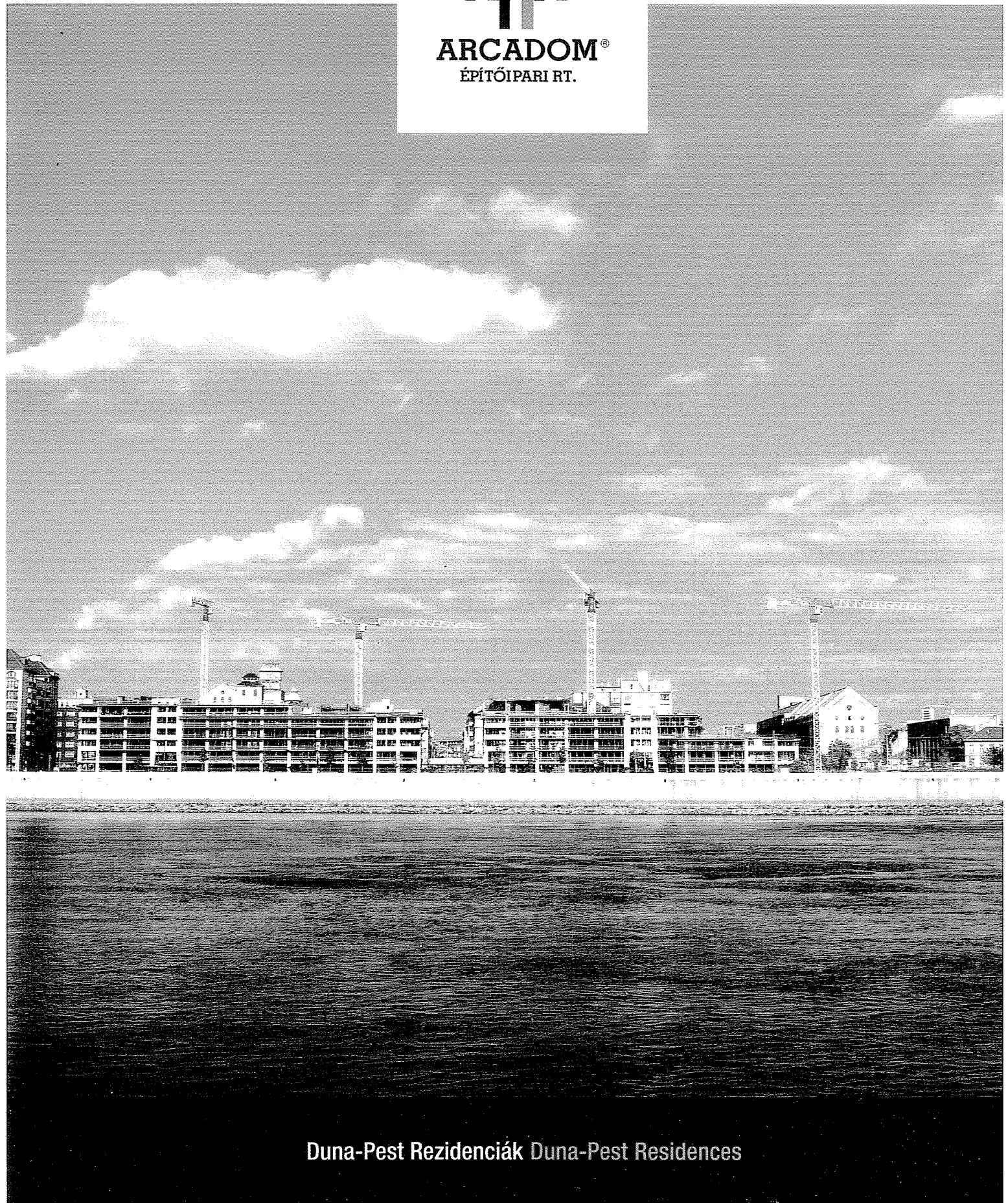
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**Cover photo:**

The first Hungarian extradosed bridge  
built at the interchange of M7 and  
M70 motorways.

Design: J. Becze and J. Barta  
Contract: Hídépítő Co.

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# "KEEP CONCRETE ATTRACTIVE"

– towards the *fib* Symposium 23-25 May 2005 in Budapest –



Prof. György L. Balázs – Dr. Adorján Borosnyói – Prof. Géza Tassi

The congresses, symposia, plenary sessions of the predecessors of *fib* were very important events for concrete construction during the second part of the 20<sup>th</sup> Century. At present time, tradition is continued. The first congress of *fib* and the symposia after the merger of CEB and FIP have proved the benefit of these international professional meetings.

The presentations of the scientific results, new techniques and outstanding structures – at sessions and on posters as well – gave always a very instructive overview to specialists who were attending. The technical exhibitions showing the progress of construction methods, development of materials, new devices serving the up-to-date concrete technology enriched the knowledge of the international society of engineers. The tours to construction sites, completed interesting structures, prefabrication plants, research laboratories, which were held in connection with the programmes, gave always deep impressions to participants. All the conventions afforded opportunity for sessions of committees, working groups, workshops. The social events offered excellent possibilities to get acquainted of colleagues coming from all over the world, to hand over awards, to impart the organising group's skill, to feel the good connection between the concrete people of different countries. The proceedings and other publications were valuable units of the technical literature. All these enriched the libraries. The delegates generally informed their countrymen who couldn't attend the convention, by calling together domestic conferences, by written reports and personal discussion.

There were in sum 63 congresses, plenary sessions and symposia of CEB, FIP and *fib* since the foundation of the international societies for concrete. These events were hosted by 44 cities in 29 countries. From among these, there are 7 cities and 9 countries which gave the venue for two such events, and 8 countries where three or more conventions took place.

It is a great honour for a member group the request for organising a congress or symposium. For the Hungarian Group of *fib*, it is a distinction that after the CEB plenary session in 1980 and the FIP Symposium in 1992, the *fib* Symposium 23-25 May 2005 will be held in the Hungarian capital. We think this can be attributed to different factors. One of them is the doubtless success of the mentioned previous events. The second is the intensive participation of Hungarian member groups in the international organisations. And last but not least, the rich tradition of a Central European country in building science, construction industry, especially in field of concrete.

Let us mention a few facts and data only.

There were approximately 120 presentations by Hungarian delegates at the events and similar number of printed contributions. Many Hungarian engineers chaired plenary and section sessions. CEB had Hungarian administrative council member for a long time, there was an intensive Hungarian presence in FIP Council over decades and *fib* has Hungarian member in its highest board. Hungarian engineers took part in sev-

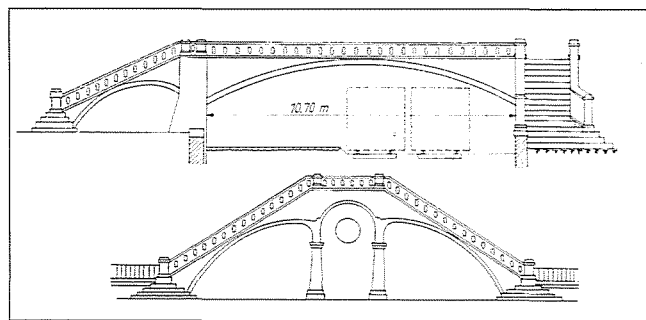
eral committees, task groups, in several cases as chairman or reporter. There were CEB administrative council meetings (also at countryside as in Pécs) and FIP council meetings in Budapest, which were continued in other cities (Szolnok, Kecskemét, Eger). Three full volumes of CEB and FIP publications (bulletins, commission reports) were edited under Hungarian authorship and execution. Hungarian authors contributed to many other volumes. Firms coming from Hungary exhibited at 11 international events. Many Hungarian specialists took part in the creation of CEB-FIP Model Codes. The Hungarian FIP, CEB, *fib* Groups invited frequently foreign specialists of high reputation to lecture at inland conferences, and there were many representatives of the Hungarian groups who had the honour to take the floor at national conventions abroad. The Hungarian achievements were regularly presented at the FIP/*fib* congresses. The leading members had permanently a very good, friendly contact to CEB/FIP/*fib* headquarters. Hungarian engineers working in the national groups did their best to represent the national colours in other related international professional organisations like IABSE, RILEM, IASS, ISO, etc. The international boards acknowledged the Hungarian activity, two members of the Hungarian group were awarded the FIP Medal, and Hungarian delegate was awarded at the first congress of *fib*.

We are proud of it that Hungarian concrete science and industry, as well as the organizer work had a good reputation among the leading personalities of the international federations.

Hungary is today a relatively small country in Central Europe. Nevertheless, during its stormy history, the country was always in the first row of development. Already, the architectural relics from the early centuries of the kingdom (from 1001 A. D.), churches, chapels and monasteries, aristocrats' palaces, noblemen's country-houses, fortresses and different other structures remained in spite of the multiple wars, invasions against the country. The modern era brought new materials, like concrete. There are noteworthy plain concrete realisations from the first part of the 19<sup>th</sup> Century. Reinforced concrete was applied shortly after its invention. Following minor structural elements, in the early 1890s the first reinforced concrete bridge was erected. The reinforced concrete footbridge in the Budapest Town Park from 1894 is shown in *Fig. 1*. The quick development is shown by the fact, that at the beginning of the 20<sup>th</sup> Century many industrial, agricultural, office and cultural buildings, residential houses were built using reinforced concrete. It is characteristic that in 1908, Hungary was "world champion" with an almost 40 m main span compound reinforced concrete girder bridge, and an arch which was the largest reinforced concrete railway bridge with its 60 m span. Between World War I and II, important and interesting structures were erected using concrete. The reinforced concrete skeleton was widely developed for industrial buildings and

dwelling houses as well. Silos, chimneys, foundations had their special character. Large span hangars, garages, storage structures were completed. The first outstanding shell structures appeared. At the same time, concrete was widely and successfully used for structures of hydraulic and transportation engineering. The number of reinforced concrete bridges increased.

World War II resulted in an awful damage of all types of structures. The late 40s and early 50s were mainly denoted for the reconstruction of destroyed buildings, bridges and other structures. Then a very forced industrialisation of the country followed. Robust concrete structures were built for new power stations and works for heavy industry. Huge sport stadium, enormous roofs for chemical and metallurgical works were erected. This was the golden age of the Hungarian site prefabrication, which was a pioneer worldwide. New types of concrete bridges were also characteristic for this period, in great part due to the application of prestressing. Improving the experience in construction of cooling towers, grain and cement silos as well as giant chimneys, Hungary became great power in slipforming. The factory prefabrication was begun very early, but the widespread development in this field belongs to this period. From small floor beams to huge bridge girders, from lamp-poles to high voltage power transmission masts, from canalisation tubes to prefabricated members for water tanks and towers etc. a very wide range of prefabricated elements were manufactured in mass production. The lion part of these factory products was pre-tensioned using improved concrete supply, machinery equipment, compacting, curing methods etc. Maybe, from among all products the prestressed concrete railway sleepers have shown the most developed technology, millions of sleepers were produced, many of them exported, also full factories. Plants for a wide range of industrial and agricultural structural members were established. Heavy columns, large span T and TT roof elements, medium and large span hollow core floor members, shell elements and many other prefabricated members were widely used. It can be said that a special chapter of Hungarian concrete prefabrication and construction industry is the mass production of panelling, mainly for residential buildings. It is a fact, that concrete solved the accommodation problems of about two million people. – The bridge construction was also developed. Besides the prefabrication of short and medium span bridge girders (up to 30 m), which were partly used for the reconstruction of highways and establishments of motorways, “under pavement” parts of the Budapest metro, up-to-date bridge construction systems were introduced, like segmental and monolithic balanced free cantilevering and incremental launching. To all of these, mainly to the latter process, Hungarian engineers contributed with original practical and economical methods and equipments. The political change in the social-financial-economic regime of the country influenced the built-up of the building industry, too. The large state owned design offices as well as huge contracting firms and prefabrication plants as well as cement factories and aggregate suppliers were divided to major-minor private firms. That way, the appearance of building industry became more variegated. Together with the further development of the international concrete technology, new results can be observed in special Hungarian concrete construction. There is a renaissance of single storey industrial/commercial buildings. Hungarian results are not only applied to fulfil domestic claims but there is an expansion abroad. On the other hand, international capital trends to take part in Hungarian construction, mainly with result and benefit. Office buildings, cultural establishments, buildings for up-to-date transportation, hydraulic structures for water sup-



**Fig. 1.** Still existing concrete pedestrian bridge over the surface line of the first European-continental underground (Budapest, 1894)

ply and wastewater treatment were improved. There was a further step in concrete bridge construction, e.g. the longest prestressed concrete railway bridge in Central Europe was completed in Hungary using advanced technology of incremental launching. The newest extradosed bridge opened in September 2004 on the crossing of M7 and M70 motorways is shown in Fig. 2. – The panelling was practically stopped, but other prefabrication procedures for apartment buildings, included one-family houses are going on. Also the mass production of sleepers and poles is continued. It cannot be neglected the effort performed for maintenance and rehabilitation of old concrete structures. Sure, there are considerable and nice tasks before the Hungarian building industry and the Hungarian engineers dealing with concrete are well prepared to fulfil the requirements of a waited boom in industry, culture, housing, transportation, agriculture, education, commerce environmental protection.

Related to its territory and population, Hungary was for a very long period at the forefront of science and education. This is continued at present time and does exist all the prerequisites for further development. We used to refer to classical Hungarian representatives of natural sciences, like the worldwide famous mathematician János Bolyai or the physicist Loránd Eötvös. The historical storms caused that from among 14 Hungarian born Nobel Prize holders only two received this highest award on their activity, which was carried out in the native country. However, the technical-scientific results of e. g. Dennis (Dénes) Gábor or Eugene (Jenő) Wigner are originating in the traditionally excellent Hungarian secondary school and university education. Other excellent Hungarian scientist (who, in our opinion would deserve award on the same level), among others Theodore (Tódor) Kármán, John (János) Neumann or Edward (Ede) Teller frequently confessed that the key of their results is to be found in the Hungarian education. We can mention excellent Hungarian engineers who are involved in concrete technology and civil engineering work, living in different countries of the world, who have brought much honour to Hungarian education, among them Thomas (Tamás) Paulay, Rudolf Szilard (Szilárd), Peter (Péter) Gergely, Ferdinand (Ferdinánd) Rostásy (indeed, these distinguished names are examples, the other outstanding specialists or their followers shouldn't be hurt because not their names were mentioned here.)

The Budapest University of Technology and Economics was founded in 1782. It had a single faculty and the name was Institutum Geometricum, which was practically the first version of the Faculty of Civil Engineering. Concrete construction was embedded into the curriculum at the beginning of the 1900s, and from 1916 it was an independent subject both for civil engineering and architecture students. Concrete as material had a wide place within the subject dealing with building materials. The extension and level of education was perma-

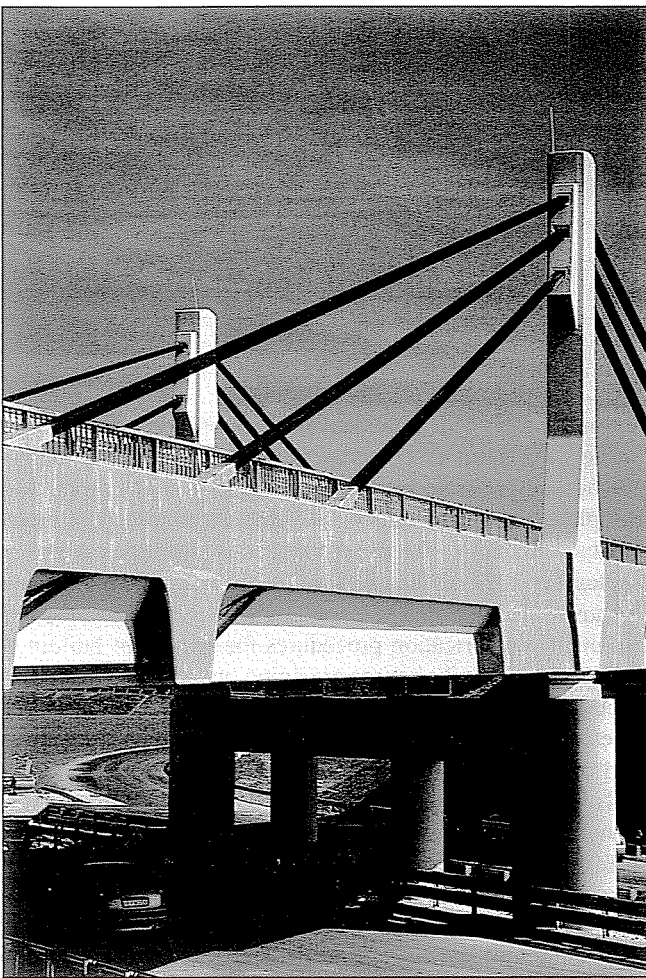


Fig. 2. The first Hungarian concrete extradosed bridge (Letenye, 2004)

nently improved. There were excellent professors of Civil Engineering as Szilárd Zielinski, Győző Mihailich, László Palotás, Elemér Bölcskei, or at the Faculty of Architecture Pál Csonka and József Pelikán. During the last century, excellent engineers were educated who fulfilled their commitments in design, construction, research, teaching and administration. The students graduated from the university in Budapest were always valuable professional people all over the world.

The research work started at the university. Besides the theoretical studies, an intensive experimental investigation started in 1931, when the Laboratory for Concrete and Reinforced Concrete was founded in the frame of the university. Late 1940s the Institute for Building Science (ÉTI) than the Quality Control Institute for Building (ÉMI) was founded, furthermore the Central Institute for Research and Design of Silicate Industry (SZIKKTI) and other. The Scientific Society for Building (ÉTE, the Scientific Society for Transport (KTE), the Scientific Society for Silicate Industry (SZTE) contributed a lot to the development of concrete technology in Hungary. There are state organisations, which support technical research, as at present time the National Scientific Research Fund (OTKA) that helps much the researchers also in the field of concrete. At the top, the Department of Technical Sciences of the Hungarian Academy of Science supports the building science, concrete included, and unites engineers having a scientific degree (D.Sc., C.Sc., Ph.D.).

It would be really difficult to give a comprehensive overview on all factors, which led to the joyful and honouring fact that the Hungarian Group of *fib* was authorised to organise the *fib* Symposium in May 2005. Sure, this event will be a new milestone for Hungarian specialists dealing with structural concrete. We are convinced that the topics of this event are promising and the 250 papers of which the abstracts are submitted will be instructive and interesting. Sure, the full programme will be pleasant and the venue will radiate an especially good atmosphere. This venue is the building of the Hungarian Academy of Science (founded in 1830, the building completed in 1862-64 in early eclectic style, designed by the German architect F. H. Stüler, supervised by the famous Hungarian architect Miklós Ybl). The view on the Széchenyi Chain Bridge and the Buda Castle with the Matthias Church will connect past and present, encouraging the participants to do much for the future construction work, too.

“KEEP CONCRETE ATTRACTIVE”

Welcome to the *fib* Symposium  
Budapest, Hungary, 23–25 May 2005.

# NEW METHOD FOR DEFLECTION CONTROL OF REINFORCED CONCRETE BEAMS AND SLABS ACCORDING TO EUROCODE 2



Prof. László P. Kollár

This paper presents a new, simple method for the deflection control of reinforced concrete slabs, rectangular beams, ribbed slabs, and T-beams. The limit span to depth ratio is presented as a function of the design load and the characteristic strength of the concrete. This paper shows that the approximation of Eurocode 2, which takes into account the effect of the provided/required steel area ratio, may not be conservative, and presents a new, simple and reliable method.

**Keywords:** deflection, reinforced concrete, Eurocode, span to depth ratio, slab, ribbed slab

## 1. INTRODUCTION

The calculation of the deflection of reinforced concrete (RC) beams and slabs is based on the assumption of cracked sections taking into account tension stiffening (ISO 4356, 1977, Dulácska, 2002, Bódi et al, 1989, Pálfalvi et al, 1996). The codes offer a simple method for deflection control based on the span to depth ( $l/d$ ) ratio (Neville et al., 1977, Litzner, 1994, Deák, 1989, Várkonyi, 2001), where  $l$  is the effective length of a simply supported beam and  $d$  is the effective depth. This method can be used in preliminary design to estimate the height of RC beams and slabs.

In this paper we propose a new method to determine the limit  $l/d$  ratio based on the "accurate" deflection calculation of the last version of Eurocode 2 (Eurocode 2, 2002). (Note that the last version gives significantly higher deflections and hence higher beams than the previous versions. For a RC beam with a reinforcement ratio of 1.5%, the limit  $l/d$  was reduced from 18 (Eurocode, 1991) to 14.)

We present the limit span to depth ratio as a function of the design load. This has two advantages over the conventional method (where the span to depth ratio is given as the function of the reinforcement ratio): (a) it is easier to use in preliminary design, when the design load is known but the reinforcement ratio is not, (b) this method enables us to correct the unconservative approximation of Eurocode, where the provided/required steel area ratio is taken into account.

## 2. CALCULATION ACCORDING TO EUROCODE 2

According to Eurocode 2, the curvatures must be determined for the uncracked ( $\kappa_1$ ) and fully cracked ( $\kappa_{II}$ ), cross sections and then the total curvature is obtained from the expression:

$$\kappa = (1-\zeta)\kappa_1 + \zeta\kappa_{II}, \quad (1)$$

where (for sustained or cyclic load)

$$\zeta = 1 - 0.5 (M_{cr}/M)^2 \epsilon \geq 0. \quad (2)$$

$M_{cr}$  is the cracking moment. Hence, the deflection is smaller than that calculated on the basis of the fully cracked section: this effect is called tension stiffening. In the calculation of  $\kappa$ , both the bending moments and the effects of shrinkage must be taken into account. (The latter is calculated by expression 7.21 of the Eurocode.) The moments are calculated from the quasi permanent load. The deflection is calculated by integrating the curvatures along the length of the beam.<sup>1</sup>

The deflection of a beam does not affect its appearance if the sag of the beam does not exceed  $l/250$ . Eurocode 2 presents an approximate expression for the limit span to depth ratio (expression 7.16), which for simply supported rectangular beams with tensile reinforcement, is as follows:

$$\frac{l}{d} = 11 + 1.5\sqrt{f_{ck}} \frac{\rho_o}{\rho} + 3.2\sqrt{f_{ck}} \left( \frac{\rho_o}{\rho} - 1 \right)^{3/2}, \text{ if } \rho \leq \rho_o \quad (3a)$$

$$\frac{l}{d} = 11 + 1.5\sqrt{f_{ck}} \frac{\rho_o}{\rho}, \text{ if } \rho \geq \rho_o \quad (3b)$$

where  $f_{ck}$  is the characteristic strength of concrete in  $N/mm^2$ ,  $\rho = A_s/bd$  is the required reinforcement ratio and  $\rho_o = \sqrt{f_{ck}} 10^{-3}$ . These expressions were determined assuming that  $f_{yk} = 500 N/mm^2$ ,  $p_{qp}/p_{Ed} = 0.5$ , and  $p_{Ed} = p_{Rd}$  (where  $p_{qp}$  is the quasi permanent load,  $p_{Ed}$  is the design load, and  $p_{Rd}$  is the ultimate load, which causes  $M_{Rd}$  at the midsection of a simply supported beam.) The last two assumptions can also be written as  $p_{qp}/p_{Rd} = 0.5$ , i.e. the quasi permanent load is 50% of the ultimate load. Table 1 was calculated from Eqs.(3a) and (3b).

When the characteristic yield strength of steel,  $f_{yk}$ , is not equal to  $500 N/mm^2$ , or more steel is provided than required, according to the Eurocode, the above values may be multiplied by  $(500/f_{yk}) (A_{s,prov}/A_{s,requ})$  where  $A_{s,requ}$  is the required, and  $A_{s,prov}$  is the provided cross sectional area of the tensile reinforcement. This approximate calculation can be unsafe. For a RC slab, the amount of reinforcement hardly influences the deflection. (See the Numerical Example.)

Precamber (up to the value of  $l/250$ ) may be applied to compensate for some of the deflections.

<sup>1</sup> As an approximation,  $\zeta$  may be calculated at one cross section only (e.g. at the midspan of a simply supported beam) and this value can be taken into account for the entire span. In this case the calculation of the deflection simplifies to  $e = (1-\zeta) e_1 + \zeta e_{II}$  where  $e_1$  and  $e_{II}$  are the deflections calculated assuming uncracked and fully cracked cross sections, respectively.

Concrete	Reinforcement ratio, $\rho$ (%)					
	1.5	1.0	0.5	0.4	0.3	0.2
C40/50	15.0	17.0	25.8	35.0	54.6	105.3
C35/45	14.5	16.3	23.0	30.4	46.6	89.1
C30/37	14.0	15.5	20.5	26.2	39.2	73.7
C25/30	13.5	14.8	18.5	22.4	32.2	59.1
C20/25	13.0	14.0	17.0	19.1	25.9	45.7
C16/20	12.6	13.4	15.8	17.0	21.5	35.8

**Table 1:** The limit span/depth ratio,  $(l/d)_{\text{lim}}$ , as a function of the reinforcement ratio (Eqs. (3a) and (3b))

### 3. NUMERICAL COMPARISONS

Numerical calculations were carried out for simply supported rectangular beams with tensile reinforcement subjected to a uniformly distributed load. In the calculation, the following creep coefficients were taken into account (Visnovitz, 2003): for concrete strength classes C40/50, 35/45, 30/37, 25/30, 20/25 and 16/20:  $\varphi_{\text{cr}} = 1.76, 1.92, 2.13, 2.35, 2.55$  and  $2.76$ , respectively; while the shrinkage strain was  $\varepsilon_{\text{sh}} = 0.04\%$ . The effective modulus was calculated as  $E_{\text{ceff}} = 22 [(f_{\text{ck}} + 8)/10]^{0.3} (1 + \varphi_{\text{cr}})$  and the tensile strength is  $f_{\text{ctm}} = 0.3 f_{\text{ck}}^{2/3}$  (Eurocode 2, 2002). We assumed that  $f_{\text{yk}} = 500 \text{ N/mm}^2$ ,  $E_s = 200 \text{ kN/mm}^2$ ,  $f_{\text{cd}} = f_{\text{ck}}/1.5$  and  $f_{\text{yd}} = f_{\text{yk}}/1.15$ .

The first row in Table 2 was calculated with the following further assumptions: the deflection was calculated with the  $\zeta$  (Eq.2) based on the mid section (see the previous footnote); the limit deflection is  $l/250$ ;  $p_{\text{qp}}/p_{\text{Rd}} = 0.5$ ; the class of concrete is C30/37; and  $d/h = 0.85$ . (For the calculated  $l/d$  ratios, when the beam is subjected to 50% of the ultimate load, the deflection is exactly  $l/250$ .) The values between reinforcement ratios 1.5 to 0.3% agree well with the values obtained by the approximate expression (Eq.3) of the Eurocode (fourth row).

The second row was calculated such that  $\zeta$  and the curvatures were calculated at 51 cross sections along the beam (assuming uniform reinforcement), and the deflection was obtained numerically. These values are slightly higher than the values in the first row. In the third row, the numerical integration was carried out by assuming that the reinforcement is not uniform, but follows exactly the (parabolic) bending moment curve.

The effect of  $d/h$  was investigated in the fifth and sixth rows. For small reinforcement ratios, the uncracked section, and hence  $h$ , plays an important role. As a consequence,  $d/h$  affects the results significantly. For high reinforcement ratios,

**Table 2:** The limit span/depth ratio,  $(l/d)_{\text{lim}}$ , as a function of the reinforcement ratio. In the first row the limit deflection is  $l/250$ ,  $p_{\text{qp}}/p_{\text{Rd}} = 0.5$ , C30/37,  $f_{\text{yk}} = 500 \text{ N/mm}^2$ ,  $d/h = 0.85$ ,  $\varepsilon_{\text{sh}} = 0.04\%$ . In rows 5-8, only one of these parameters was changed, which is listed in the first column. The deflection was calculated on the basis of the midsection (uniform  $\zeta$ ) in the first and in the 5th to 8th rows. In the second and third rows,  $\zeta$  was calculated along the beam and the deflection was calculated by numerical integration. In the second row, a uniform reinforcement was assumed along the beam, while in the third row the reinforcement varies with the bending moment.

	reinforcement ratio, $\rho$ (%)					
	1.5	1.0	0.5	0.4	0.3	0.2
1. on the basis of the midsection	13.6	15.2	20.3	24.2	37.3	113.7
2. „accurate” calculation, numerical integration	13.9	15.7	22.8	28.7	48.6	113.7
3. „accurate”, reinforcement follows the bending moment curve	12.7	14.5	21.5	27.5	48.0	115.2
4. approximate expression of EC	14.0	15.5	20.5	26.2	39.2	73.7
5. $d/h = 0.8$	13.7	15.4	21.7	27.3	53.0	136.4
6. $d/h = 0.9$	13.6	15.0	19.4	22.3	30.6	96.1
7. $\varepsilon_{\text{sh}} = 0\%$	17.0	19.2	26.1	31.2	47.5	134.2
8. limit deflection is $l/125$	27.3	30.4	40.6	48.4	74.6	227.3
9. $p_{\text{Rd}}$ (kN/m <sup>2</sup> )	234.9	134.3	39.9	22.7	7.3	0.53

Concrete	Ultimate load, $p_{\text{Rd}}$ (kN/m <sup>2</sup> )										
	300	250	200	150	100	50	25	20	15	10	5
C40/50	13.4	13.9	14.5	15.4	16.8	20.1	25.0	27.0	30.1	35.4	47.4
C35/45	13.2	13.7	14.3	15.1	16.5	19.6	24.3	26.2	29.1	34.1	45.6
C30/37	13.0	13.5	14.1	14.9	16.2	19.1	23.4	25.3	28.0	32.8	43.7
C25/30		13.3	13.8	14.6	15.8	18.5	22.6	24.3	26.9	31.3	41.5
C20/25			13.7	14.4	15.5	18.0	21.7	23.2	25.6	29.6	39.0
C16/20				14.3	15.3	17.5	20.9	22.3	24.4	28.1	36.8

**Table 3:** The limit span/depth ratio,  $(l/d)_{\text{lim}}$ , as a function of the ultimate load (kN/m<sup>2</sup>). The limit deflection is  $l/250$ ,  $p_{\text{qp}}/p_{\text{Rd}} = 0.5$ ,  $d/h = 0.85$ ,  $\varepsilon_{\text{sh}} = 0.04\%$ . The calculation is based on the midsection (uniform  $\zeta$ ), cracked cross section with tension stiffening.

Concrete	Ultimate load, $p_{\text{Rd}}$ (kN/m <sup>2</sup> )										
	300	250	200	150	100	50	25	20	15	10	5
C40/50	13.2	13.6	14.1	14.8	15.7	17.2	18.6	19.1	19.6	20.3	21.2
C35/45	13.1	13.5	14.0	14.6	15.5	17.1	18.5	18.9	19.4	20.1	21.1
C30/37	12.9	13.3	13.8	14.4	15.3	16.8	18.3	18.7	19.2	19.9	21.0
C25/30	12.8	13.1	13.6	14.2	15.1	16.6	18.1	18.5	19.1	19.8	20.8
C20/25			13.5	14.1	15.0	16.5	17.9	18.3	18.9	19.6	20.7
C16/20				14.1	14.9	16.3	17.7	18.2	18.7	19.5	20.6

**Table 4:** The limit span/depth ratio,  $(l/d)_{\text{lim}}$ , as a function of the ultimate load (kN/m<sup>2</sup>). The limit deflection is  $l/250$ ,  $p_{\text{qp}}/p_{\text{Rd}} = 0.5$ ,  $d/h = 0.85$ ,  $\varepsilon_{\text{sh}} = 0.04\%$ . The calculation is based on the midsection, fully cracked cross section without tension stiffening ( $\zeta = 1$ ).

Concrete	Ultimate load, $p_{\text{Rd}}$ (kN/m <sup>2</sup> )										
	300	250	200	150	100	50	25	20	15	10	5
C40/50	16.2	17.0	18.2	19.9	22.5	27.9	34.7	37.2	40.8	46.5	58.1
C35/45		16.6	17.8	19.4	21.9	27.1	33.7	36.2	39.7	45.2	56.5
C30/37			17.2	18.8	21.2	26.3	32.6	35.0	38.4	43.7	54.6
C25/30					20.5	25.4	31.5	33.8	37.0	42.2	52.7
C20/25						24.6	30.5	32.7	35.8	40.7	50.9
C16/20							23.8	29.5	31.7	34.7	39.4

**Table 5:** The limit span/depth ratio,  $(l/d)_{\text{lim}}$ , as a function of the ultimate load (kN/m<sup>2</sup>). The limit deflection is  $l/250$ ,  $p_{\text{qp}}/p_{\text{Rd}} = 0.5$ ,  $d/h = 0.85$ ,  $\varepsilon_{\text{sh}} = 0.04\%$ . The calculation is based on an uncracked cross section ( $\zeta = 0$ ).

the cracked cross section dominates the deflection and  $d/h$  plays a minor role.

In the seventh row, the effect of shrinkage was investigated by assuming zero shrinkage strain. It can be seen that shrinkage has a very large effect on the  $l/d$  ratio. It is worthwhile to note that these values are close to those given in a previous version of EC (Eurocode, 1991).

In the eighth row, the limit deflection was assumed to be  $l/d = 125$ .

We also determined the design value of the moment resistance,  $M_{\text{Rd}}$ , from the reinforcement ratio. Then, using  $l/d$  from the first row, a uniformly distributed load which results in  $M_{\text{Rd}}$  at the midsection was calculated. This load is called the ultimate load,  $p_{\text{Rd}}$ , and is shown in the last row of Table 2.

This calculation can be carried out such that the starting point is the ultimate load,  $p_{\text{Rd}}$ . We calculate first the reinforcement ratio, then the limit  $l/d$  ratio. The results of the calculation



Concrete	Ultimate load, $p_{Rd}$ (kN/m <sup>2</sup> )										
	300	250	200	150	100	50	25	20	15	10	5
C40/50	13.6	14.1	14.8	15.7	17.3	21.0	26.3	28.5	31.7	37.1	49.3
C35/45	13.4	13.9	14.5	15.4	17.0	20.4	25.5	27.6	30.6	35.8	47.5
C30/37	13.2	13.6	14.2	15.1	16.6	19.8	24.6	26.6	29.5	34.4	45.5
C25/30		13.4	14.0	14.8	16.2	19.2	23.7	25.5	28.3	32.9	43.3
C20/25			13.8	14.6	15.8	18.6	22.7	24.4	26.9	31.2	40.9
C16/20				14.5	15.6	18.1	21.8	23.4	25.7	29.6	38.6

**Table 6:** The limit span/depth ratio,  $(l/d)_{limit}$ , as a function of the ultimate load (kN/m<sup>2</sup>). The limit deflection is  $l/250$ ,  $p_{qp}/p_{Rd}=0.5$ ,  $d/h=0.85$ ,  $\epsilon_{st}=0.04\%$ . Uniform reinforcement was assumed along the beam. Tension stiffening was taken into account:  $\zeta$  was calculated along the beam, and the deflection was calculated by numerical integration.

Concrete	Ultimate load, $p_{Rd}$ (kN/m <sup>2</sup> )										
	300	250	200	150	100	50	25	20	15	10	5
C40/50	14.0	14.4	15.1	15.9	17.4	20.5	25.8	28.0	31.1	36.3	47.2
C35/45	13.7	14.1	14.7	15.5	16.9	19.8	24.9	27.0	30.0	34.9	45.4
C30/37	13.3	13.7	14.3	15.1	16.3	19.1	23.9	25.9	28.8	33.5	43.5
C25/30		13.3	13.8	14.5	15.7	18.3	22.8	24.7	27.4	31.8	41.3
C20/25			13.3	14.0	15.1	17.4	21.5	23.2	25.8	29.9	38.7
C16/20				13.5	14.4	16.6	20.3	21.9	24.3	28.1	36.3

**Table 7:** The limit span/depth ratio,  $(l/d)_{limit}$ , as a function of the ultimate load (kN/m<sup>2</sup>), based on the approximate expression of EC (Eq.3).

tion that follows this procedure are presented in Table 3. These calculations were repeated assuming fully cracked and uncracked cross sections (Tables 4 and 5).

We also calculated the limit  $l/d$  ratio by numerical integration of the curvatures along the beam, calculating the  $\zeta$  values at frequent cross sections. The results are given in Table 6. The approximate expression of EC (Eq.3) was used for Table 7. Note that these values are close to those of Table 3.

In all the above calculations,  $p_{qp}/p_{Rd}=0.5$  was assumed. This value may vary significantly, depending on the structural application and on the possible overstrengthening of the beam. In Tables 11 through 15 we present results for  $p_{qp}/p_{Rd}=0.7, 0.6, 0.4, 0.3$  and  $0.2$ , respectively.

When a precamber of  $(l/250)$  is applied, the allowable deflection – compared to the curved initial shape – is  $l/125$ . Results for this case are given in Table 17.

The empty places in the tables show that the tensile reinforcement was elastic under the design load.

**Table 8:** The limit span/depth ratio,  $(l/d)_{limit}$ , as a function of the design load for rectangular cross section beams and slabs. For a beam,  $p_{Ed}$  is the uniformly distributed load along the axis in kN/m, while for a slab it is the load of a strip of unit width, 1 m. When  $f_{yk}=500$  N/mm<sup>2</sup>,  $p_{qp}/p_{Ed}=0.5$ , and  $p_{Ed}=p_{Rd}$ , the values of  $b$  (and  $a$  in Eq.4) are set equal to 1.

Concrete	$\frac{\beta \times p_{Ed}}{b}$ (kN/m <sup>2</sup> ) (For a beam $b$ is the width in m, for a slab $b=1$ .)									
	300	250	200	150	100	50	25	20	15	10
C40/50	13	14	14	15	17	20	25	27	30	35
C35/45	13	14	14	15	16	19	24	26	29	34
C30/37	13	13	14	15	16	19	23	25	28	33
C25/30		13	14	14	16	18	22	24	27	31
C20/25			14	14	15	18	21	23	25	29
C16/20				14	15	17	21	22	24	28
	← „beam” →					← „slab” →				

<sup>1</sup> The values are rounded numbers from Table 3. This table was calculated assuming that  $d/h=0.85$  and the calculation is based on the midsection (uniform  $\zeta$ ). The question arises: are the values in Table 8 on the safe side, when  $d/h>0.85$ ? To answer this question we calculated Table 16, where  $d/h=0.9$  and  $\zeta$  is calculated along the beam assuming a uniform reinforcement. Comparing the values in Table 16 to those of Table 8, we can see that the values in Table 8 are on the safe side, even for  $d/h=0.9$ .

<sup>2</sup> Hence, we obtained numerically that when the ultimate load,  $p_{Rd}$ , is given,  $(l/d)_{limit}$  is inversely proportional to  $\sqrt{p_{qd}}$ . If shrinkage is neglected, and uncracked cross section is assumed  $(l/d)_{limit}$  is (approximately) inversely proportional to  $p_{qp}$ , while assuming fully cracked cross section it is inversely proportional to  $\sqrt[3]{p_{qd}}$ . Our numerical result is between these two expressions.

$p_{qp}/p_{Rd}$	0.7	0.6	0.5	0.4	0.3	0.2
$\alpha$	0.83	0.91	1.00	1.12	1.30	1.58

**Table 9:** The multiplier of  $(l/d)_{limit}$  as a function of  $p_{qp}/p_{Rd}$

## 4. DEFLECTION CONTROL IN PRACTICE

Based on the numerical calculations presented in the previous section, we suggest the following deflection control for RC beams and slabs.

The deflection of *simply supported rectangular cross section beams or slabs* should not exceed the limit of  $l/250$ , when

$$\frac{l}{d} \leq \alpha (l/d)_{limit} \quad (4)$$

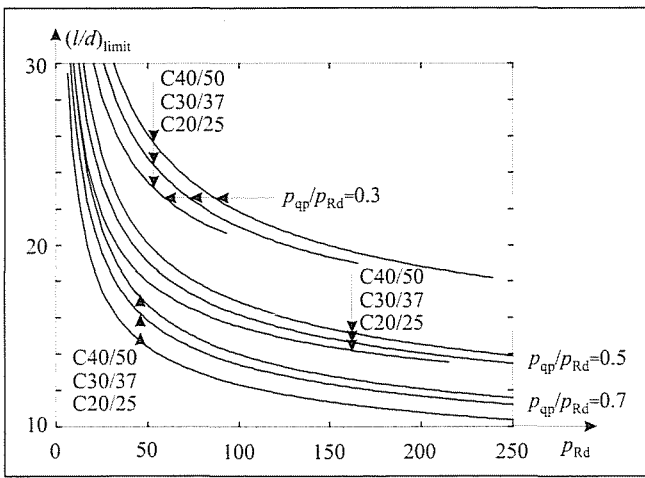
where  $l$  is the span,  $d$  is the effective depth, and the values of  $(l/d)_{limit}$  are given in Table 8.<sup>1</sup> This table and Eq.(4) can be used with  $\alpha=\beta=1$ , when  $f_{yk}=500$  N/mm<sup>2</sup>,  $p_{qp}/p_{Ed} \approx 0.5$ , and  $p_{Ed} \approx p_{Rd}$ .

The design load (over  $b$ ) of a slab is typically between 10 and 20 kN/m<sup>2</sup> and that of a beam is between 150 and 250 kN/m<sup>2</sup>. (For two-way slabs the load carried by the shorter span must be considered, which can be approximated as  $p_{Ed} l_x^4 / (l_x^4 + l_y^4)$  when  $l_x < l_y$ .)

*The effect of load ratio, overstrengthening and steel strength*

As stated before, the deflections are calculated from the quasi permanent load, which is approximately 50% of the design load, and hence of the ultimate load. When the quasi permanent load is significantly smaller or larger than 50% of the ultimate load, Tables 11 through 15 can be used. For three different ratios of  $p_{qp}/p_{Rd}$ , the results are also illustrated in Fig. 1. The values in these tables can be approximated well by multiplying the values of Table 8 by the correction factors given in Table 9.

These values can be further approximated by:  $\alpha = \sqrt{0.5 p_{Rd} / p_{qp}}$ .<sup>2</sup> Taking into account that  $p_{Rd}$  may be higher



**Fig. 1:** The limit span/depth ratio,  $(l/d)_{limit}$ , as a function of the ultimate load ( $\text{kN/m}^2$ ). [See Tables 8, 11 and 14.  $d/h = 0.85$ .  $\epsilon_{sh} = 0.04\%$ .  $f_{yk} = 500$ ]

than  $p_{Ed}$  ( $p_{Rd} > p_{Ed}$ ) and that  $f_{yk}$  can differ from  $500 \text{ N/mm}^2$ ,  $\alpha$  is written as

$$\alpha = \sqrt{\frac{1}{2} \frac{\beta p_{Ed}}{p_{qp}}}, \quad (5)$$

where

$$\beta = \frac{M_{Rd}}{M_{Ed}} \frac{500}{f_{yk}} \text{ or approximately}$$

$$\beta = \frac{A_{s,prov}}{A_{s,requ}} \frac{500}{f_{yk}} \quad (6)$$

$M_{Rd}$  is the design value of the moment resistance and  $M_{Ed}$  is the moment obtained from the design load. In the calculation of  $\beta$ , the first fraction takes into account the ratio of the ultimate load to the design load, while the second fraction provides that lower yield strength must result in a higher reinforcement ratio. In the second expression,  $A_{s,requ}$  is the required, while  $A_{s,prov}$  is the provided, cross sectional area of the tensile steel reinforcement.

If a precamber of  $l/250$  is applied, the span to depth ratio,  $(l/d)_{limit}$ , may be increased as listed in Table 17.

In wide flange T cross section beams, the compressive zone is usually in the flange. The rib plays a minor role in tension stiffening, and it is conservative to calculate the deflection on the basis of the fully cracked section neglecting tension stiffening. The result of this calculation is shown in Table 4 and the rounded values are presented in Table 10. These values are quite accurate for wide flange beams and less accurate for narrow flange beams, but the calculation is always conservative, provided that the compressive zone is in the flange.

**Table 10:** The limit span/depth ratio,  $(l/d)_{limit}$ , as a function of the design load ( $\text{kN/m}^2$ ) for T section beams. When  $f_{yk} = 500 \text{ N/mm}^2$ ,  $p_{qp}/p_{Ed} = 0.5$  and  $p_{Ed} = p_{Rd}$ , the values of  $\beta$  (and  $\alpha$  in Eq.4) are set equal to 1. The compressive zone is in the flange.

Concrete	$\frac{\beta \times p_{Ed}}{b}$ ( $\text{kN/m}^2$ ) ( $p_{Ed}$ is the design load in $\text{kN/m}$ , $b_{eff}$ is the effective width of the flange)									
	300	250	200	150	100	50	25	20	15	10
C40/50	13	13	14	15	16	17	19	19	19	20
C35/45	13	13	14	14	15	17	18	19	19	20
C30/37	13	13	14	14	15	17	18	19	19	20
C25/30	13	13	13	14	15	16	18	18	19	20
C20/25			13	14	15	16	18	18	19	19
C16/20				14	15	16	18	18	19	19

Ribbed slabs can be modelled as T section beams, with an effective width,  $b_{eff}$ , of the flange.

When the beam is not simply supported,  $l$  must be replaced by  $l/K$ , where  $K$  is given in Table 7.4 of Eurocode 2.

## 5. EFFECT OF COMPRESSION REINFORCEMENT

Compression reinforcement reduces the deflection of RC beams. Numerical calculations were carried out for rectangular cross section beams with compression reinforcement ratio,  $\rho'$  which does not exceed the tensile reinforcement ratio,  $\rho$ . The location of the compression reinforcement is  $0.15h$  from the top of the cross section. Similarly to the assumptions listed in Table 3: the limit deflection is  $l/250$ ,  $p_{qp}/p_{Rd} = 0.5$ ,  $d/h = 0.85$ ,  $\epsilon_{sh} = 0.04\%$ . The calculation is based on the midsection (uniform  $\zeta$ ), the tension stiffening was taken into account. The calculations showed that the limit span/depth ratio may be increased by up to 3.5. We calculated  $(l/d)_{limit}$  for  $\rho'/\rho = 0, 0.1, 0.2, \dots, 1.0$ ; and then an approximate expression was determined using a least square technique:

$$(l/d)_{limit} = 11 + \frac{4.1}{\sqrt{f_{ck}}} \left( \frac{p_0}{p^*} \right)^{0.6} + 0.2 \sqrt{f_{ck}} \left( \frac{p_0}{p^*} - 1 \right) + \frac{\rho'}{\rho} 18 \left( \frac{p_0}{p^*} + 5 \right)^{-0.9} \quad (7)$$

where

$$p_0 = 110 \sqrt{f_{ck}} \text{ and } p^* = \frac{\beta \times p_{Ed}}{b}$$

$\rho'$  is in  $\text{kN/m}^2$ , and  $f_{ck}$  is in  $\text{N/mm}^2$ . Expression (7) may be used if  $10 \frac{\text{kN}}{\text{m}^2} \leq p^* \leq 350 \frac{\text{kN}}{\text{m}^2}$  and  $\rho' \leq \rho$ . If  $\rho' = 0$ , Eq.(7) results very accurately the limit span/depth ratios given in Table 3. (For a beam,  $p_{Ed}$  is the uniformly distributed load along the axis in  $\text{kN/m}$ , while for a slab it is the load of a strip of unit width,  $1 \text{ m}$ . For a beam  $b$  is the width in  $\text{m}$ , for a slab  $b=1$ .)

## 6. NUMERICAL EXAMPLE

A) We consider a simply supported one-way slab, with effective span  $l=4.2 \text{ m}$  and thickness  $h=200 \text{ mm}$ . The con-

crete strength class is C20/25, the steel is B 500, the diameter of the rebars is 12 mm, and the cover is 20 mm. The dead load is  $g_k=10$  kN/m<sup>2</sup> and the live load is  $q_k=5$  kN/m<sup>2</sup>. Perform the deflection control of the beam, using the span to depth ratio.

The limit deflection is  $e_{\text{limit}} = l/250 = 16.8$  mm.

The design load of the slab is (Kollár, 1997)  $p_{\text{Ed}} = 1.35 g_k + 1.5 q_k = 1.35 \times 10 + 1.5 \times 5 = 21$  kN/m<sup>2</sup>, while the quasi permanent load is  $p_{\text{qp}} = g_k + \Psi_2 q_k = 10 + 0.3 \times 5 = 11.5$  kN/m<sup>2</sup>. Hence  $p_{\text{qp}}/p_{\text{Rd}} = p_{\text{qp}}/p_{\text{Ed}} = 0.54$ ,  $\beta=1$ ; and from

Eq.(5)  $\alpha = \sqrt{0.5\beta p_{\text{Ed}}/p_{\text{qd}}} = \sqrt{0.5/0.54} = 0.965$ . From Table 8, with  $\beta p_{\text{Ed}} = 21$ , we have:  $(l/d)_{\text{limit}} = 22.4$ ; and  $\alpha (l/d)_{\text{limit}} = 0.965 \times 22.4 = 21.6$ . The effective depth is  $d = 174$  mm, and hence  $l/d = 24.1 > 21.6$ . Consequently, the deflection of the slab exceeds the limit of  $l/250$ .

The reinforcement of the slab can be calculated from the midsection bending moment ( $M_{\text{Ed}} = p_{\text{Ed}} l^2/8 = 46.3$  kNm/m), giving:  $\phi 12/170$ . Using this reinforcement and the load  $p_{\text{qp}} = 11.5$  kN/m<sup>2</sup>, we calculated the deflection of the midsection. When  $\zeta$  was determined along the beam, and the deflection was calculated by numerical integration we obtained 19.7 mm, while using a uniform  $\zeta$  calculated at the midsection, the result is 21.7 mm. (Both are higher than the limit deflection.)

B) Apply higher reinforcement ratio in the slab to avoid the deflection problem.

According to Eurocode 2, we must increase the amount of the reinforcement by  $24.1/21.6 = 1.12$  to reduce the displacement such that it does not exceed the limit. This is wrong. We must apply about 1.7 times the original reinforcement to avoid the deflection problem. We verify this with the following calculation: the applied reinforcement is  $\phi 12/100$ , and hence the design value of moment resistance is  $M_{\text{Rd}} = 76.5$  kNm.  $\beta = 76.5/46.3 = 1.65$ , from

(5) we have  $\alpha = \sqrt{0.5\beta p_{\text{Ed}}/p_{\text{qd}}} = \sqrt{0.5 \times 1.65/0.54} = 1.23$ ;  $\beta p_{\text{Ed}} = 34.7$ , and hence from Table 8:  $(l/d)_{\text{limit}} = 19.8$ ; and  $\alpha (l/d)_{\text{limit}} = 1.23 \times 19.8 = 24.4 > 24.1$ . This slab now satisfies the deflection criterion.

An "accurate" analysis leads to the same conclusion: When  $\zeta$  was determined along the beam, and the deflection was calculated by numerical integration, we obtained 15.1 mm, while using a uniform  $\zeta$  calculated at the midsection, the result is 16.6 mm. (Both are smaller than the limit deflection of 16.8 mm.)

C) Can the original slab given in section A) be applied with a precamber?

The maximum precamber is  $l/250 = 16.8$  mm. In this case, the total limit deflection is  $l/125$ . According to Table 17  $(l/d)_{\text{limit}} = 32.2$ , and  $\alpha (l/d)_{\text{limit}} = 0.965 \times 32.2 = 31.0$ . This is larger than  $l/d = 24.1$  and hence the deflection is within the given limit.

## 7. CONCLUSIONS

In this paper, we presented a new method for the deflection control of RC beams and slabs. The span to depth ratio is given as a function of the design load instead of the reinforcement ratio. On the basis of Eurocode 2, Table 8 and Eq.(7) were determined for the limit span/depth ratio of rectangular beams and slabs, while Table 10 was determined for T beams and ribbed slabs. The method provides a more accurate means of accounting for the ratio of quasi permanent/ultimate loads. (The recommendation of Eurocode is not conservative.)

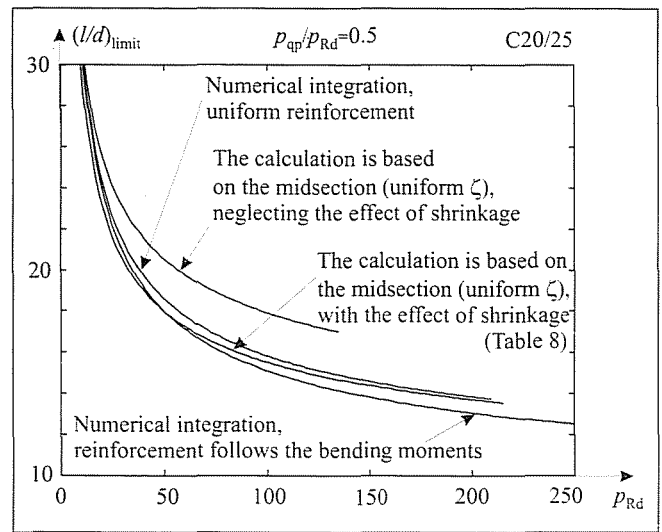


Fig. 2: The limit span/depth ratio,  $(l/d)_{\text{limit}}$ , as a function of the ultimate load (kN/m<sup>2</sup>) calculated according to the EC ( $d/h = 0.85$ ,  $\epsilon_{\text{sh}} = 0.04\%$ ,  $f_{\text{yk}} = 500$ , C20/25)

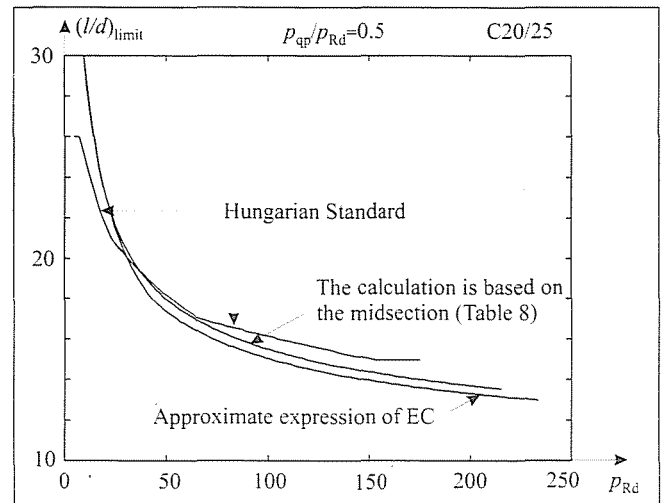


Fig. 3: The limit span/depth ratio,  $(l/d)_{\text{limit}}$ , as a function of the ultimate load (kN/m<sup>2</sup>) calculated according to the EC (Table 8 and Eq.3) and to the Hungarian Standard. (C20/25,  $p_{\text{qp}}/p_{\text{Rd}} = 0.5$ )

In Fig. 2, we compared the calculations based on the midsection (taking into account shrinkage and uniform  $\zeta$ ) with the more accurate numerical integration ( $\zeta$  is calculated along the beam) and with the calculation neglecting the effect of shrinkage. It can be seen that the effect of shrinkage is significant and that the numerical integration hardly modifies the results.

In Fig. 3, our results are compared to those obtained from the approximate expression of Eurocode, and good agreement was found. It was also compared to the results obtained from the Hungarian Standard, which is more conservative in case of slabs, and less conservative in case of beams.

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## 8. NOTATIONS

$A_s$	cross sectional area of tensile reinforcement
$b$	width of a rectangular cross section
$d$	effective depth, distance between the centre of gravity of the tensile reinforcement and the compressed outermost point of the cross section
$f_{\text{cd}} = f_{\text{ck}}/1.5$	design value of concrete cylinder compressive strength

$f_{ck}$  characteristic compressive strength of concrete  
 $f_{yd} = f_{yk} / 1.15$  design yield strength of reinforcement  
 $f_{yk}$  characteristic yield strength of reinforcement  
 $h$  height of the cross section  
 $l$  effective span  
 $M_{Rd}$  the design value of moment resistance at midspan  
 $M_{Ed}$  moment obtained from the design load at midspan  
 $p_{Ed}$  design value of the applied loads  
 $p_{Rd}$  ultimate load; which result in  $M_{Rd}$  at the midsection of a simply supported beam  
 $p_{qp}$  quasi permanent load  
 $\rho = A_s / bd$  tensile reinforcement ratio  
 $\rho' = A'_s / bd$  compression reinforcement ratio  
 $\varphi_{ef}$  the effective creep coefficient  
 $\phi$  diameter of rebars

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## 10. APPENDIX. NUMERICAL RESULTS

Concrete	Ultimate load, $p_{Rd}$ (kN/m <sup>2</sup> )										
	300	250	200	150	100	50	25	20	15	10	5
C40/50	11.2	11.6	12.1	12.8	14.0	16.7	20.9	22.6	25.2	29.7	39.8
C35/45	11.0	11.4	11.9	12.6	13.7	16.3	20.3	21.9	24.4	28.6	38.2
C30/37	10.8	11.2	11.7	12.3	13.4	15.9	19.6	21.1	23.5	27.5	36.6
C25/30	10.7	11.0	11.5	12.1	13.1	15.4	18.8	20.3	22.5	26.2	34.8
C20/25	10.6	10.9	11.3	11.9	12.8	14.9	18.0	19.4	21.3	24.8	32.6
C16/20		10.8	11.2	11.8	12.6	14.5	17.3	18.5	20.3	23.5	30.9

**Table 11:** The limit span/depth ratio,  $(l/d)_{limit}$ , as a function of the ultimate load [kN/m<sup>2</sup>]. The limit deflection is  $l/250$ ,  $p_{qp}/p_{Rd}=0.7$ ,  $d/h=0.85$ ,  $\epsilon_{st}=0.04\%$ . The calculation is based on the midsection (uniform  $\zeta$ ), cracked section with tension stiffening.

Concrete	Ultimate load, $p_{Rd}$ (kN/m <sup>2</sup> )										
	300	250	200	150	100	50	25	20	15	10	5
C40/50	12.2	12.6	13.2	14.0	15.2	18.2	22.7	24.6	27.4	32.2	43.2
C35/45	12.0	12.4	13.0	13.7	15.0	17.8	22.0	23.8	26.5	31.0	41.6
C30/37	11.8	12.2	12.7	13.5	14.6	17.3	21.3	23.0	25.5	29.8	39.8
C25/30	11.6	12.0	12.5	13.2	14.3	16.8	20.5	22.0	24.4	28.4	37.8
C20/25		11.9	12.4	13.0	14.0	16.3	19.6	21.1	23.2	26.9	35.4
C16/20			12.3	12.9	13.8	15.8	18.9	20.2	22.1	25.5	33.5

**Table 12:** The limit span/depth ratio,  $(l/d)_{limit}$ , as a function of the ultimate load [kN/m<sup>2</sup>]. The limit deflection is  $l/250$ ,  $p_{qp}/p_{Rd}=0.6$ ,  $d/h=0.85$ ,  $\epsilon_{st}=0.04\%$ . The calculation is based on the midsection (uniform  $\zeta$ ), cracked section with tension stiffening.

Concrete	Ultimate load, $p_{Rd}$ (kN/m <sup>2</sup> )										
	300	250	200	150	100	50	25	20	15	10	5
C40/50	15.1	15.6	16.3	17.3	18.9	22.6	28.1	30.4	33.8	39.6	53.1
C35/45	14.9	15.4	16.1	17.0	18.6	22.0	27.2	29.4	32.7	38.2	51.1
C30/37		15.2	15.8	16.7	18.2	21.5	26.3	28.4	31.5	36.7	48.9
C25/30			15.6	16.4	17.8	20.8	25.4	27.3	30.2	35.1	46.5
C20/25					17.5	20.3	24.4	26.1	28.7	33.2	43.7
C16/20					17.3	19.8	23.5	25.1	27.4	31.6	41.2

**Table 13:** The limit span/depth ratio,  $(l/d)_{limit}$ , as a function of the ultimate load [kN/m<sup>2</sup>]. The limit deflection is  $l/250$ ,  $p_{qp}/p_{Rd}=0.4$ ,  $d/h=0.85$ ,  $\epsilon_{st}=0.04\%$ . The calculation is based on the midsection (uniform  $\zeta$ ), cracked section with tension stiffening.

Concrete	Ultimate load, $p_{Rd}$ (kN/m <sup>2</sup> )										
	300	250	200	150	100	50	25	20	15	10	5
C40/50			18.8	20.0	21.8	26.1	32.4	35.1	39.0	45.8	61.3
C35/45			18.6	19.7	21.4	25.5	31.5	34.0	37.8	44.2	59.0
C30/37				19.3	21.0	24.8	30.5	32.8	36.4	42.5	56.5
C25/30					20.6	24.1	29.4	31.6	34.9	40.6	53.7
C20/25						23.5	28.2	30.2	33.3	38.5	50.6
C16/20						23.0	27.2	29.1	31.8	36.6	47.7

**Table 14:** The limit span/depth ratio,  $(l/d)_{limit}$ , as a function of the ultimate load [kN/m<sup>2</sup>]. The limit deflection is  $l/250$ ,  $p_{qp}/p_{Rd}=0.3$ ,  $d/h=0.85$ ,  $\epsilon_{st}=0.04\%$ . The calculation is based on the midsection (uniform  $\zeta$ ), cracked section with tension stiffening.

Concrete	Ultimate load, $p_{Rd}$ (kN/m <sup>2</sup> )										
	300	250	200	150	100	50	25	20	15	10	5
C40/50					26.4	31.7	39.5	42.8	47.6	55.9	75.0
C35/45					26.0	30.9	38.4	41.5	46.1	54.0	72.2
C30/37						30.1	37.1	40.0	44.4	51.9	69.1
C25/30							29.3	35.8	38.5	42.6	49.5
C20/25								34.4	36.9	40.6	47.0
C16/20								33.2	35.5	38.8	44.7

**Table 15:** The limit span/depth ratio,  $(l/d)_{limit}$ , as a function of the ultimate load [kN/m<sup>2</sup>]. The limit deflection is  $l/250$ ,  $p_{qp}/p_{Rd}=0.2$ ,  $d/h=0.85$ ,  $\epsilon_{st}=0.04\%$ . The calculation is based on the midsection (uniform  $\zeta$ ), cracked section with tension stiffening.

Concrete	Ultimate load, $p_{Rd}$ (kN/m <sup>2</sup> )										
	300	250	200	150	100	50	25	20	15	10	5
C40/50	13.5	14.0	14.7	15.6	17.1	20.5	25.4	27.5	30.5	35.6	47.1
C35/45	13.3	13.8	14.4	15.3	16.8	20.0	24.7	26.7	29.5	34.4	45.4
C30/37	13.1	13.6	14.2	15.0	16.4	19.4	23.9	25.7	28.5	33.1	43.6
C25/30		13.4	13.9	14.7	16.0	18.9	23.0	24.8	27.3	31.6	41.5
C20/25			13.8	14.5	15.7	18.3	22.1	23.7	26.0	30.0	39.2
C16/20				14.4	15.4	17.8	21.3	22.8	24.9	28.6	37.1

**Table 16:** The limit span/depth ratio,  $(l/d)_{limit}$ , as a function of the ultimate load [kN/m<sup>2</sup>]. The limit deflection is  $l/250$ ,  $p_{qp}/p_{Rd}=0.5$ ,  $d/h=0.9$ ,  $\epsilon_{st}=0.04\%$ . Uniform reinforcement was assumed along the beam. The tension stiffening was taken into account:  $\zeta$  was calculated along the beam, and the deflection was calculated by numerical integration.

Concrete	Ultimate load, $p_{Rd}$ (kN/m <sup>2</sup> )										
	300	250	200	150	100	50	25	20	15	10	5
C40/50					25.4	29.1	33.7	35.5	38.2	42.9	54.1
C35/45					25.1	28.6	33.0	34.8	37.3	41.8	52.4
C30/37						28.1	32.3	34.0	36.4	40.6	50.6
C25/30							27.7	31.6	33.2	35.4	39.3
C20/25								27.4	31.0	32.4	34.5
C16/20									30.6	31.9	33.7

**Table 17:** The limit span/depth ratio,  $(l/d)_{limit}$ , as a function of the ultimate load [kN/m<sup>2</sup>]. The limit deflection is  $l/125$ ,  $p_{qp}/p_{Rd}=0.5$ ,  $d/h=0.85$ ,  $\epsilon_{st}=0.04\%$ . The calculation is based on the midsection (uniform  $\zeta$ ), cracked section with tension stiffening.

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# SIMPLIFIED ANALYSIS OF REINFORCED CONCRETE COLUMNS ACCORDING TO EUROCODE 2



Prof. László P. Kollár

The paper presents a simple approximate method for the analysis of reinforced concrete columns subjected to axial loads according to the new version of Eurocode 2. Tables are determined for the calculation of the eccentricities of the axial load and of the ultimate load of centrally loaded columns with rectangular cross sections.

**Keywords:** reinforced concrete column, axial compression, centrally loaded column, Eurocode 2, approximate analysis

## 1. INTRODUCTION

Reinforced concrete columns subjected to axial compression must be designed while taking into account the imperfect shape, the second order deformations and the cracked cross sections. In the design calculation either a non-linear numerical analysis (Polgár, 2002) or an approximate analysis can be carried out. The latter, according to Eurocode, may be based on "estimation of curvatures" (Eurocode 2, 2003; Litzner, 1994), which can be applied in the analysis as the calculation of the eccentricities. Even in the approximate analysis only verification is possible, as the cross section and the reinforcement of the column are needed for the calculation of the curvatures and eccentricities.

Our task in this paper is to develop a simple method of designing centrally loaded columns which can be used in preliminary design stage. Hence, our task is to develop a formula in the form of:

$$N_{Rd} = \varphi N_u, \quad (1)$$

where  $N_{Rd}$  is the ultimate load of a centrally loaded column,  $N_u$  is the ultimate load of a centrally loaded cross section, while  $\varphi$  is a reduction factor.

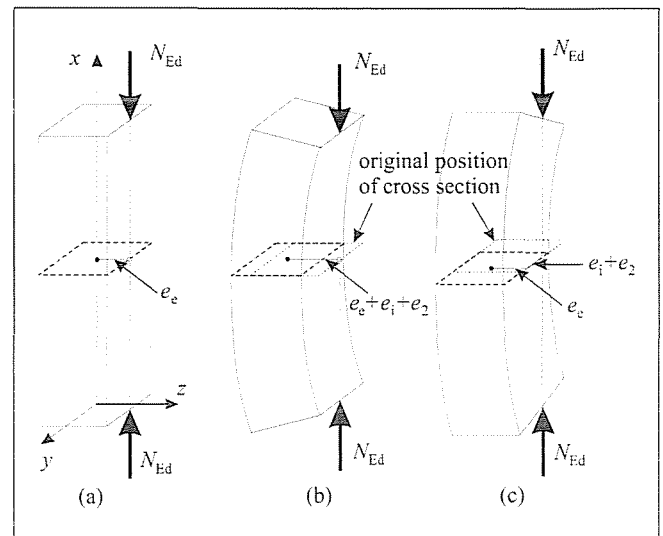
First we will very briefly summarize the analysis according to Eurocode 2. After that an approximate method will be presented for the calculation of eccentricities and the reduction factor,  $\varphi$ .

## 2. ANALYSIS OF RC COLUMNS ACCORDING TO EUROCODE 2

In the analysis, in addition to the original eccentricity of the normal force ( $e_0$ ) the eccentricities due to the imperfections ( $e_1$ ) and the second order eccentricities ( $e_2$ ) must be taken into account (Fig. 1).

### 2.1 Calculation of the eccentricities

When the original bending moment along the column is uniform, the cross section of the column must be designed for the eccentricity:



**Fig. 1:** RC column subjected to a normal force with initial eccentricity in the z direction. Straight column (a), column bent in the x-z plane (b) and column bent in the x-y plane (c). (• denotes the midpoint of the cross section, while ◦ is the intersection point of the midsection and the line of the axial load)

$$e_{tot} = \max \begin{cases} e_0 + e_1 + e_2, & \text{sum of eccentricities} \\ e_0, & \text{minimum value of eccentricities} \end{cases} \quad (2)$$

where  $e_0 = M_{0e}/N_{Ed}$  is the first order eccentricity (for uniform bending moment),  $e_1$  is due to the imperfections and  $e_2$  is the second order imperfection. (When the first order bending moment is not uniform along the column, the calculation of  $M_{0e}$  is given in Eurocode 2.) The expression for  $e_1$ , when  $l \leq 4m$  is:

$$e_1 = \frac{l_0}{400} \quad (3)$$

where  $l_0$  is the effective length and  $l$  is the length of the column in m. (For  $l > 9m$ ,  $e_1$  is the 2/3-rd of  $l_0/400$ , and for  $9m > l > 4m$  linear interpolation can be used.)

The second order eccentricity is calculated as:

$$e_2 = \frac{1}{r} \frac{l_0^2}{\pi^2} \approx \frac{1}{r} \frac{l_0^2}{10}, \quad (4)$$

where

$$\frac{1}{r} = K_r K_\sigma \frac{1}{r_0} \quad \text{the curvature,} \quad (5)$$

$$\frac{1}{r_0} = \frac{f_{yd}/E_s}{0.45 d'} \text{ the basic value of the curvature,} \quad (6)$$

$$K_e = \max\{1 + \beta\varphi_{ef}; 1\} \text{ effect of creep,} \quad (7)$$

$$\beta = 0.35 + \frac{f_{ck}}{200} - \frac{\lambda}{150}; \quad (8)$$

where  $f_{yd}$  is the design yield strength and  $E_s$  is the elastic modulus of steel,  $\varphi_{ef}$  is the effective creep coefficient,  $\lambda$  is the slenderness ratio of the homogeneous cross section (for a rectangular cross section, see Figs. 2.a and 3, when the bending is about the  $y$ -axis  $\lambda = \frac{l_0}{h}\sqrt{12}$ ), and  $f_{ck}$  is the characteristic compression strength of concrete in  $N/mm^2$ .

$$K_r = \min\left\{\frac{N'_u - N_{Ed}}{N'_u - N_{bal}}; 1\right\} \text{ effect of normal force,} \quad (9)$$

where

$$N'_u = f_{cd}bh + A_s f_{yd} \quad (10)$$

is the plastic ultimate load of the cross section (note that it is not identical to  $N_u$ , see Eq.(14)),  $A_s$  is the cross sectional area of the reinforcement:  $A_s = \sum A_{si}$ ,  $N_{bal}$  is shown in Fig. 2, and is given by Eq.(17)).

$$d' = h/2 + i_s, \quad (11)$$

where  $h$  is the height of the cross section,  $i_s$  is the radius of gyration of steel about the center of gravity of the reinforcement. Eq. (11) can be written (when the eccentricity is in the  $z$  direction, Fig. 3) as:

$$d' = \begin{cases} d_1 & \text{(reinforcement in two layers)} \\ \frac{h}{2} + z_s + \frac{A_{s1}}{2A_s} & \text{(reinforcement in three layers)} \\ \frac{h}{2} + \frac{1}{A_s} \sum \left(d_i - \frac{h}{2}\right)^2 A_{si} & \text{(general case)} \end{cases} \quad (12)$$

The minimum value of the eccentricity is:

$$e_0 = \begin{cases} 20 \text{ mm,} & \text{if } h \leq 600 \text{ mm,} \\ h/30, & \text{if } h > 600 \text{ mm.} \end{cases} \quad (13)$$

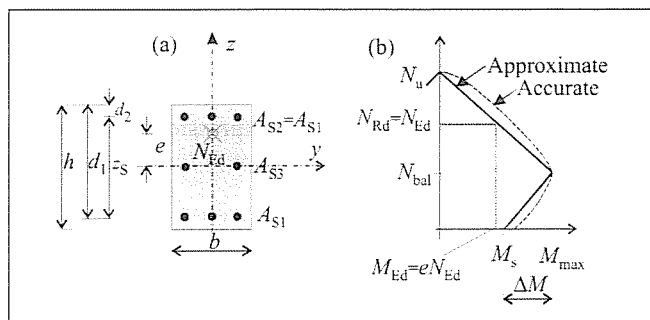


Fig. 2: Symmetrical cross section (a) and the failure envelope (b). Bending is about the  $y$ -axis.

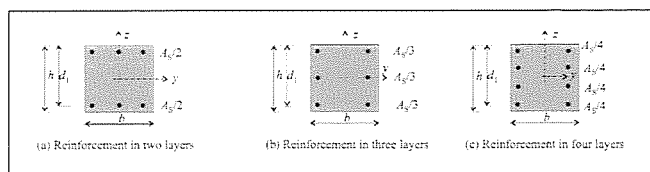


Fig. 3: RC cross section with three different steel arrangements. Bending is about the  $y$ -axis

## 2.2 Analysis of axially loaded RC columns

As we stated, the cross sections of axially loaded RC columns are designed for the eccentricities  $e_c$  and  $e_1+e_2$ , and the accidental eccentricities ( $e_1, e_2$ ) must be taken into account in both directions (Fig. 1). Design of cross sections can be performed with the aid of  $M(N)$  failure envelopes. The approximate calculation of three points of the envelope, for symmetrical cross sections is summarized below (Fig. 2, Dulácska, 2002). The reinforcement is either in two or in three layers (in the first case  $A_{s3} = 0$ ).

$$N_u = f_{cd}bh + A_s |\sigma_s|, \quad (14)$$

where

$$|\sigma_s| = \min\{f_{yd}; 400\} \text{ [N/mm}^2\text{]} \text{ and } A_s = \sum A_{si} \quad (15)$$

$$M_s \equiv A_{s1} f_{yd} z_s, \quad (16)$$

$$N_{bal} \equiv f_{cd} b x_{co}, \quad (17)$$

where  $x_{co} = \zeta_{co} d$ , and  $\zeta_{co} = 0.49$  if the characteristic yield strength of the steel is  $f_{yk} = 500 \text{ N/mm}^2$  (Kollár, 1997).

$$\Delta M \equiv N_{bal} \left( \frac{h}{2} - \frac{x_{co}}{2} \right), \quad M_{max} \equiv M_s + \Delta M. \quad (18)$$

## 3. NUMERICAL CALCULATIONS

Analyses of concentrically or eccentrically loaded RC columns require the values of  $e_2$  and  $e_1$  to be known. The appropriate expressions are given in Section 2.1. First we will calculate numerically these eccentricities (Section 3.1), then the  $\phi$  reduction factor (Section 3.2) defined in Eq. (1). The aim of these calculations is to develop approximate expressions which can be used in preliminary design. In these calculations only  $f_{ck} \leq 50$  is taken into account.

### 3.1 Eccentricities

We wish to develop approximate, conservative expressions for the calculation of eccentricities which would be independent of the arrangement of the reinforcement as well as the concrete strength class. We therefore introduce a new parameter,  $e_2'$  as:

$$e_2' = e_2 / K_r. \quad (19)$$

$e_2'$  is identical to  $e_2$  when  $N_{Ed} \leq N_{bal}$  (see Eq. (9)). In the following sections we calculate  $e_2'$  as a function of the column length and the reinforcement arrangement.

We considered three arrangements of the reinforcement as shown in Fig. 3. In every case the eccentricities were assumed to be in the  $z$  direction. Three different ratios of  $d_1/h$  were investigated:  $d_1/h = 0.9, 0.85$  and  $0.8$ , to model different concrete covers. The creep depends on the concrete strength class, as well as on the slenderness of the column. In the calculation the following creep coefficients were taken into account (Visnovitz, 2003): for concrete strength classes C40/50, 35/45, 30/37, 25/30, 20/25 and 16/20;  $\varphi_{ef} = 1.76, 1.92, 2.13, 2.35, 2.55$  and  $2.76$ ; respectively.

Cross section	$d_1/h$	$d'/d_1$	$l_0/d_1$						
			0.0	8.0	16.0	24.0	32.0	40.0	48.0
Figure 3a	0.90	1	0.000	0.053	0.171	0.316	0.487	0.760	1.095
	0.85	1	0.000	0.053	0.175	0.328	0.487	0.760	1.095
	0.80	1	0.000	0.054	0.178	0.339	0.493	0.760	1.095
Figure 3b	0.90	0.92	0.000	0.057	0.187	0.344	0.530	0.828	1.192
	0.85	0.92	0.000	0.058	0.189	0.355	0.526	0.823	1.184
	0.80	0.93	0.000	0.058	0.192	0.364	0.530	0.817	1.176
Figure 3c	0.90	0.89	0.000	0.059	0.193	0.356	0.549	0.857	1.235
	0.85	0.90	0.000	0.060	0.195	0.366	0.544	0.849	1.223
	0.80	0.90	0.000	0.060	0.197	0.375	0.545	0.841	1.211

**Table 1:** The relative eccentricity  $e_2'/d_1$  as a function of  $l_0/d_1$ .  $d'$  is given by Eq. (12).

Concrete	$l_0/d_1$											
	0	8	12	16	20	24	28	32	36	40	44	48
C50/60	0.88	0.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
C45/55	0.89	0.96	0.99	0.99	0.98	0.98	0.97	1.00	1.00	1.00	1.00	1.00
C40/50	0.90	0.96	0.99	0.98	0.97	0.95	0.93	1.00	1.00	1.00	1.00	1.00
C35/45	0.92	0.97	1.00	0.98	0.95	0.92	0.93	1.00	1.00	1.00	1.00	1.00
C30/37	0.94	0.99	1.00	0.97	0.93	0.89	0.93	1.00	1.00	1.00	1.00	1.00
C25/30	0.97	1.00	1.00	0.96	0.91	0.84	0.93	1.00	1.00	1.00	1.00	1.00
C20/25	0.98	1.00	0.99	0.94	0.87	0.84	0.93	1.00	1.00	1.00	1.00	1.00
C16/20	1.00	1.00	0.98	0.92	0.84	0.84	0.93	1.00	1.00	1.00	1.00	1.00

**Table 2:** The effect of concrete strength class on the relative eccentricity  $e_2'/d_1$ . The numbers in this table are correction factors, which can be used as multipliers of  $e_2'/d_1$  in Table 1 (or multipliers of  $e_2/d_1$  in Table 7). In Table 2 the ratios  $K_\phi/K_{\phi,max}$  are presented. For a given  $l_0/d_1$ ,  $K_\phi$  was calculated for every concrete strength class (Eq. (7)), and  $K_{\phi,max}$  is the highest of these. Italic numbers identify the strength classes, where  $K_\phi = K_{\phi,max}$  ( $d_1/h=0.85$ )

The eccentricity,  $e_2'$  (which is independent of the normal force) can be calculated from Eqs. (19) and (4-6). They yield:

$$\frac{e_2'}{d_1} = K_\phi \frac{f_{yd} / E_s}{4.5} \frac{d_1}{d'} \left( \frac{l_0}{d_1} \right)^2 \quad (20)$$

$e_2'$  depends on the strength class of concrete through the creep coefficient,  $K_\phi$ . Our aim was to develop expressions which are independent of the concrete strength class. We calculated Eq. (20) for every concrete strength class considered and corresponding creep coefficient. We considered the highest of these values. The results are given in Table 1. Note that for given  $d_1/h$  and  $l_0/d_1$  ratios,  $e_2'/d_1$  is inversely proportional to  $d'/d_1$  (Eq. (20)). (Not considering the concrete strength class as a parameter in the calculation results in a conservative design. The "accurate" calculation may give 10-15% smaller eccentricities, as shown in Table 2.)

### 3.2 Centrally loaded columns (calculation of " $\varphi$ ")

We consider a RC column with symmetrical cross section which is loaded centrally by a compression force  $N_{Ed}$ . The eccentricity of the force is  $e = e_1 + e_2$ , (see Eq.(2),  $e_e = 0$ ), which yields (Eqs. (3) and (19)):

$$\frac{e}{d_1} = \frac{e_1}{d_1} + \frac{e_2}{d_1} = \frac{1}{400} \frac{l_0}{d_1} + K_r \frac{e_2'}{d_1}, \quad (21)$$

where  $e_2'/d_1$  is given by Eq. (20). The eccentricity must be larger than the minimal eccentricity (Eq. (13))

$$\frac{e}{d_1} \geq \frac{1}{30} \frac{h}{d_1}, \quad \text{if } h > 600 \text{ mm}, \quad (22)$$

$$\frac{e}{d_1} \geq \frac{20}{d_1}, \quad \text{if } h \leq 600 \text{ mm}, \quad (23)$$

where  $d_1$  is given in mm.

<sup>1</sup> In the calculation of  $e_1$  we assumed that  $l \leq 4$  m. For longer columns Eq. (21) is on the safe side.

The failure envelope of a doubly symmetrical cross section is given in Fig. 2.b. We assume that  $N_{Ed} = N_{Rd}$  and  $N_{Rd} > N_{bal}$ , which means that the upper line of the envelope must be used. On the basis of Fig. 2.b, we can write:

$$\frac{N_{Ed}}{N_u} = \frac{1}{1 + \frac{e}{d_1} \frac{(N_u - N_{bal})d_1}{M_{max}}} \quad (24)$$

where  $N_u$ ,  $N_{bal}$  and  $M_{max}$  are given by Eqs. (14), (17) and (18). The fraction on the left hand side of Eq. (24) is identical to " $\varphi$ ". However, this expression can not be used directly because the eccentricity,  $e$ , is a function of the normal force,  $N_{Ed}$  through the parameter  $K_r$  (Eqs. (21) and (9)):

$$K_r = \frac{N_u' - N_{Ed}}{N_u' - N_{bal}}, \quad \text{if } N_{Ed} > N_{bal}, \quad (25)$$

where  $N_u'$  is given by Eq. (10). By introducing Eqs. (25) and (21) into Eq. (24) and by replacing  $N_{Ed}/N_u$  by  $\varphi$ , we obtain:

$$\varphi = \frac{1}{1 + \frac{1}{400} \frac{l_0}{d_1} \frac{(N_u - N_{bal})d_1}{M_{max}} + \frac{e_2'}{d_1} \frac{N_u - N_{bal}}{N_u' - N_{bal}} \left( \frac{N_u d_1}{M_{max}} - \frac{N_u d_1}{M_{max}} \varphi \right)}, \quad (26)$$

which yields a second order expression for  $\varphi$ . Eq. (26) is based on Eq. (21). However, the minimal eccentricity may also play a role (Eqs. (22) or (23)). Eqs. (22) and (24) or Eqs. (23) and (24) yield:

$$\varphi = \frac{1}{1 + \frac{1}{30} \frac{h}{d_1} \frac{(N_u - N_{bal})d_1}{M_{max}}}, \quad \varphi = \frac{1}{1 + \frac{20}{d_1} \frac{(N_u - N_{bal})d_1}{M_{max}}}. \quad (27a, b)$$

$\varphi$  is the lesser of the values obtained by Eqs. (26, 27a, 27b). Equation (26) can be written in the following form:

$$\begin{aligned} \varphi &= \frac{1}{1 + \frac{1}{400} \frac{l_0}{d_1} c_1 + \frac{e_2'}{d_1} \frac{N_u - N_{bal}}{N_u' - N_{bal}} (c_2 - c_2 \varphi)}, \\ &= \frac{1}{1 + \frac{e_1}{d_1} c_1 + \frac{e_2' d}{d_1^2} \frac{N_u - N_{bal}}{N_u' - N_{bal}} \left( \frac{d_1}{d} c_2 - c_3 \varphi \right)} \end{aligned} \quad (28)$$

where the parameters are defined as follows:

$$c_1 = \frac{(N_u - N_{bal})d_1}{M_{max}}, \quad c_2 = \frac{N_u d_1}{M_{max}}, \quad c_2' = \frac{N_u' d_1}{M_{max}}, \quad c_3 = \frac{c_2}{d/d_1}. \quad (29)$$

$\varphi$  was calculated for the cross sections given in Fig. 3a and b; the results are in Tables 3 to 6.

The  $\varphi$  values in Tables 3 to 5 are the lesser of the  $\varphi$ -s obtained by Eqs. (28) and (27a). These expressions are independent of the dimensions of the cross section ( $d$  or  $h$ ). The characteristic yield strength of steel was  $f_{yk} = 500$  N/mm<sup>2</sup>.

All the values in Tables 3 to 5 are independent of the concrete strength class, except  $\rho$  in the second column. The reinforcement ratio  $\rho$  is given for concrete strength class C25/30. For other strength classes,  $\rho$  must be multiplied by  $f_{ck}/25$ . (For example, for C50/60 the  $\rho$ -s in Table 3 are 0.3, 0.7, ... 13.8.) In the tables the reinforcement ratios were chosen so as to obtain  $c_3 = 6.5, 6.0, \dots, 3.0$ ; etc.

According to Eurocode the minimum reinforcement ratio is  $\rho = 0.2\%$ , and at least 10% of  $N_{Ed}$  must be carried by the reinforcement ( $N_s \geq 0.1 N_{Ed}$ ). In the 6th column of Table 3 the fraction  $N_s/N_u$  is given. For  $\varphi = 0.38$ , from the first row, we

$d'/d_1$	$\rho\%$	$c_1$	$c_2=c_3$	$c_2'$	$N_s/N_u$	$l_0/d_1$											
						0.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20.0	22.0	24.0	26.0
1	0.15	3.869	6.501	6.521	0.038	0.87	0.87	0.87	0.85	0.79	0.72	0.64	0.56	0.49	0.44	0.38	0.32
	0.35	3.675	5.994	6.035	0.084	0.87	0.87	0.87	0.86	0.81	0.74	0.66	0.59	0.52	0.47	0.42	0.38
	0.6	3.488	5.508	5.568	0.137	0.88	0.88	0.88	0.87	0.82	0.76	0.69	0.62	0.55	0.50	0.45	0.41
	0.95	3.296	5.006	5.087	0.202	0.89	0.89	0.89	0.88	0.84	0.78	0.72	0.65	0.59	0.53	0.48	0.44
	1.45	3.105	4.508	4.610	0.281	0.89	0.89	0.89	0.85	0.80	0.74	0.68	0.62	0.57	0.52	0.48	0.44
	2.25	2.911	4.001	4.123	0.381	0.90	0.90	0.90	0.90	0.86	0.82	0.77	0.71	0.66	0.61	0.56	0.51
	3.65	2.721	3.504	3.647	0.508	0.90	0.90	0.90	0.90	0.87	0.84	0.79	0.75	0.69	0.64	0.60	0.56
	6.93	2.528	3.000	3.163	0.679	0.91	0.91	0.91	0.91	0.88	0.85	0.82	0.77	0.73	0.69	0.64	0.60

Table 3:  $\varphi$  as a function of  $l_0/d_1$ ,  $d_1/h=0.85$ , the reinforcement is in two layers (Fig. 3.a)

$d'/d_1$	$\rho\%$	$c_1$	$c_2$	$c_3$	$N_s/N_u$	$l_0/d_1$											
						0.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20.0	22.0	24.0	26.0
0.92	0.325	3.955	6.474	7.003	0.079	0.87	0.87	0.87	0.84	0.77	0.69	0.61	0.53	0.47	0.41	0.34	0.29
	0.73	3.865	6.011	6.502	0.162	0.87	0.87	0.87	0.85	0.78	0.71	0.63	0.55	0.49	0.43	0.39	0.36
	1.3	3.777	5.551	6.005	0.258	0.87	0.87	0.87	0.85	0.79	0.72	0.65	0.58	0.51	0.46	0.41	0.38
	2.2	3.685	5.080	5.495	0.376	0.87	0.87	0.87	0.86	0.80	0.74	0.67	0.60	0.54	0.48	0.44	0.40
	3.7	3.598	4.625	5.003	0.511	0.88	0.88	0.88	0.86	0.81	0.75	0.69	0.62	0.56	0.51	0.46	0.42
	7.0	3.507	4.158	4.498	0.681	0.88	0.88	0.88	0.87	0.82	0.77	0.71	0.65	0.59	0.54	0.49	0.45
	19.0	3.418	3.697	3.999	0.891	0.88	0.88	0.88	0.87	0.83	0.78	0.73	0.67	0.61	0.56	0.52	0.48

Table 4:  $\varphi$  as a function of  $l_0/d_1$ ,  $d_1/h=0.85$ , the reinforcement is in three layers (Fig. 3.b)

$d_1/h$	$d'/d_1$	$\rho\%$	$c_1$	$c_2$	$c_3$	$l_0/d_1$											
						0.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20.0	22.0	24.0	26.0
0.90	0.92	1.31	3.650	5.513	6.003	0.88	0.88	0.88	0.86	0.80	0.73	0.66	0.59	0.52	0.47	0.43	0.39
	0.85	0.92	1.30	3.777	5.551	6.005	0.87	0.87	0.87	0.85	0.79	0.72	0.65	0.58	0.51	0.46	0.41
	0.80	0.93	1.33	3.916	5.588	6.001	0.86	0.86	0.86	0.85	0.79	0.72	0.64	0.56	0.50	0.45	0.40

Table 5:  $\varphi$  as a function of  $l_0/d_1$ , the reinforcement is in three layers (Fig. 3.b),  $c_3=6.0$

$h$	150	200	300	400	500	600
$\varphi$	0.62	0.68	0.76	0.81	0.84	0.87

Table 6:  $\varphi$  as a function of  $h$  (mm)

have  $N_s/N_{Rd} = N_s/\varphi N_u = 0.1$ , and hence the above requirement is met. (Note that in the first row of Table 3, when  $l_0/d_1 \leq 22$ , in the second row of Table 3, when  $l_0/d_1 \leq 10$ , and in the first row of Table 4, when  $l_0/d_1 \leq 10$ ;  $N_s < 0.1 N_{Ed}$ .)

(In a few of the cases – indicated by italics in Tables 3 and 4 –  $N_{Ed} < N_{bal}$ , and hence instead of the upper line of the failure envelope (Fig. 2.b), the lower part had to be used.)

In Table 5 the effect of  $d_1/h$  was investigated for the cross section shown in Fig. 3.b. The reinforcement ratio was chosen so that  $c_3$  is about 6.

When the height of the cross section is smaller than 600 mm ( $h < 600$  mm), the minimal eccentricity is given by

Table 8:  $\varphi$  as a function of  $l_0/d_1$  for the calculation of  $N_{Rd} = \varphi N_u$

$c_3$	$l_0/d_1$											
	0.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20.0	22.0	24.0	26.0
7.0	0.87	0.87	0.87	0.84	0.77	0.69	0.61	0.53	0.47	0.41	0.34	0.29
6.5	0.87	0.87	0.87	0.85	0.78	0.71	0.63	0.55	0.49	0.43	0.39	0.36
6.0	0.87	0.87	0.87	0.85	0.79	0.72	0.65	0.58	0.51	0.46	0.41	0.38
5.5	0.87	0.87	0.87	0.86	0.80	0.74	0.67	0.60	0.54	0.48	0.44	0.40
5.0	0.88	0.88	0.88	0.86	0.81	0.75	0.69	0.62	0.56	0.51	0.46	0.42
4.5	0.88	0.88	0.88	0.87	0.83	0.78	0.72	0.65	0.60	0.54	0.50	0.46
4.0	0.89	0.89	0.89	0.89	0.85	0.80	0.75	0.70	0.64	0.59	0.54	0.50
3.5	0.90	0.90	0.90	0.90	0.87	0.83	0.79	0.74	0.69	0.64	0.59	0.55
3.0	0.91	0.91	0.91	0.91	0.88	0.85	0.82	0.77	0.73	0.69	0.64	0.60

<sup>2</sup> Two improvements can be made: (1) when the reinforcement is in two layers (Fig. 3.a)  $e_2/d_1$  may be multiplied by 0.92; (2) when  $N_{Ed} > N_{bal}$ ,  $e_2/d_1$  may be multiplied by  $K_r$  (Eq. (25)).

$l_0/d_1$	0	6	8	10	12	14	16	18	20	22	24	26
	$e_1/d_1$	0.000	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050	0.055	0.060
$e_2/d_1$	0.000	0.034	0.058	0.085	0.116	0.151	0.189	0.229	0.271	0.313	0.355	0.395
$(e_1 + e_2)/d_1$	0.000	0.049	0.078	0.110	0.146	0.186	0.229	0.274	0.321	0.368	0.415	0.460

$l_0/d_1$	28	30	32	34	36	38	40	42	44	46	48	50
	$e_1/d_1$	0.070	0.075	0.080	0.085	0.090	0.095	0.100	0.105	0.110	0.115	0.120
$e_2/d_1$	0.434	0.471	0.526	0.594	0.666	0.742	0.823	0.907	0.995	1.088	1.184	1.285
$(e_1 + e_2)/d_1$	0.504	0.546	0.606	0.679	0.756	0.837	0.923	1.012	1.105	1.203	1.304	1.410

Table 7: The relative eccentricities as a function of  $l_0/d_1$ .

Eq. (22) and hence, Eq. (27b) instead of Eq. (27a) must be used. Results calculated from Eq. (27b) are presented in Table 6. (Equation (27b) yields  $\varphi = 1/(1+c_1 \times 20/d_1)$ , where we considered  $c_1 = c_{1,max} = 3.955$  and  $d_1/h=0.85$ .)

## 4. APPROXIMATE CALCULATION OF ECCENTRICITIES AND $\varphi$

### 4.1 Calculation of eccentricities

The eccentricities may be determined by Table 7. In the calculation of this table neither the minimal eccentricity, nor the multiplier  $K_r$  was considered. (The latter value, in case of high normal force ( $N_{Ed} \gg N_{bal}$ ) may result in a very conservative analysis.) We determined Table 7 as follows:

Table 1 shows that  $e_2$  depends on the arrangement of the reinforcement. When the reinforcement is in three layers the eccentricities are about 10% higher than in the case where the reinforcement is arranged in two layers. When the steel is in more than three layers (e.g. in 4 layers, see Table 1),  $e_2$  is practically unaffected. Hence, in Table 7 the results determined for three layers (Fig. 3.b) were taken into account. The influence of  $d_1/h$  is also shown in Table 1. The case  $d_1/h=0.85$  was considered, when  $d_1/h > 0.85$ , the values of Table 7 are on the safe side. (When the reinforcement is in two layers,  $e_2/d$  may be multiplied by 0.92.)

Table 7 was calculated with:  $f_{yk} = 500$  N/mm<sup>2</sup>,  $d_1 = 0.85 h$  and  $d' = 0.79 h = 0.92 d_1$  (the last assumption is valid for the case when the reinforcement is in three layers). When the reinforcement is in two layers,  $f_{yk} < 500$  N/mm<sup>2</sup>, or  $d_1/h > 0.85$  the relative eccentricities in Table 7 are on the safe side.



## 4.2 Calculation of centrally loaded columns

Based on *Tables 3 to 6* the ultimate load of a centrally loaded column is calculated as follows:

$$N_{Rd} = \varphi N_u \quad (30)$$

where  $\varphi$  is the lesser of the values obtained from *Tables 8 and 6*, and  $N_u$  is the ultimate load of the centrally loaded cross section (Eq. (14)). These tables are developed for bending about the  $y$ -axis. (In design of columns both the  $x$ - $y$  and the  $x$ - $z$  planes must be considered, and the smaller value of  $\varphi$  must be used in Eq. (30)).

$\varphi$  in *Table 8* depends on parameter  $c_3$ , which is defined as (Eq. (29)):

$$c_2 = \frac{N_u d_1}{M_{max}}, \quad c_3 = \frac{c_2}{d' / d_1} \quad (31)$$

When the reinforcement is in two layers  $d' / d_1 = 1$ ; when it is in three or more layers  $d' / d_1 = 0.92$  (see *Table 1*). As an approximation the first row, with  $c_3 = 7.0$  may always be used.  $N_u$  and  $M_{max}$  are given by Eqs. (14) and (18).

The challenge in determination of *Table 8* is that  $\varphi$  depends strongly on both the amount and the arrangement of the reinforcement, and a parameter had to be found on the basis of which  $\varphi$  could be approximated. This parameter is  $c_3$  (Eq. (31)). The parameter was chosen because of the following rea-

soning: Eq. (28) gives  $\varphi$  as a function of different parameters.  $N_u$  and  $N_u'$  are close to each other (Eqs. (10) and (14)), hence we introduce  $N_u = N_u'$  into Eq. (28), which yields:

$$\varphi = \frac{1}{1 + \frac{e_1}{d_1} c_1 + \frac{e_2' d'}{d_1^2} (1 - \varphi) c_3} \quad (32)$$

This expression depends only on parameters  $c_1$ ,  $c_3$ , and on ratio  $l_0 / d_1$ ; the latter affects  $e_2' d' / d_1^2$  (see Eq. (20)) and  $e_1 / d_1$  (see Eq. (21)). Parameters  $c_1$  and  $c_3$  depend both on the amount and the arrangement of the reinforcement. According to *Table 7*  $e_2'$  is considerably higher than  $e_1$ , (note also that  $2.5 < c_1 < 4$ ,  $3 < c_3 < 7$ ) and hence, in the calculation of  $\varphi$  (Eq. (32)),  $c_3$  plays a more important role than  $c_1$ . This can also be seen in *Tables 3 to 5*: when  $\varphi$  is given as a function of  $c_3$ , the arrangement of reinforcement plays only a minor role.<sup>3</sup>

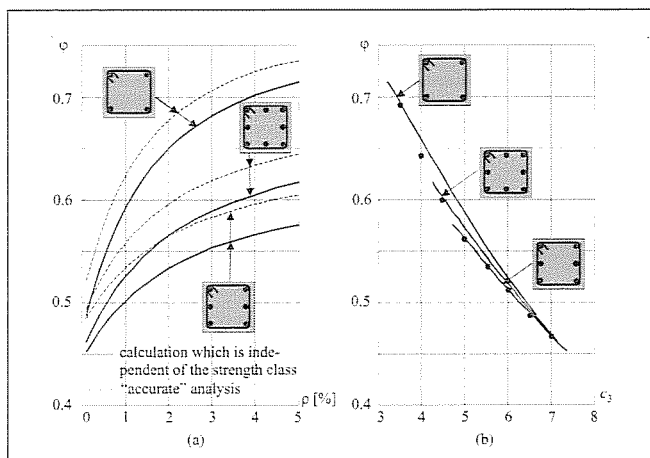
When  $c_3 \leq 5.0$ ,  $\varphi$ -s were taken from *Table 4* (reinforcement in three layers, *Figure 3b*), these values are smaller than the corresponding  $\varphi$ -s in *Table 3*. When  $c_3 \leq 4.5$  then the reinforcement ratio in *Table 4* is unrealistically high. When  $c_3 = 3$  we used the  $\varphi$ -s value from *Table 3* (reinforcement in two layers, *Fig. 3a*). For the intermediate values we used the cross section shown in *Figure 2a*, assuming that the reinforcement ratio is (for concrete strength class C25/30)  $\rho = 5\%$ .<sup>4</sup> The amounts of  $A_{s1}$  and  $A_{s3}$  were chosen so that  $c_3$  was: 4.5; 4 and 3.5. The details of the calculation can be seen in *Table 9*.

The suggested approximation is illustrated in *Fig. 4*, for  $l_0 / d_1 = 20$ . It can be seen that  $\varphi$ , which is independent of concrete strength class, may be on the safe side by up to 10%.  $\varphi$  depends strongly on both the amount and the arrangement of the reinforcement (*Fig. 4a*). However, when we use parameter  $c_3$  (instead of  $\rho$ ) the steel arrangement plays only a minor role (*Fig. 4b*). It is also illustrated that  $\varphi$ -s in *Table 8* were obtained as the lower bound of  $\varphi$  curves for different steel arrangements if  $\rho < 5\%$ .

The calculation was carried out assuming  $d_1 / h = 0.85$ , but as can be seen in *Table 5*,  $d_1 / h$  has only a minor effect on  $\varphi$ . (When  $f_{yk} < 500 \text{ N/mm}^2$  the  $\varphi$ -s in *Table 8* are on the safe side.)

$A_{s1}/A_{s3}$	$\rho$ %	$c_1$	$c_2$	$c_3$	$l_0/d_1$											
					0.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20.0	22.0	24.0	26.0
0.333	7.0	3.507	4.158	4.498	0.88	0.88	0.88	0.87	0.82	0.77	0.71	0.65	0.59	0.54	0.49	0.45
0.354	5.00	3.401	4.202	4.496	0.88	0.88	0.88	0.87	0.83	0.78	0.72	0.65	0.60	0.54	0.50	0.46
0.402	5.00	3.096	3.826	3.996	0.89	0.89	0.89	0.89	0.85	0.80	0.75	0.70	0.64	0.59	0.54	0.50
0.461	5.00	2.789	3.446	3.503	0.90	0.90	0.90	0.90	0.87	0.83	0.79	0.74	0.69	0.64	0.59	0.55
0.506	6.93	2.528	3.000	3.000	0.91	0.91	0.91	0.91	0.88	0.85	0.82	0.77	0.73	0.69	0.64	0.60

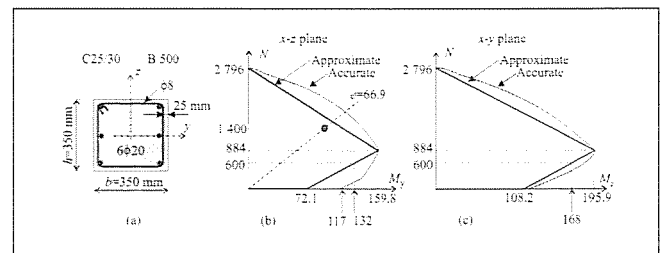
**Table 9:**  $\varphi$  as a function of  $l_0 / d_1$ , for the cross section shown in *Figure 2a*.  $d_1 / h = 0.85$



**Fig. 4:**  $\varphi$  as a function of the reinforcement ratio ( $\rho$ ) and parameter  $c_3$ . The concrete strength class is C25/30,  $f_{yk} = 500 \text{ N/mm}^2$ ,  $l_0 / d_1 = 20$ ,  $d_1 / h = 0.85$ . In (a) the dashed lines are the results of the "accurate" calculations; the solid lines in (a) and (b) are the results of the approximate calculations which are independent of concrete strength class. (Hence the solid lines are applicable for an arbitrary concrete strength class, when the reinforcement ratio is multiplied by  $f_{yk} / 25$ .) The values from *Table 8* are denoted by •.

## 5. NUMERICAL EXAMPLE

(A) A RC cantilever with length  $l = 3 \text{ m}$  is subjected centrally to a compression force  $N_{Ed} = 1\,400 \text{ kN}$ . The dimensions of the cross section are  $350 \times 350 \text{ mm}$ , the reinforcement is  $6\phi 20 \text{ mm}$ , the stirrup is  $\phi 8 \text{ mm}$  and the cover is  $25 \text{ mm}$  (*Fig. 5a*). The concrete is C25/30, while the steel is B 500. Verify the load bearing capacity of the column.



**Fig. 5:** The cross section in the numerical example (a), the failure envelope for moment about the  $y$ -axis (b) and about the  $z$ -axis (c).

<sup>3</sup> Let us consider as the example  $c_3 = 5.5$  and  $l_0 / d_1 = 24$ . According to *Table 3* (reinforcement in two layers) the reinforcement ratio is  $\rho = 0.6\%$ , and  $\varphi = 0.45$ . From *Table 4* (reinforcement in three layers)  $\rho = 2.2\%$ ,  $\varphi = 0.44$ . It can be seen that the reinforcement ratio was changed by a factor of 4, ( $c_3$ -s are the same), and the difference in  $\varphi$  is small.

<sup>4</sup> When the concrete strength class is C20/25, the reinforcement ratio is 4%. According to EC 2,  $\rho$  should not exceed 4%.

The effective length is  $l_0 = 6$  m, the effective depth is  $d = 307$  mm, and hence  $l_0/d = 19.5$ . From Eq. (14) we have  $N_u = 350 \times 350 \times 25 / 1.5 + 6 \times 10^2 \pi \cdot 400 = 2042 + 754 = 2796$  kN. The first row of Table 8 gives  $\varphi = 0.484$  (Table 6 results in a higher  $\varphi$ ), and hence  $N_{Rd} = \varphi N_u = 1352$  kN  $< N_{Ed}$ , the column is UNSAFE.

We may take into account the effect of the reinforcement to improve the calculation. In the  $x$ - $z$  plane (where we expect the smaller  $\varphi$ ):  $z_s = 264$  mm,  $x_{co} = 0.49 \times 307 = 152$  mm,  $N_{bal} = 152 \times 350 \times 25 / 1.5 = 884$  kN,  $M_s = 2 \times 10^2 \pi \cdot 500 / 1.15 \times 264 = 72.1$  kNm,  $\Delta M = N_{bal} (350/2 - 152/2) = 87.7$  kNm,  $M_{max} = M_s + \Delta M = 159.8$  kNm. Hence  $c_2 = d N_u / M_{max} = 5.37$ ;  $c_3 = c_2 / 0.92 = 5.84$ ; and from Table 8 we obtain  $\varphi = 0.535$ , which results in  $N_{Rd} = \varphi N_u = 1496$  kN  $> N_{Ed}$ , the column is SAFE.

The eccentricities were also calculated by the "accurate" expressions of Section 2.1, we obtained 66.9 mm. The "accurate" and approximate failure envelopes of the cross section are shown in Fig. 5.b when the eccentricity is in the  $z$  direction. Three points of the approximate failure envelope were determined above, while the "accurate" curve was calculated numerically (Kollár, 1997). The bending moment, from the eccentricity 66.9 mm is 94 kNm (shown by a dot in Fig. 5.b). According to the approximate failure envelope, at the normal force 1400 kN the bending moment is 117 kNm, and the ultimate eccentricity is  $117000/1400 = 83.3$  mm, the column is SAFE.

(B) The cantilever given in Section (A) is subjected to an axial compression force  $N_{Ed} = 600$  kN and to a horizontal load at the top of the column  $H_{Ed} = 30$  kN in the  $y$  direction. Verify the load bearing capacity of the column.

First the accidental eccentricities in the  $y$  direction are taken into account.

The points of the failure envelope when the bending is about the  $z$ -axis are as follows:  $N_u = 350 \times 350 \times 25 / 1.5 + 6 \times 10^2 \pi \cdot 400 = 2042 + 754 = 2796$  kN,  $N_{bal} = 152 \times 350 \times 25 / 1.5 = 884$  kN,  $M_s = 3 \times 10^2 \pi \cdot 500 / 1.15 \times 264 = 108.2$  kNm,  $\Delta M = N_{bal} (350/2 - 152/2) = 87.7$  kNm,  $M_{max} = M_s + \Delta M = 195.9$  kNm. The failure envelope is shown in Fig. 5.c.

The eccentricity due to the horizontal load is:  $e_c = l H_{Ed} / N_{Ed} = 150$  mm (we did not use the reduction suggested in Eurocode 2 for non-uniform bending moment), the minimal eccentricity is:  $e_0 = 20$  mm, the eccentricity due to the imperfections is:  $e_1 = l_0 / 400 = 15$  mm.  $l_0/d = 19.5$ , and hence from Table 7 we have  $e_2/d = 0.261$ , which results in  $e_2 = 0.261 \times 307 = 80.3$  mm. The total eccentricity is:

$$e_{tot} = 150 + 15 + 80.3 = 245.3 \text{ mm} > e_0 = 20 \text{ mm.}$$

(When we use the "accurate" expressions in Section 2.1, we obtain:  $e_{tot} = 233.1$  mm.)

According to the approximate failure envelope at  $N_{Ed} = 600$  kN the ultimate bending moment is 168 kNm, which yields the ultimate eccentricity  $e_{Rd} = 279.6$  mm  $> e_{tot}$ , the column is SAFE.

We investigate now the accidental eccentricities in the  $z$  direction.

In the  $y$  direction we have:  $e_{tot,y} = e_c = 150$  mm. In the  $z$  direction:  $e_{tot,z} = 15 + 80.3 = 95.3$  mm  $> e_0 = 20$  mm. According to the approximate failure envelope at  $N_{Ed} = 600$  kN the bending moment in the  $x$ - $z$  plane is 132 kNm and hence the ultimate eccentricity is  $e_{Rd,z} = 219.5$  mm. In the previous paragraph we determined  $e_{Rd}$  in the  $x$ - $y$  plane:  $e_{Rd,y} = 279.6$  mm. The condition of the adequate load bearing capacity is:

$$\frac{N_{Ed} e_{tot,y}}{N_{Ed} e_{Rd,y}} + \frac{N_{Ed} e_{tot,z}}{N_{Ed} e_{Rd,z}} = \frac{150}{279.6} + \frac{95.3}{219.5} = 0.54 + 0.43 = 0.97 < 1$$

The column is SAFE.

## 6. CONCLUSION

A simple, conservative and approximate method is explained and tables are presented for the calculation of the eccentricities of the axial load and the ultimate load of centrally loaded columns with rectangular cross sections. The method can be used in preliminary design and also to verify the results of more sophisticated calculations.

*Acknowledgements:* The author is thankful for the valuable comments and suggestions of Professors E. Dulácska, I. Hegedűs and Gy. Visnovitz. The presented method is based on Dulácska's approach, which he developed for the Hungarian Standard.

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# BRIDGE RECONSTRUCTION WITH EXTERNAL PRESTRESSING ON THE M7 MOTORWAY AT ÉRD



Miklós Pálossy

*The two bridges of the M7 motorway at Érd showed considerable deflections, together with a severe appearance of cracks. Beside the overall retrofit works (new waterproofing with pavement and drainage) the designers had to make suggestions for the possible elimination of the above features in order to preserve the 30-year-old bridges in service for a longer lifetime. The solution was external prestressing. This article describes the details of this intervention of a type which always requires special consideration particularly given the age of the bridge.*

**Keywords:** retrofit, external unbonded prestressing, deviator

## 1. ANTECEDENTS

The M7 Motorway connecting lake Balaton with Budapest is one of the most important motorways in Hungary. The 30–35-year-old section was fully reconstructed in 2001–2002. The retrofitting of the two overpasses at Érd (in the gate of Budapest) was of special interest due to the considerable deflection (5–10 cm !) of the reinforced concrete superstructures, and the extensive appearance of cracks in connection with that. Beside the general retrofit works, a structural analysis and proposals for the possible strengthening of the structure was also in the remit of the design engineers' work. The design was preceded by a detailed geodetic survey and material tests.

The design work was launched in April 2001 while the site works began in the following year and took ca. 4 months. Though pedestrian traffic could be continuously maintained on both bridges during the works, the road bridge was closed to vehicles during the deck reconstruction.

The main participants in the project:

Client: National Motorway Administration  
Contractor: MÁV Hídépítő Ltd. (general contractor)  
Vorspann-Technik Austria,  
Megalit Ltd. (prestressing works)  
Design: Pont-Terv Ltd.

## 2. DESCRIPTION OF THE BRIDGES

### 2.1 Road bridge at Szövő Street

The bridge was built at sta 16+944.9 km of the M7 Motorway over a ca. 10 m deep cutting in 1966–67. The spans of the cast in-situ R.C. bridge are 18.50 + 37.00 + 18.50 m long. The deck consists of a 7.0 m wide roadway flanked by 1.50 m wide sidewalks with a total width of 10.40 m. The bridge is straight in plan, with a longitudinal grade of 1.1%. The skew angle is 84°15'.

The depth of the one cell R.C. box superstructure is constant in the central span, but decreasing in the side spans. The two legs of the upwards increasing rectangular section are rigidly connected to the superstructure (frame structure). The ends of the bridge are supported by steel rail bearings allowing longitudinal movements.

The piers have 4×8 m sized flat foundations, the abutments are supported by dia. 1.70 m ca. 9 m deep well foundations bored from the upper edge of the cutting.

### 2.2 Pedestrian bridge at Tetőfedő Street

The bridge was built at sta 16+594.8 km of the M7 Motorway over a 5–9 m deep cutting in 1966–67. The spans of the cast in-situ R.C. bridge are 2 × 34.0 m long. The deck width of a single cell R.C. box structure is 3.50 m. The bridge is straight in plan, with a longitudinal grade of 5%.

The central support is a dia. 80 cm circular column standing on a 3×3 m sized foundation body. The ends of the bridge are supported by rail bearings. The abutments are based on dia. 1.70 m ~ 5 to 7 m deep well foundations at the upper edges of the cutting.

## 3. SITE ASSESSMENT AND SUGGESTIONS

The former bridge assessments indicated overall bad repair of the bridges with serious damages to the pavements and curbs, as well as a lack of a proper drainage system. Beside this, the main problem was the considerable deflection and the appearance of severe cracks as a consequence. At some parts of the footbridge, the signs of soaking were very evident as well. For the above reasons, a statical check was also necessary.

According to the geodetic measurements, the extra deflection at mid-span was 8–10 cm for the road bridge and 5–10 cm for the footbridge. The crack width reached 0.4–0.5 mm at several points. The majority of cracks appeared on the bottom edge and on the webs of the superstructures at the sagging sections, but there were some upper cracks through the deck cantilevers at the hogging sections above supports as well. In spite of that, the strength test of the structural concrete showed generally good results (C45 grade). Considerable chloride corrosion was reported at the curbs only. The main reinforcement, consisting of dia. 32 mm grade 50.36 bars, was in good condition.

The results of the site investigations and the statical check led to the following conclusions applicable to both bridges:

- the load bearing capacity of the bridges in ultimate limit state is sufficient;

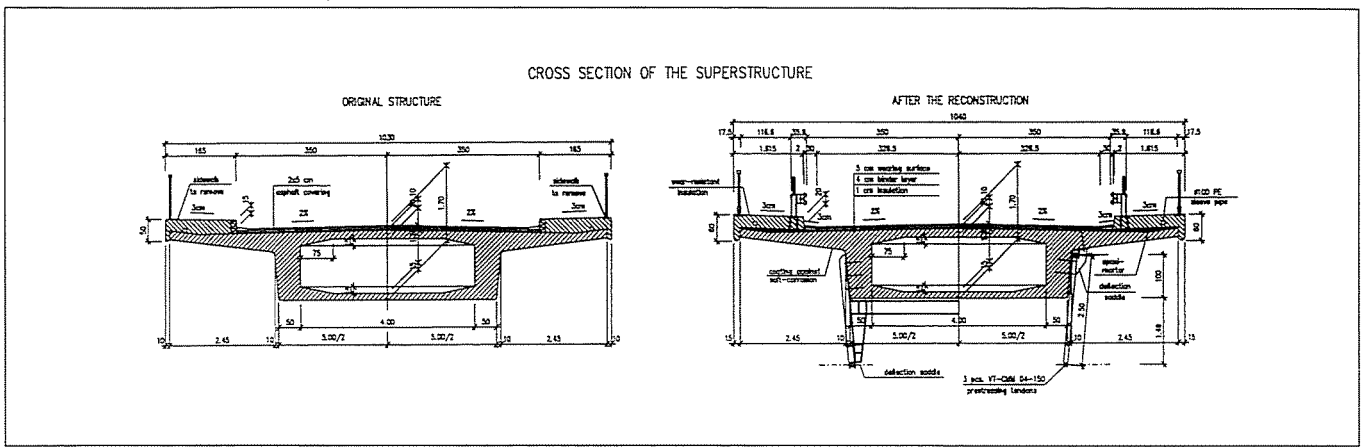


Fig. 1: Bridge at Szövő Street, cross section

- the requirements of the serviceability limit state (limitations of deflection and crack width) are not met;
- beside the overall repair works a statical intervention is advisable in order to improve the service conditions. This could be achieved by an external prestressing.

After a consultation with the client, the following measures were decided.

*Overall retrofit works:*

- new deck waterproofing and asphalt covering, reconstruction of the curbs, concrete coatings;
- new handrails;
- new drainage system;
- proper solution of the joints at the ends of the bridges.

*Strengthening works:*

- external unbonded prestressing in order to reduce the deflection and close the cracks.

It should be noted, however, that according to the calculations only one-third of the total deflection could be compensated by prestressing. Further correction of the longitudinal profile was possible with an additional equalizing cement mortar layer below the deck waterproofing and on the curbs, which determine optically the side view of the superstructure.

## 4. RETROFIT WORKS

### 4.1 Road bridge at Szövő Street

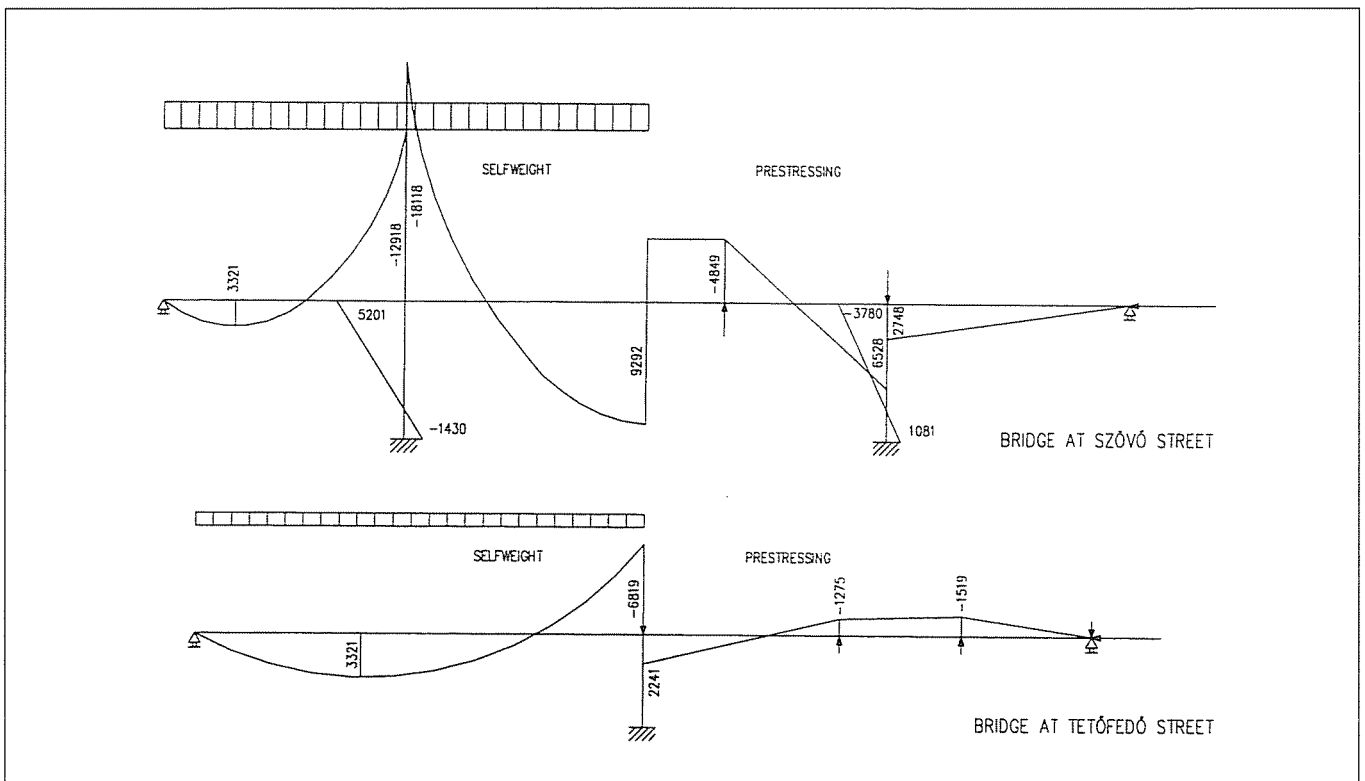
#### 4.1.1 R.C. works and repair

As the first step, the handrails, the R.C. sidewalks, the asphalt pavement and the deck were removed. In order to build the anchorage beams and the transversal drains, the areas behind the abutments were excavated as well. Based on the geodetic survey of fall after prestressing, the depth of the equalizing cement mortar layer could be determined. The proper longitudinal and transversal fall was to be proved after the expected later deflections due to the superimposed dead loads.

The height of the curb above the road level was increased to 20 cm on average. With a limited change in this height a further improvement of the external profile of the bridge was possible (Fig. 1).

As opposed to usual practice, the cracks were not grouted before the prestressing because the specific aim of the intervention was to lift the deflected spans, which would be more effective on the cracked structure of reduced stiffness. For this reason the injection was completed at the end of the works only.

Fig. 2: Bending moments from self-weight and prestressing



After the necessary cleaning and repair the concrete surfaces were coated against de-icing salts and other corrosive effects.

#### 4.1.2 Prestressing works

As the load bearing capacity of the bridge in ultimate limit state proved sufficient, the primary aim of the external prestressing was to meet the requirements of the serviceability limit state, in order to keep the bridge in service for many years to come. This required the elimination of cracks and the reduction of deflection, and could be achieved by external prestressing.

Due to the applied normal force and the bending moments arising from the eccentric cables, the cracks could be closed for the permanent loads and limited in 0.1 mm width under service conditions. In this way further corrosion damage to the structure could be avoided. Besides that, the aesthetically unfavourable deflection could be reduced. The remaining part could be further corrected with the profile of the new curb (*Fig. 1.b*).

Though the clearance height would allow ca 3.5 m free space for the cables, they are led at an optimal 1.5 m below the bottom edge of the bridge. (A bigger distance would be more effective statically, but it is unfavourable for aesthetic reasons.) The cable is centric in the side spans, breaks downward from the upper deviation saddles at the piers, and has a parallel central section between the lower deviation saddles at the thirds of the main span (*Fig. 3*).

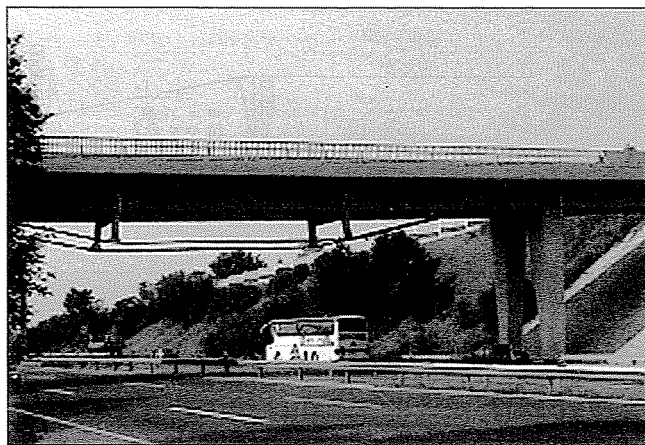
The applied external unbonded prestressing elements are Vorspann-Technik Multi-Mono-System cables with double PE cover. These elements provide long-term protection for the high-strength tendons against corrosion. A further advantage of this type of cable is the low friction coefficient ( $\mu = 0.05$ ).

Three parallel VT-CMM 04-150 bundles are combined to form a cable along both sides of the bridge. Each bundle consists of four monostrand tendons of 150 mm<sup>2</sup> steel cross section area. The bundles lie directly on each other at the deviator saddles. Though the friction between the bundles is higher, this configuration makes the installation of the cables easier: the tendon consisting of three bands can be hauled into location in one step which simplifies an otherwise complicated and time consuming procedure.

The galvanized lower deviation saddles are fixed to the supporting steel legs with HSFG bolts. The legs are connected with a 300 mm deep steel cross-girder below the bottom R.C. plate of the superstructure. As the legs are not perpendicular to the bottom edge, steel wedge plates are inserted to provide the proper force transmission.

The upper deviation saddles are also galvanized steel structures, fixed with bolts to the web of the main girder and with vertical tie rods through the deck plate. The upper ends of these ties are welded to transversal steel strips placed into the upper part of the slab, serving as additional reinforcement of the cantilever (*Fig. 4*).

Statically, the most suitable point for the cable anchorages was the reverse side of the cross-girders at the ends of the bridge. This way, the prestressing force acts along the whole structure. In order to lead the cables across the R.C. cross-girders at the abutments, dia. 200 mm bore-holes were required. Due to the high concentrated forces, an additional load distributing beam was built behind the original cross-girder. The anchorages are in the stressing pockets of this beam and protected with wax-sealed caps, an additional cement mortar fill and a waterproofing on top of that. Furthermore, a transversal drain serves to



**Fig. 3:** Bridge at Szövő Street, side view

remove the water from behind the abutments. In the case of a necessary cable replacement, the anchorages are accessible after uncovering the pavement layers of the road (*Fig. 5*).

The steps of the prestressing procedure were as follows:

The bundles were stressed separately, beginning with the lowest one. Both sides of the bridge were stressed simultaneously in order to avoid the asymmetric loading of the superstructure.

The stressing of the individual bundles followed in three steps:

The first step was a 10% prestressing in order to straighten the cables for the necessary control measurements of elongation in the following steps. In the second step, 50% of the total prestressing force was applied and the ends of the cables were fixed with wedges. Finally, the cables were stressed to the designed value at their ends, and anchored. The wedge slip was 6–7 mm. In each phase, the required force level was reached in 3–4 further steps, by gradual increase of the jacking pressure under continuous control of the cable-elongation.

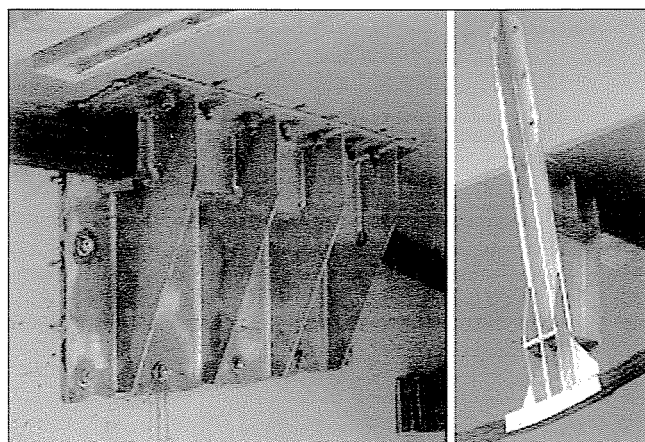
In order to reduce the horizontal forces arising on the deviator saddles due to friction, the stressing of the next bundle always began at the other end of the bridge.

The cable stresses at prestressing and after the wedge slip have been interpreted graphically in *Fig. 6*. As it can be noticed the loss due to friction at the deviators is low, in spite of the fact that the friction coefficient between the cable bandages slipping on each other is considerably higher ( $\mu = 0.16$ ) than for the individual tendons ( $\mu = 0.05$ ). The advantage of this solution is, as mentioned before, the easy installation.

#### 4.1.3 Finishing works

In order to fill cracks, the deck was coated with an epoxy-resin layer. A sprayed waterproofing with a roughened walkable surface was applied to the sidewalks.

**Fig. 4:** Bridge at Szövő Street, deviation saddles



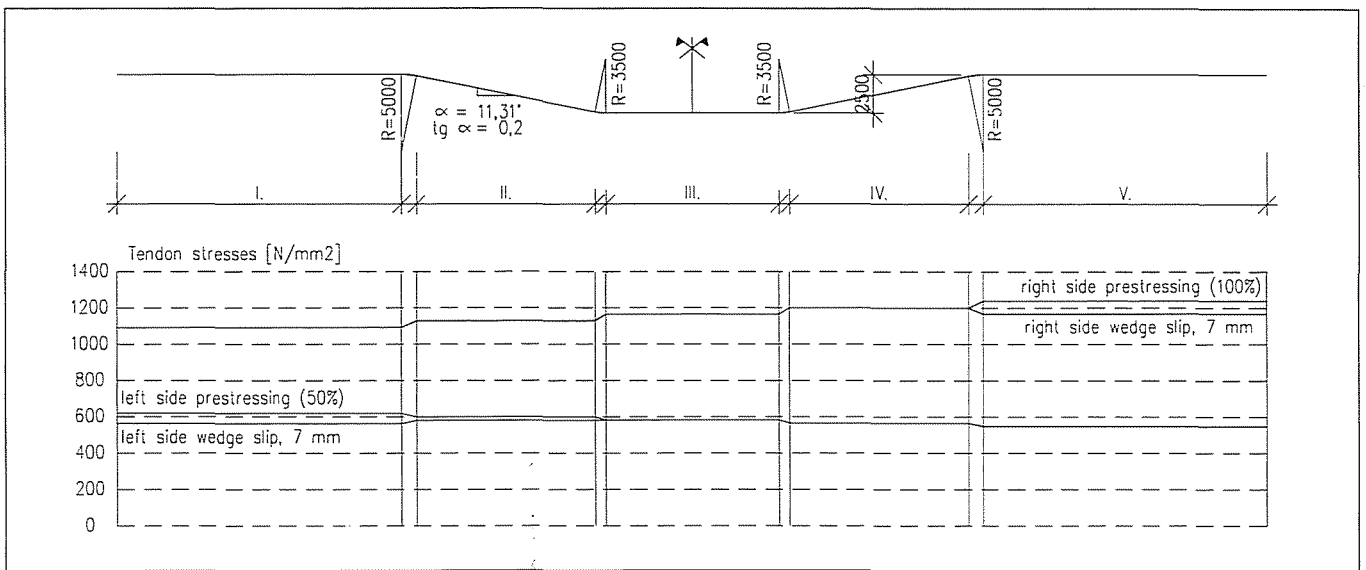


Fig. 5: Steps of prestressing and anchorage

The wearing course of the roadway is 5 cm AB-12/F asphalt on top of a binding layer of 4 cm. The adjoining road sections had to be reconstructed as well.

The corroded handrails were replaced by new, galvanized elements. New guard-barriers were mounted along the inner edges of the sidewalks as well.

The missing service stairs and chutes were also replaced at the ends of the bridge. This structure consists of precast R.C. elements.

In order to collect under-surface water, a longitudinal drain was built below the asphalt covering along the deep-line, with outlet tubes at the supports.

20 cm wide elastic bituminous expansion joints were designed at both ends of the bridge in order to avoid later transversal cracks of the covering at the ends of the superstructure.

## 4.2 Pedestrian bridge at Tetőfedő Street

### 4.2.1 R.C. works and repair

Similarly to the road bridge, the works to the pedestrian bridge began with the removal of the handrails, R.C. curbs, the as-

phalt pavement and the waterproofing. The second step was the construction of the anchorage beams at the ends of the bridge. The deviation saddles having been mounted, the superstructure was prestressed.

As the prestressing compensated only for ca. one-third of the extra deflection further improvement of the longitudinal profile was necessary with an equalizing cement mortar layer on top of the deck. The calculation of the necessary thickness required a geodetic level measurement of the deck after prestressing. In order to avoid over-weights on the bridge the thickness was limited between to 7 and 11 cm. The required 2% cross-fall was achieved with this layer as well. To prevent cracks due to shrinkage, a Ø8/200×200 mesh reinforcement was also applied.

The upper edge of the new curb was raised by a few centimetres. This way, water coming from the deck cannot flow directly down the outer side of the bridge onto the motorway below (Fig. 7).

The cracks were injected only after the deflections due to prestressing and the superimposed dead loads.

Finally, after the necessary surface repairs the structure and the supporting column was coated against salt corrosion.

Fig. 6: Bridge at Szövő Street, cable stresses

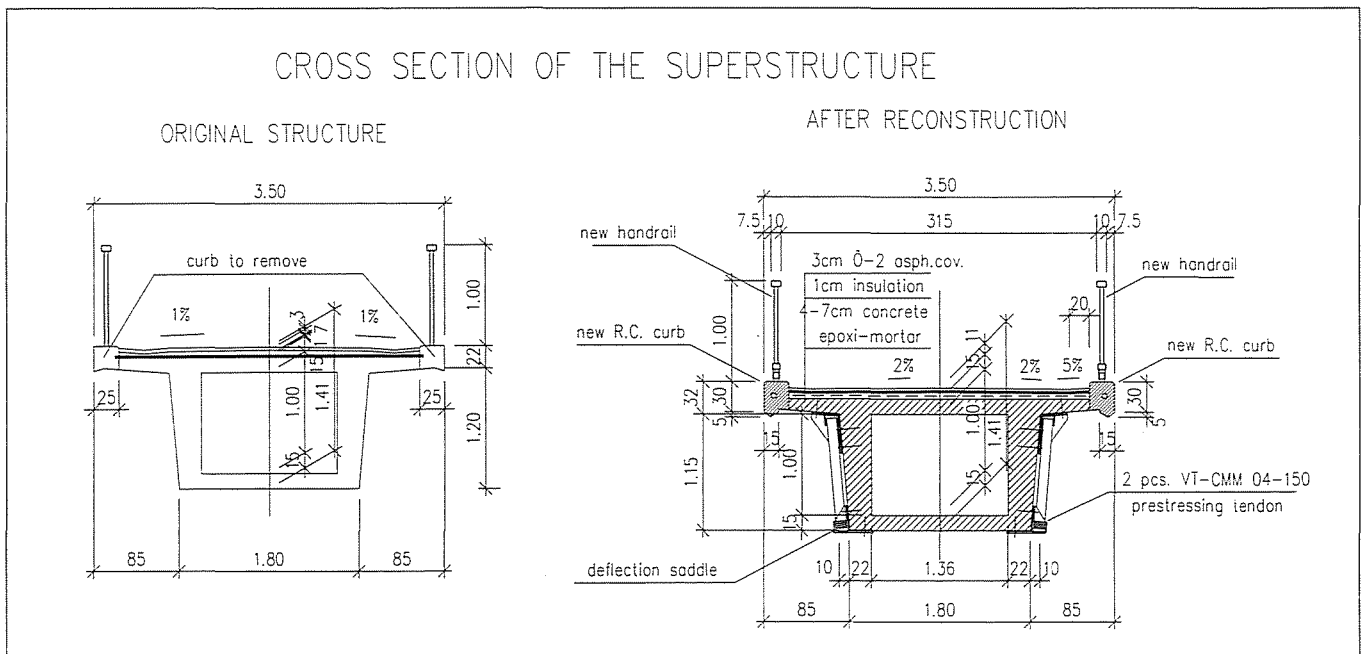




Fig. 7: Bridge at Tetőfedő Street, side view

#### 4.2.2 Prestressing works

The main goal of the intervention was – similarly to the previous bridge – the elimination of cracks and the reduction of the extra deflection. This could be achieved with the normal forces and opposite bending moments arising from the external prestressing (Fig. 2).

As the height of the clearance did not allow cables below the structure, they are led along the webs of the main girder below the cantilevers. As the deviation devices are relatively small structures in the shade of the cantilevers, they do not disturb the appearance of the bridge (Fig. 7).

The lower deflection saddles were mounted onto the main girder at the thirds of the spans (for statical reasons closer to the abutments). Due to the transversal bending, the saddles were fixed onto the bottom slab as well. The upper deviation saddles above the central support were fixed with shear bolts to the web and additional tension rods through the deck. All the saddle structures are galvanized high-strength steel elements (Fig. 8).

Two pieces of VT-CMM 04-150 bundles were combined

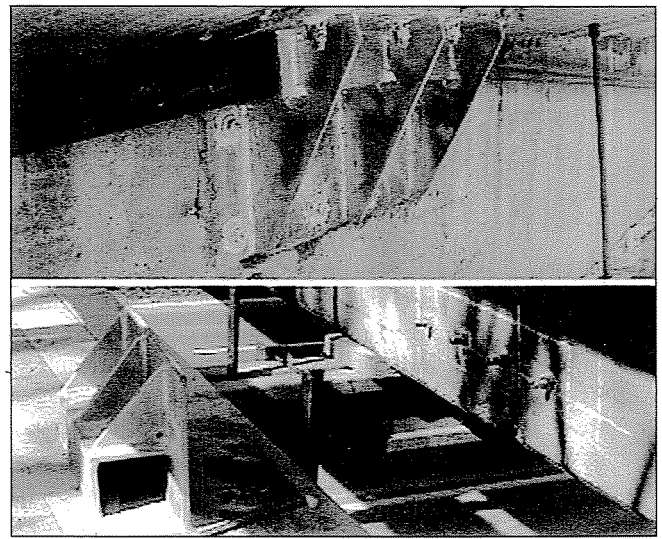
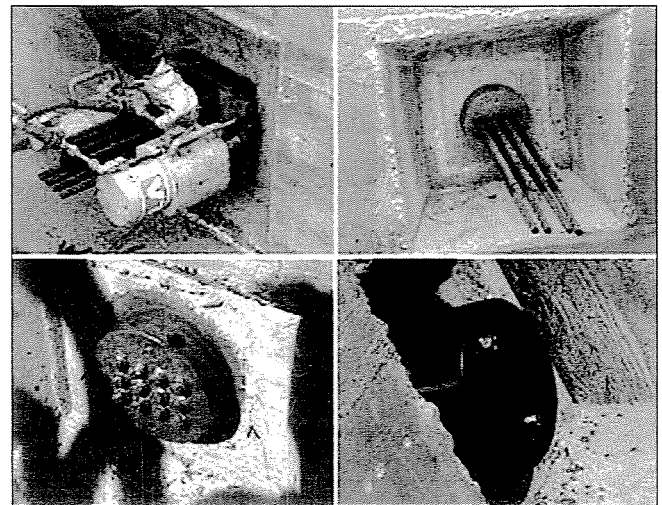


Fig. 8: Bridge at Tetőfedő Street, deviation saddles

Fig. 9: Bridge at Tetőfedő Street, anchorages



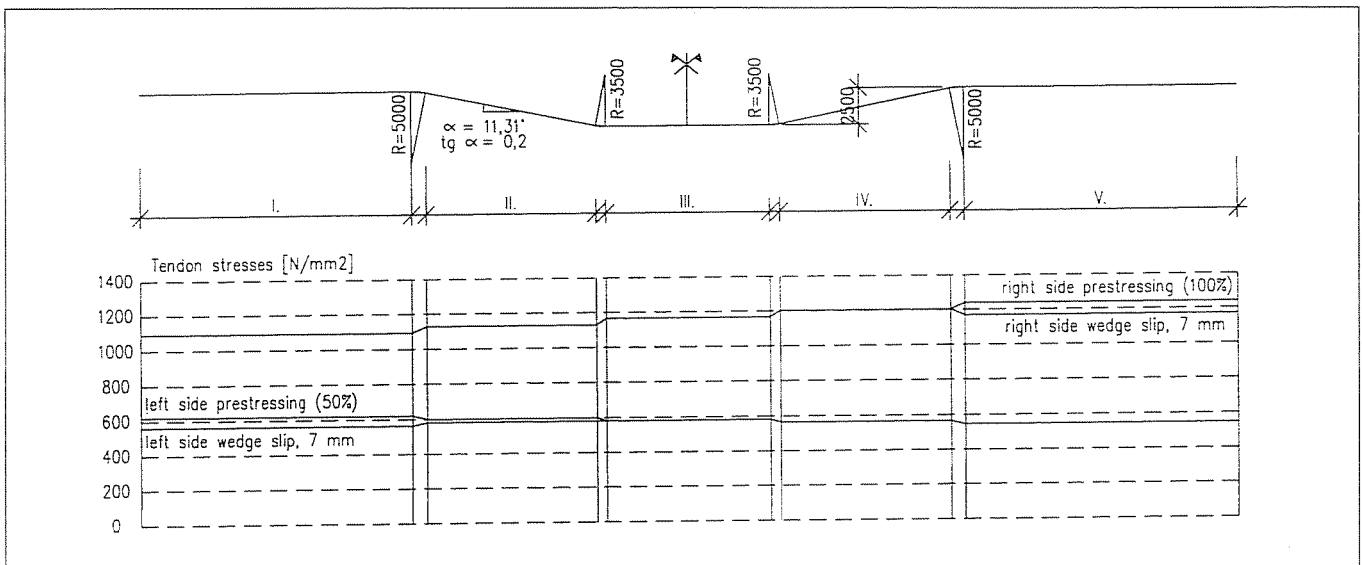
to form a tendon on each side of the bridge. The anchorage is in the new R.C. beam behind the abutments.

The prestressing procedure was similar to that of the previous bridge. The tendon-stresses are shown graphically (Fig. 10).

#### 4.2.3 Finishing works

The deck was coated with a epoxy-resin layer, with a roughened walkable surface.

Fig. 10: Bridge at Tetőfedő Street, cable stresses



As with the road bridge, the corroded handrails and the missing service stairs and chutes were replaced by new ones.

As part of the project, the adjoining section of footway was reconstructed too.

## 5. CONCLUSIONS

Using external cables in new bridge construction as well as in strengthening old ones is widely used nowadays. The method has numerous advantages, such as reliable corrosion protection, minimal friction, easy maintenance and replacement, as well as easy-to-follow stress distribution. These advantages are proved by the two retrofitted highway bridges at Érd. By adding some steel structural elements and external cables these overpasses above the heavily travelled freeway can be used for many years to come. The cable configuration clearly shows

the stress distribution on the structure without disturbing the slender aesthetic appearance of the bridge, which was the aim of the structural engineer.

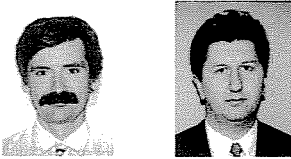
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# CSÖMÖR-KISTARCSA AUCHAN STORE INTER-CHANGE BRIDGE



György Duma – Mátyás Gyurity

*The interchange bridge at the Csömör-Kistarcsa Auchan store is a four-way raised traffic interchange with a cross-shaped ground plan, making it unique in Hungary. It represents a "rediscovery" of the slightly neglected and apparently contradictory cast-in-situ reinforced concrete structure. This turned out to be a good choice for the spatial form dictated by the bridge's unusual ground plan.*

**Keywords:** cross shaped ground plan, slab-and-beam structure, cast-in-situ RC structure, bearing, railings, design regulations

## 1. INTRODUCTION

In 2002, the French hypermarket chain Auchan opened a new shopping centre in the eastern outskirts of Budapest, near the villages of Csömör and Kistarcsa. Access to the store required construction of a new elevated road interchange, providing a link between the areas divided by the Gödöllő HÉV (suburban railway) and Highway 3.

Several options for meeting these basic demands were drawn up, including an underpass, an elevated roundabout and a sunken roundabout. Preliminary studies concluded that requirements would be met optimally with a cross-shaped ground plan, unprecedented in Hungary.

As well as serving the store, the structure also alleviates the area's awkward north-south traffic problems. No less significantly, by directing left-turning traffic on to the bridge, this arrangement also raises the throughput and enhances the safety of the T junction on Highway 30, near the county hospital at Kistarcsa.

The project started with the study plan. In this phase, several structural options were examined:

- prefabricated prestressed concrete beam superstructure,
- composite superstructure,
- cast-in-situ reinforced concrete slab superstructure,
- cable stayed superstructure with two suspension planes.

The final study plan deals only with the latter two versions in detail, based on the cross-shaped ground plan. The client chose the cast-in-situ slab version, and the initial concept was further altered in the course of producing the permission and construction plans.

## 2. THE BRIDGE

The unusual layout of the structure, a four-way interchange with cross-shaped ground plan, was designed to meet the traffic demands and the local constraints already mentioned. The superstructure comprises two merging continuous cast-in-situ slab structures with top-clamped columns similar to legs.

Traffic to the Auchan store is carried in both directions by branch A of the bridge, towards Kistarcsa, perpendicularly crossing the Gödöllő suburban railway line and Highway 3. This is also the main four-lane branch of the bridge, with an overall width of 20.63 m. As well as the through traffic, the exit from the Budapest direction and the entry to the Gödöllő

direction is also served by this branch. The main branch is approached by road embankments.

Branch B is inserted between the suburban railway line and the slightly altered route of Highway 3. This is 10.63 m wide with two lanes, and carries one-way bridge exit and entry traffic in the Budapest-Gödöllő direction. There is no through traffic on this branch, since Highway 3 itself does this. Owing to lack of space, the branch is connected by reinforced concrete ramp sections.

### 2.1 Foundation and substructures

The substructures rest on 13.00 m long CFA piles. There are 103 piles, each one of 80 cm of diameter, with an overall length of some 1.5 km. The need for the piling, apart from the considerable weight of the bridge, arises from the superstructure's sensitivity to uneven subsidence.

The columns are connected to the variable-size pile cap beams via special reinforced concrete bearings, greatly reducing the flexure stresses on the columns from heat expansion and shrinkage. These are single-axis bearings, and the pillars are fastened perpendicularly to the axis of the branch, a fact of particular significance in branch B.

The upper ends of the legs are rigidly fastened, and for a better approximation to the real internal forces, constrained joints were employed in the model.

The pillars of the main branch consist of three groups of 100 cm diameter columns. Most of the columns in the axis of branch B have 100 cm diameter round sections. The intersection of the branch axes, a key load point, is supported by a more robust pillar – B6. This has an overall cross-section of 6.00 × 2.00 m and edges rounded with a 100 cm radius. This support is an integral part of the row of columns in branch B, but is also the central pillar of the main branch.

The load transmitted by pillar B6 is 24,500 kN, transferred to 15 piles. Since pillar B6 is also essentially the centre of motion of the bridge structure, it is fixed to both the superstructure and the pile cap beam. The two slightly elongated neighbouring columns form a structural and visual transition.

The bridge has four abutments rather than the usual two. At the top of the abutments are traversable inspection passages to enable inspection access and ventilation for the key structural elements (expansion joints, bearings).

Branch B is connected by ramps on RC retaining walls –

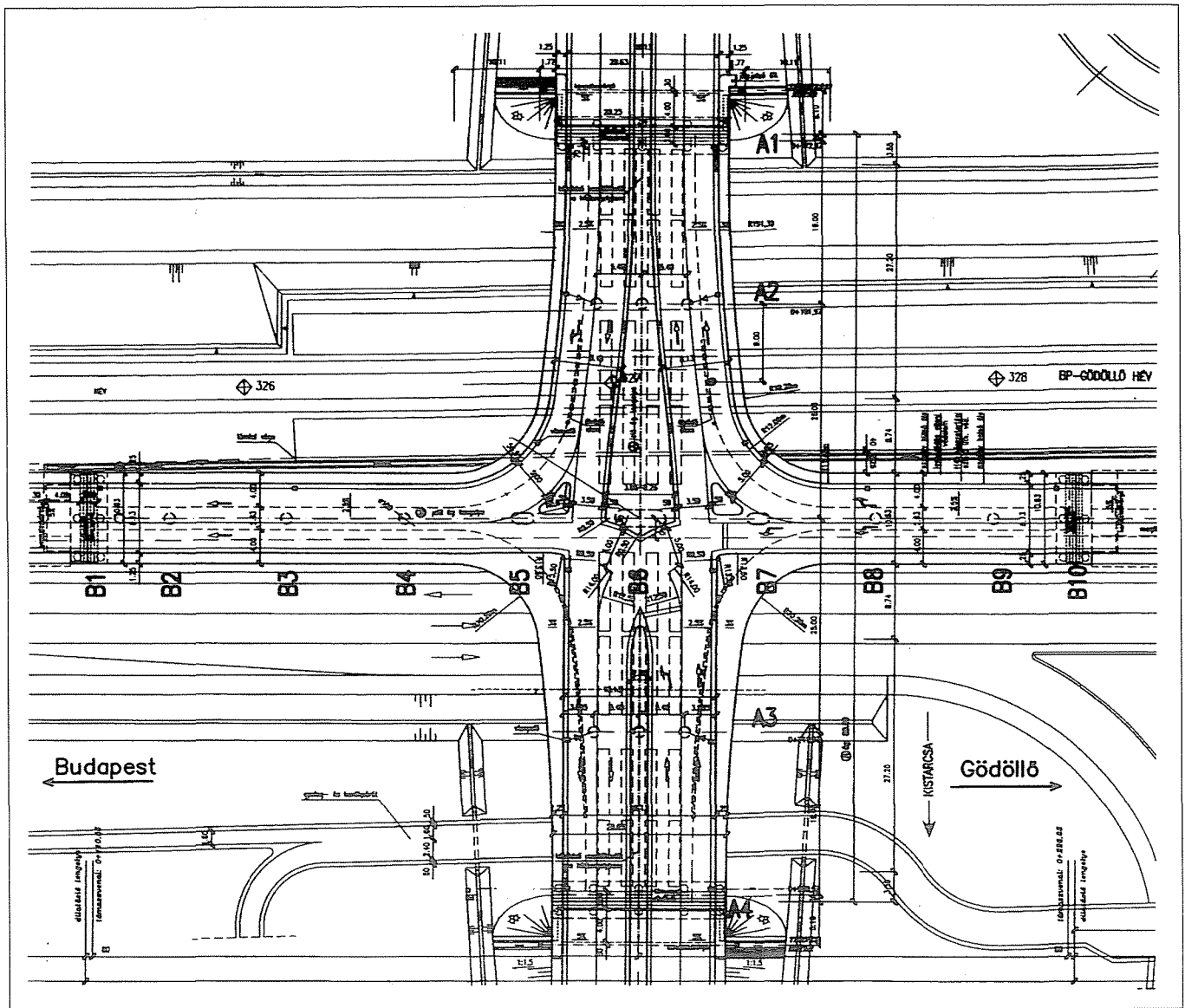


Fig. 1: Plan view

90.00 m long in the Budapest direction and 82.00 m long in that of Gödöllő. The RC retaining walls are 40 cm thick, dilatated at approximately 20.00 m intervals, and without stiffening ribs. The connection of the walls generates a framing effect which provides the appropriate load bearing capacity. To achieve this, the wall footings are tied together by 1.00 m wide cast-in-situ RC beams at approximately every 8.00 m. At the top of the retaining walls is a 30 cm thick cast-in-situ reinforced concrete deck slab up to the ends of the transition slabs behind the abutments.

Omission of the stiffening ribs made the construction of

backfilling straightforward, but necessitated a temporary “under foot” connection of the walls until the interconnecting deck slab was completed. The temporary interconnection was effected by a special tension-compression system consisting of a steel support tube (remaining in the structure) and a removable tensioning strand, arranged to avoid damaging deformations to the walls.

The free end of the transition slab behind the abutment, unlike the usual arrangement, sits on a footing that incorporates a cross-drain and also supports the free end of the deck slab linking the retaining walls. This special arrangement pre-

Fig. 2: Cross section of hinge

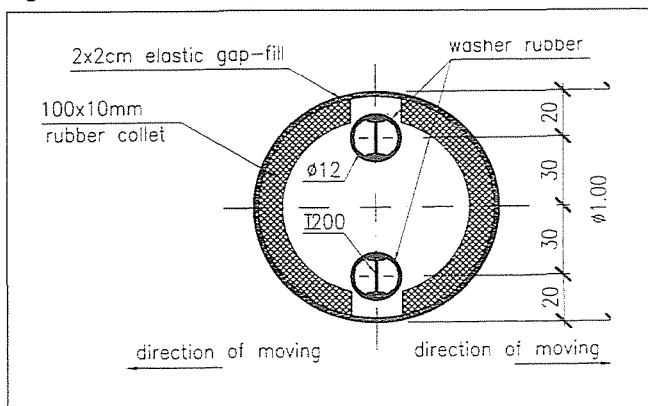


Fig. 3: Bearing prior to concreting

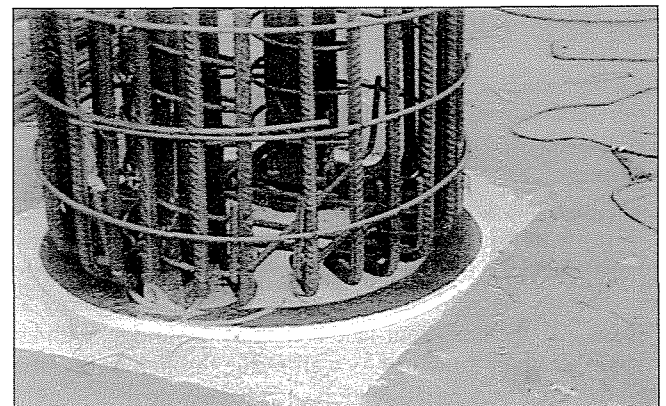




Fig. 4: Pillar B6

vents differential subsidence of neighbouring structural elements.

## 2.2 Superstructure

The bridge superstructure is a continuous, cast-in-situ RC slab structure, with 3.80 m linearly tapering slab cantilevers. The construction of the slab closely follows the planned form, and provides a low structural depth (especially in the outer zones of the bridge) and lends the span structure a lightweight appearance.

The branches of the superstructure run into each other, the four-span main branch (A) having an overall length of 99.70 m, with spans of 19.00 + 25.00 + 25.00 + 19.00 m. The beam stiffened slab form reduces weight by some 16% of the total branch weight. With a characteristic slenderness of  $L/22$ , the slab has a thickness of 1.20 m in the main branch axis and 1.04 m at the base of the slab cantilevers.

Branch B, 117.60 m long, crossing the main branch, has 9 spans and a slab thickness of 1.02 m. The spans are  $9.00\text{ m} + 7 \times 14.00\text{ m} + 9.00\text{ m}$ .

The branches run together by a curved interface in the crossroad zone. The interface of the lower surface of the branches was designed with easily manageable and visually attractive spatial geometrical forms at the crossroads. For example, the lower surfaces of the slab cantilevers in this zone form truncated cones. The mutually conformant profiles of the intersecting branches raise the cross-shaped ground plan into a spatial intersection. The cross fall of the roof section deck slab of the main branch continues in the profile of branch B, and in the same way, the profile of the main branch coincides with the cross fall of the branch B deck at in the intersection zone. Association of the profiles in this way avoids the need for the awkward construction task of reversing deck surfaces.

Fig. 5: Completed foundation

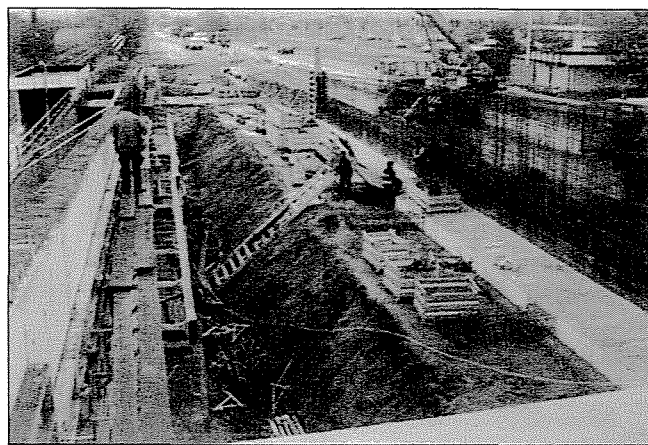
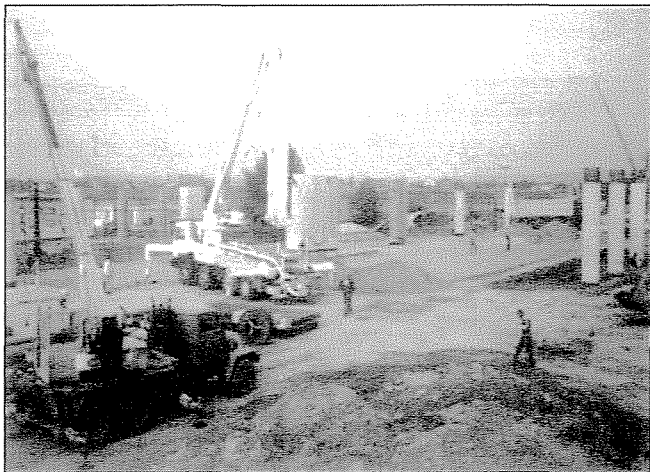


Fig. 6: Construction of retaining wall on Godöllő side

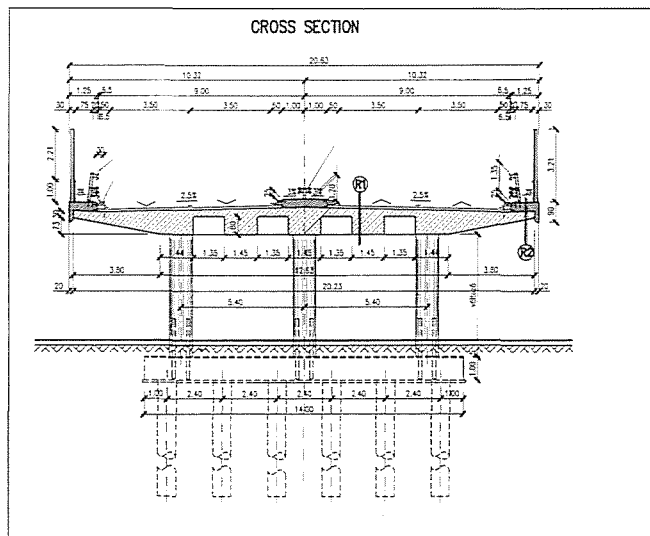
Structural calculations were carried out by realistic modeling using the LUSAS v.13 finite element program suite. Investigation covered the dead and live loads and the loads due to heat expansion and shrinkage. Owing to the structure's sensitivity to differential subsidence, support subsidence differences of the order of 5 mm were also allowed for. In addition to checking the typical cross sections of structural elements, a slab punching analysis was carried out, the bearings at the base of the pillar columns were checked, and crack width and deflection calculations were performed. The calculations conformed to the Hungarian standard MSZ-07-3700-86, assuming load class A.

At the most highly loaded point, the upper zone above pillars B 5 to 7, double-row reinforcement was required in both directions ( $\text{Ø}28/10\text{ cm}$ ).

The high load intensity is the consequence of the spatial geometrical form that caused some construction difficulties because of dense reinforcement. In anticipation of this, considerable design effort was put into simplifying the reinforcement system and make installation of the 430 tonnes of reinforcing steel as straightforward as possible. The specific steel content of the superstructure is about  $165\text{ kg/m}^3$ .

The 2600  $\text{m}^3$  of superstructure concrete was, at the initiative of the contractor, made to grade C30/37 instead of the C25/30 originally specified, so that the formwork (after a strength test on the corresponding test block) could be removed

Fig. 7: Cross section of branch A



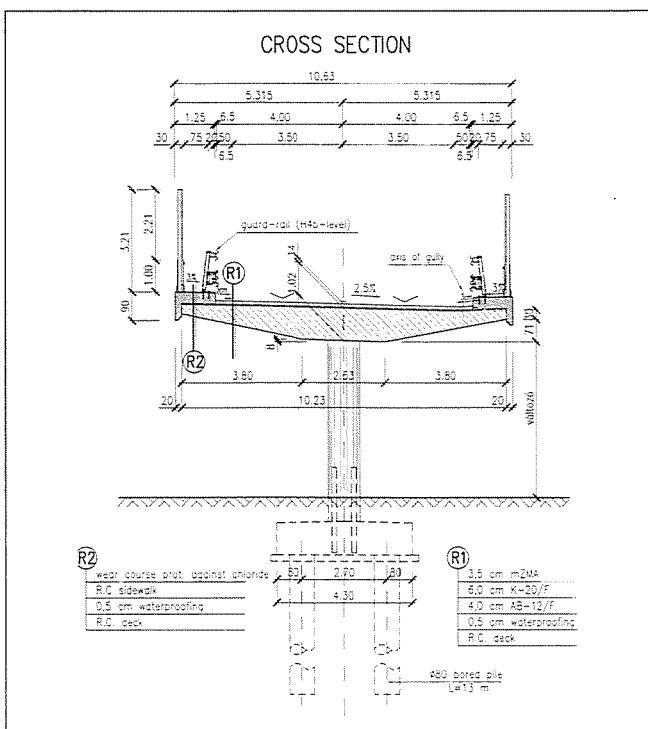


Fig. 8: Cross section of branch B

after 10 days, speeding up the course of construction. Concreting of the superstructure, rather than the originally planned 36 hours, was completed in the record time of 22 hours, with a peak rate of 130 m<sup>3</sup>/hour.

### 2.3 Waterproofing, surfacing, coatings, bridge accessories

Modified bitumen slab waterproofing was laid on some 3400 m<sup>2</sup> of the deck and 1600 m<sup>2</sup> of the interconnecting slabs on the ramp sections. Surfaces in contact with the ground were sealed with a double bitumen coating. The deck was completed with a three-layer, 13.5 cm overall thickness surface corresponding to traffic load class E.

The 1.25 m overall-width cast-in-situ RC, full length maintenance sidewalks, and the variable-size raised separating kerbs and routing islands, were surface mounted.

To prevent the serious consequences of falling from the bridge, an UT 3-1.116 conformant, H4b retention level crash barrier, and a falling cargo protection wall (also providing noise protection) were constructed along the full length of the bridge and the retaining wall beside the suburban railway platform. On the retaining walls and wing walls, grid-unit service railings were fitted. The railings have advanced 90 micron galvanised corrosion protection.

Fig. 9: Crossroads formwork

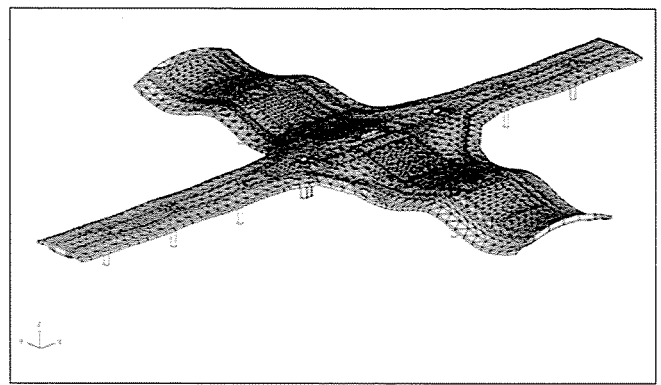


Fig. 10: Distorted deflection figure

The superstructure rests on pot bearings at the abutments, and at its ends there are bridge expansion joints embedded in steel fittings.

To improve safety and traffic management during the winter, an icing sensor station was installed on the bridge.

The exposed concrete surfaces were covered in a crack-bridging protective coating of a type suited to demands (carbonation inhibiting, salt-resistant, etc.), over an area of some 7000 m<sup>2</sup>.

## 3. CONSTRUCTION TECHNOLOGY AND ORGANISATION

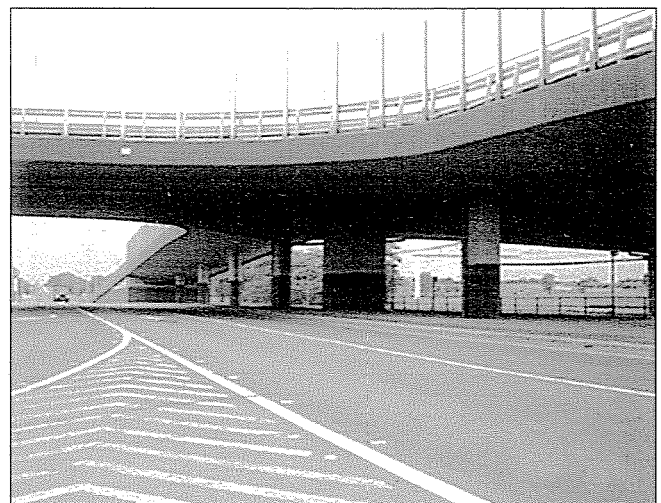
The dominant factor in bridge construction was, as usual, the short time available, which thus became the chief consideration in selecting the construction technology and organising the project. This was further influenced by the relatively cool, wet weather.

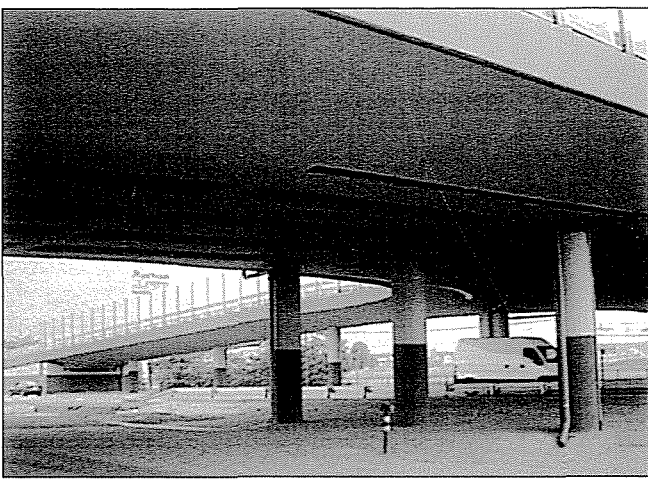
Piling work started in August 2002, and the bridge was temporarily opened to traffic on 13 December 2002, so that the contractors had just under four months to do the job.

The unprecedented short construction time demanded a high level of organisation, rapid, well-grounded decisions, and a high level of expertise from each person involved.

To speed up construction, we endeavoured to modify and improve the original working plans in collaboration with the contractors. Such was the lightening of the main branch and the alteration of the original plan for the retaining wall sections.

Fig. 11: View from below I





**Fig. 12:** View from below II

Despite the difficulties, the contractors coped with procurement of the unusually high quantities of reinforcing steel and formwork panels, mainly through imports.

What turned out to be very useful was the decision to divert Highway 30. This freed the entire building site, and traffic on Highway 30 was less disturbed by the bridge building work.

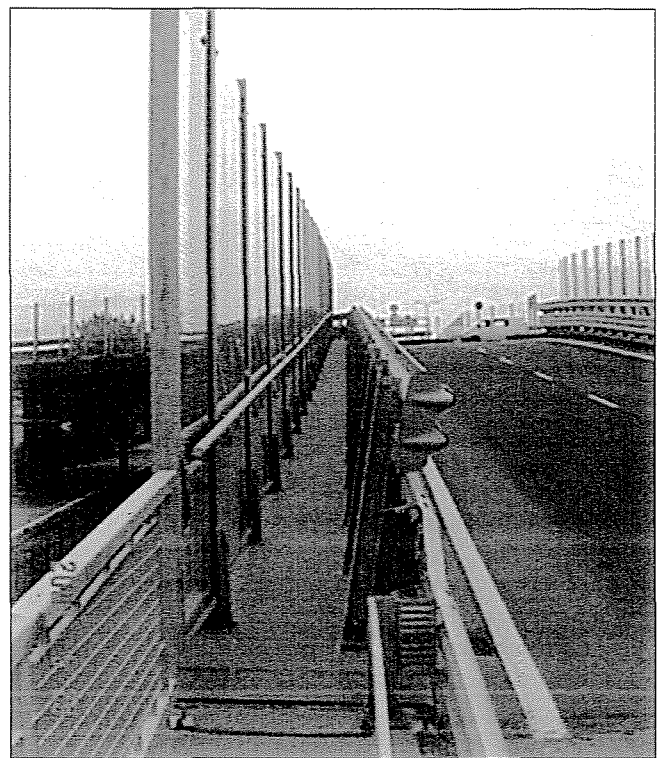
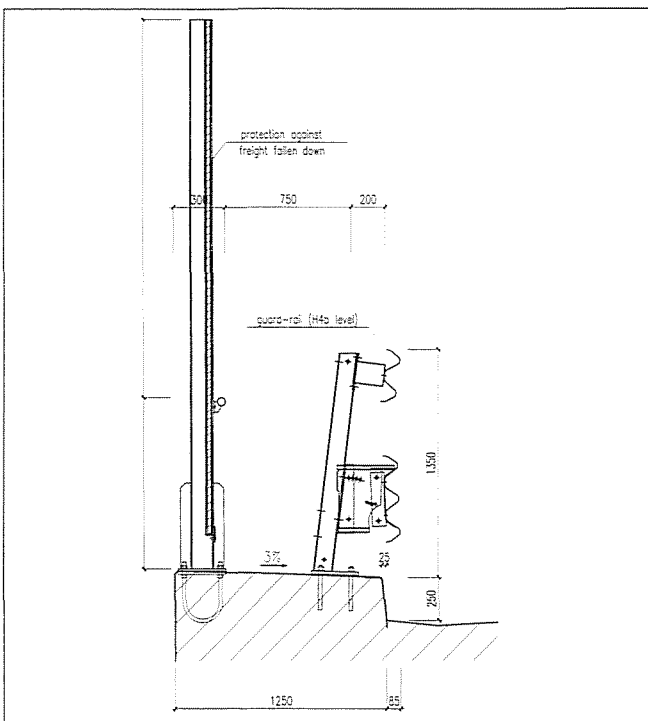
## 4. EXPERIENCES

Some experiences gained in constructing the overpass should be highlighted.

The Csömör Auchan overpass is one of the largest cast-in-situ reinforced concrete road bridges to have been built in Hungary in recent years (7000 m<sup>3</sup> concrete and 700 tonnes reinforcement steel), and was completed in less than 4 months amidst somewhat unfavourable weather conditions.

The special bridge design generated serious dilemmas during the approval process, since Hungarian regulations did not cover this type of traffic interchange, and so did not provide clear guidance for traffic management and safety. The problem remains to be solved by the line.

**Fig. 13:** Cross section of maintenance sidewalk



**Fig. 14:** Railings

There are still many unresolved and indeed contradictory issues regarding safety equipment of all kinds, railings and protection against falling cargo. It remains an important future task to settle these issues satisfactorily, striking a balance between danger to human life and unwarranted extra construction costs.

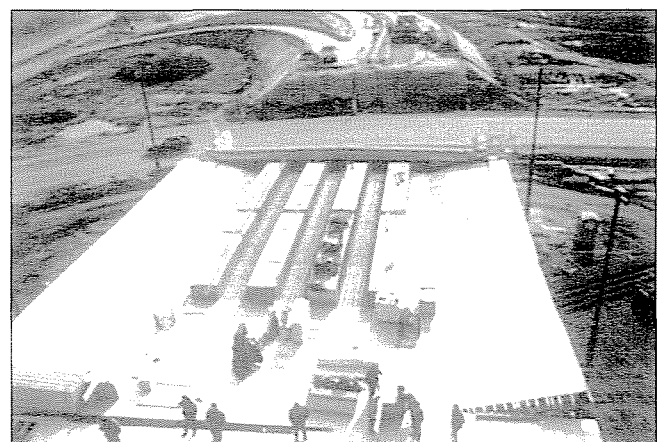
The period of design and construction of this bridge coincided with that of the change of design regulations, occasionally resulting in awkward dissonances.

Much work had to be done in a very short time, requiring a high level of organisation and expertise from all parties concerned, and eliciting praiseworthy contributions from everyone. Nonetheless, it does not set an example for the future, since such a process can generate serious risks and extra costs. To achieve top quality, the potential pitfalls must be analysed; it is an old but unavoidable truism that good work takes time.

A working relationship developed between those involved in the project which went far beyond simple working relations, and greatly contributed to its successful implementation.

Main participants in the Csömör Auchan bridge project.  
 Structural design: MSc Magyar Scetauroute Ltd.  
 Road design: Közlekedésfejlesztés Ltd.  
 Civil-Plan Ltd.

**Fig. 15:** Temporary Highway 3 diversion behind abutment

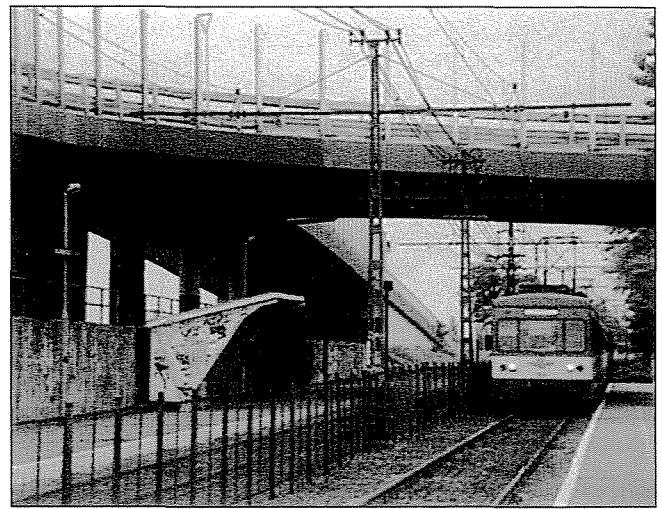




**Fig. 16:** Road junction on bridge

Public utilities design: Víziterv Ltd.  
 Bridge technical supervision: Via-Pontis Ltd.  
 Road technical supervision: Mérnök Ltd.  
 Engineering: Főber Co.  
 Prime contractor: Varpex Co.  
 Bridge prime contractor: Servico Építő Ltd.  
 Contractor: Polár-HÚSZ Ltd.  
 Contractor: MÁV Hídépítő Ltd.  
 Contractor: Egút Co.  
 Developer: Auchan Magyarország Ltd.

Many subcontractors were also involved in the project, and their contributions also merit acknowledgement.

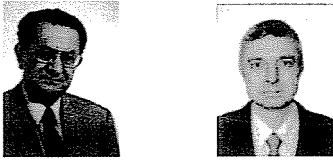


**Fig. 17:** Suburban railway station under the bridge

**György Duma** (1958) certified civil engineer (1982). After graduation, worked in the bridges office of UVATERV Co., and gained professional experience mainly in design of steel railway bridges and preliminary planning of river bridge renovations. Since 1996, has been the Deputy Technical Director of MSc Ltd, in charge of the bridge design division.

**Mátyás Gyurity** (1967). M.Sc. in Civil Engineering in 1992 from the Zagreb University of Science, Faculty of Civil Engineering. Began his design career in the Bridge and Structural Design Department of UVATERV Co., where he learned his trade from excellent teachers. Since 1996, has been an independent design engineer with MSc Kft., and since 2000, chief divisional engineer.

# CONCRETE-TO-CONCRETE INTERACTION BETWEEN THIN HORIZONTAL REINFORCED CONCRETE ELEMENTS AND IN-SITU CONCRETE



Prof. Endre Dulácska – Tamás Simon

*A key point is the interaction between the two concrete layers during the design and construction of horizontal load-bearing structures consisting of thin reinforced concrete elements (floor planks) and in-situ concrete. In 1990 the Consulting Company of Prefabricated Elements (TTI) for the order of the Hungarian Ministry of Works and Housing prepared a proposal for a Technical Advice (TA) under the title "Concrete and reinforced concrete floor planks". The proposal has passed the authorities but due to the untimely death of the organiser at the Hungarian Standard Committee the subsequent advice to designers did not occur. Not long after, the activity of the Committee was terminated and the publication of the Technical Advice was not realised. Previous to the preparation of the TA proposal wide-ranging research was conducted with numerous professional consulting organisations. Since with regard to the topic there are no regulations in force, in order to inform the profession we consider that it would be advisable to give publicity to the proceedings of the Technical Advice proposal and offer some supplement commentary to it.*

**Keywords:** reinforced concrete floor planks, prefabricated floor elements, construction joints, shear, interaction, sandwich structure

## 1. THE PUBLISHED RESULTS OF THE MAJOR RESEARCH AND REGULATIONS

The 1979 annual report of the Technical Mechanics Committee of the Hungarian Academy of Science contains the results of the local research activities completely until 1979. From this the publications regarding our topic were chosen. For the later period, until 1990, international publication research was carried out which we are currently up-dating.

We do not state that the publication list is complete but it contains the most important ones contributions to the subject ascertained following review.

For the older publications (until 1990) we also give a short summary of the papers during their review, Dulácska (1990).

Balázs and Fogarasi (1977) offer a solution for the study of the connection based on a Soviet publication. It deals with the calculation of the teeth and the reinforcement. It mentions that a small shear stress of the connection in case of a rough surface ( $0.2-0.3 \text{ N/mm}^2$ ) is allowed by most of the regulations without reinforcement.

Basler and Witta (1967) inform about experiments which examine the connection between prefabricated and in-situ concrete and offers a design method. The experiments were carried out not only on samples but also on normal size structures.

The shear load-bearing capacity of the connection of the samples, which may be on the account of concrete only was found to be about half of those having a minimum reinforcement.

Betonkalender (1966) introduces different floor solutions, where floor beams and the in-situ concrete work together at the end. The beams may be of reinforced concrete, prestressed concrete, steel lattice structure, corrugated steel plates, etc. Between the beams, filling-shuttering elements are placed onto which the in-situ concrete is placed. These elements finally

work together. (In our country such solutions are the FERT floors, the PPB floors and the prefabricated bridge beams.)

Burkhard (1990) offers a design aid for the case of lattice structured shuttering panels taking into consideration the regulations of the German standard DIN 1045.

Szederjei (1971) was investigating the dowel effect of the connecting reinforcement by the application of a slipping layer between the two concrete layers. She found that the ultimate stress of the dowel is proportional to the square root of the product of the ultimate stresses of the steel and the concrete. The experiments also showed the effect of the angle of the connecting steel reinforcement to the concrete plane.

Dulácska and Szederjei (1972) give design recommendations for the calculation of the load-bearing capacity of the connection based on experiments carried out in the BUTE laboratories together with results of French and American researches. Below 0.1% connecting reinforcement the load-bearing capacity decreases at a higher rate than would be reasoned by the decrease of the reinforcement. Due to this the application of a reducing factor is necessary in the load-bearing capacity below this limit. The appendix of the Hungarian Standard MSZ 15022/4-86 contains this advice in a simplified form.

Fouré (1970) introduces an experiment to determine the tangential load-bearing capacity in case of prefabricated and in-situ concrete having connecting reinforcement. In the load carrying both the concrete and the reinforcement take a role.

Goschy and Balázs (1960) investigate theoretically a flanged simple support beam heaving a T-section, where the flange is concreted on the site to the prefabricated web. The theoretical results were checked by experiments. The summary of their results are as follows:

- the Mohr condition of failure gives a good result for the interaction of the co-working of a reinforced concrete structure,
- the flexural and shear safety must also be checked,
- the shear (connection) safety should be at least as much as the flexural.

Hofbeck, Ibrahim and Mattock (1969) investigated experimentally the shear load-bearing capacity of elements having connecting reinforcement perpendicular to the surface. The investigation consisted of one-time concreted, cracked and concreted-in-parts elements. They concluded that the crack pattern is of 45° to the theoretical shear plane and the connecting reinforcement is in tension. In case of cracked or concreted-in-parts elements the connecting reinforcement is in flexure and the concrete suffers cohesion and friction. The ultimate stress is about 80% of the one-time concreted elements, which is much higher than the calculable value.

Mattock (1974) was examining the load-bearing capacity of the sheared and under friction surface. He was researching the behaviour of the reinforcing steel connecting two concretes at an angle. He prepared the angled surface in such a manner that the reinforcement would not be under shear bending only but would also to suffer from tension. The experiment was with one-time concreted and cracked samples. This study states that the precracked element fails by slippage along the crack, while the one-time concreted element fails through the formation of angled cracks and shear failure occurs. The safety is acceptable compared to the calculated values.

Orosz (1963) introduced laboratory experiments carried out on prefabricated (having normal or prestressing reinforcement) load-bearing elements combined with normal and lightweight concrete layer. They stated that between the two types of concrete no scaling was observable even when the state was near to failure (minimal connecting reinforcement was used). It was verified that even using weak concrete and lightweight concrete an acceptable load-bearing capacity could be attained. Fatigue tests were not carried out. The effect of shrinkage is much smaller than could theoretically be expected.

Orosz, Tassi and Ódor (1984) were examining 250 mm thick floor elements made of prestressed, core panels having a span of 5450 mm, a thickness of 50 mm with 200 mm in-situ concrete layer. The biggest slipping stress was 7.4 N/mm<sup>2</sup>. No relative movement (slipping) between the two concrete layers could be observed. The panels can be designed according to the cracked reinforced concrete cross section. The samples suffered from flexural failure where the core panel and the concrete layer worked together as one body. When the crack width was 0.2 mm the load was 25.8 kN/m<sup>2</sup>. Finally it was stated that the safety of the PR floor is acceptable.

Paulay and Loeber (1974) discusses the theory of the shear-friction of concrete. They did not take into consideration such cases when an effect wants to separate the two layers from each other. The research program also contained experimental investigation.

Pommeret (1970) experimentally examined the connection between prefabricated panel and in-situ concrete in case of looped connecting reinforcement. The result was that during the relative movement following the failure of the concrete teeth, the load bearing is constant.

Pommeret (1970) carried out experiments to investigate the load-bearing capacity of the connection between prefabricated elements and in-situ concrete. The connection, which was reinforced by using mild steel, following the failure of the concrete teeth showed a constant load-bearing capacity. In cases where the steel used was of high tension type (UTS = 670 N/mm<sup>2</sup>) following a relative movement of 10-15 mm the connecting steel bars failed. Due to these observations the capability of plastic deformation of the steel is an important condition of usability for such connections.

Regles B.A.E.L.80 (1979), the French reinforced concrete regulation, and it's explanation deals with structures made out

of in-situ concrete laid on prefabricated plate (shuttering core). Such causes do not require connecting reinforcement if:

- the load is distributed,
- there is no dynamic effect,
- there is no impact like load,
- there is no concentrated load,
- the surface of the shuttering core is rough,
- there is no tension on the connecting surfaces (there is no separating force),
- the shear stress is less than 0.35 N/mm<sup>2</sup>.

If connecting reinforcement is necessary, then it is to be designed in the same way as given in the Hungarian Standard, but it is allowed to subtract from the slipping shear stress the stress arising from loads causing compression of the two concrete layers (e.g. dead load, live load).

Schäfer and Schmidt-Kehle (1990) introduce 23 experiments for the behaviour of sandwich structures made out of a prefabricated shuttering core and an in-situ concrete layer. Nineteen samples were prepared with connecting reinforcement and in four of them no connection between the two concrete layers was applied. In 21 cases out of the 23 the structure failed due to the failure of the compressed zone, while in two cases the connecting plane suffered shear failure. Out of the samples having no connecting reinforcement none have failed due to the shear on the connecting plane. The calculated shear stress at failure was 0.8–0.86 N/mm<sup>2</sup>. In case of those samples which failed by shear on the connecting surface the connecting reinforcement rate was between 0.37 and 0.6, and the calculated shear stress at failure was 1.76 and 2.15 N/mm<sup>2</sup>.

Seiler (1989) introduces the regulations of the – at that time new – German standard DIN 1045:1988 which explains from chapter 4. the problems with respect to the connection of shear reinforcement between the prefabricated and in-situ concrete elements. Under certain conditions (e.g. if the traffic load is less than 5 kN/m<sup>2</sup>, where the concentrated load is less than 7.5 kN, the connecting surface is rough and at the end of the prefabricated element there is constructional reinforcement to avoid de-lamination of the layers, the deflection is less than 1/500 and if the shear stress is less than the half of the allowed lower shear stress) the standard allows the connection of non-reinforced co-working of the concrete layers.

Silfwerbrand (1986) in Sweden examined concrete elements made out of two layers consisting of roughened surfaced old concrete with new concrete laid on top. The simply supported two-layered elements in bending without connecting reinforcement between the layers showed the same ultimate stress as those which were homogenous. Along the connecting plane the resistance against fatigue proved to be lower. The strain caused by tension working against shrinkage proved that in case of a well-prepared accepting surface the shrinkage does not decrease the load-bearing capacity of the connection. In the case of a smooth accepting surface the co-working of the two concrete layers is decreased.

Soubret (1971) prepared experiments with shuttering core beams and panels. The connection between the two concrete layers was ensured by steel lattice. The deflection was also examined over time. The sample elements behaved properly.

J. Szalai (1967) prepared theoretical investigations of sandwich structures. He based the calculation of creep on the theory of Dischinger. According to the newer theories and experiments however, the change over time is described not by the 1/e' relation, but by 1/e √t. The Hungarian Standard was also changed to this new theory. Most probably this is the reason why during the experiments the actual effect of shrinkage was much smaller than calculated (see Orosz (1963)).



Walraven and Reinhardt (1981) were dealing with the shear load-bearing capacity of a sheared-frictioned surface. They worked out a theory to take the effect into consideration and checked it by experiments. They attempted to eliminate the dowel effect of the connecting reinforcement in two ways. Firstly by applying a rubber pipe to the bars near the sheared surface and subsequently by using not internal, but external reinforcement. According to the experiments the slipping load-bearing capacity reached a maximum when the relative movement was between 0.2–0.4 mm. The dowel effect of the connecting bars is neglectable compared to the effect of holding together the surfaces.

## 2. THE HUNGARIAN TECHNICAL ADVICE

Based on the above publications and the local (Hungarian) regulations a technical advice (TA) proposal was prepared that may indicate guidelines for present day designers even. The TA has not come to force. Since the time the proposal of the TA has been prepared further research was carried out in the topic about which we would like to offer some up to date information.

## 3. SOME IDEAS CONCERNING THE TECHNICAL ADVICE PROPOSAL

With regard to the TA proposal we would note that:

- for the case of the slipping tangential stress (choosing  $a_r$ , the adhesion coefficient) the results of the newest research (Simon (2002)) should be taken into consideration.
- the arrangement of the connecting reinforcement must be taken into consideration. It means that the shear stress diagram should be covered.

## 4. OVERVIEW OF SOME IMPORTANT RC CORE SHUTTERING ELEMENTS

Below we would like to highlight those important RC core shuttering elements which were prefabricated in Europe. We only attach figures of the surface structures applied in building engineering practice.

- *Katzenberger panel floor (Austrian)*. Has not had widespread use in Hungary. The compressed flange and the lattice was made out of steel profiles instead of hot rolled sections. It's advantage is to have more rigid reinforcement. (Fig. 1).
- *FERT beamed floor (Italian)*. Due to its easy handling (the beams can be positioned by hand) this became a very popular floor structure, used mainly for bungalow-type dwelling house construction.
- *FILIGRAN (German)*. Similar to the FERT beamed floor but without filling elements between the beams. Practically a beam floor can be constructed by using them.
- *PR floor (Hungarian)*. The first locally developed prod-

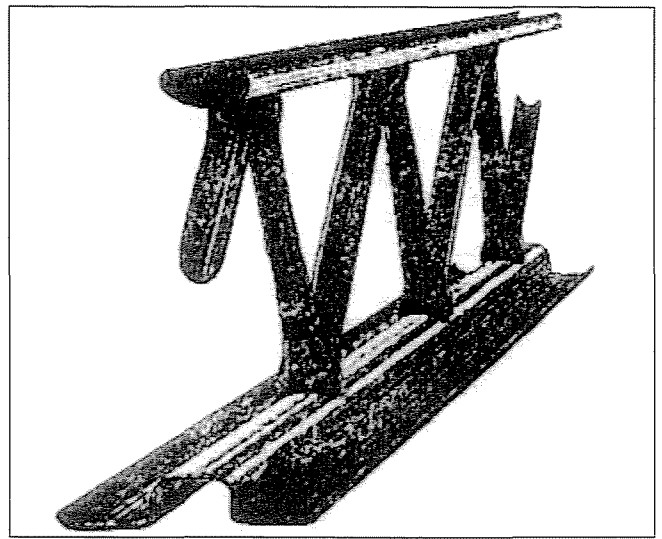
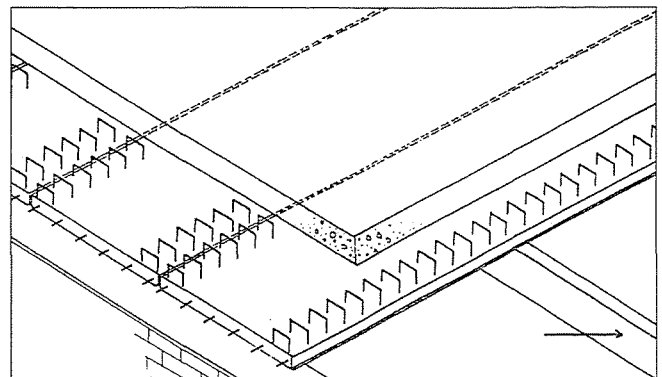


Fig. 1: The patented Katzenberger steel lattice reinforcement

uct of this type. It was made out of 60 mm thick prestressed concrete plates with a connecting stirrup system. The top concrete layer ensures that the compressed zone can be prepared by internal supports (Fig. 2).

- *Sandwich-panel structure (Hungarian)*. A core panel prepared with mild steel reinforcement which was developed for civil engineering structures because of its more robust characteristics than similar panels used in building construction.
- *IVS shuttering plank-floor (Hungarian)*. Mild steel reinforced core-shuttering which could mainly be used in building construction (Fig. 3).
- *Underside cellular concrete covered prestressed-core floor plank (Hungarian)*. Developed by YTONG Hungary and was supplied for many years until 2000. YTONG boards, on which the prestressed core was poured, covered the bottom surface. The lower layer ensured a uniform surface with YTONG masonry for rendering, and allowed a layer to lead electric cables. During construction it had to be supported and cambered before top layer concreting. The two layers of concrete are connected only by reinforcement at the ends of the planks. It was only used in building construction on a relatively small scale due to the high price (Fig. 4).
- *Masterfloor (Hungarian)*. During construction this product also needs support and cambering. The most important difference compared to FERT floors is that the bottom flange of the beams is not prepared in ceramic "slippers" but the steel bars of the bottom web are embedded only in the concrete. The compressed zone is made out of cold-formed U-section steel. The floor is very similar to FERT floors.

Fig. 2: Sketch of the PR floor



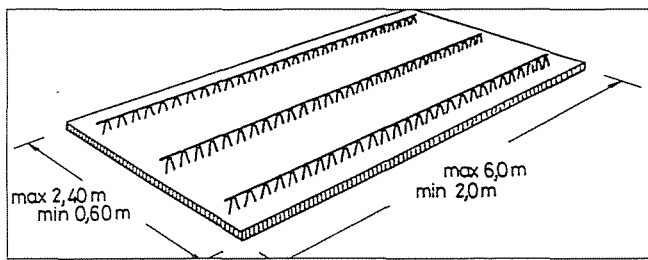


Fig. 3: Sketch of the IVS floor



Fig. 4: The YTONG prestressed-core floor-plank



Fig. 5: Profipanel, the mild steel reinforced shuttering-core

- *Profipanel (Hungarian)*. Although the factory is being produced it is only a few years old the mild steel reinforced, computer-aided production of the shuttering-core became very popular. The two concrete layers produced at different times are connected by longitudinal steel lattice (Fig. 5).

During the development of the products most probably the biggest problem was to determine quality of the co-working of the prefabricated reinforced concrete core and the concrete layer poured on it without a connecting reinforcement. This question is discussed later in more detail together with the problems which arise. One of the experiments being carried out at the Budapest University of Technology and Economics is aiming to determine the load-bearing capacity of such a concrete joint in case of different, documented and reproducible cases. (Simon (1999a), (1999b))

## 5. THE IMPROVEMENT OF THE DESIGN OF SHUTTERING-CORE FLOOR RC ELEMENTS

Due to the above-described uncertainties it is necessary to improve the concrete-to-concrete interaction calculations to be carried out in case of determination of the interacting capability of an old and a new concrete.

## 5.1 INTRODUCTION OF SAND CIRCLE DIAMETER

We can completely agree with the sandwich structure naming of such structures (Polgár and Stairits (2001)), since the prefabricated lower concrete core and the concrete layer laid on it on site can be considered as two different materials. Amongst other factors their change in size – even if exactly the same type of concrete is applied on top (which is almost impossible) – will occur at different times. The co-working of the two layers however is inevitably important from the point of structural design. The regulations and figures introduced in this paper solve this question either by a connecting reinforcement or by regularly formed anchoring teeth (MSZ 15022/4–1986). None of the regulations allow the consideration of taking into account the shear load-bearing capacity of the adjoining concrete surfaces when calculating the co-working of the two layers in case there is no normal (compressing together) force acting on the two layers. The reason for this is as follows: the shear load-bearing capacity of the joining concrete surfaces depend primarily on the surface roughness of the initially prepared concrete, the effect of which is not adequately backed up by experimental experiences (Dulácska (1990)). On the other hand the value of the shear load-bearing capacity can only be determined if we introduce a factor which defines the surface roughness of the accepting concrete. By identifying this the load-bearing capacity of the connection is calculable. Apart from the surface roughness the following factors influence the load-bearing capacity of the connection in the case of an accepting concrete surface which is clean:

- the core's:
  - strength,
  - particle size distribution of the aggregate,
  - aggregate type,
  - the age of concrete at the time of pouring the top layer,
  - porosity,
  - moisture content.
- the top layer's:
  - consistency,
  - particle size distribution of the aggregate,
  - strength,
  - shrinkage.
- application of a "trick", such as introducing an adhesive on the accepting surface, or the application of steel fibres into the top layer of the core concrete and raking the surface, etc.

From the above some of the parameters regarding the core concrete surface can be ensured during the manufacturing process by utilising quality control, can therefore be certified and consequently be used in calculations which should soon be developed. Naturally, when preparing the top concrete layer adequate quality of the concrete must also be ensured. Even wider research is needed to determine the effect of the application of any of the above mentioned "tricks".

Here would we like to refer to the articles (Simon (1999a), (1999b), (2002), (2003)) which discuss the problems regarding the co-working of the two concrete layers. From the results given therein it can be seen that the surface roughness of the prefabricated concrete is more important than any possibly applied adhesive, which only plays a role if the surface is smooth. It must be mentioned that J. Gilyén has contributed very valuable information (Gilyén (2000), (2002)). On the 3<sup>rd</sup>–6<sup>th</sup> pages of the same paper's December volume in 2000, L. Polgár also touched the topic (Polgár (2000)) to which, in the

same paper but in February of 2001, volume on pages 6–8. E. Dulácska makes a further observation regarding the topic (Dulácska (2001)). For the determination of the surface roughness of core concrete the experimental method developed with the help of the Budapest University of Technology and Economics, Department of Construction Materials and Engineering Geology, can very well be used. It is the adaptation of the sand patch method, well known from road construction. The main point is that sand with a certain fineness and volume is smoothed in a circle fashion on the surface, and from the diameter of the circle it is possible to obtain a SCD (Sand Circle Diameter) number which is characteristic of the roughness of the surface (Simon (2002)).

## 5.2 SIGNIFICANCE OF THE DISCUSSION

What is the significance of the co-working of two such concrete layers? The design and construction engineers can contribute more to the answer than most as they are work with the subject in everyday practice. Basically, the major advantages are as follows:

- the cost saving of shuttering,
- homogenisation of the floor structure, strengthening the plane effect,
- increasing rigidity and so decreasing the deflection,
- and at last but not at least, increase of the load-bearing capacity.

Furthermore, if this co-working capability can be defined then the connecting reinforcement can be decreased. The total elimination of such reinforcement would be dangerous, however, due to several reasons not discussed here (which statement does not concern the prefabricated bridge and floor beams and the floors which are constructed out of an overlay of concrete on beams and filling elements). Naturally the experience may be useful in case of construction joints prepared for any reason –not listed here.

## 6. CONCLUSIONS

In case of reinforced concrete core-shuttering panels and other such structures where we would like to take into consideration the co-working of old and young concrete, we can only do it if we have roughened the surface of the old concrete, measure and register the roughness. We have introduced a Technical Advise proposal, which, despite some criticisms, is usable for the practicing engineers. With regard to the measurement method for the surface roughness and the determination of the co-working of the two concrete layers depending on the surface roughness (SCD number) there are experiments currently undergoing in the BUTE Department of Construction Materials and Engineering Geology. Although we did not mention in the discussion the cleanness of the old concrete surface is of extreme importance. The effect of porosity and compressive strength of concrete to the shear strength of the construction joints also remains to be examined.

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# MECHANICAL MODEL CHANGES DURING LIFE TIME OF STRUCTURES



Prof. Péter Lenkei – Assoc. Prof. András Meskó

*The changes of the mechanical models for structures in use are influenced by several factors. The analyses of these factors are presented. The structures investigated are made of concrete or the necessary interventions made are using concrete structures. Special attention is given to reuse of old buildings. Short case studies are presented. The necessary interventions and the consequences are discussed. It is concluded, that each case has some specific features, the interventions are further changing the structural model, and almost in all cases several different solutions exist.*

**Keywords:** Mechanical models, material degradation, reuse of old buildings, lifetime extension, building rehabilitation and reconstruction, monument preservation.

## 1. INTRODUCTION

Over recent years the structural engineers tasks have been progressively more directed towards extension the lifetime of buildings which have reached their design life-time, or, the extended use of older buildings associated with alterations (so-called “brown field” investments).

In addition to this greater attention is now paid to the preservation of monuments which forms an important part of the national heritage. The upgrading, rehabilitation and reconstruction of historic buildings and/or infrastructure are carried out not only for reasons of patriotism but often motivated by economic concerns such as tourism. The responsibilities of the engineer are therefore often interconnected in complicated ways with these objectives.

Included in the wide range of possible actions with regard to this topic are factors attributable to the original structure such as bad quality of construction, aging of the structural materials and components, deterioration caused by non-design use and changes to the functionality of the original building. In addition other factors can be mentioned such as value added reconstruction, monument preservation, changes of the codes for urban planning and environmental protection and structural design, and last but not least the extension of lifetime of buildings.

At the same time, due to the increased aggressiveness of the atmosphere, the degradation process of the structural materials (concrete, steel, mortar, brick, stone, etc.) of old buildings, and consequently, the aging of these materials are accelerating and the effective lifetime is decreasing.

On the other hand, maintenance problems in old buildings (e.g. leakage in the Paris Pantheon) and also in comparatively younger ones, leading to serious structural problems should not be neglected either.

There are considerable uncertainties involved in the assessment of the influence of the above interconnected factors because systematic monitoring data recording the changes over time of the condition of the structures and/or of the material aging is very scarce.

The climatic changes of the world (Lowe, 2004) lead to even greater uncertainties as the statistical evaluation of the past does not necessarily determine with the required accuracy the future values of the earthquake magnitude, the levels

of high floods and groundwater or the maximum wind effects. It is probable that the wind effects will increase in the future and this would lead to some consequences in relevant codes of practice.

Another effect of the material degradation that should be taken into account is the change of the mechanical models of structures. A few such examples include the deterioration of bond between the compound structural materials/elements, the choke up of hinges, the change of fully clamped connections into partial ones and the earlier formation of plastic hinges.

## 2. THEORETICAL CONSIDERATIONS (LENKEI, MESKÓ, 2004)

From the engineering experience, the following theoretical considerations could be drawn:

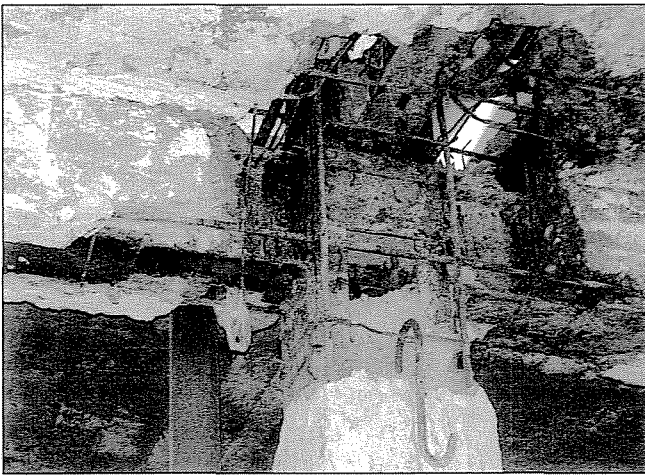
- The originally assumed linear elastic behaviour of old structures due to unfavourable effects during long use is not applicable. Often some parts exhibit plastic behaviour, hinges and/or yield lines are formed and the mechanical model has essentially changed.
- Each rehabilitation, reconstruction or upgrading is either a small or a significant intervention in the structure. These interventions usually change the mechanical model.
- What could be the extent of the structural intervention, for instance for achieving compound behaviour of the old and new structures? The answer includes minimising the induction of overstressing in the old structural elements.
- Does the original compound behaviour between the parts of the old structure still exist and to what extent? If the answer is negative, then can it be achieved?

## 3. SHORT CASE STUDIES

### 3.1 Structures of historic monuments

#### 3.1.1 Office building of Bauhaus style

The new owner of a Bauhaus style office building in Budapest, being a registered monument, decided to modify the building according to his requirements. An investigation into the



**Fig. 1:** Opened joint of the high alumina concrete structure



**Fig. 2:** Inner view of the crypt



**Fig. 3:** Detail of fresco

condition of the concrete structure made of high alumina cement ensued. *Fig. 1* shows the encasing of the steel profiles of the columns and beams which had to be investigated for bond and corrosion protection. The question was whether the original mechanical model could be assumed or could be re-created at least partly. The investigation discovered that the strength reduction of the high alumina concrete had finished. A special treatment of the concrete was proposed which included a new concrete sheathing around the columns which

would produce a constraint that would protect the steel profiles and assure the necessary simultaneous behaviour of the old and the new concrete.

Naturally the removal of the old concrete together with the surface treatment of the steel profiles followed by re-concreting would be safer, but this solution would be longer and more expensive. The decision was in the hands of the owner.

### 3.1.2 The paleo-Christian crypt

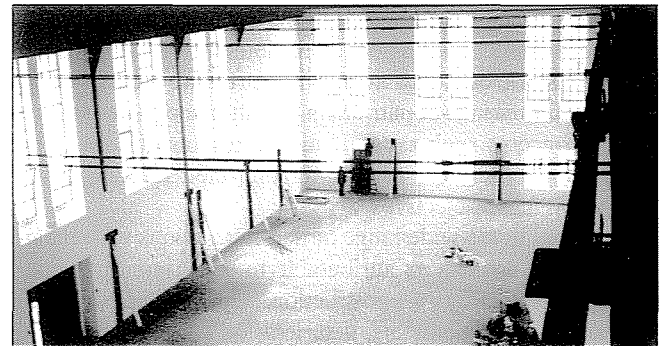
The Paleo-Christian cemetery in Pécs is from the fifth century A.D. and is designated as a World Heritage site. In the case of the crypt of Peter and Paul (*Figs. 2 and 3*) two tasks had to be solved.

One of the tasks was to clear without damage the old crypts (walls and vaults) from the 75-year old protecting and ventilating concrete cover. The other task was to erect a new, modern protecting structure and form an exhibition space. This was achieved using a flat slab structure, supported by reinforced concrete columns, encased in steel tubes.

## 3.2 Reuse of old buildings (so-called brown field investment)

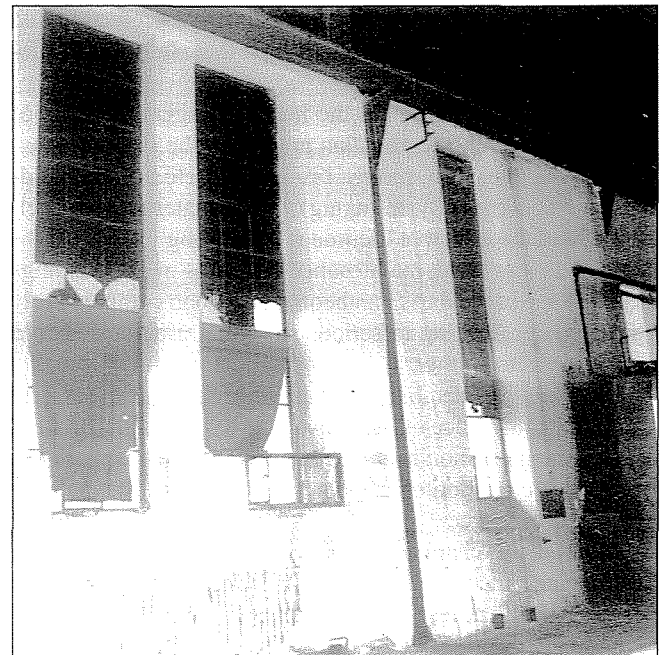
### 3.2.1. The bank-engine shop for a mining shaft

Due to the termination of coal mining in the region of the City of Pécs, as was common throughout Europe, a bank-engine shop had to be converted into a warehouse with an 11m high clearance. The original layout of the shop (*Fig. 4*) was ~18×40



**Fig. 4** The view of the old shop interior

**Fig. 5** The view of the interior after alteration



m in plan. The outside brick walls were ~650–800 mm thick. Located on the ground floor were machine and equipment foundations elevated to 3 metres on an intermediate floor which was supported by columns. Additionally this floor had openings, anchor places and mass concrete for the equipment. The roof structure and the bridge crane was supported by a double-hinged metal frame with bolted connection built in the walls and with steel ties built in the intermediate concrete floor.

The building in its original form was not suitable for an 11 m high warehouse. The intermediate floor therefore had to be removed, the ties were replaced over the bridge crane beams and the bridge crane was dismantled (Fig. 5). The horizontal connecting function of the intermediate floor was replaced by side supports.

### 3.2.2 The Camber Vaulted Floor

A beer-drinking establishment on the ground floor of a 100 year old building was converted into a bank branch office. The design load on the floor over the cellar was 12.5 kN/m<sup>2</sup> before conversion. The camber vault floor consisted of vaults supported along the longer spans by steel I- profiles and along the shorter spans by the cellar walls (Fig. 6). The steel beams were highly corroded but the load-bearing capacity, in spite of the high load, was sufficient. The reason was the following: in the diagonal direction, the section of the ~1.80\*4.80 m vault cells formed elliptical arches with a camber of about 180 mm. The approximate analysis showed that the compression stress in the arch was 14.7 MPa, which seems realistic for a good quality brickwork structure. Consequently, the ring laid vault working as an arch could significantly decrease the overload on the corroded steel beams.

To suit the new function, the building was altered. The dead and live loads were removed from the vault which resulted in free space above. A new concrete floor on corrugated steel plates was built, supported by new steel beams but not loading the vault.

## 3.3 Necessary intervention on account of bad workmanship (Lenkei, 1988)

The multi-storey building of a big consulting engineering office consists of reinforced concrete frames with 13 m span beams on the upper floors, forming large office rooms. Due to early removal of the formwork and low quality concrete, the beams developed large deflections. The contractor tried to correct this blunder but achieved only an even bigger blunder. The contractor's solution to level the deflection was to apply thick layers of concrete over the top of the beam at mid-span and thick layers of plaster to the underside of the beam near the columns. As a consequence the load-bearing capacity of

Fig. 6: The Camber vaulted floor

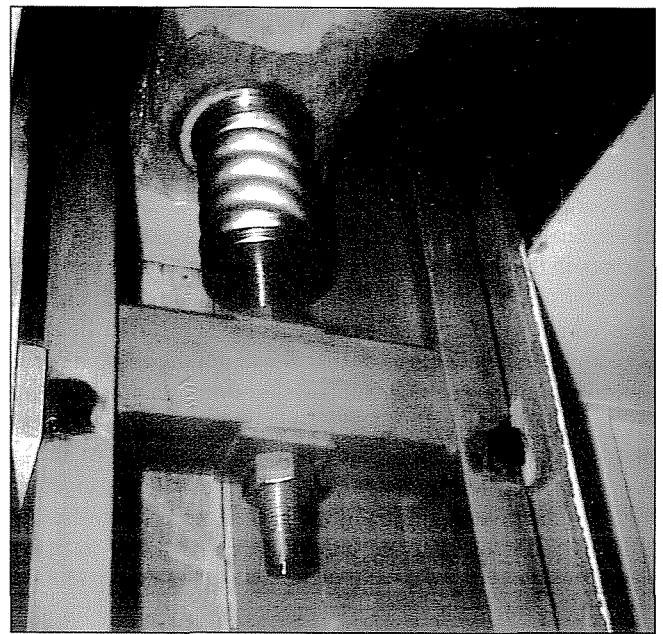
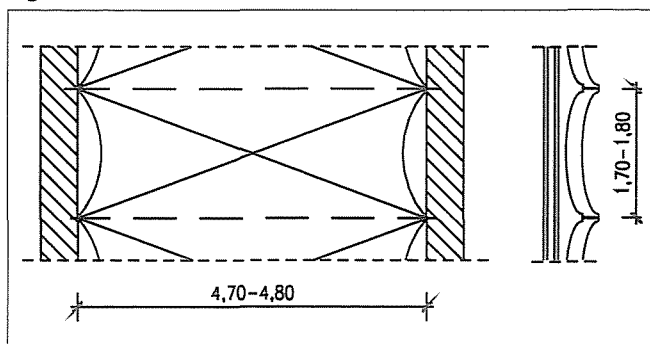


Fig. 7: The supporting spring

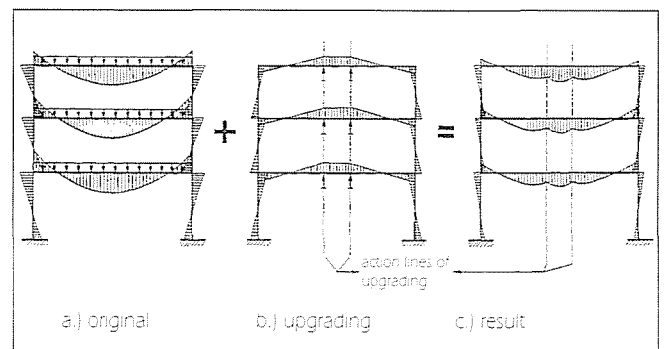


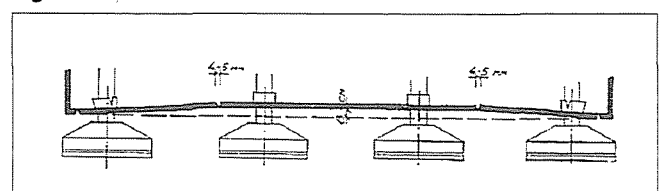
Fig. 8: The resulting moment diagram

the beams were exhausted and, due to moment redistribution in the beam-column joints, intensive cracking was detected.

The intervention was a really radical upgrading, changing completely the mechanical model. The 13 m span was divided into two lines of smaller rooms with a corridor in between. On both sides of the middle corridor at each lateral frame, two pairs of steel columns supported by new foundations were placed. A heavy spring (Fig. 7) acting on the underside of the reinforced concrete beam was installed on each small cross-beam between the twin steel columns. The spring prestressing by a bolt was controlled in order not to exert tension on the top surface of the concrete beam. This was illustrated by summing up the original bending moment diagram and the bending moment diagram from the springs' action, reducing the bending moment at the mid-span considerably, but not changing its sign (Fig. 8).

The solution has been in use for more than 30 years without any problem.

Fig. 9: Displacements and cracks due to ice lens formation



### 3.4 Non-Foreseen Changes in Natural Actions (Lenkei, 2001)

A multi-storey warehouse built very near to the Danube river embankment was left open without doors, windows and heating during the whole winter. At first sight at the end of the severe frost period it seemed that the independent column foundations in the middle column lines showed up ~80 mm of differential settlement as the collars around the middle columns showed such differential movements (*Fig. 9*). After detailed investigation it became clear that it was not the column foundations which had settled, but that the bottom reinforced concrete slab with the collars had lifted up producing the accompanying crack formation. The cause was formation of an ice lens in the soil due to the high ground water level, together with consecutive migration of the ground water to this lens.

The reinforced concrete slab had been subjected to bending moments of opposite sign to the design bending moments. After the spring melt, the cracks in the slab closed and the slab went back almost to the design geometry.

## 4. CONCLUSIONS

First of all, it should be stated that there are no two analogical cases existing. Even in case of originally analogical buildings or structures, due to differences in construction, in use and in the environmental effects, i.e. differences in histories, their states at a later time could be entirely different.

Secondly, during interventions into an old structures it is very important to know how much the mechanical model has altered during previous use, and additionally how much the proposed intervention would add further changes to that identified previous alteration.

Thirdly, during an evolution or in case of intervention into an old structure, a singular or unique solution can not be ex-

pected to emerge. There are often several solutions which may be simpler or more comprehensive.

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# STRUCTURAL MODELLING BEHIND THE ENGINEER'S CALCULATIONS – THE ROLE OF THE STRUCTURAL DESIGNER



Prof. Jenő Gilyén

*The standards of structural design are based on typical conditions of use and construction. So their prescriptions fit best to typical cases. Although a certain range of the variation in conditions is covered by the standardised regulations for safety reasons, in special (unusual) cases the designer is the only person who may consider their consequences. Such specialities may occur either in the use or in the construction of the structure. (E. g. extra long life-span, innovative techniques.) In these cases the standardised prescriptions define a “compulsory minimum” which will not properly fit to the adequate optimisation requirements of economy, safety, durability, etc. The standards are not axioms but their application gives a certain range of freedom for the conceptual as well as the detail design engineer. The task of designers is rather complicated so it is not possible to fully cover by formulae or theories the consideration of possible special requirements. Instead some examples can be elaborated to illustrate the consequences of “engineering concepts” as they influence the modelling of the materials or the structural behaviour.*

**Keywords:** safety, life-span, durability, structural model, calculation model, conditions of use, conditions of construction.

## 1. INTRODUCTION, ACTUALITY OF THE PROBLEM

The structural modelling for design calculations has been derived from test-series and measurements on existing structures – that is engineering experiences. The method (algorithm) of calculus is based on the distillate theory including the necessary simplifications for practical use. Usually, the real structure's behaviour must be dealt with by significant simplifying reductions. An attractive example is the commonly applied 2D-reduction of structural analysis.

The mechanical parameters of structural materials are not constant but vary and are seriously influenced by the range of internal forces and other physical conditions. (Pure linear elastic zone; zone with some part of remaining deformation; conditions producing cracked zones in the structure; broken status; status in contraction; so called “plastic zone”; etc.) There are many components/conditions influencing the actual status within the structural material, and moreover within the complex load-bearing structure. Picking up only some of them let us mention the speed of loading up, or the durability and variance of loads, or the actual specific physical parameters of the material itself which is random within a certain range and varies also within the same structural member along its dimensions.

The mechanical/physical parameters of the materials of load-bearing structures are considered as average values having a certain “internationally agreed” variance. So based on long-term experience the “safely expectable lower limit” of them is taken into consideration. But those standardised concepts may never refer accurately to often occurring “slightly unusual conditions” in construction technology, local conditions, meteorological impacts or specific structural conditions even if some simply worded reference is taken as, for example, “concrete quality for construction of shell structures” or “specially plasticized cast concrete mixture”, etc. Standardised prescriptions are not “forever valid axioms”. In usual situ-

ations they may replace specific engineering analysis and so serve as adequate tools in legally regulated limitations. Nevertheless, in complicated cases or with structures of extra importance, the automatic application of the simplified standard limitations/prescriptions may lead to serious losses.

The recently used design software applications only operate using common standardised concepts, applying them to all tasks they execute. Furthermore, such softwares do not give warning if some assumptions fall outside their validity range. The necessary mathematical transformations (simplifications) in the computing programme also result some neglect of the actual physical complexity. For example, the smooth and monotonous interpretation of the basic correlation between stresses and strains in extreme (but often existing) cases will cause misleadingly false results in computation analysis. (I.e. the breakdown in stiffness parameters when cracked zones occur, etc.) Any mathematical transformation/interpretation makes a (sometimes dangerous) step, increasing the difference between the real phenomena and the analysis. The limitation of the validity range for correlation functions may help together with fixed and defined limit-values, but this technique is not “a tool for everything”. Győző Mihailich wrote 80 years ago in his famous book about reinforced concrete: “The detailed and accurate examination of the experiments will save you from misinterpretation of numerical results of analyses believing them without criticism as they were simple mathematical problems” (Mihailich, 1922).

## 2. THE HISTORY OF DESIGN METHODS/STANDARDS AND THE LIMITS OF THEIR VALIDITY

In the earliest years of reinforced concrete construction, load-bearing structures were divided into simple parts for analysis as simply supported beams or columns. So the variety of prob-

lems to be covered by standard regulations was much less than nowadays. The first "Hungarian Standard for Design and Construction of Reinforced Concrete (1909/14)" consisted of only 20 pages in A/5 format. The next step, in 1931, was the "Regulation of Reinforced Concrete issued by the Hungarian Association of Engineers and Architects". It contained 51 pages and its application was compulsory in Budapest; from 1936 it was in force for the whole country. After the World War II the Institute for Building Science (ÉTI) issued in separate chapters a modernised set of regulations in 1949.

Of course just after the pioneering years of reinforced concrete, designers started utilising the knowledge available in structural behaviour; since in reality the structural members, columns and beams (first considered as separate set of simply supported beams) form complex multiple statically indeterminate structure where a lot of possible re-distribution of internal forces occur without visible consequences or damage. By observing this set of newly developed prescriptions (not so complicated ones) aspects of both safety and cost saving could be advanced. Meanwhile, the relevant regulations and prescriptions produced extended books all separately dealing with cases of prefabricated elements, plain concrete structures, etc. Each of these individual books refer to prescribed loads, etc. The time when a designer could keep in mind all regulations without handbooks and data bases is long over. The relevant set of standards require the capacity of a CD-ROM.

Although there is a rich abundance of supporting regulations and rules for the designers the engineers' particular evaluation must still take place. For example, the real distribution of internal forces (bending moments etc.) are governed by the real elasticity data of the component beams and columns. The Young's modulus and the actual moment of inertia vary significantly influenced by the actual (and former) stresses; position, quantity and quality of reinforcement; geometrical inaccuracies and, moreover, by the structure and quality of the concrete. The recently employed medium strength reinforcing bars have such elastic strain within the usual range of their stresses that the neighbouring concrete cannot bear it without producing extended zones of cracks. So, when determining the cross-section's moment of inertia, the reduced dimensions of the continuing operating concrete body should be taken into consideration which would be possible to determine only through large iteration processes. Instead of those time-consuming methods, engineering experience may help. It is very important to obtain and analyse those experiences which are collected from years of engineering practice. That "internal data-bank" may and should support the engineers' estimates and decisions. All that may be applicable only if we really analyse and understand all those specific experiences we have met during the work. The necessary decisions are usually much more complex than those based only on a limited set of data used as calculation inputs for a software. So, although human intelligence may not refer in such decisions to exact formulae, the consideration of a wide range of inputs may ensure ideal outputs – that is, if these are based on true and well interpreted experiences based on a great number of specific cases.

### 3. CONDITIONS AND FACTORS INFLUENCING SAFETY

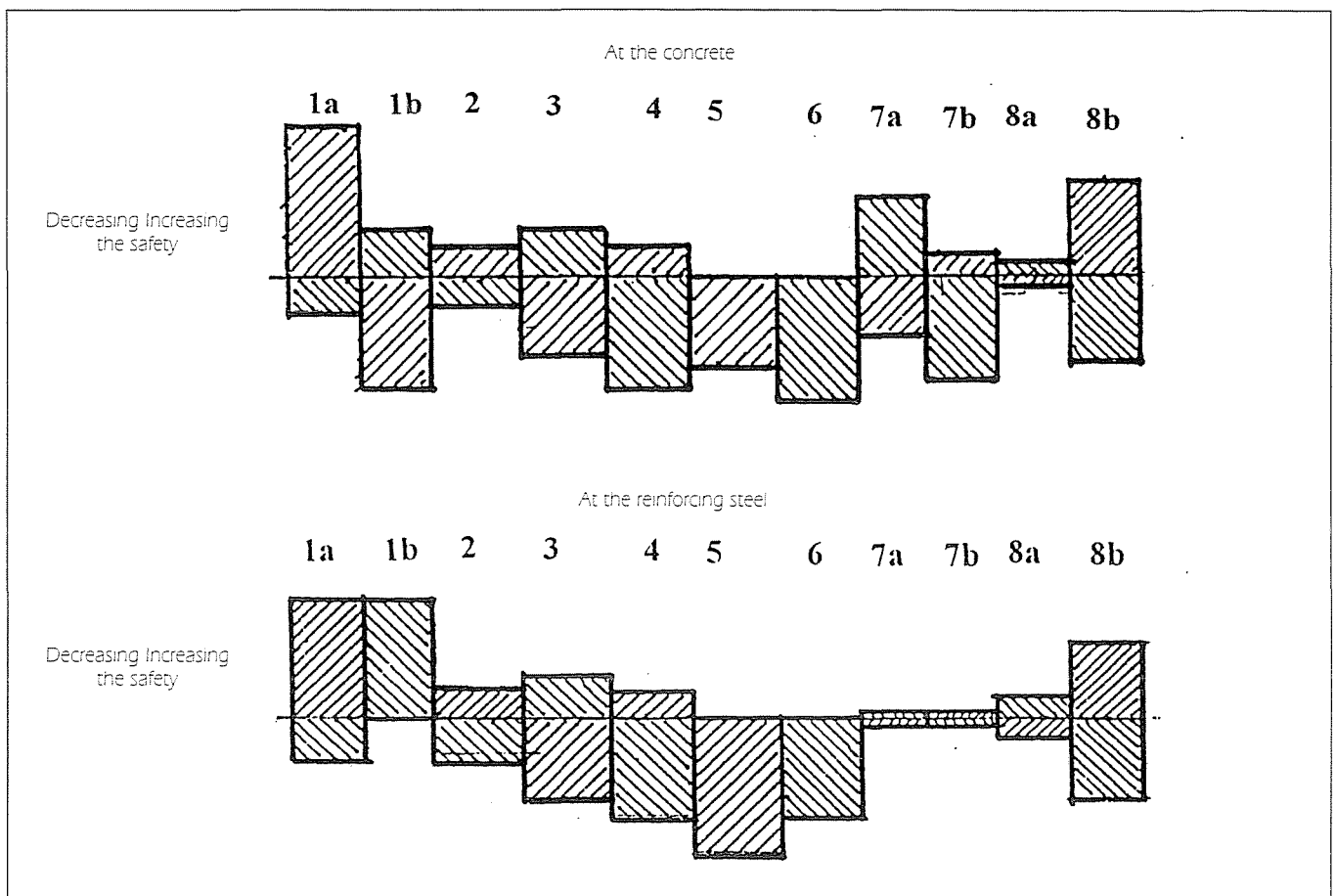
Safety itself is not a simple matter. It cannot be dealt with separately from the expected life-span, the conditions (and versatility) of planned use or accepted deviances - in condi-

tions and responses of the structure. In some cases over the past fifty years, too many optimistic opinions have been declared with reference to the acquired level of exploration of the internal characteristics of some materials. Moreover, these were aimed to ensure a life-span of no more than fifty years. The safety factors introduced referred to those basic conditions. Nowadays it is widely accepted that such a short life-span is seriously against the interests of society and can result in the wasting of energy and materials, effort and value. Even with the capacity of industry (and the financing) makes it impossible to reproduce such huge amounts of buildings within a fifty year period. There is a very attractive figure not greater than 3-5% showing the increment of the investment cost to gain a 100-120 year of life-span instead of 50, if combined with an accurate design concept. Considering the incremental increase in costs and the period of serviceability it can be considered very efficient and beneficial. Nobody should be misled by those buildings bearing a loud image of power and marketing built by international firms and neglecting solid realistic engineering concepts. The great majority of buildings are for the economical and practical use of people and with them the easy and cost saving operation for the long-term is very important. The calculated/expected safety is influenced (spoiled) by many effects as indicated in *Fig. 1*.

All involved specialists, investors and financing organs must take into consideration also that any repair or rehabilitation of the load-bearing building structure involves the consequence of removing/damaging/destroying all the covering elements – mostly expensive finishes and infrastructural systems – which will override 5-20 times the basic costs of the structural repair.

Earlier the "rigid" observance of the compulsory standards gave total legal protection for the designer and for the contractor, although the moral evaluation was more accurate even in those years. Nowadays, the responsibility for such consequential losses is more linked to the organs/experts interacting in construction and design.

Until the seventies it was widely believed that reinforced concrete would resist the elements engaged in corrosive action as the basic chemical character of the concrete body would conserve the steel bars. Only highly porous, low strength concrete was considered endangered. This false concept and the inaccurate evaluation of the conditions of use lead to serious loss and damage. However, that false concept came from previous customary practice when no free concrete surfaces were common but were usually accurately plastered by high lime-content layers. A lot of experience also supported the belief that the character of reinforced concrete was corrosion-free as many of those structures were damaged in world war and their interior showed no corrosion at all. Experts did not take into account the fact that these structures were never "naked" but covered by plaster. Furthermore, plastering was executed after the structure was loaded (up to the extent of service loads), where the consequent deformations and cracking had taken place. Many years later, as prefabrication became common, architects, designers and contractors were forced to avoid time consuming plastering since the modern precast concrete was more accurate with regular and even surfaces. Prefabrication also resulted in savings in dimension and weight and led to application of higher stresses in both steel and concrete. Therefore, cracks in tension zones became much wider while protecting plaster did not cover them. After about twenty years of service, these structures started showing serious surface degradation as a complex consequence of (1) reduced protection (no plaster); (2) increased corrosive attack (more acidic urban



**Fig.1:** Factors effecting the safety of reinforced concrete structures

1a – both the concrete and steel strengths show and upwards deviation in case of well controlled construction; 1b – In case of uncontrolled construction work the concrete shows deviation rather downward; 2 – The effect of size errors is significant mostly at the reinforcement; 3 – The permanent loads are rather increasing in case of alterations; Reconstruction; 4 – The superimposed loads exceed eventually the values in the calculation in case of Reconstruction because of material storage; 5 – The ageing of the material indicates cracking of concrete (heat expansion, shrinkage) and corrosion of steel; 6 – Damaging (cutting, . boring), increasing danger because of using up-to-date Machines; 7a – Various circumstances effecting the Young's modulus of concrete and occurrence of cracks resulting in uncertainty of the moment of inertia of cross section , even in case of well controlled construction work; 7b – Opposite to the case of reinforcement, the Young's modulus of concrete is sooner smaller; (Factors 7a and 7b can influence the accuracy of calculation by several ten percents.); 8a – In case of well controlled construction work, the mistakes are rather small; 8b – In case of uncontrolled design, large input and calculation errors can occur.

atmosphere containing also nitrous oxides/acids); and (3) increased sensitivity (more and wider cracks in concrete). These conditions differed effectively from formerly existing ones but there was no forecast of their impact since there was no ground for the usually applied speculations of seeking analogies and extrapolation methods in examining processes and consequences. Potentially any innovations bring similar unexpected risks when nothing but carefulness may ensure reserves to avoid losses. Recently a lot of accurate papers deal with the gradually extending degradation of corroded concrete and consequent decrease of pH value at different depths. But a lot of concrete surfaces have had protection added or been rehabilitated at extra costs. Only dry internal conditions are suitable without additional anticorrosion measures.

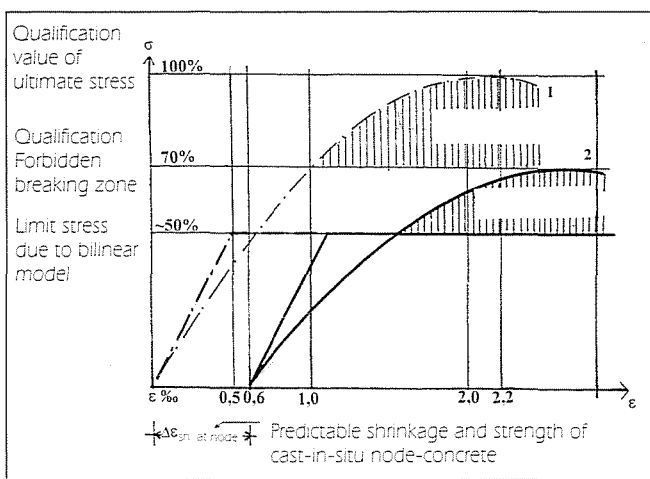
Although the common design practice dealt with reinforced concrete as working in the 2<sup>nd</sup> stress state (cracked cross-section) even before 1950, the rather low admissible values of stresses in both steel and concrete the elements mostly remained in the 1<sup>st</sup> (uncracked) state, while loads did not override significantly the genuinely permanent existing part of the service loads. If the load-bearing elements in reality are in the 2<sup>nd</sup> stress state, the danger of corrosion may occur even inside houses in wet rooms like kitchens and bathrooms where the vapour often transits towards just those relatively cooler parts of the skeleton structure. Although there are sophisticated and elaborate formulae for the calculation of crack width, their results often fail due to the large impact of uncertain physical

parameters with high deviation, like the ultimate tensile strength of concrete.

Besides the above detailed concepts in normal service conditions the appointed factor of safety must also involve acceptable margins against catastrophes, extraordinary events and possible failures in operation.

#### 4. THE PROBLEM OF THE NECESSARY SAFETY MARGIN AND THE IMPACT OF STRUCTURAL MODELLING THEORY ON IT

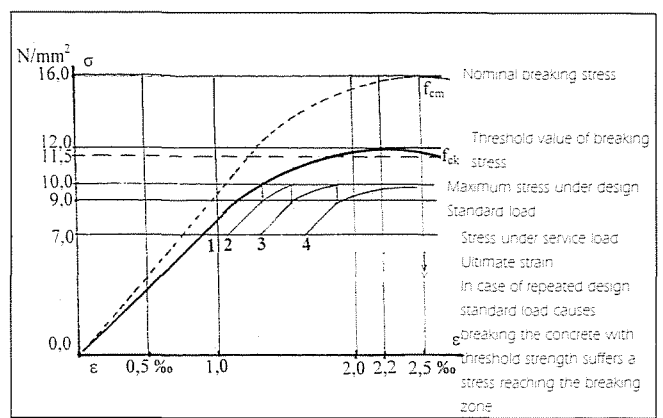
Structural design and architecture also became heavily influenced by the aggressive fashion-makers of rich consumption-societies after World War II. This refers to the great choice of attractive and expensive finishes, secondary structures, new solutions in facades, floors and ceilings. From structural point of view even layouts often became illogical in some cases. The recent high level of industrial products and design theory allows many such contradictions although their costs rise. While investors often may meet the expense of such developments, some partners have lost interest in the primary load-



**Fig. 2:** The idealised elastic-plastic material model  
 Curve 1: Stress-strain diagram of concrete having nominal strength properties  
 Curve 2: Stress-strain diagram of a concrete showing average deviation

bearing structure since its cost represents only a small fragment in the investment budget. However, the primary load-bearing structure's overall importance must be underlined since the whole committed value of the building (with the danger of consequential losses) is supported basically by the proper service of that load-bearing structure. How could be reasonable to reduce its reliability only for an expected saving of 2-5%.

Theories and estimates determining the safety factor use only a reduced set of structural materials' data and pay less attention to the effects of manufacturing conditions. Of course, we refer here basically to concrete as its production includes casting in situ and can be influenced heavily by weather conditions, human inaccuracies and the complex conditions of appertaining during the curing period. In view of these facts concrete can be considered as significantly inhomogeneous. This is taken into consideration within the regulations but not always given the appropriate emphasis. Unfortunately, accurate well-designed and evaluated trial-series referring to specific, but not unusual, site conditions are absent worldwide. The stress-strain curves commonly used in 1980's in Hungary we show in Fig. 2. Regrettably, many non-standard mixtures have been used in both the prefabrication sector and cast-in-situ sector. Their grading curves and water-cement ratios have high degrees of deviation which heavily influence basic physical parameters. Of course, there were real arguments for this circumstance: small dimensions, sometimes limited capacities during in-situ casting, for instance. Such "out-of-standard" concrete has significantly different Young's modulus and shrinkage coefficient, but those values were not properly investigated in laboratories using an extended series of tests. Instead designers have nothing but the personal experience (if they are lucky) and values obtainable from literature and prescription are in serious contradiction with those seen in real structures. Standards reported shrinkage for cast concrete as being 0-0.04%, while actual structures often showed values between 0.06 to 0.10%. Recent research on concrete (Balázs, 1994) has proved that degradation of the original internal structure of the concrete-body starts as early as when load-caused deformation reached 0.1% (Fig. 2). Since this degradation is never a fully recoverable process, this deformation-value is already beyond the serviceable, linear, purely elastic limit. In contradiction to these results the standard prescriptions (using the popular "easy-to-use" elastic/plastic stress-strain model) suggested that the normal service conditions for concrete range between 0.05 and 0.25% of deformation, and so encourage



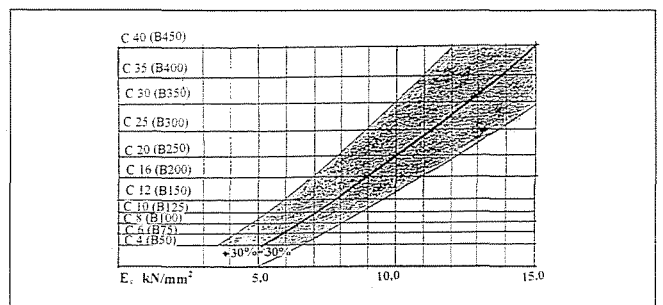
**Fig. 3:** Failure mechanism of pillar, e. g. in case of strong constraint in footing or substructure in case of  $M > M_H$  when the internal forces reach the braking (called plastic) zone

the expectation of simply calculating total additivity in load-bearing capacities for cooperating load-bearing elements which have different origin and quality (Fig. 2). As far as small dimension concrete bodies are concerned, if firmly bordered all around they have once-in-life stresses which results in such deformation, they may be considered as behaving as above, but in case of repeated loads their deformation will run out of control. Fig. 2 also illustrates that considering the allowed standard deviation of stresses (Hungarian Standard MSZ 15022/2 1986), the genuine situation may lead to the appearance of forbidden stresses. In structures planned for long-term serviceability both loads and stresses may run out of the expected probability quantilis of  $10^{-4}$ . We should also underline that if steel bars experienced once-in-life higher stresses than their strict linear limit, their deformation is causing much greater crack-width in neighbouring concrete and those cracks are also extended into wider zones.

Based on the above facts it is very dangerous to approach the so-called plasticity-zone at statically determinate structures, especially if those levels of stresses may occur repeatedly. Summarized: The so-called plastic-zone (in reality: the beginning of structural degradation/breach) may not serve as a safety-zone extension. In cases of "extra (catastrophic) loads "once-in-life", this zone must be strictly and safely avoided in cases of repeatedly occurring effects like wind-loads or other alternating loads (Fig. 3). All this refers still more strictly for statically determinate structures where there is no mechanism for redistribution of structural behaviour as with statically indeterminate structures (which may apply a force distributing internal deformation system).

The standardised qualification process of concrete testing is based on specimens cast into very rigid and reliable multiple-use forms with characteristic strength and stiffness which is much higher than those found when deploying the material in site formwork. Further dangerous regularly occurring differences in the experienced against defined ultimate stress ra-

**Fig. 4:** The Young's moduli of C4 ... C40 concretes considering a deviation of  $\pm 30\%$

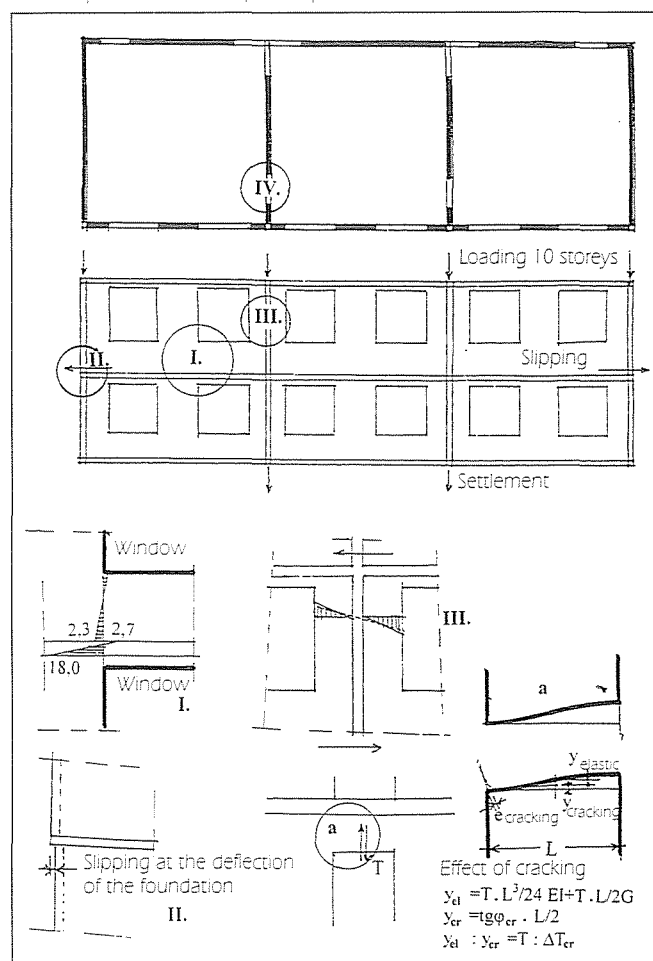


tion will come out of the differences in the manner of compacting, curing, etc. vis-à-vis the concrete of the actual structure and that of the specimen tested in the laboratory. So the ultimate stress (dealt together with its deviation) derived from a laboratory test executed on specimens may be considered true only if the dimensions of the concrete of the actual structure significantly exceeded those of dimensions of the test cylinder or cube. We refer to the experiments and reports (Leonhardt 1973) which proved that the compactness rate of the concrete gradually decreases as we consider the layers stepwise upwards from the bottom of the formwork because at the upper zones the efficiency of compacting vibration is more and more hindered by the chance that the fresh cast concrete may deviate towards the free upper surface of the formwork. In addition, there is less inertia of the surrounding still "liquid" concrete-mass. We attempt to this illustrate in Fig. 4 with the consequences in the value of the Young's modulus of concrete; and its deviation.

## 5. EXAMPLES SHOWING THE ROLE AND EFFECT OF ENGINEERING CONCEPTS APPLIED IN THE EVALUATION OF THE CONDITIONS OF USE AND/OR CONSTRUCTION

Modern headquarters of great companies, banks, attractive housing estates, shopping centres very often apply illogic load-

**Fig. 5:** Experiment to analyse the statical behaviour on a real size building part assembled of precast panels



bearing structures for the sake of gaining specific architectural features. Typically, we may therefore find a disharmonic variety of shallow piers, thin beams, rigid wall-masses, etc. Their structural model is very complicated particularly if we consider the differences in the time of completion, conditions of formwork and compacting and other issues in technology such as resulting shrinkages, etc. These "mixed character structures" also have a rather large variance in dead loads, service range of temperature alternations, all of which cause further problems in structural analysis if it is taken into consideration accurately. It is often proposed that extraordinarily extended sections are formed between expansion joint adjustments. This is sometimes even during the construction to deal with project scheduling issues. The later-cast adjustment sections widely used to handle shrinkage of extended dimension structures may help only with reduced efficiency. The difference in age/shrinkage realisation will definitely produce cracks all along those cuts (consider the parallel lengths of different age concrete structures!). Furthermore, those cracks may only be covered by secondary structures or finishes but never blocked in their effect to accelerate the devaluation of the building. Such difficulties were often experienced with buildings which utilised 3-D formwork-systems, even if they were constructed in cooler months. There were decade-long periods regarding panel-technology houses when it was theoretically denied that behaviour could be thought about as anything other than monolithic. Consideration of the serious impact of weak joint-concrete between high load-bearing capacity walls and slab elements as it breaks the monolithic model was totally neglected. This introduces rather the friction-like cooperation behaviour throughout the crack-separated concrete surfaces. It was as late as 1981 that the Scientific Institute for Building (ÉTI) could finance and execute a full-scale test with a section of a panelling structure to prove the idea of a structural technology designer; that it is impossible to expect monolith-like behaviour in multi-storey wall-discs (composed by wall panels) just because of the extended rather rigid slab-zones (Gilyén, 1974), see Fig. 5.

The applied strain gauges showed that although the panels beside each-other had joints with binding steel reinforcement and fill of joint-concrete, they did not behave as a monolith but rather as softly joining autonomous vertical cantilevers. Similarly disproved as false was the theory stating that reduced height wall-zones above doors/windows, ring beams with slabs and similarly reduced wall-zones of the next storey below the windows, may be considered as one load-bearing monolithic element (Kaliszky, Györgyi, Lovas, 1983).

It is widely known that cracks often appear in the structures at sudden changes in cross-section. As a consequence of this type of crack the effective stiffness of the part of the wall-disc located above a door/window opening decreases significantly. These are the structural zones which link the horizontally neighbouring, free of door/window vertical, pier-like disc-zones. So the binding forces arising from deformation of the whole wall will be much less than expected in monolithic behaviour. In this way, an overloaded part of the wall can hardly transfer side parts of its loads involving the capacities of neighbouring zones.

Another consequence of the very same phenomena is that these panelling walls are much less able to compensate for any mistake arising from foundation problems, i.e. non-uniform settlement along the building. That early concept, regarding the "abundant" rigidity of full-size wall-discs, insisted on the dangerous concept which resulted in not taking account of solid monolithic high-rigidity foundation-boxes below the

panelling houses. This thinking assumed that the panelling wall-discs would solve any problem arising of non-uniform functioning of foundations. Instead, the investors insisted on the introduction of fast-built foundations of precast components with rather weak shear capacities along the building. Fortunately, the danger arising from those typical inhomogeneous foundation conditions could be avoided by issuing an internal guideline for structural design of Hungarian panelling houses as early as in 1970. Later this guideline became part of the Hungarian Technical Prescription "ME-95-1972" and, unchanged, formed also the Chapter I. of the following issue "ME-95-1974" (*Gilyén, 1974*).

Another illustrative topic of engineering concepts is in correlation with thermal deformations. Traditional buildings constructed of bricks rarely had problems with the cycles of temperature variation. Usually, their horizontal dimensions were also much less than the critical length. Masonry material has thermal expansion coefficient about 25% less than concrete. Furthermore, the solid components (bricks) have small dimensions, ranging only up to 25 or 29 cm. Between these elements, there are soft mortar zones slightly adjusting themselves to obtain a certain (suitable) size of geometrical incompatibilities - especially during their week-long hardening time. After widely dealing with reinforced concrete structures, engineers first had to experience the new and important impact of thermal deformations, incompatibilities and consecutive stresses. So, regulations and traditions were developed to limit monolithic expansion zones which were not longer than about 40 m. This limitation works only if the structure's cyclic temperature-curve (with its typical daily and yearly waves) was properly smooth. That means that its total variation range could not exceed 20°C. These conditions were also fulfilled with the typical multi-flat houses where each flat operated its own autonomous heating device (formerly only stoves), producing serious daily temperature-variation but never over a wider range than 15-20°C. (Their typical structure consisted of brick walls with a thickness of 45~60 cm (never less than 38 cm) and R. C. slabs. Industrial buildings have specific daily and yearly cycles and sometimes also other technological heating/cooling cycles with specific ranges of extreme temperatures. Also the geometrical locations of their occurrence may be unusual. With these buildings nothing but a wise engineering concept, consideration of extremities including their frequency and probability may help to design proper structure with acceptable responses in thermal expansions and compressions logically supported by correct calculations. In about 1960, the standard allowed as long thermal expansion sections as 60 m as the skeleton structure was consisting of separate columns/walls with precast slabs and beams. Anyhow it is not recommended to increase the impact of crack-formation based on shrinkage of different aged cooperating structural components by extending the dimensions of thermal expansion sections. Considering the above concept, it was prescribed with taller buildings (over 30 m), that had a centrally operated, permanently working heating system applied thus ensuring a consequent homogeneous temperature distribution within the building.

All these experiences must be taken into consideration within the whole process of design, accordingly achieving a good number of interferences between architectural and structural designer and also involving the investor. The investor must also be capable of understanding his long-term interest with respect to the reliability of the future building. In an ideal case, the equivalent interest of the society (i.e. not to spoil efforts, energy and material in a short life-span structure) may

insist through the economical regulation that the partners governed by their own business interest need to fulfil this very complex optimisation issue. Let us remember the words of Iván Kotsis: "Architectural design is a kind of very complex activity of technical and economical nature, what may be done on artistic level if necessary talent was involved." This means that the practical criteria must not hinder the realisation of aims of aesthetics if proper talent and complex efforts were afforded. "Shocking architecture" may lead to primitive results where the complex requirements of simple technical concepts (functionality, durability, easy maintenance, safety, etc.) were - at least partly - spoiled.

The Author had a lot of special impressions just after World War II when many damaged structures had to be reconstructed and repaired. It was easy to observe that the buildings completed in early years of the 20<sup>th</sup> Century could localise the transition of war-damages, because of the low level of stresses - although practical guidelines for R. C. design were not available at that time. Structures completed between World War I and World War II also behaved solidly in the neighbourhood of damage. This was when more accurate utilisation of load-bearing capacities were common which led to the application of much thinner cross-sections but with due consideration to well compiled practical guidelines. These structures were also notable for having good quality concrete. This phenomenon proved the correctness and effectiveness of practical guidelines derived by a reliable series of experimental tests dealing also with non primary load-bearing reinforcement components. On the other hand, there were a lot of damaged buildings of "housing speculators" where the honest concept of reliability were given up for quick benefits. These structures were much more seriously damaged than the others. Such chain-like collapses showed that the real reserves of safety were inadequate with these "overthinned" structures. Unfortunately, high demand for construction capacity lead in the 1950's to relaxation of the regulations similar to that of "housing speculators" of former decades: regulators required inadequate safety reserves with the expectation that it would be greater than it really was. One of the greatest mistakes was the widely formulated concept of elastic-plastic-reserves which helped to explain the acceptability of significant savings in construction materials forced by political leaders.

This booming period of reconstruction and construction extended to the industrialised prefabrication technologies involving very good results in efficiency which had the effect of producing much thinner, lighter structural members (which also resulted savings in transportation costs). But we must not forget the fact that (mostly in the early years of prefabrication) the materials and concrete mixes were poor or poorly applied at site, while their stress-levels were much higher (*Gilyén, 1996*).

## 6. AN IMPORTANT CASE STUDY EXPERIENCE, ITS EVALUATION AND IMPACT IN CONSEQUENT YEARS

Through the early 1950s the Author was the leader of the structural design team for the National Stadium in Budapest (78,000 seats). The main piers were assembled on-site and consisted of precast blocks. Regardless of the tight progress schedule required and the poor supply of cement and aggregates at the

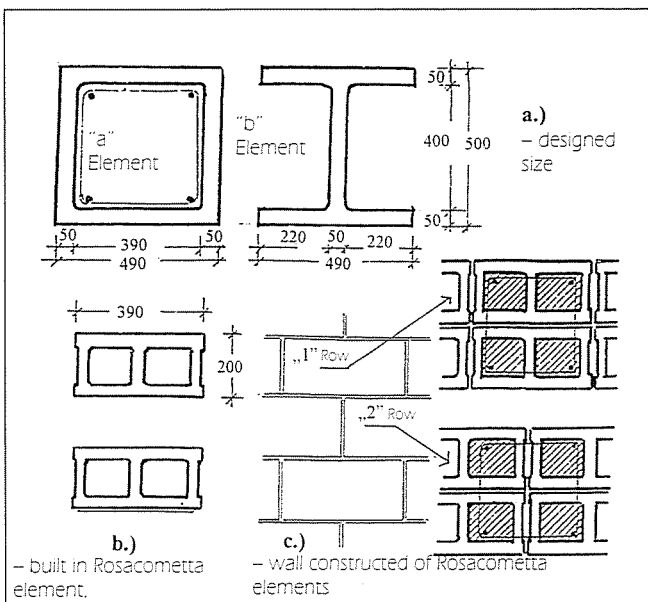


Fig. 6: Precast wall blocks of the Budapest Stadium

time, the majority of these precast blocks are in good condition after 50 years of service. This is even though the concrete was often of slightly poor quality and with a higher void ratio than required. The serviceability of this composite structure was partly the result of the fortunate fact that there was no meteorological water leakage and that the Author could apply the former R. C. standard (issued in 1931, amended in 1949 to include a greater margin of safety) and so avoided the application of R. C. code of 1951.

The concept of precasting the structural elements in the construction of the 30 m high piers (18 pieces) was decided at the very beginning. This idea seemed the best suited to fulfilling the aesthetic requirements beside the advantage of reducing time and cost in constructing formwork and temporary supporting works. The first 50×50×20 cm size blocks were designed directly for this specific purpose. However, it soon became clear that neither a contractor nor a manufacturer was available within the country with the capacity needed for the production of about 200,000 pieces of block. The size of the blocks was planned to ensure 40×40 cm internal holes for reinforcement and a final fill of concrete. Instead, a forced and urgent change of concept was applied to the only available sets of some similar concrete-block-manufacturing plant, originated from "Rosacometta". The parameters of this plant determined that the size of blocks could only be 20×40×20 cm.

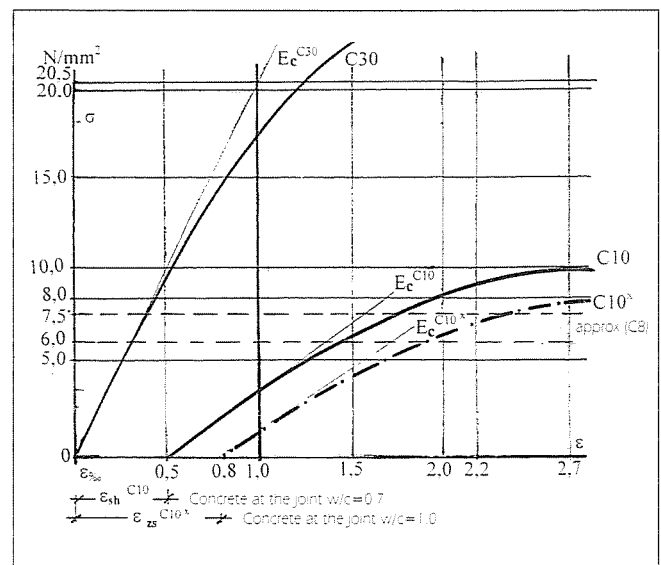
Nevertheless, the necessary capacity for 500,000 blocks of that size was obtainable, and design had to accept this fact. From this very moment there arose a considerable problem: How to ensure proper structural behaviour, how to accurately fill with reinforcement and concrete the chimney-like 12/12 cm vertical holes within this type of precast block (Fig. 6). The application of those precast blocks promised a good chance to ensure the required acceleration in construction but implied doubt in structural adequacy.

The designer combined the application of the blocks with successful and large-scale strength-tests which were executed at the Department of Bridge Construction No. II at the Technical University of Budapest. The first series of blocks manufactured were used to compose the required test walls. The tests executed by the large 5000kN capacity testing machine of the University proved much lower (composite) load-bearing capacity than expected by the conceptual design when it was added to the calculated capacities of the block-contouring shells (good quality precast concrete) and the inner cast-

in-situ reinforced concrete. The internal network of holes within the blocks consisted of long and narrow "chimneys". These were suitable to be filled correctly by nothing else but very liquid concrete with maximum aggregate size of 8 mm. (High water/cement ratio was the only way to ensure it in that time in Hungary.) This part of the cross-section therefore contributed practically nothing to the load-bearing capacity. To obtain an explanation for this surprising result two ideas were raised: 1. The concrete fill has only a very poor ultimate compression strength. 2. The fill concrete's shrinkage is so high that practically no stresses occur while the compressive deformations of the block-skeleton already starts to destroy the blocks, exhausting their final load-bearing capacity. The arithmetic analysis showed that even a very poor ultimate stress value of the concrete fill would have contributed more to composite load-bearing capacity since its greater cross-sectional area. So, the result of the tests and their analysis remained nothing else but the evidence that with such a type of "fill-in-later" composite load-bearing structure it is proscribed to add together the component parts of the calculated load-bearing capacities. This fact was ascertained through the experience and was supported by accurate test series which changed the belief in elastic-plastic behaviour of reinforced concrete - just at the time when this theory was to be extended and introduced into the standards. These facts were difficult to accept for the partners but were unquestionable following evidence of the test data of achieved by an eminent professional laboratory. This in turn helped the Author to manage the design permission standards in order to neglect the application of the new standard "cost saving" which was treated as a "... singular and special case, ... applied to this representative building of national importance". The Author solved the issue of the lower load-bearing capacity of the piers as derived from the tests by the introduction of three semi-hidden frames and by binding the 2-2 piers to each other and so reducing the buckling effect. The handling of the uncertainties with regard to elastic-plastic behaviour has been continued, and after more than 10 years of elaboration of national prescriptions for panelling houses (design of joints), some partial consent has been achieved.

A certain mixture of concrete composed of fine-type aggregate (small maximum gravel diameter) mixed with a water/cement ratio value of about 1.0 will attain as high a value

Fig. 7: Deformation curves of concrete C10× made with w/c=1.0 in case of composition with precast element of concrete C30 with shifted curves considering the afterwards occurring shrinkage



of shrinkage as 0.10 % (Fig. 7). So permanent loads (dead loads) will hardly cause such a load-bearing situation within composite structures wherein this component of structure (finally poured-in joint-concrete) even starts answering to any kind of compressive stress – the foregoing based on the fact that compatible elastic deformations of the composite will not reach that 0.1% with this “basic” load. (Its tensile stress capacity is just zero because of cracks of shrinkage.) In case of bearing shearing-type loads only a reduced contribution, (aggregate interface phenomenon) will occur within the later poured-in joint-concrete. It was reported (Leonhardt, 1970) that poured, mechanically non-compacted concrete will reach its ultimate stress at only about 30% of the same concrete properly compacted – this with 0.5 water/cement ratio. A very similar handling of this problem was published in Hungary very early (Mihailich, Schwertner, Gyengő, 1946). This book proposed to reduce the calculated/expected strength of high water/cement ratio concrete by 50% having called this reduction a “factor reflecting the special conditions of construction”. Opposite to this, those theoreticians of reinforced concrete who dealt mostly with mathematical approaches and had less industrial experience could hardly accept the large impact of applied technological conditions. They rather cited instead the commonly used simple approach stating that a plastic-elastic behaviour ensures a uniformly calculated response of any concrete within the deformation-range between 0.05 and 0.25%, where the admissible stress (the ultimate stress reduced by its safety factor reflecting the variance of the expected actual value of real ultimate stress) can be taken into consideration. This popular calculation method was believed to be as realistic as it was easy to use. That approach suggested that there are practically no problems of load-bearing compatibility at all. This dangerous belief was only overcome as late as in 1981 when a full scale model of a part of a panel house was loaded up to failure and measured results indicated that there is no such easy additive application of the load-bearing capacities as was calculated on the basis of elastic-plastic theory.

## 7. CONSEQUENCES, RECOMMENDATIONS

The accounts cited in the former chapters show conclusively that it is very difficult to overcome widespread and given opinions even if considerable experience act against them. The explanation and the applied consequences of load-bearing capacity problems in R. C. structures influence very definitely the safety of more than 550,000 panel structures in Hungary. The obsolete concept of considering the precast R. C. structures as monolithic has caused many problems and/or damage in commonly occurring load-situations generated by usual wind speed (e.g. wide cracks, increasing stepwise).

It is also very important to design proper, suitable cross-sectional dimensions for the structural elements considering the realistic conditions of the actual construction site. Codes of practice usually define only limitations reflecting common combinations of typical conditions. The designer engineer is required to rationalise the design and adapt it to the specific conditions. We must never forget the real characteristics of concrete as it is composed of particles of aggregates being bonded to each other by small arches of hardened cement. Particles of the aggregate consist of high strength undamaged mineral pieces while cement arches are thin and have much more limited strength. If the grading curve of the aggregate is

not ideal then a greater amount of cement-pulp is necessary to fill somehow the greater percentage of voids. The same tendency works also if only smaller size of aggregate particles is used thus forcing a higher amount of cement to apply. But on the other hand, the extremely large particles - although promising savings in cement use - would cause another problem, that of hindering the manageable compaction process. So, the concrete body may contain holes and inadequately compacted shadow-zones which all act against proper density and against appropriate strength. Moreover, the randomly located failure-zones will never form symmetrically and so cross-sectional inhomogeneity and an extended risk of buckling also will occur. That means that the value of maximum grain-size must be determined as a wise compromise between both the economy in cement-use and the expected safety parameters, while considering manageable conditions of pouring into the formwork and compaction. With the recent smaller dimensions of structures denser reinforcement is common; the usual maximum grain diameter value turned into 16 mm instead of the former 40 mm. As a global hint: never use a maximum gravel diameter greater than 1/10 of the smaller dimension of the cross-section.

In closing, we would remark that all the above examples demonstrate that there is sometimes no way to elaborate generally valid formulae for special cases. Furthermore, engineer's well-developed accurate considerations may lead to quality solutions in all fields of design. Recommendations also are merely illustrative examples of the promotion of the development of a design philosophy dealing with all important details. Experience and professional information gathered may also help the designer not to forget that sometimes the consideration of individual details are as important as the overall conceptual design.

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## REMARK BY THE EDITORIAL BOARD

The Author of this paper achieved wide range experience during a many decades structural designing practice. Sure, it is a consequence of the great oeuvre spanning over various periods of design codes, that members of the younger engineer generation who were educated on the basis of new design principles interpret many things otherwise from it as the Author understands them. This refers to the safety concept, to the design method based on theory of plasticity, several questions of relation between design and practical construction,

etc. Therefore, the Editorial Board recommends this article to Readers that they should overlook the formation of the design codes over many decades. The Editors fully agree with the statement of the Author, that *the engineering considerations cannot be neglected in structural design and construction*. Many examples dealt with in the paper, which originate from the rich practice of Prof. Gilyén help to follow the hint due to the statement. It is known that specialists of design philosophy intend to find in all aspects better solutions. They improve for this target the design methods using the results of the technical mechanics, the material science, mathematics and other joining sciences.

# FLEXURAL STIFFNESS OF CRACKED REINFORCED CONCRETE SLABS WITH SKEW REINFORCEMENT PERPENDICULAR TO THE CRACK



Assoc. Prof. Ferenc Németh

Modern computing technology together with the finite element method has made it possible that the following factors could be taken into consideration in the calculation of a moment field related to the subject: The in-homogeneity of the reinforced concrete plate and the circumstance that after cracking in the direction perpendicular to the crack the flexural stiffness decreases significantly. This also has an effect on the further developing of moment field. The main goal of this article is to develop a general case formula to determine the flexural stiffness coefficient in the direction perpendicular to the crack when the reinforcing bars are skewed.

**Keywords:** cracked slab, flexural stiffness, skewed reinforcing bars

## 1. SOME STATEMENTS BASED ON LABORATORY TESTS

Flexural stiffness is a two-dimensional, tensorial quantity. However, this paper only deals with one element of the quantity, specifically, with the stiffness perpendicular to the crack.

Tests are proving that the flexural stiffness of a slab decreases in the direction perpendicular to the crack. Otherwise, the stiffness remains constant as long as the slab does not begin to yield.

From among concrete slab tests there are few which are able to analyse the flexural stiffness tensor. The only appropriate tests are where, within a measuring area, it can be distinctly measured and where the flexural state is constant. Furthermore, to meet the test criteria it is necessary to measure in the same position; the state of deformation and the curvature tensor together with its principal directions. Such experimental work has been carried out with model-sized slabs with both perpendicular and skew reinforcement (Németh, 1968, 1974), while full-sized slabs have only been studied with perpendicular reinforcement (Lenschow, Sozen 1966 and Cardenas, Sozen, 1968).

Additionally, Cardenas, Lenschow, Sozen (1972), have published detailed research results concerned with the problem of flexural stiffness. In this work, they derived a formula referring to a slab with perpendicular reinforcement under single direction bending. The formula was supported by the test results.

These tests of Lenschow, Cardenas and Sozen have convinced specialists that the moment-curvature diagram is linear in a crackled state. However, the tangent of the diagram can be considered equal in cases of arbitrary reinforcement and the cracking moment doesn't depend on the arrangement of reinforcement.

After cracking – maybe somewhat surprisingly for many researchers – a linear moment-curvature diagram was also found, but its tangent (i.e. the flexural stiffness) depends significantly on the direction and arrangement of reinforcing bars.

Wegner (1974) also considers the linear moment-curvature line to be acceptable. In the case of tests published by Karpenko (1976), a linear rule can be found for the I. and II. stress states.

The reinforced concrete slab can be considered to be approximately isotropic in stress state I. Furthermore, as the stiffness decreases significantly after cracking, it seems reasonable to deal with the slab as orthotropic in stress state II.

The kinking of bars in the cracks can be neglected. According to the mentioned experiments, if any bending of the reinforcement exists, it is not to be reckoned with.

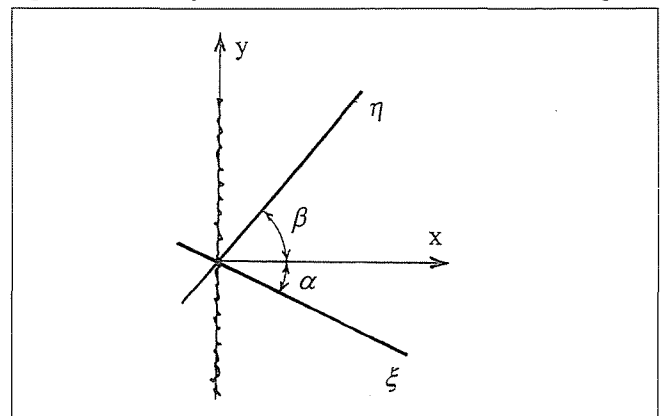
## 2. ANALYSIS OF THE CRACKED REINFORCED CONCRETE SLAB

Let us assume that the crack of the slab constitutes the y-direction (Fig. 1) and desired data is the flexural stiffness coefficient  $D_x$  in the perpendicular direction to the crack. The reinforcing bars are located at  $\xi$  and  $\eta$  directions, their unit area being:  $A_\xi$ ,  $A_\eta$ , and their distance to the compression surface:  $h_\xi$ ,  $h_\eta$  are given. Similarly, materials strength dates are given in the text.

Fig. 2 shows the cross section of the cracked slab, diagrams of the strain  $\varepsilon_x$ , of the stress  $\sigma_x$  and all so the characteristic sizes. The usual assumptions are used for the derivation of the stiffness coefficient:

- The slab is cracked at the considered point. The tensile strength of the concrete is neglected so the tensile and

**Fig. 1:** Coordinate system connected to the crack and the reinforcing bars



shear forces in the tension zone are resisted only by the reinforcement which passes through the crack.

- The crack ensues because of the yield of the reinforcing steel. All the steel bars going through the crack have the ultimate stress  $\sigma_u$ .
- There is only tensile force in the steel bars and the direction of this force is not changed by the crack.
- The resisting moment in any direction can be calculated by the superposition of the contribution of steel bars existing in several directions.

The main goal of this article is to develop a general method to determine the flexural stiffness factor of the cracked reinforced concrete slab in the direction perpendicular to the crack.

Let us start with the supposition that the strain state is linear:  $\varepsilon_x \neq 0, \varepsilon_y = 0, \gamma_{xy} = 0$ . The strain values in the x-direction appertaining to the layer of the reinforcing bars are  $\varepsilon_{\xi x}$  and  $\varepsilon_{\eta x}$ . Then, in the direction of the reinforcing bars,

$$\varepsilon_{\xi} = \varepsilon_{\xi x} \cos^2 \alpha, \quad \varepsilon_{\eta} = \varepsilon_{\eta x} \cos^2 \beta \quad (1)$$

strains ensue, according to well known formulae. Stresses in the steel bars can be calculated using Hooke's law

$$\sigma_{\xi} = \varepsilon_{\xi} E_s, \quad \sigma_{\eta} = \varepsilon_{\eta} E_s \quad (2)$$

The resisting unit moments at the section perpendicular to the reinforcing bars are,

$$\bar{m}_{\xi} = q_{\xi} A_{\xi} \sigma_{\xi}, \quad \bar{m}_{\eta} = q_{\eta} A_{\eta} \sigma_{\eta} \quad (3)$$

Substituting equations (2) and (1) for (3), then,

$$\bar{m}_{\xi} = q_{\xi} A_{\xi} E_s \varepsilon_{\xi x} \cos^2 \alpha, \quad \bar{m}_{\eta} = q_{\eta} A_{\eta} E_s \varepsilon_{\eta x} \cos^2 \beta \quad (4)$$

At the cracked cross section, the  $m_x$  applied bending moment and the resisting moment should be in equilibrium, so

$$m_x = \bar{m}_{\xi} \cos^2 \alpha + \bar{m}_{\eta} \cos^2 \beta \quad (5)$$

Replacing the moments in eq. (5) with eq. (4), then

$$m_x = E_s (q_{\xi} A_{\xi} \varepsilon_{\xi x} \cos^4 \alpha + q_{\eta} A_{\eta} \varepsilon_{\eta x} \cos^4 \beta) \quad (6)$$

The curvature at cracking in the x-direction can be written from the  $\varepsilon_x$  diagram of the Fig. 2, where  $k_x = \text{tg} \vartheta$ , thus

$$k_x = \frac{\varepsilon_{\xi x}}{c_{\xi}}, \quad \text{or} \quad k_x = \frac{\varepsilon_{\eta x}}{c_{\eta}} \quad (7)$$

The stiffness coefficient can be formulated by

$$D_x = \frac{m_x}{k_x} \quad (8)$$

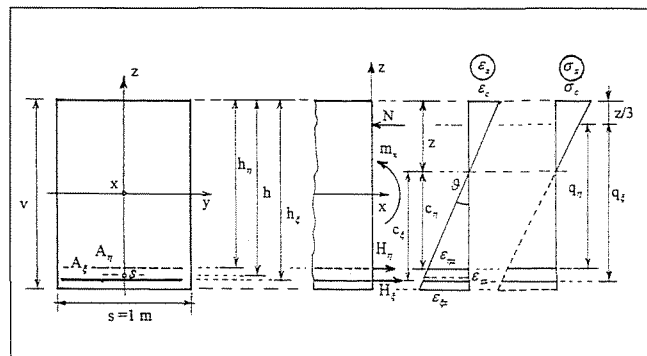
This is an approximate formula, because the exact connection is  $m_x = D_x (k_x + \nu_c k_y)$ , where  $\nu_c$  is the Poisson-factor of the concrete. But in case of an x-direction cracking  $k_x \gg \nu_c k_y$ , so eq. (8) is acceptable.

Let us substitute eq. (6) and (7) for (8), so,

$$D_x = E_s (q_{\xi} A_{\xi} c_{\xi} \cos^4 \alpha + q_{\eta} A_{\eta} c_{\eta} \cos^4 \beta) \quad (9)$$

formula can be obtained for the stiffness of the cracked reinforced concrete slab in the x-direction.

It is remarkable that the trigonometric function is raised to the fourth power. In this way the stiffness coefficient strongly



depends on the direction of the reinforcing bars. Also note that Lenschow and Sozen (1966) have found formulae for flexibility in case of perpendicular reinforcing bars in which the trigonometric functions are raised to the fourth power. Further, Cardenas, Lenschow, Sozen (1972) have published similar formula to (9) above for the stiffness coefficient in case of perpendicular reinforcement in a plate which is bent in one direction.

Now, let us transform the formula (9). The specific moment is,

$$\mu = \frac{m_x}{\sigma_c s \cdot h^2} \quad (10)$$

with the thickness of the compression concrete zone being

$$z = \xi h \quad (11)$$

Here h is the effective thickness of the slab, measured from the centre of gravity of the  $A_{\xi} \cos^4 \alpha$  and  $A_{\eta} \cos^4 \beta$  quantities. The coefficient  $\xi$  can be expressed by  $\mu$ .

From eq. (10)

$$m_x = \mu \sigma_c s \cdot h^2,$$

and from Fig. 2

$$m_x = \frac{1}{2} \sigma_c s \cdot z \left( h - \frac{z}{3} \right).$$

From these two equations

$$\mu = \frac{1}{2} \frac{z}{h} \left( 1 - \frac{z}{3h} \right),$$

respectively eq. (11)

$$\mu = \frac{1}{2} \xi \left( 1 - \frac{\xi}{3} \right).$$

From this equation of second order

$$\xi = 1,5 - \sqrt{2,25 - 6\mu} \quad (12)$$

This is the specific thickness of the compression concrete zone. The effective thickness is  $z = \xi h$ , and the arm of the internal forces and the distances c are,

$$q_{\xi} = h_{\xi} - \frac{z}{3} \quad q_{\eta} = h_{\eta} - \frac{z}{3} \quad (13)$$

$$c_{\xi} = h_{\xi} - z \quad c_{\eta} = h_{\eta} - z \quad (14)$$

Introducing some relations,

$$\kappa = \frac{q_\eta}{q_\xi}, \quad \lambda = \frac{A_\eta}{A_\xi}, \quad \nu = \frac{c_\eta}{c_\xi} \quad (15)$$

and using these, eq. (9) takes the form,

$$D_x = E_s q_\xi c_\xi A_\xi (\cos^4 \alpha + \kappa \lambda \nu \cos^4 \beta) \quad (16)$$

This is the stiffness coefficient of the cracked reinforced concrete slab in the x-direction.

### 3. NUMERICAL EXAMPLE

For geometrical and strength of materials data see Figs. 1, 2 and 3.

Concrete: C20,  $\sigma_c = 14.5$  MPa,  $E_c = 20$  GPa,  $\nu = 20$  cm

Steel:

$$(\xi): \alpha = 30^\circ, \quad \phi 14/10, \quad A_\xi = 15.4 \text{ cm}^2/\text{m}, \quad h_\xi = 17.3 \text{ cm}$$

$$(\eta): \beta = 45^\circ, \quad \phi 12/20, \quad A_\eta = 5.6 \text{ cm}^2/\text{m}, \quad h_\eta = 16.0 \text{ cm}.$$

Let us assume an applied moment  $m_x = 80$  kNm/m, occurring at the crack which is not yet yielding. Computing the effective thickness of the plate, measured from the centre of gravity of the  $A_\xi \cos^4 \alpha$  and  $A_\eta \cos^4 \beta$  quantities, it will be  $h = 17.1$  cm.

By further calculation, the number at the beginning of the row refers to the formula which is used, eq. (10), specific moment:

$$\mu = \frac{80 \cdot 10^3}{14.5 \cdot 10^6 \cdot 1 \cdot 0.171^2} = 0.1887$$

eq. (12), specific thickness of the compression concrete:

$$\xi = 1.5 - \sqrt{2.25 - 6 \cdot 0.1887} = 0.443$$

eq. (11), thickness of the compression zone:

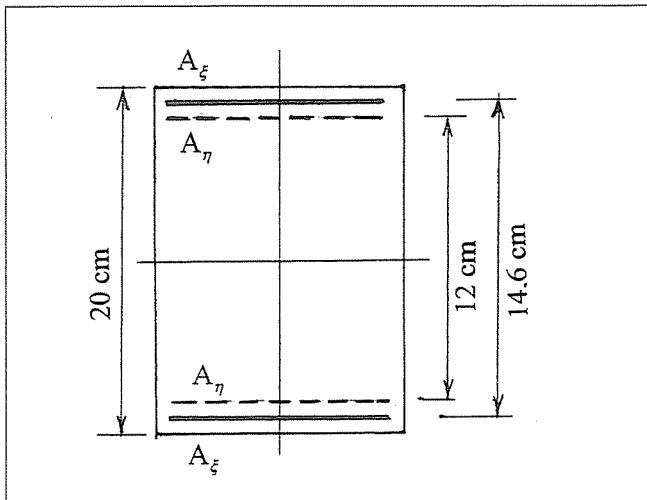
$$z = 0.443 \cdot 17.1 = 7.6 \text{ cm}$$

eq. (14):  $c_\xi = 17.3 - 7.6 = 9.7$  cm  $c_\eta = 16.0 - 7.6 = 8.4$  cm

eq. (13), lever arm of the internal forces:

$$q_\xi = 17.3 - 2.5 = 14.8 \text{ cm} \quad q_\eta = 16.0 - 2.0 = 13.5 \text{ cm}$$

Fig. 3: Cross section data of the numerical example.



$$\text{eq. (15)} \quad \kappa = \frac{13.5}{14.8} = 0.912 \quad \nu = \frac{8.4}{9.7} = 0.866$$

$$\lambda = \frac{5.6}{15.4} = 0.364 \quad \kappa \lambda \nu = 0.287$$

eq. (16) flexural stiffness factor:

$$D_x = 210 \cdot 10^9 \cdot 0.148 \cdot 0.097 \cdot 15.4 \cdot 10^{-4} (\cos^4 30^\circ + 0.287 \cos^4 45^\circ) = 2.945 \cdot 10^6 \frac{\text{Nm}^2}{\text{m}}$$

$$\text{i.e.: } D_x = 2.945 \frac{\text{MNm}^2}{\text{m}}$$

This is the value of the flexural stiffness factor in the direction perpendicular to the crack (II. stress state).

For comparison let us calculate the flexural stiffness of the uncracked slab (I. stress state).

Only from concrete:

$$D_{x0} = \frac{\nu^3 E_c}{12} = \frac{0.2^3 \cdot 20 \cdot 10^3}{12} = 13.333 \frac{\text{MNm}^2}{\text{m}}$$

Taking into consideration the reinforcement, having a double reinforcing net (bottom and top), the ideal moment of inertia is:

$$I_{xi} = \frac{\nu^3}{12} + 2n(d_\xi^2 A_\xi \cos^4 \alpha + d_\eta^2 A_\eta \cos^2 \beta),$$

where

$$n = \frac{E_s}{E_c} = \frac{210}{20} = 10.5, \quad d_\xi = 7.3 \text{ cm}, \quad d_\eta = 6.0 \text{ cm}.$$

Numerically

$$I_{xi} = \frac{0.2^3}{12} + 2 \cdot 10.5 (0.073^2 \cdot 15.4 \cdot 10^{-4} \cos^4 30^\circ +$$

$$+ 0.06^2 \cdot 5.6 \cdot 10^{-4} \cos^4 45^\circ) =$$

$$= 6.667 \cdot 10^{-4} + 1.075 \cdot 10^{-4} = 7.742 \cdot 10^{-4} \text{ m}^4/\text{m}$$

The ideal flexural stiffness coefficient:

$$D_{xi} = I_{xi} E_c = 7.742 \cdot 10^{-4} \cdot 20 \cdot 10^3 = 15.484 \frac{\text{MNm}^2}{\text{m}}$$

In case of only one set of reinforcing bars (at the bottom), the inertia moment will be  $I_{xi} = 7.713 \cdot 10^{-4} \text{ m}^4/\text{m}$ , and the stiffness coefficient  $D_{xi} = 14.734 \text{ MNm}^2/\text{m}$ .

### 4. CONCLUSIONS

The flexural stiffness of the uncracked reinforced concrete slabs depends mainly on the thickness and the modulus of elasticity of the concrete, while the influence of the steel bars is small. Using the introduced examples:

$$\text{Only from concrete} \quad D_{x0} = 13.333 \frac{\text{MNm}^2}{\text{m}}$$

$$\text{Reinforcement at the bottom} \quad D_{xi} = 14.734 \frac{\text{MNm}^2}{\text{m}}$$

Reinforcement at the bottom and at the top

$$D_{xi} = 15.484 \frac{MNm^2}{m}$$

Following the cracking of the reinforced concrete slabs, the flexural stiffness considerably decreases perpendicular to the crack:

$$D_x = 2.945 \frac{MNm^2}{m}$$

This effect depends considerably on the reinforcing bars and in particular on their angle to the crack.

## 5. NOTATIONS

- $m_x$ : applied bending unit moment in the x-direction (Nm/m)
- $\xi, \eta$ : directions of the reinforcement
- $\bar{m}_\xi$ : resisting unit cracking moment in the  $\xi$ -direction given only by the reinforcement in the  $\xi$ -direction
- $\varepsilon_x$ : concrete strain in the x-direction
- $\varepsilon_{\xi s}, \varepsilon_{\eta s}$ : strains in the x-direction at the layer of the reinforcing bars and
- $E_s$ : modulus of elasticity of steel
- $E_c$ : modulus of elasticity of concrete
- $A_\xi, A_\eta$ : area of reinforcement per unit width in the  $\xi$  and  $\eta$  direction
- $\sigma_\xi, \sigma_\eta$ : stresses in the reinforcement
- $k_x$ : curvature at cracking in the x-direction, perpendicular to the crack
- $D_x$ : stiffness coefficient of the reinforced concrete slab in the x-direction
- $D_{x0}$ : stiffness of the unreinforced concrete slab
- $D_{xi}$ : stiffness of the uncracked reinforced concrete slab
- $z$ : thickness of the compression concrete zone
- $c_\xi, c_\eta$ : depth of the reinforcing bars under the neutral axis
- $q_\xi, q_\eta$ : arm of the internal forces
- $\mu$ : specific moment
- $\xi$ : specific thickness of the compression concrete:  $\xi = z/h$
- $h_\xi, h_\eta$ : effective thickness of the slab

- $\kappa, \lambda, \nu$ : relation numbers connecting with the  $\xi, \eta$  reinforcing bars
- $I_{xi}$ : ideal moment of inertia of the reinforced concrete slab in the x-direction
- $d_\xi, d_\eta$ : distance from the centre of gravity of the cross section to the reinforcing bars
- $s$ : width of the cross section
- $\nu_c$ : Poisson-factor of the concrete

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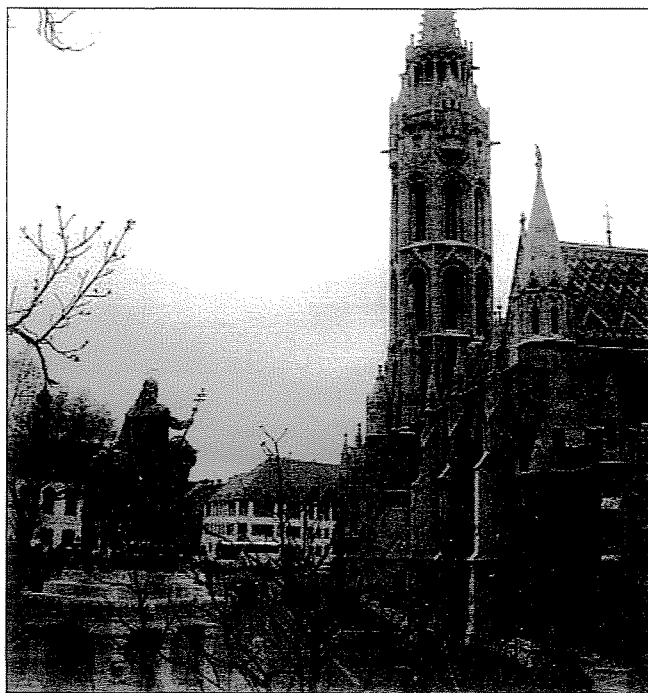
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# THE MOST UP-TO-DATE CONSTRUCTION TECHNICS IN HUNGARY

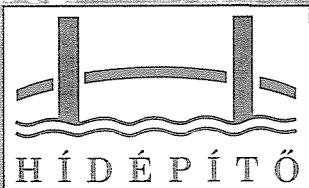
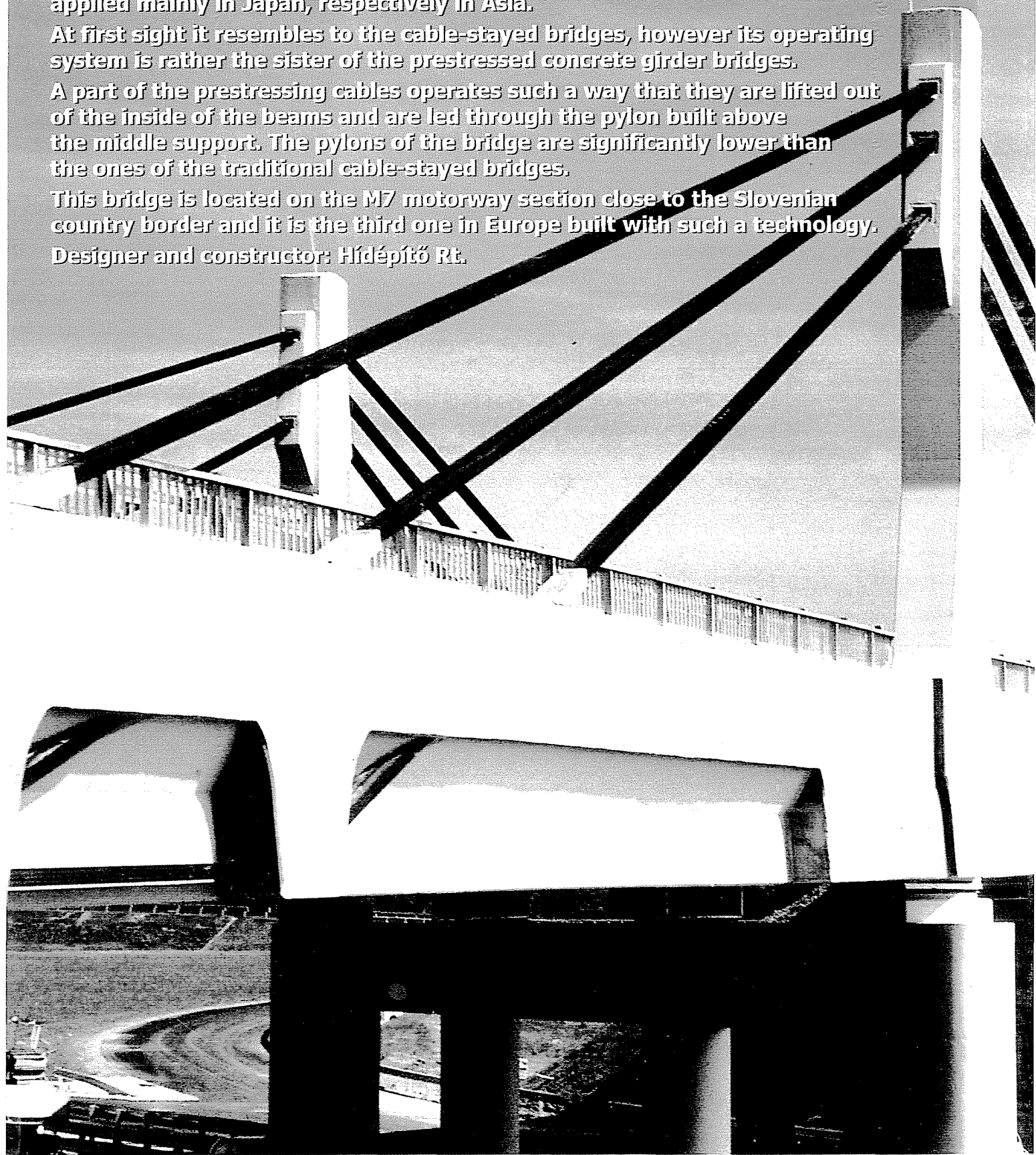
Preceding the most European countries and even more USA the first extradosed bridge in Hungary is completed. So far this new type of bridge structure has been applied mainly in Japan, respectively in Asia.

At first sight it resembles to the cable-stayed bridges, however its operating system is rather the sister of the prestressed concrete girder bridges.

A part of the prestressing cables operates such a way that they are lifted out of the inside of the beams and are led through the pylon built above the middle support. The pylons of the bridge are significantly lower than the ones of the traditional cable-stayed bridges.

This bridge is located on the M7 motorway section close to the Slovenian country border and it is the third one in Europe built with such a technology.

Designer and constructor: Hídépítő Rt.



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